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Differential Swerve State Space Controller Design

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1 Introduction

Swerve is a type of drivetrain that is characterized by having at least two wheels that can each rotate and spin independently of each other (here rotate is define as a rotation around the robots up axis and spin is defined as a rotation around the modules y axis). This allows the robot to move in any direction and turn in place. Differential Swerve is a type of swerve drivetrain in which the rotation of the wheel is controlled by the difference of the motor velocities and the spin if the wheel is controlled by the average of the motor velocities. This allows for the combination of the torque produced by both motors when driving and turning. This is the main advantage over conventional (coaxial) swerve drivetrains in which rotation and spin are controlled by separate motors. The main disadvantage of differential swerve is that it is significantly more complex to control.

2 Designing the State Space Model

State space controller in the form of

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

2.1 Inputs, Outputs, and States

The States will be in the form of

$$x = \begin{bmatrix} v_t \\ v_b \\ v_w \\ \theta \end{bmatrix} \tag{3}$$

Where v_w is the spin velocity of the wheel and θ is the angle of the wheel. v_t and v_b are the top and bottom motor velocities respectively.

The inputs will be in the form of

$$u = \begin{bmatrix} V_t & V_b \end{bmatrix} \tag{4}$$

Where V_t is the top motor voltage and V_b is the bottom motor voltage.

Our output matrix is the same as the state matrix as we can measure all states.

$$y = \begin{bmatrix} v_t \\ v_b \\ v_w \\ \theta \end{bmatrix} \tag{5}$$

Where v_t is the top motor velocity, v_b is the bottom motor velocity, and θ is the angle of the wheel.

2.2 DC Motor Model

We know that a permanent magnet DC motor follows the general equation of

$$V = K_v\dot{x} + K_a\ddot{x} \tag{6}$$

And we can rewrite this as

$$\begin{aligned} V &= K_v\dot{x} + K_a\ddot{x} \\ V - K_a\ddot{x} &= K_v\dot{x} \\ -K_a\ddot{x} &= K_v\dot{x} - V \\ \ddot{x} &= \frac{-K_v\dot{x} + V}{K_a} \\ \ddot{x} &= \frac{-K_v\dot{x}}{K_a} + \frac{V}{K_a} \end{aligned}$$

We can also substitute v as $\dot{x}$ to create

$$\dot{v} = \frac{-K_v v}{K_a} + \frac{V}{K_a} \tag{7}$$

This equation can then be written in state space form as

$$\dot{x} = \begin{bmatrix} \frac{-K_v}{K_a} \end{bmatrix} x + \begin{bmatrix} \frac{1}{K_a} \end{bmatrix} u \tag{8}$$

$$y = 1x + 0u \tag{9}$$

2.3 A and B Matrices

Now that we know how to calculate the angular velocity of a motor based on its constants and the input voltage we can start to create the formulas needed to compute the different velocity components of the system

$$\dot{v_t} = \frac{-K_{v_{drive}}}{K_{a_{drive}}} v_t + \frac{1}{K_{a_{drive}}} V_t \tag{10}$$

$$\dot{v_b} = \frac{-K_{v_{drive}}}{K_{a_{drive}}} v_b + \frac{1}{K_{a_{drive}}} V_b \tag{11}$$

$$\dot{v_w} = \frac{1}{2} K_{dr} (\dot{v_t} - \dot{v_b}) \tag{12}$$

$$\dot{\theta} = \frac{v_t + v_b}{2} * K_{turn\_ratio} \tag{13}$$

Equation 12 is the angular acceleration of the wheel but it needs to be rewritten to not use $\dot{v_t}$ or $\dot{v_t}$ This can be achieved by combining the three equations.

$$\dot{v_w} = \frac{K_{dr}}{2} (\dot{v_t} - \dot{v_b}) \tag{14}$$

$$\dot{v_w} = \frac{K_{dr}}{2} (\frac{-K_{v_{drive}}}{K_{a_{drive}}} v_t + \frac{1}{K_{a_{drive}}} V_t - \frac{-K_{v_{drive}}}{K_{a_{drive}}} v_b - \frac{1}{K_{a_{drive}}} V_b) \tag{15}$$

$$\dot{v_w} = \frac{-K_{dr} K_{v_{drive}}}{2K_{a_{drive}}} v_t + \frac{K_{dr}}{2K_{a_{drive}}} V_t - \frac{-K_{dr} K_{v_{drive}}}{2K_{a_{drive}}} v_b - \frac{K_{dr}}{2K_{a_{drive}}} V_b \tag{16}$$

$$\dot{v_w} = -\frac{K_{dr} K_{v_{drive}}}{2K_{a_{drive}}} v_t + \frac{K_{dr} K_{v_{drive}}}{2K_{a_{drive}}} v_b + \frac{K_{dr}}{2K_{a_{drive}}} V_t - \frac{K_{dr}}{2K_{a_{drive}}} V_b \tag{17}$$

Where K_{dr} is the gear ratio of the wheel to the motor shaft

We can now rewrite these equations as a system that has coefficients for every state and input

$$\dot{v_t} = \frac{-K_{v_{drive}}}{K_{a_{drive}}} v_t + 0v_b + 0v_w + 0\theta + \frac{1}{K_{a_{drive}}} V_t + 0V_b \tag{18}$$

$$\dot{v_b} = 0v_t + \frac{-K_{v_{drive}}}{K_{a_{drive}}} v_b + 0v_w + 0\theta + 0V_t + \frac{1}{K_{a_{drive}}} V_b \tag{19}$$

$$\dot{v_w} = -\frac{K_{v_{drive}}}{2K_{a_{drive}}} v_t + \frac{K_{v_{drive}}}{2K_{a_{drive}}} v_b + 0v_w + 0\theta + \frac{1}{2K_{a_{drive}}} V_t - \frac{1}{2K_{a_{drive}}} V_b \tag{20}$$

$$\dot{\theta} = \frac{K_{turn\_ratio}}{2} v_t + \frac{K_{turn\_ratio}}{2} v_b + 0v_w + 0\theta + 0V_t + 0V_b \tag{21}$$

Now that we have these linear equations we can turn them into the A and B matrix

$$A = \begin{bmatrix} \frac{-K_{v_{drive}}}{K_{a_{drive}}} & 0 & 0 & 0 \\ 0 & \frac{-K_{v_{drive}}}{K_{a_{drive}}} & 0 & 0 \\ -\frac{K_{dr} K_{v_{drive}}}{2K_{a_{drive}}} & \frac{K_{dr} K_{v_{drive}}}{2K_{a_{drive}}} & 0 & 0 \\ \frac{K_{turn\_ratio}}{2} & \frac{K_{turn\_ratio}}{2} & 0 & 0 \end{bmatrix} \tag{22}$$

$$B = \begin{bmatrix} \frac{1}{K_{a_{drive}}} & 0 \\ 0 & \frac{1}{K_{a_{drive}}} \\ \frac{K_{dr}}{K_{a_{drive}}} & -\frac{K_{dr}}{K_{a_{drive}}} \\ 0 & 0 \end{bmatrix} \tag{23}$$

2.4 C and D matrices

With these, we just need the C and D matrices. Because the output matrix is the same as the state matrix C is just the identity matrix.

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

$$D = 0 \tag{25}$$

Now that we have our full state space representation of the system we need to create a Linear Quadratic Regulator to control the system.

3 Designing the LQR

The LQR is designed to minimize the cost function