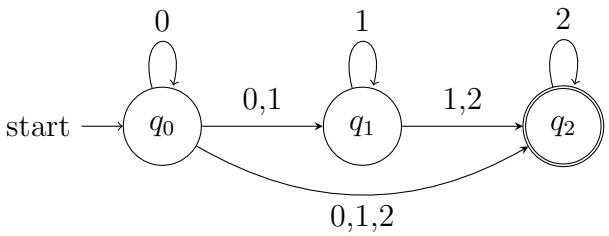
AC32008 Theory of Computation

Tutorial Sheet 2 - NFAs, Equivalence Between NFAs and DFAs and NFAs With Epsilon Transitions

1. Describe nondeterministic finite automata (with or without ϵ-transitions) which accept each of the following languages over $\Sigma = \{0, 1\}$.
   (a) The set of all strings that contain the substring 0101.
   (b) The set of all strings that start with 00 or end with 11 (or both).
   (c) The set that contains just two strings - 010 and 11.
   (d) The set of all strings of zero or more 0's, followed by zero or more 1's, followed by one or more 0's (e.g. 0110, 0000, 1000, but not 0011 or 0101).
   (e) The set of all strings starting with a sequence of zero or more 1's, followed by zero or more sequences, each of which is of the form 00 followed by one or more 1's (e.g. $\epsilon, 11, 111001, 100100111, 001$, but not $010, 000, 1101$).

2. Prove that every NFA can be converted to an NFA (possibly with ϵ-transitions) that has only one accepting state.

3. Given below is an NFA that accepts the language of strings comprising a sequence of zero or more 0's, followed by a sequence of zero or more 1's, followed by a sequence of zero or more 2's. Using a standard construction, find the DFA that accepts the same language.



4. For a language L over some alphabet Σ, we can define L^C (L complement) as $L^C = \{w | w \in \Sigma^*, w \notin L\}$.
   - Prove formally that, if we have a DFA M that accepts language L, then swapping accepting and non-accepting states will give us a new automaton M' that accepts L^C.
   - The same is **not** true for NFAs. Find an example of an NFA that accepts some language L, such that swapping accepting and non-accepting states will result in an automaton M' that will accept a language other than L^C.

5. (**Harder**) Suppose L is any regular language, and let L^R be the set of all strings in L reversed, i.e.:

$$L^R = \{w^R \mid w \in L\},$$

where w^R is the string w written backwards (e.g. $0001^R = 1000$).

Show that there is a NFA with ϵ-transitions that accepts L^R.
[**Hint**: Consider a DFA which accepts the language L and consider how it might be modified to get a finite automaton (DFA, NFA, or NFA with ϵ-transitions) which accepts L^R.]

6. (**Harder**) Suppose L is any regular language, and let Prefixes(L) be the set of all prefixes of strings in L, i.e.:

$$\text{Prefixes}(L) = \{x \mid \text{for some string } y,\ xy \in L\}.$$

Show that Prefixes(L) is regular also.
[**Hint**: You should not need to change the transition function, just the final states.]