

Seam-carving is a content-aware image resizing technique where the image is reduced in size by one pixel of height (or width) at a time. A *vertical seam* in an image is a path of pixels connected from the top to the bottom with one pixel in each row; a *horizontal seam* is a path of pixels connected from the left to the right with one pixel in each column. Below left is the original 505-by-287 pixel image; below right is the result after removing 150 vertical seams, resulting in a 30% narrower image. Unlike standard content-agnostic resizing techniques (such as cropping and scaling), seam carving preserves the most interest features (aspect ratio, set of objects present, etc.) of the image.

Although the [underlying algorithm](#) is simple and elegant, it was not discovered until 2007. Now, it is now a core feature in Adobe Photoshop and other computer graphics applications.



In this assignment, you will create a data type that resizes a W -by- H image using the seam-carving technique.

Finding and removing a seam involves three parts and a tiny bit of notation:

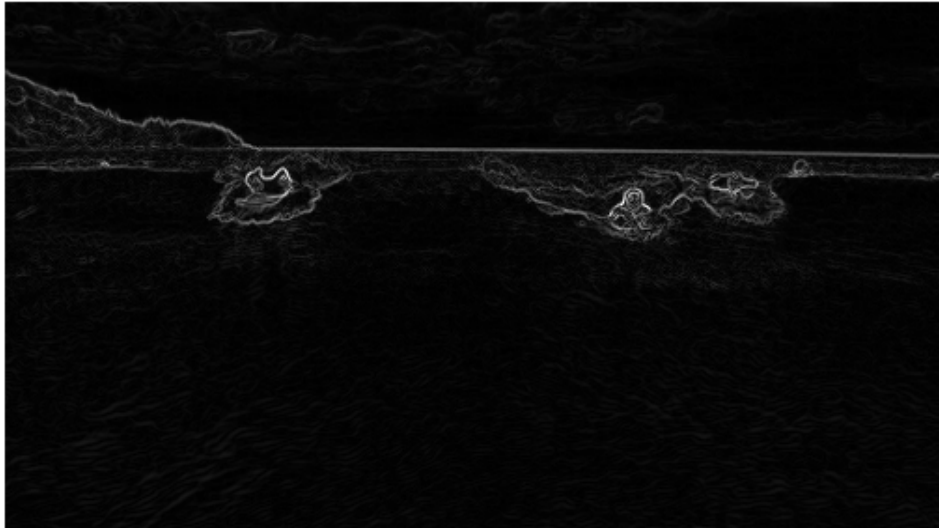
0. *Notation.* In image processing, pixel (x, y) refers to the pixel in column x and row y , with pixel $(0, 0)$ at the upper left corner and pixel $(W - 1, H - 1)$ at the bottom right corner. This is consistent with the [Picture](#) data type in `algs4.jar`. *Warning:* this is the opposite of the standard mathematical notation used in linear algebra where (i, j) refers to row i and column j and with Cartesian coordinates where $(0, 0)$ is at the lower left corner.

a 3-by-4 image

(0, 0)	(1, 0)	(2, 0)
(0, 1)	(1, 1)	(2, 1)
(0, 2)	(1, 2)	(2, 2)
(0, 3)	(1, 3)	(2, 3)

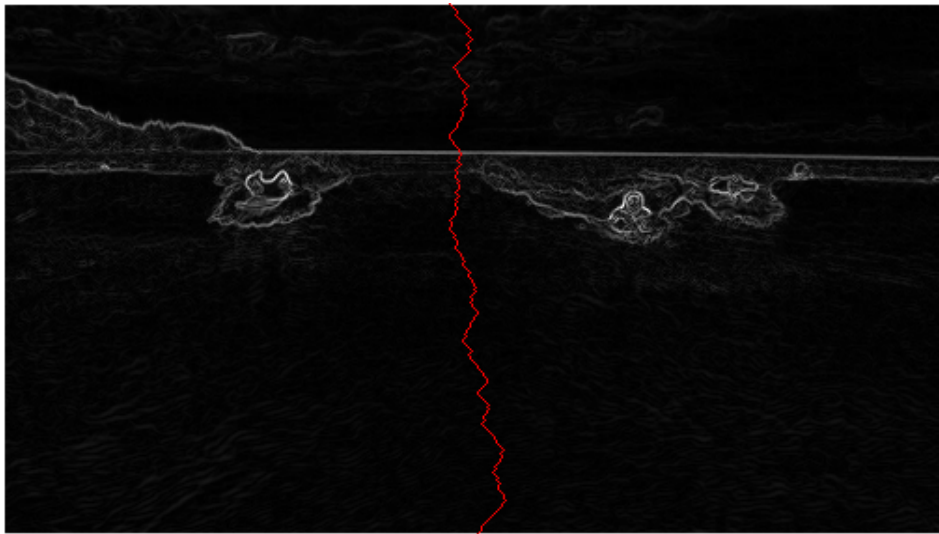
We also assume that the color of a pixel is represented in RGB space, using three integers between 0 and 255. This is consistent with the [java.awt.Color](#) data type.

1. *Energy calculation.* The first step is to calculate the *energy* of each pixel, which is a measure of the importance of each pixel—the higher the energy, the less likely that the pixel will be included as part of a seam (as we'll see in the next step). In this assignment, you will implement the *dual-gradient energy function*, which is described below. Here is the dual-gradient energy function of the surfing image above:



The energy is high (white) for pixels in the image where there is a rapid color gradient (such as the boundary between the sea and sky and the boundary between the surfing Josh Hug on the left and the ocean behind him). The seam-carving technique avoids removing such high-energy pixels.

2. *Seam identification.* The next step is to find a vertical seam of minimum total energy. This is similar to the classic shortest path problem in an edge-weighted digraph except for the following:
 - The weights are on the vertices instead of the edges.
 - We want to find the shortest path from any of the W pixels in the top row to any of the W pixels in the bottom row.
 - The digraph is acyclic, where there is a downward edge from pixel (x, y) to pixels $(x - 1, y + 1)$, $(x, y + 1)$, and $(x + 1, y + 1)$, assuming that the coordinates are in the prescribed range.



3. *Seam removal*. The final step is to remove from the image all of the pixels along the seam.

The SeamCarver API. Your task is to implement the following mutable data type:

```
public class SeamCarver {

    // create a seam carver object based on the given picture
    public SeamCarver(Picture picture)

    // current picture
    public Picture picture()

    // width of current picture
    public int width()

    // height of current picture
    public int height()

    // energy of pixel at column x and row y
    public double energy(int x, int y)

    // sequence of indices for horizontal seam
    public int[] findHorizontalSeam()

    // sequence of indices for vertical seam
    public int[] findVerticalSeam()

    // remove horizontal seam from current picture
    public void removeHorizontalSeam(int[] seam)

    // remove vertical seam from current picture
    public void removeVerticalSeam(int[] seam)

    // unit testing (optional)
    public static void main(String[] args)

}
```

- **Corner cases.** Your code should throw an `IllegalArgumentException` when a constructor or method is called with an invalid argument, as documented below:
 - By convention, the indices x and y are integers between 0 and $width - 1$ and between 0 and $height - 1$ respectively, where $width$ is the width of the current image and $height$ is the height. Throw an `IllegalArgumentException` if either x or y is outside its prescribed range.

- Throw an `IllegalArgumentException` if the constructor, `removeVerticalSeam()`, or `removeHorizontalSeam()` is called with a null argument.
 - Throw an `IllegalArgumentException` if `removeVerticalSeam()` or `removeHorizontalSeam()` is called with an array of the wrong length or if the array is not a valid seam (i.e., either an entry is outside its prescribed range or two adjacent entries differ by more than 1).
 - Throw an `IllegalArgumentException` if `removeVerticalSeam()` is called when the width of the picture is less than or equal to 1 or if `removeHorizontalSeam()` is called when the height of the picture is less than or equal to 1.
- **Constructor.** The data type may not mutate the `Picture` argument to the constructor.
 - **Computing the energy of a pixel.** You will use the *dual-gradient energy function*: The energy of pixel (x, y) is $\sqrt{\Delta_x^2(x, y) + \Delta_y^2(x, y)}$, where the square of the x -gradient $\Delta_x^2(x, y) = R_x(x, y)^2 + G_x(x, y)^2 + B_x(x, y)^2$, and where the central differences $R_x(x, y)$, $G_x(x, y)$, and $B_x(x, y)$ are the differences in the red, green, and blue components between pixel $(x + 1, y)$ and pixel $(x - 1, y)$, respectively. The square of the y -gradient $\Delta_y^2(x, y)$ is defined in an analogous manner. We define the energy of a pixel at the border of the image to be 1000, so that it is strictly larger than the energy of any interior pixel.

As an example, consider the 3-by-4 image (supplied as [3x4.png](#)) with RGB values—each component is an integer between 0 and 255—as shown in the table below:

RGB values for pixel (1, 2)			energy of pixel (1, 2)		
(255, 101, 51)	(255, 101, 153)	(255, 101, 255)	1000	1000	1000
(255, 153, 51)	(255, 153, 153)	(255, 153, 255)	1000	$\sqrt{52225}$	1000
(255, 203, 51)	(255, 204, 153)	(255, 205, 255)	1000	$\sqrt{52024}$	1000
(255, 255, 51)	(255, 255, 153)	(255, 255, 255)	1000	1000	1000
a 3-by-4 image (RGB values)			dual-gradient energies		

The ten border pixels have energy 1000. Only the pixels (1, 1) and (1, 2) are nontrivial. We calculate the energy of pixel (1, 2) in detail:

$$\begin{aligned}
 R_x(1, 2) &= 255 - 255 = 0, \\
 G_x(1, 2) &= 205 - 203 = 2, \\
 B_x(1, 2) &= 255 - 51 = 204, \\
 \text{yielding } \Delta_x^2(1, 2) &= 2^2 + 204^2 = 41620.
 \end{aligned}$$

$$\begin{aligned}
 R_y(1, 2) &= 255 - 255 = 0, \\
 G_y(1, 2) &= 255 - 153 = 102, \\
 B_y(1, 2) &= 153 - 153 = 0, \\
 \text{yielding } \Delta_y^2(1, 2) &= 102^2 = 10404.
 \end{aligned}$$

Thus, the energy of pixel (1, 2) is $\sqrt{41620 + 10404} = \sqrt{52024}$. Similarly, the energy of pixel (1, 1) is $\sqrt{204^2 + 103^2} = \sqrt{52225}$.

- **Finding a vertical seam.** The `findVerticalSeam()` method returns an array of length H such that entry y is the column number of the pixel to be removed from row y of the image. For example, the dual-gradient energies of a 6-by-5 image (supplied as [6x5.png](#)).

$$\text{energy of seam} = 1000.00 + 107.89 + 133.07 + 174.01 + 1000.00 = 2414.97$$

1000.00	1000.00	1000.00	1000.00	1000.00	1000.00
1000.00	237.35	151.02	234.09	107.89	1000.00
1000.00	138.69	228.10	133.07	211.51	1000.00
1000.00	153.88	174.01	284.01	194.50	1000.00
1000.00	1000.00	1000.00	1000.00	1000.00	1000.00

a minimum energy vertical seam in a 6-by-5 image

The minimum energy vertical seam is highlighted in blue. In this case, the method `findVerticalSeam()` returns the array `{ 3, 4, 3, 2, 2 }` because the pixels in a minimum energy vertical seam are (3, 0), (4, 1), (3, 2), (2, 3), and (2, 4). When there are multiple vertical seams with minimal total energy, your method can return any such seam.

- **Finding a horizontal seam.** The behavior of `findHorizontalSeam()` is analogous to that of `findVerticalSeam()` except that it returns an array of length *width* such that entry *x* is the row number of the pixel to be removed from column *x* of the image. For the 6-by-5 image, the method `findHorizontalSeam()` returns the array `{ 2, 2, 1, 2, 1, 2 }` because the pixels in a minimum energy horizontal seam are (0, 2), (1, 2), (2, 1), (3, 2), (4, 1), and (5, 2).

$$\text{energy of seam} = 1000.00 + 138.69 + 151.02 + 133.07 + 107.89 + 1000.00 = 2530.68$$

1000.00	1000.00	1000.00	1000.00	1000.00	1000.00
1000.00	237.35	151.02	234.09	107.89	1000.00
1000.00	138.69	228.10	133.07	211.51	1000.00
1000.00	153.88	174.01	284.01	194.50	1000.00
1000.00	1000.00	1000.00	1000.00	1000.00	1000.00

a minimum energy horizontal seam in a 6-by-5 image

- **Performance requirements.** The `width()`, `height()`, and `energy()` methods should take constant time in the worst case. All other methods should run in time at most proportional to $\text{width} \times \text{height}$ in the worst case. For faster performance, do not construct explicit `DirectedEdge` and `EdgeWeightedDigraph` objects.

Analysis of running time (optional and not graded).

- Give the worst-case running time to remove *r* rows and *c* columns from a *width*-by-*height* image as a function of *r*, *c*, *width*, and *height*.
- Estimate empirically the running time (in seconds) to remove *r* rows and *c* columns from a *width*-by-*height* image as a function of *r*, *c*, *width*, and *H*. Use tilde notation to simplify your answer.

Web submission. Submit a .zip file containing `SeamCarver.java` and any other supporting files (excluding `algs4.jar` and `SCUtility.java`). You may not call any library functions except those in `java.lang`, `java.util`, `java.awt.Color`, and `algs4.jar`.

