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nth Fibonacci number

# description

## The Fibonacci numbers are a sequence of integers defined by the recurrence relation Fn = Fn-1 + Fn-2, where Fn is the nth number in the series and F0 = 0 and F1 = 1. This sequence appears frequently in mathematics, computer science, and even biology, and has been described in mathematical texts for centuries.

# applications

## Golden Ratio – Fn /Fn-1 approaches φ as n approaches ∞

## Fibonacci heap – data structure for priority queues

## Hilbert’s Tenth Problem – Fibonacci numbers used to show unsolvability

## Bee ancestry – bee reproduction creates an unusual number of ancestors

## Brock-Mirman model – a generalized sequence is used in an optimal control function

## Fibonacci Quarterly & the Fibonacci Association – publishing scholarly work since 1963

# Competing algorithms

## Recursion

### Directly implement recurrence relation Fn = Fn-1 + Fn-2, base cases F0 = 0 and F1 = 1

### Creates a recursion tree of height n where each level, L, has at most 2L sub problems

### T(n) = T(n-1) + T(n-2) 🡪 O(2n)

## Dynamic Programming

### Store the previously calculated Fn-1, Fn-2 in an array, starting with F0 = 0 and F1 = 1

### Add Fn to the array by summing the top 2 elements only

### 1 for loop of n – 1 elements 🡪 O(n)

# experiments

## Direct comparison

### Because the runtimes of the recursive method grow exponentially, the two algorithms can only be directly compared for relatively small values of n. The graph below shows the two algorithms’ runtimes in milliseconds from n = 1 up to n = 50. The recursive calculation has a theoretical runtime of O(2n), while dynamic programming has a theoretical runtime of O(n). Experimental runtimes are shown on the graph below.

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## Recursive calculation

### Fibonacci numbers up to F50 (n = 50) were calculated and the runtime for each calculation was recorded in milliseconds. For theoretical runtimes, F0 and F1 are assumed to take 2n = 1 and 2ms, respectively. By comparing the experimental runtime to the theoretical runtime, the hidden constant c = (experimental RT / theoretical RT) was calculated. This was then used to bound the experimental runtime, such that cmin2n <= experimental runtime <= cmax2n. This function was then graphed for n = 1 to 50.

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## Dynamic Programming calculation

### The same methods were applied to the dynamic programming algorithm, but because runtimes grew slowly, change would not be evident unless very large values of n were used. In this case, Fibonacci numbers up to F150,000 (n = 150,000) were calculated, starting at n = 3,000 and stepping by 3,000, giving a total of 50 values. For theoretical runtimes, F3,000 and F6,000 are assumed to take n = 3,000 and 6,000ms, respectively. All other calculation and graphing methods are the same as those used for the recursive calculation, however Fibonacci values are thousands of digits long, so they are truncated in the table.

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# programming language

## Java

### For handling extremely large numbers, the Long and BigInteger data types must be used

### Output to console is too cumbersome, results are saved in a .csv file

### Runtimes are determined by taking the current time before and after the calculation, then recording the difference

### Smallest runtime granularity is milliseconds

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