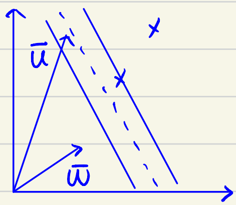
# SVM訓練推導




$\bar{w}\cdot\bar{u} \geq C \quad C=-b$

$\triangle\ \bar{w}\cdot\bar{u}+b \geq 0 \Rightarrow$ 正樣本

Decision Rule

$\bar{w}\cdot\bar{x}_{+}+b \geq 1 \Longrightarrow y_i(\bar{x}_i\cdot\bar{w}+b)\geq 1 \Rightarrow y_i(\bar{x}_i\cdot\bar{w}+b)-1\geq 0$

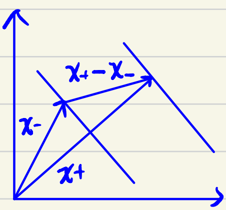
$\bar{w}\cdot\bar{x}_{-}+b \leq 1 \Longrightarrow y_i(\bar{x}_i\cdot\bar{w}+b)\geq 1 \Rightarrow \triangle\ \underline{y_i(\bar{x}_i\cdot\bar{w}+b)-1\geq 0}$

假設$y_i$在正樣本時為1

負樣本時為-1

for $x_{in}$ in Gutter

代表在2條實線內




$\text{width} = (x_+ - x_-)\cdot\frac{\bar{w}}{\|w\|} = \frac{2}{\|w\|}$

$x_+ \to 1-b \qquad x_- \to 1+b$

$\text{MAX}\ \frac{2}{\|w\|} \Rightarrow \text{MAX}\ \frac{1}{\|w\|} \Rightarrow \min\|w\| \Rightarrow \min\frac{1}{2}\|w\|^2$

$L=\frac{1}{2}\|w\|^2-\sum\alpha_i[y_i(\bar{w}\cdot\bar{x}_i+b)-1]$

$\frac{\partial L}{\partial \bar{w}} = \bar{w}-\sum\alpha_i y_i x_i = 0 \Rightarrow \underline{\bar{w}=\sum_i \alpha_i y_i \bar{x}_i}$

$\frac{\partial L}{\partial b} = -\sum\alpha_i y_i = 0 \Rightarrow \sum\alpha_i y_i = 0$

將 $\bar{w}=\sum_i \alpha_i y_i x_i$ 代入L

$L=\frac{1}{2}(\sum\alpha_i y_i \bar{x}_i)(\sum\alpha_j y_j \bar{x}_j)-\sum\alpha_i y_i x_i\cdot(\sum\alpha_j y_j x_j)\underbrace{-\sum\alpha_i y_i b}_{0}+\sum\alpha_i$

$L=\sum\alpha_i-\frac{1}{2}\sum_i\sum_j\alpha_i\alpha_j y_i y_j \bar{x}_i\cdot\bar{x}_j$

將 $\bar{w}=\sum_i \alpha_i y_i x_i$ 代入 $y_i(\bar{w}+\bar{x}_i+b)-1=0$

$\sum\alpha_i y_i \bar{x}_i\cdot\bar{u}+b\geq 0 \Rightarrow$ 正樣本

$\phi(\bar{x})$ 將向量進行轉換

$\phi(\bar{x}_i)\cdot\phi(\bar{x}_j) \Rightarrow \phi(\bar{x}_i)\cdot\phi(\bar{u})$

$K(x_i, x_j)=\phi(\bar{x}_i)\cdot\phi(\bar{x}_j)$

$\Rightarrow$ ① $(\bar{u}\cdot\bar{v}+1)^n$ ② $e^{-\frac{|(x_i-x_j)|}{\sigma}}$