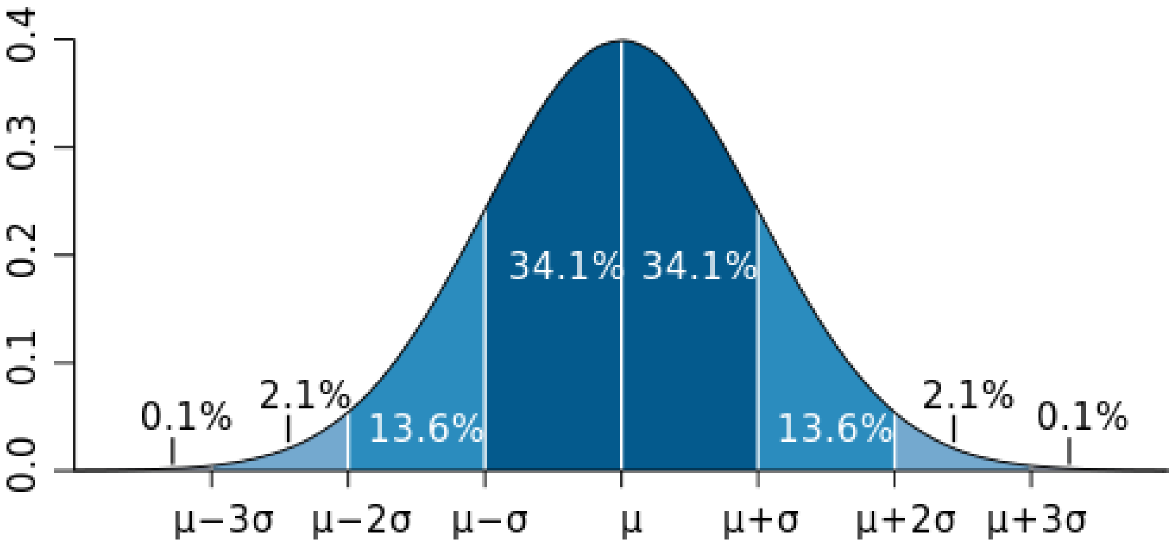
Th1: The LLN Meaning, Proof, Simulations
Law of large numbers LLN

The law of large numbers, in probability and statistics, states that as a sample size grows, its mean gets closer to the average of the whole population. This is due to the sample being more representative of the population as the sample becomes larger.
Every additional data point gathered has the potential to increase the likelihood that the outcome is a true measure of the mean, but this doesn't mean that a given sample or group of successive samples will always reflect the true population characteristics, especially for small samples.

In statistical analysis, the law of large numbers is related to the central limit theorem. The central limit theorem states that as the sample size increases, the sample mean will be evenly distributed. This is often represented as a bell-shaped curve where the peak of the curve depicts the mean and the central limit theorem relates to the distribution of a curve, we can see an example in the following graph..



In statistical analysis, the law of large numbers is important because it gives validity to your sample size.
When working with a small amount of data, the assumptions you make may not appropriately translate to the actual population. Therefore, it is important to make sure enough data points are being captured to adequately represent the entire data set.

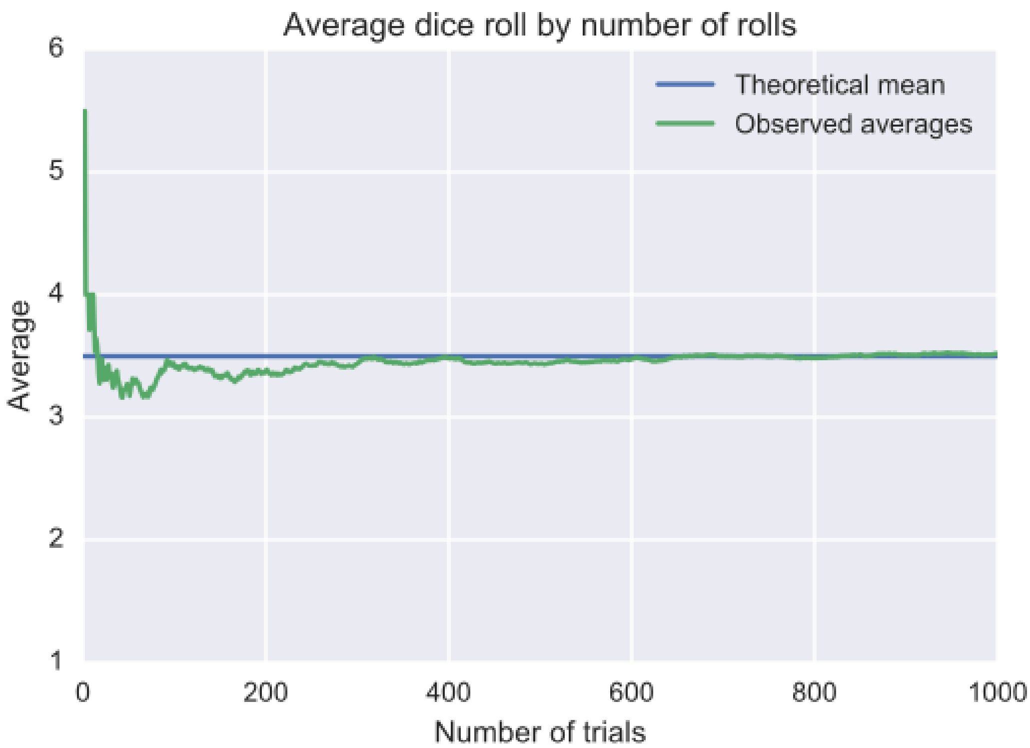
The weak law of large numbers states that, as the number of trials or observations increases, the average of the results will tend to converge on the expected value. In other words, the more trials or observations you make, the more accurate the average will be in predicting the actual value.

The strong law of large numbers states that, with probability one, the average of the results of a large number of trials or observations will converge on the expected value. In other words, the average will be exactly equal to the expected value with probability one as the number of trials or observations increases.

Both the strong and weak laws are valid for independent random variables that are identically distributed under the sole requirement that the mean exists. If the mean does not exist, then the sample mean does not have a distribution that becomes narrower as the sample size N increases. An example of a distribution without a mean is the Cauchy distribution.

As an example, consider the tossing of a coin whose probability of a head is p. The law of large numbers does not imply that the number of heads in N tosses necessarily deviate little from the expected number of heads Np; rather it states that the relative frequency of heads to flips is close to p. Nor does this law imply that, if the ratio of heads to flips is >p after N flips, the probability of tails on subsequent flips becomes larger to compensate for the surplus of heads.

The following illustration of the law of large numbers using a particular run of rolls of a single die is another example. As the number of rolls in this run increases, the average of the values of all the results approaches 3.5. Although each run would show a distinctive shape over a small number of throws (at the left), over a large number of rolls (to the right) the shapes would be extremely similar.



A single roll of a fair, six-sided die produces one of the numbers 1, 2, 3, 4, 5, or 6, each with equal probability. Therefore, the expected value of the average of the rolls is:

$$\frac{1+2+3+4+5+6}{6} = 3.5$$

According to the law of large numbers, if a large number of six-sided dice are rolled, the average of their values will approach 3.5, with the precision increasing as more dice are rolled.

It follows from the law of large numbers that the empirical probability of success in a series of Bernoulli trials will converge to the theoretical probability. For a Bernoulli random variable, the expected value is the theoretical probability of success, and the average of *n* such variables (assuming they are independent and identically distributed (i.i.d.)) is precisely the relative frequency.

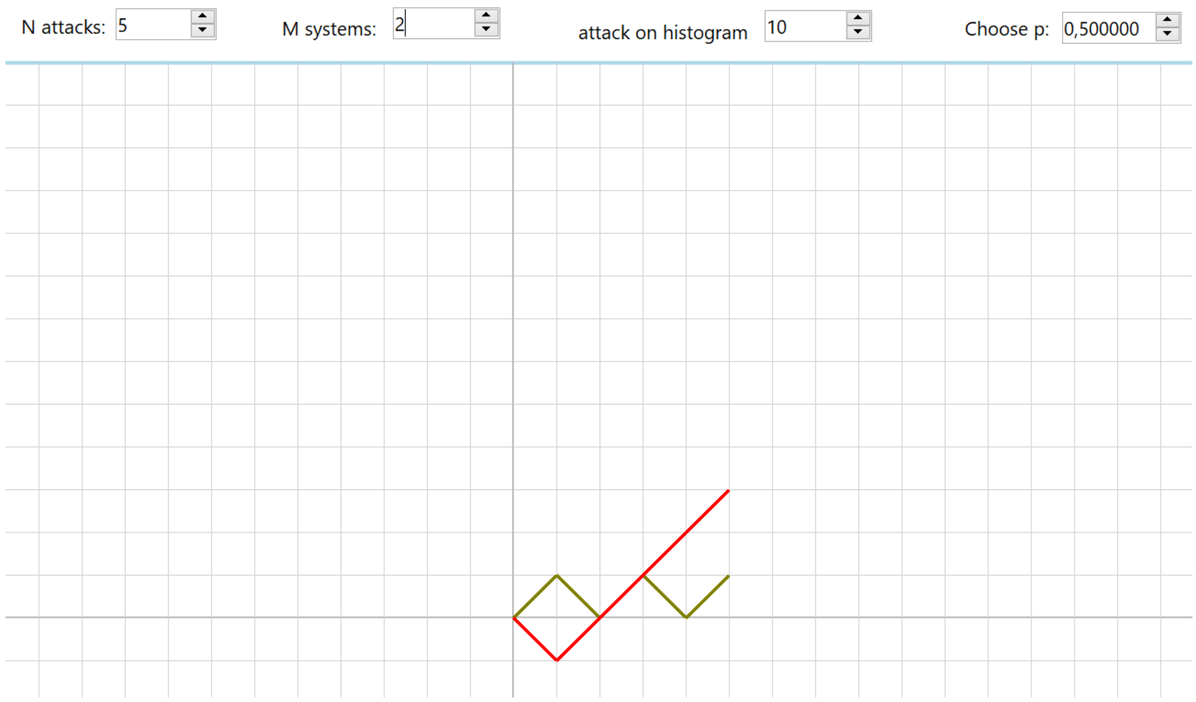
The formal proof of the weak law of large numbers can be found on this [page](#).

The formal proof of the strong law of large numbers can be found on this [page](#).

The law of large numbers in my homework 3 can be seen using the following graph

We can see that if we choose the probability of success of the attack p = 0.5, the average value of the curves is near y=0, so it is near the expected value.

Instead if we choose a smaller number of attacks and a smaller number of systems, we can see by the following graph that the average value is not close to y=0;



So we can see that incrementing the number of samples the average is more precise.

Bibliography

-https://www.investopedia.com/terms/l/lawoflargenumbers.asp
-https://builtin.com/data-science/law-of-large-numbers
-https://www.sciencedirect.com/topics/mathematics/laws-of-large-number
-https://en.wikipedia.org/wiki/Law_of_large_numbers