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nth Fibonacci number

# description

## The Fibonacci numbers are a sequence of integers defined by the recurrence relation Fn = Fn-1 + Fn-2, where Fn is the nth number in the series and F0 = 0 and F1 = 1. This sequence appears frequently in mathematics, computer science, and even biology, and has been described in mathematical texts for centuries.

# applications

## Golden Ratio – Fn /Fn-1 approaches φ as n approaches ∞

## Fibonacci heap – data structure for priority queues

## Hilbert’s Tenth Problem – Fibonacci numbers used to show unsolvability

## Bee ancestry – bee reproduction creates an unusual number of ancestors

## Brock-Mirman model – a generalized sequence is used in an optimal control function

## Fibonacci Quarterly & the Fibonacci Association – publishing scholarly work since 1963

# Competing algorithms

## Recursion

### Directly implement recurrence relation Fn = Fn-1 + Fn-2, F0 = 0 and F1 = 1

### Creates a recursion tree of height n

### Each level, L, has at most 2L sub problems

### T(n) = T(n-1) + T(n-2) 🡪 O(2n)

## Dynamic Programming

### Store the previously calculated numbers in an array

### Add new numbers to the array by summing the last 2 elements only

### 1 for loop of n – 1 elements 🡪 O(n)

# experiments

## Implement both algorithms such that run times are reported in milliseconds

## Graph run times on y axis, value of n on x axis

## Run times will be very large for large values of n with recursion, but possibly imperceptible with dynamic programming

## Data type size limitations may prevent large values of n from being found

## Special implementations of the dynamic programming algorithm may be needed to observe changes in run time

# programming language

## Java

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