

**Is what you see what you expected? What about the averages of the distributions and the shapes of the histograms:**

**Do you see regularities, differences and can you attempt to explain what you see or guess what is the "theoretical" limit distribution, when as N increases, and you can make the distribution simulation "more detailed" by increasing M ?**

The resulting distribution depends on the value of N (number of attacks), M (number of systems) and P (probability of success). More precisely if N is high and M is high too I will see a distribution that is similar to the normal one. If I start decreasing N the distribution tends to be more discrete with the columns of the histogram that tend to assume the same height.

If I fix N and change M the histogram tends to be more discrete.

The mean of the binomial distribution is  $N * P$ , and the standard deviation is the square root of  $N * P * (1 - P)$ . However, when N is large, the binomial distribution can be approximated by a normal distribution with the same mean and standard deviation. This approximation is based on the Central Limit Theorem.

**Recall briefly the definition and math notions relevant to "probability space" and make some simple examples, indicating among the triple of the space the meaning of each element in your particular example.**

In probability theory, a probability space or a probability triple  $(\Omega, F, P)$  is a mathematical construct that provides a formal model of a random process or "experiment".

A probability space consists of three elements:

- A sample space,  $\Omega$ , which is the set of all possible outcomes.
- An event space, which is a set of events,  $F$ , an event being a set of outcomes in the sample space.
- A probability function,  $P$ , which assigns each event in the event space a probability, which is a number between 0 and 1.

Example 1:

Suppose that the probabilistic experiment consists in extracting a ball from an urn containing two balls, one red (R) and one blue (B). The sample space is  $\Omega = \{R, B\}$ .

A possible sigma-algebra of events is  $F = \{\emptyset, \Omega, \{R\}, \{B\}\}$  where  $\emptyset$  is the empty set.

The four events could be described as follows:

- $\emptyset$ : nothing happens;
- $\Omega$ : either a blue ball or a red ball is extracted;
- $\{R\}$ : a red ball is extracted;
- $\{B\}$ : a blue ball is extracted.

A possible probability measure  $P$  on  $F$  is

$$P(F) = \begin{cases} 0 & \text{if } F = \emptyset \\ 1/2 & \text{if } F = \{R\} \\ 1/2 & \text{if } F = \{B\} \\ 1 & \text{if } F = \Omega \end{cases}$$

Another possibility would be to define the so-called trivial sigma-algebra  $F' = \{\emptyset, \Omega\}$  and specify a probability measure  $P$  on  $F'$  as

$$P(F) = \begin{cases} 0 & \text{if } F = \emptyset \\ 1 & \text{if } F = \Omega \end{cases}$$

## Example 2

Question: You choose a card from a standard deck. What is the probability space for choosing a two?

Solution:

Step 1: Create the sample space. The sample space for this question is a list of all possible cards you could choose (H=hearts, D=diamonds, C=clubs, S=spades):

$\Omega = \{AH, 2H, 3H, 4H, 5H, 6H, 7H, 8H, 9H, 10H, JH, QH, KH, AD, 2D, 3D, 4D, 5D, 6D, 7D, 8D, 9D, 10D, JD, QD, KD, AC, 2C, 3C, 4C, 5C, 6C, 7C, 8C, 9C, 10C, JC, QC, KC, AS, 2S, 3S, 4S, 5S, 6S, 7S, 8S, 9S, 10S, JS, QS, KS\}$ .

Step 2: Assign probabilities. There are 52 choices in the sample space in Step 1. Each card has a  $1/52$  chance of being chosen.

Note that the question asked for the probability space and not the solution (i.e. it didn't ask for the "probability of choosing a two."). You could use the probability space for answering any question though. Just add up the probabilities. For example, the probability of choosing a two would be  $1/52 + 1/52 + 1/52 + 1/52 = 4/52 = 1/13$ .

**If you wanted to model probabilistically the homework Exercise 1, explain what are the 3 sets of your probability space and their elements, in this case.**

To model probabilistically the homework 1 the three sets of my probability space and their elements are:

- Sample Space ( $\Omega$ ) is represented by all the possible lines that can be represented in the graph.
- Event Space ( $F$ ) is the set of events that takes place during the drawing.
- Probability Function ( $P$ ) is the probability of success of the event.

## **Bibliography**

<https://www.statisticshowto.com/probability-and-statistics/binomial-theorem/find-the-mean-of-the-probability-distribution-binomial/>

[https://en.wikipedia.org/wiki/Probability\\_space](https://en.wikipedia.org/wiki/Probability_space)

<https://www.statlect.com/glossary/probability-space>

<https://www.statisticshowto.com/probability-space/>