Lesson 2 Naive Bayes

Supervised Classification

- learning from labeled data. After understanding the data, the algorithm determines which label should be given to new data by associating patterns to the unlabeled new data.
- Examples
    - Identifying someone from a set of pictures
    - Song recommendation based on previous liked songs

Features and Labels

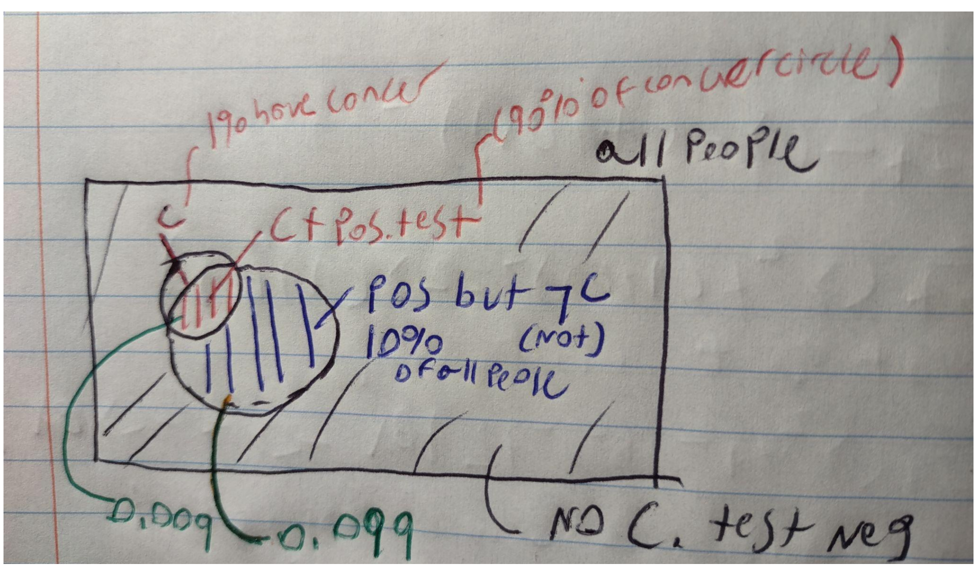
- Song example
- Features
    - Intensity, Temp, Genre, Voice gender
- Labels
    - Like, Dislike

Naive Bayes

$p(c) = 0.01$, c = Cancer

Test: 90% it is positive if you have C (Sensitivity)
90% it is negative if you don't have C (Specitivity)

Question: Test = Positive



What is the Probability of Having cancer?
about 8%, see diagram in written notes

- **Sensitivity** (True Positive Rate) refers to the proportion of those who received a positive result on this test out of those who actually have the condition (when judged by the 'Gold Standard').
- **Specificity** (True Negative Rate) refers to the proportion of those who received a negative result on this test out of those who do not actually have the condition (when judged by the 'Gold Standard').

Bayes Rule

Prior probability * test evidence → posterior probability

Prior: p(c) = 0.01 = 1% p(not c) = 0.99 = 99%
p(pos | c) = .9 = 90%
p(neg | not c) = 0.9 p(pos | not c) = 0.1

Posterior (joint probability):
p(c , pos.) = p(c) * p(pos | c) = 0.01 * .9 = 0.009
p(not c , pos) = p(not c) * p(pos | not c) = .99 * 0.1 = 0.099

Normalizer
p(c , pos.) + p(not c , pos) = 0.009 + 0.099 = 0.108

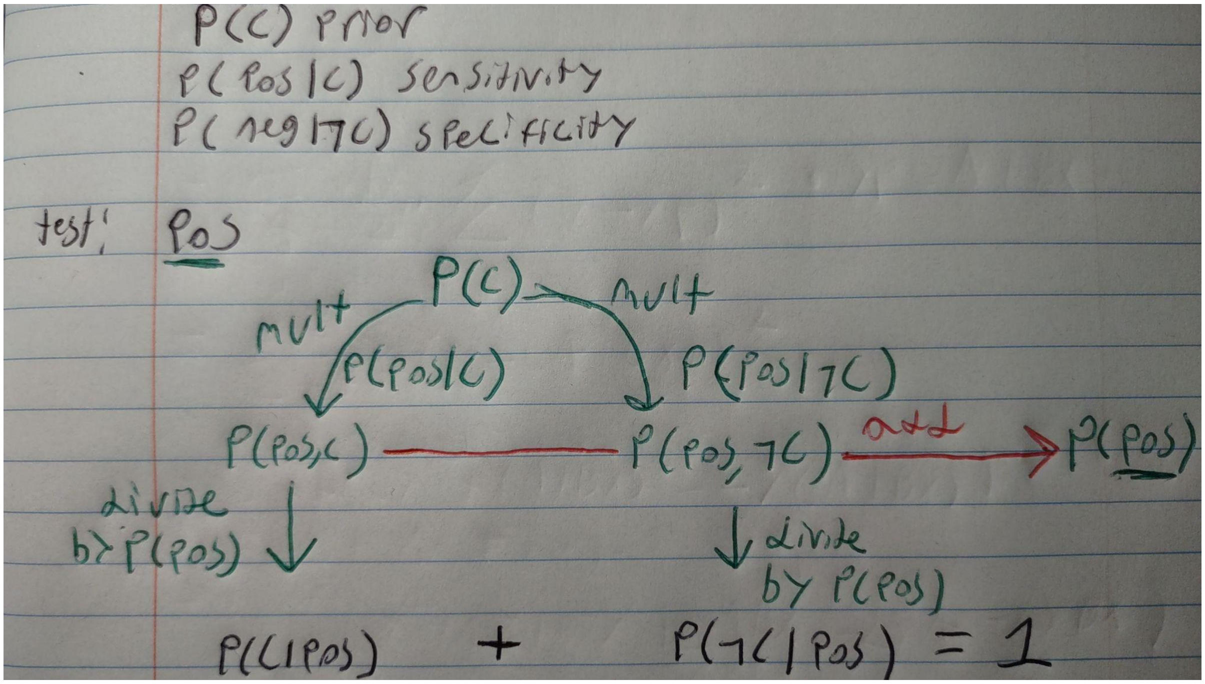
Posterior (actual):

p(c | pos) = $\frac{0.009}{0.108}$ = 0.0833

p(not c | pos) = $\frac{0.099}{0.108}$ = 0.9167

p(c | pos) + p(not c | pos) = 0.0833 + 0.9167 = 1

Bayes Rule Diagram



Bayes Rule for Classification

$$\underbrace{P(c|x)}_{\text{Posterior Probability}} = \frac{\overbrace{P(x|c)}^{\text{Likelihood}}\,\overbrace{P(c)}^{\text{Class Prior Probability}}}{\underbrace{P(x)}_{\text{Predictor Prior Probability}}}$$

$$P(c|X) = P(x_1|c) \times P(x_2|c) \times \cdots \times P(x_n|c) \times P(c)$$

<u>Random email identification example</u>:
Naive Bayes allows us to determine who is likely to have sent an email if given at random

Probabilities of using word in email

- Chris - love (.1), Deal(.8), life(.1)
- Sara - love (.5), Deal(.2), life(.3)

Prior probabilities:

- p(chris) = 0.5
- p(sara) = 0.5

Who is likely to have sent the following emails given the probabilities?

Email 1: Love Life! - A: Sara

chris : p(chris | love) x p(chris | life) x p(chris) = .1 x .1 x .5 = 0.005
sara : p(sara | love) x p(sara | life) x p(sara) = .5 x .3 x .5 = <u>0.075</u>

Email 2: Life Deal! - A: Chris

chris : p(chris | life) x p(chris | deal) x p(chris) = .1 x .8 x .5 = <u>0.04</u>
sara : p(sara | life) x p(sara | deal) x p(sara) = .3 x .2 x .5 = 0.03

Calculate the following posterior probabilities of the following

1)

p(chris | “Life Deal”) = $\frac{0.04}{0.04 + 0.03}$ = 0.57

p(sara | “Life Deal”) = $\frac{0.03}{0.04 + 0.03}$ = 0.43

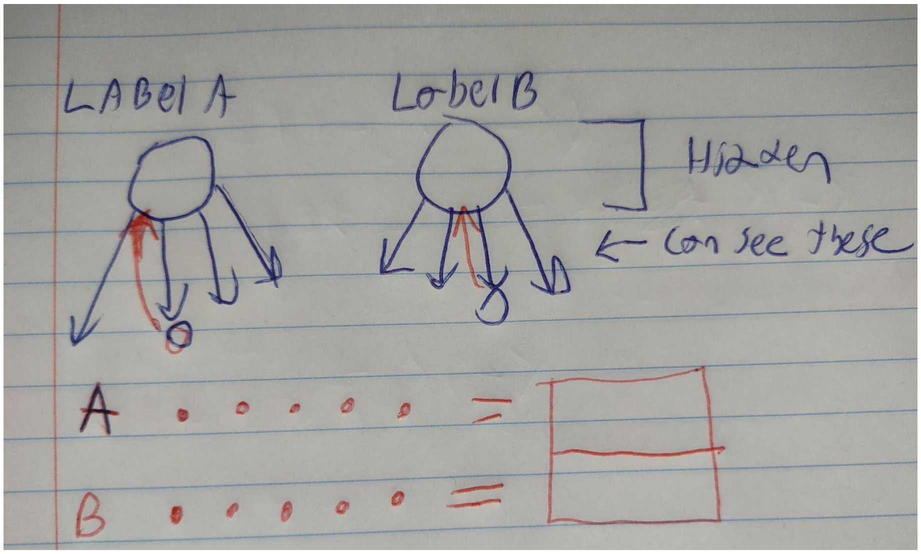
2)

chris : p(chris | love) x p(chris | deal) x p(chris) = .1 x .8 x .5 = 0.04
sara : p(sara | love) x p(sara | deal) x p(sara) = .5 x .2 x .5 = 0.05

p(chris | “Love Deal”) = $\frac{0.04}{0.04 + 0.05}$ = 0.444

p(sara | “Love Deal”) = $\frac{0.05}{0.04 + 0.05}$ = 0.555

Naive Bayes



- Target labels a and b are hidden, you don't get to see them
- What you see are things they do like words,
    - each with different probabilities
    - Each one you see gives you evidence as to whether it is A or B
- Multiply evidences for all the things you see
    - The product gives you the ratio whether you believe it is A or B
- Naive bayes lets you identify from a text source which label is more likely
- Called naive because it ignores one thing
    - Order of the words / Order of evidences

Bayes rule (pros / cons)

Pros

- Easy to implement and efficient to run
- Deals well with very large feature sets
    - Example set being the 20,000 - 200,00 words in the english language

Cons

- Breaks in “funny” ways
- Phrases that incorporate multiple words do not work really well in naive bayes
    - Phrase “Chicago Bulls” could bring up pictures of an the animal bull and pictures of city of Chicago rather than pictures of the Chicago Bulls sports team