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CXII. *On the Analysis of τ-Meson Data and the Nature of the τ-Meson*

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Abstract

A convenient method of representation is proposed (§ 2) for data on τ-meson decay configurations, applicable when the unlike outgoing π-meson is not distinguished. The relation between the spin and parity of the τ-meson and the distribution of decay configurations is obtained for some simple cases. The hypothesis that the τ- and χ-mesons are identical requires a non-zero spin for this particle and the available data on τ-meson decay does not exclude this possibility. However, observations in which the unlike outgoing π-meson is not distinguished are relatively ineffective in discriminating between the various possibilities. The distortions which strong meson–meson attraction may produce in τ-decay configurations are discussed ; the present data offers no evidence on this effect.

§ 1. Introduction

The existence of a τ-meson whose mass is $978 \pm 6\ m_e$ and which decays into three π-mesons is now well established (Brown *et al.* 1949). Although mass measurements on the outgoing particles in high energy cosmic ray stars suggest (Daniel and Perkins 1953) that only a small proportion ($<20\%$) of the heavy mesons produced could possibly be τ-mesons, the number of τ-decay events observed in photographic emulsions has become quite considerable, presumably because of the very characteristic τ-decay pattern. About fourteen clear examples of the process $\tau^{\pm} \rightarrow \pi^{\pm} + \pi^{+} + \pi^{-}$ are now available. In each event the angles between the outgoing meson tracks, and also the individual meson energies, have been measured, but it is not known which track is produced by the meson whose charge is opposite to that of the τ-meson (we shall generally refer to this π-meson as the ' unlike ' meson). In § 4, it will be seen that the frequency distribution predicted for the decay configuration depends rather less sensitively on the nature of the τ-meson when this additional information is lacking ; however, since such a complete identification of the decay process may well not be generally possible until τ-mesons may be artificially produced, it seems worthwhile to present an analysis appropriate to the data available at the present stage and to discuss what one may hope to learn concerning the τ- and π-mesons when more

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events have been observed. The τ-meson properties of first interest are those related to the conservation laws of physics, the mass, charge, intrinsic spin, parity and (possibly) isotopic spin. The charge of the τ-meson is known to have magnitude $|e|$ and the experimental data on its mass has recently been discussed by Rochester and Butler (1953). Concerning the isotopic spin properties, one type of observation which could lead to some information has been discussed recently (Dalitz 1953). A phenomenological description of the decay for assumed spin and parity may also allow the determination of other parameters relevant to the τ-meson, which may provide guidance in a more detailed theory of the τ-meson. The existence of any strong interactions between the outgoing π-mesons would be expected to influence the decay configuration considerably (Brueckner and Watson 1952), so that a study of the decay events may also provide some evidence concerning the meson–meson interaction.

§2. The Specification and Analysis of τ-Meson Decay Events

In the decay of a τ-meson at rest, the three outgoing π-mesons have zero total momentum and a total kinetic energy of $E=(m_\tau-3m_\pi)c^2$, so that, apart from the spatial orientation, the specification of the decay configuration requires two parameters. A convenient choice would be the energy of the unlike meson and the magnitude of the difference between the energies of the like mesons. However, it is not possible to distinguish between the outgoing mesons in the present data, so that a choice of parameters which is symmetrical between the three π-mesons is more appropriate. Denoting the total kinetic energy of the π-mesons by E and the individual kinetic energies by ϵ_1, ϵ_2, ϵ_3, define

$$\alpha_i=\epsilon_i-E/3 \quad (i=1,\,2,\,3). \qquad (2.1)$$

These α_i then have zero sum, thus $\Sigma_i\alpha_i=0$. If E and the α_i are known, the decay configuration is specified. If the unlike meson is unidentified, two symmetrical functions,

$$Y=-(\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1)=-\Sigma\alpha_1\alpha_2,\ Z=\alpha_1\alpha_2\alpha_3, \qquad (2.2)$$

may conveniently be used to specify the configuration uniquely, since, for given (Y, Z) the α_1, α_2, α_3 are the three roots of the cubic equation

$$\alpha^3-Y\alpha-Z=0. \qquad (2.3)$$

Y and Z are limited firstly by the condition that (2.3) has three real solutions, which requires that $Z^2\leqslant 4Y^3/27$, and secondly by the requirement that the outgoing mesons have total momentum zero. A necessary and sufficient condition that the momenta p_1, p_2, p_3 satisfy the triangular inequalities $p_1+p_2\geqslant p_3$, $p_2+p_3\geqslant p_1$, $p_3+p_1\geqslant p_2$, which are necessary and sufficient to allow a choice of $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ satisfying $\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3=0$, is that

$$(p_1+p_2+p_3)(p_1+p_2-p_3)(p_1-p_2+p_3)(p_1+p_2-p_3)=16m_\pi^2\left(\frac{E^2}{12}-Y\right)\geqslant 0.$$

so that Y, Z are limited by the inequalities

$$Y \leqslant E^2/12 \text{ and } Z \leqslant \sqrt{(4Y^3/27)} \leqslant E^3/108. \quad . \quad . \quad . \quad . \quad (2.4)$$

As the uncertainty in the measurement of individual meson energies is often quite large, it will be more appropriate to use the ratios of the meson energies, which may be deduced from the more accurately known angles between the meson tracks. Accordingly, co-ordinates (λ, θ) which are functions of these ratios, may be defined by

$$Y = \lambda E^2/12, \; Z = \lambda^{3/2} \sin\theta \, E^3/108 = \eta E^3/108, \quad . \quad . \quad . \quad (2.5)$$

where it is convenient to define the quantity $\eta = \lambda^{3/2} \sin\theta$. The inequalities (2.4) then correspond to $0 \leqslant \lambda \leqslant 1$ and $-\pi/2 \leqslant \theta \leqslant \pi/2$.

The kinetic energy of an outgoing π-meson cannot exceed $2E/3$, which is close to 50 MEV since the experimental data gives $E = 76{\cdot}5 \pm 3$ MEV. Consequently it is reasonable to adopt a non-relativistic description, since the resulting errors are at most a few per cent, small compared with other uncertainties in the data. The expression for the energy distribution of the three π-mesons may be written in the general form

$$F(\epsilon_1, \epsilon_2, \epsilon_3) \delta(\epsilon_1 + \epsilon_2 + \epsilon_3 - E) \delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) d^3p_1 \, d^3p_2 \, d^3p_3,$$

where $F(\epsilon_1, \epsilon_2, \epsilon_3) = F(\epsilon_2, \epsilon_1, \epsilon_3)$ since the like mesons (1, 2) are indistinguishable. Non-relativistically this may be reduced to

$$8\pi^2 m_\pi{}^2 F(\epsilon_1, \epsilon_2, \epsilon_3) \delta(\epsilon_1 + \epsilon_2 + \epsilon_3 - E) \, d\epsilon_1 \, d\epsilon_2 \, d\epsilon_3.$$

If the unlike particle cannot be identified the same configuration may occur in three ways and the total frequency is then the sum of the three separate frequencies, thus

$$\begin{aligned}[8\pi^2 m_\pi{}^2 (F(\epsilon_1, \epsilon_2, \epsilon_3) &+ F(\epsilon_2, \epsilon_3, \epsilon_1) \\ &+ F(\epsilon_3, \epsilon_1, \epsilon_2))] \, \delta(\epsilon_1 + \epsilon_2 + \epsilon_3 - E) \, d\epsilon_1 \, d\epsilon_2 \, d\epsilon_3. \quad . \quad . \quad . \quad (2.6)\end{aligned}$$

The function within the square brackets is a symmetrical function of $\epsilon_1, \epsilon_2, \epsilon_3$ and thus of $\alpha_1, \alpha_2, \alpha_3$, so that it may be expressed as a function of E and of Y, Z and hence of λ, θ, say $\phi(\lambda, \theta) \equiv \Phi(\lambda, \eta)$. If we write $\zeta = \alpha_1 + \alpha_2 + \alpha_3$, then (2.6) may be written as

$$\phi(\lambda, \theta) \delta(\zeta) \, d\alpha_1 \, d\alpha_2 \, d\alpha_3 = \phi(\lambda, \theta) \, \delta(\zeta) \frac{\partial(\alpha_1, \alpha_2, \alpha_3)}{\partial(\zeta, \lambda, \eta)} d\zeta \, d\lambda \, d\eta. \quad . \quad . \quad . \quad (2.7)$$

The ratio of the volume elements $d\alpha_1 \, d\alpha_2 \, d\alpha_3$ and $d\zeta \, d\lambda \, d\eta$, namely $\partial(\alpha_1, \alpha_2, \alpha_3)/\partial(\zeta, \lambda, \eta)$, is equal to the magnitude of

$$(\alpha_1 - \alpha_2)(\alpha_2 - \alpha_3)(\alpha_3 - \alpha_1) 1296/E^5,$$

which equals $\sqrt{(4Y^3 - 27Z^2)} \, 1296/E^5 = 36\sqrt{3}\lambda^{3/2} \cos\theta/E^2$. Integrating over ζ, (2.7) becomes, apart from some numerical factors,

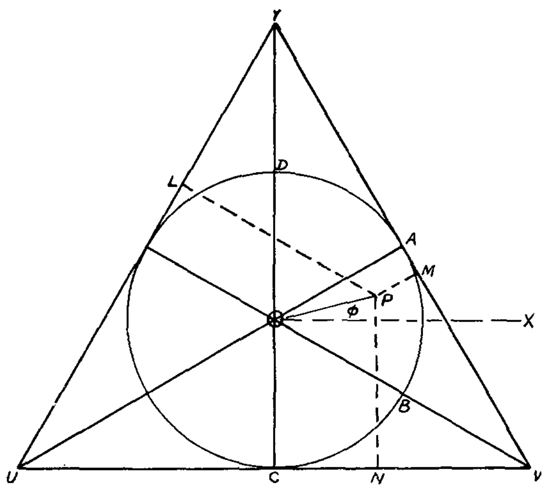
$$\phi(\lambda, \theta) \, d\lambda \, d\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.8)$$

The following considerations lead to a very convenient realization of the co-ordinates defined above. For an equilateral triangle YUV (see fig. 1), the sum PL+PM+PN of the perpendiculars to the sides from any point P within the triangle equals the altitude of the triangle. Hence a decay event may be uniquely specified by a point P such that the perpendiculars (PL, PM, PN) are proportional to the meson energies (ϵ_1, ϵ_2, ϵ_3). Interchange of two of these energies corresponds to the reflection of P in a corresponding altitude of the triangle so that, corresponding to the incomplete identification of the mesons, the event is represented a point in each of the six sub-triangles of YUV. The distribution in any sub-triangle is obtainable from that in AOV by successive reflections in the altitudes, so that one need consider only that ordering of the meson energies for which P lies within AOV. The Cartesian co-ordinates (x, y) with respect to the axes OX, OY are given by

$$x=\sqrt{3}(\epsilon_1-\epsilon_2)/E, \quad y=(2\epsilon_3-\epsilon_1-\epsilon_2)/E,$$

where the altitude of the triangle has been taken as 3 units. It may now be readily verified that the polar co-ordinates (r, ϕ) of P are $(\lambda^{1/2}, -\theta/3)$. The points representing decay events are therefore confined

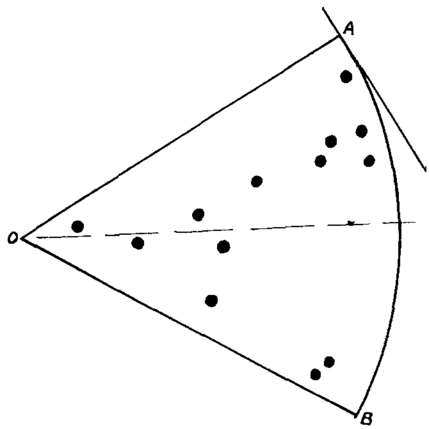
Fig. 1



to the interior of the circle inscribed to the triangle YUV, since momentum conservation requires $\lambda \leqslant 1$. In plotting decay events on this diagram, the momenta have been calculated from the observed angles between the meson tracks, and the energy ratios from the squares of these momenta, so that the representation expresses angular relations in the decay and is not modified by relativistic effects. When the co-ordinates are given, the corresponding configuration of the decay process may be quickly obtained by reference to this diagram.

If the function $\phi(\lambda, \theta)$ is independent of λ, θ which is the case, for example, when F is independent of ϵ_1, ϵ_2, ϵ_3, the experimental points should be distributed randomly over the area AOB, since the element of area is $r\,dr\,d\phi=d\lambda\,d\theta/6$. The limited data available is displayed in fig. 2 and although there appears to be some tendency to favour the upper and outer part of AOB the distribution obtained is not yet significantly different from a random distribution. The corner A corresponds to decay events in which one π-meson is rather slow and a tendency to favour this corner may perhaps result from some initial experimental bias favouring detection of this particular decay configuration. The distribution to be expected according to various hypotheses will be obtained in § 4, where it will be shown that the present statistics are insufficient to allow discrimination between the alternatives considered.

Fig. 2



The data from 13 τ-meson decay events.

§ 3. Considerations on the Nature of the τ-Meson

The questions of most direct interest concerning the τ-meson relate to its intrinsic spin, its parity and to the applicability of the notion of isotopic spin. If the τ-meson is some kind of meson–nucleon complex occurring in consequence of the interaction between the meson and nucleon fields, it would follow that the τ-meson would have a definite isotopic spin, if charge independence be established for the meson–nucleon interaction. If the τ-meson needs specification by some new field coupled directly to the meson and nucleon fields, these interactions may still satisfy charge independence and the τ-meson may still have a definite isotopic spin. Some evidence concerning the applicability of isotopic spin to the τ-meson may be obtainable from the study of the branching ratio for the decay processes $\tau^{\pm}\rightarrow\pi^{\pm}+\pi^{+}+\pi^{-}$ and $\tau^{\pm}\rightarrow\pi^{\pm}+\pi^{0}+\pi^{0}$, as discussed earlier (Dalitz 1953). At the present, there is certainly no evidence requiring that the τ-meson should satisfy charge independence : nevertheless, in this and the following section, we shall indicate what additional consequences would follow if this notion were applicable.

The configuration of the three mesons resulting from τ-meson decay will depend on the spin j and parity w of the τ-meson. The configuration is completely specified by the momenta $\mathbf{p}_1$, $\mathbf{p}_2$ of the like particles and the matrix element for decay to this configuration must be an irreducible tensor of rank j and parity $(-1)^3 w$, formed from $\mathbf{p}_1$, $\mathbf{p}_2$ and symmetrical for interchange of the suffices 1, 2, since the π-meson satisfies Bose statistics. If $w=-(-1)^j$ such a tensor is easily found (e.g. $Y_m{}^j(\mathbf{p}_1)+Y_m{}^j(\mathbf{p}_2)$) so that the decay is then not forbidden by angular momentum and parity conservation. If $w=+(-1)^j$, then for every j except $j=0$, such a tensor may be formed, for example, by multiplying the tensor just formed for $(j-1,\ w'=-(-1)^{j-1})$ by the pseudovector $\mathbf{p}_1\ \ \mathbf{p}_2(p_1{}^2-p_2{}^2)$ and extracting the irreducible tensor of rank j from the product. For $j=0$, however, we are required to form a pseudoscalar from the vectors $\mathbf{p}_1$, $\mathbf{p}_2$, which is not possible. The 3 π-decay, therefore, is forbidden by angular momentum and parity conservation only for a scalar heavy meson.

The selection rules for decay of a heavy meson into (*a*) two π-mesons or (*b*) one π-meson and a photon, which are allowed as far as energy and momentum conservation are concerned, are well known. For case (*a*), they may be simply obtained by noting that, since the final state is specified by the relative momentum $\mathbf{p}$ of the mesons, the matrix element for decay of a particle of spin j must be given by $Y_m{}^j(\mathbf{p})$. Since this tensor has parity $(-1)^j$, the 2π-decay is therefore forbidden by angular momentum and parity conservation for $w=-(-1)^j$. If isotopic spin is a good quantum number, then since interchange of the mesons corresponds to $\mathbf{p}\rightarrow-\mathbf{p}$ and the mesons satisfy Bose statistics, the 2π-decay is allowed only for even or odd j according as the isotopic spin is even or odd. For states with even j and odd isotopic spin, the 2 π-decay may still proceed unfavourably through electromagnetic interactions which violate charge independence, though one might then expect that the decay $\tau^\pm\rightarrow\pi^\pm+\pi^0+\gamma$ would be more probable. For case (*b*), the final state is specified by the relative momentum $\mathbf{p}$ and the photon polarization vector $\mathbf{e}$ which satisfy $\mathbf{e}\,.\,\mathbf{p}=0$ since the photon is a transverse wave, and the matrix element for the decay to this state must be a linear function of $\mathbf{e}$. For every j (except $j=0$) such a matrix element may be formed for parity $w=(-1)^j$ or $-(-1)^j$ by multiplying $Y_m{}^{j-1}(\mathbf{p})$ by $\mathbf{e}\times\mathbf{p}$ or $\mathbf{e}$ respectively and extracting the irreducible tensor of rank j. However, no non-zero scalar or pseudoscalar can be formed, so that angular momentum conservation forbids (π,γ) decay for a heavy meson of spin $j=0$. In view of the number of 3π-decay events observed, it has been natural to suppose that the τ-meson is pseudoscalar, since the 3 π-decay would then be the only allowed decay process involving π-mesons alone and other decay processes involving photons (e.g. $\tau^\pm\rightarrow\pi^\pm+\pi^0+\gamma$) might well be less probable.

Recently, however, a number of events described as χ-meson decay have been found from which a charged π-meson of unique energy 110 MEV results (Menon and O'Ceallaigh 1953). If it is supposed that the neutral particle produced in this decay is a π^0-meson, these events may be interpreted as the decay of a particle of mass about 980 m_e. Also the analysis of V^0-particle decay (Armenteros *et al.* 1951, Barker 1953, Thompson *et al.* 1953) strongly suggests that there exists a V_2^0-meson which decays into two π-mesons with a Q-value corresponding to a V_2^0 mass of $962\,m_e$. It is very plausible that this V_2^0 meson is the neutral counterpart of the χ-meson and an attractively simple representation of the data is offered by the further hypothesis (Menon and O'Ceallaigh 1953) that the τ- and χ-mesons are identical, the events observed representing two alternative decay schemes of the (τ, χ) particle. In this case the (τ, χ) particle could not have spin $j=0$ but must be a particle of parity $(-1)^j$ if its spin is $j(\geqslant 1)$—if isotopic spin is rigorously applicable, j is restricted to odd values. On the other hand, the hypothesis that the neutral particle in χ-decay is a photon is still compatible with the hypothesis of a (τ, χ) particle and also requires that this particle have $j\neq 0$. If the parity of this particle is $(-1)^j$, then both 2π- and (π, χ)-decay may compete with 3π-decay, although, for j even, the 2π-decay will be relatively unfavoured if charge independence is a good approximation. If the parity is $-(-1)^j$, the χ-decay would have to be (π, γ)-decay. The hypothesis that the τ-meson is a particle of spin 1 is therefore of particular interest, and if the hypothesis $\chi\equiv\tau$ is to be maintained, we must examine whether the consequences for the 3π-decay pattern are then in agreement with the data.

§4. Discussion of Some Particular Hypotheses

The decay of the τ-meson into the π-meson states of momenta $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ (in this and the next section, the suffix 3 will refer to the unlike meson) will be described by a matrix element $M(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$. Corresponding to the analysis of the final state into partial waves of definite angular momentum, this matrix element may be written as the sum of a number of terms of definite form with coefficients depending on the energies of the mesons. It will be assumed that only low angular momenta are important and that these coefficients may be expanded in powers of the meson energies. This supposes the region of production to be small compared with the meson wavelengths and neglects the effects of any strong interactions between the mesons after the decay. These effects will be discussed briefly in § 5, following the viewpoint of Brueckner and Watson (1951) that the main features of these interaction effects will not depend on the details of the production process.

(1) *The Hypothesis of a Pseudoscalar τ-Meson*

Since there is no change in intrinsic parity, $M(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$ is a scalar whose general form is

$$\begin{aligned}M(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) &= M_0(p_1, p_2, p_3)+\Sigma\mathbf{p}_1\cdot\mathbf{p}_2 M_3'(p_1, p_2, p_3)\\ &\quad+\Sigma(\mathbf{p}_1\cdot\mathbf{p}_3\mathbf{p}_2\cdot\mathbf{p}_3-\tfrac{1}{3}\mathbf{p}_1\cdot\mathbf{p}_2 p_3^2)M_3''(p_1, p_2, p_3)+\ldots\ . \qquad (4.1)\end{aligned}$$

of which the successive terms describe the emission of s-wave mesons, of one s-wave and two p-wave mesons, and of one d-wave and two p-wave mesons respectively. The summations are over all permutations of (123) and $M(\mathbf{p}_1, \mathbf{p}_2 ; \mathbf{p}_3)$ is required to be symmetric for interchange of (12), because of the Bose statistics. By expanding the scalar coefficients in powers of p_1^2, p_2^2, p_3^2, making use of the energy conservation relation $(p_1^2+p_2^2+p_3^2)=2m_\pi E$, momentum balance $\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3=0$ and the symmetry of M, (4.1) may be reduced to the form

$$M=A+a\alpha_3+b\alpha_1\alpha_2+c\alpha_3^2+ \quad \ldots\ldots \quad (4.2)$$

A, a, b, c being constants and α_1, α_2, α_3 being defined by (2.1). In (4.2), A contains contributions from all partial waves, a arises from the energy dependence of the s-waves and from $s-p$ interference, b and c from the p-waves alone or from higher effects, and so on. The function of interest is $\phi(\lambda, \theta)$ which is proportional to

$$\begin{aligned}\Sigma(AA^*+\alpha_3 2\mathscr{R}(aA^*)+\alpha_3^2(aa^*+2\mathscr{R}(cA^*))+2\alpha_1\alpha_2\mathscr{R}(bA^*))\\ =3AA^*+\lambda(2aa^*+2\mathscr{R}(2cA^*-bA^*))E^2/12+. \quad \ldots \quad (4.3)\end{aligned}$$

Although the term a in (4.2) allows a considerable range of variation in the energy distribution of the unlike meson, the cross terms between a and A vanish in the symmetrized function $\phi(\lambda, \theta)$ whose (λ, θ) dependence arises from higher order terms. The fact that the unlike meson cannot be identified leads here to an insensitive dependence of the observable phenomena on the details of the theory. Consequently, apart from meson–meson interaction effects, the decay may be expected to be isotropic. Also, if a is negligible, the orbital state of the outgoing mesons is totally symmetric (of the representation [3]) which implies a branching ratio of $\frac{1}{4}$ for the τ-decay involving neutral mesons if isotopic spin is a good quantum number (Dalitz 1953). The term a, and possibly the effects of meson–meson interactions, will lead to an admixture of [21] orbital symmetry which would increase this branching ratio.

(2) *The Hypothesis of a Vector τ-Meson*

The matrix element must be a pseudovector and, including the possibilities up to a final (p^2d) or (sd^2) configuration, its form is uniquely

$$M(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)=B\mathbf{p}_1\times\mathbf{p}_2(p_1^2-p_2^2). \quad \ldots\ldots \quad (4.4)$$

Averaging over the orientations of the initial τ-meson then leads to

$$F(\epsilon_1, \epsilon_2 ; \epsilon_3)=4m_\pi^4B^2(\epsilon_1-\epsilon_2)^2[\epsilon_3(2E-3\epsilon_3)-(\epsilon_1-\epsilon_2)^2]/3, \quad . \quad (4.5)$$

which corresponds to a distribution function

$$\phi(\lambda, \theta)\sim\lambda(1-\lambda). \quad \ldots\ldots\ldots \quad (4.6)$$

Since the matrix element (4.4) is zero if two of the π-mesons have parallel paths or if the like mesons have equal energies, it may be easily understood why the centre and edge of the circle in fig. 1 are unfavoured. The matrix element (4.4) leads to a final state of orbital symmetry [21] which corresponds to equal probability for the two modes of 3π-decay for the τ-meson if the notion of isotopic spin is applicable. In this case, since the only decay observed with certainty to yield a single charged

π-meson is the χ-decay, the hypothesis of (τ, χ) identity would be compatible only if this decay is the predominant mode of decay for the particle.

(3) *The Hypothesis of a Pseudovector τ-Meson*

The matrix element must be a vector. The simplest decay is emission of two s-wave and one p-wave particle for which $M(\mathbf{p}_1, \mathbf{p}_2 ; \mathbf{p}_3)$ is proportional to $\mathbf{p}_3$, giving

$$F(\epsilon_1, \epsilon_2 ; \epsilon_3)=C\epsilon_3, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.7)$$

so that the distribution function is the constant

$$\phi(\lambda, \theta)=8\pi^2 m_\pi{}^2 CE. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.8)$$

This matrix element leads to a final state of orbital symmetry [21], its symmetric part $\frac{1}{3}(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)$ being zero owing to momentum conservation. Remarks similar to those at the end of the last paragraph apply also to this case except that here the χ-decay must be of the (π, γ) type.

(4) *Higher Spin Values*

For spin j, even the simplest forms for M rapidly increase in number with j, so that the predicted distributions offer a considerable range of possibilities. For the cases $2+$, $3-$, however, the simplest M is unique and leads to the expressions $(1-\lambda)$ and $(1-\lambda)(1-\frac{1}{5}\lambda)$ for the function ϕ, respectively.

For cases (1), (2) and (3) the energy distribution of the unlike meson is plotted in fig. 3. To emphasize the loss of sensitivity resulting from the incomplete identification of the decay, the energy distributions predicted* for the slowest and fastest meson and for all mesons, irrespective of charge, are plotted in fig. 4 and compared with the available data. This effect is particularly marked in the pseudovector case, where the distributions of fig. 4 agree with those for the pseudoscalar case, despite the marked difference in the distribution for the unlike meson. The data available at present are clearly inadequate to discriminate between the various possibilities discussed here.

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* Given the function $\Phi(\lambda, \eta)$, the energy distribution of all particles is given by

$$P(\epsilon)=\int_0^{\sqrt{(1-y^2)}} \Phi(x^2+y^2, y(y^2-3x^2))\, dx,$$

where $y=(3\epsilon-E)/E$. The distribution $Q_s(\epsilon)$ of the slowest particle is given by $Q_s(\epsilon)=P(\epsilon)$ for $\epsilon<E/6$, since ϵ is then necessarily the slowest particle, whereas for $E/6<\epsilon<E/3$,

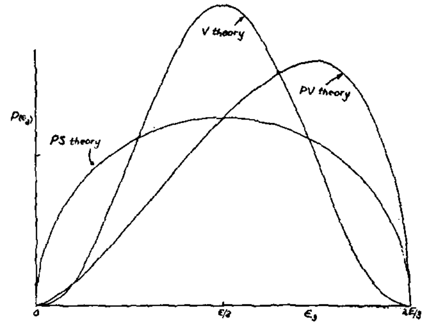
$$Q_s(\epsilon)=\int_0^{-y\sqrt{3}} \Phi(x^2+y^2, y(y^2-3x^2))\, dx,$$

corresponding to integration over the sector OCB of fig. 1. The distribution $Q_f(\epsilon)$ of the fastest particle is given by $Q_f(\epsilon)=P(\epsilon)$ for $\epsilon>E/2$, whereas for $E/3<\epsilon<E/2$, $Q_f(\epsilon)$ is found by integrating Φ from 0 to $+y\sqrt{3}$. Q_f and Q_s differ in form only in virtue of any η-dependence of Φ.

§ 5. Some Possible Meson–Meson Interaction Effects

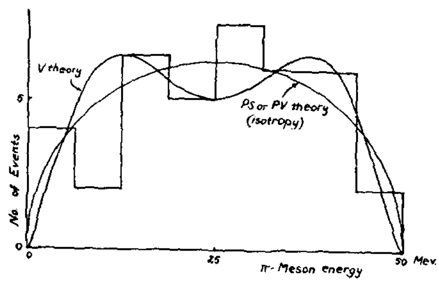
In the emission of π-mesons in cosmic ray stars, a marked correlation in the directions and energies of pairs of π-mesons has been found by Danysz, Lock and Yekutieli (1952), who tentatively suggested that these meson pairs may result from the decay of a short lived ζ^0-meson (see

Fig. 3



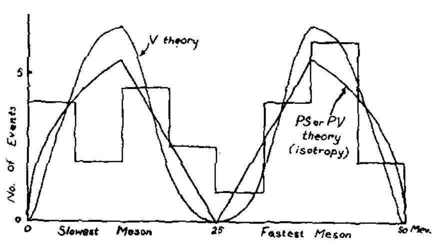
The energy distribution of the unlike π-meson in τ-meson decay for several τ-meson spin values.

Fig. 4



(*a*)

Energy distribution for all π-mesons from τ-meson decay: comparison with histogram for 13 events.



(*b*)

Energy distribution of the slowest and fastest π-meson in τ-meson decay. Experimental histogram based on 13 events.

also Danysz *et al.* 1953). However, Brueckner and Watson (1952) have remarked that the existence of a strong attractive interaction between π-mesons could provide an alternative explanation for these correlations and have pointed out that, in any case, the study of correlation effects near the threshold for multiple meson production would provide quantitative evidence on the nature of the meson–meson interaction. A similar point of view has already lead to some information on the neutron–neutron interaction (Watson and Stuart 1951) from the distortion of the γ-ray spectrum emitted in the capture of π^--mesons by deuterium due to the neutron–neutron correlations resulting from their interaction.

In τ-meson decay, three π-mesons are produced with low kinetic energy and the existence of strong attractive meson–meson interactions might be expected to distort the distribution of decay patterns in a characteristic way. The kinetic energy of the relative motion of two mesons (1 and 2, say) is given by

$$\Delta=(\mathbf{p}_1-\mathbf{p}_2)^2/4M=E-3\epsilon_3/2,$$

so that, since the interaction is most effective between mesons of low relative momentum, this effect would favour a configuration in which the third meson has energy near to the maximum $2E/3$. The corresponding angle between the paths of the interacting mesons is about $\sqrt{(3\Delta)}/5$, or less, i.e. about $30°$ for $\Delta\sim 2$ MeV (a value for Δ which is typical of the cosmic ray data at very much higher total energies). The parameters (λ, θ) of these configurations have values close to $\lambda=1$, $\theta=\pi/2$, so that the distribution of the decay events would then crowd towards the corner B of fig. 1.

Following the discussion of Watson (1951), the matrix element leading to a state in which two mesons have small relative momentum k has a k-dependence essentially given by a factor $(\sin\delta)/k$, when it is supposed that meson–meson scattering is strong for small k in only one partial wave (assumed here, for simplicity, to be the s-wave) of phase-shift δ. If, to illustrate the effects quantitatively, it is supposed that the basic process is otherwise isotropic and that only unlike mesons attract strongly, the matrix element is essentially $C(\sin\delta_{13}/k_{13})(\sin\delta_{23}/k_{23})$, and

$$F(\epsilon_1, \epsilon_2\,;\,\epsilon_3)=C^2/(\alpha^2+(\mathbf{p}_1-\mathbf{p}_3)^2)(\alpha^2+(\mathbf{p}_2-\mathbf{p}_3)^2), \quad . \quad . \quad (5.1)$$

if it is supposed further that $k\cot\delta=\alpha=\hbar/a$, a being the zero-energy scattering length. With $\mu=\alpha^2/2ME$, the corresponding symmetrized function is then

$$\phi(\lambda, \theta)=C^2(1+\mu)/M^2E^2(\mu(3+2\mu)^2+3(1+\mu)(1-\lambda)+(1-\lambda^{3/2}\sin\theta))$$
$$\simeq C^2/M^2E^2(9\mu+3(1-\lambda)+1-\lambda^{3/2}\sin\theta), \quad . \quad . \quad . \quad (5.2)$$

since the values of μ of interest are small. If the interaction effect is strong for relative energies of up to $\sim\Delta$, then $\mu\sim 2\Delta/E$, which is 0·05 for $\Delta=2$ MeV. The expression (5.2) will not be a good approximation for points not close to B since both k_{13} and k_{23} will then be large and the approximations leading to (5.1) are not valid. Even for a point close to B, one of the three corresponding configurations will have both k_{13} and k_{23} large, but the error thus made will be small compared with the large increase in the probability of the other two configurations. Hence if the τ-meson is pseudoscalar and there is an attractive interaction between mesons comparable with that necessary to explain the correlations in cosmic ray stars, a considerable fraction of the τ-decay events should lie quite close to B. The present data (fig. 2) shows no evidence for any very strong effect of this kind.

The vector meson deserves separate consideration since its matrix element favours configurations in which the relative momentum of each meson pair is high. For this case,

$$\left.\begin{aligned} F(\epsilon_1, \epsilon_2\,;\,\epsilon_3) &= \frac{[\epsilon_3(2E-3\epsilon_3)-(\epsilon_1-\epsilon_2)^2](\epsilon_1-\epsilon_2)^2}{\left(\dfrac{\alpha^2}{2M}+2E-3\epsilon_1\right)\left(\dfrac{\alpha^2}{2M}+2E-3\epsilon_2\right)}, \\ \phi(\lambda, \theta) &= 4(1-\lambda)(\lambda+\tfrac{1}{2}\eta)/(9\mu+3(1-\lambda)+(1-\eta)), \end{aligned}\right\} . \tag{5.3}$$

assuming μ is small. This expression remains bounded as $\mu \to 0$; though the meson–meson interaction increases the probability of an event near to B, the effect does not crowd a high proportion of events into this corner. Away from this corner, (5.3) shows a considerable θ-dependence, which depends, however, on the incorrect assumptions underlying (5.1) when k_{13}, k_{23} are large; a genuine θ-dependence will, of course, occur near the outer edge of AOB. The present limited data of τ-meson decay does not conflict with the hypothesis of a strong meson–meson attraction in the case of a vector τ-meson (or of a $(2+)$ or $(3-)$ τ-meson). It should finally be noted that in all cases, the upper half of the region AOB should suffer very little distortion as a result of this interaction effect.

§ 6. Conclusions

The data available at present on the decay of the τ-meson offers no significant evidence for any lack of isotropy in the decay process nor for any distortion of the decay patterns as would be characteristic of a strong attraction between π-mesons. On the other hand, the data does not at present exclude the possibility that the τ-meson is a vector particle, in which case the absence of evidence for any correlations characteristic of strong π–π attraction would not be significant, their effect not being particularly marked in the vector theory. However, theories giving quite different matrix elements have been found to lead finally to very similar results, so that a comparison with the evidence would not be sensitive to the τ-meson spin and parity. This insensitivity results from the need for averaging over three mesons whose energies are limited by energy and momentum conservation—a knowledge of the energy distributions for the unlike meson alone would provide a more sensitive indication of the meson spin. In this respect, the τ-meson decay contrasts sharply with the decay event $\mu \rightarrow e+2\nu$ in which less quantitative data is available from each event but in which the electron spectrum has a fair dependence on quite detailed μ-meson properties. If it should prove possible to analyse τ-meson decay events in more detail (e.g. if the τ-mesons which decay were all of positive charge, as is conceivable, identification of the π^--meson in each event would suffice) a more direct comparison with the functions $F(\epsilon_1, \epsilon_2; \epsilon_3)$ will become possible and a more sensitive analysis of the data be feasible. At that stage a more detailed study of these functions will be justified.

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