Computational Logic Lab

30/10/2024

Use the SMT-solver z3 to solve the following problems.

(1) Check whether the following inferences are correct or not:

You can find a ticket only if you you make a reservation in advance.
You never make a reservation.

You cannot find a ticket.

If Paul joins our group, Mary and John will join too.
If John joins the group, neither Peter nor Andrew will join.

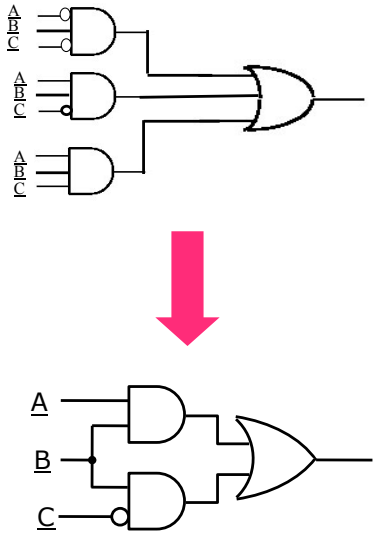
If either Peter or Andrew join the group, Paul does not.

(2) *Portia's Caskets, an enigma from Shakespeare.* One of the most famous enigmas in history is the one called Portia's treasure chests. It is even recounted in 'The Merchant of Venice' by Shakespeare. Portia, one of the characters, is to be married, she is beautiful and has many suitors, but she doesn't want just any guy, she wants her future husband to be smart too, and here's what she does. Portia had three caskets, one of gold, one of silver and one of lead, and in one was her portrait. Portia wanted to choose her groom on the basis of his intelligence and had the following inscriptions engraved on the chests:

(a) Golden casket: "the portrait is in this casket"
(b) Silver casket: "the portrait is not in this casket"
(c) Lead casket: "the portrait is not in the gold casket".

Portia explained to her suitor that of these three statements at most one was true. Which casket contains Portia's portrait?

(3) Show that the two circuits below are equivalent.



(4) In a pigeon-hole problem,[1] we are given n pigeons and k holes. A pigeon cannot stay inside two holes; each hole can contain only one pigeon and each pigeon must stay inside some hole. In addition there are specific requirements, to be tested for consistency. In our case, we have $n = 4$, $k = 4$ and the following requirements:

- pigeon 1 does not like holes 2 and 3;
- pigeon 2 does not like holes 3 and 4;
- pigeon 3 does not like holes 1 and 2:
- pigeon 4 does not like holes 3 and 4.

Check that the above problem has a solution and supply the solution. *[Hint: use propositional letters Pij to express that pigeon i is in hole j (for $i, j = 1, \ldots, 4$). Then assert that each pigeon must stay in exactly one hole and that each hole can contain at most one pigeon. Finally, formalize the remaining constraints.]*

[1] This is a classical challenging problem per SAT solvers (it is practically impossible to solve it for large n, k).

(5) We must schedule the lectures of the following courses:

C=calculus, L=logic, P=programming, H=physics, A=algebra

We have many available classrooms, however only three time slots can be used:

9am, 10am, 11am

So some classes will be held in parallel. We want to accomplish (if possible) the following desiderata:

- student X wants to attend all C, H, A;
- student Y wants to attend both L and P;
- student Z wants to attend C and H but cannot come before 10am.

Check whether this is possible or not and - in the affirmative case - show how to schedule the above five courses.

[*Hint: use propositional letters C9, C10, C11 to say that Calculus is at 9am, 10am, 11am. Then write down formulae saying that exactly one among C9, C10, C11 hold.Do the same for Logic, Programming, Physics and Algebra. Finally, formalize the students desiderata: to this aim, notice that a student cannot attend two different courses if they are held in parallel. Finally, run z3 to see whether all this is consistent or not. If it is, ask z3 to give you a model.*]