

**I collaborated with: TA**

**Solution**

The algorithm will turn this problem into a network flow and use the Ford-Fulkerson algorithm to determine whether every client can be connected simultaneously to a base station. We start by making a graph  $G$  with a source  $s$  and a sink  $t$  vertex. We make every client and base station a vertex of  $G$ . Now, we go through every client and check if its distance from every base station is within  $r$ ; if it is, we make an edge with capacity 1 from the client to the base station within distance. We then make an edge of capacity 1 from the source  $s$  to every client. We also make edges from every base station to  $t$  with capacities  $L$ . We have now turned this problem into a network flow. We now run the Ford-Fulkerson algorithm and if the max flow equals  $n$  we return true, and return false if otherwise.

Time Complexity: Add every client and base station to  $G$  :  $O(n + k)$  + Determine edges between clients and base stations  $O(nk)$  + adding edges from  $s$  to clients and from base stations to  $t$  :  $O(n + k)$  + Ford-Fulkerson:  $O(|E||V|C) = O(|E||V|C)$

Space Complexity:  $G = (V, E)$  where  $|V| = (n + k + 2)$  and  $|E| = (n + 2n + k)$  in the worst case.

*Proof.* Claim: If the maxflow equals  $n$ , then every client can be connected simultaneously to a base station.

If the maxflow equals the number of clients  $n$ , then we know that edges from  $s$  to every client has a flow of 1. We know that for every vertex, the flow coming into it has to equal the flow leaving it. Thus, having a max flow of  $n$  means that every client is connected to a base station by the nature of our graph. This obeys the range and load conditions as clients are only connected to base stations within  $r$  distance away and base stations have edges of capacity  $L$  coming out of them.  $\square$