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Assignment 1 - Electronics

Q1]
a) Given : $V = 4V$
$l = 2m$
$A = 10^{-6} m^2$
$\rho = 1.7 \times 10^{-8} \Omega\text{-}m$
(density) $d = 8920\ kg/m^3$
atomic wt. $= 63.5\ gm/mol$
$V_{drift} = ?$
$J = ?$

By unitary method,
$63.5gm \longrightarrow 6.023 \times 10^{23}$ electrons [since per mole]
$8920 \times 10^3\ gm/m^3 \longrightarrow ?(n)$ electrons$/m^3$

$$\therefore n = \frac{8920 \times 10^3 \times 6.023 \times 10^{23}}{63.5}$$
$$= 846.065 \times 10^{26} \text{ electrons}/m^3$$

Now $\rho = \frac{1}{ne\mu}$

$$\mu = \frac{1}{ne\rho}$$
$$= \frac{1}{1.6 \times 10^{-19} \times 846.065 \times 10^{26} \times 1.7 \times 10^{-8}}$$
$$= \frac{10^{27}}{10^{26} \times 1.6 \times 846.065 \times 1.7}$$
$$= \frac{10}{2301.297}$$
$$= 4.34 \times 10^{-3}\ m^2 V^{-1} s^{-1}$$

a) continue...

$$V_{drift} = \mu E$$

(where $E = \frac{V}{l}$

$= \frac{4V}{2m}$

$= 2V/m$)

$\therefore v_{drift} = 4.34 \times 10^{-3} \times 2$

$= 8.68 \times 10^{-3}\ ms^{-1}$

$J = ?$

$R = \rho \frac{l}{A}$

$= \frac{1.7 \times 10^{-8} \times 2}{10^{-6}}$

$= 1.7 \times 2 \times 10^{-2}$

$= 3.4 \times 10^{-2}$.

$J = \frac{I}{A}$ $\left[\begin{array}{l} V = IR \quad \text{ohm's Law} \\ \frac{V}{R} = I \end{array} \right]$

$= \frac{V}{R \times A}$

$= \frac{4}{3.4 \times 10^{-2} \times 10^{-6}}$

$= \frac{4}{3.4 \times 10^{-8}}$

$= 1.176 \times 10^{8}\ Am^{-2}$.

b) Given: $E = 2\,V/cm$

$n = 4.4 \times 10^{28}$ electrons/m^3

$\rho = 1.54 \times 10^{-8}\ \Omega\text{-}m$

$\mu = ?$

$$\rho = \frac{1}{ne\mu}$$

$$1.54 \times 10^{-8} = \frac{1}{4.4\times10^{28} \times 1.6\times10^{-19} \times \mu}$$

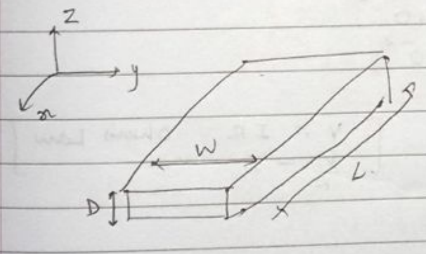
$$\mu = \frac{1}{10^{-27}\times10^{28} \times 4.4\times1.6\times1.54}$$

$$= \frac{1}{10 \times 4.4\times1.6\times1.54}$$

$$= 9.2237 \times 10^{-3}$$

$$= 9.22 \times 10^{-3}\ m^2 V^{-1} s^{-1}$$

Q2)



Given:
- $I = 200\,mA$
- in the x direction
- $V_y = 0.3\,mV$
- $V_x = 3\,mV$
- $B_z = 0.4\,T$
- $\mu = ?$
- $W = 6\,mm$
- $L = 20\,mm$
- $D = 2\,mm$

From derivation we know,

$$0 = \frac{dp}{dt} = -\frac{p}{\tau} + F \quad \text{(equation of motion)}$$

(Steady current)

$$0 = \frac{m}{ne\tau}\vec{J} - e[\vec{E} + \vec{v}\times\vec{B}]$$

$$\vec{E} = \frac{m}{ne^2\tau}\vec{J} + \frac{\vec{J}\times\vec{B}}{ne}$$

Since force & field balance each other
no current in $\hat{y}$

$$E_x\hat{x} + E_y\hat{y} = \frac{M}{ne^2\tau}(J_x\hat{x} + \cancel{J_y\hat{y}}) + \frac{1}{ne}[\cancel{J_y}B_z\hat{x} - J_xB_z\hat{y}]$$

2] continue

$$E_x \hat{x} + E_y \hat{y} = \frac{m}{ne^2\tau} J_x \hat{x} - \frac{1}{ne} J_x B_z \hat{y}$$

$\therefore E_x = \frac{m}{ne^2\tau} J_x$ $\qquad$ $E_y = \frac{-1}{ne} J_x B_z$ — (ii)

$= \frac{1}{\sigma} J_x$ — (i)

$\sigma = \frac{J_x}{E_x}$ — (iii)

$\therefore$ Hall's coefficient $R_H = \frac{E_y}{J_x B_z}$

Substituting (i) & (ii)

$$R_H = \frac{-1}{ne}$$

we know that $\mu = \frac{\sigma}{ne}$

$$\therefore \mu = \sigma R_H$$

substituting (iii),

$$\mu = \frac{-J_x}{E_x} \times \frac{E_y}{J_x B_z}$$

$$= \frac{-E_y}{E_x B_z}$$

$$= \frac{+V_y \times L}{+w \times V_x \times 0.4}$$

$$= \frac{0.3}{6} \times \frac{20}{3 \times 0.4}$$

$$= \frac{10}{12}$$

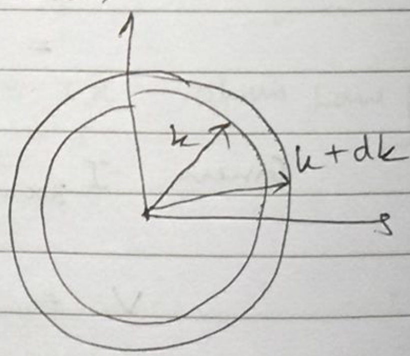
$$= 0.833 \ m^2 V^{-1} s^{-1}$$

Q3] To show : [in two dimension]

$$n = \frac{m}{\pi\hbar^2}\left[\mu + k_BT\log\left(1+\exp\left(-\frac{\mu}{k_BT}\right)\right)\right]$$

We know the relation between Fermi-Dirac statistics $f(E)$ - Probability an electron will occupy an energy state having energy E, and $g(E)$ - density of states per unit vol in 3D / area in 2D per unit energy range and n (conc. of electrons). as :

$$n = \int g(E)f(E)\cdot dE \quad \text{— (a)}$$



Finding no. of allowed k states in area between k & $k+dk$,

$$A = \pi k^2$$

$$dA = 2\pi k\,dk$$

$$g^{2D}(k)\cdot dk = \frac{2\pi k\,dk}{\left(\frac{2\pi}{L}\right)^2}$$

(since $L^2 = A$)

$$= \frac{2\pi k\,dk\,A}{4\pi^2}$$

$$= \frac{kA\cdot dk}{2\pi}$$

3] continued

$$g(k) = \frac{g^{2D}(k)}{A}$$

$$= \frac{k}{2\pi} \quad \text{— (iii)}$$

$$g(E) \cdot dE = 2\,g(k)\,dk$$

(since each k state involve 2 electrons with diff spin)

$$g(E) = \frac{2\,g(k)}{\frac{dE}{dk}} \quad \text{— (ii)}$$

(since $E = \frac{\hbar^2 k^2}{2m}$ since given parabolic energy dispersion)

$$\left[\frac{dE}{dk} = \frac{2\hbar^2 k}{2m} \right] \text{— (i)}$$

substituting (iii) & (i) in (ii),

$$g(E) = \frac{\cancel{2}\,\cancel{k}}{\cancel{2}\pi} \times \frac{\cancel{2}\,m}{\cancel{2}\,\hbar^2 \cancel{k}}$$

$$= \frac{m}{\pi \hbar^2}$$

$$f(E) = \frac{1}{e^{(E-\mu)k_BT} + 1}$$

Now back to equation (a),

$$n = \int_0^\infty g(E)\,f(E) \cdot dE$$

$$= \int_0^\infty \frac{m}{\pi \hbar^2} \left(\frac{1}{e^{(E-\mu)k_BT} + 1} \right)$$

Now we know that at $E > \mu$ $f(E) = 0$

(Assuming $T = 0\,K$)

3) continued

$$\therefore \quad n = \int_0^{\mu} \frac{m}{\pi\hbar^2}\left(\frac{1}{e^{(E-\mu)/k_BT}+1}\right)dE + \overbrace{\cancel{\int_{\mu}^{\infty} \frac{m}{\pi\hbar^2}\left(\frac{1}{e^{(E-\mu)/k_BT}+1}\right)dE}}^{0}$$

since $E > \mu$

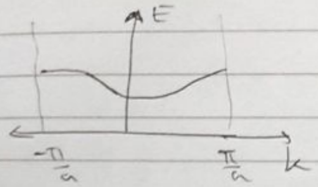
$$= \int_0^{\mu} \frac{m}{\pi\hbar^2}\left(\frac{1}{e^{(E-\mu)/k_BT}+1}\right)dE$$

$$= \frac{m}{\pi\hbar^2}\left[\left(\frac{E-\mu}{k_BT}\right) - \log\left(1+\exp\left(\frac{E-\mu}{k_BT}\right)\right)\right]_0^{\mu}$$

$$n = \frac{m}{\pi\hbar^2}\left[\mu + k_BT\log\left(1+\exp\left(-\frac{\mu}{k_BT}\right)\right)\right]$$

Hence Proved

4)



We know that by Bloch's theorem, wave function for an electron in a periodic potential is a plane wave modulated by a periodic function

$$\psi(x+na) = e^{ikna}\psi_k x$$

Since by energy schrodinger equation,

$$-\frac{\hbar^2}{2\underbrace{m_c^*}_{\text{effective mass}}}\frac{d^2\psi(x)}{dx^2} + U_0\psi(x) = E\psi(x)$$

At node, j: $\quad -\frac{\hbar^2}{2m^*}\left.\frac{d^2\psi(x)}{dx^2}\right|_{x_j} + U_0\psi(x_j) = E\psi(x_j)$.

4) continued

Taking the second order partial derivative,

$$m = \frac{\hbar^2}{2m^* a^2}$$

$a \rightarrow$ lattice parameter.

$$\therefore E\psi_j = (U_0 + 2t)\psi_j - t\psi_{j-1} - t\psi_{j+1}$$

$j, j-1, j+1 \rightarrow$ periodic nodes

$$\therefore \psi(x) = e^{ikx_j}, \quad \psi_{j-1} = e^{ik(x_j - a)}, \quad \psi_{j+1} = e^{ik(x_j + a)}$$

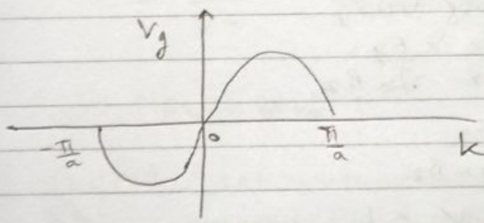
$$\therefore E e^{ikx_j} = (U_0 + 2m) e^{ikx_j} - m e^{ik(x_j - a)} - m e^{ik(x_j + a)}$$

$$E = (U_0 + 2m) - m e^{(-ika)} - m \exp^{(ika)}$$

$$E(k) - U_0 = 2m \left[1 - \cos(ka)\right]$$

Now we know group velocity $V_G = \frac{1}{\hbar}\frac{dE}{dk} = \frac{1}{\hbar} am \sin ka$

$\therefore$ Relation between V_G & k is a sine function



Q5) $R_{20^\circ c} = 16\,\Omega$

$\alpha_{20^\circ c} = 3 \times 10^{-3}$ /K

$T = 3t + 300$ K

At $t = 0$ sec, $T = (3\times 0) + 300$

$= 300$K

At $t = 66$ sec, $T = (3 \times 66) + 300$

$= 498$ K

Now $\rho_T = \rho_0 [1 + \alpha_0 \Delta T]$

$\frac{R_T A}{l} = \frac{R_0 A}{l} [1 + \alpha_0 \Delta T]$

[Since physical parameters remain constant]

$R_T = R_0 [1 + \alpha_0 \Delta T]$

Let reference temp. be $20^\circ C = 293$ K

$R_{300k} = R_{293k} [1 + \alpha_{293k} (300 - 293)]$

$= 16 [1 + 3 \times 10^{-3} (7)]$

$= 16 [1.021]$

$= 16.336\,\Omega$

Given $I_{300k} = 45$ mA

$= 45 \times 10^{-3}$ A

$V = I_{300k} \times R_{300k}$

$= 45 \times 10^{-3} \times 16.336$

$= 735.12 \times 10^{-3}$

$= 0.735$ V

$R_{498k} = R_{293k} [1 + \alpha_{293k} (498 - 293)]$

$= 16 [1 + 3 \times 10^{-3} (205)]$

$= 16 \times 1.615$

$= 25.84\,\Omega$

5) continue...

$I_{498K} = ?$ (After 66 sec)

$V = I_{498K} \times R_{498K}$ [Potential difference constant]

$I_{498K} = \dfrac{0.735}{25.84}$

$= 0.02844 \text{ A}$

$= 28.44 \text{ mA}$