**I collaborated with: TA**

**Solution**
Using FordFulkerson on a network flow $G = (V, E)$, we can build a residual graph using the following rules, where $e = (u, v) \in E, e^R = (v, u)$, flow $f(e)$, capacity $c(e)$:

$$c_f(e) = \left\{ \begin{array}{ll} c(e) - f(e), & \text{if } e \in E \\ f(e), & \text{if } e^R \in E \end{array} \right\}$$

Now, grab all the vertices reachable from $s$ in the residual graph. These vertices are $upstream$. Then, switch the direction of every edge in $G$, and grab all the vertices reachable from $t$ in the residual graph. These vertices are $downstream$. All the remaining vertices are $central$.
Time Complexity: $O(|E||V|C)$
Space Complexity: $O(|E| + |V|)$

*Proof.* Let $A$ be the set of all vertices reachable from $s$ in the residual graph of a network flow $G$. Let $U$ be the set of all $upstream$ vertices.
Claim: $A = U$.
In order to show $A = U$, we must show $U \subseteq A$ and $A \subseteq U$. We know $A$ is a min-cut because all edges not reachable from $s$ have maximum flow and are not part of a min-cut, thus all $upstream$ vertices are in $A$ (7.9 in the book). Therefore, $U \subseteq A$. Now, assume a vertex $v \in A$ is $downstream$. This means that $v$ is always on the sink side of every min-cut. However, we know $v$ is reachable from $s$. This is a contradiction, so $v$ is $upstream$ and $A \subseteq U$. We have showed that $U \subseteq A$ and $A \subseteq U$ which implies $A = U$. □