1. (a) If NP $\neq$ coNP AND P = NP then since any language $L \in NP$ is a member of P, its complement $\overline{L}$ must belong to P and therefore $\overline{L} \in NP$ and finally $L = \overline{\overline{L}} \in coNP$. Thus $NP \subseteq coNP$. By similar reasoning, if $L \in coNP \Rightarrow \overline{L} \in NP \Rightarrow \overline{L} \in P \Rightarrow L \in P \Rightarrow L \in NP$ and therefore $coNP \subseteq NP \Rightarrow NP = coNP$.

   (b) If X is NP-complete, then $X \in NP$ and therefore $\overline{X} \in coNP$. In addition, for any $L \in coNP$, we know that $\overline{L} \in NP$ and therefore $\overline{L} \leq_p X$. This means we can find a polynomial computable function f_L such that $f_L(w) \in X$ iff $w \in \overline{L}$. This same function will clearly have the property that $f_L(w) \in \overline{X}$ iff $w \in L$. Therefore, for any $L \in coNP$, $L \leq_p \overline{X}$. Therefore $\overline{X}$ is coNP-complete.

2.

We will show that 3-SAT $\leq_p$ SCOVER by describing a mapping $f(w) = \langle (S_1, S_2, ...S_n), k\rangle$ where if w is not a valid 3-SAT formula, f simply returns a pair containing 2 disjoint sets with $k = 1$. If ϕ is a valid $3 - SAT$ formula then f returns a pair with k equal to the number of variables in ϕ and $(S_1, S_2, ...S_n)$ describing the following $2k$ sets. We assume that the variables and clauses of ϕ are numbered counting from 0. If x is the ith variable in ϕ we include the set $x_T = \{i\} \cup \{k + j \mid j$ is the number of a clause containing x $\}$ and the set $x_F = \{i\} \cup \{k + j \mid j$ is the number of a clause containing $\overline{x}$ $\}$. The first set includes the number of the variable and the number of all clauses that would be satisfied if the variable were assigned true. The second set contains the number of the variable x and thenumbers corresponding to all the clauses that would be satisfied if x were assigned false.

Given a satisfying assignment for ϕ, if for each x we include x_T in our cover if x is assigned true and x_F if x is assigned false then the resulting collection of sets forms a k-cover. Clearly the collection is of size k. It includes the number corresponding to each variable. Finally, since the assignment satisfies all the clauses in ϕ, for each clause there must be some x_T or x_F included in the collection such that the clause's number is included in that set.

Similarly, if we are given a collection of k of the x_Ts and x_F that forms a cover the collection must include exactly one of the pair x_T and x_F for each variable since each variable's number only appears in that pair and each variable number must be covered. If we create a truth assignment by setting x based on whether x_T or x_F is in the cover, then the truth assignment will satisfy ϕ since for each clause, there must be a variable whose setting makes the clause true or that clause's number would not have been covered.

3.

We will show that 3-COLOR is NP-complete using a reduction from $\neq$-3SAT.

Following the hints, given a 3-CNF formula ϕ, first, create a set of 3 independent nodes that form a clique. In any coloring, these nodes will have to be assigned three different colors. Call one of these colors TRUE, another FALSE, and the third WHITE.[1]

Next, create a pair of connected nodes for each variable x and its negation $\overline{x}$ and connect both nodes in each pair to the WHITE node in the 3-clique. This ensures that in any 3-coloring of the final graph, for each $x/\overline{x}$ pair, one will be painted TRUE and the other will be painted FALSE.

Finally, for each clause in the formula ϕ, create a 3-clique with one node corresponding to each literal in the clause. Connect each node in the clique to the corresponding node in the set of pairs created for variables.

Now, suppose that we have a $\neq$-assignment for the formula ϕ. To see that the graph we have created will then have a 3-coloring, first color each of the nodes in the variable pairs TRUE or FALSE to correspond to its color in the $\neq$-assignment. Now, consider one of the 3-cliques included for a clause of ϕ. Since we have an $\neq$-assignment, we know that two of the nodes in the clique are connected to variable nodes that are painted the same color (without loss of generality, we can say TRUE). Call these nodes A and B. The third node, C, is connected to a variable node painted FALSE. We can color the clique with 3 colors by making C TRUE, A FALSE, and B WHITE.

On the other hand, suppose we are given a 3-coloring of the graph. As suggested in the construction, we will name the colors TRUE, FALSE, and WHITE based on which colors are used for the first clique described in the construction. This ensures that all the variable nodes (both positive and negated versions) will be painted TRUE and FALSE in such a way that we can use the colors as the basis for a truth assignment. Now, if all of the nodes for the literals in any of the clause cliques were all painted TRUE or all painted FALSE, it would be impossible to paint the clauses 3-clique using just the three colors WHITE, TRUE, and FALSE. Therefore, if we have been given a 3-coloring, we know that the literals used in each clause have been assigned some mixture of the values true and false. Thus, the assignment induced by the coloring is an $\neq$-assignment.

3-COLOR is clearly polynomial verifiable, Therefore 3-COLOR $\in NP$. The construction we have described can certainly be accomplished in polynomial time. Thus, $\neq$-SAT $\leq_p$ 3-COLOR and therefore 3-COLOR is NP-complete.

[1] Following Sipser's hint for this part of the construction leads to two unnecessary nodes. If you eliminate all but the WHITE node in the palette, the fact that this node is still included in one triple of interconnected nodes for each variable ensures that the combination of this node and the variable "dumbbells" will still required three colors.