

ARJUNA 2.0

FOR IIT/JEE 2024 CLASS 11TH



LET'S STUDY

Logarithm

Lecture-06



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Characteristic Mantissa continued

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Recap :



★ log of a number any base always has

Two parts ① Integer part / Integral part

② Decimal part

♥ Integral part is called as characteristic

♥ Decimal part is called as mantissa.



♥ Characteristic can be +ve, -ve or 0

♥ Mantissa is never negative

$$\text{mantissa} \in [0, 1)$$

$$2 < \log_2 5 < 3 \Rightarrow \log_2 5 = \underbrace{2}_{\text{characteristic} = 2} \cdot \underbrace{.321920}_{\text{mantissa} = 0.321920}$$



$$2^3=8, 2^4=16$$

$$2^? = 9$$

$$3 < \log_2 9 < 4$$

$$3^2=9 \quad 3^3=27$$

$$2 < \log_3 19 < 3$$

$$2^7=128$$

$$2^8=256$$

$$\log_2 9 = 3 + f, \quad f \in [0, 1)$$

$$\log_3 19 = 2 + f, \quad f \in [0, 1)$$

$$\log_2 134 = 7 + f, \quad f \in [0, 1)$$

$$\log_8 75 = 2 + f, \quad f \in [0, 1)$$

$$\log_2 27 \quad \textcircled{>} \quad \log_4 65$$

" "

4. 3.

$$\begin{array}{r} 1.000 \\ 322 \\ \hline 0.678 \end{array}$$

$$\log_2 \frac{1}{10} = \log_2 10^{-1} = -\log_2 10$$

$$= -(3 + f) = -3 - f$$

$$= -\overline{3} - f + 1 - \overline{1} = -3 - 1 + 1 - f = -4 + 1 - f$$

$$= -4 + (1 - f)$$

$$= -4 + g \quad \text{let } 1 - f = g$$

$$\begin{array}{l} \text{Ex: } \log_2 \frac{1}{10} \\ = -(3.322) \\ = -(3 + 322) \\ = -3 - 0.322 \\ = -3 - 1 + 1 - 0.322 \\ = -4 + 0.678 \\ \quad \quad \quad \downarrow \\ \quad \quad \quad \text{Mantissa} \end{array}$$

Popular Doubts



Calculate : $4^{5 \log_4 \sqrt{2} (3 - \sqrt{6}) - 6 \log_8 (\sqrt{3} - \sqrt{2})}$

$$\log_a y^m = m \log_a y$$

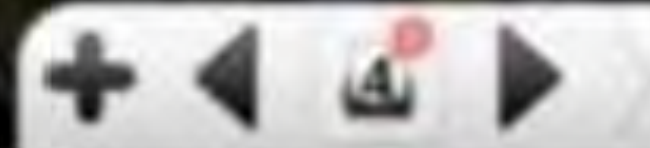
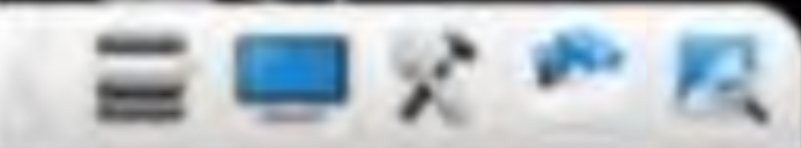
⑤ $P = 5 \log_{4\sqrt{2}} (3 - \sqrt{6}) - 6 \log_8 (\sqrt{3} - \sqrt{2})$

$$2 + \frac{1}{2} = \frac{5}{2}$$

$$2^{2 + \frac{1}{2}} = 2^{\frac{5}{2}}$$

$$= 5 \log_{2^{\frac{5}{2}}} (3 - \sqrt{6}) - 6 \log_{2^3} (\sqrt{3} - \sqrt{2})$$

$$= 5 \log_{2^{\frac{5}{2}}} (3 - \sqrt{6}) - 6 \log_{2^3} (\sqrt{3} - \sqrt{2}) = \cancel{5} \times \frac{2}{5} \log (3 - \sqrt{6}) - \frac{2}{3} \log (\sqrt{3} - \sqrt{2})$$



3 rd step smaj main nahi aya

$$= 5 \log_{2^{5/2}} (3 - \sqrt{6}) - 6 \log_{2^3} (\sqrt{3} - \sqrt{2})$$

$$3 - \sqrt{6} = (\sqrt{3})^2 - \sqrt{6}$$

$$= \sqrt{3} (\sqrt{3} - \sqrt{2})$$

$$= 5 \times \frac{1}{5/2} \times \log_2 (3 - \sqrt{6}) - 6 \cdot \frac{1}{3} \log_2 (\sqrt{3} - \sqrt{2})$$

$$= 5 \times \frac{2}{5} \log_2 (3 - \sqrt{6}) - 2 \log_2 (\sqrt{3} - \sqrt{2})$$

$$= 2 \log_2 (3 - \sqrt{6}) - 2 \log_2 (\sqrt{3} - \sqrt{2})$$

$$= 2 (\log_2 (3 - \sqrt{6}) - \log_2 (\sqrt{3} - \sqrt{2}))$$

$$= 2 \log_2 \left(\frac{3 - \sqrt{6}}{\sqrt{3} - \sqrt{2}} \right) = 2 \log_2 \frac{\sqrt{3} (\sqrt{3} - \sqrt{2})}{\sqrt{3} - \sqrt{2}} = 2 \log_2 \sqrt{3} = \log_2 (\sqrt{3})^2 = \log_2 3 = p.$$

$$\begin{aligned}
 4^P &= 4^{\log_2 3} \\
 &= 3^{\log_2 4} = 3^2 = \underline{9}
 \end{aligned}$$

$\log_5(5^{1/x} + 125) = \log_5 6 + 1 + \frac{1}{2x}$ then x is

$$5^{\frac{1}{x}} + 125 = 5^{\log_5 6 + 1 + \frac{1}{2x}} = 5^{\log_5 6} \cdot 5^1 \cdot 5^{\frac{1}{2x}} = 6 \cdot 5 \cdot 5^{\frac{1}{2x}}$$

$$5^{1/x} + 125 = 5^{\log_5 6} \cdot 5^1 \cdot 5^{\frac{1}{2x}}$$

$$5^{1/x} + 125 = 6 \cdot 5 \cdot 5^{1/2x}$$

$$\text{let } 5^{1/2x} = t \Rightarrow (5^{1/2x})^2 = t^2 \Rightarrow 5^{1/x} = t^2$$

$$t^2 + 125 = 30t$$

$$t^2 - 30t + 125 = 0$$

$$\begin{matrix} a^{m+n+p} \\ a^m \cdot a^n \cdot a^p \end{matrix}$$

$$5^{1/2x} = t$$

squaring

$$(5^{1/2x})^2 = t^2$$

$$5^{1/x} = t^2$$

$$5^{1/x} = t^2$$

sir square karne se $1/4x^2$ ho jaye naki $1/x$ iske liye 2 se multiply karna padega

$$5^{1/x} + 125 = 30 \cdot 5^{\frac{1}{5x}}$$

$$5^{\frac{1}{5x}} = t$$

Squaring

$$\left(5^{\frac{1}{5x}}\right)^2 = t^2$$

$$5^{\frac{1}{x}} = t^2$$

$$\left(5^{\frac{1}{5x}}\right)^2 \neq 5^{\frac{1}{4x2}}$$

$$(3^3)^2 = 3^{3 \times 2} = 3^6$$

$$(3^3)^2 \neq 3^9$$



Common & Natural Logarithm

- ★ $\log_{10}x$ is called Common Logarithm
- ★ $\log_e x$ is called Natural Logarithm

$e = \text{Napierian Const} \approx 2.718$



Characteristic of $\log_{10} x$

$$\log_{10} 1 = 0, \log_{10} 10 = 1$$

$$1 \leq x < 10 \Rightarrow 0 \leq \log_{10} x < 1 \Rightarrow \log_{10} x = 0 + f \quad f \in [0, 1)$$

$$10 \leq x < 100 \Rightarrow 1 \leq \log_{10} x < 2 \Rightarrow \log_{10} x = 1 + f \quad f \in [0, 1)$$

$$100 \leq x < 1000 \Rightarrow 2 \leq \log_{10} x < 3 \Rightarrow \log_{10} x = 2 + f \quad f \in [0, 1)$$

Ex: $\log_{10} \overleftarrow{80} = \textcircled{1} + f$
b/w 10 & 100

$\log_{10} \overleftarrow{79.68} = \textcircled{1} + f$
b/w 1 & 100

$\log_{10} \overleftarrow{997.35} = \textcircled{2} + f$



$$\log_{10} \overleftarrow{7285.67} = 3 + f$$

balke

$$\log(00\overleftarrow{765}.32) = 2 + f$$

$$\log_{10} \overleftarrow{3.65} = 0 + f$$

Therefore if $x \geq 1$ the characteristic of $\log_{10} x$ is

No: of significant digits before decimal - 1

i.e characteristic = No: of significant digits - 1.

Ex: find no. of digits in

(a) 2^{64}

given $\log_{10} 2 = 0.3010$

$\log_{10} 3 = 0.4771$

$\log_{10} 7 = 0.8451$

$2^7 = \underline{128}$

$\log_{10} \overleftarrow{9876.4} = (3) + f$

let $x = 2^{64}$

Taking log on both sides
to base 10.

$$\log_{10} x = \log_{10} 2^{64} = 64 \cdot \log_{10} 2 = 64 \times 0.3010 = 19.264$$

$$\log_{10} x = (19) + 0.264$$

\downarrow
 characteristic

mantissa



Using $\log 2 = 0.3010$ and $\log 3 = 0.4771$ and $\log 7 = 0.8451$

Find the number of digits

(i) $(2.5)^{200}$

KTK ①

(ii) $3^{12} \times 2^8$

(iii) 5^{25}

136048896

$X = 3^{12} \times 2^8$

Taking \log on both
sides to base 10.

$$\log_{10} X = \log_{10} (3^{12} \times 2^8)$$

$$= \log_{10} 3^{12} + \log_{10} 2^8$$

$$= 12 \log_{10} 3 + 8 \log_{10} 2$$

Let $X = 5^{25}$

$$\log_{10} X = \log_{10} 5^{25} = 25 \log_{10} 5$$

$$= 25 \log_{10} (10/2)$$

$$= 25 (\log_{10} 10 - \log_{10} 2)$$

$$= 25 (1 - 0.3010)$$

$$= 25 \times 0.6990 = 17.4725$$

$$= 12 \times 0.4771 + 8 \times 0.3010 = 8.1332$$
$$= 8 + 0.1332$$

No. of digit in
 $3^{12} \times 2^8 = 9$

$\log_{10} x = 17 + 0.425$
 No. of digit in $5^{25} = \underline{18}$

$$\text{if } \frac{1}{10} \leq x < 1 \Rightarrow -1 \leq \log_{10} x < 0 \Rightarrow \log_{10} x = -1 + f, \quad f \in [0, 1)$$

\downarrow
 $0.1 \leq x < 1$

$$\frac{1}{100} \leq x < \frac{1}{10} \Rightarrow -2 \leq \log_{10} x < -1 \Rightarrow \log_{10} x = -2 + f, \quad f \in [0, 1)$$

\downarrow
 $0.01 \leq x < 0.1$

$$\frac{1}{1000} \leq x < \frac{1}{100} \Rightarrow -3 \leq \log_{10} x < -2 \Rightarrow \log_{10} x = -3 + f.$$

\downarrow
 $0.001 \leq x < 0.01$

Ex: $\log(0.\overrightarrow{003}) = -3 + f$

$\log(0.\overrightarrow{0567}) = -2 + f$

$\log_{10}(0.\overrightarrow{00005678}) = -5 + f$

$\log_{10}(0.\overrightarrow{573}) = -1 + f$

If $0 < x < 1$

then characteristic of $\log_{10} x$ is given by

Characteristic = $-(\text{No. of zeros immediately after decimal before significant digit starts} + 1)$

Ex

$$\log(0.00\overrightarrow{5}7000) = -3 + p$$

$$\log(0.0\overrightarrow{00}5076) = -4 + \underline{p}$$

Ex: find no. of zeros immediately after decimal before
significant digit starts in

(a) 2^{-100} given $\log_{10} 2 = 0.3010$

$$x = 2^{-100}$$

Take log to base 10.

$$\log_{10} x = \log_{10} 2^{-100}$$

$$= -100 \log_{10} 2$$

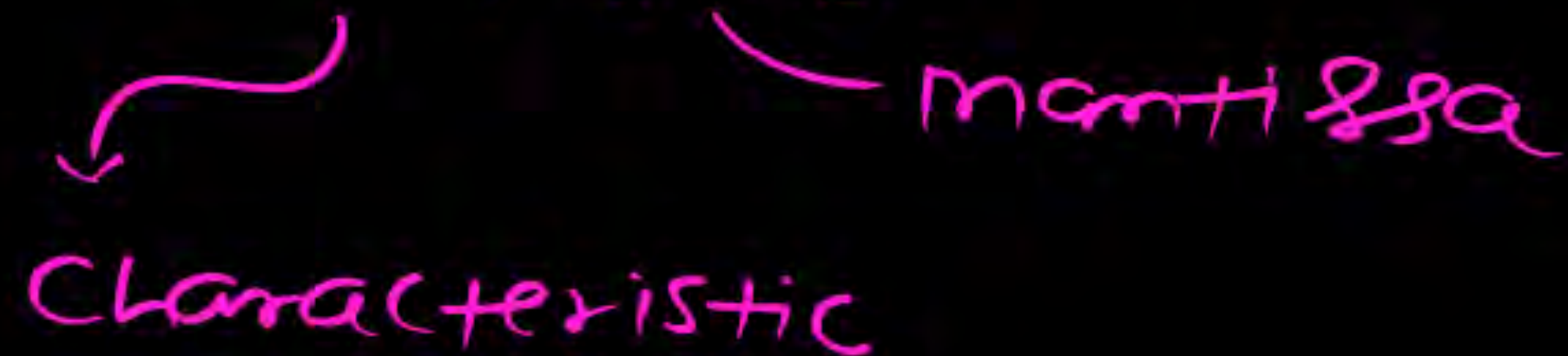
$$= -100 (0.3010) = -30.10$$

$$\log_{10} x = -30.10$$

$$= -30 - 0.10$$

$$= -30 - 1 + 1 - 0.10$$

$$= -31 + 0.90$$





No: of zero immediately
 after decimal before
 significant starts = 30



Find the number of zeroes after decimal before a significant figures start in



(i) $\left(\frac{9}{8}\right)^{-100}$
(KTK - 2)

(ii) $(0.35)^{12}$

(iii) $\frac{1}{2^{40}}$
(KTK - 1)

$$x = (0.35)^{12}$$

$$\log_{10} x = 12 \log_{10} (0.35)$$

$$= 12 \log_{10} \frac{35}{100} = 12 \log_{10} \frac{7}{20}$$

$$= 12 (\log_{10} 7 - \log_{10} 20)$$

$$= 12 (\log_{10} 7 - (\log_{10} 2 + \log_{10} 10)) = 12 - (\log_{10} 7 - (\log_{10} 2 + 1))$$

$$= 12 (\log_{10} 7 - \log_{10} 2 - 1) = 12 (0.8451 - 0.3010 - 1) = -5.4078$$



$$\log_{10} x = -5.4078 = -5 - 0.4078 = -5 - 1 + 1 - 0.4078$$

$$= \textcircled{-6} + 0.5922$$

Characteristic

No. of zero after
decimal before
significant digit

$$\ln (0.35)^{12} = 5.$$

↓

0000033792



Let $\log_3 N = \alpha_1 + \beta_1$, $\log_5 N = \alpha_2 + \beta_2$ and $\log_7 N = \alpha_3 + \beta_3$ where $\alpha_1, \alpha_2, \alpha_3$ are integers and $\beta_1, \beta_2, \beta_3 \in [0, 1)$.

- $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{I}$ (i) Find the number of integral values of N if $\alpha_1 = 4$ and $\alpha_2 = 2$.
 $\beta_1, \beta_2, \beta_3 \in [0, 1)$ (ii) Find the largest integral values of N if $\alpha_1 = 5$ and $\alpha_2 = 3$ and $\alpha_3 = 2$.

$$\log_3 N = \alpha_1 + \beta_1$$

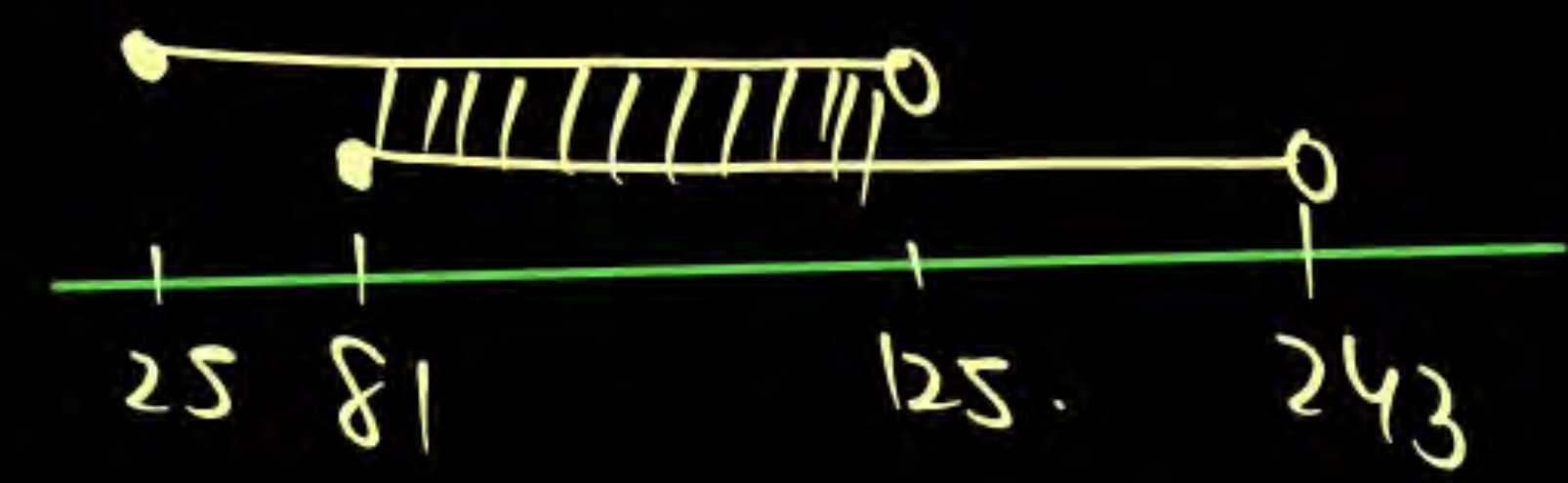
$$\log_5 N = \alpha_2 + \beta_2$$

$$\log_7 N = \alpha_3 + \beta_3$$

(i) $\alpha_1 = 4, \alpha_2 = 2$

$$4 \leq \log_3 N = 4 + \beta_1 < 5 \Rightarrow 3^4 \leq N < 3^5 \Rightarrow 81 \leq N < 243$$

$$2 \leq \log_5 N = 2 + \beta_2 < 3 \Rightarrow 5^2 \leq N < 5^3 \Rightarrow 25 \leq N < 125$$

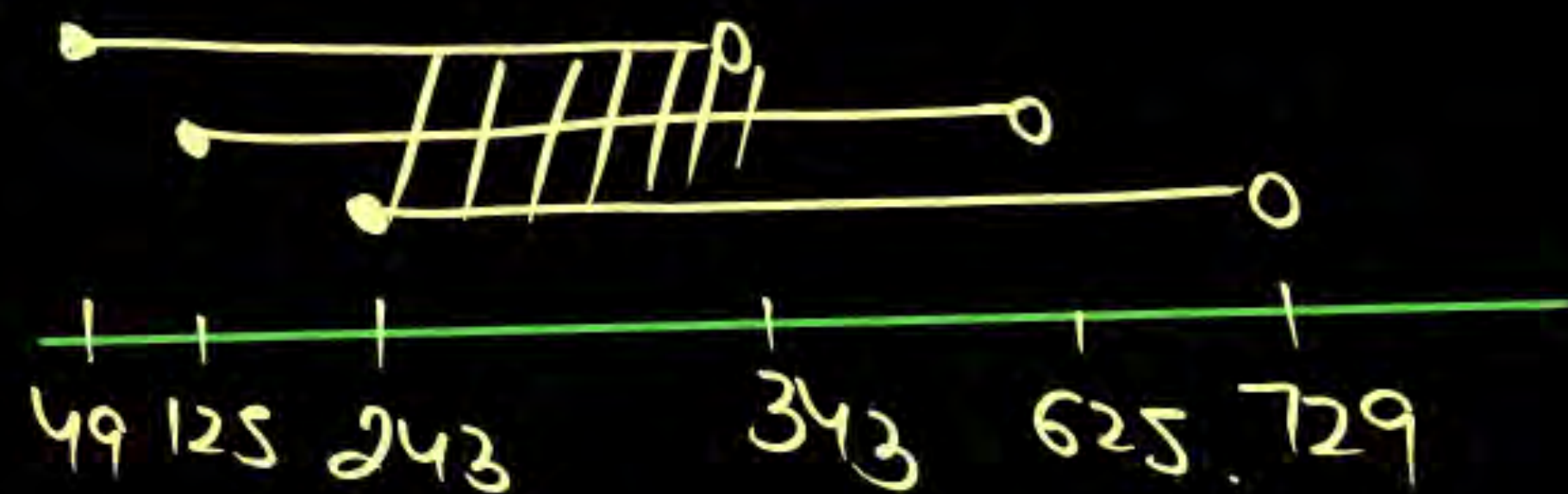


$\Rightarrow N \in [81, 125)$
 \Downarrow
 44 integral values

$$5 \leq \log_3 N = 5 + \beta_1 < 6 \Rightarrow 3^5 \leq N < 3^6 \Rightarrow 243 \leq N < 729$$

$$3 \leq \log_5 N = 3 + \beta_2 < 4 \Rightarrow 5^3 \leq N < 5^4 \Rightarrow 125 \leq N < 625$$

$$2 \leq \log_7 N = 2 + \beta_3 < 3 \quad 7^2 \leq N < 7^3 \Rightarrow 49 \leq N < 343$$



$N \in [243, 343)$ largest possible
integer value = 342.

Simplify: $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

A $\log_{10} 100$

B $\log_{10} 10$

~~**C** $\log_{10} 1 = 0$~~

~~**D** $\log_2 8 - \log_3 27$
 $\overset{||}{3} - \overset{||}{3} = 0$~~

$$\cancel{5^{\log_3 7}} + \cancel{7^{\log_5 3}} - \cancel{5^{\log_3 7}} - \cancel{7^{\log_5 3}} = 0$$

If $(4)^{\log_9 3} + (9)^{\log_2 4} = (10)^{\log_x 83}$, then x is equal to :

A 2

~~**B** 10~~

C 3

D 30

$$4^{\frac{1}{2}} + 9^2 = 10^{\log_x 83}$$

$$2 + 81 = 10^{\log_x 83}$$

$$\underline{10}^{\log_x 83} = 83$$

$$x = 10$$



Let $P = \frac{5}{\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x}}$ and $(120)^P = 32$, then the value of x be



A 1

~~**B** 2~~

C 3

D 4

$$P = \frac{5}{\log_x 2 + \log_x 3 + \log_x 4 + \log_x 5}$$

$$= \frac{5}{\log_x (2 \times 3 \times 4 \times 5)} = \frac{5}{\log_x 120}$$

$$P = 5 \log_{120} x$$

$$(120)^P = (120)^{5 \log_{120} x}$$

$$= x^{5 \log_{120} 120} = x^5 = 32 \Rightarrow x^5 = 2^5 \Rightarrow x = 2$$



Mathematical Gyaan



Sum of first n Natural Numbers :

$$\begin{array}{l} \text{3 no: 8} \\ \overbrace{1+2+3} = 6 = \frac{3 \times 4}{2} \\ \underbrace{1+2+3+4}_{\text{4 no: 8}} = 10 = \frac{4 \times 5}{2} \end{array}$$

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

Q. find the value of n if $\log_2 4 + \log_2 4^2 + \log_2 4^3 + \dots + \log_2 4^n = 20$

$$\begin{array}{l} 1+2+3+\dots+100 \\ \text{"} \\ \frac{100 \times 101}{2} = 50 \times 101 \\ = 5050 \end{array}$$

$$\log_2 4 + \log_2 4^2 + \log_2 4^3 + \dots + \log_2 4^n = 20$$

$$\log_2 4 + 2\log_2 4 + 3\log_2 4 + \dots + n\log_2 4 = 20$$

$$(\log_2 4) [1+2+3+\dots+n] = 20$$

$$\cancel{2} \times \frac{n(n+1)}{\cancel{2}} = 20 \Rightarrow n(n+1) = 20 \Rightarrow n^2 + n - 20 = 0$$

$$(n+5)(n-4) = 0 \Rightarrow n = 4, -5$$



Today's KTK & Home Challenge



Home Challenge-9



Passage

Let A be the sum of the roots of

$$\frac{1}{5 - 4 \log_4 x} + \frac{4}{1 + \log_4 x} = 3,$$

B be the product of m and n , where $2^m = 3$ and $3^n = 4$, and C be the sum of the integral roots of

$$\log_{3x} \left(\frac{3}{x} \right) + (\log_3 x)^2 = 1.$$

1. The value of $A + B$ is

- (a) 10 (b) 6 (c) 8 (d) 4

2. The value of $B + C$ is

- (a) 6 (b) 2 (c) 4 (d) 8

3. The value of $(A + C \div B)$ is

- (a) 5 (b) 8 (c) 7 (d) 4



(KTK2) Solve for x : $7^{\log_2 x} = 98 - x^{\log_2 7}$.

(KTK3) Solve for x : $4^{\log_3 x} = 32 - x^{\log_3 4}$.

(KTK4) If α and β be the solutions of
 $|x - 2|^{\log_2(x^3) - 3\log_x 4} = (x - 2)^3$,
 find the value of $(\alpha + 2\beta + 3)$.



Solution to Previous KTKs & Home Challenge





Simplify: $\frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409} \cdot \left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6} \right)$

(KTK1)



KTk01

$$Q. \frac{81 \log_5^9 + 3 \log_5^3}{409} \cdot \left((\sqrt{7})^{\frac{2}{\log_5 7}} - (125)^{\log_5 6} \right)$$

$$\Rightarrow \frac{81 \log_9^5 + 3 \log_3^{\sqrt{6}}}{409} \cdot \left((\sqrt{7})^{2 \log_7 25} - (125)^{\log_5 6} \right)$$

$$\Rightarrow \frac{5 \log_9^{81} + \sqrt{6} \log_3^3}{409} \cdot \left(25^{2 \log_7 \sqrt{7}} - 6^{\log_5 125} \right)$$

$$\Rightarrow \frac{5 \log_9^{9^2} + \sqrt{6}^3}{409} \cdot \left(25^{2 \times \frac{1}{2}} - 6^{\log_5 5^3} \right)$$

$$\Rightarrow \frac{5^2 + \sqrt{6}^3}{409} \cdot \left(25 - 6^{\frac{3}{2}} \right)$$

$$= \frac{(25 + 6\sqrt{6}) \cdot (25 - 6\sqrt{6})}{409}$$

$$= \frac{(25)^2 - (6\sqrt{6})^2}{409}$$

$$= \frac{625 - 216}{409} = \frac{409}{409} = \underline{\underline{1}} \text{ Answer}$$

→ SAKSHI

from U.P (Agra)

KTK-01

Simplify $\frac{81^{\frac{1}{\log_3 5}} + 3^{\frac{9}{\log_3 \sqrt{6}}}}{409} \cdot \left((\sqrt{7})^{\frac{2}{\log_7 25}} - (25)^{\frac{\log_6 4}{\log_6 25}} \right)$

$$\Rightarrow \frac{81^{\frac{\log 5}{\log 3}} + 3^{\frac{3 \log \sqrt{6}}{\log 3}}}{409} \cdot \left(\sqrt{7}^{\frac{2 \log 25}{\log 7}} - (25)^{\frac{\log 4}{\log 25}} \right)$$

$$\Rightarrow \frac{5^{\frac{\log 81}{\log 3}} + 27^{\frac{\log \sqrt{6}}{\log 3}}}{409} \cdot \left(7^{\frac{\log 25}{\log 7}} - (25)^{\frac{\log 4}{\log 25}} \right)$$

$$\Rightarrow \frac{5^{\frac{\log 81}{\log 3}} + \sqrt{6}^{\frac{\log 27}{\log 3}}}{409} \cdot \left(25^{\frac{\log 7}{\log 7}} - 6^{\frac{\log 25}{\log 25}} \right)$$

$$\Rightarrow \frac{5^2 + \sqrt{6}^3}{409} \cdot (25 - 6^{\frac{3}{2}})$$

$$\Rightarrow \frac{(25 + 6\sqrt{6})(25 - 6\sqrt{6})}{409}$$

$$\Rightarrow \text{Now, } \frac{25^2 - 6\sqrt{6}^2}{409}$$

$$\Rightarrow \frac{625 - 216}{409} \Rightarrow \frac{409}{409} = 1$$

ANS : 1



Solve for x:

(KTK2) $\log_4 (x - 1) = \log_2 (x - 3)$

(KTK3) $\log_4 (x - 2) = \log_2 (x - 2)$

(KTK4) $\log_9 (x - 1) = \log_3 (x - 1)$

(KTK5) $\log_2 x + \log_2 (x + 3) = 1/4$

(KTK6) $\log_4 (x^2 + x) - \log_4 (x + 1) = 2$



KTK 2

$$\log_4(x-1) = \log_2(x-3)$$

$$\Rightarrow \log_{2^2}(x-1) = \log_2(x-3) \Rightarrow \frac{1}{2} \log_2(x-1) = \log_2(x-3)$$

$$\Rightarrow 2^{-1} \log_2(x-1) = \log_2(x-3) \Rightarrow \cancel{\log_2}(x-1)^{\frac{1}{2}} = \cancel{\log_2}(x-3)$$

$$\Rightarrow (x-1)^{\frac{1}{2}} = (x-3)$$

On squaring both side we get

$$(x-1) = (x-3)^2 \Rightarrow x-1 = x^2+9-6x$$

$$\Rightarrow x^2-7x+10=0$$

$$\Rightarrow (x-5)(x-2)=0$$

$$\Rightarrow \boxed{x=5}, \boxed{x=2}$$

$x=2$ is rejected because if we put $x=2$ in $\log_2(x-3)$ then the log is not define

Vaibhav Panchal from Samawali (Dist-Shamli)

KTK-2

ANAND MANI TIWARI
GORAKHPUR (U.P.)

Q. Solve for x :-

$$\text{Q.1) } \log_4(x-1) = \log_2(x-3)$$

$$\text{Sol: } \log_{2^2}(x-1) = \log_2(x-3)$$

$$\frac{1}{2} \log_2(x-1) = \log_2(x-3)$$

$$\log_2(x-1) = 2 \log_2(x-3)$$

$$\cancel{\log_2}(x-1) = \log_2(x-3)^2$$

$$(x-1) = (x^2-6x+9)$$

$$x^2-6x+9-x+1=0$$

$$x^2-7x+10=0$$

$$x^2-5x-2x+10=0$$

$$x(x-5)-2(x-5)=0$$

$$(x-2)(x-5)=0$$

$$x=2 \text{ or } \boxed{x=5}$$

(Rejected $x=2$ as $(x-3)$ cannot be a negative number).

KTK:-3

ANAND MANI TIWARI
GORAKHPUR (U.P.)

Q. Solve for x :-

Q. $\log_4(x-2) = \log_2(x-2)$

Sol: $\log_{2^2}(x-2) = \log_2(x-2)$

$$\frac{1}{2} \log_2(x-2) = \log_2(x-2)$$

$$\log_2(x-2) = 2 \log_2(x-2)$$

$$\log_2(x-2) = \log_2(x-2)^2$$

$$(x-2) = (x^2 - 4x + 4)$$

$$x^2 - 4x + 4 - x + 2 = 0$$

$$x^2 - 5x + 6 = 0$$

$$x^2 - 3x - 2x + 6 = 0$$

$$x(x-3) - 2(x-3) = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2, x = 3$$

$x = 3$ Ans
 \uparrow
rejected
because $x-2$ becomes 0

K T 4

$$\Rightarrow \log_3(x-1) = \log_3(x-1)$$

$$\Rightarrow (x-1) = 3^{\log_3(x-1)}$$

$$\Rightarrow (x-1) = (x-1)^{\log_3 3}$$

$$\Rightarrow (x-1) = (x-1)^{\log_3 3^2}$$

$$\Rightarrow (x-1) = (x-1)^{2 \log_3 3}$$

$$\Rightarrow (x-1) = (x-1)^2$$

$$\Rightarrow x-1 = x^2 - 2x + 1$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x = 2 \text{ or } x = 1$$

Rejected

$$\therefore x = 2 \text{ Ans}$$

Dipanshu U.P

KTK-5

MAANU RAW



$$Q) \log_2(x) + \log_2(x+3) = \frac{1}{4}$$

$$\log_2(x(x+3)) = \frac{1}{4}$$

$$\log_2(x^2+3x) = \frac{1}{4}$$

$$x^2+3x = (2)^{\frac{1}{4}}$$

$$x^2+3x = \sqrt[4]{2}$$

$$x^2+3x - \sqrt[4]{2}$$

$$-3 \pm \sqrt{(3)^2 - 4(1)(-\sqrt[4]{2})}$$

$$-3 \pm \sqrt{9 - 4\sqrt[4]{2}}$$

$$x = -3 + \sqrt{9 - 4\sqrt[4]{2}} \quad \checkmark$$

$$x = -3 - \sqrt{9 - 4\sqrt[4]{2}} \quad (\text{rejected})$$



KTK 6 $\log_4(x^2+x) - \log_4(x+1) = 2$

we can write it as $\log_4\left(\frac{x^2+x}{x+1}\right) = 2$ $\because \left(\log_a^m - \log_a^n = \log_a\left(\frac{m}{n}\right)\right)$

$$\Rightarrow 4^2 = \frac{x^2+x}{x+1} \Rightarrow 16(x+1) = x^2+x$$

$$\Rightarrow 16x + 16 = x^2 + x \Rightarrow x^2 - 15x - 16 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{15 \pm \sqrt{(-15)^2 - 4(1)(-16)}}{2}$$

$$\Rightarrow x = \frac{15 \pm \sqrt{225 + 64}}{2} \Rightarrow x = \frac{15 \pm \sqrt{289}}{2}$$

$$\Rightarrow x = \frac{15 \pm 17}{2} \Rightarrow \boxed{x = 16}, \boxed{x = -1}$$

$x = -1$ is rejected by if we put $x = -1$ in $\log_4(x+1)$

then its value is not define

Hence $x = 16$ is final answer



Correction in KTK sent by Students



Solve the following equations :

(i) $\log_{x-1} 3 = 2$ (KTK 1)

(ii) $\log_4 (2 \log_3 (1 + \log_2 (1 + 3 \log_3 x))) = \frac{1}{2}$ (KTK 2)

(iii) $\log_3 (1 + \log_3 (2^x - 7)) = 1$ (KTK 3)

(iv) $\log_3 (3^x - 8) = 2 - x$ (KTK 4)



Vaibhav Panchal from U.P

KTK 2

$$\log_4 [2 \log_3 (1 + \log_2 (1 + 3 \log_3 x))] = \frac{1}{2}$$

We can write it as $(4)^{\frac{1}{2}} = [2 \log_3 (1 + \log_2 (1 + 3 \log_3 x))]$

$$\Rightarrow 2 = \log_3 [1 + \log_2 (1 + 3 \log_3 x)]^2$$

$$\Rightarrow \text{we can write it as } [1 + \log_2 (1 + 3 \log_3 x)]^2 = 3^2 = 9$$

$$\Rightarrow 1 + \log_2 (1 + 3 \log_3 x) = \sqrt{9} = 3$$

$$\Rightarrow \log_2 (1 + 3 \log_3 x) = 3 - 1 = 2$$

$$\Rightarrow \text{we can write it as } 1 + 3 \log_3 x = 2^2$$

$$\Rightarrow 3 \log_3 x = 3 \Rightarrow \log_3 x^3 = 3$$

$$\Rightarrow \text{we can write it as } (3)^3 = x^3$$

$$\Rightarrow \boxed{x = 3} \text{ Ans}$$

KTK 2
(ii) $\log_4 (2 \log_3 (1 + \log_2 (1 + 3 \log_3 x))) = \frac{1}{2}$

Solve:- $2 \log_3 (1 + \log_2 (1 + \log_3 x)) = 2$

$$\Rightarrow \log_3 (1 + \log_2 (1 + \log_3 x)) = 1$$

$$\Rightarrow \log_3 (1 + \log_2 (1 + \log_3 x)) = 1$$

$$\Rightarrow 1 + \log_2 (1 + \log_3 x) = 3$$

$$\Rightarrow \log_2 (1 + \log_3 x) = 3 - 1$$

$$\Rightarrow \log_2 (1 + \log_3 x) = 2$$

$$\Rightarrow 1 + \log_3 x = 2^2$$

$$\Rightarrow \log_3 x = 4 - 1$$

$$\Rightarrow \log_3 x = 3$$

$$\Rightarrow 3^3 = x$$

$$\Rightarrow x = 27 \text{ Ans:-}$$

$$(iii) \log_3 (1 + \log_3 (2^n - 7)) = 1$$

Ans:-

$$1 + \log_3 (2^n - 7) = 3$$

$$\log_3 (2^n - 7) = 3 - 1$$

$$\Rightarrow \log_3 (2^n - 7) = 2$$

$$\Rightarrow 2^n - 7 = 3^2$$

$$\Rightarrow 2^n - 7 = 9$$

$$\Rightarrow 2^n = 9 + 7$$

$$\Rightarrow 2^n = 16$$

$$\Rightarrow 2^n = 2^4$$

$$\Rightarrow n = 4 \text{ Ans:-}$$

$$(iv) \log_3 (3^n - 8) = 2 - n$$

Solve:-

$$\log_3 (3^n - 8) = 2 - n$$

$$\Rightarrow 3^n - 8 = 3^{2-n}$$

$$\Rightarrow 3^n - 8 = 3^2 \cdot 3^{-n}$$

$$\Rightarrow 3^n - 8 = 3^2 \times \frac{1}{3^n}$$

$$\text{Let } 3^n = x$$

$$\Rightarrow x - 8 = 3^2 \times \frac{1}{x}$$

$$\Rightarrow \frac{x^2 - 8x - 9}{x} = 0$$

$$\Rightarrow x^2 - 8x - 9 = 0$$

$$\Rightarrow x^2 - 9x + x - 9 = 0$$

$$\Rightarrow (x-9)(x+1) = 0$$

$$\Rightarrow x = 9, x = -1$$

$$\Rightarrow 3^n = 9, 3^n = -1$$

$$\Rightarrow 3^n = 3^2, 3^n = -1$$

$$\Rightarrow n = 2, n = \text{rejected}$$

$$\Rightarrow n = 2$$

$3^x = -1$ (reject)
b'coz 3^x can
never be -ve



PW Maharathi

Because Practice makes a Maharathi

You have a TEST !!!

Rewards for Students who attempt all the tests regularly on Sunday/ Monday!

- Top 3 Students in each month will receive a Gift Voucher worth Rs 500.
- Top 20 Students in each month will receive a Gift Voucher worth Rs 200.
- Lucky 20 students in each month will be selected on a random basis who will receive a Gift Voucher worth Rs 100 for attending regular tests.

MEGA MAHARATHI (FEBRUARY-2023)

- Top 5 Students will receive a Gift Voucher worth Rs 1000
- Lucky 100 Students in September 2022 will be given a Gift Voucher worth Rs 500 under MEGA MAHARATHI.

** Eligibility and Rules are covered in the next slide.

MAHARATHI MONTHS

- SEPTEMBER 2022
- NOVEMBER 2022
- JANUARY 2023
- FEBRUARY 2023

MEGA MAHARATHI MONTHS

- FEBRUARY 2023

DO's AND DON'Ts TO BECOME A PW MAHARATHI:-

- 1). You have to **attempt all the test** (except short tests) occurring between the last Maharathi and till the next one to be eligible for Maharathi at all.
- 2). Maharathi will be announced only in the batch for the month if more than one test has occurred in that last month. For eg- If only one test occurred in June then that test would be considered in July's Month Maharathi Results.
- 3). The **combined performance of all the tests** would ensure your selection in the **toppers prizes**.
- 4). The selection of **Lucky students** will be done using our **Random Selection Algorithm** and not on the basis of marks, names and past maharathi results, you just have to be **eligible by attempting all tests** and yes, if you have been **selected once it can happen again** :D
- 5). You are eligible in MEGA MAHARATHI if you have given **more than 9 TESTS** out of the **13 TESTS** occurring throughout the batch i.e May-Aug and **not on marks or any other factor**.
- 6). All the tests need to be attempted on Day of test (Sunday) or the next day (Monday).
- 7). You will have to attempt all tests genuinely and completely to be eligible for any scheme. We will use our Fraud check algorithms before identifying the award winners.

THANK-YOU

