Solution

a) Claim: We want to show that if a DFS tree T_1 and a BFS tree T_2 of a connected, undirected graph $G = (V, E)$ are equal, then $G = T_1 = T_2 = T$.

Proof. Suppose, for the sake of contradiction, the claim is false. That is, $G \neq T$ even if $T_1 = T_2$. This means that there exists an edge e that is in G but does not belong to T. A BFS traverses the vertices of a graph a level at a time. Meaning that vertices at level i are of distance i from the root. Thus, an edge e in G that is not in T connects vertices at consecutive levels or at the same level of T_2. On the other hand, a DFS traversal of G would follow the path of e up or accross the level considered by the BFS traversal. This is because DFS traverses down edges until it reaches an end before going back on any other unexplored path. Therefore, $T_1 \neq T_2$. We have reached a contradiction because it is given that $T_1 = T_2$. We have shown that if a DFS tree is the same as a BFS tree then the graph is equal to said tree.

□