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**Homework 2 Node Similarity and Community Detection**

**Question 1 [1.5 pts]** Please use your own language to briefly explain the following concepts:

PageRank score: a way of determining the importance of a web page by counting the number and quality of links to that page.

Rooted PageRank: a modification to PageRank that keeps the random walker focused on a particular area of the graph. This is done by randomly resetting the walker back to the root node being investigated.

Network community: a group of nodes in a network which are densely connected to each other compared to their connections with the rest of the network.

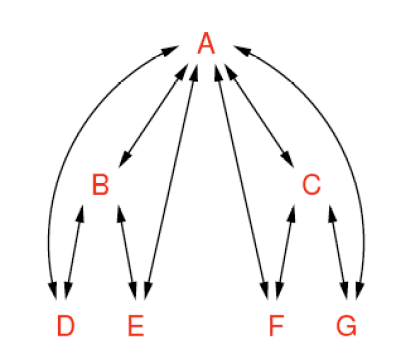
Clique: a subset of nodes in a network that forms a complete subgraph. Each node in the group is connected to all other nodes in the group.

k-Clique: a subgraph where the shortest path between any two nodes is <= k, and which cannot maintain this feature if more nodes are added.

Low-rank approximation: by computing the rank-k matrix of a graph, choosing a low value of k, and using another community detection method, the noise in a graph can be reduced. This is also used in conjunction with Singular Value Decomposition to find significant data amid noise.

**Question 2 [2 pts]:** Given seven web pages with the following link structure,

1. Please use “Power Iteration” (a.k.a simple iteration) to calculate the PageRank scores for each website. (You only need to show the first and the second iterations results, with the initial PageRank scores for each node being set as 1/n=0.15) [1 pt].
2. Please also use Eigenvector based approach to calculate PageRank scores for each web page [1 pt] (please show your solutions.)



1. The adjacency matrix is normalized and the PageRank Power Iteration is applied.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F | G |  |  | A | B | C | D | E | F | G |
| A | 0 | 1 | 1 | 1 | 1 | 1 | 1 |  | A | 0.0000 | 0.3333 | 0.3333 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |
| B | 1 | 0 | 0 | 1 | 1 | 0 | 0 |  | B | 0.1667 | 0.0000 | 0.0000 | 0.5000 | 0.5000 | 0.0000 | 0.0000 |
| C | 1 | 0 | 0 | 0 | 0 | 1 | 1 |  | C | 0.1667 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.5000 | 0.5000 |
| D | 1 | 1 | 0 | 0 | 0 | 0 | 0 |  | D | 0.1667 | 0.3333 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| E | 1 | 1 | 0 | 0 | 0 | 0 | 0 |  | E | 0.1667 | 0.3333 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| F | 1 | 0 | 1 | 0 | 0 | 0 | 0 |  | F | 0.1667 | 0.0000 | 0.3333 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| G | 1 | 0 | 1 | 0 | 0 | 0 | 0 |  | G | 0.1667 | 0.0000 | 0.3333 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Iteration 1 | Iteration 2 | Iteration 3 | Iteration 4 | Iteration 5 | Iteration 6 | Iteration 7 |
| A | 0.1500 | 0.4000 | 0.2667 | 0.3444 | 0.2963 | 0.3272 | 0.3070 |
| B | 0.1500 | 0.1750 | 0.1417 | 0.1694 | 0.1491 | 0.1633 | 0.1536 |
| C | 0.1500 | 0.1750 | 0.1417 | 0.1694 | 0.1491 | 0.1633 | 0.1536 |
| D | 0.1500 | 0.0750 | 0.1250 | 0.0917 | 0.1139 | 0.0991 | 0.1090 |
| E | 0.1500 | 0.0750 | 0.1250 | 0.0917 | 0.1139 | 0.0991 | 0.1090 |
| F | 0.1500 | 0.0750 | 0.1250 | 0.0917 | 0.1139 | 0.0991 | 0.1090 |
| G | 0.1500 | 0.0750 | 0.1250 | 0.0917 | 0.1139 | 0.0991 | 0.1090 |

2. The eigenvalues and eigenvector were calculated with bluebit.gr, with the highest value highlighted to indicate the corresponding eigenvector. The first table is from the adjacency matrix; the second table is from the normalized adjacency matrix.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Eigenvalues | Eigenvector |  | Eigenvalues | Eigenvector |
| 3.2361 | 0.6015 |  | 1.0000 | 0.7171 |
| 2.0000 | 0.3717 |  | 0.3333 | 0.3586 |
| 1.2361 | 0.3717 |  | 0.6667 | 0.3586 |
| 1.4142 | 0.3008 |  | 0.5773 | 0.2391 |
| 1.4142 | 0.3008 |  | 0.5773 | 0.2391 |
| 0.0000 | 0.3008 |  | 0.0000 | 0.2391 |
| 0.0000 | 0.3008 |  | 0.0000 | 0.2391 |

In both methods, the nodes are ranked as follows:

Rank 1 – A

Rank 2 – B, C

Rank 3 – D, E, F, G

**Question 3 [1 pt]:** In Quesiton 2, please use rooted PageRank to calculate similarity between each pair of nodes. Each time, the random walker has a probability 1-*α*  (where *α=0.2*)to return back toan original node*.* (Please show your solutions).

The formula for rooted PageRank is (1 - α)(I - α \* (D-1\*A))-1 and is calculated below.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | A | B | C | D | E | F | G |  | I | A | B | C | D | E | F | G |  | D | A | B | C | D | E | F | G |
| A | 0 | 1 | 1 | 1 | 1 | 1 | 1 |  | A | 1 | 0 | 0 | 0 | 0 | 0 | 0 |  | A | 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 1 | 0 | 0 | 1 | 1 | 0 | 0 |  | B | 0 | 1 | 0 | 0 | 0 | 0 | 0 |  | B | 0 | 3 | 0 | 0 | 0 | 0 | 0 |
| C | 1 | 0 | 0 | 0 | 0 | 1 | 1 |  | C | 0 | 0 | 1 | 0 | 0 | 0 | 0 |  | C | 0 | 0 | 3 | 0 | 0 | 0 | 0 |
| D | 1 | 1 | 0 | 0 | 0 | 0 | 0 |  | D | 0 | 0 | 0 | 1 | 0 | 0 | 0 |  | D | 0 | 0 | 0 | 2 | 0 | 0 | 0 |
| E | 1 | 1 | 0 | 0 | 0 | 0 | 0 |  | E | 0 | 0 | 0 | 0 | 1 | 0 | 0 |  | E | 0 | 0 | 0 | 0 | 2 | 0 | 0 |
| F | 1 | 0 | 1 | 0 | 0 | 0 | 0 |  | F | 0 | 0 | 0 | 0 | 0 | 1 | 0 |  | F | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| G | 1 | 0 | 1 | 0 | 0 | 0 | 0 |  | G | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  | G | 0 | 0 | 0 | 0 | 0 | 0 | 2 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| D-1 | A | B | C | D | E | F | G |
| A | 0.167 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| B | 0.000 | 0.333 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| C | 0.000 | 0.000 | 0.333 | 0.000 | 0.000 | 0.000 | 0.000 |
| D | 0.000 | 0.000 | 0.000 | 0.500 | 0.000 | 0.000 | 0.000 |
| E | 0.000 | 0.000 | 0.000 | 0.000 | 0.500 | 0.000 | 0.000 |
| F | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.500 | 0.000 |
| G | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.500 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| D-1\*A | A | B | C | D | E | F | G |
| A | 0.000 | 0.167 | 0.167 | 0.167 | 0.167 | 0.167 | 0.167 |
| B | 0.333 | 0.000 | 0.000 | 0.333 | 0.333 | 0.000 | 0.000 |
| C | 0.333 | 0.000 | 0.000 | 0.000 | 0.000 | 0.333 | 0.333 |
| D | 0.500 | 0.500 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| E | 0.500 | 0.500 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| F | 0.500 | 0.000 | 0.500 | 0.000 | 0.000 | 0.000 | 0.000 |
| G | 0.500 | 0.000 | 0.500 | 0.000 | 0.000 | 0.000 | 0.000 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| α \* (D-1\*A) | A | B | C | D | E | F | G |
| A | 0.000 | 0.033 | 0.033 | 0.033 | 0.033 | 0.033 | 0.033 |
| B | 0.067 | 0.000 | 0.000 | 0.067 | 0.067 | 0.000 | 0.000 |
| C | 0.067 | 0.000 | 0.000 | 0.000 | 0.000 | 0.067 | 0.067 |
| D | 0.100 | 0.100 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| E | 0.100 | 0.100 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| F | 0.100 | 0.000 | 0.100 | 0.000 | 0.000 | 0.000 | 0.000 |
| G | 0.100 | 0.000 | 0.100 | 0.000 | 0.000 | 0.000 | 0.000 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I - α \* (D-1\*A) | A | B | C | D | E | F | G |
| A | 1.000 | -0.033 | -0.033 | -0.033 | -0.033 | -0.033 | -0.033 |
| B | -0.067 | 1.000 | 0.000 | -0.067 | -0.067 | 0.000 | 0.000 |
| C | -0.067 | 0.000 | 1.000 | 0.000 | 0.000 | -0.067 | -0.067 |
| D | -0.100 | -0.100 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 |
| E | -0.100 | -0.100 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 |
| F | -0.100 | 0.000 | -0.100 | 0.000 | 0.000 | 1.000 | 0.000 |
| G | -0.100 | 0.000 | -0.100 | 0.000 | 0.000 | 0.000 | 1.000 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| (I - α \* (D-1\*A))-1 | A | B | C | D | E | F | G |
| A | 1.020 | 0.041 | 0.041 | 0.037 | 0.037 | 0.037 | 0.037 |
| B | 0.083 | 1.017 | 0.003 | 0.071 | 0.071 | 0.003 | 0.003 |
| C | 0.083 | 0.003 | 1.017 | 0.003 | 0.003 | 0.071 | 0.071 |
| D | 0.110 | 0.106 | 0.004 | 1.011 | 0.011 | 0.004 | 0.004 |
| E | 0.110 | 0.106 | 0.004 | 0.011 | 1.011 | 0.004 | 0.004 |
| F | 0.110 | 0.004 | 0.106 | 0.004 | 0.004 | 1.011 | 0.011 |
| G | 0.110 | 0.004 | 0.106 | 0.004 | 0.004 | 0.011 | 1.011 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| (1 - α)(I - α \* (D-1\*A))-1 | A | B | C | D | E | F | G |
| A | 0.816 | 0.033 | 0.033 | 0.029 | 0.029 | 0.029 | 0.029 |
| B | 0.066 | 0.813 | 0.003 | 0.056 | 0.056 | 0.002 | 0.002 |
| C | 0.066 | 0.003 | 0.813 | 0.002 | 0.002 | 0.056 | 0.056 |
| D | 0.088 | 0.085 | 0.004 | 0.809 | 0.009 | 0.003 | 0.003 |
| E | 0.088 | 0.085 | 0.004 | 0.009 | 0.809 | 0.003 | 0.003 |
| F | 0.088 | 0.004 | 0.085 | 0.003 | 0.003 | 0.809 | 0.009 |
| G | 0.088 | 0.004 | 0.085 | 0.003 | 0.003 | 0.009 | 0.809 |

**Question 4 [1.5 pts]:** The following networks show connections between 8 individuals in a small community. For node pairs (1, 7) and (1, 6), please use following measures to calculate their similarity (or distance) value and conclude which pair is more likely to form a link.

* 1. Jacarrd’s Coefficient (0.25 pt)
  2. Adamic/Adar (0.25 pt)
  3. Preferential attachment (0.25 pt)
  4. Katz (with β=0.05) (0.25 pt)
  5. SimRank score with C=1 (please show the SimRank score after the 1st iteration). (0.5 pt)

1

2

4

5

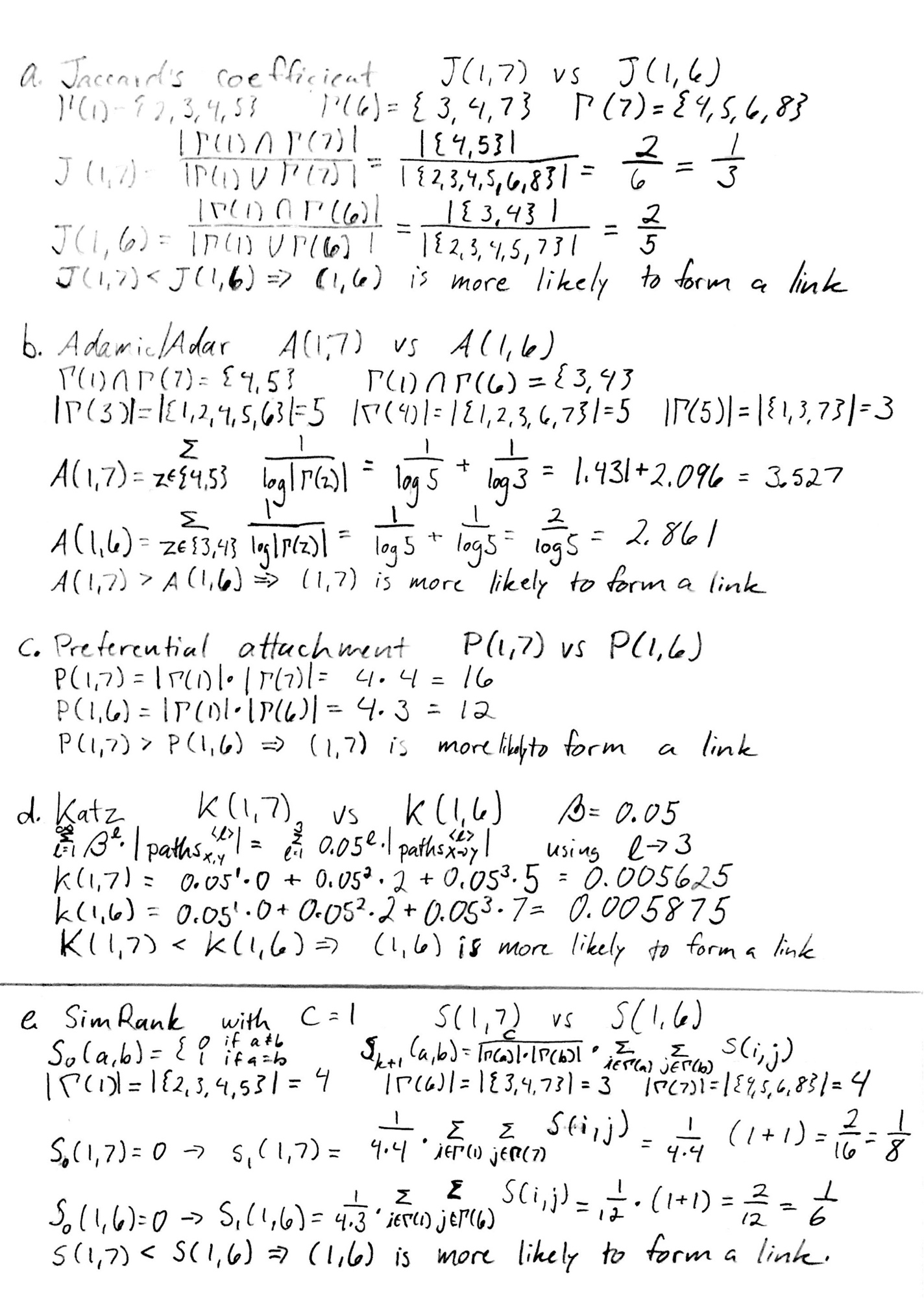
6

7

8

3

Solutions are in the following image:



**Question 5 [4 pts]:** In the following network,

1. Please find the complete set of communities by using 3-clquie [0.25pt], 3-club [0.25pt], and 3-core [0.25pt], respectively (If there are multiple sets, please just report the top three sets with the maximum number of nodes).
2. Please calculate the Geodesic distance between each pair of nodes, and use Multidimensional Scaling (MDS) to convert the network into a two dimensional space. Please report the values of all nodes in the two dimensional space and draw all nodes in the two dimensional space [1.25 pt].
3. Implement a k-means clustering algorithm (selecting k=2 and using node 18 and node 1 as the initial centers), and report the community structures after 10 iterations (You may use any other third party tools for k-means clustering. Or you can follow the k-means Excel implementation in the following URL to calculate the results) [2 pts]

k-means: <http://www.csse.monash.edu.au/courseware/cse5230/2004/assets/clustering.pdf>

3

1

2

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18

16

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14

11

8

12

10

15

1. 3-cliques:

{17, 16, 15, 14, 13, 11, 10, 9, 8, 7, 6, 5} = 12 nodes

{18, 17, 16, 15, 14, 13, 11, 10, 9, 8, 7} = 11 nodes

{1, 2, 3, 4, 5, 6, 7, 8} = 8 nodes

3-clubs:

{16, 15, 14, 13, 11, 10, 9, 8, 7, 6, 5} = 11 nodes, substructure diameter <= 3

{18, 17, 16, 15, 14, 13, 11, 10, 9, 8} = 10 nodes, substructure diameter <= 3

{1, 2, 3, 4, 5, 6, 7, 8} = 8 nodes, substructure diameter <= 3

3-cores:

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18} = 18 nodes

All nodes have degree >= 3, so they all connect to at least 3 other nodes in the same substructure, and are therefore in the same 3-core.

2. Several matrices are needed for this calculation, the first is the geodesic distance and square of geodesic distance, D:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 1 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 5 | 5 | 5 | 5 | 6 |
| 2 | 1 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 5 | 5 | 5 | 5 | 6 |
| 3 | 1 | 1 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 4 | 4 | 3 | 4 | 4 | 4 | 4 | 5 |
| 4 | 1 | 1 | 1 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 5 |
| 5 | 2 | 2 | 1 | 1 | 0 | 1 | 1 | 2 | 3 | 3 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 4 |
| 6 | 2 | 2 | 2 | 1 | 1 | 0 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 4 |
| 7 | 3 | 3 | 2 | 2 | 1 | 1 | 0 | 2 | 3 | 3 | 3 | 3 | 1 | 2 | 2 | 2 | 2 | 3 |
| 8 | 3 | 3 | 3 | 2 | 2 | 1 | 2 | 0 | 1 | 1 | 1 | 1 | 3 | 2 | 3 | 2 | 3 | 3 |
| 9 | 4 | 4 | 4 | 3 | 3 | 2 | 3 | 1 | 0 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 2 |
| 10 | 4 | 4 | 4 | 3 | 3 | 2 | 3 | 1 | 1 | 0 | 1 | 1 | 3 | 2 | 3 | 2 | 3 | 3 |
| 11 | 4 | 4 | 4 | 3 | 3 | 2 | 3 | 1 | 1 | 1 | 0 | 1 | 3 | 2 | 3 | 2 | 3 | 3 |
| 12 | 4 | 4 | 4 | 3 | 3 | 2 | 3 | 1 | 2 | 1 | 1 | 0 | 4 | 3 | 4 | 3 | 4 | 4 |
| 13 | 4 | 4 | 3 | 3 | 2 | 2 | 1 | 3 | 2 | 3 | 3 | 4 | 0 | 1 | 1 | 1 | 1 | 2 |
| 14 | 5 | 5 | 4 | 4 | 3 | 3 | 2 | 2 | 1 | 2 | 2 | 3 | 1 | 0 | 1 | 2 | 2 | 1 |
| 15 | 5 | 5 | 4 | 4 | 3 | 3 | 2 | 3 | 2 | 3 | 3 | 4 | 1 | 1 | 0 | 1 | 2 | 2 |
| 16 | 5 | 5 | 4 | 4 | 3 | 3 | 2 | 2 | 1 | 2 | 2 | 3 | 1 | 2 | 1 | 0 | 1 | 1 |
| 17 | 5 | 5 | 4 | 4 | 3 | 3 | 2 | 3 | 2 | 3 | 3 | 4 | 1 | 2 | 2 | 1 | 0 | 1 |
| 18 | 6 | 6 | 5 | 5 | 4 | 4 | 3 | 3 | 2 | 3 | 3 | 4 | 2 | 1 | 2 | 1 | 1 | 0 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| D | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 1 | 0 | 1 | 1 | 1 | 4 | 4 | 9 | 9 | 16 | 16 | 16 | 16 | 16 | 25 | 25 | 25 | 25 | 36 |
| 2 | 1 | 0 | 1 | 1 | 4 | 4 | 9 | 9 | 16 | 16 | 16 | 16 | 16 | 25 | 25 | 25 | 25 | 36 |
| 3 | 1 | 1 | 0 | 1 | 1 | 4 | 4 | 9 | 16 | 16 | 16 | 16 | 9 | 16 | 16 | 16 | 16 | 25 |
| 4 | 1 | 1 | 1 | 0 | 1 | 1 | 4 | 4 | 9 | 9 | 9 | 9 | 9 | 16 | 16 | 16 | 16 | 25 |
| 5 | 4 | 4 | 1 | 1 | 0 | 1 | 1 | 4 | 9 | 9 | 9 | 9 | 4 | 9 | 9 | 9 | 9 | 16 |
| 6 | 4 | 4 | 4 | 1 | 1 | 0 | 1 | 1 | 4 | 4 | 4 | 4 | 4 | 9 | 9 | 9 | 9 | 16 |
| 7 | 9 | 9 | 4 | 4 | 1 | 1 | 0 | 4 | 9 | 9 | 9 | 9 | 1 | 4 | 4 | 4 | 4 | 9 |
| 8 | 9 | 9 | 9 | 4 | 4 | 1 | 4 | 0 | 1 | 1 | 1 | 1 | 9 | 4 | 9 | 4 | 9 | 9 |
| 9 | 16 | 16 | 16 | 9 | 9 | 4 | 9 | 1 | 0 | 1 | 1 | 4 | 4 | 1 | 4 | 1 | 4 | 4 |
| 10 | 16 | 16 | 16 | 9 | 9 | 4 | 9 | 1 | 1 | 0 | 1 | 1 | 9 | 4 | 9 | 4 | 9 | 9 |
| 11 | 16 | 16 | 16 | 9 | 9 | 4 | 9 | 1 | 1 | 1 | 0 | 1 | 9 | 4 | 9 | 4 | 9 | 9 |
| 12 | 16 | 16 | 16 | 9 | 9 | 4 | 9 | 1 | 4 | 1 | 1 | 0 | 16 | 9 | 16 | 9 | 16 | 16 |
| 13 | 16 | 16 | 9 | 9 | 4 | 4 | 1 | 9 | 4 | 9 | 9 | 16 | 0 | 1 | 1 | 1 | 1 | 4 |
| 14 | 25 | 25 | 16 | 16 | 9 | 9 | 4 | 4 | 1 | 4 | 4 | 9 | 1 | 0 | 1 | 4 | 4 | 1 |
| 15 | 25 | 25 | 16 | 16 | 9 | 9 | 4 | 9 | 4 | 9 | 9 | 16 | 1 | 1 | 0 | 1 | 4 | 4 |
| 16 | 25 | 25 | 16 | 16 | 9 | 9 | 4 | 4 | 1 | 4 | 4 | 9 | 1 | 4 | 1 | 0 | 1 | 1 |
| 17 | 25 | 25 | 16 | 16 | 9 | 9 | 4 | 9 | 4 | 9 | 9 | 16 | 1 | 4 | 4 | 1 | 0 | 1 |
| 18 | 36 | 36 | 25 | 25 | 16 | 16 | 9 | 9 | 4 | 9 | 9 | 16 | 4 | 1 | 4 | 1 | 1 | 0 |

Next is I – (1/n)eeT, which is also denoted as J:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| J | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 1 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 2 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 3 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 4 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 5 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 6 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 7 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 8 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 9 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 10 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 11 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 12 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 13 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 |
| 14 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 | -0.056 |
| 15 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 | -0.056 |
| 16 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 | -0.056 |
| 17 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 | -0.056 |
| 18 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | -0.056 | 0.944 |

From this, we calculate (-1/2)J\*D\*J = P:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| P | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 1 | 9.515 | 9.015 | 7.293 | 6.293 | 3.710 | 3.154 | 0.821 | 0.654 | -2.401 | -1.762 | -1.762 | -0.623 | -2.123 | -5.985 | -5.290 | -6.068 | -5.290 | -9.151 |
| 2 | 9.015 | 9.515 | 7.293 | 6.293 | 3.710 | 3.154 | 0.821 | 0.654 | -2.401 | -1.762 | -1.762 | -0.623 | -2.123 | -5.985 | -5.290 | -6.068 | -5.290 | -9.151 |
| 3 | 7.293 | 7.293 | 6.071 | 4.571 | 3.488 | 1.432 | 1.599 | -1.068 | -4.123 | -3.485 | -3.485 | -2.346 | -0.346 | -3.207 | -2.512 | -3.290 | -2.512 | -5.373 |
| 4 | 6.293 | 6.293 | 4.571 | 4.071 | 2.488 | 1.932 | 0.599 | 0.432 | -1.623 | -0.985 | -0.985 | 0.154 | -1.346 | -4.207 | -3.512 | -4.290 | -3.512 | -6.373 |
| 5 | 3.710 | 3.710 | 3.488 | 2.488 | 1.904 | 0.849 | 1.015 | -0.651 | -2.707 | -2.068 | -2.068 | -0.929 | 0.071 | -1.790 | -1.096 | -1.873 | -1.096 | -2.957 |
| 6 | 3.154 | 3.154 | 1.432 | 1.932 | 0.849 | 0.793 | 0.460 | 0.293 | -0.762 | -0.123 | -0.123 | 1.015 | -0.485 | -2.346 | -1.651 | -2.429 | -1.651 | -3.512 |
| 7 | 0.821 | 0.821 | 1.599 | 0.599 | 1.015 | 0.460 | 1.127 | -1.040 | -3.096 | -2.457 | -2.457 | -1.318 | 1.182 | 0.321 | 1.015 | 0.238 | 1.015 | 0.154 |
| 8 | 0.654 | 0.654 | -1.068 | 0.432 | -0.651 | 0.293 | -1.040 | 0.793 | 0.738 | 1.377 | 1.377 | 2.515 | -2.985 | 0.154 | -1.651 | 0.071 | -1.651 | -0.012 |
| 9 | -2.401 | -2.401 | -4.123 | -1.623 | -2.707 | -0.762 | -3.096 | 0.738 | 1.682 | 1.821 | 1.821 | 1.460 | -0.040 | 2.099 | 1.293 | 2.015 | 1.293 | 2.932 |
| 10 | -1.762 | -1.762 | -3.485 | -0.985 | -2.068 | -0.123 | -2.457 | 1.377 | 1.821 | 2.960 | 2.460 | 3.599 | -1.901 | 1.238 | -0.568 | 1.154 | -0.568 | 1.071 |
| 11 | -1.762 | -1.762 | -3.485 | -0.985 | -2.068 | -0.123 | -2.457 | 1.377 | 1.821 | 2.460 | 2.960 | 3.599 | -1.901 | 1.238 | -0.568 | 1.154 | -0.568 | 1.071 |
| 12 | -0.623 | -0.623 | -2.346 | 0.154 | -0.929 | 1.015 | -1.318 | 2.515 | 1.460 | 3.599 | 3.599 | 5.238 | -4.262 | -0.123 | -2.929 | -0.207 | -2.929 | -1.290 |
| 13 | -2.123 | -2.123 | -0.346 | -1.346 | 0.071 | -0.485 | 1.182 | -2.985 | -0.040 | -1.901 | -1.901 | -4.262 | 2.238 | 2.377 | 3.071 | 2.293 | 3.071 | 3.210 |
| 14 | -5.985 | -5.985 | -3.207 | -4.207 | -1.790 | -2.346 | 0.321 | 0.154 | 2.099 | 1.238 | 1.238 | -0.123 | 2.377 | 3.515 | 3.710 | 1.432 | 2.210 | 5.349 |
| 15 | -5.290 | -5.290 | -2.512 | -3.512 | -1.096 | -1.651 | 1.015 | -1.651 | 1.293 | -0.568 | -0.568 | -2.929 | 3.071 | 3.710 | 4.904 | 3.627 | 2.904 | 4.543 |
| 16 | -6.068 | -6.068 | -3.290 | -4.290 | -1.873 | -2.429 | 0.238 | 0.071 | 2.015 | 1.154 | 1.154 | -0.207 | 2.293 | 1.432 | 3.627 | 3.349 | 3.627 | 5.265 |
| 17 | -5.290 | -5.290 | -2.512 | -3.512 | -1.096 | -1.651 | 1.015 | -1.651 | 1.293 | -0.568 | -0.568 | -2.929 | 3.071 | 2.210 | 2.904 | 3.627 | 4.904 | 6.043 |
| 18 | -9.151 | -9.151 | -5.373 | -6.373 | -2.957 | -3.512 | 0.154 | -0.012 | 2.932 | 1.071 | 1.071 | -1.290 | 3.210 | 5.349 | 4.543 | 5.265 | 6.043 | 8.182 |

Finally, S(x, y) = V\*λ1/2, where S is the 2D coordinates of each node, λ is the diagonal matrix of the 2 top eigenvalues of P, and V is the corresponding 2 eigenvectors of P:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| V | X | Y |  | λ1/2 | 1 | 2 |  | S | X | Y |
| 1 | -0.420 | -0.024 |  | 1 | 7.353 | 0.000 |  | 1 | -3.088 | -0.108 |
| 2 | -0.420 | -0.024 |  | 2 | 0.000 | 4.514 |  | 2 | -3.088 | -0.108 |
| 3 | -0.297 | -0.273 |  |  |  |  |  | 3 | -2.184 | -1.232 |
| 4 | -0.283 | 0.007 |  |  |  |  |  | 4 | -2.081 | 0.032 |
| 5 | -0.159 | -0.165 |  |  |  |  |  | 5 | -1.169 | -0.745 |
| 6 | -0.137 | 0.050 |  |  |  |  |  | 6 | -1.007 | 0.226 |
| 7 | -0.029 | -0.247 |  |  |  |  |  | 7 | -0.213 | -1.115 |
| 8 | -0.019 | 0.246 |  |  |  |  |  | 8 | -0.140 | 1.111 |
| 9 | 0.141 | 0.193 |  |  |  |  |  | 9 | 1.037 | 0.871 |
| 10 | 0.081 | 0.349 |  |  |  |  |  | 10 | 0.596 | 1.576 |
| 11 | 0.081 | 0.349 |  |  |  |  |  | 11 | 0.596 | 1.576 |
| 12 | -0.010 | 0.502 |  |  |  |  |  | 12 | -0.074 | 2.266 |
| 13 | 0.114 | -0.343 |  |  |  |  |  | 13 | 0.838 | -1.548 |
| 14 | 0.245 | -0.017 |  |  |  |  |  | 14 | 1.802 | -0.077 |
| 15 | 0.231 | -0.234 |  |  |  |  |  | 15 | 1.699 | -1.056 |
| 16 | 0.252 | -0.037 |  |  |  |  |  | 16 | 1.853 | -0.167 |
| 17 | 0.235 | -0.241 |  |  |  |  |  | 17 | 1.728 | -1.088 |
| 18 | 0.395 | -0.091 |  |  |  |  |  | 18 | 2.905 | -0.411 |

From S, the 2D Space graph can be created:

3. 10 iterations of a k-means clustering algorithm are shown in the tables below. There are 2 clusters, labeled K1 and K2, with centroids at (K1\_X, K1\_Y) and (K2\_X, K2\_Y), respectively. After each iteration, the centroids are updated to (NK1\_X, NK1\_Y) and (NK2\_X, NK2\_Y). In this graph, the centroids are stable by the second iteration. The members of the clusters are:

K1 = {1, 2, 3, 4, 5, 6, 7, 8, 12}

K2 = {9, 10, 11, 13, 14, 15, 16, 17, 18}

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| I1 | X | Y | D\_K1 | D\_K2 | Clust. | K1 | | K2 | |
| 1 | -3.088 | -0.108 | 0.000 | 6.001 | K1 | -3.088 | -0.108 |  |  |
| 2 | -3.088 | -0.108 | 0.000 | 6.001 | K1 | -3.088 | -0.108 |  |  |
| 3 | -2.184 | -1.232 | 1.443 | 5.154 | K1 | -2.184 | -1.232 |  |  |
| 4 | -2.081 | 0.032 | 1.017 | 5.005 | K1 | -2.081 | 0.032 |  |  |
| 5 | -1.169 | -0.745 | 2.022 | 4.087 | K1 | -1.169 | -0.745 |  |  |
| 6 | -1.007 | 0.226 | 2.108 | 3.963 | K1 | -1.007 | 0.226 |  |  |
| 7 | -0.213 | -1.115 | 3.046 | 3.196 | K1 | -0.213 | -1.115 |  |  |
| 8 | -0.140 | 1.111 | 3.191 | 3.403 | K1 | -0.140 | 1.111 |  |  |
| 9 | 1.037 | 0.871 | 4.240 | 2.265 | K2 |  |  | 1.037 | 0.871 |
| 10 | 0.596 | 1.576 | 4.051 | 3.046 | K2 |  |  | 0.596 | 1.576 |
| 11 | 0.596 | 1.576 | 4.051 | 3.046 | K2 |  |  | 0.596 | 1.576 |
| 12 | -0.074 | 2.266 | 3.838 | 4.004 | K1 | -0.074 | 2.266 |  |  |
| 13 | 0.838 | -1.548 | 4.182 | 2.359 | K2 |  |  | 0.838 | -1.548 |
| 14 | 1.802 | -0.077 | 4.890 | 1.152 | K2 |  |  | 1.802 | -0.077 |
| 15 | 1.699 | -1.056 | 4.880 | 1.368 | K2 |  |  | 1.699 | -1.056 |
| 16 | 1.853 | -0.167 | 4.942 | 1.079 | K2 |  |  | 1.853 | -0.167 |
| 17 | 1.728 | -1.088 | 4.915 | 1.357 | K2 |  |  | 1.728 | -1.088 |
| 18 | 2.905 | -0.411 | 6.001 | 0.000 | K2 |  |  | 2.905 | -0.411 |
|  |  |  |  |  |  |  |  |  |  |
|  | K1\_X | K1\_Y | K2\_X | K2\_Y |  | NK1\_X | NK1\_Y | NK2\_X | NK2\_Y |
|  | -3.088 | -0.108 | 2.905 | -0.411 |  | -1.449 | 0.036 | 1.450 | -0.036 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| I2 | X | Y | D\_K1 | D\_K2 | Clust. | K1 | | K2 | |
| 1 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 2 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 3 | -2.184 | -1.232 | 1.466 | 3.826 | K1 | -2.184 | -1.232 |  |  |
| 4 | -2.081 | 0.032 | 0.632 | 3.532 | K1 | -2.081 | 0.032 |  |  |
| 5 | -1.169 | -0.745 | 0.830 | 2.714 | K1 | -1.169 | -0.745 |  |  |
| 6 | -1.007 | 0.226 | 0.481 | 2.472 | K1 | -1.007 | 0.226 |  |  |
| 7 | -0.213 | -1.115 | 1.689 | 1.983 | K1 | -0.213 | -1.115 |  |  |
| 8 | -0.140 | 1.111 | 1.694 | 1.960 | K1 | -0.140 | 1.111 |  |  |
| 9 | 1.037 | 0.871 | 2.623 | 0.997 | K2 |  |  | 1.037 | 0.871 |
| 10 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 11 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 12 | -0.074 | 2.266 | 2.620 | 2.761 | K1 | -0.074 | 2.266 |  |  |
| 13 | 0.838 | -1.548 | 2.783 | 1.631 | K2 |  |  | 0.838 | -1.548 |
| 14 | 1.802 | -0.077 | 3.253 | 0.354 | K2 |  |  | 1.802 | -0.077 |
| 15 | 1.699 | -1.056 | 3.332 | 1.050 | K2 |  |  | 1.699 | -1.056 |
| 16 | 1.853 | -0.167 | 3.309 | 0.424 | K2 |  |  | 1.853 | -0.167 |
| 17 | 1.728 | -1.088 | 3.370 | 1.088 | K2 |  |  | 1.728 | -1.088 |
| 18 | 2.905 | -0.411 | 4.377 | 1.502 | K2 |  |  | 2.905 | -0.411 |
|  |  |  |  |  |  |  |  |  |  |
|  | K1\_X | K1\_Y | K2\_X | K2\_Y |  | NK1\_X | NK1\_Y | NK2\_X | NK2\_Y |
|  | -1.449 | 0.036 | 1.450 | -0.036 |  | -1.449 | 0.036 | 1.450 | -0.036 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| I3 | X | Y | D\_K1 | D\_K2 | Clust. | K1 | | K2 | |
| 1 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 2 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 3 | -2.184 | -1.232 | 1.466 | 3.826 | K1 | -2.184 | -1.232 |  |  |
| 4 | -2.081 | 0.032 | 0.632 | 3.532 | K1 | -2.081 | 0.032 |  |  |
| 5 | -1.169 | -0.745 | 0.830 | 2.714 | K1 | -1.169 | -0.745 |  |  |
| 6 | -1.007 | 0.226 | 0.481 | 2.472 | K1 | -1.007 | 0.226 |  |  |
| 7 | -0.213 | -1.115 | 1.689 | 1.983 | K1 | -0.213 | -1.115 |  |  |
| 8 | -0.140 | 1.111 | 1.694 | 1.960 | K1 | -0.140 | 1.111 |  |  |
| 9 | 1.037 | 0.871 | 2.623 | 0.997 | K2 |  |  | 1.037 | 0.871 |
| 10 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 11 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 12 | -0.074 | 2.266 | 2.620 | 2.761 | K1 | -0.074 | 2.266 |  |  |
| 13 | 0.838 | -1.548 | 2.783 | 1.631 | K2 |  |  | 0.838 | -1.548 |
| 14 | 1.802 | -0.077 | 3.253 | 0.354 | K2 |  |  | 1.802 | -0.077 |
| 15 | 1.699 | -1.056 | 3.332 | 1.050 | K2 |  |  | 1.699 | -1.056 |
| 16 | 1.853 | -0.167 | 3.309 | 0.424 | K2 |  |  | 1.853 | -0.167 |
| 17 | 1.728 | -1.088 | 3.370 | 1.088 | K2 |  |  | 1.728 | -1.088 |
| 18 | 2.905 | -0.411 | 4.377 | 1.502 | K2 |  |  | 2.905 | -0.411 |
|  |  |  |  |  |  |  |  |  |  |
|  | K1\_X | K1\_Y | K2\_X | K2\_Y |  | NK1\_X | NK1\_Y | NK2\_X | NK2\_Y |
|  | -1.449 | 0.036 | 1.450 | -0.036 |  | -1.449 | 0.036 | 1.450 | -0.036 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| I4 | X | Y | D\_K1 | D\_K2 | Clust. | K1 | | K2 | |
| 1 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 2 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 3 | -2.184 | -1.232 | 1.466 | 3.826 | K1 | -2.184 | -1.232 |  |  |
| 4 | -2.081 | 0.032 | 0.632 | 3.532 | K1 | -2.081 | 0.032 |  |  |
| 5 | -1.169 | -0.745 | 0.830 | 2.714 | K1 | -1.169 | -0.745 |  |  |
| 6 | -1.007 | 0.226 | 0.481 | 2.472 | K1 | -1.007 | 0.226 |  |  |
| 7 | -0.213 | -1.115 | 1.689 | 1.983 | K1 | -0.213 | -1.115 |  |  |
| 8 | -0.140 | 1.111 | 1.694 | 1.960 | K1 | -0.140 | 1.111 |  |  |
| 9 | 1.037 | 0.871 | 2.623 | 0.997 | K2 |  |  | 1.037 | 0.871 |
| 10 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 11 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 12 | -0.074 | 2.266 | 2.620 | 2.761 | K1 | -0.074 | 2.266 |  |  |
| 13 | 0.838 | -1.548 | 2.783 | 1.631 | K2 |  |  | 0.838 | -1.548 |
| 14 | 1.802 | -0.077 | 3.253 | 0.354 | K2 |  |  | 1.802 | -0.077 |
| 15 | 1.699 | -1.056 | 3.332 | 1.050 | K2 |  |  | 1.699 | -1.056 |
| 16 | 1.853 | -0.167 | 3.309 | 0.424 | K2 |  |  | 1.853 | -0.167 |
| 17 | 1.728 | -1.088 | 3.370 | 1.088 | K2 |  |  | 1.728 | -1.088 |
| 18 | 2.905 | -0.411 | 4.377 | 1.502 | K2 |  |  | 2.905 | -0.411 |
|  |  |  |  |  |  |  |  |  |  |
|  | K1\_X | K1\_Y | K2\_X | K2\_Y |  | NK1\_X | NK1\_Y | NK2\_X | NK2\_Y |
|  | -1.449 | 0.036 | 1.450 | -0.036 |  | -1.449 | 0.036 | 1.450 | -0.036 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| I5 | X | Y | D\_K1 | D\_K2 | Clust. | K1 | | K2 | |
| 1 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 2 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 3 | -2.184 | -1.232 | 1.466 | 3.826 | K1 | -2.184 | -1.232 |  |  |
| 4 | -2.081 | 0.032 | 0.632 | 3.532 | K1 | -2.081 | 0.032 |  |  |
| 5 | -1.169 | -0.745 | 0.830 | 2.714 | K1 | -1.169 | -0.745 |  |  |
| 6 | -1.007 | 0.226 | 0.481 | 2.472 | K1 | -1.007 | 0.226 |  |  |
| 7 | -0.213 | -1.115 | 1.689 | 1.983 | K1 | -0.213 | -1.115 |  |  |
| 8 | -0.140 | 1.111 | 1.694 | 1.960 | K1 | -0.140 | 1.111 |  |  |
| 9 | 1.037 | 0.871 | 2.623 | 0.997 | K2 |  |  | 1.037 | 0.871 |
| 10 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 11 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 12 | -0.074 | 2.266 | 2.620 | 2.761 | K1 | -0.074 | 2.266 |  |  |
| 13 | 0.838 | -1.548 | 2.783 | 1.631 | K2 |  |  | 0.838 | -1.548 |
| 14 | 1.802 | -0.077 | 3.253 | 0.354 | K2 |  |  | 1.802 | -0.077 |
| 15 | 1.699 | -1.056 | 3.332 | 1.050 | K2 |  |  | 1.699 | -1.056 |
| 16 | 1.853 | -0.167 | 3.309 | 0.424 | K2 |  |  | 1.853 | -0.167 |
| 17 | 1.728 | -1.088 | 3.370 | 1.088 | K2 |  |  | 1.728 | -1.088 |
| 18 | 2.905 | -0.411 | 4.377 | 1.502 | K2 |  |  | 2.905 | -0.411 |
|  |  |  |  |  |  |  |  |  |  |
|  | K1\_X | K1\_Y | K2\_X | K2\_Y |  | NK1\_X | NK1\_Y | NK2\_X | NK2\_Y |
|  | -1.449 | 0.036 | 1.450 | -0.036 |  | -1.449 | 0.036 | 1.450 | -0.036 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| I6 | X | Y | D\_K1 | D\_K2 | Clust. | K1 | | K2 | |
| 1 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 2 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 3 | -2.184 | -1.232 | 1.466 | 3.826 | K1 | -2.184 | -1.232 |  |  |
| 4 | -2.081 | 0.032 | 0.632 | 3.532 | K1 | -2.081 | 0.032 |  |  |
| 5 | -1.169 | -0.745 | 0.830 | 2.714 | K1 | -1.169 | -0.745 |  |  |
| 6 | -1.007 | 0.226 | 0.481 | 2.472 | K1 | -1.007 | 0.226 |  |  |
| 7 | -0.213 | -1.115 | 1.689 | 1.983 | K1 | -0.213 | -1.115 |  |  |
| 8 | -0.140 | 1.111 | 1.694 | 1.960 | K1 | -0.140 | 1.111 |  |  |
| 9 | 1.037 | 0.871 | 2.623 | 0.997 | K2 |  |  | 1.037 | 0.871 |
| 10 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 11 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 12 | -0.074 | 2.266 | 2.620 | 2.761 | K1 | -0.074 | 2.266 |  |  |
| 13 | 0.838 | -1.548 | 2.783 | 1.631 | K2 |  |  | 0.838 | -1.548 |
| 14 | 1.802 | -0.077 | 3.253 | 0.354 | K2 |  |  | 1.802 | -0.077 |
| 15 | 1.699 | -1.056 | 3.332 | 1.050 | K2 |  |  | 1.699 | -1.056 |
| 16 | 1.853 | -0.167 | 3.309 | 0.424 | K2 |  |  | 1.853 | -0.167 |
| 17 | 1.728 | -1.088 | 3.370 | 1.088 | K2 |  |  | 1.728 | -1.088 |
| 18 | 2.905 | -0.411 | 4.377 | 1.502 | K2 |  |  | 2.905 | -0.411 |
|  |  |  |  |  |  |  |  |  |  |
|  | K1\_X | K1\_Y | K2\_X | K2\_Y |  | NK1\_X | NK1\_Y | NK2\_X | NK2\_Y |
|  | -1.449 | 0.036 | 1.450 | -0.036 |  | -1.449 | 0.036 | 1.450 | -0.036 |

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| I7 | X | Y | D\_K1 | D\_K2 | Clust. | K1 | | K2 | |
| 1 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 2 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 3 | -2.184 | -1.232 | 1.466 | 3.826 | K1 | -2.184 | -1.232 |  |  |
| 4 | -2.081 | 0.032 | 0.632 | 3.532 | K1 | -2.081 | 0.032 |  |  |
| 5 | -1.169 | -0.745 | 0.830 | 2.714 | K1 | -1.169 | -0.745 |  |  |
| 6 | -1.007 | 0.226 | 0.481 | 2.472 | K1 | -1.007 | 0.226 |  |  |
| 7 | -0.213 | -1.115 | 1.689 | 1.983 | K1 | -0.213 | -1.115 |  |  |
| 8 | -0.140 | 1.111 | 1.694 | 1.960 | K1 | -0.140 | 1.111 |  |  |
| 9 | 1.037 | 0.871 | 2.623 | 0.997 | K2 |  |  | 1.037 | 0.871 |
| 10 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 11 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 12 | -0.074 | 2.266 | 2.620 | 2.761 | K1 | -0.074 | 2.266 |  |  |
| 13 | 0.838 | -1.548 | 2.783 | 1.631 | K2 |  |  | 0.838 | -1.548 |
| 14 | 1.802 | -0.077 | 3.253 | 0.354 | K2 |  |  | 1.802 | -0.077 |
| 15 | 1.699 | -1.056 | 3.332 | 1.050 | K2 |  |  | 1.699 | -1.056 |
| 16 | 1.853 | -0.167 | 3.309 | 0.424 | K2 |  |  | 1.853 | -0.167 |
| 17 | 1.728 | -1.088 | 3.370 | 1.088 | K2 |  |  | 1.728 | -1.088 |
| 18 | 2.905 | -0.411 | 4.377 | 1.502 | K2 |  |  | 2.905 | -0.411 |
|  |  |  |  |  |  |  |  |  |  |
|  | K1\_X | K1\_Y | K2\_X | K2\_Y |  | NK1\_X | NK1\_Y | NK2\_X | NK2\_Y |
|  | -1.449 | 0.036 | 1.450 | -0.036 |  | -1.449 | 0.036 | 1.450 | -0.036 |

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| I8 | X | Y | D\_K1 | D\_K2 | Clust. | K1 | | K2 | |
| 1 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 2 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 3 | -2.184 | -1.232 | 1.466 | 3.826 | K1 | -2.184 | -1.232 |  |  |
| 4 | -2.081 | 0.032 | 0.632 | 3.532 | K1 | -2.081 | 0.032 |  |  |
| 5 | -1.169 | -0.745 | 0.830 | 2.714 | K1 | -1.169 | -0.745 |  |  |
| 6 | -1.007 | 0.226 | 0.481 | 2.472 | K1 | -1.007 | 0.226 |  |  |
| 7 | -0.213 | -1.115 | 1.689 | 1.983 | K1 | -0.213 | -1.115 |  |  |
| 8 | -0.140 | 1.111 | 1.694 | 1.960 | K1 | -0.140 | 1.111 |  |  |
| 9 | 1.037 | 0.871 | 2.623 | 0.997 | K2 |  |  | 1.037 | 0.871 |
| 10 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 11 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 12 | -0.074 | 2.266 | 2.620 | 2.761 | K1 | -0.074 | 2.266 |  |  |
| 13 | 0.838 | -1.548 | 2.783 | 1.631 | K2 |  |  | 0.838 | -1.548 |
| 14 | 1.802 | -0.077 | 3.253 | 0.354 | K2 |  |  | 1.802 | -0.077 |
| 15 | 1.699 | -1.056 | 3.332 | 1.050 | K2 |  |  | 1.699 | -1.056 |
| 16 | 1.853 | -0.167 | 3.309 | 0.424 | K2 |  |  | 1.853 | -0.167 |
| 17 | 1.728 | -1.088 | 3.370 | 1.088 | K2 |  |  | 1.728 | -1.088 |
| 18 | 2.905 | -0.411 | 4.377 | 1.502 | K2 |  |  | 2.905 | -0.411 |
|  |  |  |  |  |  |  |  |  |  |
|  | K1\_X | K1\_Y | K2\_X | K2\_Y |  | NK1\_X | NK1\_Y | NK2\_X | NK2\_Y |
|  | -1.449 | 0.036 | 1.450 | -0.036 |  | -1.449 | 0.036 | 1.450 | -0.036 |

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| I9 | X | Y | D\_K1 | D\_K2 | Clust. | K1 | | K2 | |
| 1 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 2 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 3 | -2.184 | -1.232 | 1.466 | 3.826 | K1 | -2.184 | -1.232 |  |  |
| 4 | -2.081 | 0.032 | 0.632 | 3.532 | K1 | -2.081 | 0.032 |  |  |
| 5 | -1.169 | -0.745 | 0.830 | 2.714 | K1 | -1.169 | -0.745 |  |  |
| 6 | -1.007 | 0.226 | 0.481 | 2.472 | K1 | -1.007 | 0.226 |  |  |
| 7 | -0.213 | -1.115 | 1.689 | 1.983 | K1 | -0.213 | -1.115 |  |  |
| 8 | -0.140 | 1.111 | 1.694 | 1.960 | K1 | -0.140 | 1.111 |  |  |
| 9 | 1.037 | 0.871 | 2.623 | 0.997 | K2 |  |  | 1.037 | 0.871 |
| 10 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 11 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 12 | -0.074 | 2.266 | 2.620 | 2.761 | K1 | -0.074 | 2.266 |  |  |
| 13 | 0.838 | -1.548 | 2.783 | 1.631 | K2 |  |  | 0.838 | -1.548 |
| 14 | 1.802 | -0.077 | 3.253 | 0.354 | K2 |  |  | 1.802 | -0.077 |
| 15 | 1.699 | -1.056 | 3.332 | 1.050 | K2 |  |  | 1.699 | -1.056 |
| 16 | 1.853 | -0.167 | 3.309 | 0.424 | K2 |  |  | 1.853 | -0.167 |
| 17 | 1.728 | -1.088 | 3.370 | 1.088 | K2 |  |  | 1.728 | -1.088 |
| 18 | 2.905 | -0.411 | 4.377 | 1.502 | K2 |  |  | 2.905 | -0.411 |
|  |  |  |  |  |  |  |  |  |  |
|  | K1\_X | K1\_Y | K2\_X | K2\_Y |  | NK1\_X | NK1\_Y | NK2\_X | NK2\_Y |
|  | -1.449 | 0.036 | 1.450 | -0.036 |  | -1.449 | 0.036 | 1.450 | -0.036 |

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| I10 | X | Y | D\_K1 | D\_K2 | Clust. | K1 | | K2 | |
| 1 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 2 | -3.088 | -0.108 | 1.645 | 4.539 | K1 | -3.088 | -0.108 |  |  |
| 3 | -2.184 | -1.232 | 1.466 | 3.826 | K1 | -2.184 | -1.232 |  |  |
| 4 | -2.081 | 0.032 | 0.632 | 3.532 | K1 | -2.081 | 0.032 |  |  |
| 5 | -1.169 | -0.745 | 0.830 | 2.714 | K1 | -1.169 | -0.745 |  |  |
| 6 | -1.007 | 0.226 | 0.481 | 2.472 | K1 | -1.007 | 0.226 |  |  |
| 7 | -0.213 | -1.115 | 1.689 | 1.983 | K1 | -0.213 | -1.115 |  |  |
| 8 | -0.140 | 1.111 | 1.694 | 1.960 | K1 | -0.140 | 1.111 |  |  |
| 9 | 1.037 | 0.871 | 2.623 | 0.997 | K2 |  |  | 1.037 | 0.871 |
| 10 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 11 | 0.596 | 1.576 | 2.560 | 1.824 | K2 |  |  | 0.596 | 1.576 |
| 12 | -0.074 | 2.266 | 2.620 | 2.761 | K1 | -0.074 | 2.266 |  |  |
| 13 | 0.838 | -1.548 | 2.783 | 1.631 | K2 |  |  | 0.838 | -1.548 |
| 14 | 1.802 | -0.077 | 3.253 | 0.354 | K2 |  |  | 1.802 | -0.077 |
| 15 | 1.699 | -1.056 | 3.332 | 1.050 | K2 |  |  | 1.699 | -1.056 |
| 16 | 1.853 | -0.167 | 3.309 | 0.424 | K2 |  |  | 1.853 | -0.167 |
| 17 | 1.728 | -1.088 | 3.370 | 1.088 | K2 |  |  | 1.728 | -1.088 |
| 18 | 2.905 | -0.411 | 4.377 | 1.502 | K2 |  |  | 2.905 | -0.411 |
|  |  |  |  |  |  |  |  |  |  |
|  | K1\_X | K1\_Y | K2\_X | K2\_Y |  | NK1\_X | NK1\_Y | NK2\_X | NK2\_Y |
|  | -1.449 | 0.036 | 1.450 | -0.036 |  | -1.449 | 0.036 | 1.450 | -0.036 |