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Notes to Instructor or TA: If needed, include here any special notes for TAs or instructor; delete if no notes

Problem
Consider the following variant of the stable marriage problem. There are $2n$ people, each of whom completely ranks the other $2n - 1$ people in order of preference (with no ties). For example, Alice, Bob, Carlos, and Don might have the following rankings:

Name	Preference		
Alice	Bob	Carlos	Don
Bob	Alice	Carlos	Don
Carlos	Don	Alice	Bob
Don	Carlos	Alice	Bob

The goal is to find a stable perfect matching: *i.e.,* n pairs of people such that no two people prefer each other to their current matches. We saw in class that a stable perfect matching always exists for instances of the stable marriage problem. Do stable matchings always exist for this problem? If so, provide a proof. If not, give a counterexample.
Solution
No, stable matchings do not always exist for this problem. Here is a counterexample:

Name	Preference		
Alice	Bob	Carlos	Don
Bob	Carlos	Don	Alice
Carlos	Don	Bob	Alice
Don	Bob	Carlos	Alice

There exists a love triangle between Bob, Carlos, and Don. It is inevitable that two members of the love triangle end up matched to each other. Say Bob matches with Carlos, then Don is matched with Alice. In this case, Don would rather be with Carlos than with Alice, and Carlos would rather be with Don than with Bob. Furthermore, if we tried breaking this instability by matching Carlos and Don together, then Alice would be matched with Bob. An instability is then formed as Bob would rather be with Don than with Alice, and Don would rather be with Bob than with Carlos.