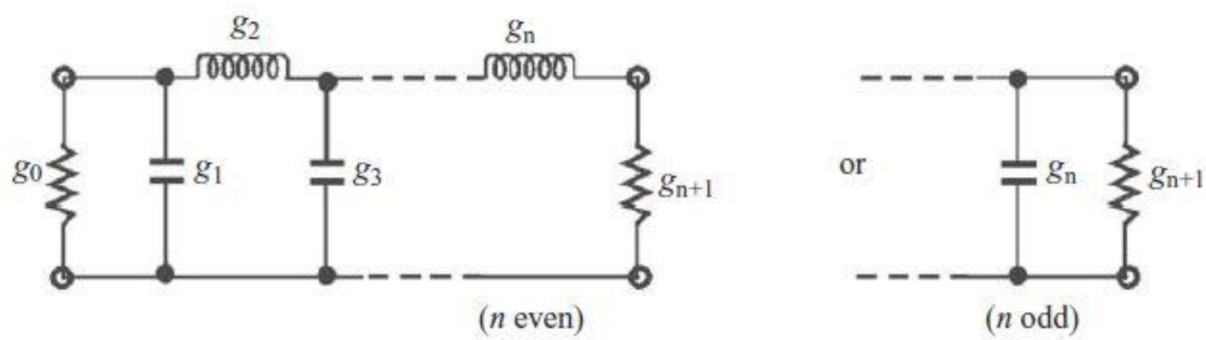
Part 1: Design Stepped-Impedance Low-Pass Filter

1.1. Choosing the Filter Type

Microwave filter are two-port network, reciprocal, passive, linear device which attenuate heavily the unwanted signal frequencies while permitting transmission of wanted frequencies. There are several types of microwave filters.

- Butterworth filter – no ripple in pass band, medium roll-off, maximally flat response
- Chebyshev filter – ripples in pass band, fast roll-off, amplitude of ripples are the same for all frequencies.
- Inverse chebyshev filter – flat pass band, rippled stop band
- Elliptic filter – fastest roll-off, ripples in both pass band and stop band
- Bessel filter – no ripple in both pass band and stop band, slowest roll-off

As I mentioned the characteristics of each filter, Butterworth filter design is suitable for the given criteria. The main reason for selecting the Butterworth filter is its maximally flat response and no ripples in the pass band. More than that, we can design the butterworth filter easier than other filters.



1.2. Cut-off Frequency

For the given criteria, they haven't mentioned the cut-off frequency of the filter. So we have to estimate the cut-off frequency from the given parameters. We have the transfer function for butterworth filter design as given below.

$$|H(j\omega)| = \frac{G_0}{\sqrt{1+(\frac{\omega}{\omega_c})^{2n}}}$$

Where; n=order of filter, ω_c=cut-off frequency

At the cut-off frequency, the gain is -3dB. It's $1/\sqrt{2}$ times of the gain at the beginning. But we have to find the order of the filter before we find the cut-off frequency. I have found the order of the filter as 6. I have explained how I found the order of the filter in the next sub-chapter in the report. Then we can find the cut-off frequency. We know that the maximum gain we can have for frequencies below 1GHz is -2dB. We can apply this to our equation.

$$10\ \log\frac{1}{\sqrt{1+\left(\frac{1}{\omega_c}\right)^{12}}}\leq -2$$

By solving this equation, we can have a minimum cut-off frequency of 1.05509 GHz. I choose a cut-off frequency of 1.2 GHz because of the ripples obtain in the stop band.

1.3. Calculate Order of the Filter

We can obtain the order of the filter by using the stop band attenuation and stop band frequency.

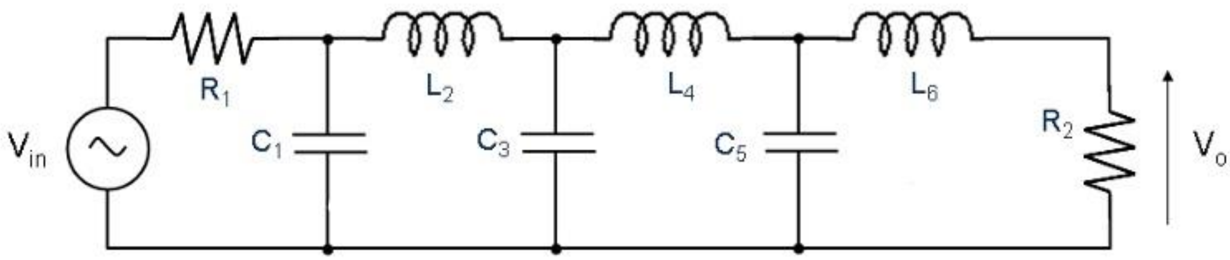
$$n\ \geq\ \frac{\log[10^{0.1L_{AS}}-1]}{2\ \log\omega_s}$$

Where; n=order, L_{AS}=stop band attenuation, ω_s= stop band frequency/cut-off frequency.

We have L_{AS}= 20dB and ω_s= 1.5(cut-off frequency=1.2 GHz, stop band frequency=1.8 GHz). Then by applying these values for above equation we have,

$$n\geq 5.67$$

Therefore, we can select the order of the filter as 6. Then the filter will look as follow.



1.4. Calculate Element Values

In order to design the stepped impedance low-pass filter, we have to determine the element values of the butterworth filter of 6th order. We can calculate the element values as given below.

$$g_0=\ g_{n+1}=1,\qquad\qquad g_i=2\ \sin[\frac{(2i-1)\pi}{2n}]\ for\ i=1,2,3,\dots\dots..,n$$

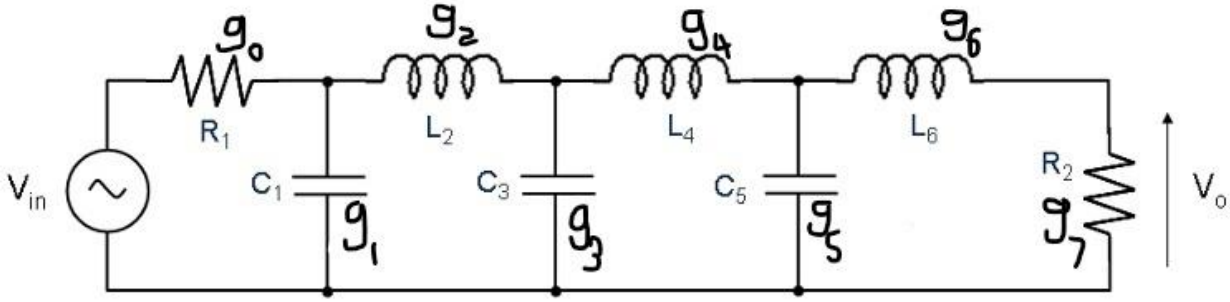
But we can have the elements easily by obtaining them from the table according to the order of the filter given below.

TABLE 3.1 Element Values for Butterworth Lowpass Prototype Filters ($g_0 = 1.0$, $\Omega_c = 1$, $L_{Ar} = 3.01$ dB at Ω_c)

n	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	2.0000	1.0								
2	1.4142	1.4142	1.0							
3	1.0000	2.0000	1.0000	1.0						
4	0.7654	1.8478	1.8478	0.7654	1.0					
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0				
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0			
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0		
8	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902	1.0	
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0

Since the order of the filter is equal to 6, we can obtain the element values as given below.

$g_0 = 1\,, g_1 = 0.5176\,, g_2 = 1.4142\,, g_3 = 1.9318\,, g_4 = 1.9318\,, g_5 = 1.4142, g_6 = 0.5176\,, g_7 =$



1.5. <u>Calculate Component Values</u>

Since we know the element values, we can calculate the component values C_1, L_2, C_3, L_4, C_5 and L_6 by using the following formulas.

$$L = \frac{Z_0}{g_0} \times \frac{\omega_c}{2\pi f_c} \times g \;\; Where;\; \omega_c = normalized\ cutoff, f_c = cutoff, Z_0 = filter\ impedance$$

$$C = \frac{g_0}{Z_0} \times \frac{\omega_c}{2\pi f_c} \times g \;\; Where;\; \omega_c = normalized\ cutoff, f_c = cutoff, Z_0 = filter\ impedance$$

Normalized cut-off frequency is 1. Calculated the component values are given below.

Component	Value
C_1	1.372pF
L_2	9.378nH
C_3	5.124pF
L_4	12.810nH
C_5	3.751pF
L_6	3.432nH

1.6. Calculate Electrical Lengths

Since I have calculated the element values of each component, next step is to calculate electrical lengths. We can calculate the electrical lengths for capacitors and inductors as shown below.

$$for\ capacitor;\ \beta_l = \frac{g}{R_0} \times Z_l$$

$$Where; g = element\ value, R_0 = filter\ impedance, Z_l = impedance\ of\ capacitor$$

$$for\ inductor;\ \beta_l = \frac{g}{Z_h} \times R_0$$

Before calculating the electrical lengths you should know the highest (inductor) and the lowest (capacitor) transmission line impedances. But we have some limitations for highest and lowest transmission line impedances.

- Minimum impedance = 7 ohm

  We are using the FR4 substrate ($\varepsilon = 4.4$) with thickness of 1.58mm and a cut-off frequency of 1.2 GHz. Impedances below 7 ohm allow any transverse resonance to occur at frequencies.

- Maximum impedance = 145 ohm

  Minimum copper strip width can be produced is 0.203 mm. Therefore we cannot have impedances greater than 145 ohm since the width of the copper will be lower than 0.203 mm.

Therefore I selected the lowest transmission impedance and the highest transmission line impedance as follows which gives a better maximally flat response for the filter.

Lowest transmission line impedance = 7 ohm

Highest transmission line impedance = 145 ohm

Then I calculated the electrical lengths for each component. I have shown the values in the table given below (electrical lengths are in degrees).

Element Value	Component	Electrical Length (β_l)
g_1 = 0.5176	C_1	4.1518^0
g_2 = 1.4142	L_2	27.9405^0
g_3 = 1.9318	C_3	15.4957^0
g_4 = 1.9318	L_4	38.1668^0
g_5 = 1.4142	C_5	11.3438^0
g_6 = 0.5176	L_6	10.2263^0

1.7. Calculate widths and lengths

Since we have found the electrical lengths, we can determine the lengths and widths of the stepped impedance microstrip line filter. We can obtain the physical widths (W) of low impedance and high impedance lines as shown below.

For inductor,

$$\left(\frac{W}{h}\right) = \frac{8e^{A}}{e^{2A}-2} \quad Where; A = \frac{Z_{0L}}{60}\sqrt{\frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{\varepsilon_r+1}\left(0.23+\frac{0.11}{\varepsilon_r}\right)} \ , h = height\ of\ substrate$$

For capacitor,

$$\left(\frac{W}{h}\right) = \frac{2}{\pi}\left[B-1-\ln(2B-1)+\frac{\varepsilon_r-1}{2\varepsilon_r}\left\{\ln(B-1)+0.39-\frac{0.61}{\varepsilon_r}\right\}\right] \quad Where; B = \frac{377\pi}{2Z_{0C}\sqrt{\varepsilon_r}}$$

We need the effective dielectric constant and the effective wavelength in order to calculate the physical lengths of the impedance lines. We can find those values from the formulas given below.

$$(\varepsilon_{eff}) = \frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{2}\left[\frac{1}{\sqrt{1+12\frac{h}{W}}}\right] \quad , \quad lambda = \frac{c}{f_c\sqrt{\varepsilon_{eff}}} \quad where; c = speed\ of\ light$$

Then I can calculate the lengths of impedance lines from the equation given below.

$$l = \frac{\beta_l}{2\pi}\times lambda$$

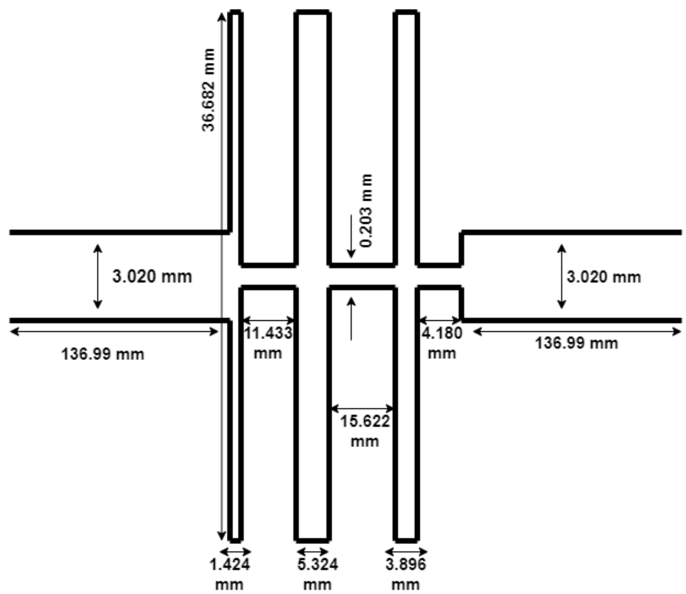
$$where;\ \beta_l = electrical\ length(radians), lambda = effective\ wave\ length$$

I calculated the widths and lengths of the transmission line as given in the table below.

Component	Impedance(ohm)	Electrical Length(degrees)	Width(mm)	Length(mm)
C_1	7	4.1518^0	36.6825	1.4247
L_2	145	27.9405^0	0.2031	11.4338
C_3	7	15.4957^0	36.6825	5.3241
L_4	145	38.1668^0	0.2031	15.6225
C_5	7	11.3438^0	36.6825	3.8966
L_6	145	10.2263^0	0.2031	4.1805
R_0	50	360^0	3.020	136.99

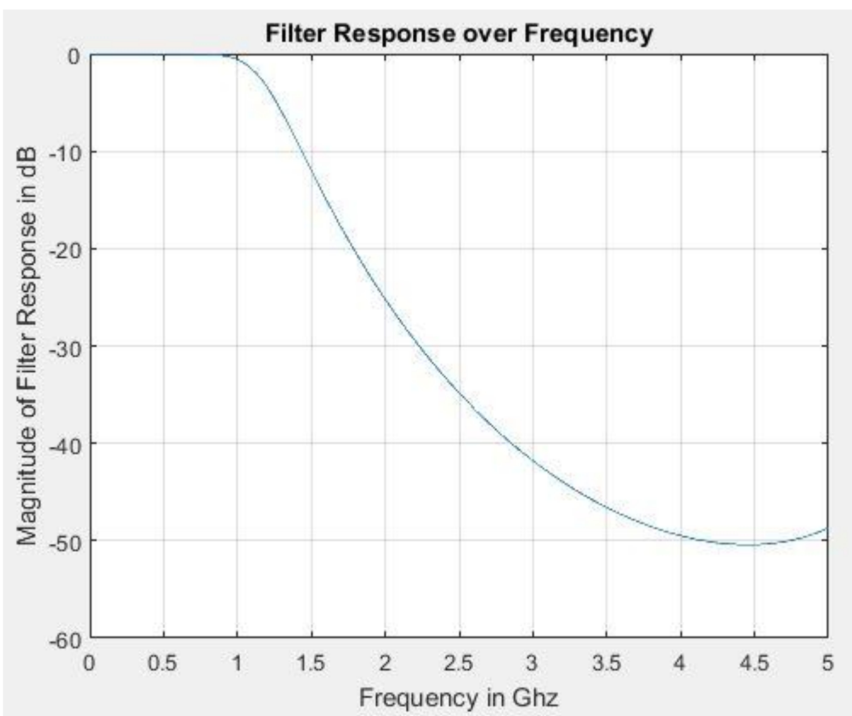
1.8. Design of the Filter

I have calculated the widths and lengths of both lowest and highest impedance transmission lines. Therefore I can design the stepped-impedance low pass microstrip filter.



1.9. Simulation in MATLAB

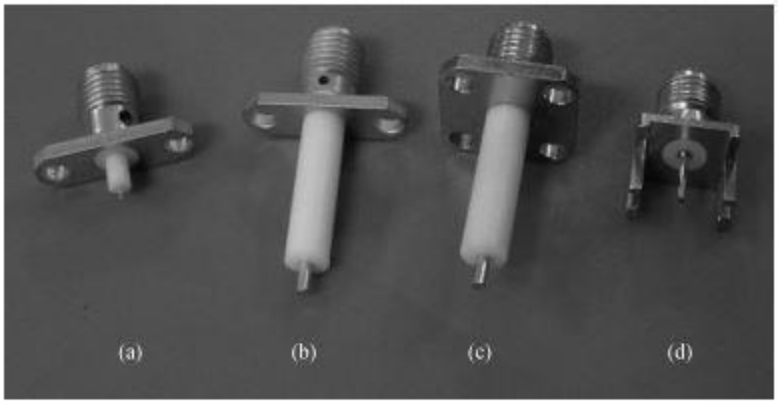
I simulated the stepped-impedance filter which I designed above for given parameters. I have plotted the Insertion loss over the frequency range of 0 – 5 GHz. The graph is shown below (I have attached the code at ANNEX 1).



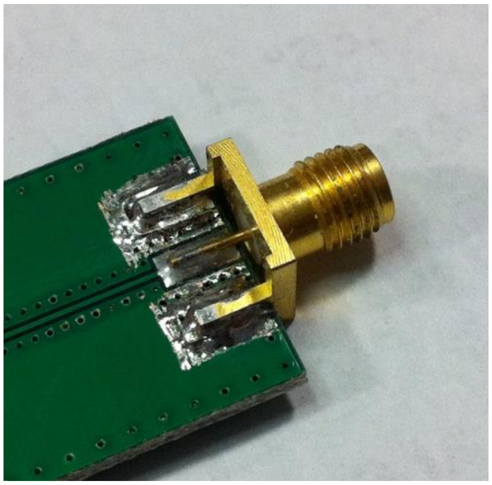
In here maximum loss when the frequency is less than 1 GHz is -0.9 dB and the minimum loss when the frequency is greater than 1.8 GHz is -20.43 dB. Therefore it fits for our conditions.

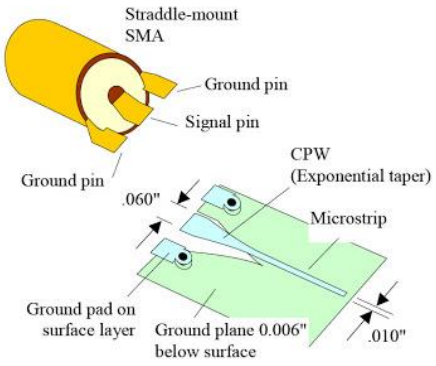
1.10. Connectors for Microstrip Board

SMA cables will be used to connect to the microstrip board. SMA is a very common, popular and easily available connector, which is used to excite a microstrip line or a microstrip antenna. They are a mainly 50 ohm probe with a central conductor extended to connect the microstrip element. Depending on the size of the substrate, the connector dimensions may be different. Following figure shows different SMA probes for microstrip circuits. All of these are male connectors.



They have different dimensions of central conductor and surrounding dielectric sleeve. We have a substrate with thickness of 1.58mm, a copper strip with thickness 35μm and an impedance line with width of 3.020 mm. Square flange male connector which is shown in above figure as type (d) is specially designed for microstrip circuits. Therefore, I recommend square flange male connector for the filter I designed above. Connectors are attached to the microstrip board as in the figure given below.





1.11. Substrates for Filter

I have used FR4 substrate for my filter design which have a dielectric constant of 4.4 but there are other dielectric substrates as well. There are several things we should consider when we select dielectric substrates.

- Surface wave excitation
- Dispersion of the dielectric constant and loss tangent of the substrate

- Copper loss
- Effects of temperature, humidity and aging

When we are choosing the type of substrate, surface wave excitation is the most important thing we should consider about. Surface waves can be excited at the dielectric-to-air interface. Surface waves give rise to end fire radiation. In addition they can lead to unwanted coupling between array elements. The phase velocity of surface waves is strongly dependent on the dielectric constant ε_r and thickness h of the substrate. The excited transverse electric mode (TE) frequency of filter can be calculated from the following formula.

$$f_c^{(n)} = \frac{n\,C}{4h\sqrt{\varepsilon_r - 1}} \quad Where;\ n = order$$

Therefore you can see that if we have a substrate of low dielctric constant we can have a high excited transverse electric mode (TE) frequency.

Therefore I select the **RT Duroid ($\boldsymbol{\varepsilon_r = 2.32}$)** material as dielectric substrate. There are some other reasons as well for selecting RT Duroid.

- Lowest electrical loss
- Low moisture absorption
- Uniform electrical properties over frequency

Since it has uniform electrical properties over frequency, it will give a maximum flat response for the filter.

Part 2: Vector Network Analyzer

Task 1

(i) Figure-8 (The 50 ohm terminations of Figure 5 are connected to the VNA test cable SMA ends.)

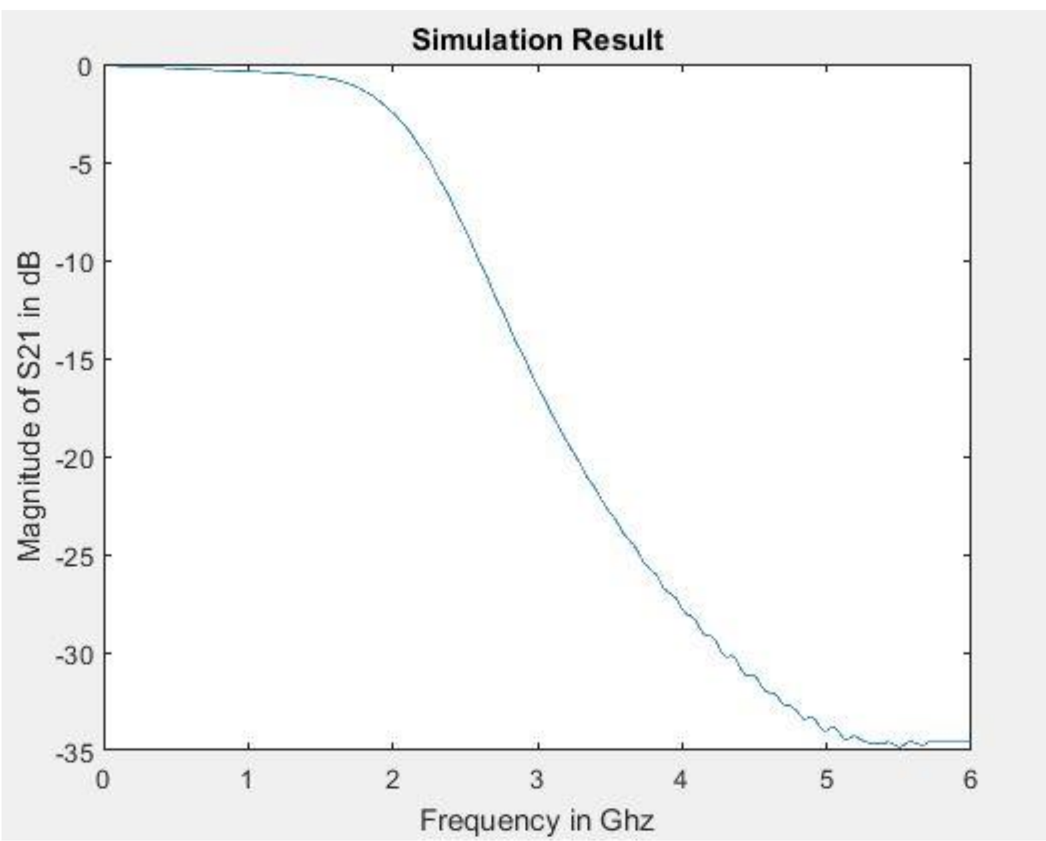
(ii) Figure-6 (The SMA ends of the two VNA test cables are connected together via the SMA adapter of Figure 4.)

(iii) 'Apply' button

1. Set the bandwidth and test power (**Enhancement button**)
2. Set the frequency range and measurement steps (**Calibration button**)
3. Select S_{21} calibration (in **Calibration window**)
4. Connect adaptors and test cable 2 to port 1 of the instrument
5. Connect ends of test cable together (the 'through' connection) and select **Through** (in **Calibration** window)
6. Connect 50 Ω load to the ends of test cables (to perform the isolation step) and select **Isolation** (in **Calibration window**)
7. Click **Apply** (in **Calibration** window)
8. If desired, for later use, save calibration (**File > save cal**)
9. Connect device to be tested and click **Start**

Task 2

I have plot the measured results in MATLAB. Results are shown below.



We can observe some important characteristics of this microwave component. Cut-off frequency of the response is 2.065 GHz which gives -3 dB attenuation. If we are operating pass band gain of 0 to -2 dB as we used in the designed filter, we have a pass band frequency of 1.926 GHz. If the minimum attenuation at the stop band is -20 dB as we used in the designed filter, this component have a stop band frequency of 3.265 GHz.

In the graph you can observe that there are some ripples in the measurements for high frequencies (> 3.5 GHz). Ripple has been increased with the increasing of frequency as well. By considering the characteristics of the types of low pass filter types I mentioned in the section 1.1, this microwave component fits with the characteristics of Inverse-Chabyshev low pass filter.

REFERENCES

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8. https://rf-tools.com/lc-filter/

ANNEX 1 (MATLAB code)

```
% Impedances
Z1=7; Z2=145; Z0=50;
% Cut-off frequency
fc=1.2;

j=sqrt(-1);
% Element Values
g1=0.517; g2=1.414; g3=1.932; g4=1.932; g5=1.414;
g6=0.517;g7=1.000;
% Electrical Lengths in radian
BL1=g1*Z1/Z0; BL2=g2*Z0/Z2; BL3=g3*Z1/Z0; BL4=g4*Z0/Z2; BL5=g5*Z1/Z0;
BL6=g6*Z0/Z2; BL7=g7*Z1/Z0;

k=0;
for f=[0:0.001:5]

A1=cos(BL1*f/fc); B1=j*Z1*sin(BL1*f/fc);
C1=(j/Z1)*sin(BL1*f/fc); D1=cos(BL1*f/fc);
A2=cos(BL2*f/fc); B2=j*Z2*sin(BL2*f/fc);
C2=(j/Z2)*sin(BL2*f/fc); D2=cos(BL2*f/fc);
A3=cos(BL3*f/fc); B3=j*Z1*sin(BL3*f/fc);
C3=(j/Z1)*sin(BL3*f/fc); D3=cos(BL3*f/fc);
A4=cos(BL4*f/fc); B4=j*Z2*sin(BL4*f/fc);
C4=(j/Z2)*sin(BL4*f/fc); D4=cos(BL4*f/fc);
A5=cos(BL5*f/fc); B5=j*Z1*sin(BL5*f/fc);
C5=(j/Z1)*sin(BL5*f/fc); D5=cos(BL5*f/fc);
A6=cos(BL6*f/fc); B6=j*Z2*sin(BL6*f/fc);
C6=(j/Z2)*sin(BL6*f/fc); D6=cos(BL6*f/fc);
M1=[A1 B1; C1 D1]; M2=[A2 B2; C2 D2];
M3=[A3 B3; C3 D3]; M4=[A4 B4; C4 D4];
M5=[A5 B5; C5 D5]; M6=[A6 B6; C6 D6];
T=M1*M2*M3*M4*M5*M6;
A=T(1,1); B=T(1,2); C=T(2,1); D=T(2,2);
Amp=2/(A+B/Z0+C*Z0+D);
k=k+1;
Y1(k)=Amp;

end;

plot([0:0.001:5], 20*log10(abs(Y1))); grid
xlabel('Frequency in Ghz');
ylabel('Magnitude of Filter Response in dB');
title('Filter Response over Frequency');
```