**I collaborated with:** went to TA sessions

**Question**
Chapter 10, Problem 2. (a) Hint: See if you can figure out how to break the problem into subproblems that give the recursion $T(n,d) = 3T(n,d-1)+p(n)$ for some polynomial $p()$; then solve the recurrence. (b) Hint: Think about the two assignments that set every variable to 0 and every variable to 1, respectively. How far can an arbitrary assignment $\Phi$ simultaneously be from these two assignments? Divide and conquer....

**Solution**
a) Run Time: $O(3^d * p(n))$

*Proof.* Base Case: Let $d = 0$. If Explore($\Phi$,d) returns "yes" then $\Phi$ is a satisfying assignment that is at most $d$ distance away from $\Phi$. Also, if Explore($\Phi$,d) returns "no" then $\Phi$ is not a satisfying assignment.
Inductive Hypothesis: Let $k \geq 0$ and suppose the algorithm returns "yes" if and only if there exists a satisfying assignment $\Phi$ such that the distance from $\Phi$ to $\Phi$' is at most $k$.
Inductive Sep: If Explore($\Phi, k + 1$) returns "yes," then one of the recursive calls to Explore($\Phi_i$,k) returned "yes." Using our inductive hypothesis, we know that $\Phi_i$ has distance $k$ to a satisfying assignment. Thus, $\Phi$ has distance at most $k + 1$ to a satisfying assignment. Conversely, suppose $\Phi$ has distance $k + 1$ to a satisfying assignment $\Phi$'. Let $C$ be a clause unsatisfied by $\Phi$. Since $\Phi$' satisfies $C$, then it must disagree with $\Phi$ in at least one variable. Thus, one of the assignments $\Phi_i$ that changes this variable is $k$ distance away from $\Phi$'. By the inductive hypothesis, this call returns "yes" so the call Explore($\Phi, k + 1$) returns "yes." □

b) Let $\Phi_0$ be an assignment of variables such that they are all set to 0 and $\Phi_1$ be an assignment of variables such that they are all set to 1. A satisfying assignment will be at a distance of at most $n/2$ from one of these two assignments. We call Explore($\Phi_0$,n/2) and Explore($\Phi_1$,n/2) and see if either returns "yes." The running time of each call is $O(p(n) * 3^{n/2}) = O(p(n) * (\sqrt{3})^n)$