Th6: Algorithms for random variates generation

The basic ingredient needed for every method of generating random variates from any distribution is a source of IID U(0,1) random variates.
Hence, it is essential that a statistically reliable U(0,1) random number generator be available.

Generating random variates, or random numbers from a specific probability distribution, is a crucial aspect of many simulation and statistical modeling applications. There are various algorithms for generating random variates from different distributions.
The following are the most commonly used algorithms.

1. Uniform Distribution:

   - Linear Congruential Generator (LCG): This is a simple and widely used method. It generates a sequence of numbers using a linear congruential recurrence relation. The formula is Xn+1 = (a * Xn + c) mod m, where Xn is the current value, a is a multiplier, c is an increment, and m is the modulus.

2. Normal Distribution:

   - Box-Muller Transform: This method generates a pair of independent standard normal variates from two independent uniform variates. These normal variates can then be scaled and shifted to obtain a normal distribution with the desired mean and standard deviation.

   - Marsaglia's Polar Method: Similar to the Box-Muller method, Marsaglia's Polar Method generates pairs of standard normal variates but uses polar coordinates. It is computationally efficient and avoids the need for trigonometric functions.

3. Exponential Distribution:

   - Inverse Transform Sampling: If U is a uniform random variable between 0 and 1, then the inverse transform sampling method transforms U into an exponential random variable with parameter λ using the formula X = -1/λ * log(1 - U).

4. Poisson Distribution:

   - Poisson Random Number Generation: The Poisson distribution can be generated using various methods, such as the Poisson process or the rejection method. The Poisson process involves simulating the number of events in a fixed interval based on a known average rate.

5. Binomial Distribution:

   - Inverse Transform Sampling: Similar to the exponential distribution, the binomial distribution can be generated using the inverse transform sampling method.

These are just a few examples, and there are many other algorithms for generating random variates from different distributions.

A very important aspect related to algorithms for random variates generation is the execution time.
Execution time has two components: set-up time and marginal execution time.
Set-up time is the time required to do some initial computing to specify constants or tables that depend on the particular distribution and parameters.

Marginal execution time is the incremental time required to generate each random variate. Since in a simulation experiment, we typically generate thousands of random variates, marginal execution time is far more than the set-up time.

Bibliography

-https://www-eio.upc.es/~lmontero/lmm_tm/MESIO-SIM%20-%20%28US%29%20Generation%20of%20random%20variables.pdf

-https://www.omscs-notes.com/simulation/random-variate-generation/#illustration

-https://www.cyut.edu.tw/~hchorng/downdata/1st/SS8_Random%20Variate.pdf