AC32008 Theory of Computation
Tutorial Sheet 7 - Questions from Previous Second Class Tests

1. (i) Let M be a Turing Machine. Describe informally what do we mean by some string w being accepted by M.

   (ii) What does it mean for a language L to be partially-decidable, but not totally decidable?

2. Suppose we are given a Turing Machine with

   - $Q = \{q_0, q_1, q_2, q_3, q_4\}$,
   - $\Sigma = \{0, 1\}$,
   - $\Gamma = \{0, 1, X, Y, B\}$,
   - $F = \{q_4\}$.

δ	q_0	q_1	q_2	q_3	q_4
0	(q_1, X, R)	$(q_1, 0, R)$	$(q_2, 0, L)$	$-$	$-$
1	$-$	(q_2, Y, L)	$-$	$-$	$-$
X	$-$	$-$	(q_0, X, R)	$-$	$-$
Y	(q_3, Y, R)	(q_1, Y, R)	(q_2, Y, L)	(q_3, Y, R)	$-$
B	$-$	$-$	$-$	(q_4, B, R)	$-$

   Simulate the computation of this Turing Machine on a string 0011 by writing a serious of instantaneous descriptions the machine goes through. Is the string accepted by the machine?

3. Let M be a Turing Machine with states q_1, q_2, where q_1 is the initial state, $F = \{q_2\}$, input alphabet $\{0, 1\}$ and tape alphabet $\{0, 1, B\}$. The transition function δ for M is as follows:

δ	q_1	q_2
0	$(q_1, 0, R)$	$-$
1	$-$	$-$
B	(q_2, B, L)	$-$

   Let w be the string 00000.

   (a) Determine a code for the machine M.

   (b) What is $\langle M, w\rangle$?

   (c) Is $\langle M, w\rangle \in L_{\text{halt}}$?

   [Recall that $X_1 = 0$, $X_2 = 1$, $X_3 = B$, $D_1 = L$, $D_2 = R$.]

4. Let $\langle M, w\rangle = 111010101010110100101001001111111011$, where M is a Turing Machine and w is a string.

   (i) Give the transition table for M.

(ii) Is $\langle M, w \rangle \in L_{\text{halt}}$? Give a very brief reason for your answer.

5. Let L be the language given by

$$L = \{w_i | M_i \text{ does not accept } w_i\}.$$

Then it can be shown (by diagonalisation) that L is not partially decidable and (by simulation) that $\overline{L}$ is partially decidable. Given these facts, show that there is a language which is partially decidable but not totally decidable.

6. (**Harder**) Design a Turing Machine M that accepts the language

$$L = \{ww | w \in \{0, 1\}^*\}.$$

You do not need to specify the whole transition table. An informal description of M will do, provided it is precise and convincing enough. For example, an informal description of the purpose of each state that M has, along with the outline of transitions in that state, is enough.