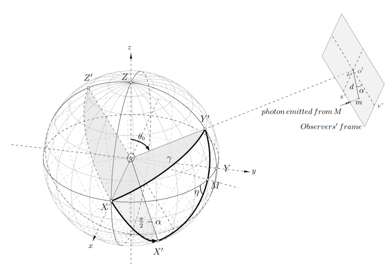
LuminetCpp: The maths behind the code.
C++ version of the Python code by bgmeulem
//https://github.com/bgmeulem/Luminet/



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Status: DRAFT

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2.1 Derivation of equation (2).

Derivation of

$$\left\{\frac{1}{r^2}\left(\frac{dr}{d\phi}\right)\right\}^2 + \frac{1}{r^2}\left(1-\frac{2M}{r}\right) = \frac{1}{b^2}$$

from

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$

The metric form Φ is (considering the invariance of a rotation along the θ generating axis)

$$\Phi = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2 d\phi^2 \tag{1}$$

For geodesic null lines we have

$$\begin{cases} \frac{d^2x^r}{du^2} + \Gamma^r_{mn}\frac{dx^m}{du}\frac{dx^n}{du} = 0 \\ \\ g_{mn}\frac{dx^m}{du}\frac{dx^n}{du} = 0 \end{cases} \tag{2}$$

or (2.448 page 46)

$$\begin{cases} \frac{d^2x^r}{dv^2} + \Gamma^r_{mn}\frac{dx^m}{dv}\frac{dx^n}{dv} = \lambda(v)\frac{dx^r}{dv} \\ a_{mn}\frac{dx^m}{dv}\frac{dx^n}{dv} = 0 \end{cases} \tag{3}$$

where by suitable choice of the parameter v , $\lambda(v)$ can be made any preassigned function of v.

$$(2) \quad \Rightarrow \quad \frac{d^2x^r}{dv^2} = \lambda\frac{dx^r}{dv} - \Gamma^r_{mn}\frac{dx^m}{dv}\frac{dx^n}{dv} \tag{4}$$

$$\Rightarrow \quad \frac{d^3x^r}{dv^3} = \frac{d\lambda}{dv}\frac{dx^r}{dv} + \lambda\frac{d^2x^r}{dv^2} + A^r_{.mns}\frac{dx^m}{dv}\frac{dx^n}{dv}\frac{dx^s}{dv} \tag{5}$$

$$\text{with} \quad \Rightarrow \quad A^r_{.mns} = -\partial_s\Gamma^r_{mn} + 2\Gamma^r_{sp}\Gamma^p_{mn} \tag{6}$$

For the considered metric form (Schwarzschild), the connection functions are:

$$\left[\begin{bmatrix} 0 & \frac{M}{r(-2M+r)} & 0 \\ \frac{M}{r(-2M+r)} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} \frac{M(-2M+r)}{r^3} & 0 & 0 \\ 0 & \frac{M}{r(2M-r)} & 0 \\ 0 & 0 & 2M-r \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r} \\ 0 & \frac{1}{r} & 0 \end{bmatrix}\right] \tag{7}$$

$$\tag{8}$$

$$\Gamma^t\, rt = \frac{M}{r\left(-2M+r\right)} \tag{9}$$

$$\tag{10}$$

$$\Gamma^r\, tt = \frac{M\left(-2M+r\right)}{r^3} \tag{11}$$

$$\tag{12}$$

$$\Gamma^r\, rr = \frac{M}{r\left(2M-r\right)} \tag{13}$$

$$\tag{14}$$

$$\Gamma^r\, \phi\phi = 2M - r \tag{15}$$

$$\tag{16}$$

$$\Gamma^\phi\, \phi r = \frac{1}{r} \tag{17}$$

and all the others are zero.

♦

VectorFields

$$\nabla_t V^\alpha = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\nabla_r V^\alpha = \begin{bmatrix} 0 & 0 & \frac{c}{r} & \frac{d}{r} \end{bmatrix}$$

$$\nabla_\theta V^\alpha = \begin{bmatrix} 0 & -cr & \frac{b}{r} & \frac{d}{\tan(\theta)} \end{bmatrix}$$

$$\nabla_\phi V^\alpha = \begin{bmatrix} 0 & -dr\sin^2(\theta) & -\frac{d\sin(2\theta)}{2} & \frac{b}{r} + \frac{c}{\tan(\theta)} \end{bmatrix}$$

3. Image of a clothed black hole

3.1

The coordinate system:

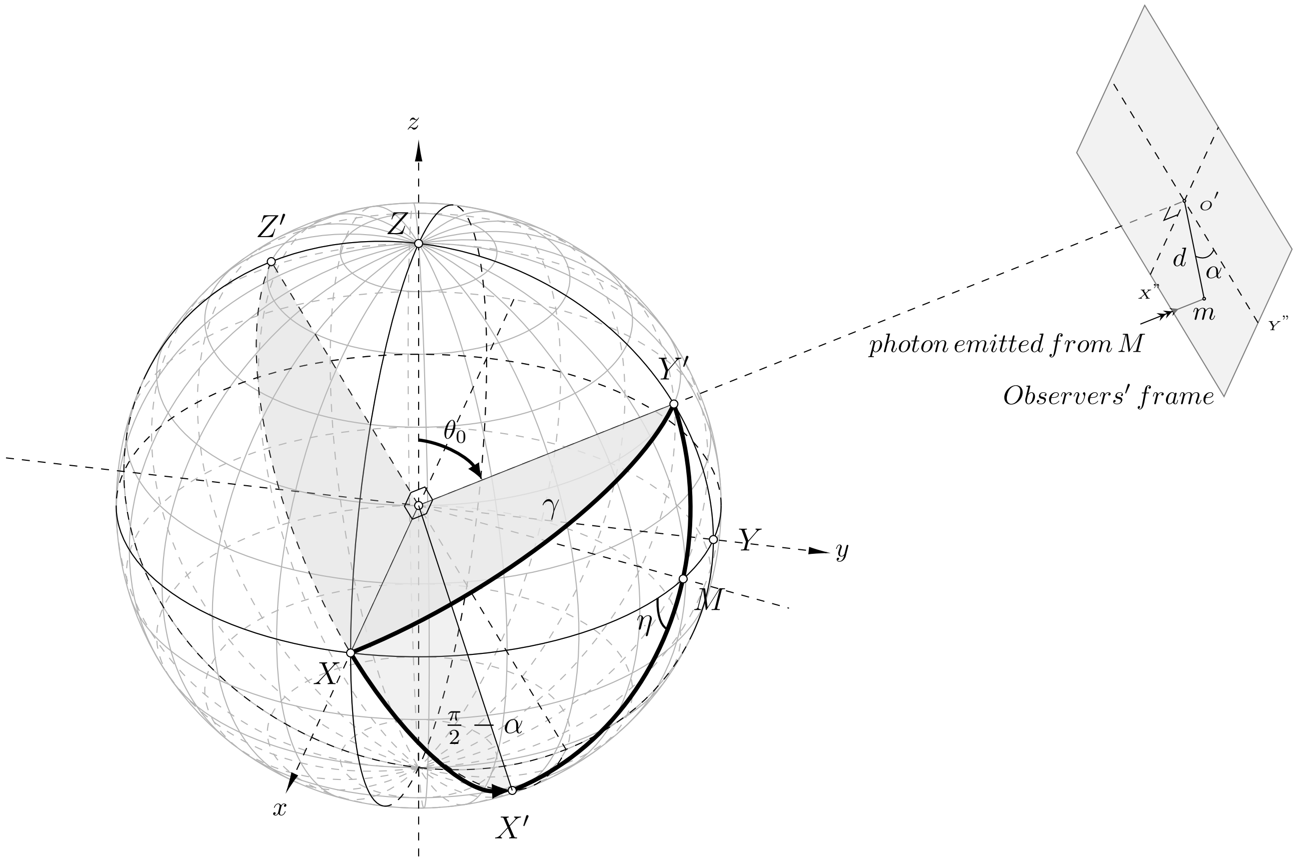

Figure 3.1: Fig.3 The coordinate system (see text).

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