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N^{TH} FIBONACCI NUMBER

1. DESCRIPTION

The Fibonacci numbers are a sequence of integers defined by the recurrence relation $F_n = F_{n-1} + F_{n-2}$, where F_n is the n^{th} number in the series and $F_0 = 0$ and $F_1 = 1$. This sequence appears frequently in mathematics, computer science, and even biology, and has been described in mathematical texts for centuries.

2. APPLICATIONS

A. Golden Ratio – F_n / F_{n-1} approaches φ as n approaches ∞

B. Fibonacci heap – data structure for priority queues

C. Hilbert's Tenth Problem – Fibonacci numbers used to show unsolvability

D. Bee ancestry – bee reproduction creates an unusual number of ancestors

E. Brock-Mirman model – a generalized sequence is used in an optimal control function

F. Fibonacci Quarterly & the Fibonacci Association – publishing scholarly work since 1963

3. COMPETING ALGORITHMS

A. Recursion

i. Directly implement recurrence relation $F_n = F_{n-1} + F_{n-2}$, $F_0 = 0$ and $F_1 = 1$

ii. Creates a recursion tree of height n

iii. Each level, L, has at most 2^L sub problems

iv. $T(n) = T(n-1) + T(n-2) \rightarrow O(2^n)$

B. Dynamic Programming

i. Store the previously calculated numbers in an array

ii. Add new numbers to the array by summing the last 2 elements only

iii. 1 for loop of n – 1 elements → O(n)

4. EXPERIMENTS

A. Implement both algorithms such that run times are reported in milliseconds

B. Graph run times on y axis, value of n on x axis

C. Run times will be very large for large values of n with recursion, but possibly imperceptible with dynamic programming

D. Data type size limitations may prevent large values of n from being found

E. Special implementations of the dynamic programming algorithm may be needed to observe changes in run time

5. PROGRAMMING LANGUAGE

A. Java

6. SOURCES

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