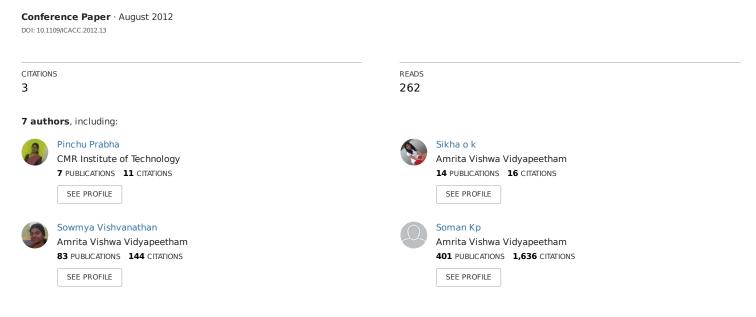
# Visualization of OFDM Using Microsoft Excel Spreadsheet in Linear Algebra Perspective



Some of the authors of this publication are also working on these related projects:



# Visualization of OFDM using Microsoft Excel Spreadsheet in Linear Algebra Perspective

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Abstract—Orthogonal Frequency Division Multiplexing (OFDM) is one of the leading technology that is ruling the communication field. But unfortunately, it is shrouded in mystery. A good knowledge in Linear Algebra is required to appreciate the technology in a better way. So the work focuses on explaining OFDM system from linear algebra point of view. Also, OFDM model communication system is simulated using Excel which makes ease for anyone experiment with OFDM and understand the underlying principle. The paper aims to provide strong foundation on the concept behind OFDM without the need of having much knowledge in electronics field.

Keywords-OFDM, FFT, IFFT, Excel Computation.

#### I. INTRODUCTION

In the modern communication system, OFDM is one of the most used technologies and has wide ranges of applications. OFDM has developed into a popular scheme for wideband digital communication, whether wireless or over copper wires, used in applications such as digital television and audio broadcasting, DSL broadband internet access, wireless networks, and 4G mobile communications. OFDM is an elegant divide-and-conquer approach to high-speed transmission [7], where the basic idea of OFDM is to load data into several orthogonal sub-carriers.

While many details of OFDM systems are very complex, the basic concept of OFDM is quite simple [6]. The whole OFDM technology is built under the foundation of Linear algebra. So there lies difficulty in understanding this technology, when viewed with abstract spectral based communication point of view. The complete understanding of the underlying technology of OFDM requires adequate footing in Linear Algebra, which can be thought as generalization of coordinate geometry to higher dimension.

The work focuses on understanding the OFDM system with linear algebra point of view. The theoretical foundation has been simplified and written in a very lucid way such that any person who has some knowledge in coordinate geometry

and trigonometry can understand the underlying principle.

Excel is an excellent tool for the students to get practical learning experience from high school onwards. It is a computational platform that has the power to do any mathematical recursive computation with ease. The students can work on Excel without any guidance as it is a user-friendly learning platform. The present education system focus merely on delivering the concepts without demanding the students to simulate and experiment with the technology. The Excel can serve as a valuable and versatile tool that helps the students to compute and visualize mathematical expressions and algorithms. The concept behind OFDM can be understood in a better way using Excel based approach. Also, practical feel about the communication part can be provided to the students using the step by step implementation in Excel.

The organization of the paper is as follows: Section II provides Linear Algebra concepts for understanding this work. Section III gives short introduction to Signal analysis and Fourier transform. Section IV provides theoretical explanation of OFDM in linear algebra point of view. Section V explains step by step implementation of the work in Excel. Section VI discusses briefly about the results and the work is concluded in Section VII.

# II. LINEAR ALGEBRA-BACKGROUND

In order to understand OFDM, the prerequisites are some basic coordinate geometry and linear algebra. These concepts are discussed below.

A vector is the representation of a point in space with reference to a coordinate system. Consider a point (2, 3) in two dimensional space (X-Y plane which we use in graph sheet). It becomes a vector with reference to origin(0, 0).

Now (2, 3) can be expressed as linear combination of two independent and orthonormal vectors (1, 0) and (0, 1) as shown



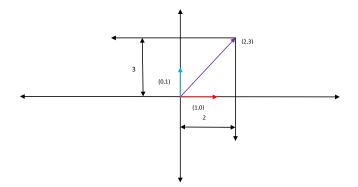


Fig. 1. Co-ordinate representation of point (2, 3)

in Fig.1

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

Here

$$\left(\begin{array}{c}1\\0\end{array}\right),\left(\begin{array}{c}0\\1\end{array}\right)$$

are known as basis. Basis is a collection of linearly independent vectors that spans the space they come from. Any vector in the space can be represented as the linear combination of basis.

## Note:

1. The basis defined in above case are orthogonal, that is, angle between the basis is 90°. Mathematically, two vectors are orthogonal if its inner product is zero.

$$\left(\begin{array}{c} 1\\0 \end{array}\right) \cdot \left(\begin{array}{c} 0\\1 \end{array}\right) = 0 \text{ or } \left(\begin{array}{c} 1\\0 \end{array}\right) \cdot \left(\begin{array}{c} 0&1 \end{array}\right) = 0$$

2. (1) can be rewritten as:

$$\left(\begin{array}{c}2\\3\end{array}\right)=\left(\begin{array}{cc}1&0\\0&1\end{array}\right)\left(\begin{array}{c}2\\3\end{array}\right)$$

which is of the form y = Ax where,

$$2 = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

and

$$3 = \left(\begin{array}{cc} 0 & 1 \end{array}\right) \left(\begin{array}{c} 2 \\ 3 \end{array}\right)$$

- 3. To represent a point in  $\Re^2$  plane, we require 2 basis, each coming from  $\Re^2$ .
- 4. The point  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$  was represented using canonical basis  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ . This is not the only way.

Infinite basis set are possible to represent the above point. All that has to be done is rotate the canonical base, to obtain new basis set.

The point  $\begin{pmatrix} 2\\3 \end{pmatrix}$  can be represented using the basis  $\begin{pmatrix} \frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}} \end{pmatrix}$ ,  $\begin{pmatrix} \frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}} \end{pmatrix}$  as  $\begin{pmatrix} 2\\3 \end{pmatrix} = x \begin{pmatrix} \frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}} \end{pmatrix} + y \begin{pmatrix} \frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}} \end{pmatrix}$   $\begin{pmatrix} 2\\3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}&\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} x\\y \end{pmatrix}$   $\begin{pmatrix} x\\y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}&\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}} \end{pmatrix}^{-1} \begin{pmatrix} 2\\3 \end{pmatrix}$  Since A is orthonormal,  $A^{-1} = A^T$   $\begin{pmatrix} x\\y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}&\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}} \end{pmatrix}^T \begin{pmatrix} 2\\3 \end{pmatrix}$  This reduces lot of computation.

The Solution set is  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{5}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$  So,

$$\left(\begin{array}{c}2\\3\end{array}\right) = \frac{5}{\sqrt{2}} \left(\begin{array}{c}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{array}\right) + \frac{1}{\sqrt{2}} \left(\begin{array}{c}\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{array}\right)$$

The basis  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$  can be represented using the basis  $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ ,  $\begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ , which are obtained by rotating the basis  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  by  $45^{\circ}$ . The newly formed basis are orthonormal.

5. In general, any vector Y can be represented as:

$$Y = (Y^T Base0) Base0 + (Y^T Base1) Base1 + \cdots + (Y^T Base(N-1)) Base(N-1)$$

If the bases are complex, Y can be represented as:

$$Y = (Y^T \overline{Base0}) Base0 + (Y^T \overline{Base1}) Base1 + \cdots + (Y^T \overline{Base(N-1)}) Base(N-1)$$

# III. SIGNAL ANALYSIS USING LINEAR ALGEBRA

The concept of orthogonal signal is very essential for understanding OFDM. In the normal sense, it may look miracle that one can separately demodulate overlapping carrier. The concept of orthogonality unveils the miracle. To understand these concepts, it is very helpful to interpret signals as vectors. Like vectors, signal can be added and they

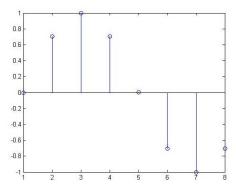


Fig. 2. Sampled Sine signal

can be expanded into a base. So signals can be visualized by geometric objects and many conclusion can be drawn by simple geometric argument without formal derivation. So it is worthwhile to become familiar with this point of view [3].

Signal analysis is intimately related with change of basis. That is why linear algebra (matrix algebra) is considered essential to understand modern signal processing [1].

The signal can be analysed using Linear Algebra Concepts. For instance, a sine signal sampled at 8 samples/sec is considered. The signal is shown in Fig 2.

The sampled values obtained is

$$\begin{bmatrix} 0 & 0.7 & 1 & 0.7 & 0 & -0.7 & -1 & -0.7 \end{bmatrix}^T$$

This is a point in  $\Re^8$ . So to represent this point, 8 orthonormal basis are required, each from  $\Re^8$ .

The standard way of representing the signal is by using canonical basis. There is infinite ways of representing the above signal using change of basis (some are obtained by rotating the canonical basis). Fourier Series, Discrete Cosine Transform (DCT), Wavelet transform, which plays a vital role in modern signal processing, are all change of basis to represent the same information(signal).

Fourier analysis is a family of mathematical techniques, all based on decomposing signals into sinusoids. The discrete Fourier transform (DFT) is the family member used with digitized signals.

Fourier Basis are widely used for OFDM communication. In fourier series, orthogonal basis can be created by equispaced samples of Complex Exponentials.

Let the  $\theta$  be defined as a vector

$$\theta = 0 \times \frac{2\pi}{N}, 1 \times \frac{2\pi}{N}, \dots (N-1) \times \frac{2\pi}{N}$$

The N orthogonal basis are created as follows

$$\begin{array}{lll} \operatorname{Base0} & \to & \exp^{i\times 0\times \theta} \\ \operatorname{Base1} & \to & \exp^{i\times 1\times \theta} \\ \operatorname{Base2} & \to & \exp^{i\times 2\times \theta} \\ \operatorname{Base3} & \to & \exp^{i\times 3\times \theta} \\ \vdots & & & \vdots \\ & \vdots & & & \vdots \\ \operatorname{Base(N/2)+1} & \to & \exp^{i\times (N/2)\times \theta} \\ \operatorname{Base(N/2)+2} & \to & \exp^{i\times (-(N/2)+1)\times \theta} \\ \vdots & & \vdots & & \vdots \\ \vdots & & & \vdots \\ \operatorname{BaseN} & \to & \exp^{i\times (-1)\times \theta} \end{array}$$

The DFT is basically about finding the projection of signal onto basis set. So the DFT operation can be defined as follows:

$$\begin{pmatrix} X(0) \\ X(1) \\ \vdots \\ \vdots \\ X(N-1) \end{pmatrix} = \begin{pmatrix} - & \overline{Base0} & - \\ - & \overline{Base1} & - \\ - & \vdots & - \\ - & \overline{BaseN-1} & - \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ \vdots \\ x(N-1) \end{pmatrix}$$

Where x represents the sampled signal and X represents the projection values.  $\overline{Base}$  represents the complex conjugate of DFT Base.

So Inverse Discrete Fourier transform (IDFT) involves representing the signal as linear combination of DFT basis.

$$\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ \vdots \\ x(N-1) \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} & & & | \\ & & & | \\ Base0 & \cdots & BaseN-1 \\ & & & | \\ & & & | \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ \vdots \\ \vdots \\ X(N-1) \end{pmatrix}$$

The above equation should be interpreted as

$$\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{pmatrix} = X(0) \begin{pmatrix} \begin{vmatrix} 1 \\ Base0 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} \begin{vmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix} 1 \\ Base1 \\ | \end{vmatrix} \\ +X(1) \begin{pmatrix}$$

In the above equation, if X(0), X(1), ..., X(N-1) are taken complex symbols, each of the symbol changes the phase(also magnitude) of each of the orthogonal base. That is, symbol X(i) changes the phase of the base Base i. This

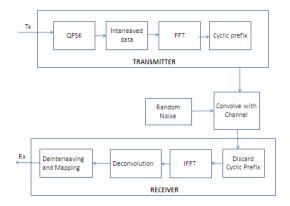


Fig. 3. OFDM - Block Diagram

is the core concept used in OFDM to load information into sub carriers. So a deep understanding of DFT and IDFT is required to understand OFDM.

Normally Fast Fourier Transform (FFT) and Inverse Fast Fourier transform (IFFT) are used to compute DFT and IDFT faster respectively.

#### IV. OFDM

Any communication system broadly consists of three blocks Transmitter, Channel and Receiver. The user information, which is in digital form, is generated, modulated and transmitted. The transmitted message gets affected due to channel parameters and noise. The receiver receives signal, demodulate and interprets the transmitted signal. This is the simplest explanation of any communication system.

OFDM is a method of encoding digital data on multiple carrier frequencies. This section focuses on explaining the OFDM system in linear algebra point of view.

A simple OFDM communication system can be represented using the block diagram shown in Fig 3. The transmitter loads the data into orthogonal sub carriers forming a composite signal. The transmitted signal passes through the channel which gets affected due to noise and the receiver extracts data from the composite signal.

The channel in real time, is very noisy and affects the transmitted signal. We don't have any control over the channel. But efficient algorithms are implemented in transmitter and receiver to account for channel parameters and for reliable communication. There are lot of parameters that affect the data. In this work, we have focussed on two parameters in channel that make reliable communication challenging: Inter Symbol Interference (ISI) and Random noise.

Each of the block is analysed with the aid of a simple example. Let the data that need to be transmitted is **00111101**.

1. The first step is to convert the user data into Quadratic Phase Shift Keying (QPSK) scheme. In QPSK, two bits are taken and are assigned a complex

symbol based on the value of the two bits as shown in Fig.4.

After mapping the input data to complex symbols, the user data is converted into  $4\times 1$  data.

$$\begin{pmatrix} 1+i\\1-i\\1-i\\-1+i \end{pmatrix}_{4\times 1}$$

2. The next step is interleaving of data. That is, the 4 bit symbol is repeated 4 times to make the system more resistant to error. So the data has been converted into data as shown below

$$\begin{pmatrix} 1+i \\ 1-i \\ 1-i \\ 1-i \\ -1+i \\ 1-i \\ 1-i \\ -1+i \end{pmatrix}_{4\times 1} \Rightarrow \begin{pmatrix} 1+i \\ 1-i \\ 1$$

It can be noted that  $1^{st}$ ,  $5^{th}$ ,  $9^{th}$  and  $13^{th}$  symbols contains the same information. This helps in effective estimation of symbols in the receiver side. For instance, if the  $1^{st}$  symbol is affected by noise, it can be recovered with the aid of rest of the three symbols  $(5^{th}, 9^{th} \text{ and } 13^{th} \text{ symbols})$ .

There are various ways of interleaving data. But this way of interleaving increases the robustness of the system. Interleaving can also be done by repeating  $1^{st}$  symbol 4 times, then  $2^{nd}$  symbol 4 times etc. But this is not resistent to burst error. So this type of interleaving is not done.

3. The next is the most crucial step. The  $16 \times 1$  data is loaded into 16 orthogonal DFT basis (subcarriers) as shown below

$$\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ \vdots \\ x(15) \end{pmatrix}_{16 \times 1} = \begin{pmatrix} \begin{vmatrix} & & & | \\ | & & & | \\ | & & & | \\ | & & & | \\ | & & & | \end{pmatrix} \begin{pmatrix} 1+i \\ 1-i \\ \vdots \\ \vdots \\ -1+i \end{pmatrix}$$

So the vector X left side is written as linear

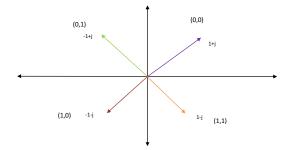


Fig. 4. QPSK mapping scheme

combination of each complex symbol with its base. Each complex symbol changes the phase of DFT subcarrier (also the magnitude is changed by  $\sqrt{2}$ ). The change of phase is taken as information in OFDM. Here, the serial data stream is passed through a serial to parallel converter, which splits the data into number of parallel channels [2]. The total number of the subcarriers in the OFDM signal and its carrier frequency depends on the information of the available spectrum from the spectrum sensing unit [5].

The above process is IFFT. So in order to carry out this step, IFFT of the input sequence is performed in transmitter. In the receiver side, FFT is taken to retrieve the original message.

One of the primary problem faced while understanding OFDM lies in this step. Since most of the people don't have strong foundation in Linear algebra, the above process is not visualized properly. So taking Inverse Fourier transform in transmitter creates a big mental block in understanding OFDM.

So in order to address this problem, in this work, we have taken FFT in the transmitter and IFFT in the receiver. This is possible because the rows of the IDFT matrix also acts like orthonormal subcarrier. So swapping the operation produces same result as that of previous method. But it eliminates all the confusions and barrier in understanding OFDM process. So, in this step, FFT of the  $16\times 1$  data is taken.

4. The next step is the addition of cyclic prefix. This step is performed in order to reduce Inter Symbol Interference(ISI). In telecommunication, ISI is defined as the cross talk between signals with the same sub channel of consecutive FFT frames, which are separated in time domain [2]. This is an unwanted phenomenon as the previous symbols have similar effect as noise, thus making

the communication less reliable. Also, ISI affects high bit rate transmission [4]. ISI is usually caused by multipath propagation resulting in interference of symbols. The presence of ISI in the system introduces errors in the decision device at the receiver output. The multipath problem that occur in the channel is shown in Fig 5. The channel shown contains three path, one straight path and two curved path. Here  $h_0$ ,  $h_1$ ,  $h_2$  are called channel coefficients.

Here the case assumed is there is no delay through direct Path 1, one time delay through Path 2 and two time delay through Path 3. It is obvious that multipath leads to ISI.

Cyclic prefix are also known as Guard Interval [3], which are added in order to reduce ISI. If there is N channel path, last (N-1) values are added as prefix to the transmitting data. This considerably reduces ISI. So, in this case 2 symbols are added as cyclic prefix as shown below, which makes the data size  $18 \times 1$ .

$$\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(14) \\ x(15) \end{pmatrix}_{16 \times 1} \implies \begin{pmatrix} x(14) \\ x(15) \\ x(0) \\ x(1) \\ \vdots \\ x(14) \\ x(15) \end{pmatrix}_{18 \times 1}$$

- 5. The transmitted signal is send. The signal passes through the channel. In channel, the signal gets convolved with the channel coefficient. For each channel path, there is a channel coefficient associated with it. This channel coefficient affects the signal passing through that path. In this example three channel coefficient are considered. So after convolution, the resultant vector is of size  $20 \times 1$ .
- 6. In channel, Random noise gets added to the transmitted signal. Some of the common noise that occurs in channel are Shot noise, Johnson noise, Partition noise and White noise etc [7]. In this paper, White Gaussian noise is considered. The noise is added manually to the signal.
- 7. The noisy signal is received at the receiver. The algorithm at receiver just inverses the process done in transmitter. In transmitter, 2 symbols has been added in front as cyclic prefix and 2 symbols is added at end by channel due to convolution. So these symbols are ignored by the receiver and only  $16 \times 1$  symbols in between is considered for further

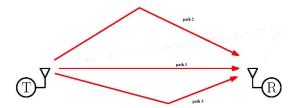


Fig. 5. Multipath Propogation in Channel

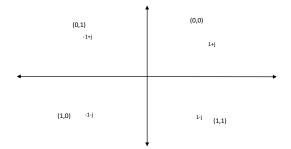


Fig. 6. Reverse Mapping

processing.

- 8. The next step is to retrieve the message from the subcarriers. For doing so, IFFT is calculated for the received signal.
- 9. The next step is to perform deconvolution on the data. If H represents the FFT of channel coefficients, then multiplying the signal with conjugate of H performs deconvolution of the data.
- 10. The step involves De-interleaving, that is, from  $16\times 1$ , the transmitted signal  $4\times 1$  has to be estimated. It is done as shown below

$$\left( \begin{array}{cccc} x (\hat{0}) & x (\hat{1}) & \dots & x (\hat{1}5) \end{array} \right)_{1 \times 16}^T$$

$$\begin{pmatrix} \frac{x(\hat{0}) + x(\hat{4}) + x(\hat{8}) + x(\hat{1}2)}{4} \\ \frac{x(\hat{1}) + x(\hat{5}) + x(\hat{9}) + x(\hat{1}3)}{4} \\ \frac{x(\hat{2}) + x(\hat{6}) + x(\hat{1}0) + x(\hat{1}4)}{4} \\ \frac{x(\hat{3}) + x(\hat{7}) + x(\hat{1}1) + x(\hat{1}5)}{4} \end{pmatrix}_{4 \times 1}$$

- 11. The final step involves estimating the transmitted signal from the received signal. This is done by mapping the De-interleaved data in complex plane and estimating the transmitted bits as shown in Fig 6.
- 12. Now the estimated signal is compared with the

actual transmitted signal. Bit Error Rate (BER) is defined as the number of received bit that has been altered due to noise in the channel. The number of bits wrongly estimated by the system contribute to BER and it measures the efficiency of the communication system.

#### V. OFDM ALGORITHM AND EXCEL IMPLEMENTATION

The algorithm for implementing OFDM is given separately for transmitter, channel and receiver.

#### A. Transmitter

- Declare number of carriers (Nc) and the number of taps (Nch) within the channel impulse response.
- 2. Generate data by using random numbers.
- 3. Convert to QPSK.
- Interleave the data to avoid the burst error while transmission.
- Take FFT of the interleaved data and then normalize it.
- 6. Do cyclic prefix of length Nch-1.

#### B. Channel

- Generate random channel coefficients (Rayleigh fading).
- 2. Convolve the data with the channel coefficients.
- Generate White Gaussian noise and add along with the generated data.

# C. Receiver

- 1. Remove the cyclic prefixes added while transmission.
- 2. Find IFFT of the received sequence.
- 3. Do de-convolution.
- 4. Find the error rate.

Using the above algorithm, step by step implementation in excel has been provided below taking no of transmitting bit (N) as 64, Nc=128, Nch=10.

### A. Transmitter

- Step 1.Generate 64 random numbers using RAND() function. Copy those numbers and paste it in the column A [A1 to A64] using the "paste special" option(since we are only interested in one set of such random numbers).
- Step 2.Convert the 64 random numbers in to QPSK and store it in the column "B" . Type expression in B1 = COMPLEX(IF(A1 < 0.5, -1, 1), IF(A33 < 0.5, -1, 1))then select and drag up to B32.
- Step 3.Interleave the data to make it as 128 symbols (C1 to C128): [Copy the 32 symbols QPSK data and paste four times ].
- Step 4.Take FFT of the interleaved data (loading on to the sub carriers)
  - Data→data analysis→Fourier analysis

Select input (C1 to C128) and output range (D1 to D128), click OK

Note: In order to carry out Fourier operation in Excel, "Analysis Toolpak" Add In has to be enabled.

Step 5.Normalize the output of STEP 4 by multiplying it with square root of the length of data (128).

Step 6.Do cyclic prefixing: [Copy last 9(Nch-1) symbols and paste it in to first], then copy the rest of the values in to the cells [F9 to F 137].

#### B. Channel

- Step 1.Store the following variables in any cell in the Excel sheet
  - Channel attenuation, att = 2
  - •Number of carriers, Nc = 16
  - •Number of taps, Nt=2
- Step 2.Enter the numbers 0-9 in the cells [G1-G10]. Calculate variances of channel taps according to exponential decaying power profile using the expression, [= EXP(-G1/ att)] in the column H1 then select and drag upto H10.
- Step 3.Normalize the channel variance by dividing each element with its sum.
- Step 4.Calculation of channel coefficients [h]. using following code
  - •COMPLEX(NORMINV(RAND(),0,1),

NORMINV(RAND(),0,1))

Click and drag and do special paste make 10 values •Multiply this with square root of normalized variance and square root of 0.5.

Step 5.Convolving channel coefficient with the data

- 5.1 Do zero padding to the cyclic prefixed data . [Add zeros to make it 256 symbols ( = COMPLEX(0,0) ].
- 5.2 Zero pad the channel coefficients and make it of length 256 [Add zeros to make it 256 symbols].
- 5.3 Take FFT of data of length 256.
- 5.4 Take FFT of channel coefficient of length 256(H).
- 5.5 Multiply the two [convolution of data and channel coefficients].
- 5.4 Take IFFT of the result.
- Step 6.Generate white Gaussian noise and add along with the data. (Or randomly generate noise).
- Step 7. Consider only first 146 symbols.

# C. Receiver

- Step 1.Remove first 9 symbols which contribute to Cyclic prefix. Also last 9 symbols which was added to data while convolution. So Consider 128 symbols in between.
- Step 2.Take IFFT of the received data (Y).
- Step 3.Perform De-convolution on the data using the formula: conjugate(H)\*Y.
- Step 4.Perform de-interleaving of the data.

Step 5.Take sign of the real and imaginary part of the complex number.

Step 6.Map the symbols to the corresponding bits.

#### VI. RESULTS AND DISCUSSIONS

In this paper, we have discussed the Orthogonal Frequency Division Multiplexing (OFDM) based on linear algebra point of view. The simulation of it is done in Excel. We have taken a 32 symbols for transmission, and it is interleaved to 128 symbols data in order to avoid the burst error that may occur while transmission. The normalised FFT of the interleaved data is then taken and cyclic prefix is done to avoid the multi path errors in the channel.

The channel coefficients are generated randomly based on Rayleigh Fading. The data to be transmitted is then convolved with the generated channel coefficients. Then, the White Gaussian noise that is being added to the transmitted data. In the receiver part, the cyclic prefix added is discarded and IFFT of the received data is taken. The original transmitted data is then retrieved back by deconvolving it with the conjugate of the channel coefficients. The step by step OFDM simulation done in Excel can be downloaded from the link: "nlp.amrita.edu:8080/sisp/wavelet/ofdm/ofdm.xls".

#### VII. CONCLUSION

In this paper, effort has been made to bring down the abstract concept of OFDM. The technology was explained using simple coordinate geometry concepts in linear algebra, thus making anyone understand the underlying principle. Also a practical simulation of OFDM communication system is done using Excel, thus providing opportunity for anyone to get a feel of the technology. The Excel-based implementation of OFDM helps anyone to understand communication part in easy way.

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