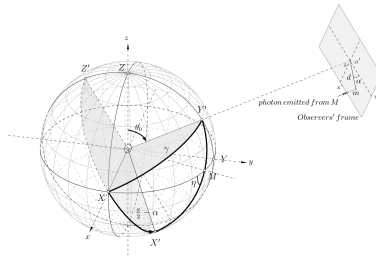


LuminetCpp: The maths behind the code.
C++ version of the Python code by bgmeulem
[//https://github.com/bgmeulem/Luminet/](https://github.com/bgmeulem/Luminet/)



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Status: DRAFT

Contents

2	2. Image of a bare black hole	3
2.1	Derivation of equation (2)	4
3	3. Image of a clothed black hole	6
3.1	7

List of Figures

3.1 Fig.3 The coordinate system (see text).	7
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2. Image of a bare black hole

2.1 Derivation of equation (2).

Derivation of

$$\left\{ \frac{1}{r^2} \left(\frac{dr}{d\phi} \right) \right\}^2 + \frac{1}{r^2} \left(1 - \frac{2M}{r} \right) = \frac{1}{b^2}$$

from

$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \left(1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

The metric form Φ is (considering the invariance of a rotation along the θ generating axis)

$$\Phi = - \left(1 - \frac{2M}{r} \right) dt^2 + \left(1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 d\phi^2 \quad (1)$$

For geodesic null lines we have

$$\begin{cases} \frac{d^2 x^r}{du^2} + \Gamma_{mn}^r \frac{dx^m}{du} \frac{dx^n}{du} = 0 \\ g_{mn} \frac{dx^m}{du} \frac{dx^n}{du} = 0 \end{cases} \quad (2)$$

or (2.448 page 46)

$$\begin{cases} \frac{d^2 x^r}{dv^2} + \Gamma_{mn}^r \frac{dx^m}{dv} \frac{dx^n}{dv} = \lambda(v) \frac{dx^r}{dv} \\ a_{mn} \frac{dx^m}{dv} \frac{dx^n}{dv} = 0 \end{cases} \quad (3)$$

where by suitable choice of the parameter v , $\lambda(v)$ can be made any preassigned function of v .

$$(2) \Rightarrow \frac{d^2 x^r}{dv^2} = \lambda \frac{dx^r}{dv} - \Gamma_{mn}^r \frac{dx^m}{dv} \frac{dx^n}{dv} \quad (4)$$

$$\Rightarrow \frac{d^3 x^r}{dv^3} = \frac{d\lambda}{dv} \frac{dx^r}{dv} + \lambda \frac{d^2 x^r}{dv^2} + A_{.mns}^r \frac{dx^m}{dv} \frac{dx^n}{dv} \frac{dx^s}{dv} \quad (5)$$

$$\text{with } \Rightarrow A_{.mns}^r = -\partial_s \Gamma_{mn}^r + 2\Gamma_{sp}^r \Gamma_{mn}^p \quad (6)$$

For the considered metric form (Schwarzschild), the connection functions are:

$$\left[\begin{bmatrix} 0 & \frac{M}{r(-2M+r)} & 0 \\ \frac{M}{r(-2M+r)} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{M(-2M+r)}{r^3} & 0 & 0 \\ 0 & \frac{M}{r(2M-r)} & 0 \\ 0 & 0 & 2M-r \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r} \\ 0 & \frac{1}{r} & 0 \end{bmatrix} \right] \quad (7)$$

(8)

$$\Gamma^t r t = \frac{M}{r(-2M+r)} \quad (9)$$

(10)

$$\Gamma^r t t = \frac{M(-2M+r)}{r^3} \quad (11)$$

(12)

$$\Gamma^r r r = \frac{M}{r(2M-r)} \quad (13)$$

(14)

$$\Gamma^r \phi \phi = 2M - r \quad (15)$$

(16)

$$\Gamma^\phi \phi r = \frac{1}{r} \quad (17)$$

and all the others are zero.



3. Image of a clothed black hole

3.1

The coordinate system:

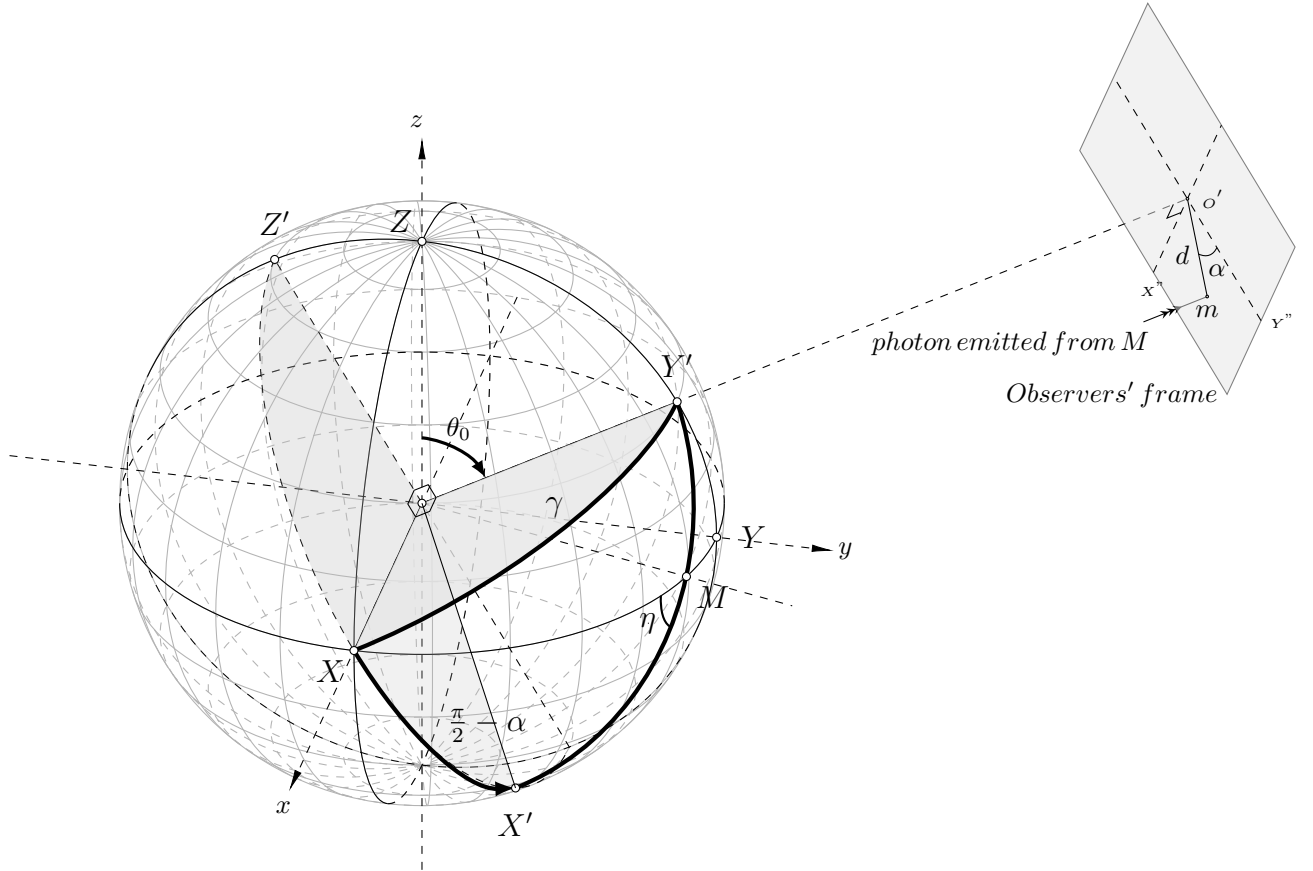


Figure 3.1: Fig.3 The coordinate system (see text).

