

I collaborated with: TA

Solution

Let G be a node-capacitated network. Let G^* be an empty graph. For every node v in G add two vertices v_{in} and v_{out} to G^* and an edge connecting them with capacity equal to c_v . If two vertices in G are connected, then connect the corresponding pairs in G^* with an edge of capacity equal to the capacity of the node where the original edge comes out of. Now, run the FordFulkerson algorithm on G^* to find an $s - t$ maximum flow in G^* which corresponds to such flow in G .

Time Complexity: Building $G^* = O(|V| + |E|) + \text{FordFulkerson algorithm } O(|E||V|CC) = O(|E||V|CC)$

Space Complexity: $O(|V| + |E|)$

Proof. Claim: The analogue of the Max-Flow Min-Cut Theorem holds true.

We can find a minimum $s - t$ cut in an edge-capacitated network using the FordFulkerson algorithm. This cuts through edges of G^* and separates the network. These edges that are cut correspond to vertices in G . By removing these vertices, we have found a minimum $s - t$ cut for the node-capacitated network G . We can now see there is a one-to-one correspondence between G and G^* . The sum of the edges across the minimum $s - t$ cut in G^* corresponds to the maximum flow of G^* , so the sum of the capacities of the vertices in G that correspond to these edges results in a maximum flow in G . \square