Solution

The algorithm terminates since at least one vertex becomes tight after each iteration of the while loop. Given sufficient iterations of the while loop, all vertices will be tight and we will have some properly colored edges. Let C be the set of all properly colored edges produced upon the termination of the algorithm. In order to show that Approx3Color is a 3/2-approximation algorithm for MAX-3-COLOR, we can show that Approx3Color properly colors at least 2/3 of all edges. We show $|C| \geq \frac{2}{3}|E|$. We can derive the following statements:

- $|C| = \frac{1}{2}\sum_{v \in V} deg_j(v)$, where $j \in \{r, g, y\}, j \neq f(v)$.

- $\sum_{v \in V} deg_j(v) \geq \sum_{v \in V} deg_{f(v)}(v)$, because all of our vertices are tight.

- $\frac{1}{2}\sum_{v \in V} deg_{f(v)}(v) \geq \frac{1}{3}\sum_{v \in V} deg(v)$, because there are three possible colors and nonproperly colored edges must form at least a third of the edges.

- $\sum_{v \in V} deg(v) = 2|E|$

We can put it all together as such:

$$|C| = \frac{1}{2}\sum_{v \in V} deg_j(v) \geq \frac{1}{2}\sum_{v \in V} deg_{f(v)}(v) \geq \frac{1}{3}\sum_{v \in V} deg(v) = \frac{2}{3}|E| \tag{1}$$

$$2|C| \geq \frac{2}{3}\sum_{v \in V} deg(v) = \frac{4}{3}|E| \tag{2}$$

$$|C| \geq \frac{2}{3}|E| \tag{3}$$

Time Complexity: $O(|E|)$ because the algorithm has to find a color j such that $deg_j(v) < deg_{f(v)}(v)$ and at most $|E|$ edges need to be checked * $O(|V|^2)$ as some formerly tight neighborhood might no longer be tight after an iteration of the while loop = $O(|V|^2|E|)$.
Space Complexity: $O(|V| + |E|)$