

**I collaborated with:**

**Solution**

Claim: Any instance of Vertex Cover can be solved using Hitting Set.

*Proof.* First, we must show a certificate for Hitting Set. Given a set  $A$ , and collection of sets  $B_1, B_2, \dots, B_m$  we can check if  $H$  is a hitting set by showing  $H \cap B_i \neq \{\}$ . We can simply check if for all elements  $h \in H$ , some element  $h$  is in  $B_i$  for  $0 \leq i \leq m$ . If no element  $h$  is in a subset  $B_i$  then  $H$  is not a Hitting Set. This takes time proportional to the size of  $H$  and to the size of all subsets  $B_i$ .

Let  $G = (V, E)$  be a graph. We want to show that we can find a vertex cover for  $G$  of size at most  $k$  by solving the Hitting Set problem. First, we must transform an instance of vertex cover to an instance of the hitting set problem. We can make every vertex of  $G = \{v_1, v_2, \dots, v_n\}$  an element of a set  $A = \{v_1, v_2, \dots, v_n\}$ . Vertices  $u$  and  $v$  are in the same subset of the collection if they are connected by an edge. Thus, we have converted a vertex cover problem to a hitting set problem.

A vertex cover  $C$  of  $G$  gives us a hitting set for this reduces problem, because vertices in  $C$  are incident to every edge. Elements of  $C$  are elements of a hitting set because subsets of  $A$  are made up of incident vertices, and elements of  $C$  are incident to every edge. Now, we can find a hitting set  $H$  of  $A$  of size at most  $k$  in polynomial time. This set  $H$  contains at least one element from each subset in the collection, so an element  $h_i$  is connected to all vertices of subset  $B_i$ . This means that the elements of  $H$  are incident to all edges of  $G$ , because they connect to all vertices of our collection.  $\square$