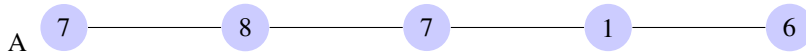
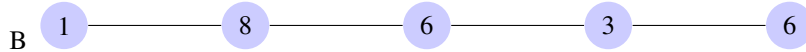


I collaborated with:**Problem**

Answer Exercise 1 (page 312) in Chapter 6 of your text. For part (c) include a justification of correctness and a time/space complexity analysis.

Solution

The algorithm will return the set of nodes $\{8,6\}$, while the independent set of maximum total weight is $\{7,7,6\}$.



The algorithm will return the set of nodes $\{1,6,6\}$, while the independent set of maximum total weight is $\{8,6\}$.

C The recurrence $opt(v_i) = \max(w_i + opt(v_{i-2}), opt(v_{i-1}))$ tells us a dynamic programming approach for solving this problem.

Algorithm 1 returns an independent set of maximum total weight

Start with A equal to the empty array of size n

function OPT(v)

if v is adjacent to only one node of greater index or v is the last node **then**

$A[i] = w_i$

return $A[1]$

else if $A[i] > 0$ **then**

return $A[i]$

else

$A[i] = \max(w_i + opt(v_{i-2}), opt(v_{i-1}))$

return $A[i]$

end if

end function

This algorithm builds the array of maximum total weights for independent sets for all nodes from v_1 to v_n . We can then just find the maximum entry of A and trace subsequent maximum values of the remaining array to build up our answer. We will now prove why it works.

Proof. Base Case: Let g be a graph of only one node v_0 . The independent set of maximum total weight is that only containing v_0 . Our algorithm will see that v_0 is the last node of g and will update $A[0] = w_0$. It will then return $\{v_0\}$ as it corresponds to the maximum weight.

Inductive Hypothesis: Assume our algorithm works for all graphs that have up to k nodes where $k \geq 1$.

Inductive Step: We need to show our algorithm always returns an independent set of maximum total weight for a graph of $k + 1$ nodes. Let G be a graph of k nodes labeled v_1 to v_k . If we add another node v_0 then we will need to see if v_0 is part of our independent set of maximum total weight. We will either include its weight to a maximum set of vertices not adjacent to v_0 (Note that this existing set corresponds to a graph of less than $k + 1$ nodes and our inductive hypothesis assures the set is of maximum weights). Otherwise, we do not consider w_0 in our maximum set and we use the independent set of maximum total weight of G (by our inductive hypothesis, the algorithm returns the proper set for G as it has k nodes). \square

Time Complexity: $O(n)$ to build array + $O(n)$ to find set = $O(n)$

Space Complexity: Array of size n $O(n)$ + n recursive calls for stack frame = $O(n)$