Transmission System Restoration:

Co-optimization of Repairs, Load Pickups, and Generation Dispatch

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***Abstract*—This paper studies the restoration of a transmission system after a significant disruption (e.g., a natural disaster). It considers the co-optimization of repairs, load pickups, and generation dispatch to produce a sequencing of the repairs that minimizes the size of the blackout over time. The core of this process is a Restoration Ordering Problem (ROP), a non-convex mixed-integer nonlinear program that is outside the capabilities of existing solver technologies. To address this computational barrier, the paper examines two approximations of the power flow equations: The DC model and the recently proposed LPAC model. Systematic, large-scale testing indicates that the DC model is not sufficiently accurate for solving the ROP. In contrast, the LPAC power flow model, which captures reactive power and voltage magnitudes, is sufficiently accurate to obtain restoration plans that can be converted into AC-feasible power flows. Experiments also suggest that the LPAC model provides a robust and appealing tradeoff of accuracy and computational performance for solving the ROP.**

Keywords*—*Power System Restoration, Load Pickup, AC Power Flow, LPAC Power Flow, Optimization

NOMENCLATURE

$\widetilde{V} = v + i\theta$	AC voltage
$\widetilde{S} = p + iq$	AC power
$\widetilde{Y} = g + ib$	Line admittance
$\widetilde{V} = \lvert\widetilde{V}\rvert\angle\theta^\circ$	Polar form
$\mathcal{PN}$	Power network
N	Set of buses in a power network
L	Set of lines $\langle n, m\rangle$ in a power network where n is the from node
L^r	Set of lines $\langle n, m\rangle$ in a power network where n is the to node
$\mathcal{D}$	Set of damaged network components
$\mathcal{R}$	Set of damaged network components to be repaired
s	Slack bus
$\lvert\widetilde{V}^t\rvert$	Voltage magnitude for normal operation
μ	Average
$\overline{x}$	Upper bound of x
$\underline{x}$	Lower bound of x

I. INTRODUCTION

Restoring a power system after a significant disruption (e.g., a natural disaster) is an important task with consequences on both human and economic welfare. To mitigate the consequences of such events, the next generation of power system is expected to be more resilient and *self healing* [1]. This work focuses on the restoration of the transmission system, which is computationally challenging for a variety of reasons. First, since no *typical* operating point is known for the damaged network, it is often difficult to determine a steady state for the network, i.e., a solution to the AC power flow problem [2]. Second, restoration plans must jointly optimize the routing of repair crews, the scheduling of component energizing, load pickups, and generation dispatch. The resulting optimization problem is a non-convex mixed-integer nonlinear program which is extremely hard from a computational standpoint.

These difficulties are addressed in the power restoration algorithm proposed in [3] by decomposing the problem into several steps. The algorithm decouples the power system and logistics aspects, first scheduling the component energizings and then routing the repair crews. The two subproblems are linked through precedence constraints that are derived from the power schedule and injected into the crew routing. This paper focuses on the first step of this decomposition, the so-called Restoration Order Problem (ROP). The goal of the ROP is to find a high-quality restoration schedule, i.e., a sequence of steady-state power flows for the network that minimizes the size of the blackout over time. Each steady state corresponds to a restoration action (e.g., repairing a line) and may increase the served load and change the generation dispatch compared to earlier steady states.

To find a high-quality restoration plans, the ROP formulation in [3] relies on the DC power flow approximation, which is widely used in power system optimization (e.g., [2], [4], [5], [6]). However, the accuracy of the DC power flow is the topic of much discussion: Some papers take an favorable outlook, (e.g., [2], [7]), while others (e.g., [8], [9]) are more cautious. The accuracy of the DC power flow is particularly important in power restoration as a feasible AC base-point solution is often not available and it is preferable to operate the network near its design limits.

This paper investigates the use of the DC power flow model and the recent LPAC power flow model [10] for transmission system restoration optimization. The LPAC model enhances the DC approximation by capturing line losses, voltage magnitudes and reactive power. Moreover, the LPAC model is a linear program and can utilize the computational benefits of mixed-integer programs [11]. Our key findings can be summarized as follows.

1) The DC power flow is overly optimistic on the quantity of load pickups the network can support. This results in generator dispatches that have AC-feasible flows in only 78.0% of cases studied. In contrast, the additional constraints in the LPAC lead to AC-feasible flow solutions in 99.5% of the cases studied.
2) In the 78.0% of cases where the generator dispatch from the DC model leads to AC-feasible solutions, line loading, reactive injection and voltage drops are significantly underestimated producing considerable constraint violations. Meanwhile, the dispatched obtained by the LPAC model have only negligible violations.
3) Prioritizing repairs with the DC and LPAC models leads to significantly different restoration plans in the 18 real-world case studies considered. LPAC-based restorations significantly reduce the size of the blackouts over time compared to DC-based plans, indicating that the DC approximation is also not sufficient for prioritizing repairs.

These three findings are demonstrated experimentally with three case studies of increasing complexity. A study of 170,000 damage contingencies of the IEEE30 network highlights point (1). Points (2) and (3) are demonstrated using 18 real-life restoration problems stemming from natural disasters. These benchmarks were provided by Los Alamos National Laboratories and are based on a U.S. transmission system with 266 components. A detailed analysis of restoration plans with fixed repair schedules illustrates point (2). Finally, point (3) is demonstrated by a careful comparison of repair schedules produced by both the DC and LPAC models. The resounding conclusion of these studies is that the DC power flow model is not reliable for planning the restoration of transmission systems. The LPAC model, in contrast, seems to provide a reliable and accurate alternative.

The rest of the paper is organized as follows. Section II reviews the relevant aspects in the restoration of transmission systems and formalizes the core computational problems considered in this paper. Section III illustrates the challenges of solving the generator dispatch problem on damaged IEEE30 networks. Section IV extends the findings of Section III to real-world power system restoration benchmarks and Section V concludes the paper.

II. Power System Restoration Formalization

Power system restoration (PSR) has been an area of active research for at least 30 years (see [12] for a comprehensive collection of work). Its goal is to find fast and reliable ways of restoring a power system to its normal operational state after many components have been damaged or disconnected (e.g., after a cascading blackout). Due to the complex nature of power systems, PSR remains challenging. Practitioners must delicately manipulate many aspects of the network including power balance, line thermal limits, power quality, generator ramp rates, and transient effects. In general, these challenges form a loose priority list: Power balance must be ensured first before considering line thermal limits and so on. This paper focuses on power balance, line limits, and power quality, leaving additional aspects for future consideration.[1]

Drawing inspiration from the joint repair and restoration algorithm proposed in [3], this paper restricts its attention to the steady-state behavior of the network and focuses on scheduling repairs, generator dispatch, and load pickups in order to reduce the size of the blackout over time. This task is the so-called Restoration Ordering Problem (ROP). We introduce the ROP by first defining a core single-step optimization sub-problem and then extend it to the multi-step ROP.

The maximum generator dispatch problem (GDP) is a core sub-problem of the ROP consisting of supplying as much active power as possible to the load points in a damaged power network. A formulation of the GDP using the steady-state AC power flow equations is presented in Model 1. The AC-GDP departs from classic AC power optimization problems in that the loads are decision variables: The algorithm thus determines how much load is served and where. This model assumes that the loads are continuous quantities and that their power factor must be maintained during restoration. The input data and decision variables are discussed in Model 1. The objective (M1.1) maximizes the active power reaching the load points. Constraints (M1.2) force all of the network components to be active, while Constraints (M1.3–M1.4) define the operational relationship of components (e.g., lines are non-operational unless all connected buses are operational). Constraint (M1.5) fixes the slack-bus phase angle to 0 to compare various models of the power system easily. Constraints (M1.6–M1.7) model energy conservation (i.e., Kirchhoff's Current Law). Constraints (M1.8–M1.9) prevent power from flowing in non-operational components. Constraints (M1.10–M1.11) capture the flow of power via Ohm's Law (when a line is operational) and constraints (M1.12) enforce the thermal line-loading limits. This AC-GDP formulation can be naturally generalized to include discrete loads or prioritized power demands: It suffices to discretize the l_n variable and/or weight the objective function appropriately. The activation and operation variables (y, z respectively) are not stickily necessary in a GDP formulation, and hence they are fixed to 1 in Model 1. They are included here so that the ROP can then be presented as a generalization of the GDP.

Let $\mathcal{D} \subseteq N \cup L$ be a damaged subset of the power network and let $\mathcal{R} \subseteq \mathcal{D}$ be the components to be repaired. The ROP can be viewed as searching for a sequence of the damaged components in $\mathcal{R}, r_1 \rightarrow r_2 \rightarrow \ldots \rightarrow r_{|\mathcal{R}|}$ and their associated steady states $GDP_0 \rightarrow GDP_1 \rightarrow GDP_2 \rightarrow \ldots \rightarrow GDP_{|\mathcal{R}|}$, where steady state GDP_i corresponds to the GDP solution for the damaged network with components $\{r_1, \ldots, r_i\}$ repaired. The goal is to find a repair sequence that minimizes the blackout over time. Model 2 formalizes the ROP as a sequence of GDPs linked by the network topology changes made in each restoration step. The objective (M2.1) maximizes the active power reaching the load points over all of the time steps, hence minimizing the size of the blackout. Constraints (M2.2) force all of the working network components to be active, while Constraints (M2.3) activate one new network component in each restoration step. Constraints (M2.4) ensure that repaired components stay repaired.

[1] Note that extensions of the work presented here to account for some aspects of transient stability are proposed in [13].

Model 1 AC Generator Dispatch Problem (AC-GDP).

Inputs:

$\mathcal{PN} = \langle N, L, s \rangle$	the power network
g_{nm}, b_{nm}	admittance between bus n and bus m
$\underline{p}^g_n, \overline{p}^g_n$	active generation bounds at bus n
$\underline{q}^g_n, \overline{q}^g_n$	reactive generation bounds at bus n
$\overline{p}^l_n, \overline{q}^l_n$	desired active and reactive load at bus n
$\underline{\lvert\widetilde{V}_n\rvert}, \overline{\lvert\widetilde{V}_n\rvert}$	voltage magnitude limits at bus n
$\overline{\lvert\widetilde{S}_k\rvert}$	line loading limit on line k

Variables:

$y_i \in \{0, 1\}$	- item i is activated
$z_i \in \{0, 1\}$	- item i is operational
$\theta^\circ_n \in (-\infty, \infty)$	- phase angle of bus n (radians)
$\lvert\widetilde{V}_n\rvert \in (\underline{\lvert\widetilde{V}_n\rvert}, \overline{\lvert\widetilde{V}_n\rvert})$	- voltage magnitude of bus n
$p^g_n \in (\underline{p}^g_n, \overline{p}^g_n)$	- active generation at bus n
$q^g_n \in (\underline{q}^g_n, \overline{q}^g_n)$	- reactive generation at bus n
$l_n \in (0, 1)$	- percentage load served at bus n
$p_{nm} \in (-\overline{\lvert\widetilde{S}_k\rvert}, \overline{\lvert\widetilde{S}_k\rvert})$	- line active power $\forall \langle k : n, m \rangle \in L \cup L^r$
$q_{nm} \in (-\overline{\lvert\widetilde{S}_k\rvert}, \overline{\lvert\widetilde{S}_k\rvert})$	- line reactive power $\forall \langle k : n, m \rangle \in L \cup L^r$

Maximize:

$$\sum_{n \in N} \overline{p}^l_n l_n \tag{M1.1}$$

Subject to:

$$y_n = 1 \quad \forall n \in N \cup L \tag{M1.2}$$
$$z_n = y_n \quad \forall n \in N \tag{M1.3}$$
$$z_k = y_k \wedge y_n \wedge y_m \quad \forall \langle k : n, m \rangle \in L \tag{M1.4}$$
$$\theta^\circ_s = 0 \tag{M1.5}$$
$$p^g_n - \overline{p}^l_n l_n = \sum_{\substack{m \in N \\ n \neq m}} p_{nm} \quad \forall n \in N \tag{M1.6}$$
$$q^g_n - \overline{q}^l_n l_n = \sum_{\substack{m \in N \\ n \neq m}} q_{nm} \quad \forall n \in N \tag{M1.7}$$
$$\neg z_n \rightarrow q^g_n, l_n = 0 \quad \forall n \in N \tag{M1.8}$$
$$\neg z_k \rightarrow p_{nm}, q_{nm} = 0 \quad \forall \langle k : n, m \rangle \in L \cup L^r \tag{M1.9}$$
$$z_k \rightarrow p_{nm} = \lvert\widetilde{V}_n\rvert^2 g_{nm} - \lvert\widetilde{V}_n\rvert\lvert\widetilde{V}_m\rvert g_{nm} \cos(\theta^\circ_n - \theta^\circ_m) - \lvert\widetilde{V}_n\rvert\lvert\widetilde{V}_m\rvert b_{nm} \sin(\theta^\circ_n - \theta^\circ_m) \quad \forall \langle k : n, m \rangle \in L \cup L^r \tag{M1.10}$$
$$z_k \rightarrow p_{nm} = -\lvert\widetilde{V}_n\rvert^2 b_{nm} + \lvert\widetilde{V}_n\rvert\lvert\widetilde{V}_m\rvert b_{nm} \cos(\theta^\circ_n - \theta^\circ_m) - \lvert\widetilde{V}_n\rvert\lvert\widetilde{V}_m\rvert g_{nm} \sin(\theta^\circ_n - \theta^\circ_m) \quad \forall \langle k : n, m \rangle \in L \cup L^r \tag{M1.11}$$
$$p^2_{nm} + q^2_{nm} \leq \overline{\lvert\widetilde{S}_k\rvert}^2 z_k \quad \forall \langle k : n, m \rangle \in L \cup L^r \tag{M1.12}$$

Model 2 AC Restoration Order Problem (AC-ROP).

Inputs:

$\mathcal{R}$ - the set of items to restore
$\mathcal{D}$ - the set of damaged items
All inputs from Model 1

Variables:

Variables of Model 1 replicated $|\mathcal{R}|$ times

Maximize

$$\sum_{k=1}^{|\mathcal{R}|} \sum_{n \in N} \overline{p}^l_n l_{kn} \tag{M2.1}$$

Subject to: $(1 \leq k \leq |\mathcal{R}|)$

$$y_{kn} = 1 \quad \forall n \in (N \cup L) \setminus \mathcal{D} \tag{M2.2}$$
$$\sum_{n \in \mathcal{R}} y_{kn} = k \tag{M2.3}$$
$$y_{(k-1)n} \leq y_{kn} \quad \forall n \in \mathcal{R} \tag{M2.4}$$

$|\mathcal{R}|$ replicates of constraints (M1.3-M1.12) from Model 1

Solution Methods: The AC-GDP is a non-convex nonlinear program (NLP) and the AC-ROP is a non-convex mixed integer nonlinear program (MINLP). Both present significant computational challenges. For example, the AC-GDP may be solved by interior-point methods, such as IPOPT [14] or the Newton-Raphson method. However, these methods have two drawbacks: (1) They are not easily generalized to mixed (integer and continuous) models such as those in [3]; (2) Their convergence is not guaranteed and, when the system is heavily loaded, the solution space is riddled with infeasible low-voltage solutions which are not usable in practice [15]. In fact, finding a solution to the AC power flow when a base-point solution is not available can be "maddeningly difficult" [2]. Therefore, we consider more computationally tractable variants of the power flow equations which admit reliable algorithms, such as, the DC approximation [16], LPAC approximation [10], SOCP relaxation [17], QC relaxation [18], and the SDP relaxation [19]. To stay within the context of the MIP models proposed in [3], the best choice for comparison is the DC power flow, as used previously, and the LPAC approximation.

The DC power flow is the most common linear approximation of AC power flow. This approximation is incredibly simple and consists of replacing constraints (M1.10–M1.12) with

$$p_{nm} = -b_{nm}(\theta^\circ_n - \theta^\circ_m)$$

From a computational standpoint, the DC model is much more appealing than the AC model: It forms a system of linear equations which can be embedded in Mixed Integer Programs (MIP), a technology widely used for power system applications [3], [2], [4], [5], [6]. However, each new application domain must verify that the assumptions of the DC model hold in a new context. A major shortcoming of the DC model is that it does not capture reactive power or voltage magnitudes. As we will see, these constraints are very important for the GDP and ROP. This paper denotes the approximations of the GDP and ROP based on the DC model by DC-GDP and DC-ROP.

Recently, a Linear-Programming approximation of the AC power flow equations (LPAC) was proposed [10] to bridge the gap between the DC and AC power flow models. The LPAC model approximates the AC power flow with a linear program and hence it can still be solved reliably and embedded in MIP solvers, making it a natural extension to the methods of [3]. The LPAC model is derived from the following assumptions: (1) $\sin(\theta^\circ_n - \theta^\circ_m) \approx \theta^\circ_n - \theta^\circ_m$; (2) the voltage magnitude at each bus is expressed as a deviation from a nominal operating voltage, i.e., $|\widetilde{V}| = |\widetilde{V}^t| + \phi$; (3) the non-convex cosine function is replaced with a polyhedral relaxation denoted by $\widehat{\cos}_{nm}$; (4) the remaining nonlinear terms are factored and approximated with a first-order Taylor expansion. These modifications yield the following power flow equations, replacing constraints (M1.10–M1.11):

$$p_{nm} = g_{nm} - g_{nm}\widehat{\cos}_{nm} - b_{nm}(\theta^\circ_n - \theta^\circ_m)$$
$$q_{nm} = -b_{nm} + b_{nm}\widehat{\cos}_{nm} - g_{nm}(\theta^\circ_n - \theta^\circ_m) - b_{nm}(\phi_n - \phi_m)$$

As mentioned in [10], the constraints (M1.12) can be incorporated in the LPAC model via a polyhedral outer approximation similar to $\widehat{\cos}$. Although the LPAC model is an approximation, it captures line losses, reactive power flows, and bus voltage magnitudes, which greatly improves its accuracy over the DC model. This paper uses LPAC-GDP and LPAC-ROP to denote the GDP and ROP approximations based on the LPAC model.

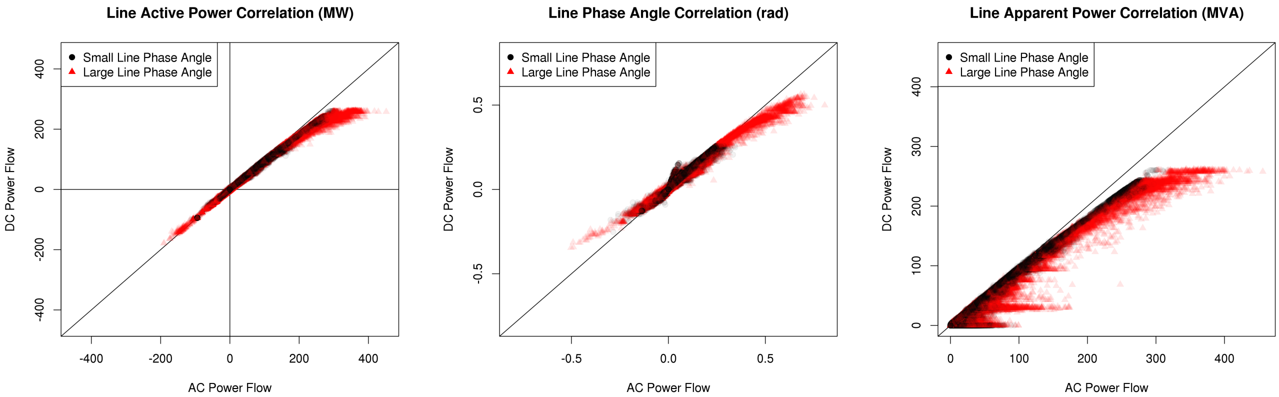

Fig. 1. Accuracy of the DC-GDP Algorithm with N–10 Contingencies on Active Power (left), Line Phase Angles (center), and Apparent Power (right).

Evaluating the Quality of the Approximations: AC power flow problems are used to compare the solution of various approximations in a consistent way. The approach is the following:

1) Solve an approximate GDP (DC or LPAC).
2) Construct an AC load flow problem with the generator dispatch and load pickups from step (1) to obtain an AC-GDP solution.

Note that this approach easily extends to the ROP as it is simply a sequence of GDPs. Once a solved AC load flow is available, constraint violations can be measured to test the quality of the approximation. Because the DC and LPAC approximations are linear, the first step of this approach is guaranteed to converge to a solution, if one exits. The AC power flow problem in the second step of this algorithm amounts to solving a system of non-convex nonlinear equations (a feasibility problem) and is not guaranteed to produce a feasible AC solution. The study of this convergence, or the absence thereof, is one of the key points of the next section.

III. Damaged IEEE30 Networks

This section answers a simple question: *Can the DC-GDP and LPAC-GDP effectively solve arbitrary damage cases of the IEEE30 bus network?* It considers 170,000 damage contingencies of the IEEE30 to evaluate the DC-GDP and LPAC-GDP algorithms. The contingencies are obtained by randomly selecting 10,000 damaged networks for each of the N–3, N–4, …, N–20 cases, where N–k denotes a network where k lines have been damaged. Due to the extensive damage in these networks, the maximum power that can be delivered is not immediately clear.

Reliability of the DC-GDP: Our first experiment investigates how often the DC-GDP algorithm converges to an AC solution. The result are presented in Table I. The second and the third columns show the number of solved models and the average active power dispatched, grouped by contingency size. For small contingencies (i.e., N–3 and N–4) where less than 10% of the network is damaged, the DC-GDP algorithm almost always converges to an AC solution. However, for large contingencies (i.e., N–10 and N–14) where more than 25% of the network is damaged, the DC-GDP algorithm only finds an AC solution in about 78% of the contingencies.

TABLE I. IEEE30 Contingencies for DC-GDP and LPAC-GDP.

	DC-GDP		LPAC-GDP	
Damage	Solved	μ(% delivered)	Solved	μ(% delivered)
N–3	9945	98.76	10000	93.14
N–4	9846	97.48	9999	90.37
N–5	9652	95.96	9998	87.35
N–6	9401	93.79	9995	84.25
N–7	9048	91.02	9991	80.8
N–8	8720	87.2	9983	77.07
N–9	8291	83.4	9971	73.58
N–10	8022	78.67	9971	69.88
N–11	7801	73.56	9956	65.87
N–12	7683	68.82	9951	62.44
N–13	7832	63.06	9965	57.94
N–14	7920	58.06	9946	53.95
N–15	8154	52.19	9955	49.22
N–16	8472	48.23	9962	45.83
N–17	8724	44.18	9962	42.21
N–18	8973	40.84	9974	39.06
N–19	9197	36.58	9989	35.17
N–20	9463	33.65	9988	32.26

To understand why the DC-GDP is producing infeasible generator dispatch and load pickups, we look at the N–10 contingency case in detail. The DC-GDP accuracy is measured by comparing the DC flow obtained in Step (1) with the AC flow produced in Step (2). In general, we measure various quantities (e.g., active power, phase angles) in both steps of the GDP algorithm and visualize them as a correlation plot. Specifically, for some measurement (e.g., the active power of a line), the x-axis gives the data value in the AC solution from Step (2) and the y-axis gives the data value in the DC solution in Step (1). As a result, the closer the plot is to the line $x = y$, the more the AC and DC solutions agree. Figure 1 presents the correlation for active power, line phase angles, and apparent power for the 8022 converged N–10 contingencies on the IEEE30 network. Each data point in the plots represents the active power, the phase angle difference, or the apparent power on a line. For brevity, the results from all contingencies are superimposed onto the same correlation graph. The plots also use red triangles for data points obtained from networks that feature *large* line phase angles (i.e., $|\theta^\circ_n - \theta^\circ_m| > \pi/12$). This makes it possible to understand the link between large line phase angles and discrepancies in model accuracy (as observed in [9]). Figure 3 presents histograms of the bus reactive injection and voltage magnitudes of the AC solutions obtained by DC-GDP. The histograms indicate that many of the AC solutions have extreme reactive injection values (e.g.,

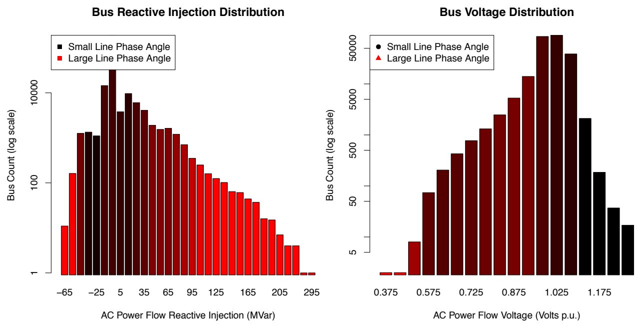

Fig. 3. Histograms of Reactive Injection (left) and Voltage Magnitudes (right) of N–10 Contingencies using the DC-GDP Algorithm.

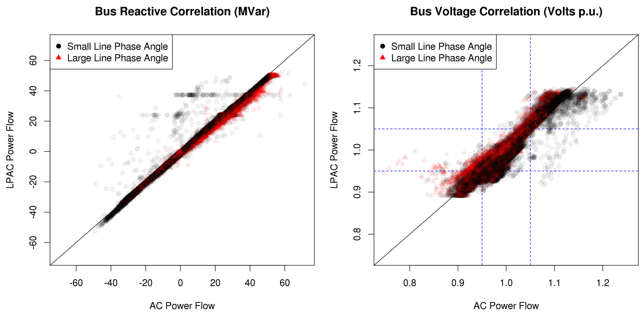

Fig. 4. The Accuracy of Reactive Injection (left) and Voltage Magnitudes (right) of N–10 Contingencies using the LPAC-GDP Algorithm.

over 100 MVArs) and very low voltage magnitudes (e.g., below 0.8 Volts p.u.). These results indicate that many of the 8022 solved instances are producing extreme and inoperable AC solutions. The remaining unsolved contingencies suffer from unreasonable generator dispatch. Figure 3 indicates that high reactive injection and low voltage magnitudes are a major problem for DC-GDP solutions and suggests that constraints on these values must be incorporated into the GDP solution method.

Quality of the LPAC-GDP: Contrary to the DC model, the LPAC model incorporates aspects of reactive power and voltage magnitudes and can impose operational constraints on these values. These additional constraints remove the extreme AC solutions observed in Figure 3 and produce more reasonable AC solutions at the cost of dispatching slightly less active power. Consider Table I again: The fourth and the fifth columns present the number of solved models and the average active power dispatched grouped by contingency size for the LPAC-GDP algorithm. The results indicate that LPAC-GDP produces an AC solution in 99.5% of the contingencies. Furthermore, this significant increase in solved models is achieved with only a 10% reduction in dispatched loads on average. The robustness of LPAC-GDP is particularly appealing as computational reliability is an essential criterion in the disaster-management contexts studied in [3]. Consider now the correlation plots in Figures 2 and 4. Figure 2 presents the correlation of active power, line phase angles, and apparent power for the N–10 contingencies in the format described previously. The plots are encouragingly accurate for the 9971 solved contingencies, even in the presence of large line phase angles. Figure 4 investigates the accuracy of reactive power and bus voltages, once again using correlation plots between the flows in Steps (1) and (2) of the LPAC-GDP algorithm. Again, these are quite accurate for the vast majority of contingencies. The largest approximation errors occur for the bus voltages, which is not surprising since the LPAC model only uses a linear approximation of voltage magnitudes. The reactive injection and voltage magnitudes remain within reasonable bounds in the LPAC-GDP algorithm (e.g., ± 60 MVAr, 1.0 ± 0.2 Volts p.u. respectively). Furthermore, these results demonstrate that the operational constraints on voltage magnitudes and reactive injection, enabled by the LPAC model, effectively constrain the generator dispatch to produce feasible AC power flows.

IV. Power Restoration for Disaster Management

The previous section demonstrated the shortcomings of the DC-GDP algorithm and the benefits of LPAC-GDP on artificial contingencies generated on the IEEE30 network. This section extends those experiments to the ROP problem on 18 real-life PSR problems and demonstrates similar findings. The 18 PSR benchmarks were provided by the Los Alamos National Laboratory: They are based on a U.S. transmission system with 266 components (including loads, generators, lines, and buses) and use state-of-the-art disaster simulation tools [20] for damage generation. The network damage varies significantly across the benchmarks and the most significant damage removes 23% of the network (61 components). This study is divided into two parts: (1) a detailed study of the DC-ROP and LPAC-ROP problem on the same fixed restoration order; and (2) a comparison of the complete restoration plans produced by both methods.

A. Fixed Restoration Order

To compare the DC-GDP and LPAC-GDP algorithms consistently over a sequence of restoration actions, the ROP is instrumented with an ordering heuristic based on operating procedures recommended by the U.S. Department of Homeland Security. With the restoration order fixed, this case study focuses on solving the induced sequence of GDP problems. The violations in line capacities, reactive injection, and voltage bounds are measured across both models throughout the proposed restoration plans to study their solution quality.

Restoration Plan Details: Figure 5 presents a detailed analysis of a significant damage scenario that requires 61 repairs. The first plot shows the amount of power served after each repair action. In this plot, the dashed lines reflect the amount of served power projected by the DC-GDP. The solid black line reflects the restoration steps that converge to an AC solution. The dotted line is the best *interpolation* of the DC-GDP solution using only converged AC solutions. No dash or dotted lines appear for the LPAC-GDP model because it produces an AC solution at every step. Note that the generator dispatch of the LPAC-GDP algorithm is always strictly less than the *projected* dispatch of the DC-GDP algorithm (dashed line). However, the dotted line indicates that the best interpolation of the DC-GDP algorithm may be much worse than the LPAC-GDP restoration. The significant gaps in DC-GDP feasibility suggest that the feasibility issues presented in Section III carry over to real-world disaster

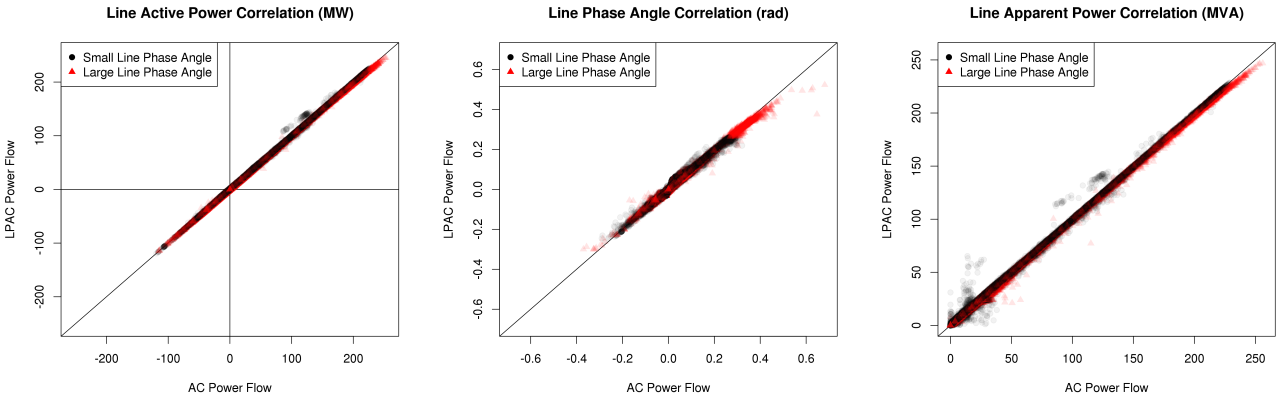

Fig. 2. Accuracy of the LPAC-GDP Algorithm with N–10 contingencies on Active Power (left), Line Phase Angles (center), and Apparent Power (right).

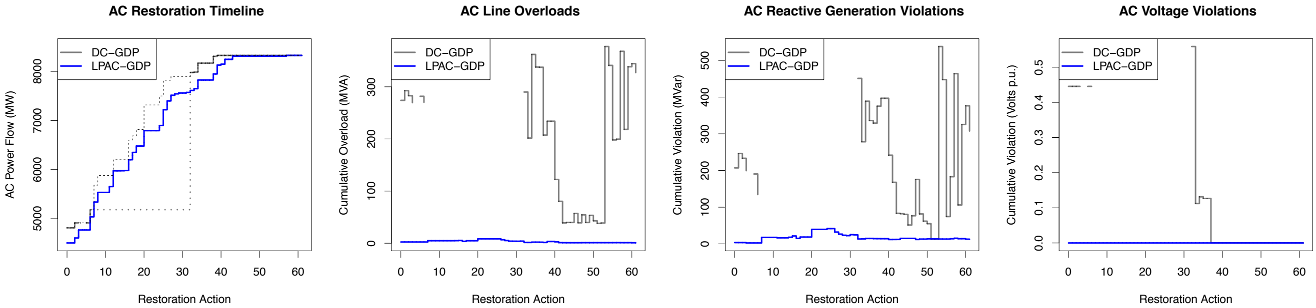

Fig. 5. Benchmark 1, a Characteristic Example of a Significant Damage Scenario (Group C).

scenarios. Moreover, the remaining plots indicate that, when the DC-GDP produces an AC solution, it is too optimistic and has significant constraint violations. The second plot presents the cumulative line overloads, which is the sum of line capacity violations in the entire network. The plot shows significant violations (between 50–300MVA) in the DC-GDP algorithm. These violations are so extreme that they could cause self-protection faults and cascading failures. The third plot presents the cumulative violations on reactive injection at the generators. The plot shows significant violations (between 100–500MVAr) in the DC-GDP algorithm. Once again, such extreme violations could damage the generation units or cause them to disconnect from the network. Finally, the fourth plot presents the cumulative violations for bus voltages (i.e., when the voltages exceed ± 0.05 p.u. from their nominal state). The LPAC model does incur some small violations on the lines and reactive injection due to its approximation of the power flow AC equations. A detailed study of these violations revealed that they were very small individually both in relative and absolute terms, and were spread evenly throughout the network. Hence they are likely acceptable for short-term operations.

Aggregate Performance: To extend the results in Figure 5, aggregate metrics are given for all 18 disaster management scenarios Since Step (2) of the DC-GDP algorithm fails to converge in numerous cases, the instances are divided into three groups: *Group A* where the DC-GDP algorithm always converges to an AC solution; *Group B* where the DC-GDP algorithm converges to an AC solution most of the time; *Group C* where the DC-GDP algorithm converges so rarely to an AC solution that the interpolated restoration plan is worse than the LPAC-GDP restoration plan. Table II summarizes the performance of the fixed order DC-ROP and LPAC-ROP algorithms on all of the benchmarks for each of the instance categories. The columns in the table are as follows.

1) *Instances solved*: The number of benchmarks where the algorithm converge to an AC solution for all GDPs in the sequence;
2) *Models solved*: The total number of GDPs for which the algorithm converges to an AC solution;
3) *Service Lost over Time (SLT)*: The size of the blackout scaled by the projected dispatch of the DC-GDP algorithm as it provides a optimistic bound on the size of the blackout;
4) *Maximum Cumulative Line Capacity Violation (MLCV)*: The largest cumulative line capacity violation for each instance (MVA);
5) *Maximum Cumulative Reactive Injection Violation (MRIV)*: The largest cumulative generator reactive injection bound violation in each instance (MVAr);
6) *Maximum Cumulative Voltage Violation (MVV)*: The largest cumulative voltage violation in each instance (Volts p.u.).

We use maximum values for the last three measures because the summation and the mean are not particularly meaningful, since the DC-GDP algorithm fails to converge to an AC solution on numerous occasions. Table II reports the average value of these *maximum-cumulative* measures over all of the instances in each category.

The LPAC-ROP algorithm appears to increase the size of the blackout significantly (8.5 times worse on average). However, the violations in the DC-ROP solutions are significant:

TABLE II. RESTORATION PLAN QUALITY FOR THE DC-ROP AND LPAC-ROP ALGORITHMS WITH A FIXED RESTORATION ORDER.

Model	Inst. Solved	Models Solved	Average SLT	MLCV	MRIV	MVV
All Instances						
DC	13/18	333/404	1.178	301.2	360.6	0.06073
LPAC	18/18	404/404	8.544	1.656	13.81	0
Group A - All DC-GDP generation dispatch are AC feasible.						
DC	13/13	196/196	1	337.7	330.8	0.0005223
LPAC	13/13	196/196	12.58	1.261	12.41	0
Group B - Most DC-GDP generation dispatch are AC feasible.						
DC	0/3	75/105	1.233	203.3	435.1	0.08339
LPAC	3/3	105/105	1.599	1.508	14.29	0
Group C - Few DC-GDP generation dispatch are AC feasible.						
DC	0/2	62/103	1.745	331.3	341.4	0.1522
LPAC	2/2	103/103	1.376	2.561	15.98	0

They indicate that the trends observed in the IEEE30 networks and Figure 5 continue to hold in the 18 instances studied here. The DC-ROP algorithm produces extreme AC solutions that significantly violate many of the capacity and voltage constraints. It is interesting to observe that the violations in the DC-ROP and LPAC-ROP solutions are consistent and of a similar magnitude across all instances, i.e., very significant for the DC-ROP algorithm and small for the LPAC-ROP algorithm. Moreover, the distance in objective value between the DC-ROP and LPAC-ROP solutions decreases substantially from Group A to Group C. In fact, for the largest disruptions, the LPAC-ROP model only increases the size of the blackout by 37.6% over the (very optimistic) DC-ROP bound.

B. Effects on Restoration Prioritization

Section III and Section IV-A have shown the shortcomings of the DC-GDP algorithm and the DC-ROP for scheduling load pickups and generator dispatch under a fixed scheduling order. This section investigates *whether the DC-ROP is sufficient for selecting a high-quality restoration order.* This would allow the DC-ROP to be used as a first step before turning to a more accurate model for a fixed ordering. This section answers this question by comparing the restoration prioritization plans produced by the DC-ROP and LPAC-ROP. Table III presents the results. To compare the restoration orders, the DC-ROP and LPAC-ROP prioritizations are evaluated with the DC-GDP and the LPAC-GDP dispatch algorithms. Since the LPAC-ROP always converges to an AC-feasible flow, Columns (3) and (6) give the blackout sizes for the DC-ROP and LPAC-ROP and the last column gives the relative differences between these blackout sizes. Column (4) shows the quality of the DC schedule when evaluated under the LPAC model, and Column (5) gives the "approximated" quality of the LPAC-ROP when evaluated by the DC-GDP.

The key insight is that the LPAC-ROP prioritization is significantly better than the DC-ROP prioritization: The blackout size is reduced more than half on benchmark 4 and by significant percentages on almost all benchmarks. The LPAC-ROP is also always better than the DC-ROP schedule. Column (3) also indicates that DC-GDP significantly underestimates the blackout size as Column (4) shows that the actual blackout sizes are much higher than what the DC-GDP projects. These inaccuracies of the DC approximation lead to incorrect repair prioritizations. This is confirmed by Column (5) which shows that the LPAC-ROP prioritizations are almost always not optimal for the DC-ROP. *Overall, these results demonstrates*

TABLE III. COMPARISON OF BLACKOUT SIZES PRODUCED BY ROP-DC AND ROP-LPAC.

ID	$\lvert\mathcal{R}\rvert$	ROP-DC DC-GDP	ROP-DC LPAC-GDP	ROP-LPAC DC-GDP	ROP-LPAC LPAC-GDP	Relative Difference
1	61	35,320	73,807	30,406	47,687	64.61%
16	53	33,104	60,655	38,340	53,138	87.61%
3	41	12,906	31,581	13,061	22,963	72.71%
13	40	14,655	35,468	20,233	31,267	88.15%
12	36	16,954	41,442	22,934	41,415	99.94%
2	32	6,508	20,515	7,132	13,459	65.60%
4	24	2,245	11,956	2,836	5,230	43.75%

that the LPAC-ROP is a significantly superior prioritization algorithm compared to the DC-ROP. It is critical to use a more accurate power flow model for the co-optimization of repair prioritization, load pickups, and generation dispatch.

V. CONCLUSION

This paper considered the restoration of transmission system after significant disruptions and, in particular, the joint optimization of repair prioritization, load pickups, and generation dispatch. It studied both the the Restoration Ordering Problem (ROP) and its core sub-problem the Generation Dispatch Problem (GDP), which are the key steps of the power restoration algorithms in [3]. The paper used 170,000 damage scenarios on IEEE30 bus network and 18 real-life natural disaster scenarios to compare restoration algorithms based on the DC and LPAC models. The results shows that the DC model is not adequate for solving the GDP and ROP. Its generation dispatch and load pickups cannot be converted to AC power flows or feature significant violations of thermal limits or voltage constraints, primarily because of its inability to model reactive power and voltage magnitudes. In contrast, the LPAC model retains the computational benefits of the DC model but provides a robust and appealing tradeoff between accuracy and efficiency. With the LPAC model, the restoration plans almost always converge to AC solutions and exhibit minimal constraint violations. Moreover, the LPAC prioritization significantly reduces the blackout sizes compared to the DC schedules.

ACKNOWLEDGMENT

The authors would like to thank Scott Backhaus, Russell Bent, Misha Chertkov, David Hill, Ian Hiskens, and George Matthews for stimulating discussions and insights on this work. This work was conducted in part at NICTA and is funded by the Australian Government as represented by the Department of Broadband, Communications and the Digital Economy and the Australian Research Council through the ICT Centre of Excellence program.

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