Solution

a) If there are more clubs than students, then no such set T exists because no individual person can be a treasurer for more than one group by the pigeonhole principle.
First make a bipartide graph $G = (S, Y, E)$ where $S = \{x_1, ..., x_k\}$ is the set of students part of a club, Y is a collection of subsets of S denoting clubs and where elements of Y_i are students that are members of the club, and $E = \{(x, Y_i) : x \in Y_i\}$, meaning that an edge is made from a student in S to a club in Y if the student is part of the club. A successful set of tresurers corresponds to a set $T = \{s_1, ..., s_n\} \subseteq S$ such that each $s_i \in Y_i$ and there exists only one tresurer per group.
To determine whether such a set T exists, transform G into a flow network G' by:

1. Directing all edges of G from S to Y.
2. Adding a vertex s with edges to all $x \in S$, and a vertex t with edges from all $y \in Y$ to t.
3. Giving every edge in G' capacity 1.

Claim: The n clubs can each get a treasurer if and only if G' has a flow of value n.

Proof. $\Rightarrow$ Assume the set of treasurers exists and let A be the set of edges representing the assignment of students to the club they are treasuring. Let f be a flow on G' with all edges having flow 0. Augment f as follows: For each edge $\{x, y\} \in A$, add 1 to the flow of the directed edges $(s, x), (x, y)$, and (y, t) in G'. Note that doing so preserves flow conservation, and increases the value of the flow f by 1. At the end of this process, no edge has flow exceeding capacity since no edges ever receive more than 1 unit of flow. Moreover, a student in S receiving a flow of 1 must not treasure another club because of flow conservation. Also, a student being treasurer means that they have an edge in A. The value of this flow is n since the set of treasurers exists so $|A| = n$, meaning that every club received a unit of flow. Thus, all edges from Y to t have 1 unit of flow.
$\Leftarrow$ Assume there is an integer flow f of value n. Then each edge (y, t) must have 1 unit of flow, and so each club y must have an incoming edge (from some $x \in S$ with 1 unit of flow). Each $x \in S$ can have at most 1 flow leaving it since edges (s, x) have a capacity of 1. Thus, the flow of n from clubs to the sink correspond to n students being treasurers of said club. □

Space Complexity: $|S||Y| + 2$ vertices * at most every student is is every club $|S||Y| + n + |S| = O((|S||Y|)^2)$
Time Complexity: $O(|V||E|) = O((|S||Y|)^2)$
b) The bipartide graph G we created might not always have a perfect matching. A certificate we can use to verify that there does not exists a perfect matching is one similar to Hall's Theorem, that is that for some $X \subseteq S$, $|\Gamma(X)| < |X|$. This means that for some subsets of S, there are not enough conncetions to Y. Assume there is a sub-collection $Y_{i_1}, ..., Y_{i_k}$ such that $|Y_{i_1}, ..., Y_{i_k}| < k$. This means that in this sub-collection of groups, there are less members than groups. In other words, some students in S do not have sufficient edges to Y. In fact, this means that a perfect matching cannot be made from S to Y. Conversely, if no such a set $T \subseteq S$ exists, then some students do not have sufficient edges to a set of groups. Meaning that some groups are short of students.