1.Arithmetic Brownian Motion:

The equation for arithmetic Brownian motion is given by:
$dX_t = \mu dt + \sigma dW_t$
X_t represents the value of the process at time t.
μ is the drift term, representing the average rate of return per unit time.
σ is the volatility, representing the standard deviation of the random fluctuations.
dW_t is a Wiener process or Brownian motion, representing random, continuous, and unpredictable movements.

Arithmetic Brownian Motion (ABM) is a mathematical model that describes the random movement of a variable over time. It is a specific form of Brownian motion, which is a stochastic process named after the Scottish botanist Robert Brown, who observed the erratic motion of pollen particles in water.

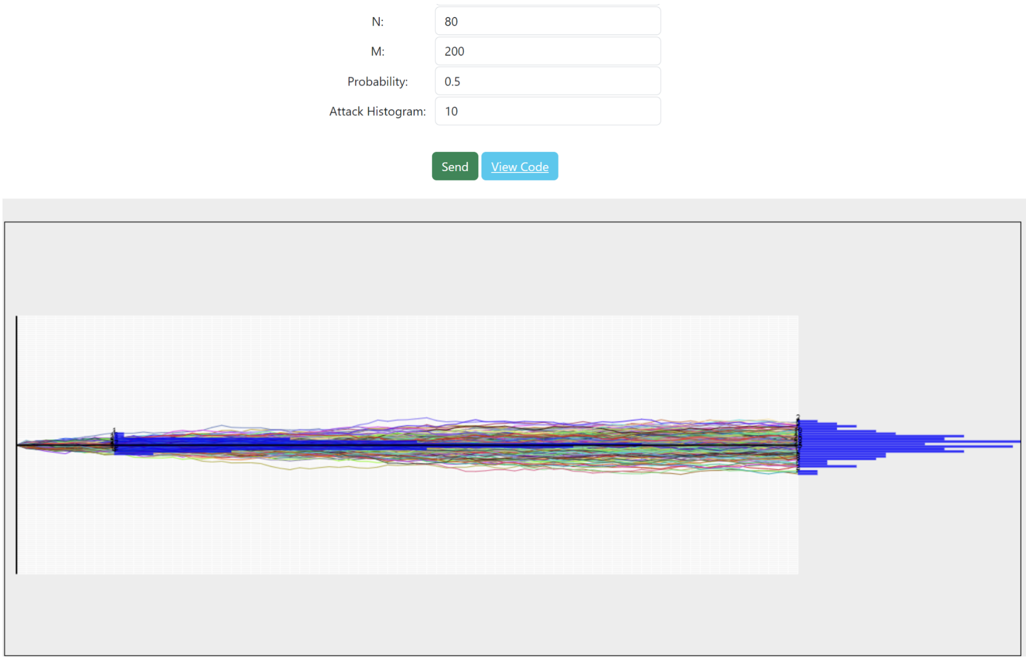
The key characteristics of ABM include:

Randomness: The movement of the variable is driven by random factors, making it a stochastic process.

Constant drift: ABM includes a constant drift term, which represents the average rate of return or growth of the variable over time. This term reflects the trend in the variable.

Volatility: The variable also has a random component, known as volatility. This reflects the degree of variation or dispersion of the variable's values.

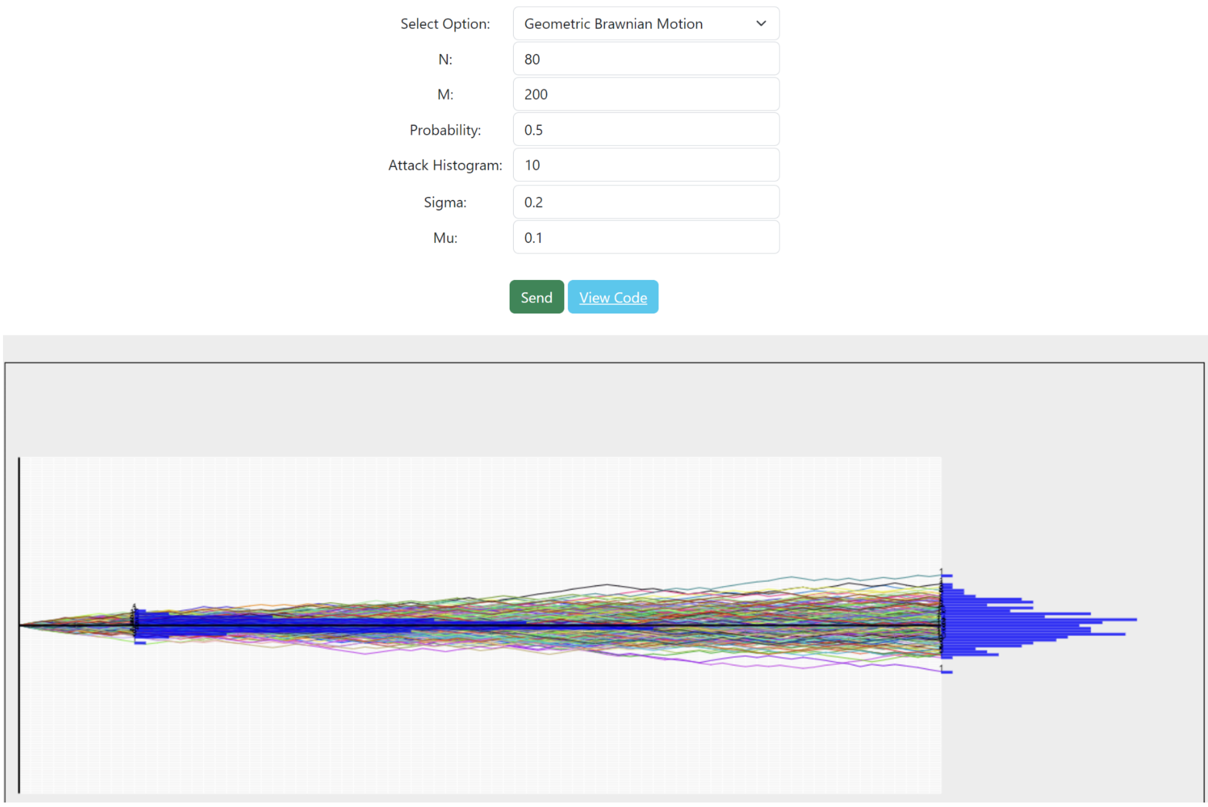
The basic stochastic differential equation (SDE) for Arithmetic Brownian Motion is given by:



2. Geometric Brownian Motion (Black–Scholes Model):

The equation for geometric Brownian motion is:
$dS_t = \mu S_t dt + \sigma S_t$
dW_t is the stock price at time t.
μ is the average rate of return.
σ is the volatility.
dW_t is a Wiener process.

GBM is an extension of Arithmetic Brownian Motion (ABM), and it incorporates the concept of compounding.
One notable difference from Arithmetic Brownian Motion is that the drift and volatility are now multiplied by the current value of the variable (S_t). This multiplication introduces a compounding effect, which means that the percentage returns are proportional to the current value of the asset. This makes GBM more suitable for modeling the continuous compounding of returns over time.

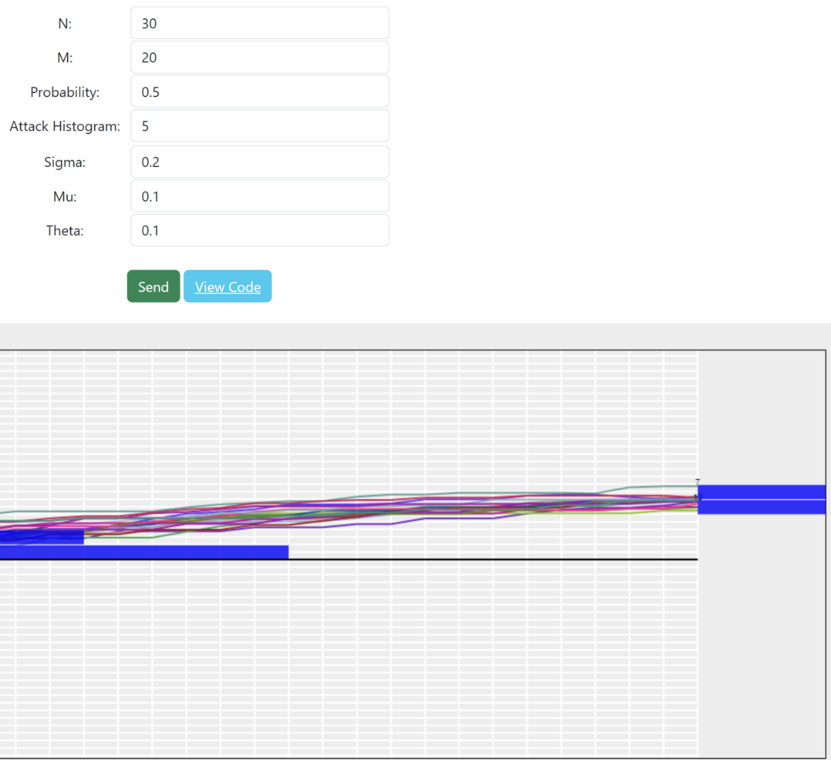


3. Ornstein–Uhlenbeck Process (Mean-Reverting Process):

The equation is:
$dX_t = \theta(\mu - X_t)dt + \sigma dW_t$
X_t is the value of the process at time t.
θ is the speed of mean reversion.
μ is the mean to which the process reverts.

σ is the volatility.
dW_t is a Wiener process.

The Ornstein–Uhlenbeck (OU) process, also known as the mean-reverting process, is a stochastic process that describes the continuous-time evolution of a variable that tends to revert towards a mean or equilibrium level. It is often used in finance to model the behavior of interest rates, exchange rates, and other economic variables that exhibit mean-reverting tendencies.
The mean-reverting nature of the process is evident in the term $\theta(\mu - X_t)$, which acts as a force pulling X_t towards the mean μ.
The random term σdW_t introduces variability and ensures that the process does not deterministically converge to the mean.
The stationary distribution of the Ornstein–Uhlenbeck process is normally distributed with mean μ and variance $(\sigma^2)/(2\theta)$. This means that as t approaches infinity, the process tends to fluctuate around the mean with a certain level of volatility.
The OU process contrasts with geometric Brownian motion, which does not exhibit mean-reverting behavior and is often used to model assets like stocks that may follow a trend without a tendency to revert to a specific level.

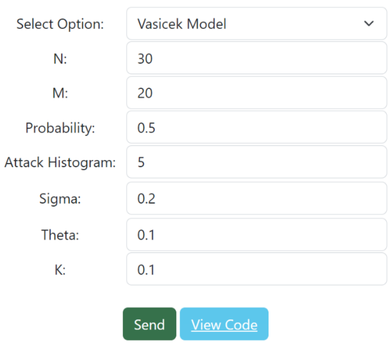


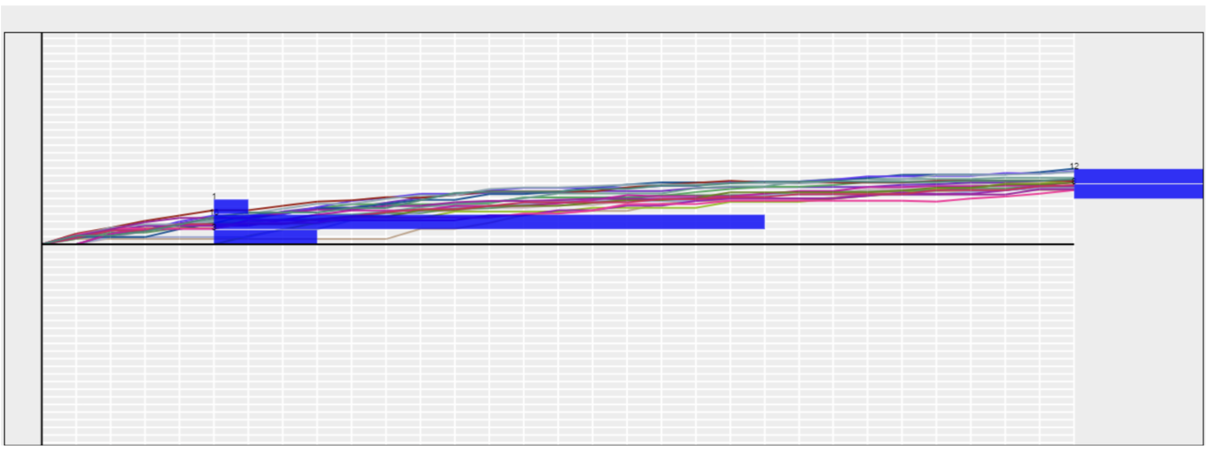
4. Vasicek Model:

The equation is:
$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t$
X_t is the value of the process at time t.
κ is the speed of mean reversion.

θ is the mean.
σ is the volatility.
dW_t is a Wiener process.

The Vasicek model is a mathematical model used in finance to describe the evolution of interest rates over time. It was introduced by Oldřich A. Vasicek in 1977 and is a type of stochastic model that incorporates mean-reverting behavior. The model is particularly used to analyze and simulate the term structure of interest rates.
The term $\kappa(\theta - r_t)$ acts as a mean-reverting force, pulling the interest rate towards the long-term mean θ. The random term σdW_t introduces variability, ensuring that the interest rate does not deterministically converge to the mean.
The Vasicek model provides a closed-form solution for the expected value of the short-term interest rate and the variance of interest rate changes.





5. Hull–White Model:

It extends the Vasicek model by making the mean reversion and volatility parameters time-dependent.

The Hull–White Model, also known as the Hull–White One-Factor Model, is a mathematical model used in finance to describe the evolution of interest rates over time. It is named after John Hull and Alan White, who introduced the model in the early 1990s. The Hull–White Model is an extension of the Vasicek model and is designed to address some of its limitations.
In the Hull–White Model, the mean-reverting level θ(t) can be a function of time, allowing for more flexibility compared to the constant mean in the original Vasicek model. This feature

makes the Hull–White Model more suitable for capturing time-varying mean levels in interest rates.
One key advantage of the Hull–White Model is that it allows for calibration to the initial term structure of interest rates, providing a more realistic representation of the market conditions.

6. Cox–Ingersoll–Ross (CIR) Model:

The equation is:
$dX_t = \kappa(\theta - X_t)dt + \sigma (X_t)$^½ dW_t
It is similar to the Vasicek model but includes a term proportional to the square root of the process.
The Cox–Ingersoll–Ross (CIR) model is a mathematical model used in finance to describe the evolution of interest rates over time. It is a type of stochastic model that, like the Vasicek model, incorporates mean-reverting behavior. The CIR model was introduced by John C. Cox, Jonathan E. Ingersoll, and Stephen A. Ross in 1985.
The term $a(\theta - r_t)$ acts as a mean-reverting force, pulling the interest rate towards the long-term mean θ. The random term $\sigma (r_t)$^½ dW_t introduces variability and ensures that the interest rate does not deterministically converge to the mean.
One distinctive feature of the CIR model is the presence of the square root of the interest rate r_t in the volatility term. This feature allows the model to capture the fact that volatility tends to decrease as interest rates approach zero, a characteristic often observed in interest rate data.

7. Black–Karasinski Model:
The Black–Karasinski model is an extension of the Cox–Ingersoll–Ross (CIR) model. It introduces stochastic volatility, allowing the volatility parameter to vary over time. The equation is given by:

$dX_t = (\theta(t) - aX_t)dt + \sigma(t) (X_t)$^½ dW_t
X_t is the short rate at time t.
θ(t) is the mean reversion function.
a is the speed of mean reversion.
σ(t) is the volatility function.
dW_t is a Wiener process.
The Black–Karasinski model is commonly used in interest rate modeling, especially when considering the term structure of interest rates and how it evolves over time.

8. Heston Model:
The Heston model is widely used in option pricing and describes the dynamics of the volatility of an asset. It consists of two stochastic differential equations:

$dS_t = \mu S_t\, dt + (V_t)$^½ $S_t\, dW_t^1$
$dV_t = \kappa(\theta - V_t)dt + \sigma (V_t)$^½ dW_t^2
S_t is the asset price at time t.
V_t is the instantaneous variance (volatility squared) at time t.
μ is the average rate of return.

θ is the long-term average volatility.
κ is the speed of mean reversion of volatility.
σ is the volatility of volatility.
dW_t^1 and dW_t^2 are two correlated Wiener processes.
The Heston model is known for capturing the volatility smile observed in financial markets, where implied volatilities vary with strike prices and expiration dates.

9. Chen Model:
The Chen model is a more complex stochastic volatility model that incorporates jumps in addition to continuous-time processes. It is expressed as:
$dX_t = (\mu - \lambda X_t)dt + \sigma(X_t)$^½ $dW_t + dJ_t$
X_t is the volatility of the asset at time t.
μ is the average growth rate.
λ is the mean-reverting speed.
σ is the volatility of the process.
dW_t is a Wiener process.
dJ_t is a jump process.
The Chen model is used to capture both the continuous and jump components in the volatility of financial assets, providing a more realistic representation of market dynamics.