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Notes to Instructor or TA: If needed, include here any special notes for TAs or instructor; delete if no notes

Problem
Decide whether you think the following statements are true or false. If a statement is true, give a short explanation why it's the case. If it's false, give a counterexample.

(a) In every instance of the Stable Matching Problem there is a stable matching containing a pair (m, w) such that m was ranked first on the preference list of w and w was ranked first on the preference list of m.

(b) Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then the pair (m, w) belongs to every possible stable matching for this instance.

Solution

1. False. Let the following table be an input to the Stable Matching Problem

Name	Preference	
Alice	Bob	Don
Bertha	Don	Bob
Bob	Bertha	Alice
Don	Alice	Bertha

   In this case, a stable matching will be made even though no two people have each other as their top preference.

2. True, a stable matching is defined as one where for every unmatched $\{m, w\}$, either m prefers his match to w, or w prefers her match to m. Using the P-R algorithm, the man will propose to his first choice, and the woman will be engaged and never trade down, or trade up to get with him if she is taken already.