Solution

This problem is in NP since a claimed set $\mathcal{P}$ of directed paths in G can be checked to have k pairwise vertex-disjoint paths in polynomial time. Since this appears to be a packing problem, lets try a reduction from Set Packing. An instance of Set Packing is a collection $S_1, ..., S_m$ of subsets of a set $S = s_1, ..., s_n$ and an integer k; a solution is a collection $S_{i_1}, ..., S_{i_k}$ of the sets, no two of which intersect.

To reduce this to the problem posted, imagine that we build a strongly connected graph $G = (V, E)$ where $V = S$. The collection of subsets $S_1, ..., S_m$ corresponds to a collection of vertices $V_1, ..., V_m$, where vertices in the same subset form a directed path. In this reduction a set $\mathcal{P}$ can be found.

We claim that a solution to Set Packing is "yes" if and only if a solution to the reduction is also "yes." Suppose we have a Set Packing solution Y of at least k subsets. Since the subsets are disjoint, the path they map to are pairwise vertex-disjoint. Conversely, each path $\mathcal{P}$ corresponds to a subset in an instance of Set Packing. These paths are vertex-disjoint, so the subsets must be disjoint as well.