No-cloning theorem

- Suppose that there exists a (joint) unitary evolution that copies an arbitrary state $|\psi\rangle$ to an empty "scratchpad" $U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$

  But by linearity, $U(|\psi\rangle + |\varphi\rangle) \otimes |0\rangle = |\psi\rangle|\psi\rangle + |\varphi\rangle|\varphi\rangle$. This is not what we want ! What we want is $(|\psi\rangle + |\varphi\rangle) \otimes (|\psi\rangle + |\varphi\rangle)$. Thus, there doesn't exist such a copying unitary. $\square$

Quantum operations

Now that we have seen that the notion of state vector should be replaced by a more general description of a density operator, this opens up the possibility of dynamics that are not unitary because, after all, unitary dynamics can only bring pure states to pure states; there must be a more general description of dynamics when a system evolves as a part of a larger, entangled system.

$$\begin{tikzcd}
{|\psi\rangle_{AB}} \arrow[r, "U"] \arrow[d, "\text{Partial trace}"'] & {|\psi'\rangle_{AB}} \arrow[d] \\
\rho_A \arrow[r, "?"'] & \rho_A'
\end{tikzcd}$$

Example

$$\begin{tikzcd}
{|+0\rangle} \arrow[r, "CNOT"] \arrow[d] & {\frac{|00\rangle+|11\rangle}{\sqrt{2}}} \arrow[d] \\
{|+\rangle\langle+|} \arrow[r, "?"'] & \frac{\mathbb{1}}{2}
\end{tikzcd}$$

How can this lose of coherence, or decoherence, be described at the level of a subsystem?

ONB $\{|f_\alpha\rangle\}_\alpha$ for $\mathcal{H}_B$

Partial inner product

$$\rho_A' = \mathrm{tr}_B\left(U \rho_A \otimes |0\rangle_B\langle 0| U^\dagger\right) = \sum_\alpha {}_B\langle f_\alpha | U |0\rangle_B \, \rho_A \, {}_B\langle 0| U^\dagger |f_\alpha\rangle_B$$

$$= \boxed{\sum_\alpha K_\alpha \rho_A K_\alpha^\dagger} \qquad \boxed{\sum_\alpha K_\alpha^\dagger K_\alpha = \mathbb{1}}$$

(Kraus operators)

$$\sum_\alpha K^\dagger K = \sum_\alpha {}_B\langle 0| U^\dagger |f_\alpha\rangle_B \langle f_\alpha | U |0\rangle_B$$

$$= {}_B\langle 0| U^\dagger U |0\rangle_B = {}_B\langle 0| \mathbb{1}_{AB} |0\rangle_B = \mathbb{1}_A \langle 0| \mathbb{1}_B |0\rangle = \mathbb{1}_A$$

Measurement: Suppose that we looked at system B and saw one of the $|f_\alpha\rangle$

$$\Pr(\alpha) = \mathrm{tr}\left[\mathbb{1} \otimes |f_\alpha\rangle\langle f_\alpha| U \rho_A \otimes |0\rangle_B\langle 0| U^\dagger\right]$$

$$= \mathrm{tr}_A\left[\langle f_\alpha | U |0\rangle \rho_A \langle 0| U^\dagger |f_\alpha\rangle\right] = \mathrm{tr}\left(K_\alpha \rho_A K_\alpha^\dagger\right)$$

POVM/Generalized measurement

$$= \mathrm{tr}(K_\alpha^\dagger K_\alpha \rho) =: \mathrm{tr}(E_\alpha \rho) \qquad \boxed{\{E_\alpha\}_\alpha \ \forall E_\alpha \geq 0, \ \sum_\alpha E_\alpha = \mathbb{1}}$$

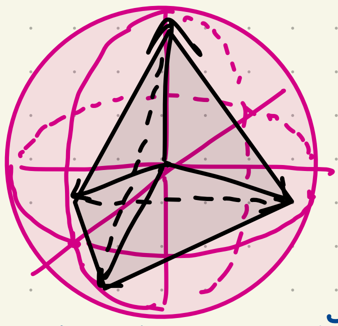
Notice that this is a positive operator

POVMs = Positive-operator-valued measures are so named as to contrast to PVM projector-valued measures / projective measurements (measurements of observables)

Comments

① A POVM may have more outcomes than the dimension of the Hilbert space, something that is not possible with standard measurements.

Tetrahedron measurement, SIC-POVM



$$|\psi_1\rangle = |0\rangle$$

$$|\psi_2\rangle = \frac{|0\rangle + \sqrt{2}|1\rangle}{\sqrt{3}}$$

$$|\psi_3\rangle = \frac{|0\rangle + e^{2\pi i/3}\sqrt{2}|1\rangle}{\sqrt{3}}$$

$$|\psi_4\rangle = \frac{|0\rangle + e^{4\pi i/3}\sqrt{2}|1\rangle}{\sqrt{3}}$$

$$E_j = |\psi_j\rangle\langle\psi_j|$$

② POVMs only give measurement statistics and not the post-measurement states. The reason is that many quantum dynamics can give rise to the same POVM.

Given a POVM $\{E_\alpha\}$, we can define their positive square roots to be Kraus operators $E_\alpha = \sum_j \lambda_{\alpha j} |e_{\alpha j}\rangle\langle e_{\alpha j}| \Rightarrow K_\alpha = \sum_j \sqrt{\lambda_{\alpha j}}\, |e_{\alpha j}\rangle\langle e_{\alpha j}| = \sqrt{E_\alpha}$

Then
$$\sum_\alpha K_\alpha^\dagger K_\alpha = \sum_{\alpha j k} \sqrt{\lambda_{\alpha j}\lambda_{\alpha k}}\, |e_{\alpha j}\rangle\langle e_{\alpha j}|e_{\alpha k}\rangle\langle e_{\alpha k}|$$
$$= \sum_{\alpha j} \lambda_{\alpha j} |e_{\alpha j}\rangle\langle e_{\alpha j}| = \sum_\alpha E_\alpha = \mathbb{1}$$

However, there is a unitary freedom we have ignored: $K_\alpha = U\sqrt{E_\alpha}$ works equally well for any arbitrary unitary U since $K_\alpha^\dagger K_\alpha = \sqrt{E_\alpha}\, U^\dagger U \sqrt{E_\alpha} = E_\alpha$.

In fact, this ambiguity readily appears in what we thought are "projective measurements"

$|0\rangle$ = no photon

$|1\rangle$ = one photon

$|1\rangle \longrightarrow D\sim$ Photon counting absorbs and destroy photon so the post-measurement state is not $|1\rangle$ but the vacuum $|0\rangle$

In other words, the Kraus operators are $\{|0\rangle\langle 0|, |0\rangle\langle 1|\}$ not $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$

$= X|1\rangle\langle 1|$

$$K_0^\dagger K_0 + K_1^\dagger K_1 = |0\rangle\langle 0|0\rangle\langle 0| + |1\rangle\langle 0|0\rangle\langle 1|$$
$$= |0\rangle\langle 0| + |1\rangle\langle 1| = \mathbb{1} \checkmark$$

This may looks strange but K_α needs not even be normal!

The assumption that measurements are repeatable is not always true!

③ von Neumann's model of (noisy) measurements

Suppose we want to measure an observable A of our system $\Rightarrow$ Coupling to a continuous "meter" M, a canonically quantized system with $[x, p] = i\hbar$

$$A = \sum_j \lambda_j |a_j\rangle\langle a_j|$$

$$e^{i\varphi \hat{A}\otimes\hat{p}} |x\rangle|\psi\rangle_M = e^{i\varphi\hat{A}\otimes\hat{p}} \sum_j c_j |a_j\rangle|\psi\rangle$$

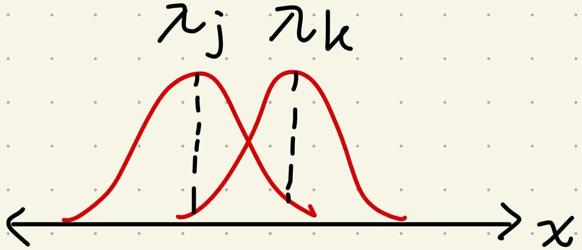
$$= \sum_j c_j \left[1 + i\varphi \hat{A}\otimes\hat{p} + \frac{(i\varphi)^2}{2!} \hat{A}^2\otimes\hat{p}^2 + \cdots \right] |a_j\rangle|\psi\rangle$$

$$= \sum_j c_j |a_j\rangle \left[1 + i\varphi\lambda_j \hat{p} + \frac{(i\varphi\lambda_j)^2}{2!} \hat{p}^2 + \cdots \right] |\psi\rangle$$

$$= \sum_j c_j |a_j\rangle \underbrace{e^{i\varphi\lambda_j p}|\psi\rangle}_{} \xrightarrow[\text{function}]{\text{wave}} \sum_j c_j \psi(x - \varphi\lambda_j)$$

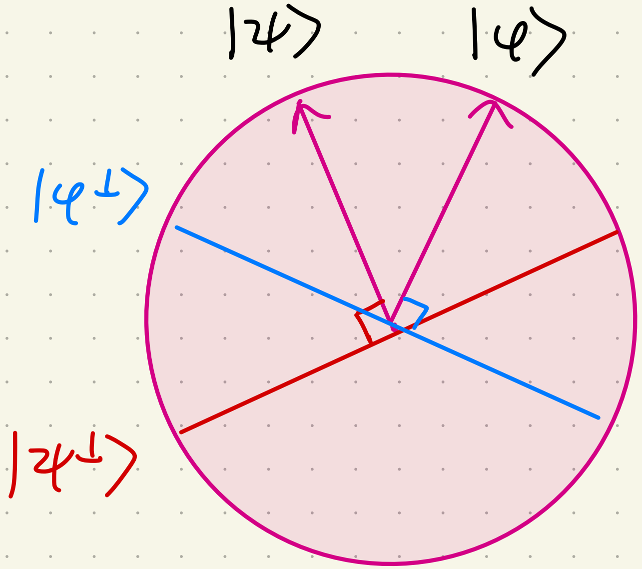
$$\langle x| e^{i\varphi\lambda_j p}|\psi\rangle = \psi(x - \varphi\lambda_j)$$

Say ψ is a Gaussian wave function. The distinguishability of distinct outcomes are limited by the width of the Gaussian.



④ Since E_α is positive and can be fine-grained into rank-one operators, there is a tendency to think of a POVM as just a more noisy quantum measurements. This is incorrect as sometimes a (non-projective) POVM is the optimal measurement.

Unambiguous state discrimination



Suppose that we want to tell apart a pair of non-orthogonal states $|\psi\rangle$ and $|\varphi\rangle$ without ever mistaking one for the other. This seems impossible with standard projective measurements. But armed with POVMs, we can do the following:

$$E_1 = a|\varphi^\perp\rangle\langle\varphi^\perp|, \quad E_2 = b|\psi^\perp\rangle\langle\psi^\perp|$$

If we have the state $|\psi\rangle$, we will never obtain result "2"! So if we obtain "2", we know that we have the state $|\varphi\rangle$ for sure. The price to pay is that we have to add a third outcome $E_3 = \mathbb{1} - E_1 - E_2$ where we are unsure. But this is possible precisely because $E_1 E_2$ can be subnormalized (not projectors).

You will find in the tutorial the constants a, b that make E_3 positive.

- Since the task is symmetric between $|\psi\rangle$ and $|\varphi\rangle$ (we can specify, say, that we are given each state with equal probability), we may choose $a=b$.

$$E_3 = \mathbb{1} - a\left(|\psi^\perp\rangle\langle\psi^\perp| + |\varphi^\perp\rangle\langle\varphi^\perp|\right)$$
$$= (1-2a)\mathbb{1} + a\left(|\psi\rangle\langle\psi| + |\varphi\rangle\langle\varphi|\right)$$

We can assume without loss of generality that $|\psi\rangle$ and $|\varphi\rangle$ are planar and have real inner product. So let us define for convenience, $\Delta = \langle\psi|\varphi\rangle \in \mathbb{R}$

$$\Rightarrow |\varphi\rangle = \langle\psi|\varphi\rangle|\psi\rangle + \langle\psi^\perp|\varphi\rangle|\psi^\perp\rangle = \Delta|\psi\rangle + \sqrt{1-\Delta^2}\,|\psi^\perp\rangle$$

Therefore, $|\psi\rangle\langle\psi| + |\varphi\rangle\langle\varphi| = (1+\Delta^2)|\psi\rangle\langle\psi| + (1-\Delta^2)|\psi^\perp\rangle\langle\psi^\perp|$
$$+\Delta\sqrt{1-\Delta^2}\left(|\psi\rangle\langle\psi| + |\psi^\perp\rangle\langle\psi^\perp|\right)$$
$$= \mathbb{1} + \Delta^2 Z + \Delta\sqrt{1-\Delta^2}\,X \text{ in the appropriate basis.}$$

$$\Rightarrow E_3 = (1-a)\mathbb{1} + a\Delta\underbrace{\left(\Delta Z + \sqrt{1-\Delta^2}\,X\right)}_{\text{This has eigenvalues } \pm 1 \text{, so the eigenvalue of } E_3 \text{ is}}$$

$$1 - a(1\pm\Delta) \Rightarrow a \leq \frac{1}{1\pm\Delta} \Rightarrow a \leq \frac{1}{1+\Delta}$$

□

Completely positive maps

We have seen that the new kind of dynamics is decribed by a set of so-called Kraus operators $\{K_\alpha\}_\alpha$, $\sum_\alpha K_\alpha^\dagger K = \mathbb{1}$ (and no other restriction)

Quantum operation $\rightarrow \mathcal{E}(\rho) = \sum_\alpha K_\alpha \rho K_\alpha^\dagger$ (Kraus representation of a quantum operation)

Are assumptions on the form of the initial state $\rho \otimes |0\rangle\langle 0|$ that lead to the Kraus form reasonable?

- If the ancilla's fiducial state is mixed, one can purify it.
- If the system and the ancilla has pre-existing correlation, it will not be possible in general to write the output $\mathcal{E}(\rho)$ as a function of the system's state ρ (since that can be backflow of information from the ancilla).

An alternative, but equivalent characterization of quantum operations as completely positive (CP) maps also suggests that there are a physically reasonable class of linear maps of density operators.

As they are linear maps that send operators to operators, they are superoperators. The mathematics of superoperators and CP maps (which form a subset of superoperators) are rich and extensive. At a basic level, the theory of CP maps and their Kraus representation have many parallels to that of positive operators and their ensemble decompositions. But rather than going into the general theory, let me just give examples and non-examples of qubit CP maps.

Every qubit quantum operations assigns an operator basis element to a linear combination of the basis elements. $\{\mathbb{1}, X, Y, Z\}$

Some terminologies:

("Not looking at the measurement result")

- A trace-preserving Q op can't map a Pauli to $\mathbb{1}$
- A unital Q op maps $\mathbb{1}$ to $\mathbb{1}$.

Qubit unital = random unitary
Not true in general.

Non-example

$$\mathcal{E}(\mathbb{1}) = \mathbb{1}, \quad \mathcal{E}(X) = X, \quad \mathcal{E}(Y) = -Y, \quad \mathcal{E}(Z) = Z.$$

$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ Transposition / Complex conjugation / "Time reversal"

Positive map $\quad \mathcal{E}(\mathbb{1} + \vec{r}\cdot\vec{\sigma}) = \mathbb{1} + r_x X - r_y Y + r_z Z \geq 0$

Just inverting the y component of the Bloch vector

$$\mathcal{E}\otimes\mathbb{1}(|\Omega\rangle\langle\Omega|) = \frac{1}{2}\mathcal{E}\otimes\mathbb{1}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)$$

$$= \frac{1}{2}\left(|00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 10| + |11\rangle\langle 11|\right)$$

$$|1\rangle\langle 0| \xleftrightarrow{\mathcal{E}} |0\rangle\langle 1|$$

$$|1\rangle\langle 0| = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = X + iY$$

$$|0\rangle\langle 1| = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = X - iY$$

$$= \frac{1}{2}\begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix}$$

X has a negative eigenvalue

Not completely positive (CP)

Partial transpose/time reversal is not a physically reasonable operation

Examples

① Depolarizing channel $\mathcal{E}(\rho) = p\frac{\mathbb{1}}{2} + (1-p)\rho$

Replace the DM with the maximally mixed state with probability p.
The fixed point $(\mathbb{1}/2)$ is invariant under any rotation, so the Kraus operators can be realized by a sufficiently uniform random rotations

$$\int dU\, U\rho U^\dagger = \frac{\mathbb{1}}{2} \Rightarrow \mathcal{E}(\rho) = (1-p)\rho + p\int dU\, U\rho U^\dagger$$

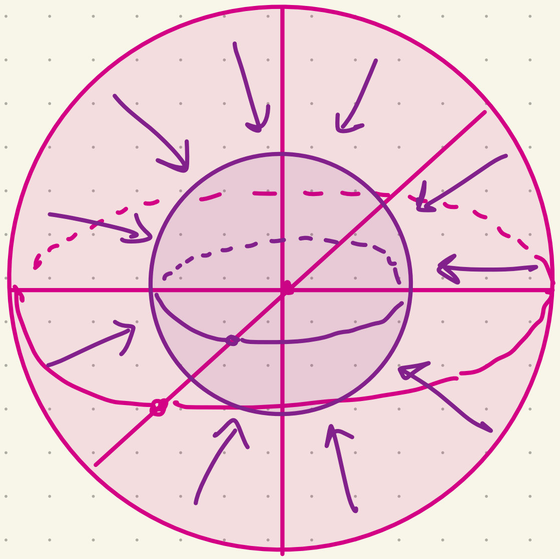
More economical

$$\frac{\rho + X\rho X + Y\rho Y + Z\rho Z}{4} = \frac{\mathbb{1}}{2} \Rightarrow \mathcal{E}(\rho) = \left(1 - \frac{3p}{4}\right)\rho + \frac{p}{4}(X\rho X + Y\rho Y + Z\rho Z)$$

More convenient to scale $\frac{3p}{4} \to p$ $\Rightarrow (1-p)\rho + \frac{p}{3}(X\rho X + Y\rho Y + Z\rho Z)$

$$\Rightarrow K_\alpha = \sqrt{1-p}\,\mathbb{1},\ \sqrt{\frac{p}{3}}\,\sigma_j,\ j=1,2,3$$

Already see non-uniqueness of Kraus operators. In fact, Kraus ops have the "same" freedom as EDs/purifications of DMs



Shrink the Bloch ball isotropically

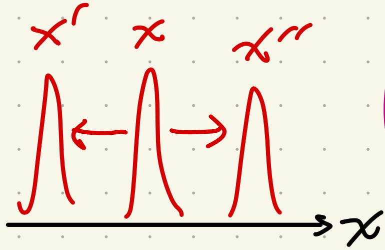
Can't be undone by an operation that deflates the Bloch ball ← Not a positive Œ operation

② Dephasing/phase-damping channel ("How Schrödinger's cat becomes dead OR alive")

There is a non-unitary process that picks out and destroys the coherence in a particular basis. Can be modeled as a von Neumann's measurement

$$|0\rangle_A \mapsto \sqrt{1-p}\,|0\rangle_A |x\rangle_M + \sqrt{p}\,|0\rangle_A |x'\rangle_M$$

$$|1\rangle_A \mapsto \sqrt{1-p}\,|1\rangle_A |x\rangle_M + \sqrt{p}\,|1\rangle_A |x''\rangle_M$$



(Meter state)

A particle interacts with our quantum system and gets kicked depending on the state of our system.
scatters, say

$$\begin{array}{c|cccccc|}
 & 0x & 0x' & 0x'' & 1x & 1x' & 1x'' \\
\hline
0x & \sqrt{1-p} & & & 0 & & \\
0x' & \sqrt{p} & & & 0 & & \\
0x'' & 0 & & & 0 & & \\
1x & 0 & & & \sqrt{1-p} & & \\
1x' & 0 & & & 0 & & \\
1x'' & 0 & & & \sqrt{p} & & \\
\hline
\end{array},$$

$U|x\rangle$

Kraus operators (Tutorial)

$$K_0 = \langle x|U|x\rangle = \begin{pmatrix} \sqrt{1-p} & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}$$

$$K_1 = \langle x'|U|x\rangle = \begin{pmatrix} \sqrt{p} & 0 \\ 0 & 0 \end{pmatrix}$$

$$K_2 = \langle x''|U|x\rangle = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{pmatrix}$$

Not linearly independent (e.g. $\frac{K_0}{\sqrt{1-p}} - \frac{K_1}{\sqrt{p}} = \frac{K_2}{\sqrt{p}}$) $\Rightarrow$ Suggests that the # of Kraus ops. can be reduced

$$\mathcal{E}(\rho) = (1-p)\rho + p\left(\frac{\mathbb{1}+Z}{2}\rho\frac{\mathbb{1}+Z}{2} + \frac{\mathbb{1}-Z}{2}\rho\frac{\mathbb{1}-Z}{2}\right)$$

$K_1 = \sqrt{p}\,\frac{\mathbb{1}+Z}{2}$, $K_2 = \sqrt{p}\,\frac{\mathbb{1}-Z}{2}$

$$= (1-p)\rho + \frac{p}{4}\left(\rho + \cancel{Z\rho} + \cancel{\rho Z} + Z\rho Z + \rho - \cancel{Z\rho} - \cancel{\rho Z} + Z\rho Z\right)$$

$$= (1-p)\rho + \frac{p}{2}(\rho + Z\rho Z) = \left(1-\frac{p}{2}\right)\rho + \frac{p}{2}Z\rho Z \Rightarrow K_0' = \sqrt{1-\frac{p}{2}}\,\mathbb{1},\ K_1' = \sqrt{\frac{p}{2}}\,Z$$

$$\mathcal{E}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & (1-p)b \\ (1-p)c & d \end{pmatrix} \xrightarrow[n \text{ times}]{\text{Apply channel}}$$ The off-diagonal terms decay exponentially $\left(1-\frac{\Gamma t}{n}\right)^n \xrightarrow[n\to\infty]{} e^{-\Gamma/t}$ where $\boxed{T_2 = \frac{1}{\Gamma}}$

$ZXZ = -X$

$ZYZ = -Y$

Poisson process with rate $\Gamma \Rightarrow$ Exponential decay $e^{-\Gamma/t}$

(Expectation value at any time step is Γt)

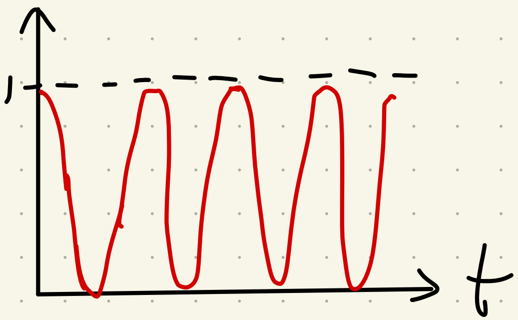
This channel models the process of decoherence, a transition from the quantum world to the classical world. Why do we observe definite properties of things in the classical world whereas we can have superposition in the quantum world?

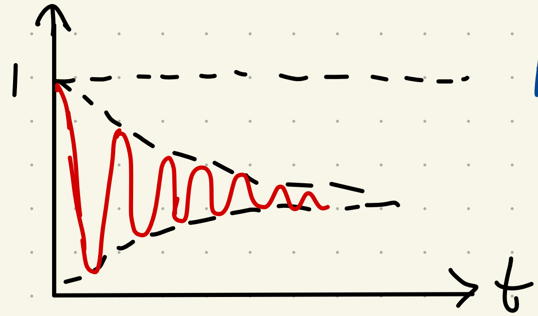
In our phase-damping model, the initial state $|0\rangle_A, |1\rangle_A$ could signify an initial position of a grain of dust, and the meters are photons that scatter off of the dust at a rate Γ and end up in a final state that depends on the position of the dust particle. The distance-dependent interaction Hamiltonian (scattering) picks out the position basis of the dust particle as the basis in which the decay of the off-diagonal terms occur.

This a generic feature of decoherence; the physical nature of the interaction is what picks out the "classical" basis and this decoherence time T_2 is typically extremely short How short?

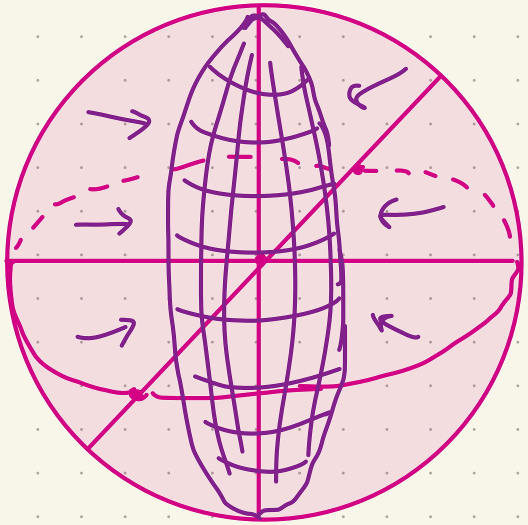
$$P_r(t) = [1+\cos(\omega t)]/2 \qquad\qquad P_r(t) = [1+e^{-\Gamma t}\cos(\omega t)]$$

Rabi oscillation





Fit the curve to an exponential function to find T_2



The channel shrinks the Bloch ball on the x and y axes but leaves points on the z axis unchanged. (The kraus operators commute with Z).

Is there a channel that shrinks only one of the Cartesian axis? No, because such a channel would be unitarily equivalent to a channel that leaves the state untouched with probability $1-p$ and take the transpose ($Y \mapsto -Y$) with probability p, the latter of which we have seen to be unphysical (not CP). ("No pancake theorem")

③ Amplitude-damping channel (Spontaneous emission)

In the presence of an electromagnetic field, an atom can spontaneously relax to the ground state with probability p and emits a photon

vacuum

$$|0\rangle_A |0\rangle_{EM} \mapsto |0\rangle_A |0\rangle_{EM}$$

$$|1\rangle_A |0\rangle_{EM} \mapsto \sqrt{1-p}\, |1\rangle_A |0\rangle_{EM} + \sqrt{p}\, |0\rangle_A |1\rangle_{EM}$$

Not only that this channel picks out a preferred basis, it is also not symmetric between the ground and the excited state.

$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 00 & 1 & & 0 & \\ 01 & 0 & & \sqrt{p} & \\ 10 & 0 & & \sqrt{1-p} & \\ 11 & 0 & & 0 & \end{array}$$

$U|0\rangle$

$$K_0 = \langle 0|U|0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \quad K_1 = \langle 1|U|0\rangle = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}$$

$$\mathcal{E}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+pd & \sqrt{1-p}\, b \\ \sqrt{1-p}\, c & (1-p)d \end{pmatrix}$$

$\downarrow$ n times

From ρ_{11} and the trace-preserving condition

$$\begin{pmatrix} a+(1-e^{-\Gamma t})d & e^{-\Gamma t/2} b \\ e^{-\Gamma t/2} c & e^{-\Gamma t} d \end{pmatrix}$$

T_2 $\qquad$ T_1

Energy transfer also causes decoherence but only by "half" an amount

$$\boxed{T_2 = \frac{2}{\Gamma} = 2T_1}$$

Typically, $T_2 \ll T_1$ but not always, as we just saw that for the amplitude-damping channel, $T_2 = 2T_1$. In fact, it is sometimes mistakenly stated that $T_2 \leq T_1$ always.

http://hobbieroth.blogspot.com/2020/08/can-t2-be-longer-than-t1.html

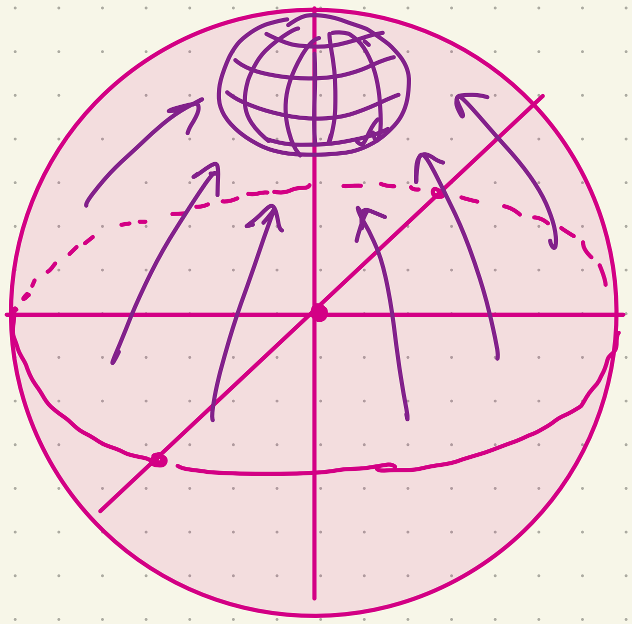
Is T_2 necessarily shorter than T_1?

Olaf Dietrich,* Munich

2018-01-17

In fact, it can be shown (see appendix if interested) that the maximum T_2 value, for which the red curve stays always below the black line, is exactly $T_2 = 2\,T_1$. And as so often, almost everything that is physically possible is also realized in nature (although the case $T_1 < T_2 < 2\,T_1$ is really extremely rare), as described by Malcolm H. Levitt in his highly recommendable NMR text book "Spin dynamics" (2nd ed., section 11.9.2, note 13):

> The case where $T_2 > T_1$ is encountered when the spin relaxation is caused by fluctuating microscopic fields that are predominantly transverse rather than longitudinal. One mechanism which gives rise to fields of this form involves the *antisymmetric component of the chemical shift tensor* (not to be confused with the CSA). [...] Molecular systems in which this mechanims is dominant are exceedingly rare (see F. A. L. Anet, D. J. O'Leary, C. G. Wade and R. D. Johnson, *Chem. Phys. Lett.*, **171**, 401 (1990)).



$$\boxed{T_2 \leq 2T_1}$$

T_1 = relaxation time = "qubit lifetime"

T_2 = decoherence time = "qubit coherence time"

Sometimes there is also T_2^* which is a "decoherence" time observed from a certain kind of experiment (Ramsey). Not a true decoherence time since it includes an apparently irreversible, but in fact reversible effect (via spin echo).

Quantum error correction

Dephasing channel

A crucial insight from the previous lecture is that decoherence can be modeled as a discrete phase-flip process, the insight which made the development of quantum error correction (QEC) possible (Nielsen & Chuang p. 385)

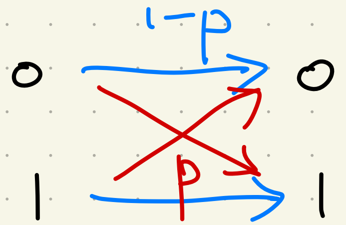
In particular, Kraus operators can be thought of as discretized errors, and if $\{K_\alpha\}_\alpha$ can be corrected, any linear combination $\sum_\alpha c_\alpha K_\alpha$ of the correctable errors can also be corrected.

Essentially what we want to do in QEC is inverting a non-unitary evolution caused by relaxation (decay to ground state) and decoherence. It turns out that a quantum dynamics is invertible on the whole subspace iff it is unitary. But we can still partially invert the dynamics on a specifically designed entangled subspace called a code space, on which we encode our qubit state $\alpha|0\rangle + \beta|1\rangle$

Preskill's "We can fight entanglement with entanglement."

Classical repetition code

Protect against bit flip error by copying the bit into many bits, then after some time has passed, look at the value of every bit and take majority vote.



$0 \to 000$
$1 \to 111$

Corruption

p	p^2
0 0 1	0 1 1
0 1 0	1 0 1
1 0 0	1 1 0
1 1 0	0 0 1
1 0 1	0 1 0
0 1 1	1 0 0

Guess that the original is '000' if we see '0' more than '1'

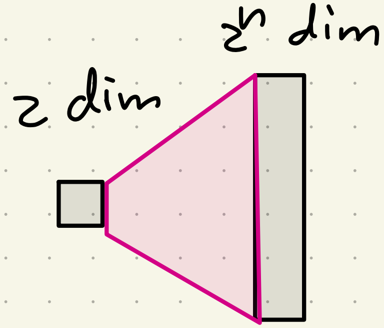
Correctable error (single bit flip)

If we take majority vote here we will be wrong 100%

Uncorrectable error (> single bit flip)

This is the property of all error correcting codes (a feature, not a bug!) We don't aim to correct "all" possible errors (imagine an asteroid hitting our computer) only the highly probable ones.

Transition to QEC



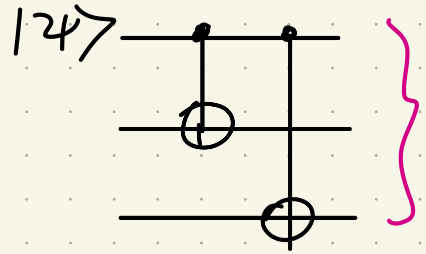
$|0\rangle \mapsto |000\rangle$

$|1\rangle \mapsto |111\rangle$

Measuring all qubits also destroy the information. Instead, we only make partial measurements of parity Z_1Z_2, Z_2Z_3, Z_1Z_3

Can't copy an arbitrary state due to no-cloning

$$|\psi\rangle \not\mapsto |\psi\rangle|\psi\rangle|\psi\rangle$$



$$\alpha|000\rangle + \beta|111\rangle$$

$$|x_1 x_2\rangle \xrightarrow[Z_1Z_2]{\text{Measure}} (-1)^{x_1 \oplus x_2} \begin{cases} +1 & \text{if } x_1 = x_2 \\ -1 & \text{if } x_1 \neq x_2 \end{cases}$$

Clearly can't correct a phase-flip error $|1\rangle \mapsto -|1\rangle$