I collaborated with: went to TA sessions

Question
Chapter 10, Problem 3. Hint: There is a Hamiltonian path from v1 to vn if and only if, for some vi there is an edge from vi to vn (final edge) and a Hamiltonian path (in G vn) from v1 to vi. This suggests dynamic programming: if we had a table H[S, 1, j] to tell us if there is a Hamiltonian path from v1 to vj using only the vertices of S, wed be set!
Solution
Consider a triplet (H, i, j) where $1 \leq i, j \leq n$ and S is a subset of V. Let $H[S, i, j]$ tell us if there is a Hamiltonian path from v_i to v_j using only the vertices of S. Our base case is when $i = j$, and there is a path from a vertex to itself. Given a larger subset we can spend $O(n)$ to check if there is a path using those vertices and using the answer to smaller problems. There are 2^n subsets, so our runtime is $O(2^n * p(n))$.

Proof. $H[S, i, j]$ is true if and only if for some v_k there is an edge from v_k to v_j (final edge) and a Hamiltonian path (in $G - \{v_j\}$) from v_i to v_k. Thus, we set $H[S, i, j]$ to be true if and only if $(v_k, v_j) \in E$ and $H[S - \{v_j\}, i, k]$ is also true. □