

# Dispersive Quantum Interface for Nanofiber-Trapped Atoms: QND Measurement and Spin Squeezing

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## dispersive interaction: phase shift

An off-resonant probe of detuning  $\Delta$  induces an atomic dipole  $\mathbf{d}(\mathbf{r}') = \hat{\alpha} \cdot \mathbf{E}_0(\mathbf{r}')$

the re-radiated field interferes with the probe, producing a **phase shift**

$$\delta\phi_{\{H,V\}} = \frac{\Gamma_{\{H,V\}}^{1D}}{\Delta}$$

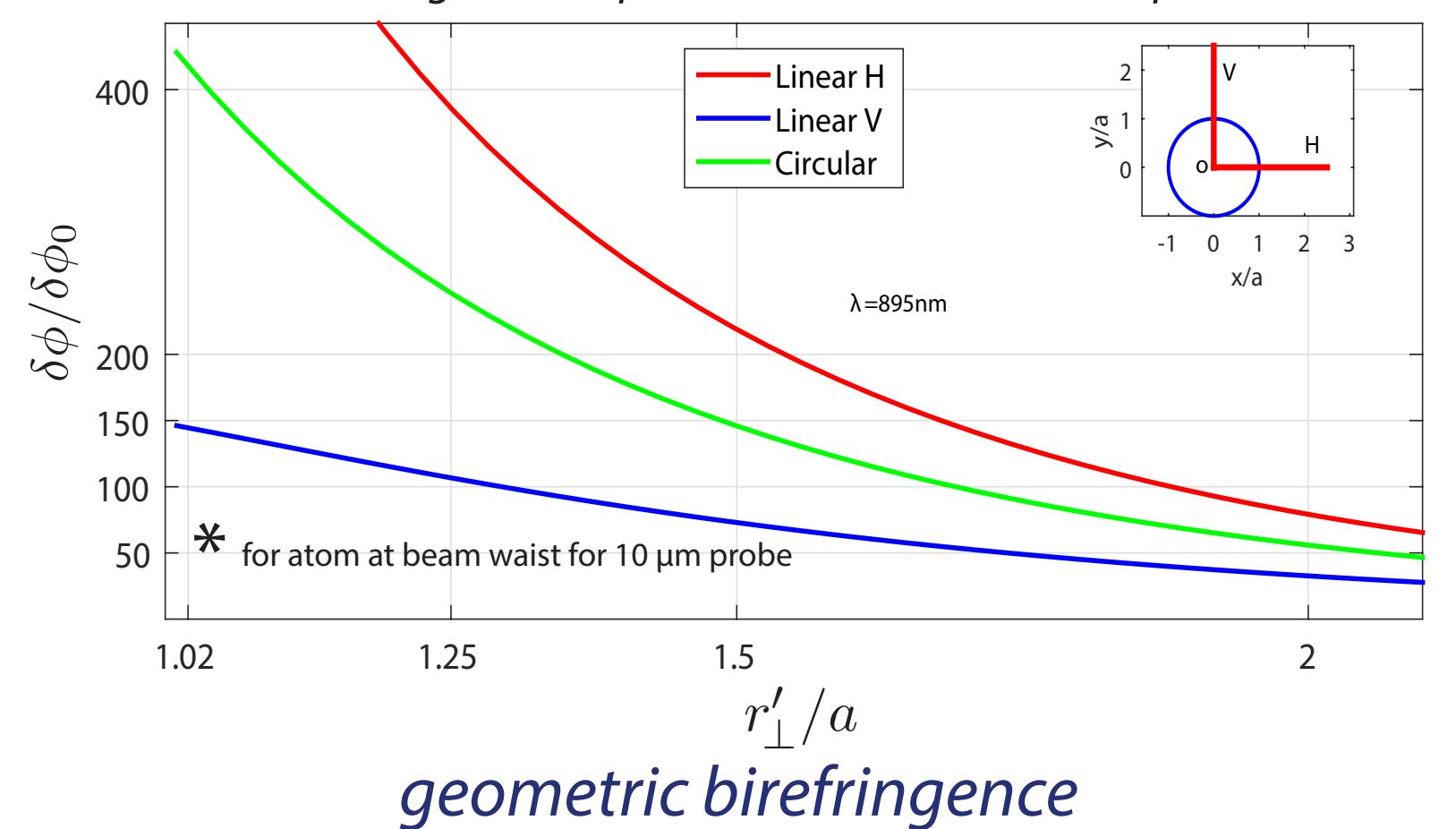
that depends on the coupling rate to the guided modes H and V:

$$\Gamma_{\{H,V\}}^{1D} = 2\pi \frac{\omega_{eg}}{v_g} |\mathbf{d}_{eg} \cdot \mathbf{u}_{\{H,V\}}(\mathbf{r}')|^2$$

**Comparing to free space phase shift:**

$$\frac{\delta\phi_{fiber}}{\delta\phi_{vac}} = \frac{A_{in}^{vac}}{A_{in}^{fiber}} = A_{in} |\mathbf{u}_{in}^{fiber}(\mathbf{r}'_\perp)|^2$$

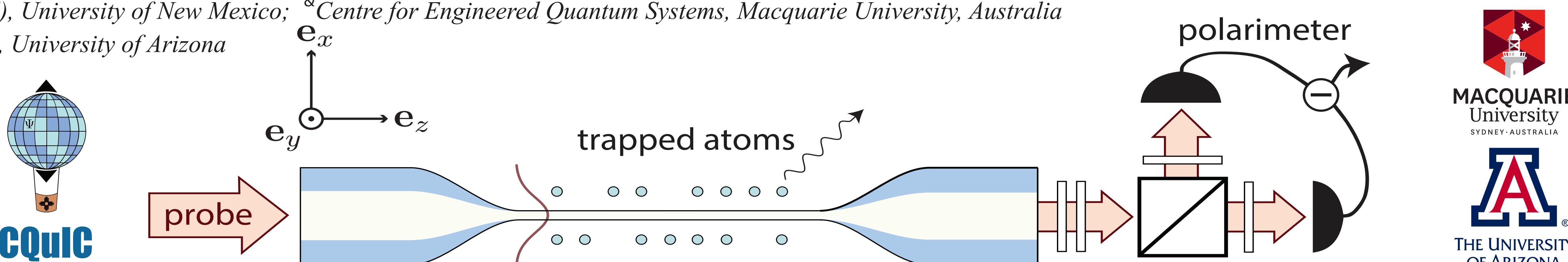
single-atom phase shift relative to free-space\*



even a scalar polarizable particle produces position-dependent

**birefringence** given by the differential phase shift

$$\delta\phi_H - \delta\phi_V = \sigma_0 \frac{c}{v_g} \frac{\Gamma_0}{4\Delta} (|\mathbf{u}_H(\mathbf{r}')|^2 - |\mathbf{u}_V(\mathbf{r}')|^2)$$



## quantum interface on the atomic clock states

off-resonant probe dispersively couples the **collective atomic pseudospin**

$$\left| \uparrow \right\rangle \equiv |F=4, m_f=0\rangle \quad \left| \downarrow \right\rangle \equiv |F=3, m_f=0\rangle \quad \begin{cases} \text{ground hyperfine clock states} \\ \text{to the polarization (Stokes operator) of the guided modes} \end{cases}$$

$$\hat{H}_{J_3} = \Delta\chi \hat{J}_3 \hat{S}_0 + \chi_{J_3} \hat{J}_3 \hat{S}_1$$

$$\hat{J}_3 = \frac{1}{2} \sum_{n=1}^{N_A} (|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|)^{(n)} \quad \hat{S}_1 = \frac{1}{2} (\hat{a}_H^\dagger \hat{a}_H - \hat{a}_V^\dagger \hat{a}_V)$$

the coupling strengths:

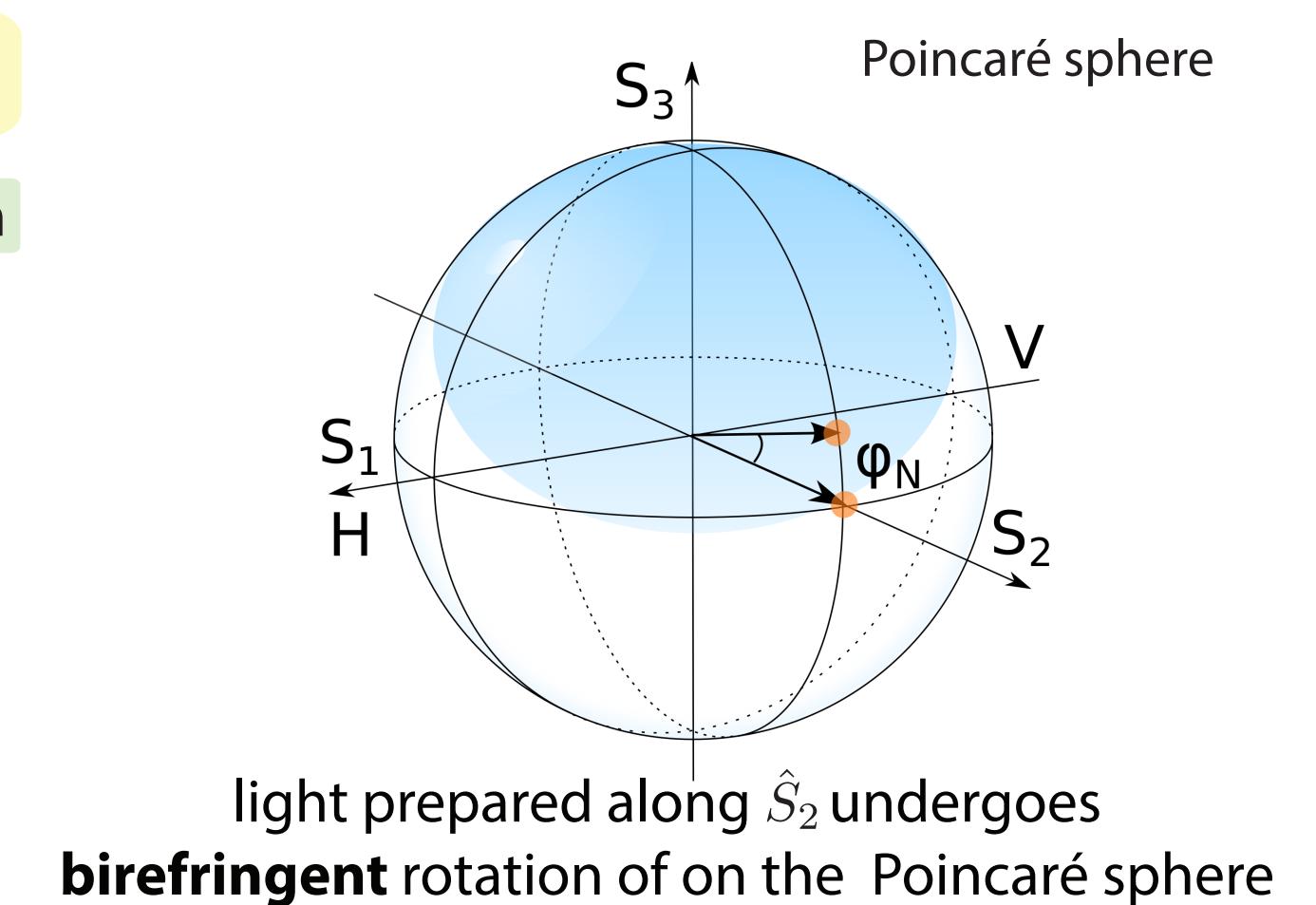
$$\chi_{J_3} = (\chi_{H,\uparrow} - \chi_{H,\downarrow}) + (\chi_{V,\uparrow} - \chi_{V,\downarrow})$$

$$\Delta\chi = (\chi_{H,\uparrow} - \chi_{H,\downarrow}) - (\chi_{V,\uparrow} - \chi_{V,\downarrow})$$

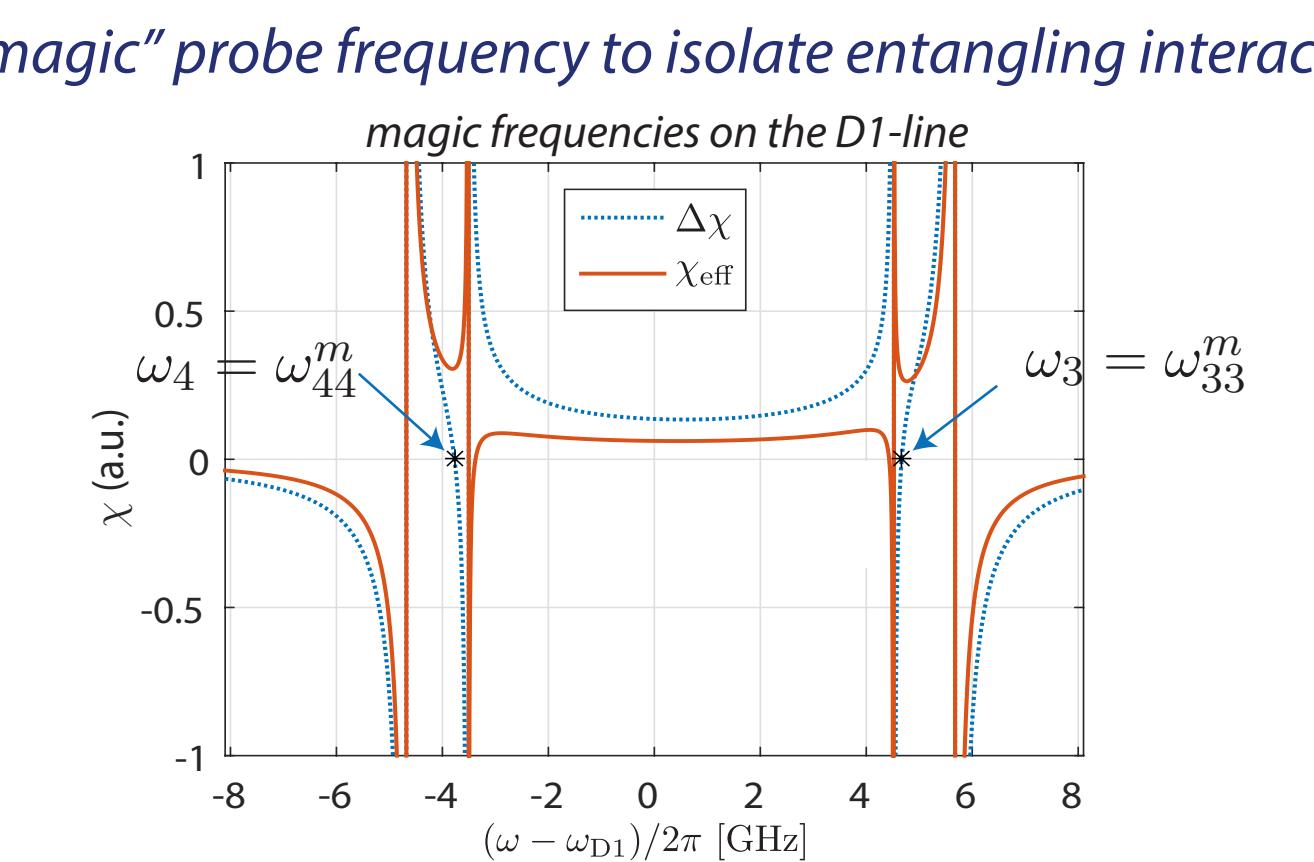
depend on the atomic position and its internal state

$$\chi_{H,\uparrow} = -\frac{2\pi\omega_0}{v_g} \langle \uparrow | \mathbf{u}_H^*(\mathbf{r}') \cdot \hat{\alpha} \cdot \mathbf{u}_H(\mathbf{r}') | \uparrow \rangle$$

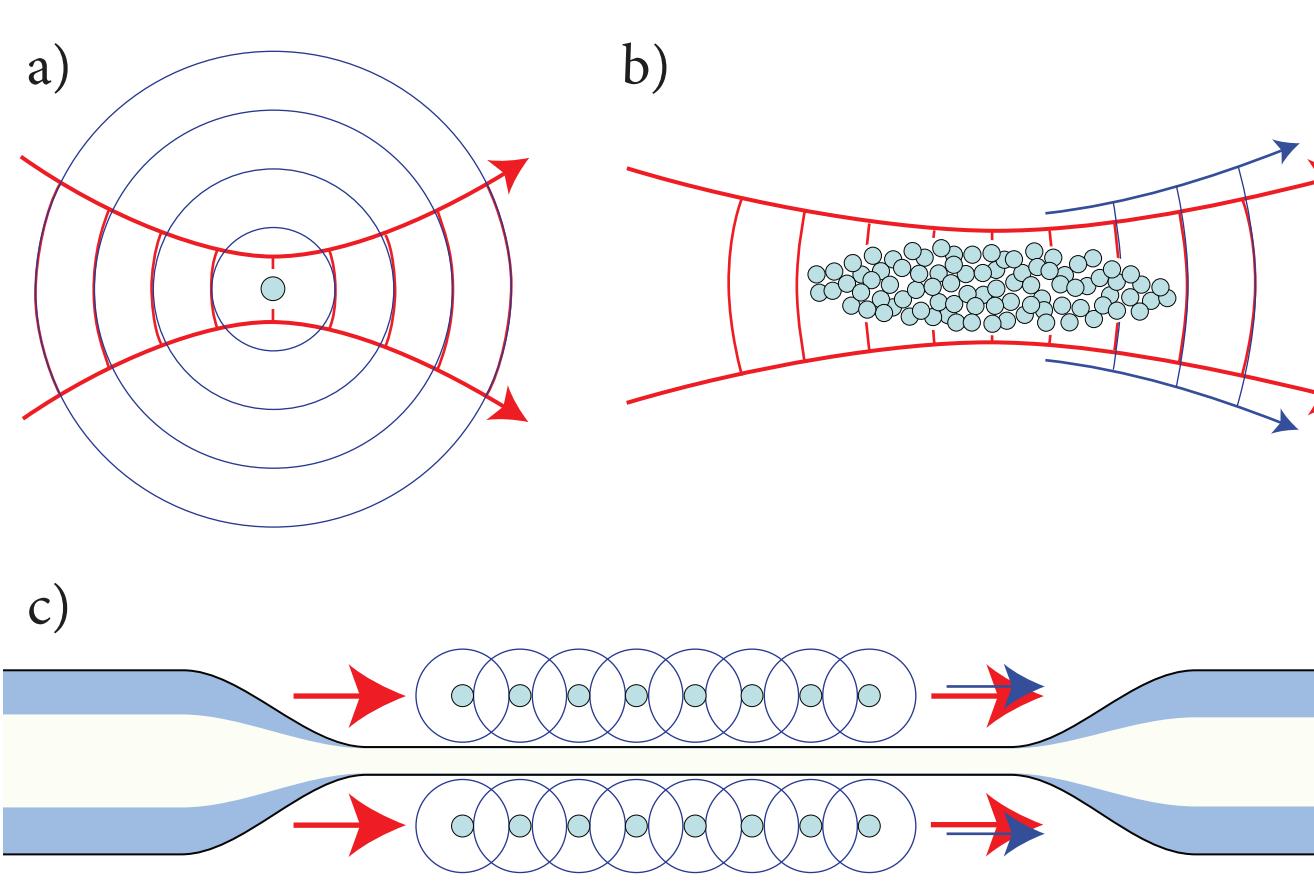
atomic polarizability tensor



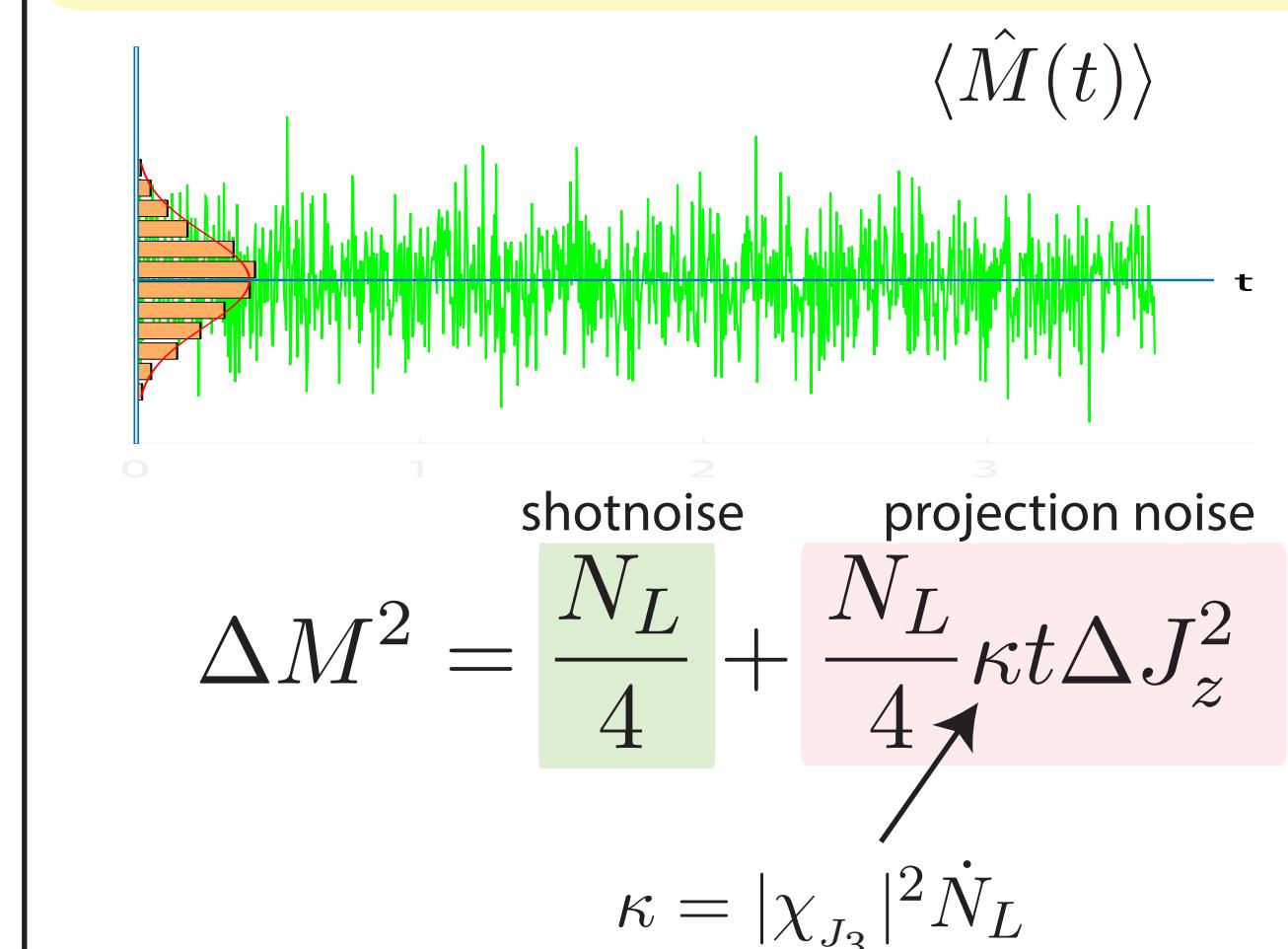
light prepared along  $S_2$  undergoes birefringent rotation of  $\phi_N$  on the Poincaré sphere



mode-matching in different atomic systems



## shotnoise-limited atom number detection



the resolution of an atom number measurement is set by the fluctuations in the measurement of  $\hat{S}_3$ .

When the **atomic projection noise** is equal to the **shotnoise**:

$$\Delta M_{PN}^2 = \Delta M_{SN}^2$$

the minimum detectable atom number is

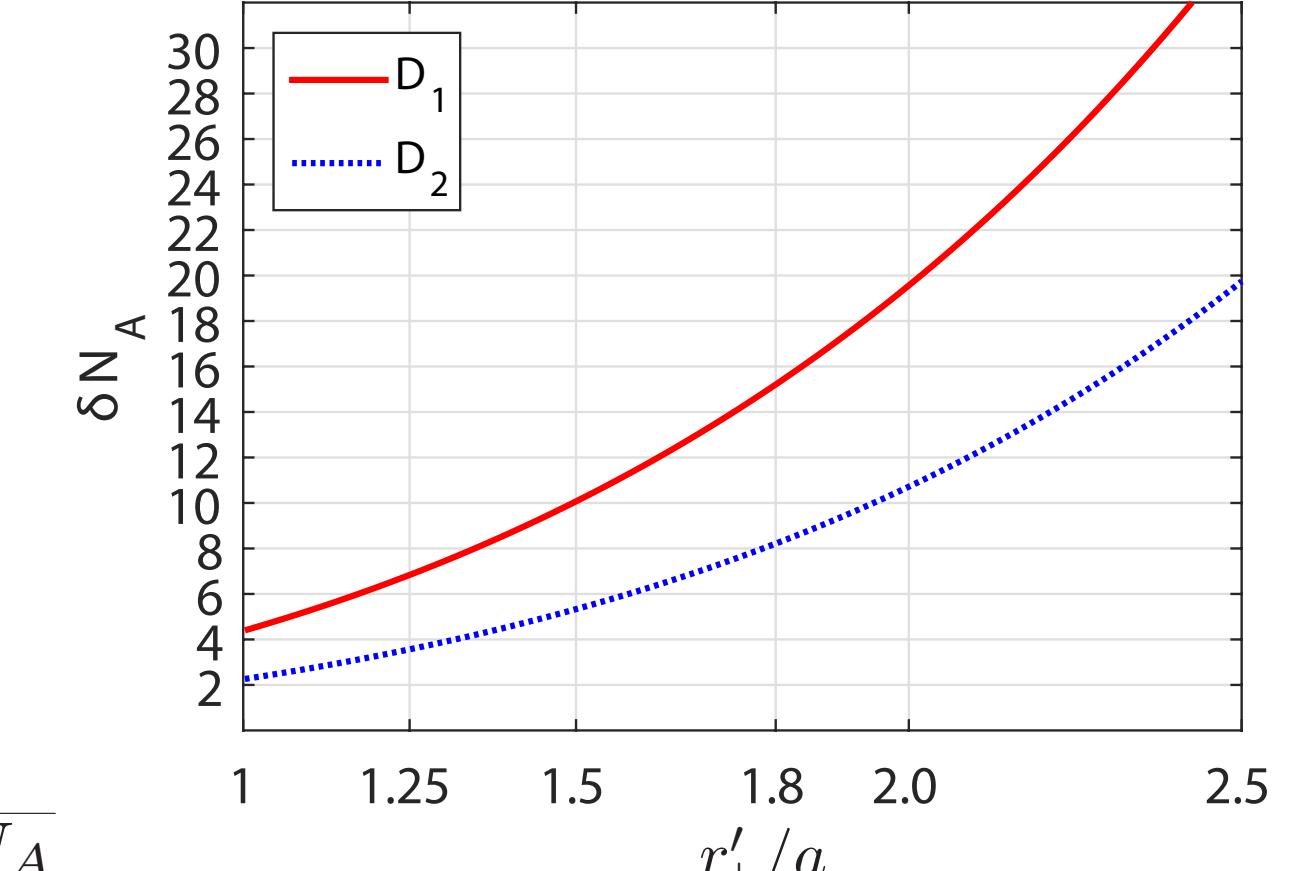
$$\delta N_A \sim \frac{1}{C_j^{(0)}} \sqrt{\frac{A_N^2}{A_{in}\sigma_0}}$$

$\kappa = |\chi_{J_3}|^2 \dot{N}_L$

measurement strength

Two effective mode areas:

$$A_{in} \equiv \frac{1}{n_g} |\mathbf{u}_in(\mathbf{r}'_\perp)|^2, \quad A_N \equiv \frac{2}{n_g} (|\mathbf{u}_H(\mathbf{r}'_\perp)|^2 + |\mathbf{u}_V(\mathbf{r}'_\perp)|^2)$$



## Heisenberg-Langevin equations and input-output relationships

$$\begin{aligned} \frac{d}{dt} \hat{a}_\mu &= -i\omega \hat{a}_\mu + i \sum_{e,g} g_{\mu,e,g}^* \hat{\sigma}_{ge}, & \frac{d}{dt} \hat{a}_\nu &= -i\omega \hat{a}_\nu + i \sum_{e,g} g_{\nu,e,g}^* \hat{\sigma}_{ge}, \\ \frac{d}{dt} \hat{\sigma}_{ge} &= -i\omega_{eg} \hat{\sigma}_{ge} + i \int_0^\infty d\omega \sum_{e',g'} (\delta_{ee'} \hat{\sigma}_{gg'} - \delta_{gg'} \hat{\sigma}_{e'e}) \left[ \sum_{b,p} g_{\mu,e',g'} \hat{a}_\mu + \sum_{m,p} \int_{-kn_2}^{kn_2} d\beta g_{\nu,e',g'} \hat{a}_\nu \right] \end{aligned}$$

Input-output relationships of the guided mode photon creation operators

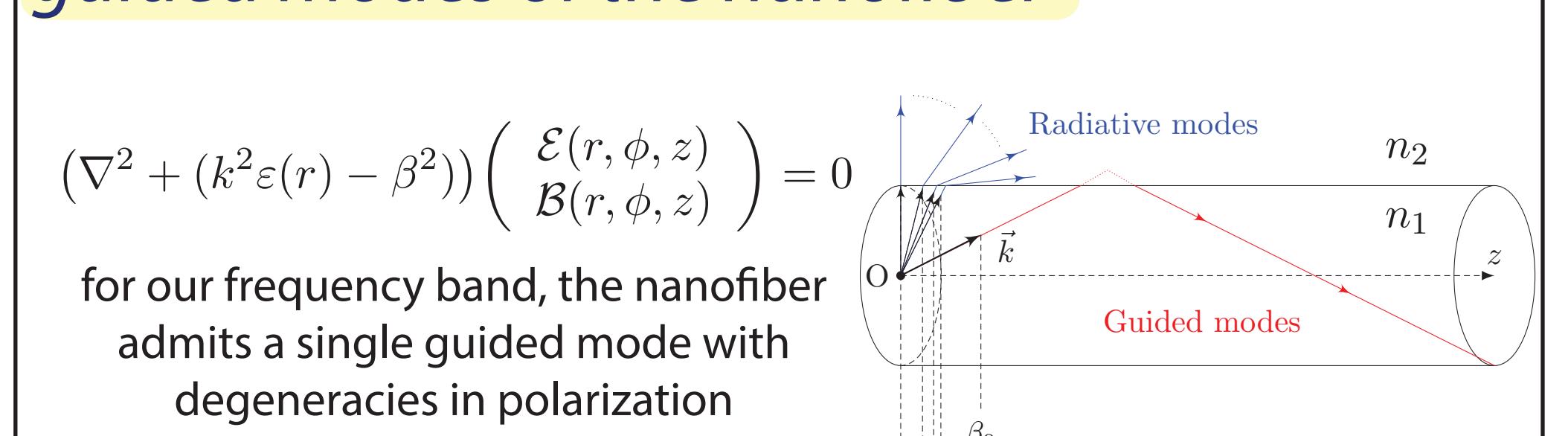
$$\hat{a}_\mu^{out}(\omega) = \hat{a}_\mu^{in}(\omega) - 2\pi i \sum_{b,p} \sum_{e,g} \hat{\sigma}_{ge} \frac{g_{\mu,e,g}^* g_{\mu',e',g'}}{\omega - \omega_{eg} + i\Gamma_e/2} \hat{a}_{\mu'}^{in}(\omega)$$

Phase-shift for a given guided mode

$$\delta\phi_{p,g} = -\sum_e \frac{\Gamma_p^e}{\Delta_{eg}}$$

$$\Gamma_p^e = \frac{2}{\hbar} \sum_g \langle g | \hat{d} | e \rangle \cdot \text{Im} [\mathbf{G}_{pp}^{(+)}(\mathbf{r}', \mathbf{r}'; \omega_{eg})] \cdot \langle e | \hat{d} | g \rangle$$

## guided modes of the nanofiber

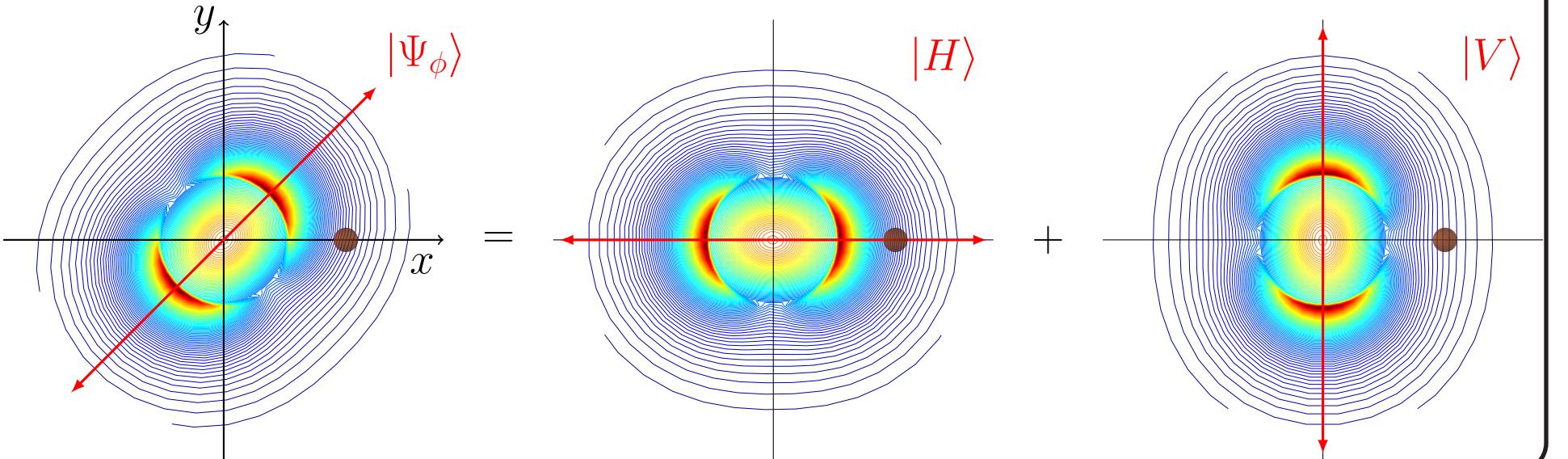


### quasilinear guided modes

placing the atom at  $\phi = 0$ , the two "quasilinear" forward-propagating guided modes are

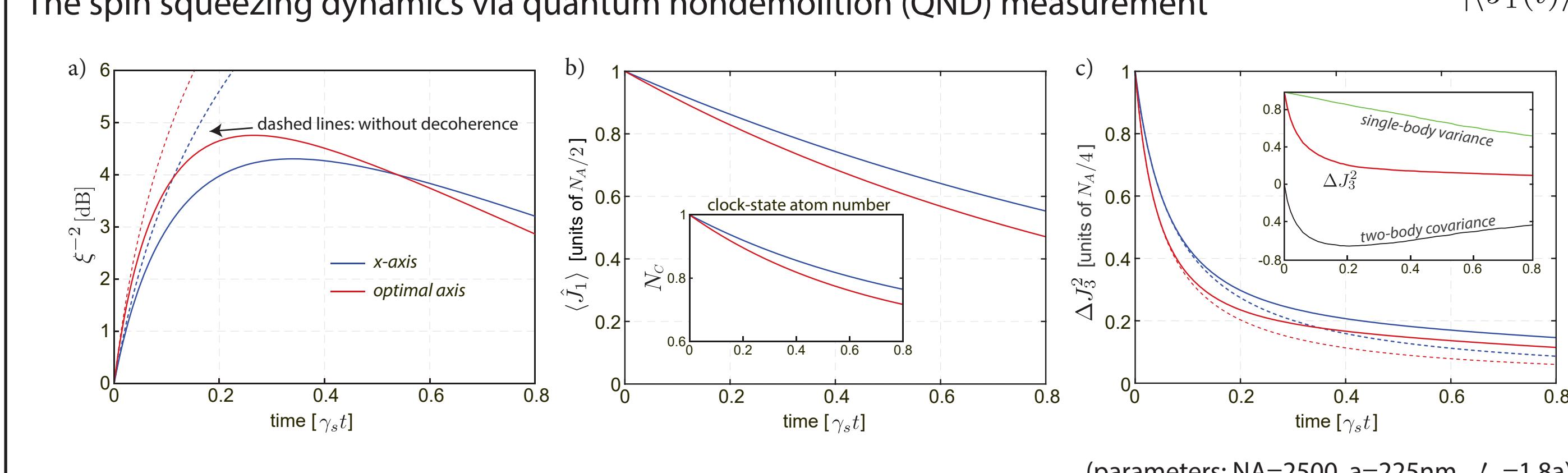
$$\mathbf{u}_{b,H}(r_\perp, \phi = 0) = \sqrt{2} [\mathbf{e}_x u_r(r_\perp) + i \mathbf{e}_z u_z(r_\perp)]$$

$$\mathbf{u}_{b,V}(r_\perp, \phi = 0) = \sqrt{2} \mathbf{e}_y u_\phi(r_\perp)$$



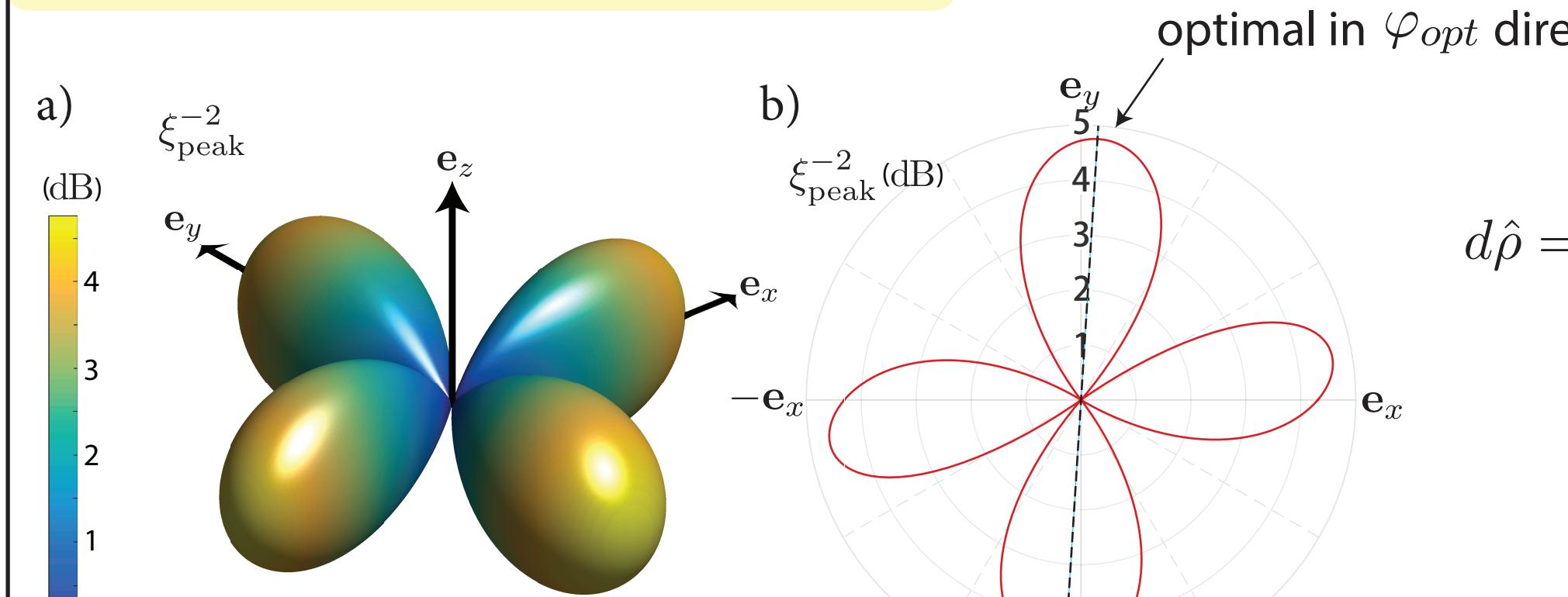
## spin squeezing via QND measurement

The spin squeezing dynamics via quantum nondemolition (QND) measurement



(parameters: NA=2500, a=225nm,  $r'_\perp=1.8a$ )

The optimal choice of quantization axis



## squeezing parameter:

$$\xi^2 = N_A \frac{\Delta J_3^2(t)}{|\langle \hat{J}_1(t) \rangle|}$$

with  $s = \text{sign}(\chi_{J_3})$ .

Alternative expressions of **cooperativity** or **Purcell enhancement factor**

$$\text{cavities: } C \equiv \frac{2g^2}{\kappa_c \Gamma} = \mathcal{F}_A^{\sigma_0} = \mathcal{F}_A^{\Gamma_{1D}}$$

$$\text{nano-fiber: } C = N_A \frac{\Omega_D}{N_A} = N_A \frac{\Gamma_{1D}}{\Gamma_0}$$

Other characteristic parameters

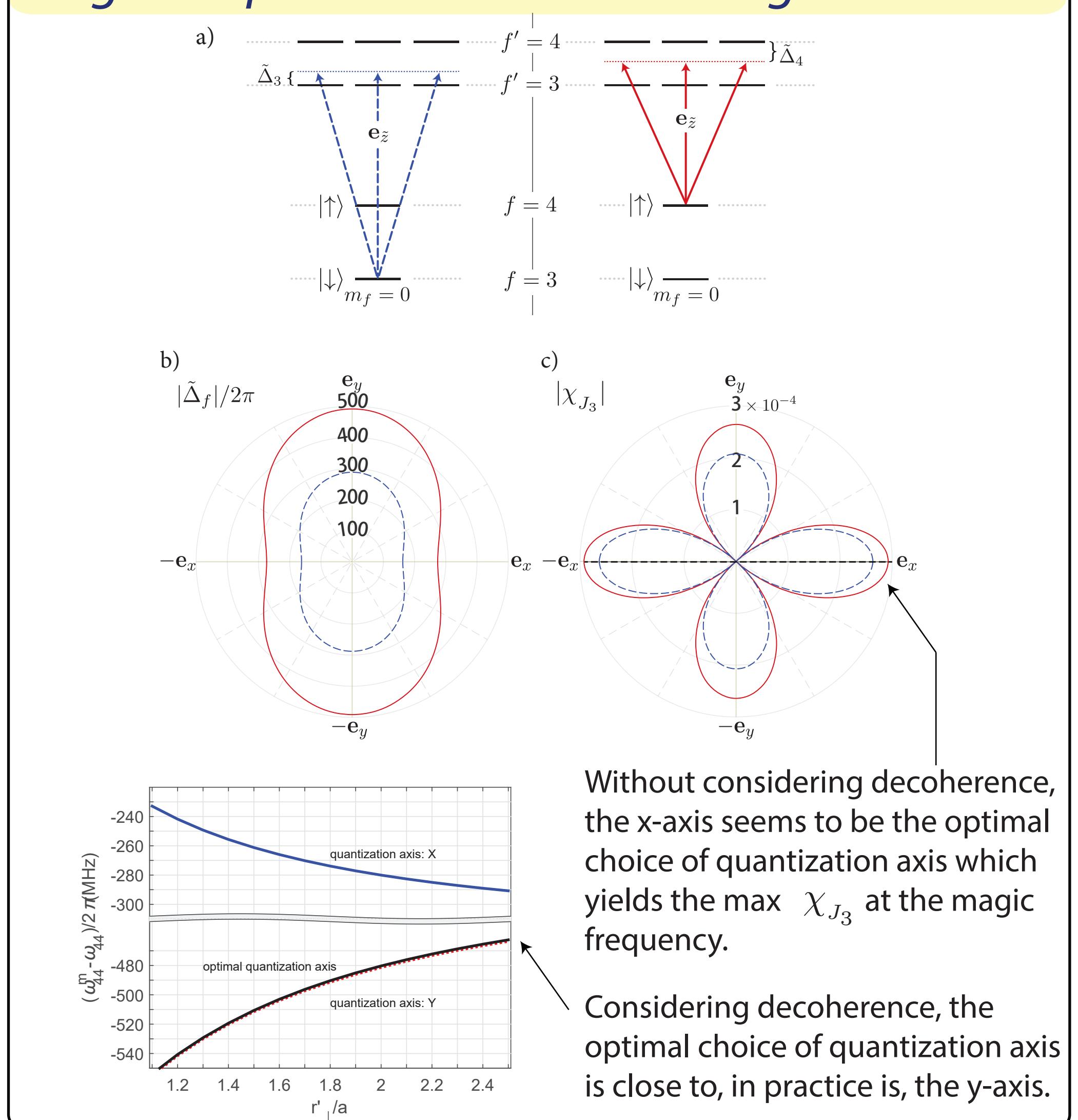
$$\text{coupling strength } \chi_{J_3} = \frac{\sigma_0}{A_{J_3}} \frac{\Gamma}{2\Delta_{J_3}}$$

$$\text{OD/NA} = \frac{\kappa}{\gamma_s} = \frac{\sigma_0 A_{in}}{A_{J_3}^2}$$

$$\text{photon scattering rate } \gamma_s = \frac{\Gamma \Omega^2}{4\Delta_{J_3}^2} = \frac{\sigma_0}{A_{in}} \frac{\Gamma^2}{4\Delta_{J_3}^2} \dot{N}_L$$

$$\frac{1}{A_{J_3}} = n_g \frac{|\mathbf{e}_z \cdot \mathbf{u}_H(\mathbf{r}')|^2 |\mathbf{u}_V(\mathbf{r}')|^2 - |\mathbf{e}_z \cdot \mathbf{u}_V(\mathbf{r}')|^2 |\mathbf{u}_H(\mathbf{r}')|^2}{|\mathbf{u}_H(\mathbf{r}')|^2 + |\mathbf{u}_V(\mathbf{r}')|^2}$$

## magic frequencies for the birefringence effect



Without considering decoherence, the x-axis seems to be the optimal choice of quantization axis which yields the max  $\chi_{J_3}$  at the magic frequency.

Considering decoherence, the optimal choice of quantization axis is close to, in practice is, the y-axis.

## Green's function for the nanofiber

The Green's function is a tensor that characterizes the **scattering** from a point source

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) + \hat{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{d}_{eg}$$

Expression of the guided mode dyadic Green's function:

$$\hat{\mathbf{G}}_g(\mathbf{r}, \mathbf{r}'; \omega_0) = 2\pi i \frac{\omega_0}{v_g} \sum_{b,p} \mathbf{u}_{b,p}(\mathbf{r}_\perp) \mathbf{u}_{b,p}^*(\mathbf{r}'_\perp) e^{ib\beta_0(z-z')} \Theta(b(z-z'))$$

sum over all propagation directions, b, and polarizations, p.

The imaginary part of the guided-mode Green's function tensor is

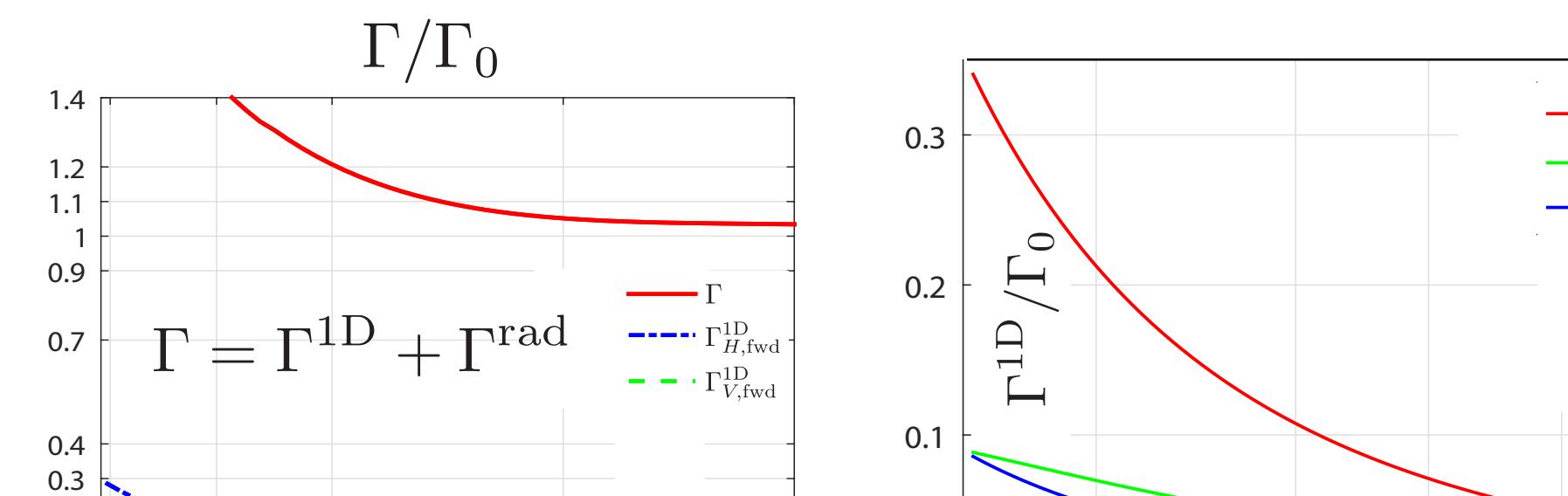
$$\text{Im} [\hat{\mathbf{G}}_g(\mathbf{r}', \mathbf{r}')] = \pi \frac{\omega_{eg}}{v_g} \sum_{b,p} \mathbf{u}_{b,p}(\mathbf{r}') \mathbf{u}_{b,p}^*(\mathbf{r}')$$

## modification of the atomic decay rate

the **total decay rate** of an atom is modified by the presence of the dielectric nanofiber

$$\Gamma \propto \text{Im} [\mathbf{d}_{eg} \cdot \hat{\mathbf{G}}^*(\mathbf{r}', \mathbf{r}') \cdot \mathbf{d}_{eg}^*]$$

$$\Gamma/\Gamma_0$$



the **decay rate into the guided modes**  $\Gamma^{1D}$  depends on the orientation of the dipole

## emission surfaces

show relative guided-mode decay rate as dipole orientation changes



## References:

- [1] X. Qi, B. Q. Baragiola, P. S. Jessen, and I. H. Deutsch, Phys. Rev. A **93**, 023817 (2016)
- [2] S. Dawkins, R. Mitsch, D. Reitz, E. Vetsch, and A. Rauschenbeutel, Phys. Rev. Lett. **107**, 243601 (2011)
- [3] I. Deutsch and P. Jessen, Opt. Comm. **283**, 681 (2010)

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# *Quantum Interface for Nanofiber-Trapped Atoms: QND Measurement and Spin Squeezing*

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