I collaborated with: TA

Solution
This algorithm will use the algorithm from problem 4 (problem 23 in the book) to determine the set of $upstream, downstream$, and $central$ vertices of the graph G. Now, if the set of $central$ vertices is empty, then G has a unique minimum s-t cut.
Time Complexity: Problem 4 algorithm: $O(|E||V|C)$ + Check if there are $central$ vertices: $O(1) = O(|E||V|C)$
Space Complexity: The tree sets of vertices $upstream, downstream,$ and $central$, along with the graph.
Claim: Having no $central$ vertices ensure that there is a unique s-t cut of a network flow G.

Proof. $Central$ vertex v is defined such that at least one minimum s-t cut (A, B) for which $v \in A$, and at least one minimum s-t cut (A, B) for which $v \in B$. If there are no such vertices, then that means that all vertices are either $upstream$ or $downstream$. Furthermore, this means that all vertices are always part of the same s-t cut and always on their corresponding side; thus, there is only one s-t cut. □