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Solution

Claim: Any instance of Set Packing can be solved using Multiple Interval Scheduling.

Proof. Multiple Interval Scheduling is in NP because we can check if jobs overlap given a scheduling of at least k jobs in polynomial time. We show that Multiple Interval Scheduling is NP-complete by showing a reduction from Set Packing to Multiple Interval Scheduling.

Suppose we have a universe $U = \{u_1, u_2, \dots, u_j\}$ and disjoint subsets B_1, B_2, \dots, B_n of the universe and a number k . We can map elements of U to disjoint intervals from 9 A.M to 5 P.M and subsets correspond to jobs taking up some intervals. This takes time proportional to the size of our universe.

We claim that a solution of Set Packing is "yes" iff the answer to Multiple Interval Scheduling is also "yes." Suppose we have a set packing solution S of size at least k subsets. Since the subsets are disjoint, then the jobs they map to do not overlap. Conversely, if we have a solution to the Multiple Interval Scheduling problem laid out, then each job corresponds to a disjoint subset of our universe U . There are at least k nonintersecting jobs, so it must be the case that there are at least k disjoint subsets of U . \square