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Notes to Instructor or TA: If needed, include here any special notes for TAs or instructor; delete if no notes

Problem

Take the following list of functions and arrange them in ascending order of growth rate. That is, if function $g(n)$ immediately follows function $f(n)$ in your list, then it should be the case that $f(n)$ is $O(g(n))$. Please prove your claims.

1. $f_1(n) = 2^{\sqrt{(\log n)}}$
2. $f_2(n) = 2^n$
3. $f_3(n) = n^{4/3} = 2^{4/3 \log n}$
4. $f_4(n) = n(\log n)^3$
5. $f_5(n) = n^{\log n} = 2^{(\log n)^2}$
6. $f_6(n) = 2^{2^n}$
7. $f_7(n) = 2^{n^2}$

Solution

1. $f_1(n) = 2^{\sqrt{(\log n)}}$
2. $f_4(n) = n(\log n)^3$: $\sqrt{(\log n)}$ grows very slowly, so the 2 in f_1 will be raised to small exponents.
3. $f_3(n) = n^{4/3} = 2^{4/3 \log n}$: $\lim_{n\to\infty} \frac{n(\log n)^3}{n^{4/3}} = \lim_{n\to\infty} \frac{(\log n)^3}{n^{1/3}} = \lim_{n\to\infty} \frac{(\log n)}{n^{1/9}} = 0$ as exponentials grow faster than logarithms (shown in class).
4. $f_5(n) = n^{\log n} = 2^{(\log n)^2}$: Assuming this is in base 2, $(\log n)^2$ grows faster than $4/3 \log n$ when $n > 2^{4/3}$.
5. $f_2(n) = 2^n$: n grows faster than $(\log n)^2$.
6. $f_7(n) = 2^{n^2}$: n^2 grows faster than n.
7. $f_6(n) = 2^{2^n}$: 2^n grows faster than n^2 (discussed in class).