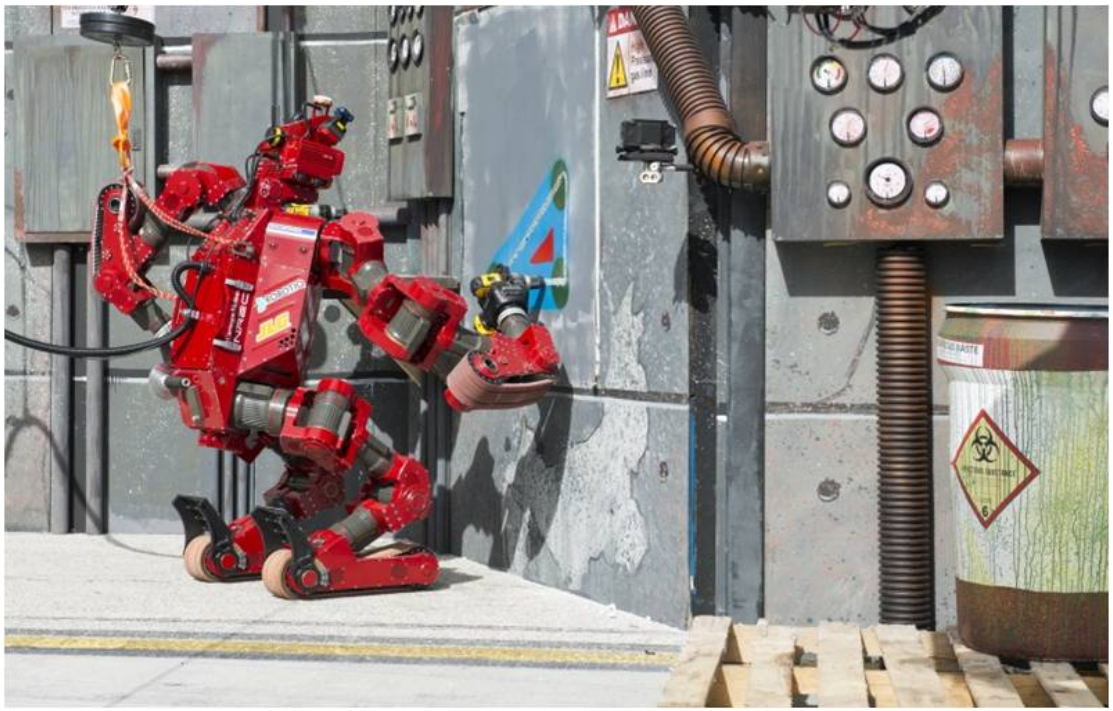
Algorithmic Motion Planning (236610)
Lecture 11—Task Space Planning

Oren Salzman

Computer Science Department, Technion

Today's lecture—Following Paths in Task Space

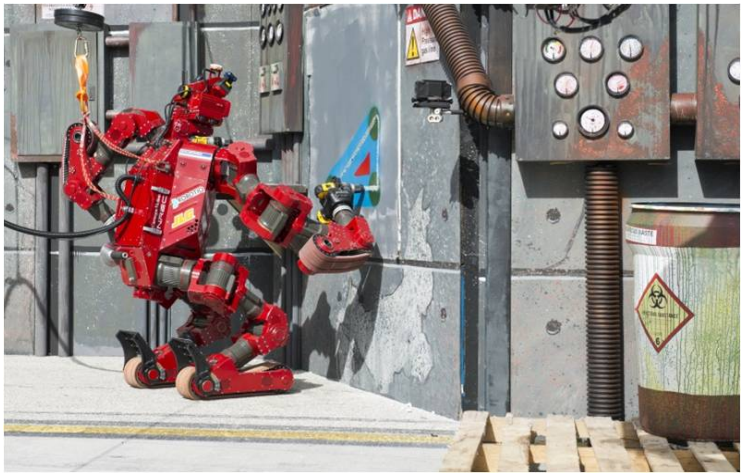
- Motivation & Problem definition
- Algorithmic approach
- Design optimization



Motivation & Problem definition

(informal) definition of Task space

The space describing all poses of the robot's end effector



Following task-space paths

Given a path ζ_{ref} in task space, compute a collision-free C-space path that maps "as closely" as possible to ζ_{ref}

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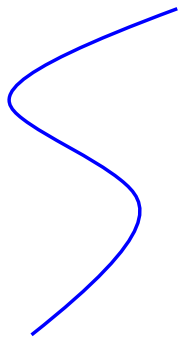

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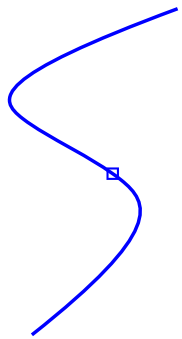

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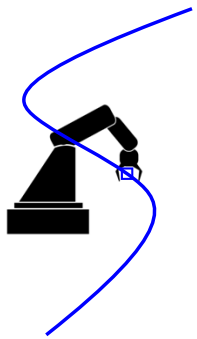

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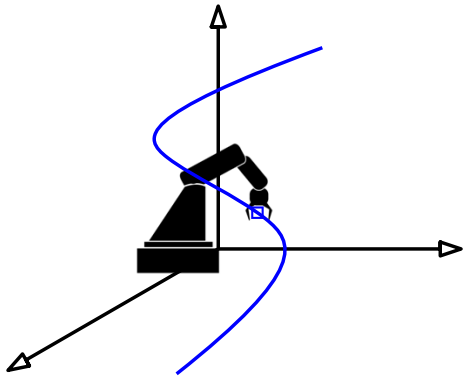

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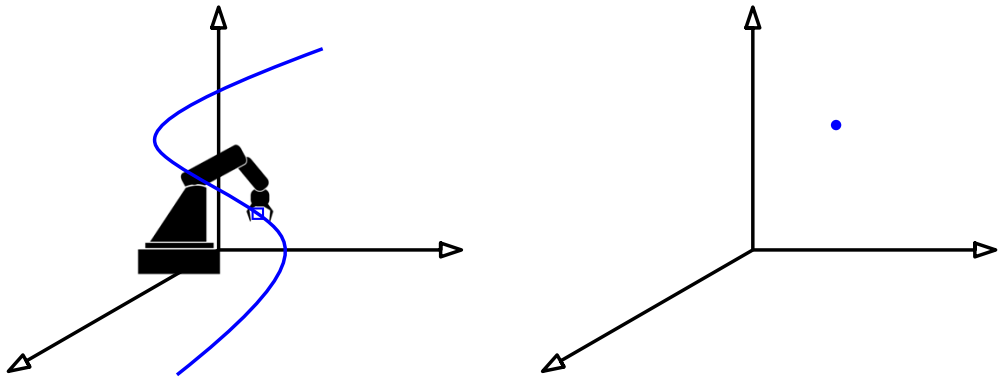

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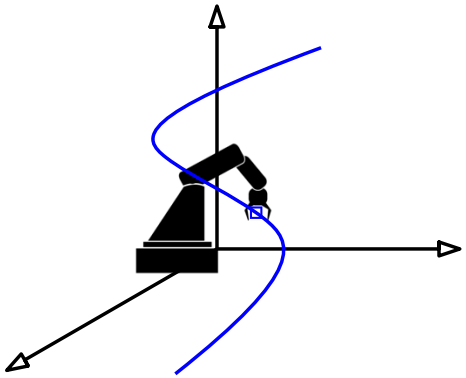

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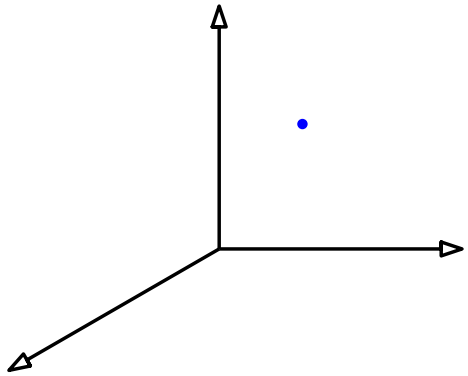

Configuration Space

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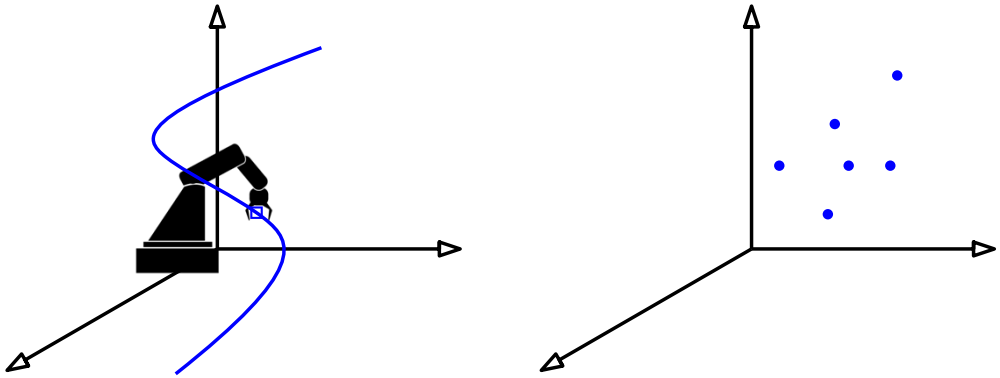

Task Space | Configuration Space

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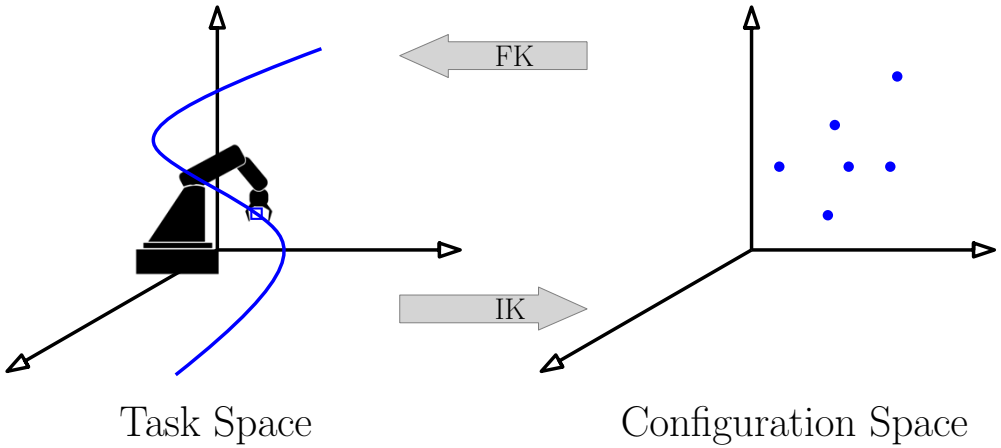

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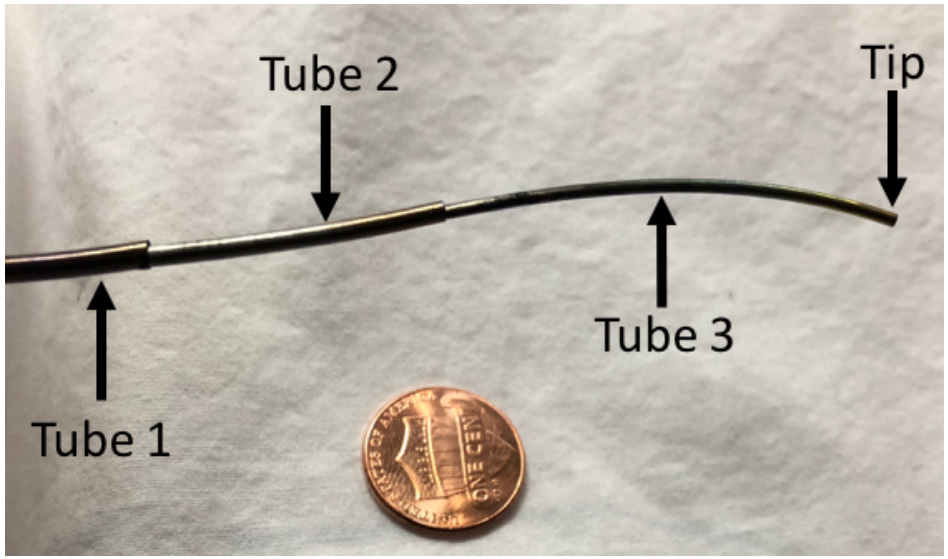
Motivation & Problem definition

Concentric-tube robots—tentacle-like medical devices composed of thin, pre-curved, telescoping tubes

- Controlled by axially rotating and/or telescopically translating individual tubes



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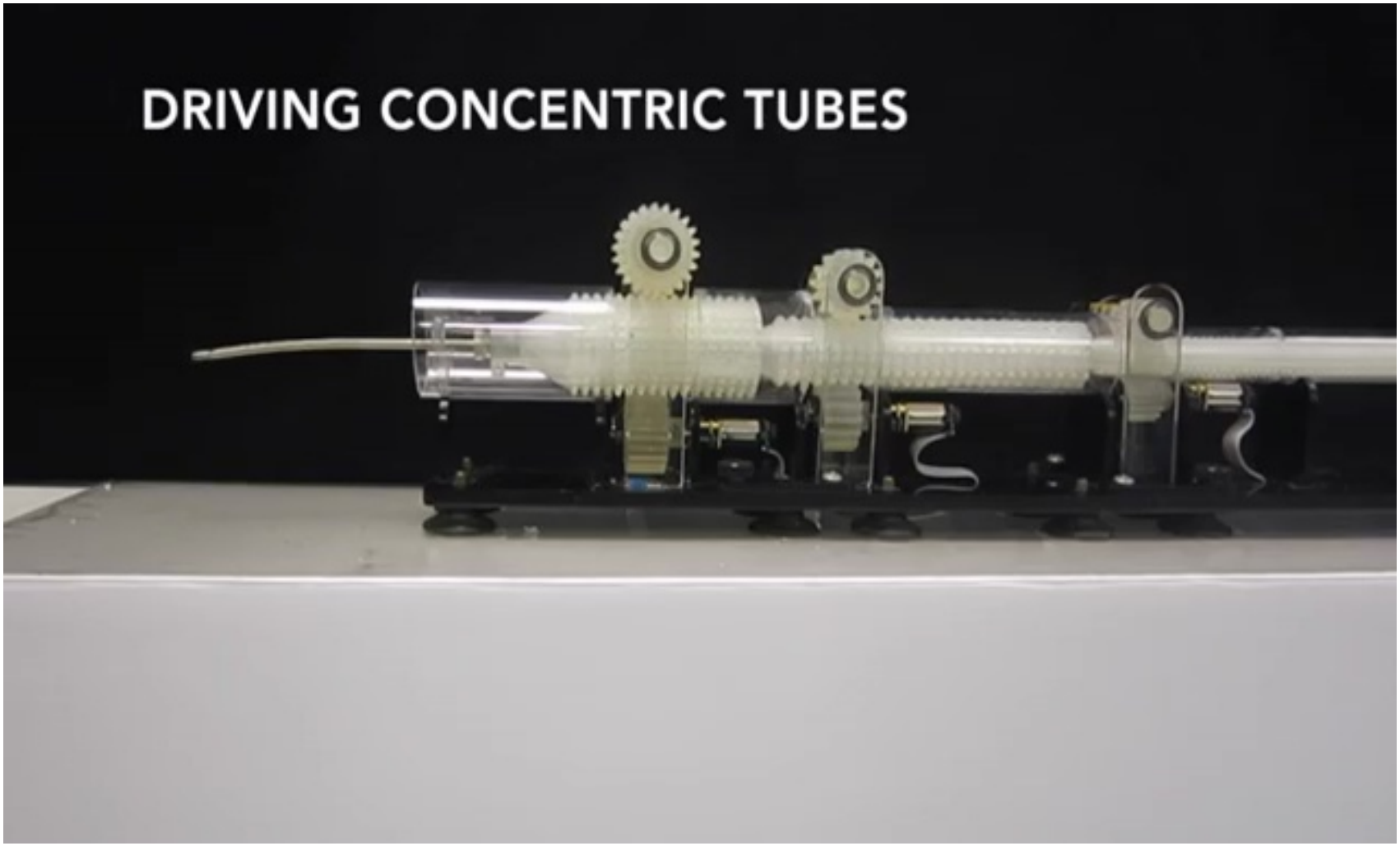

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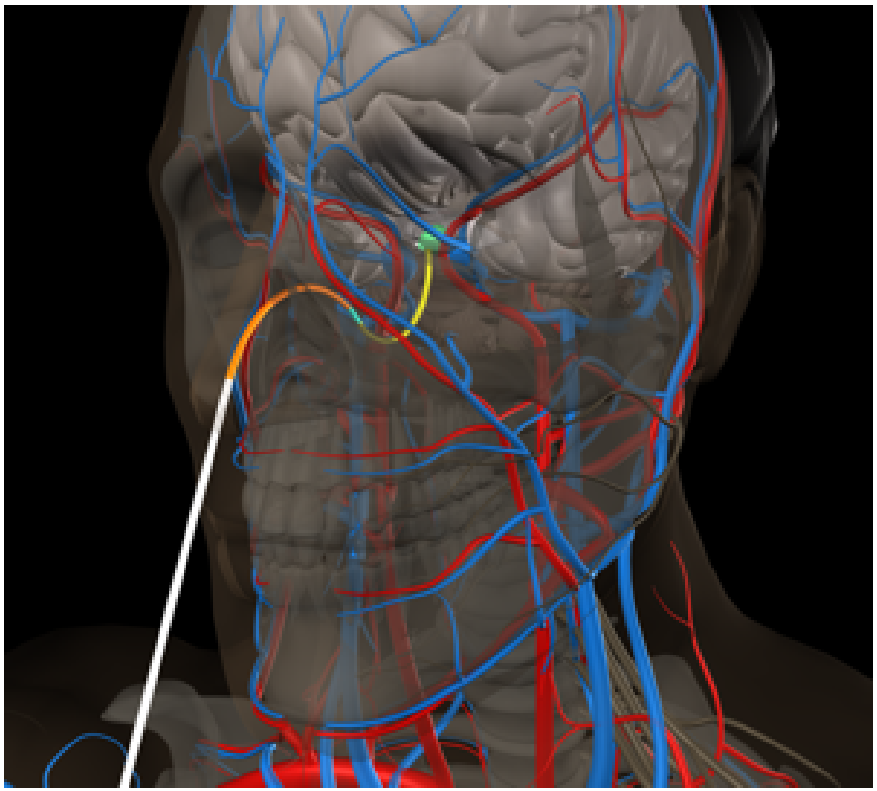
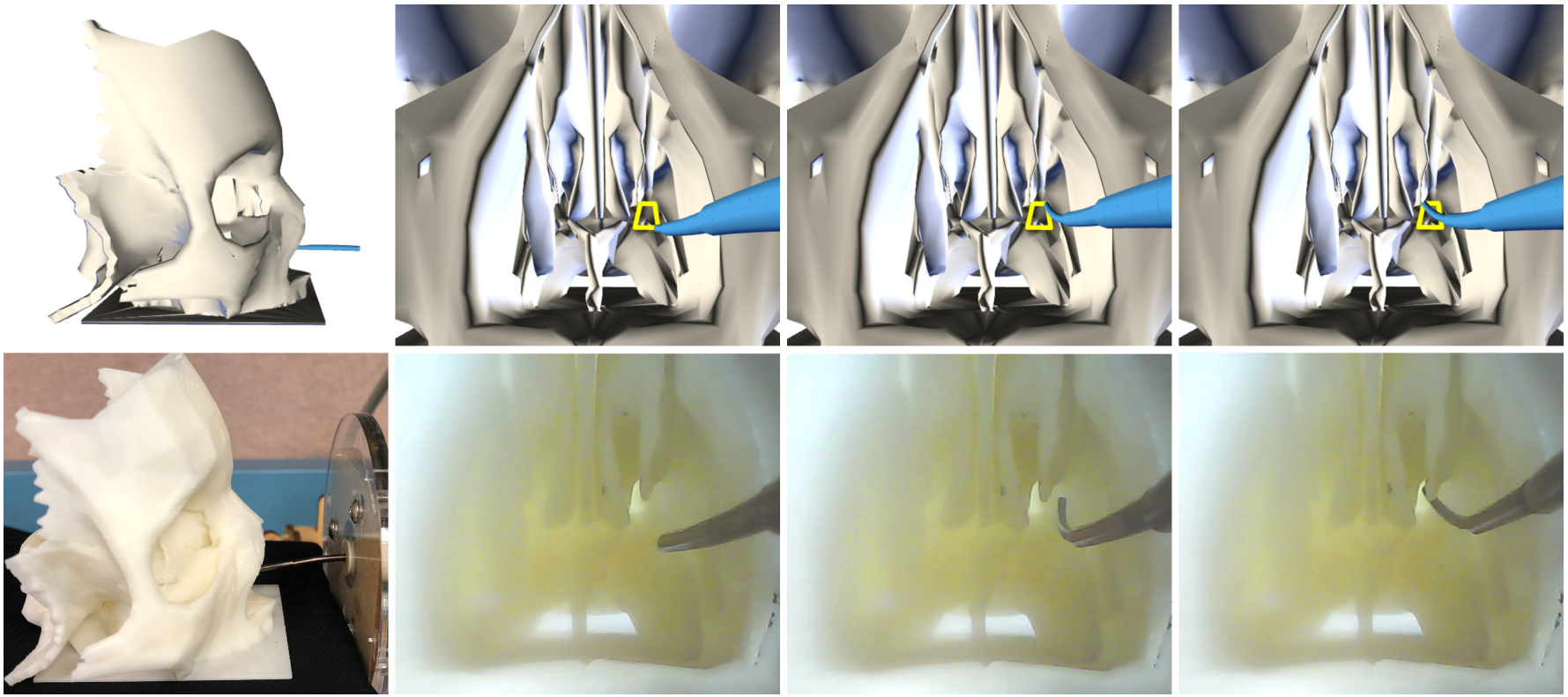


Use motion planning to follow surgeon's desired path as "closely-as-possible"

Motivation & Problem definition



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How do we measure distances ?

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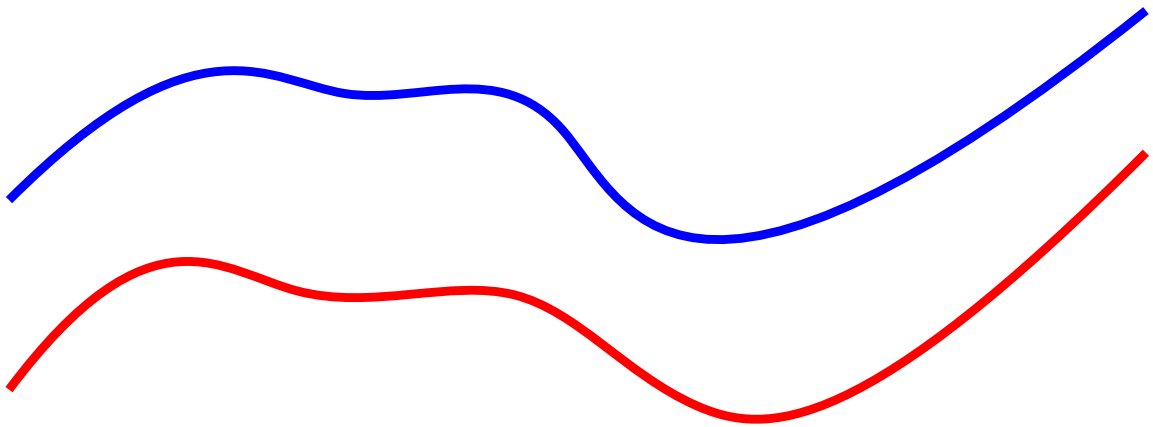
Measuring distances between paths

Key idea–borrow tools from Computational Geometry



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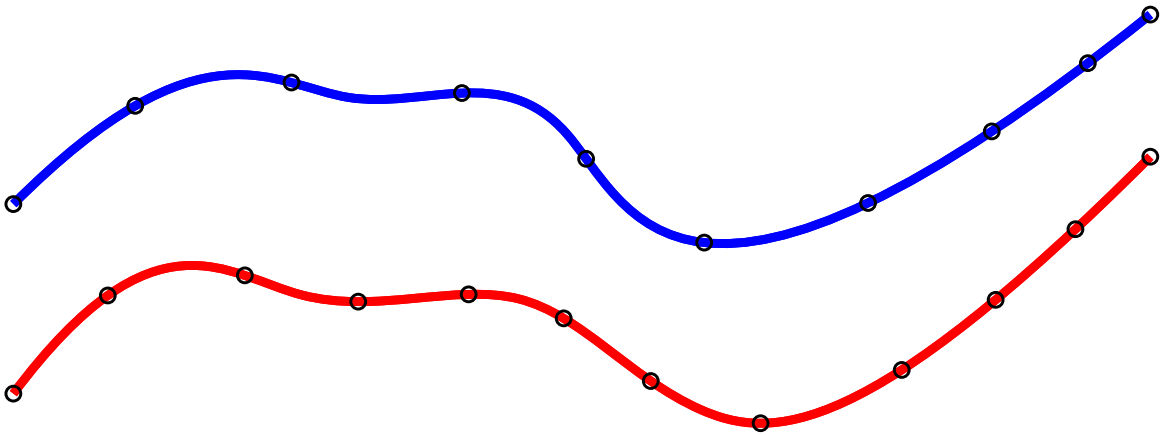

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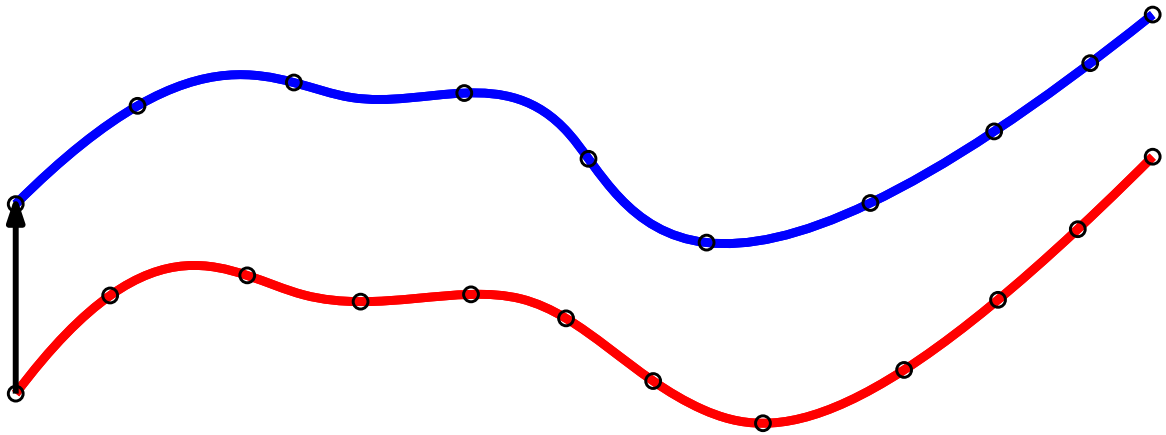

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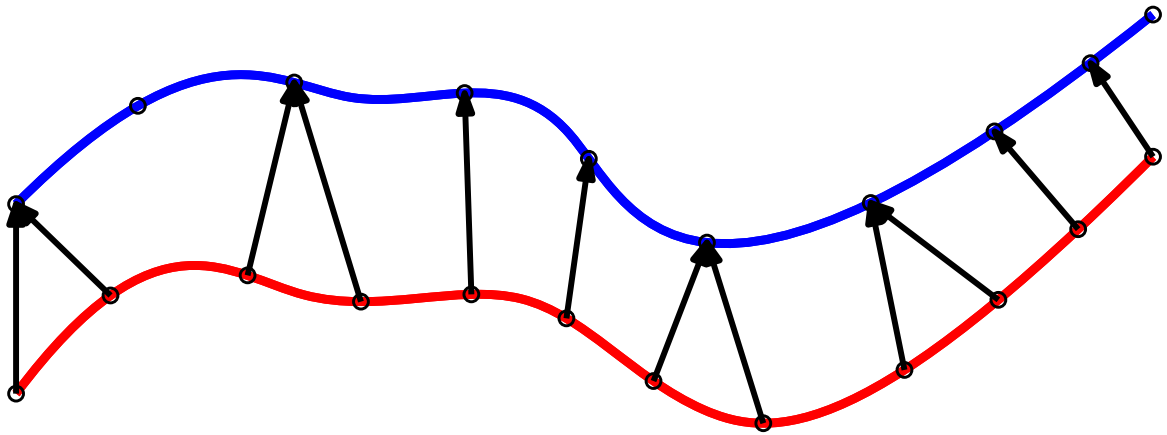

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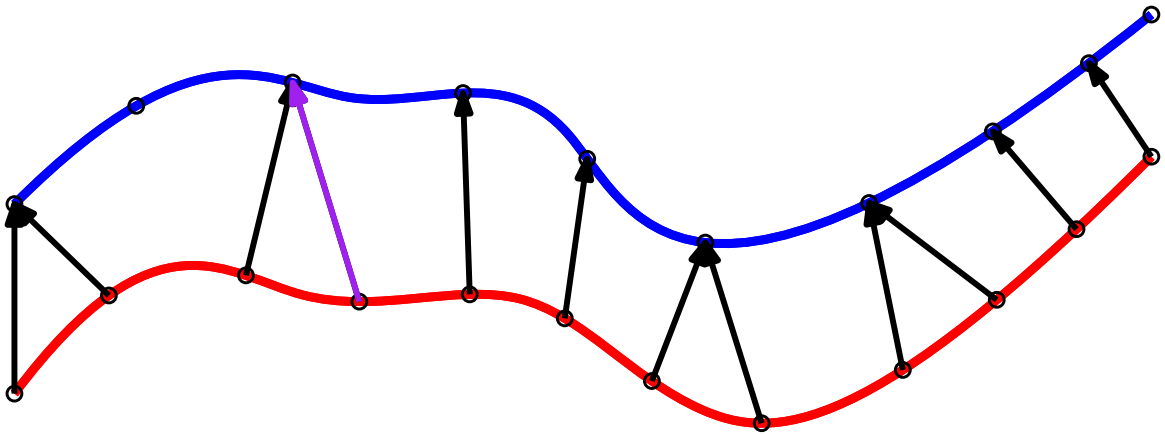

One-way Hausdorff distance

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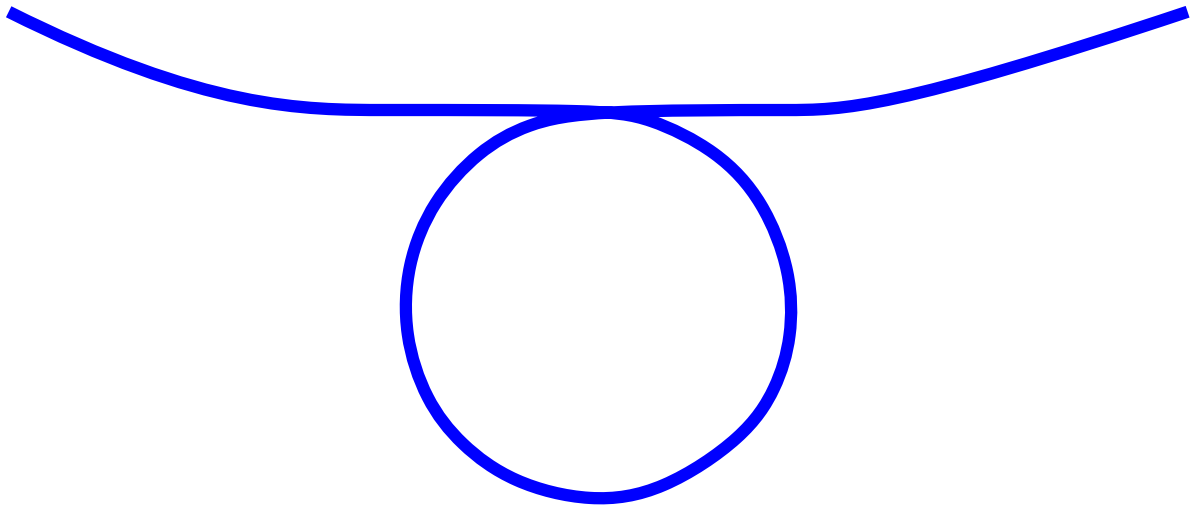

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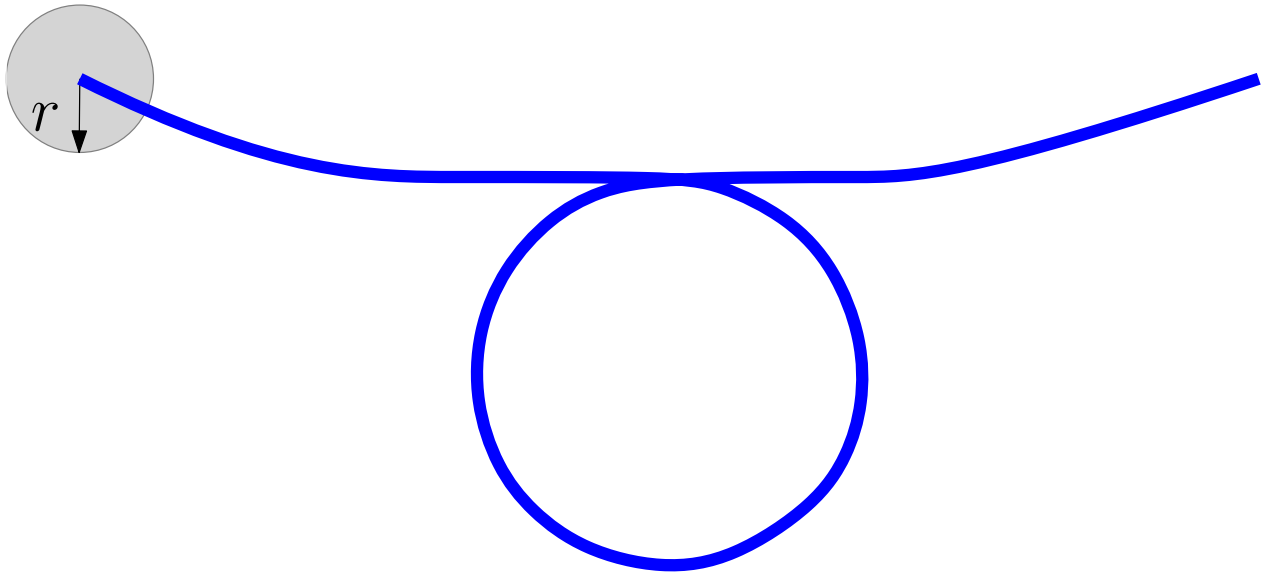

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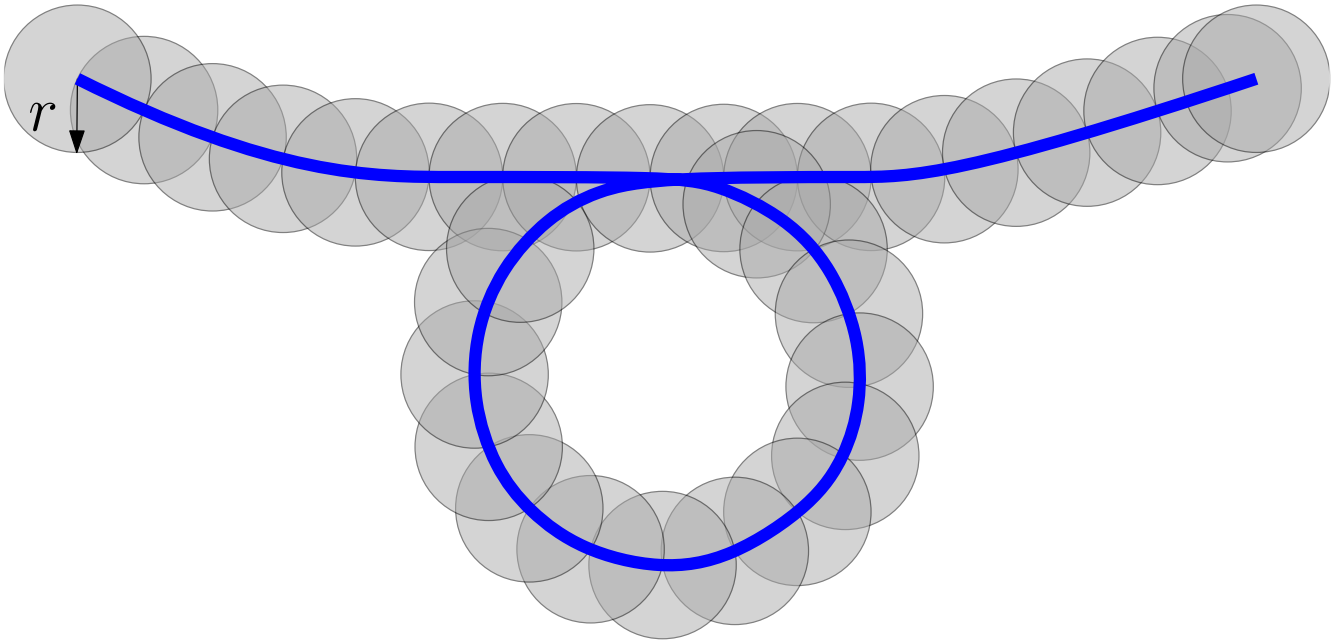

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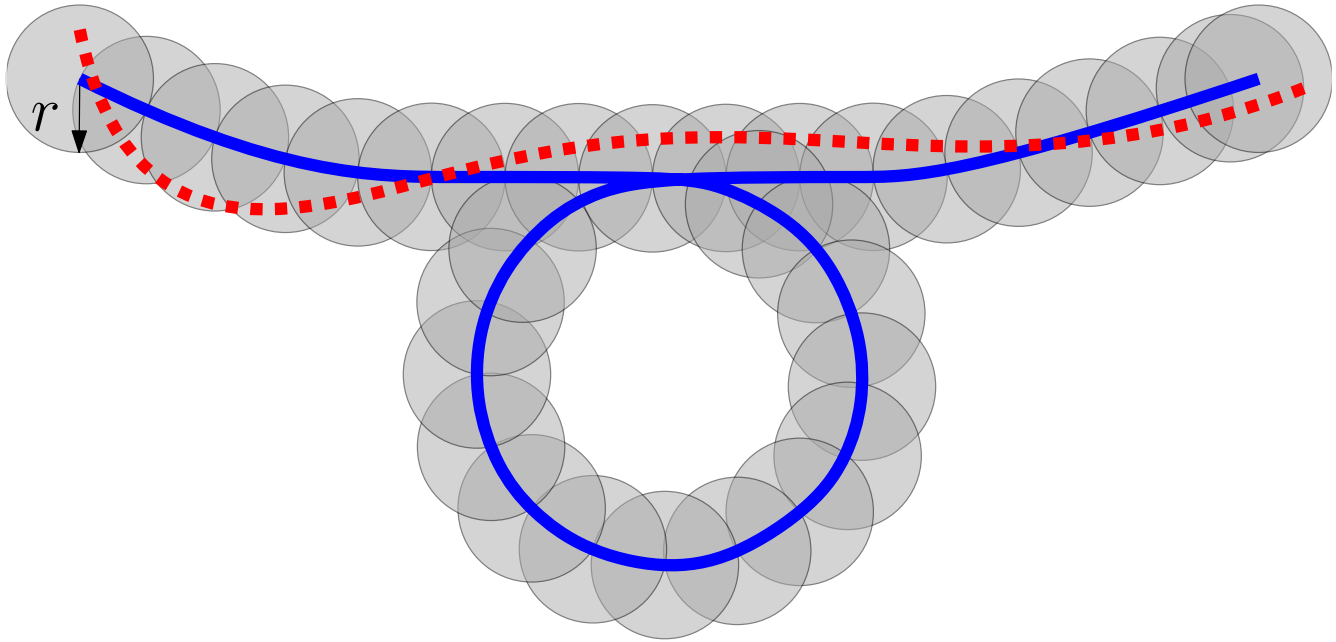

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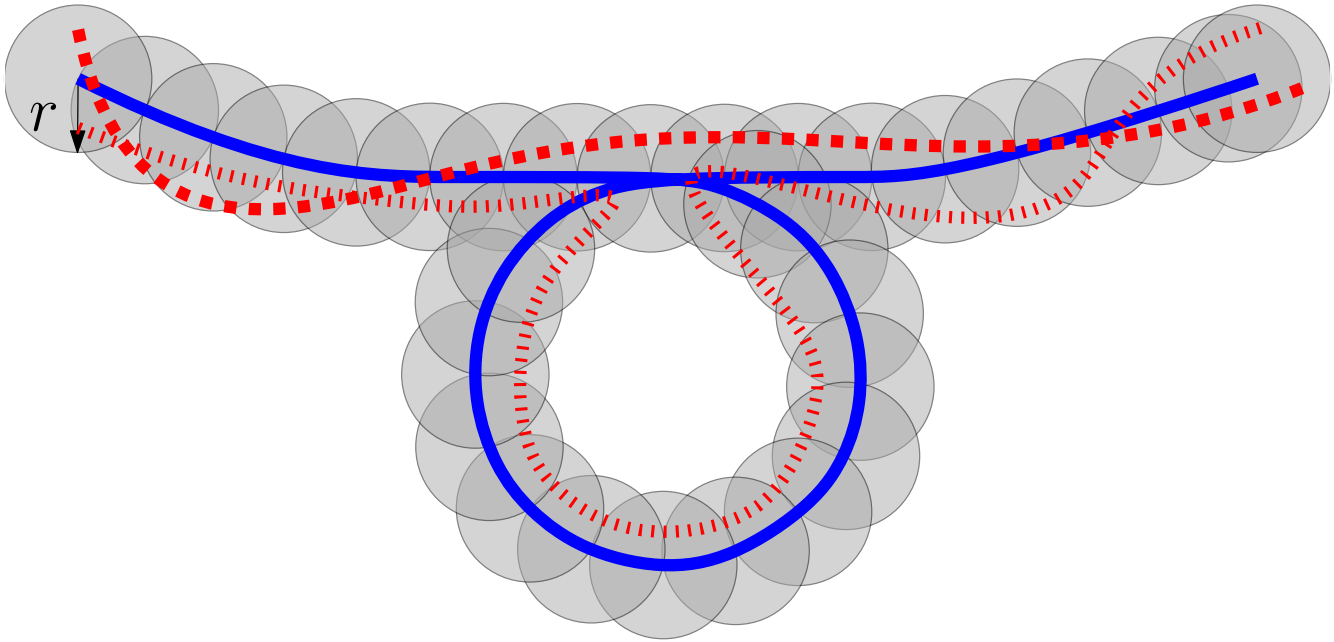

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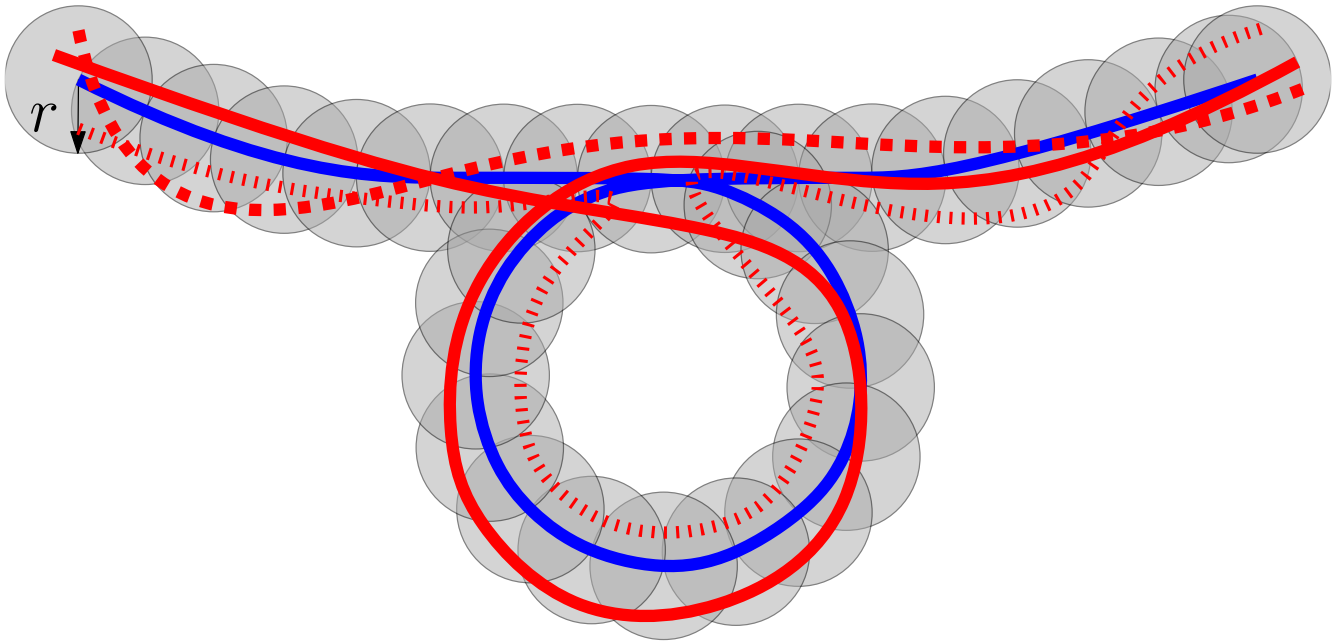

We want to follow the balls in order—Frechet distance

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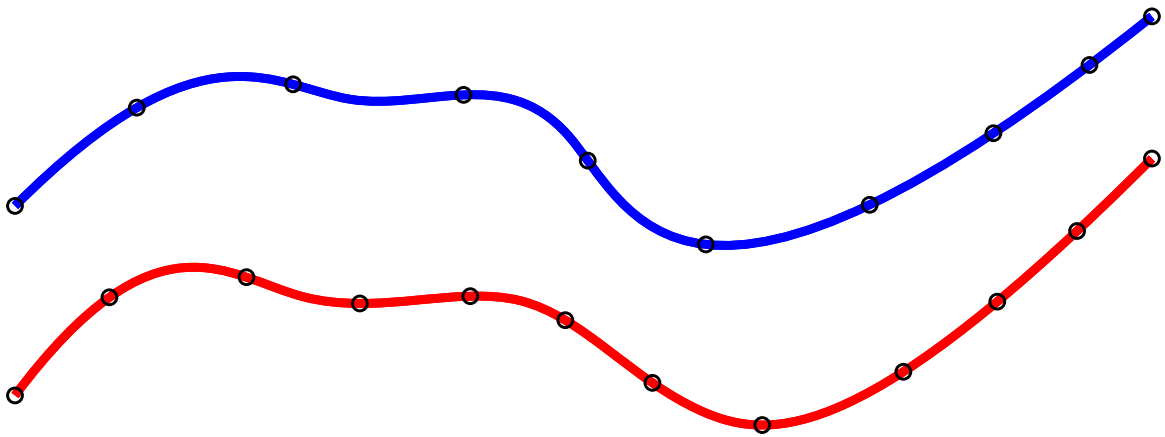

(discrete) Frechet distance—Similarity measure that accounts for location and ordering

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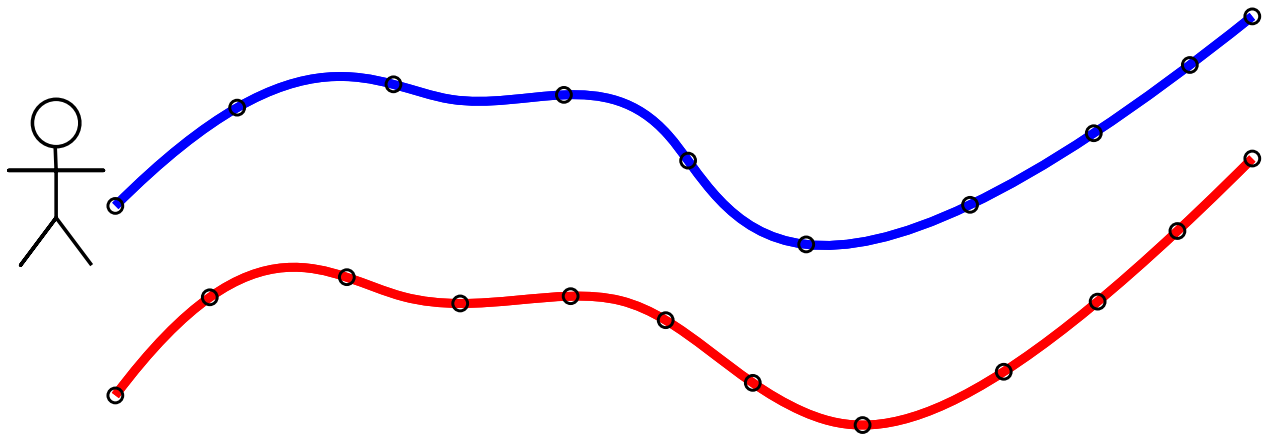

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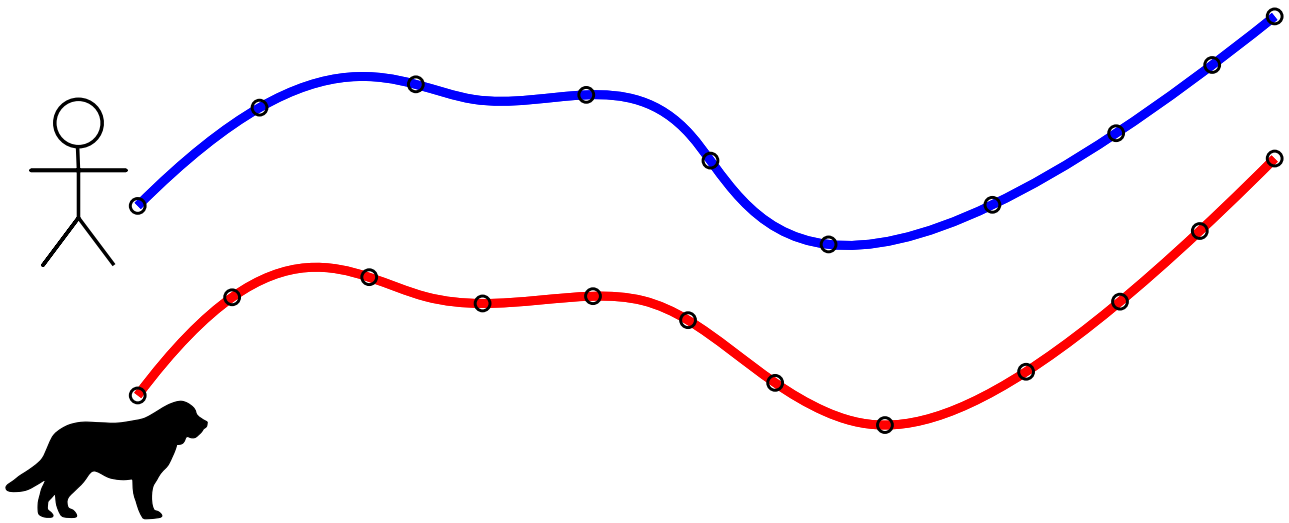

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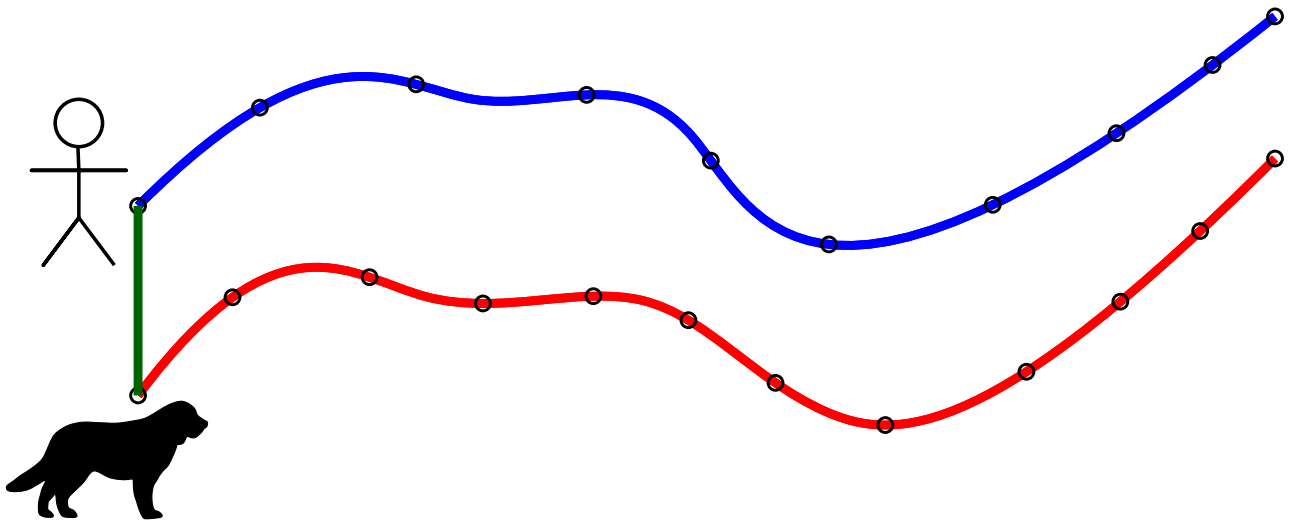

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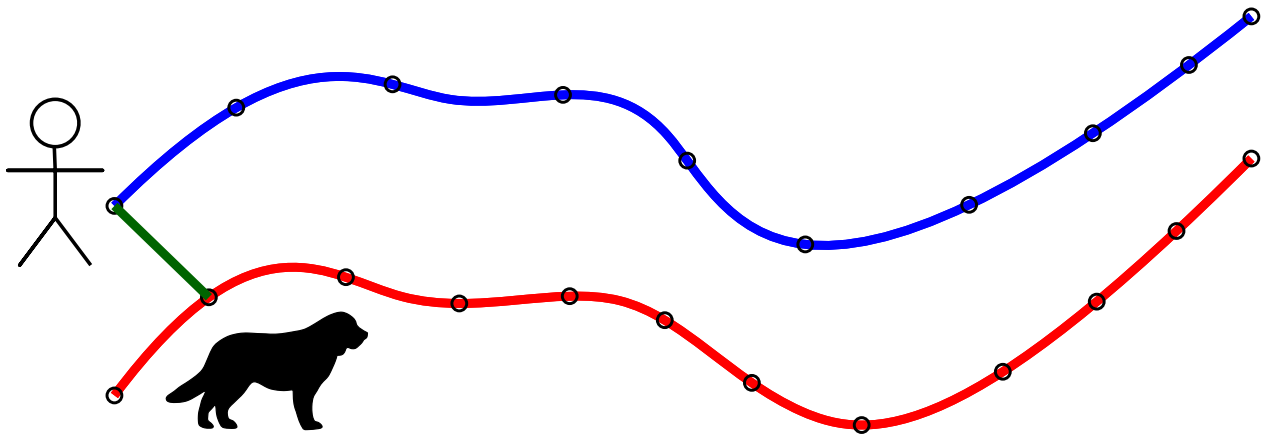

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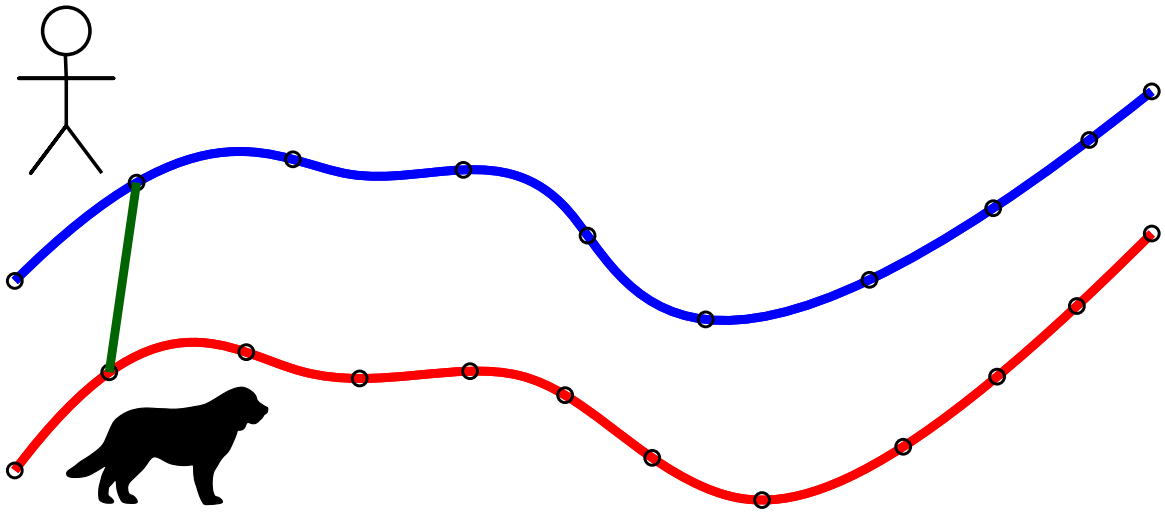

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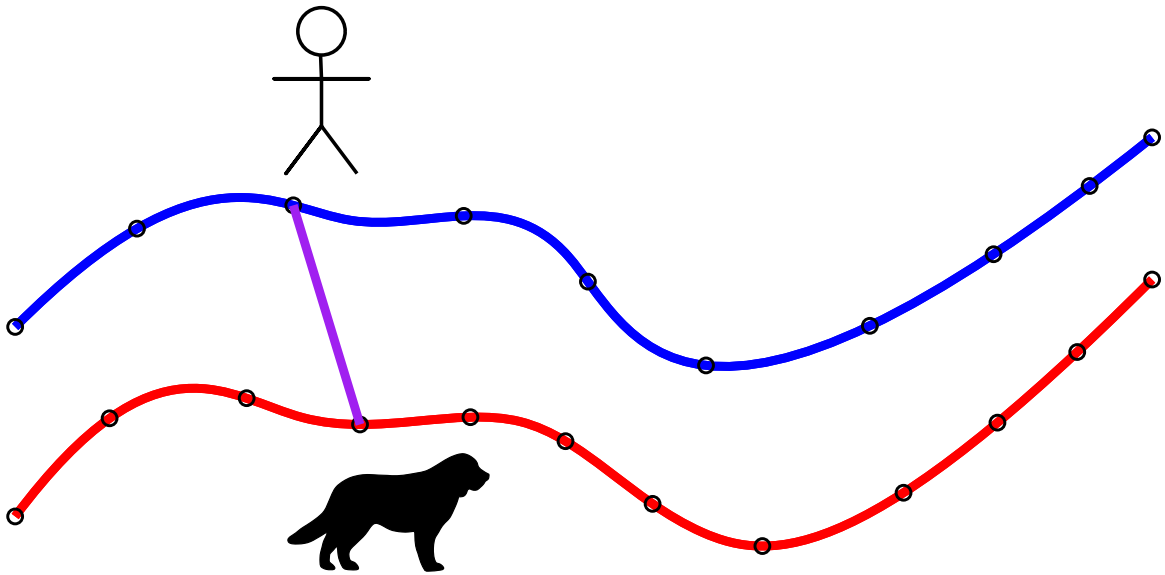

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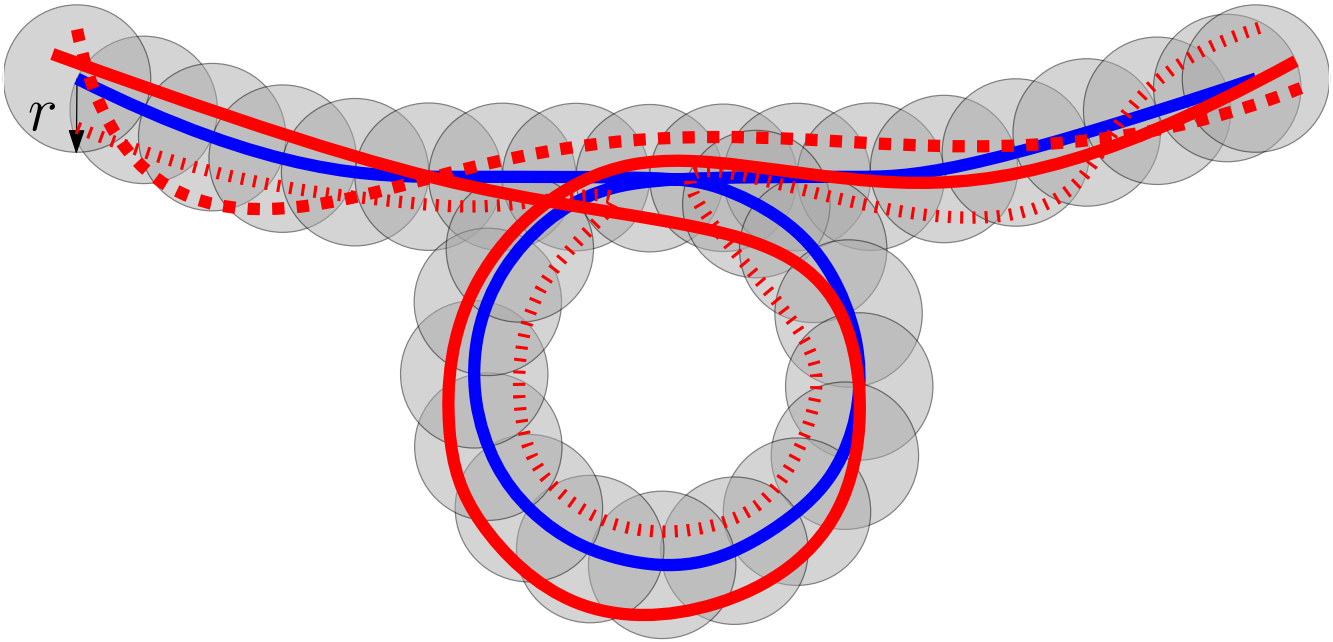

(discrete) Frechet distance—Similarity measure that accounts for location and ordering

(discrete) Frechet distance—computation

- Let $P := \{p_1, \ldots p_n\}$ and $Q := \{q_1, \ldots q_n\}$ be sequences of points
- $f(x, y)$—discrete Frechet distance between $\{p_1, \ldots p_x\}$ and $\{q_1, \ldots q_y\}$
- The last move in the optimal path was
  - $(x-1, y) \to (x, y)$ or
  - $(x, y-1) \to (x, y)$
- $\Rightarrow f(x, y) = \max(|p_x - q_y|, \min(f(x-1, y), f(x, y-1)))$
- Using dynamic program, $f(n, m)$ can be computed in $O(nm)$
- Slightly better algorithms exist [Agarwal et al. 14]

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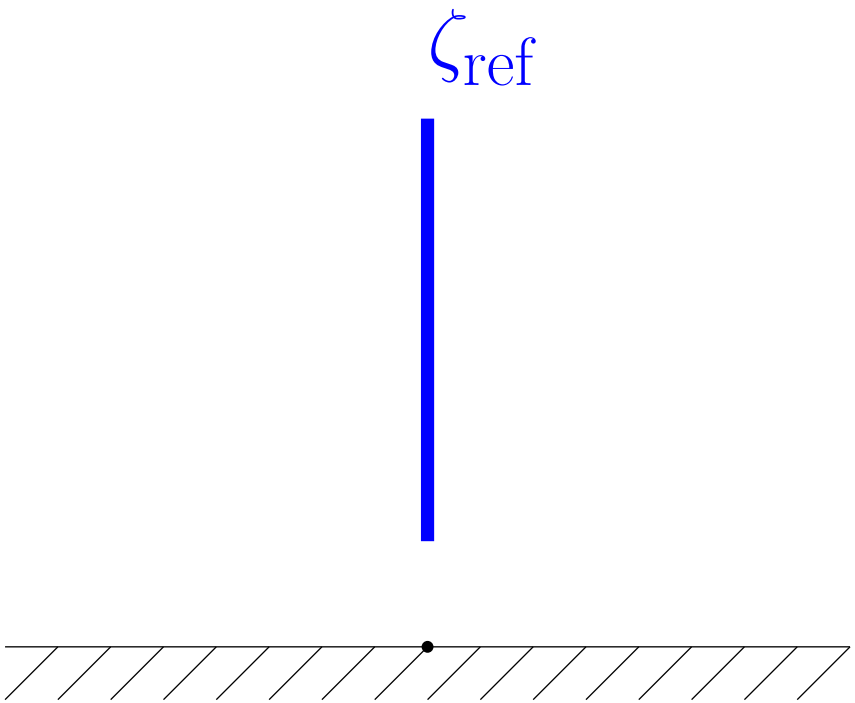
dFrećhet distance

Following reference paths—Take (1)

Optimization-based approach [Holladay, Srinivassa 16]

- Sample along ζ_{ref}
- Compute IK of each sample
- Connect using local planner
- Refine samples



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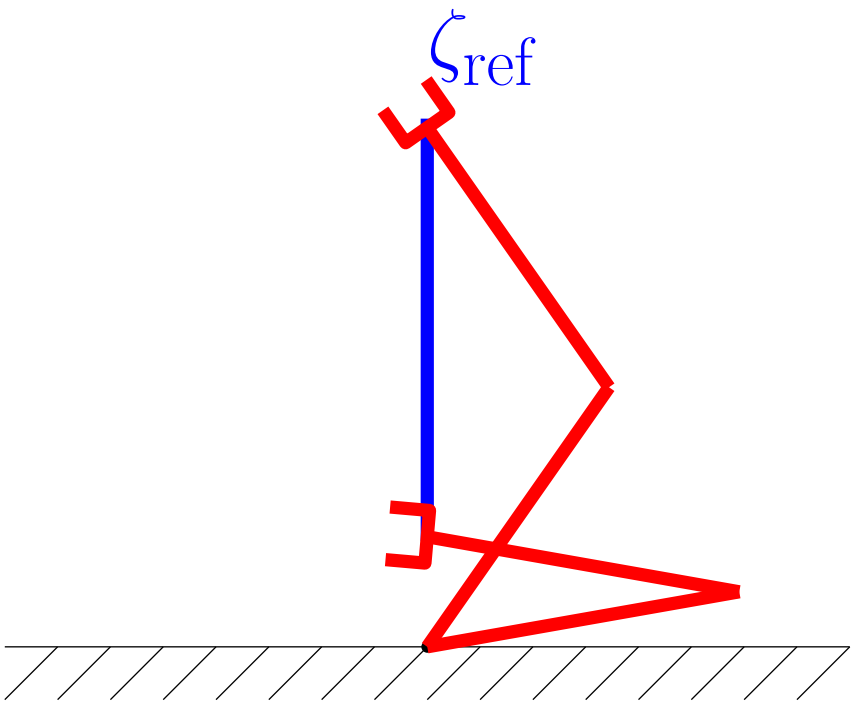

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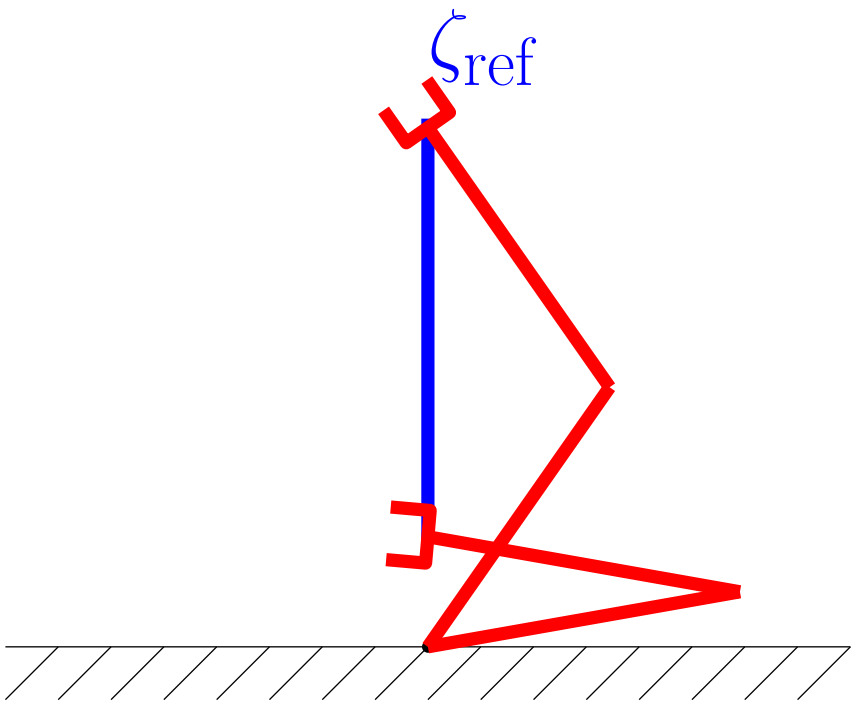

Following reference paths—Take (1)

Optimization-based approach [Holladay, Srinivassa 16]

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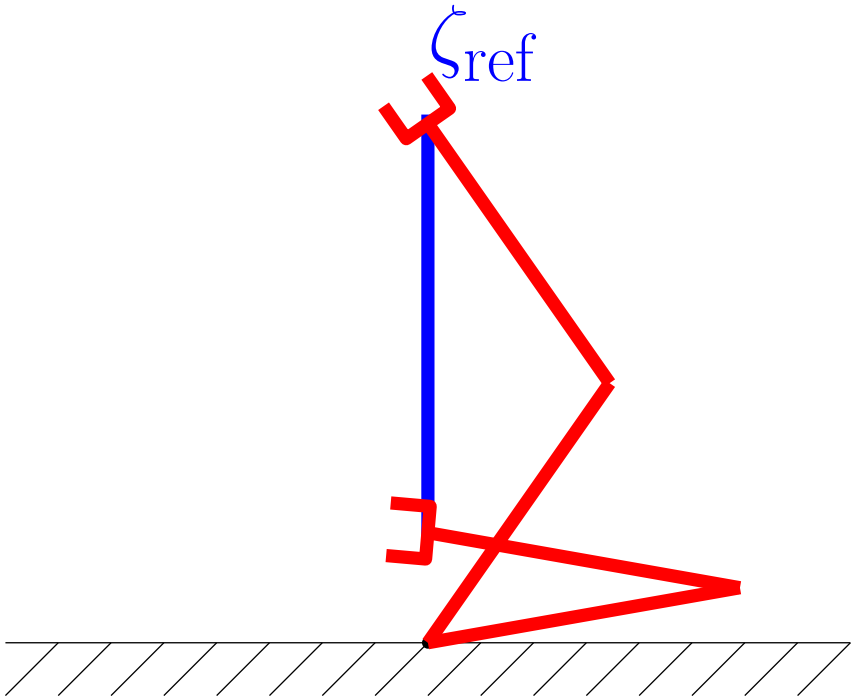

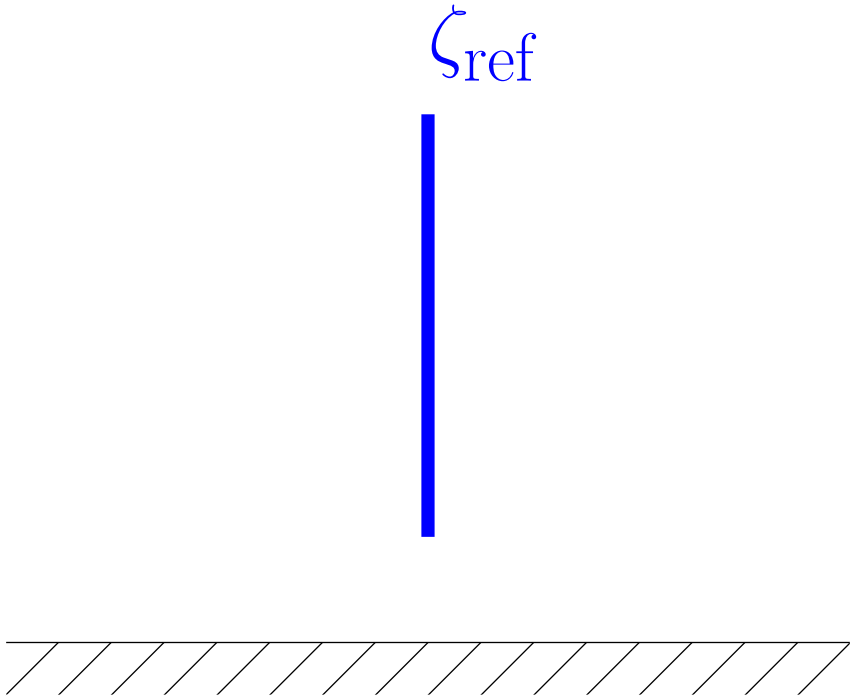

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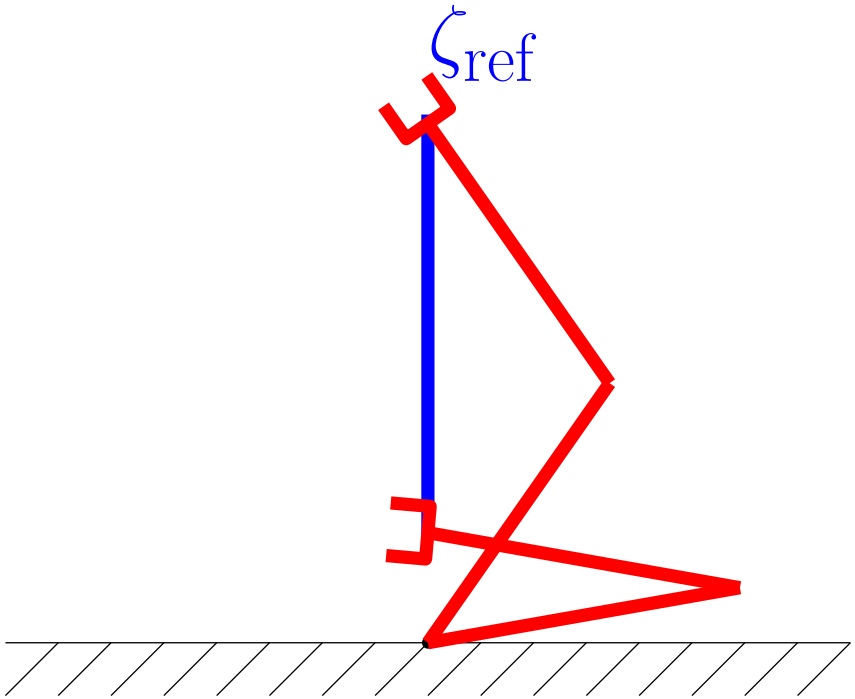

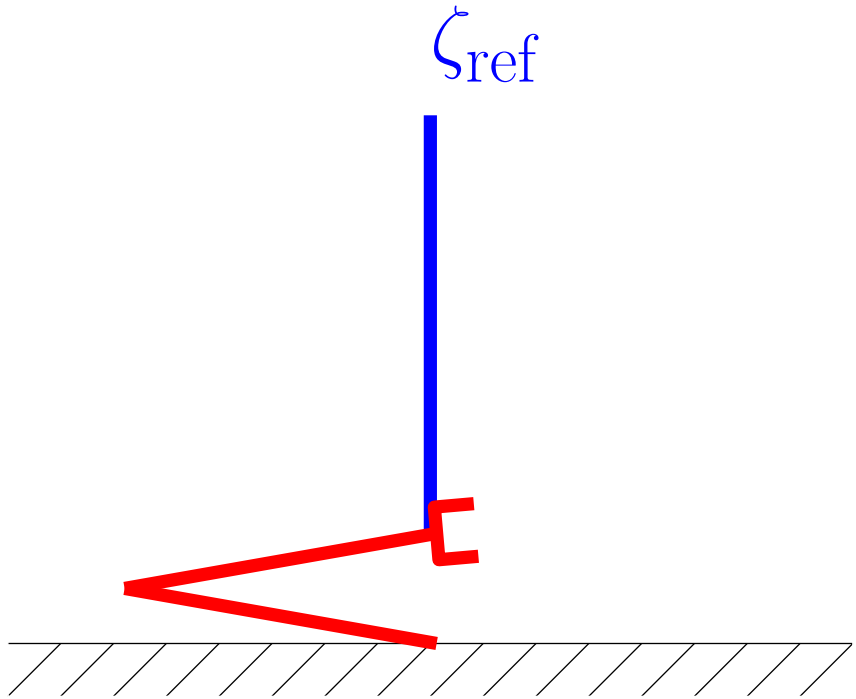

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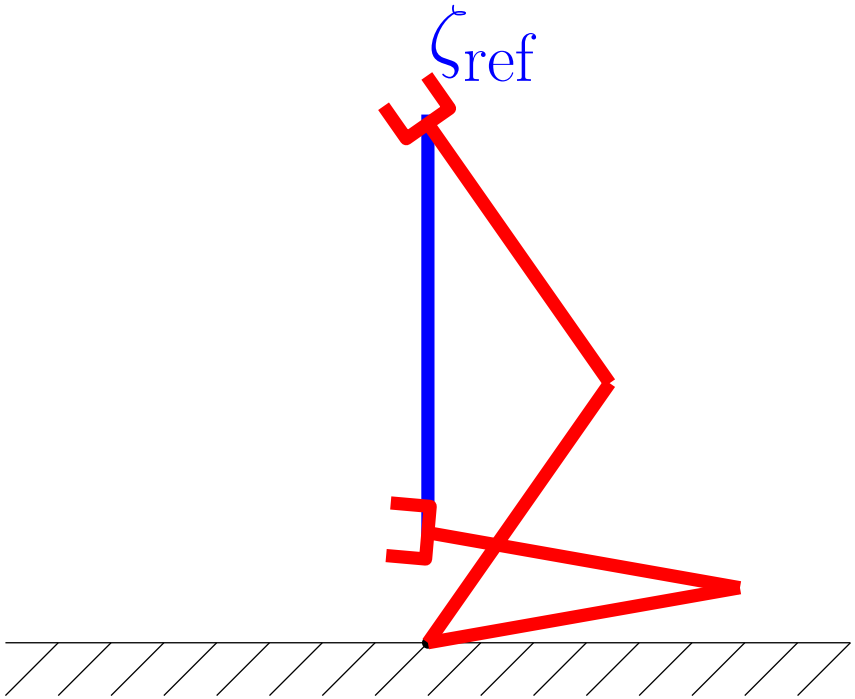

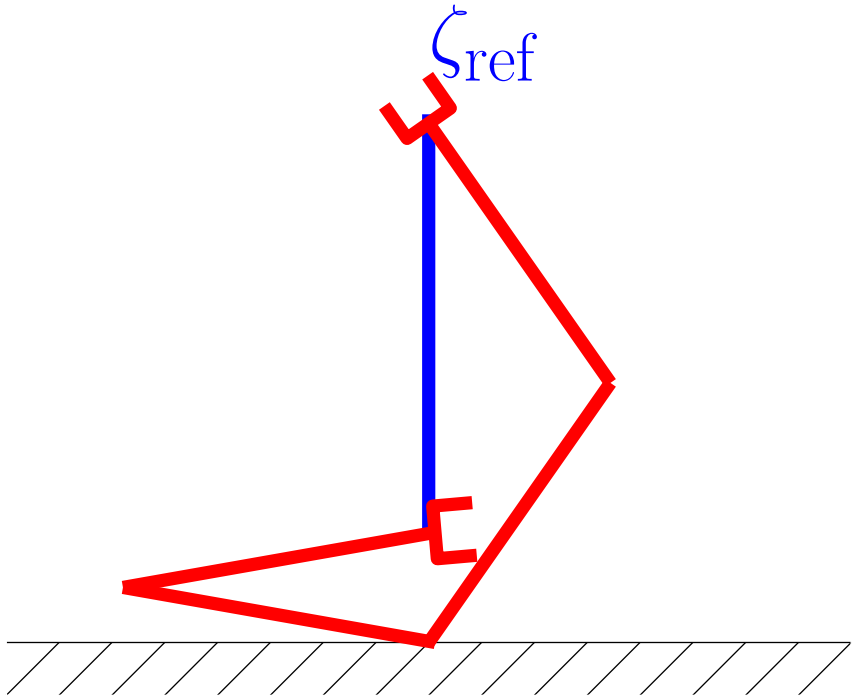

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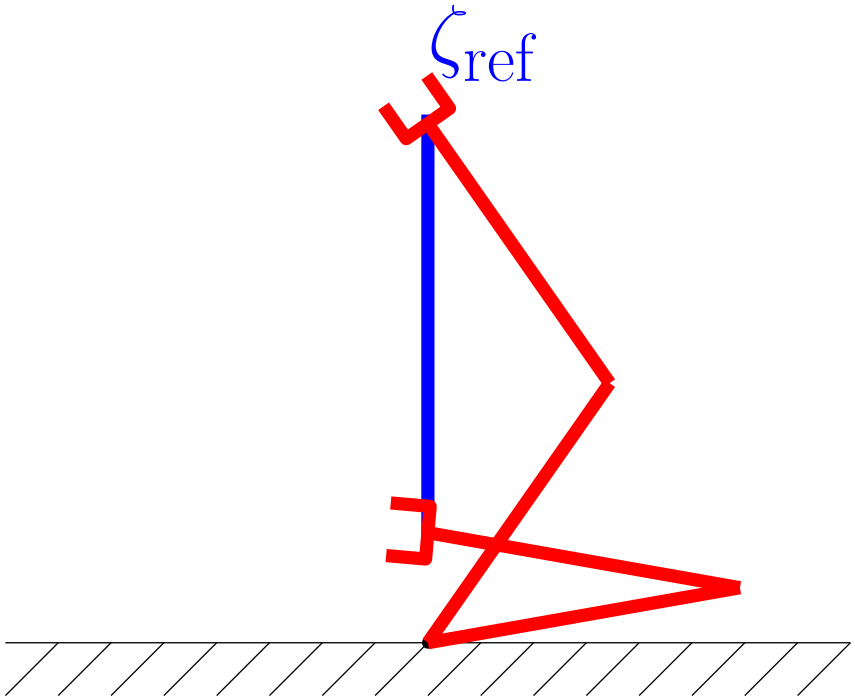

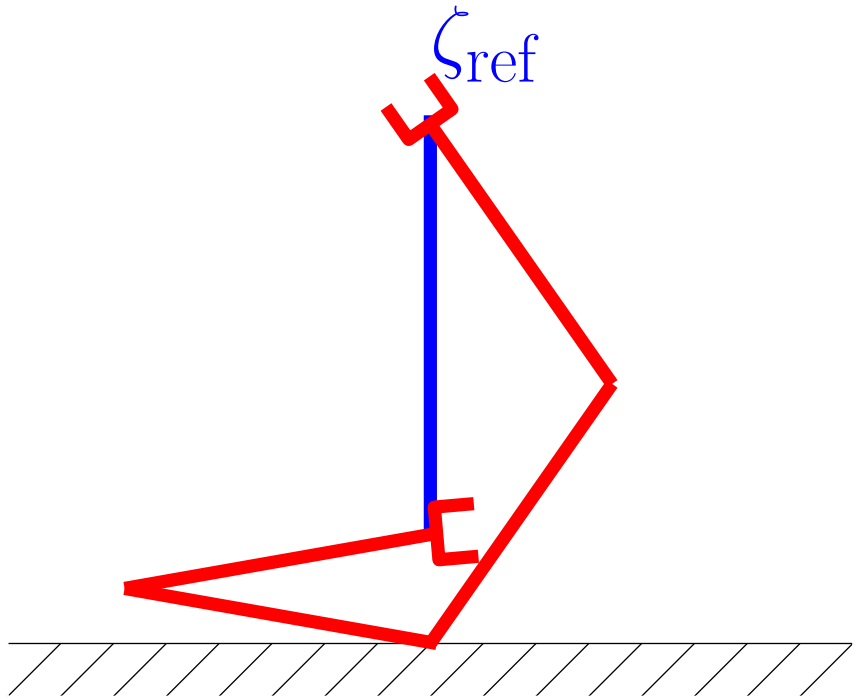

Following reference paths—Take (2)

Planning [Holladay, S., Srinivasa19]

- Sample along ζ_{ref}
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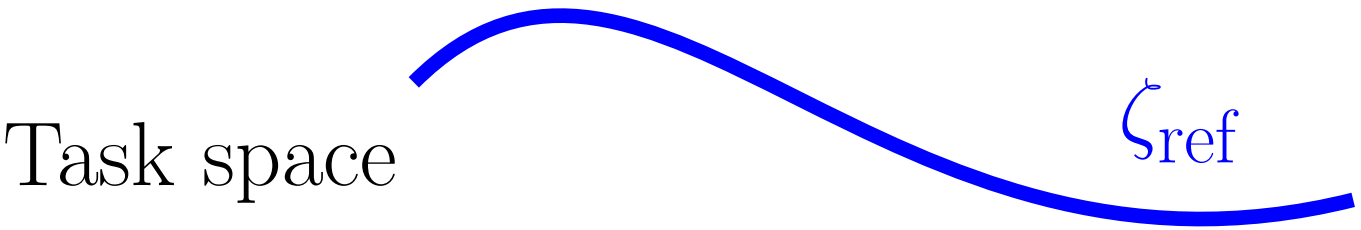
- Construct a layered graph L
- Find minimal-cost path in L



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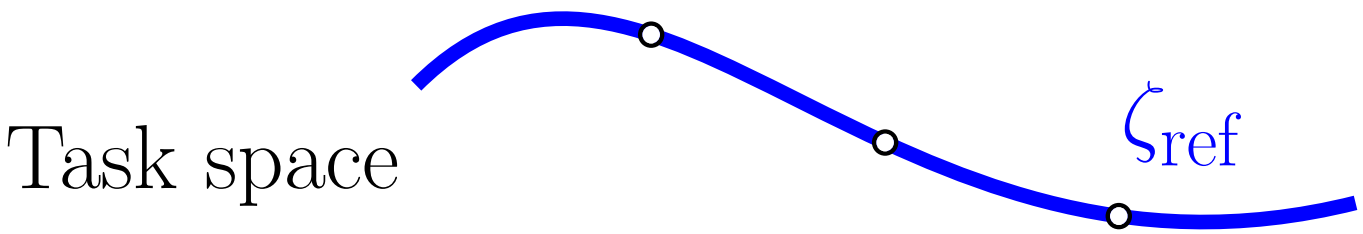

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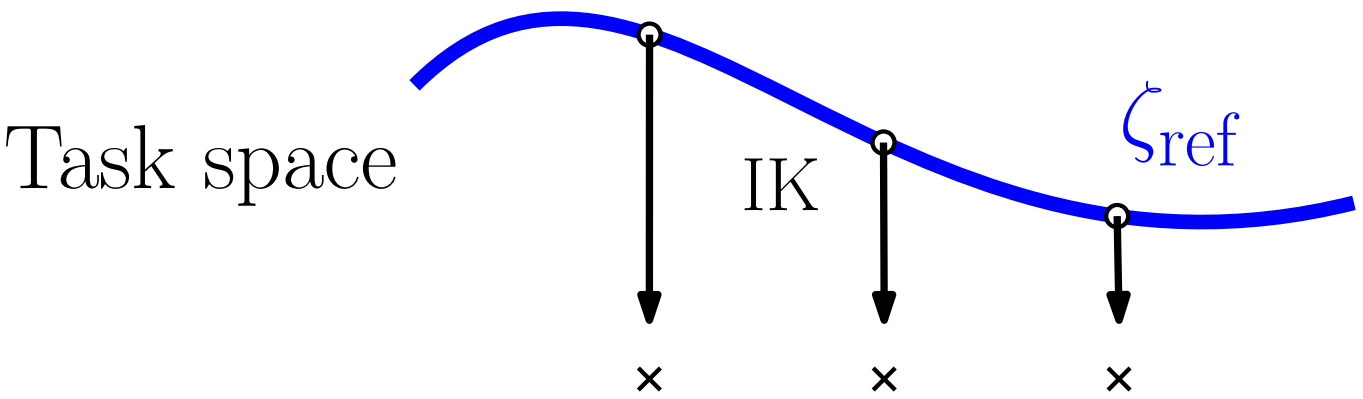

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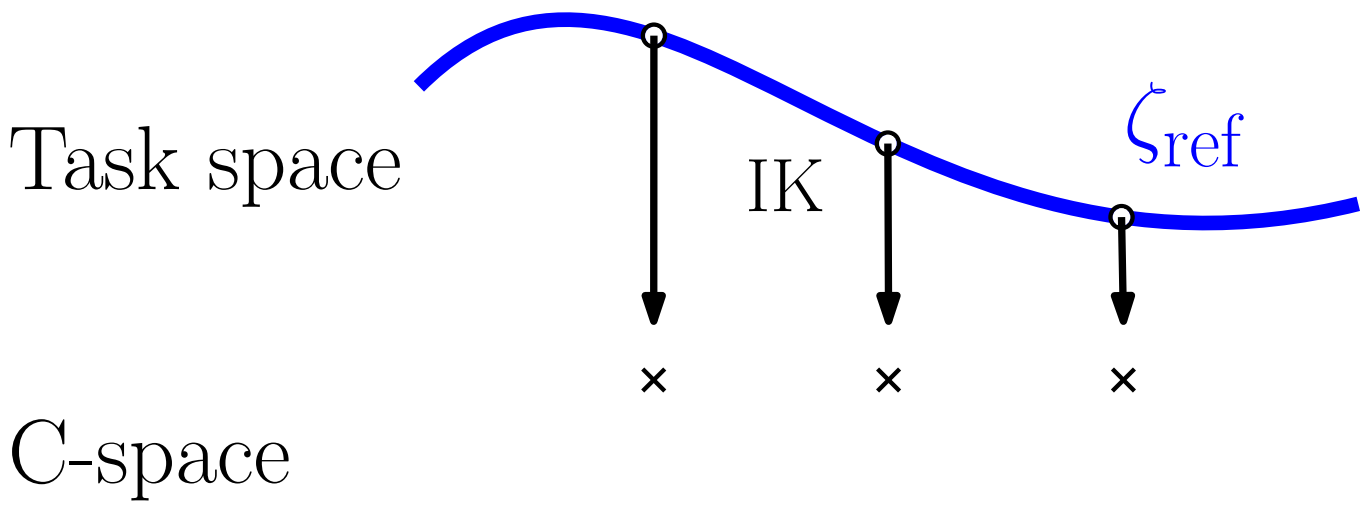

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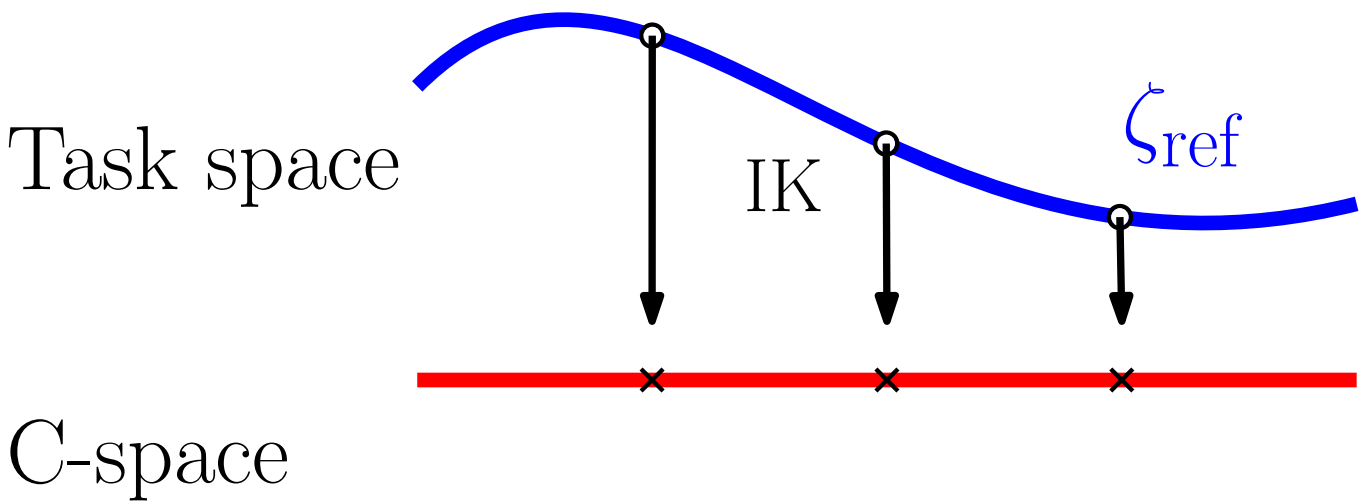

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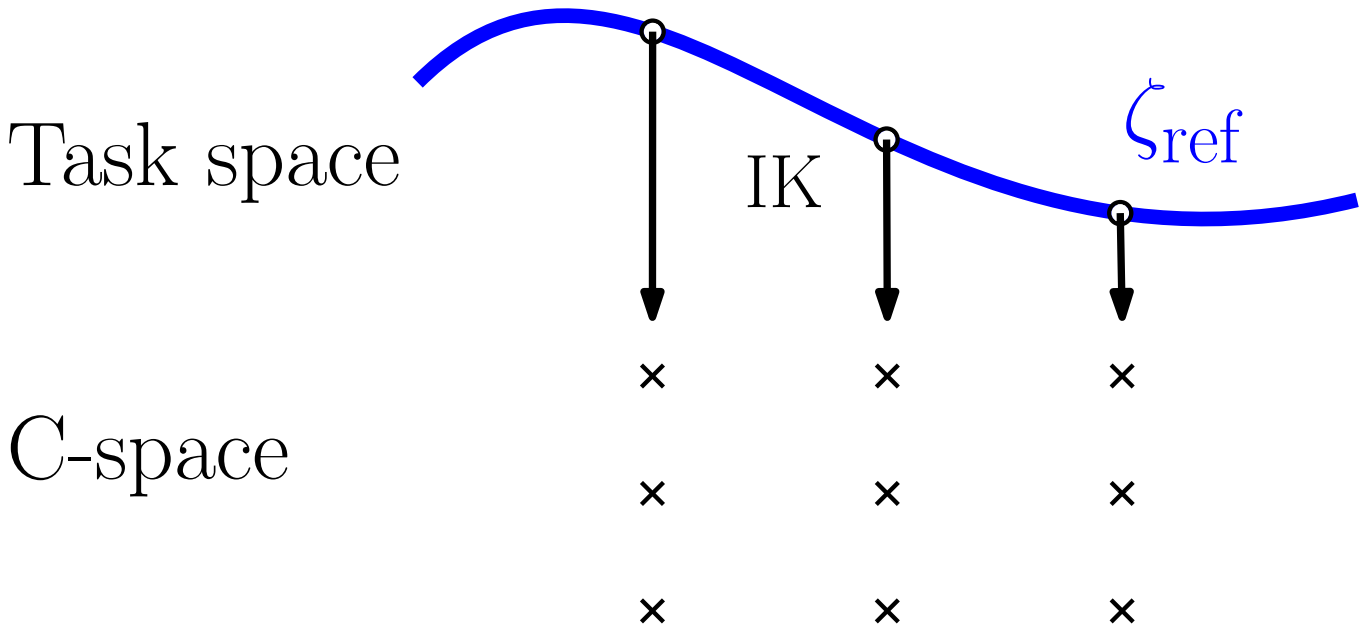

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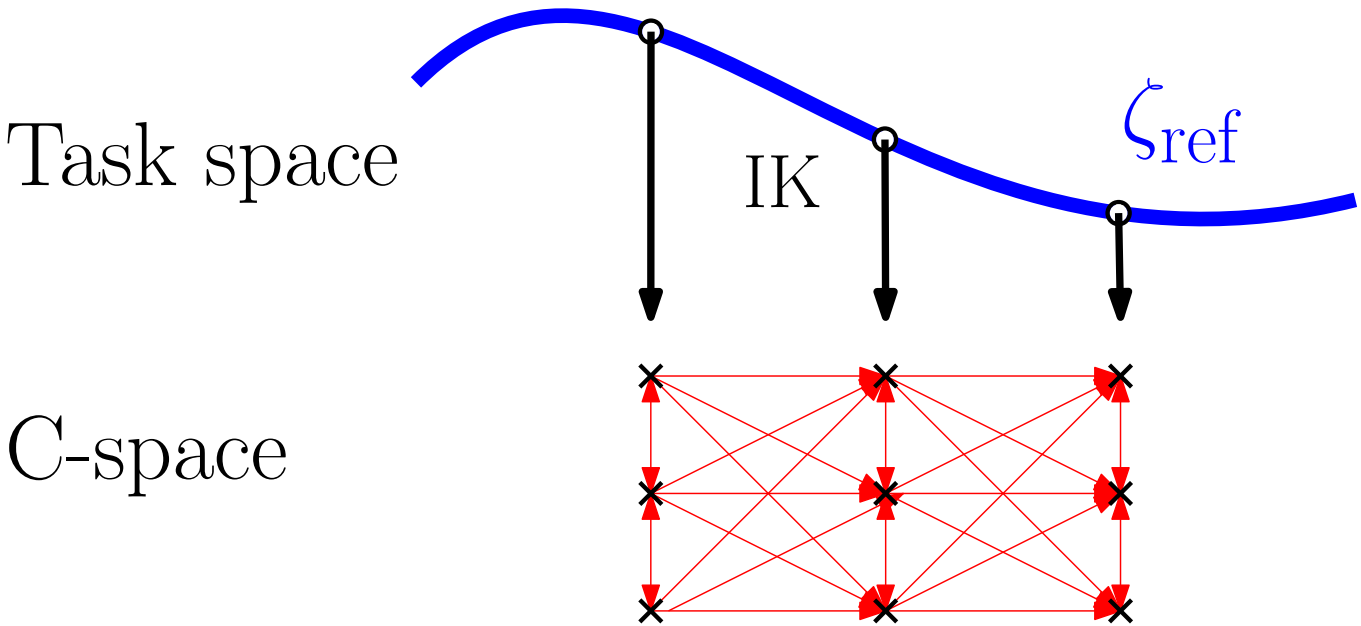

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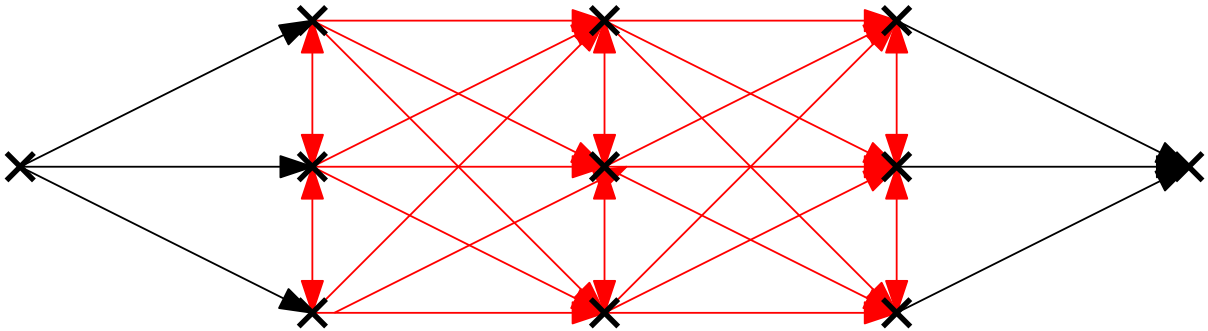

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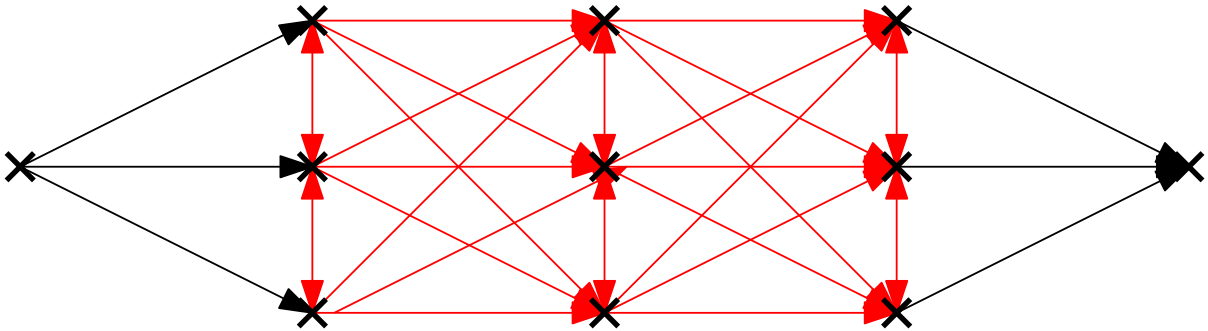
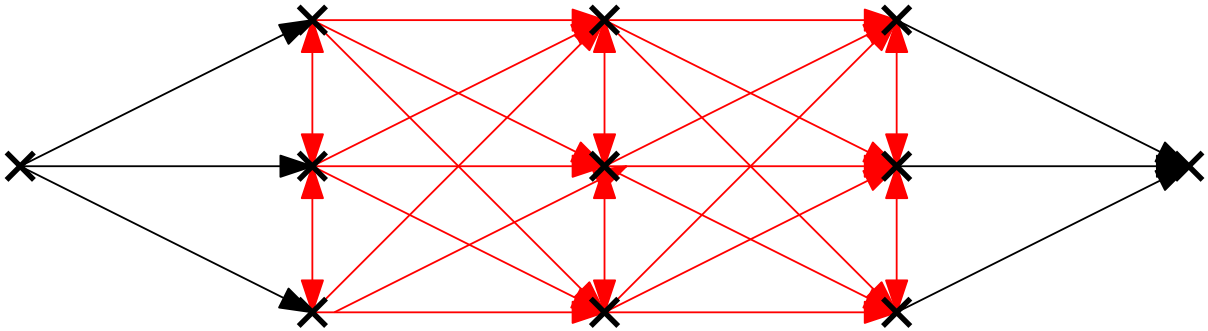


$$\zeta^* = \arg\min_{p \in L} \text{dFrećhet}(\text{FK}(p), \zeta_{\text{ref}})$$

Domain-Specific challenges—following reference paths

How do we efficiently find minimal-cost path in L?



$$\zeta^* = \arg\min_{p \in L} \mathrm{dFrećhet}(\mathrm{FK}(p), \zeta_{\mathrm{ref}})$$

- Number of paths in L is $O(n^k)$
  - n is the number of layers
  - k is the maximal number of IK solutions
- By adapting existing machinery [Har-Peled, Raichel14] we can find ζ^* in $O(n^2k^2)$

Efficiently computing minimal-cost path in L

- We define the cross-product graph $\Phi = (V_\Phi, E_\Phi)$ of $L = (V_L, E_L)$ and $\zeta_{\text{ref}} = (V_{\zeta_{\text{ref}}}, E_{\zeta_{\text{ref}}})$

$$
\begin{aligned}
V_\Phi &= V_{\zeta_{\text{ref}}} \times V_L. \\
E_\Phi &= \{((w_1, q_1), (w_2, q_2)) \mid \text{ if} \\
&\quad ((w_1 = w_2) \text{ and } (q_1, q_2) \in E_L) \text{ or} \\
&\quad ((w_1, w_2) \in E_{\zeta_{\text{ref}}} \text{ and } (q_1 = q_2)) \text{ or} \\
&\quad ((w_1, w_2) \in E_{\zeta_{\text{ref}}} \text{ and } (q_1, q_2) \in E_L)\}. \\
c((w_1, q_1)) &= ||(w_1, \text{FK}(q_1))||. \\
c(u, v) &= \max\{c(u), c(v)\}.
\end{aligned}
$$

Definition

The bottleneck cost of a path is the cost of the edge with the maximal cost along the path

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Thm (adapted from [Raichel Har-Peled14])

Let $P = \{(w_1, q_1), \ldots, (w_k, q_k)\}$ be the bottleneck shortest path in Φ. Then the FK of $P_q = \{q_1, \ldots, q_k, \}$ minimizes the Frechet distance to $\{w_1, \ldots, w_k\}$ (discretization of ζ_{ref})

Computing the bottleneck shortest path

We can compute the bottleneck shortest path by a simple adaptation of Dijkstra's algorithm

Input:

- Graph $G = (V, E)$
- Start & target vertices s, t
- Cost function $c : V \to \mathbb{R}$

Output: Cost b^* of bottleneck shortest path connecting s and t

---

1: Open $\leftarrow s$ with b(s) = 0; ▷ Initialization
2: **while** Open is not empty **do**
3: $u \leftarrow$ Open.min_key() ▷ u has minimal $b(u)$ in Open
4: **if** $u = t$ **then**
5: **return** $b(t)$
6: **for** each v such that $(u, v) \in E$ **do**
7: $b' = \max(c(v), b(u))$ ▷ new bottleneck cost
8: **if** $v \in$ Open with $b(v) < b'$ **then continue**
9: **else** insert v to Open with $b(v) = b'$ ▷ updates $b(v)$ if in Open
10: **return**

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- What is the complexity of the algorithm?

$$O(|V| \log |V| + |E|)$$

- Can we do better?

Computing the bottleneck shortest path (cont.)

We can actually compute the bottleneck shortest path in $O(|V| + |E|)$ [Har-Peled, Raichel14]

1: remove isolated vertices ▷ $O(|V|)$
2: check that s and t are connected ▷ $O(|E|)$
3: **if** G has constant size **then**
4: **compute** bottleneck shortest path brute force ▷ $O(1)$
5: **loop**
6: $e_{\text{med}} \leftarrow$ median weight edge ▷ $O(|E|)$
7: $E_{\leq \text{med}} \leftarrow \{e \in E | c(e) \leq c(e_{\text{med}})\}$; $G_{\leq \text{med}} \leftarrow (V, E_{\leq \text{med}})$ ▷ $O(|E|)$
8: compute the connected components of $G_{\leq \text{med}}$ ▷ $O(|E|)$
9: **if** s and t are in the same component of $G_{\leq \text{med}}$ **then**
10: recurse on $G_{\leq \text{med}}$ ▷ $b^* \leq c(e_{\text{med}})$
11: **else**
12: contract each connected component of $G_{\leq \text{med}}$ to a vertex
13: recurse on the graph with these vertices and the edges in $E_{>\text{med}} = E \setminus E_{\leq \text{med}}$

- In practice, Dijkstra's adaptation runs often faster than this (theoretically)-optimal variant

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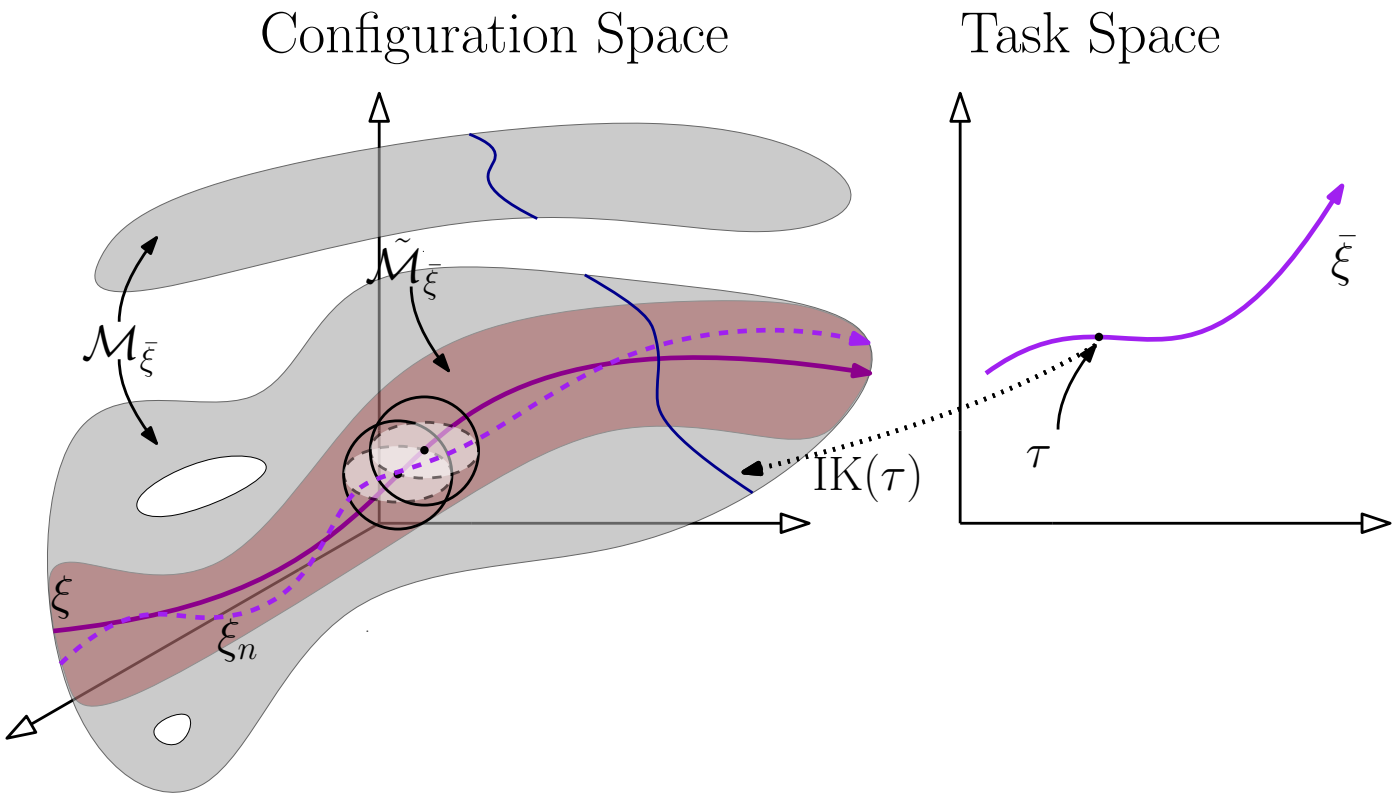
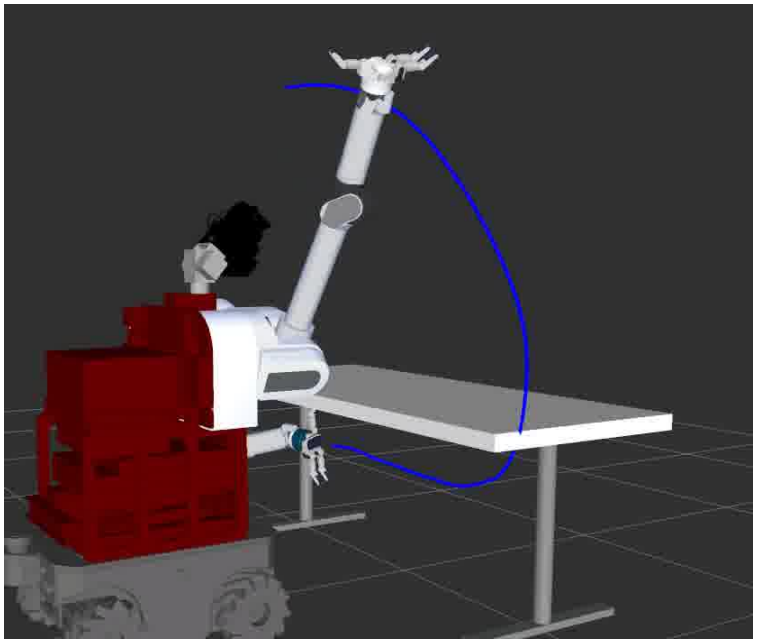
Properties [Holladay, S., Srinivassa19]

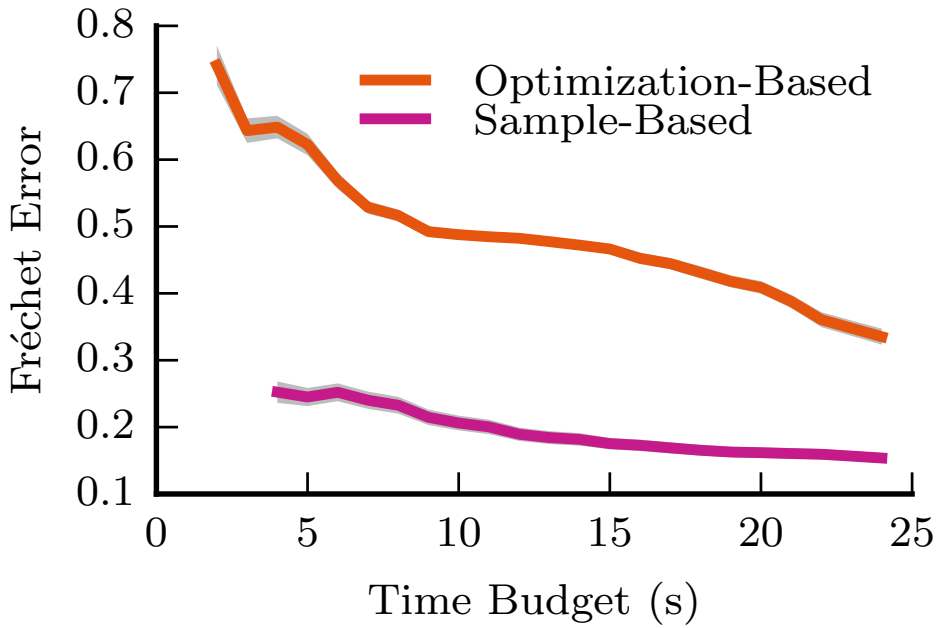
Thm

Let $M_{\zeta^*} \subset C_{\text{free}}$ be the the set of all configurations that directly map to ζ^*. If M_{ζ^*} contains a "well-behaved" portion, then the algorithm is asymptotically optimal (with respect to n and k)

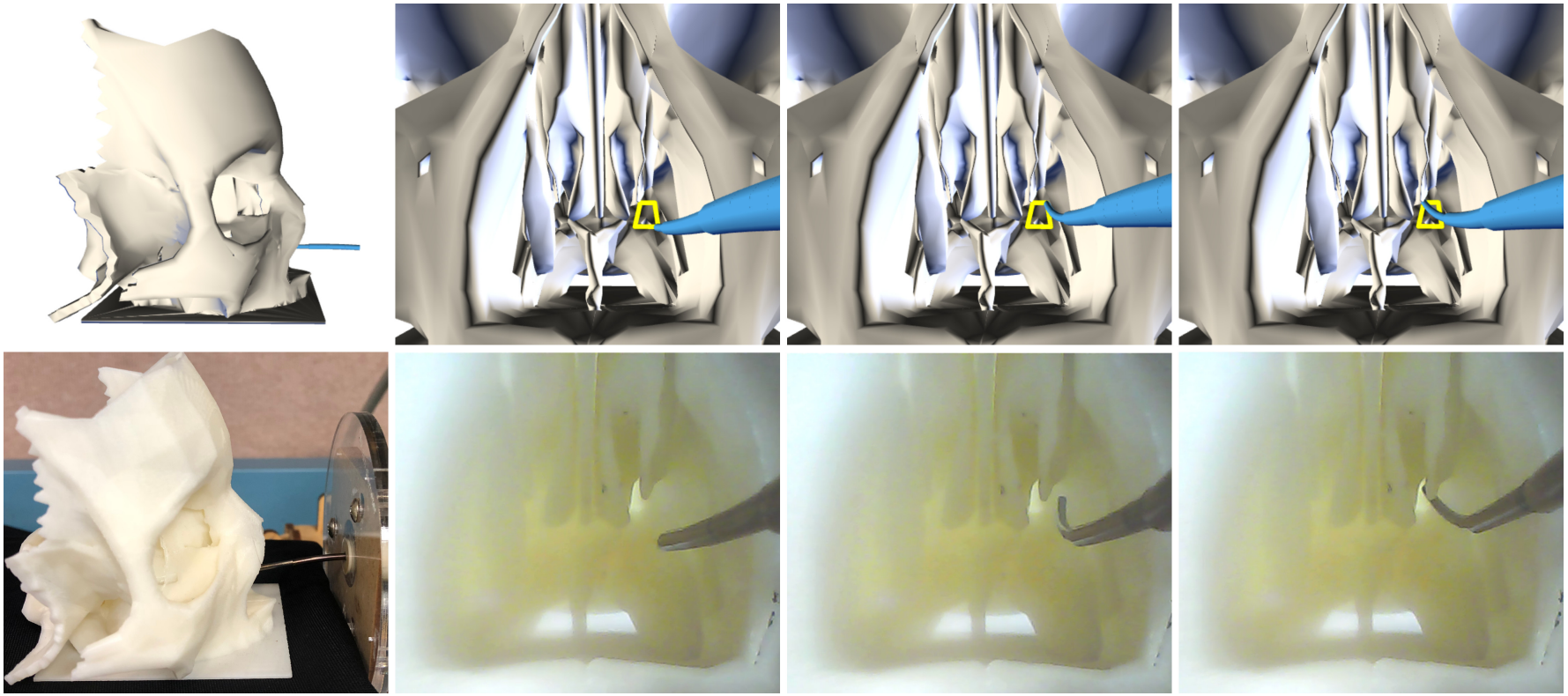


Evaluation [Holladay, S., Srinivassa19]





Key challenge—obstacles along the reference path



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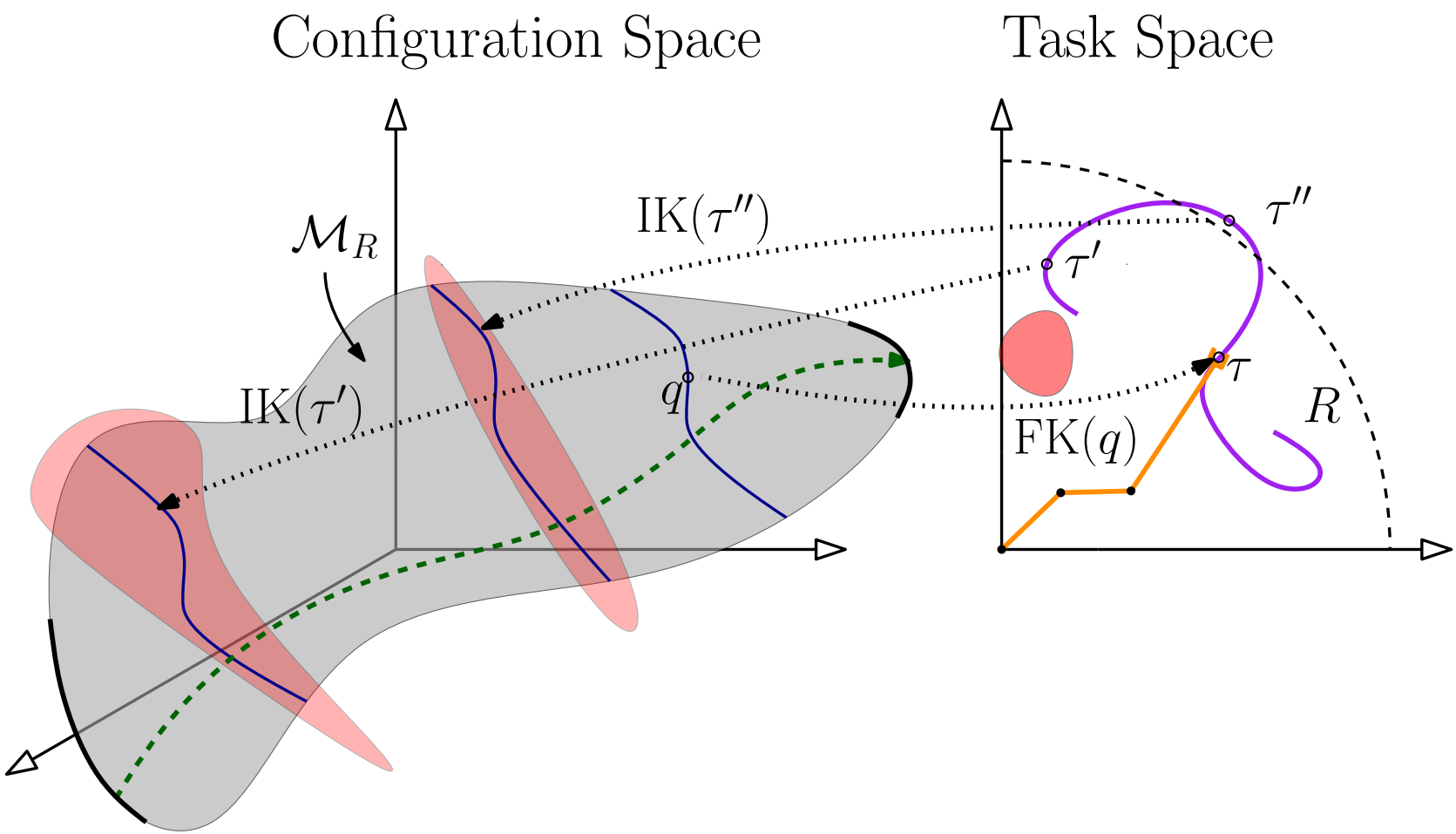

Approach—from layered graph to NN graph

Planning, take (2) [Holladay, Salzman, Srinivasa17]†

- Sample along ζ_{ref}
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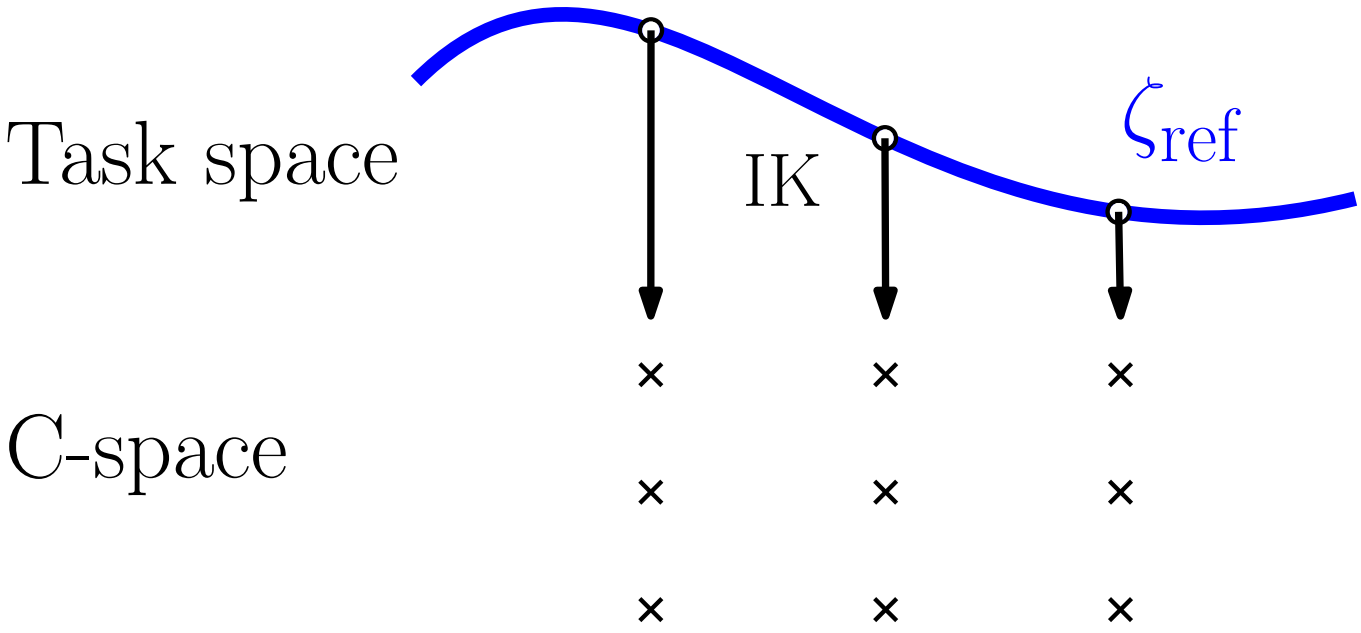

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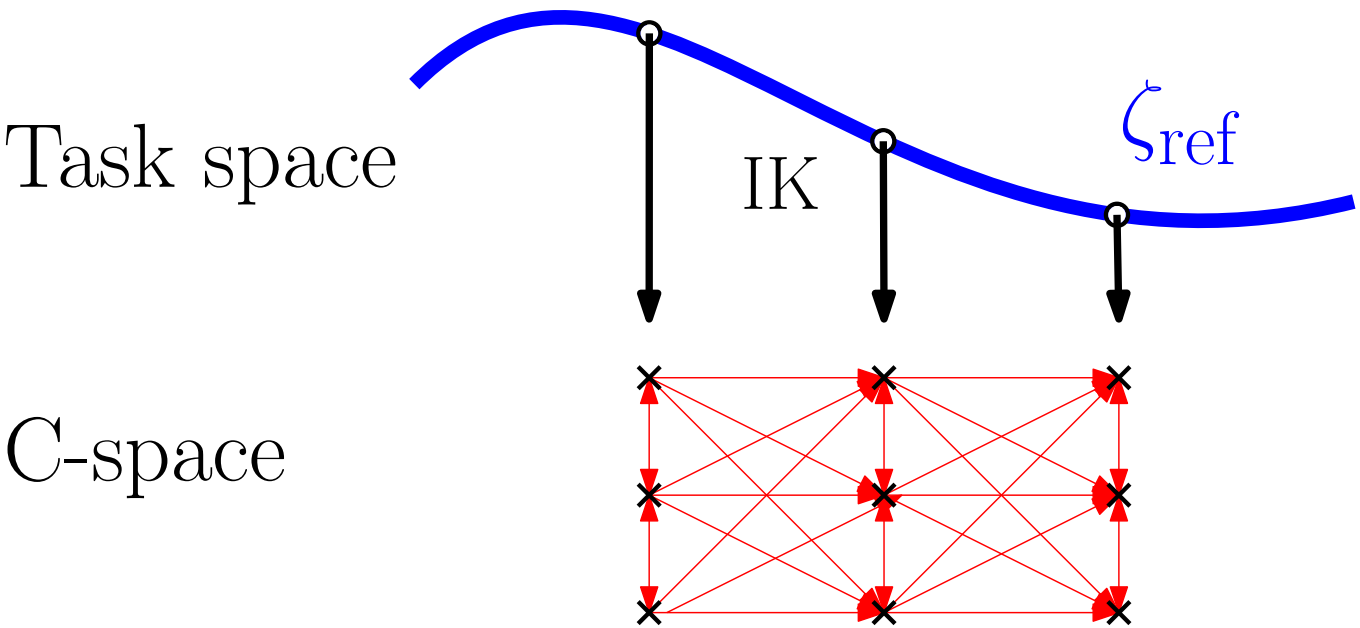

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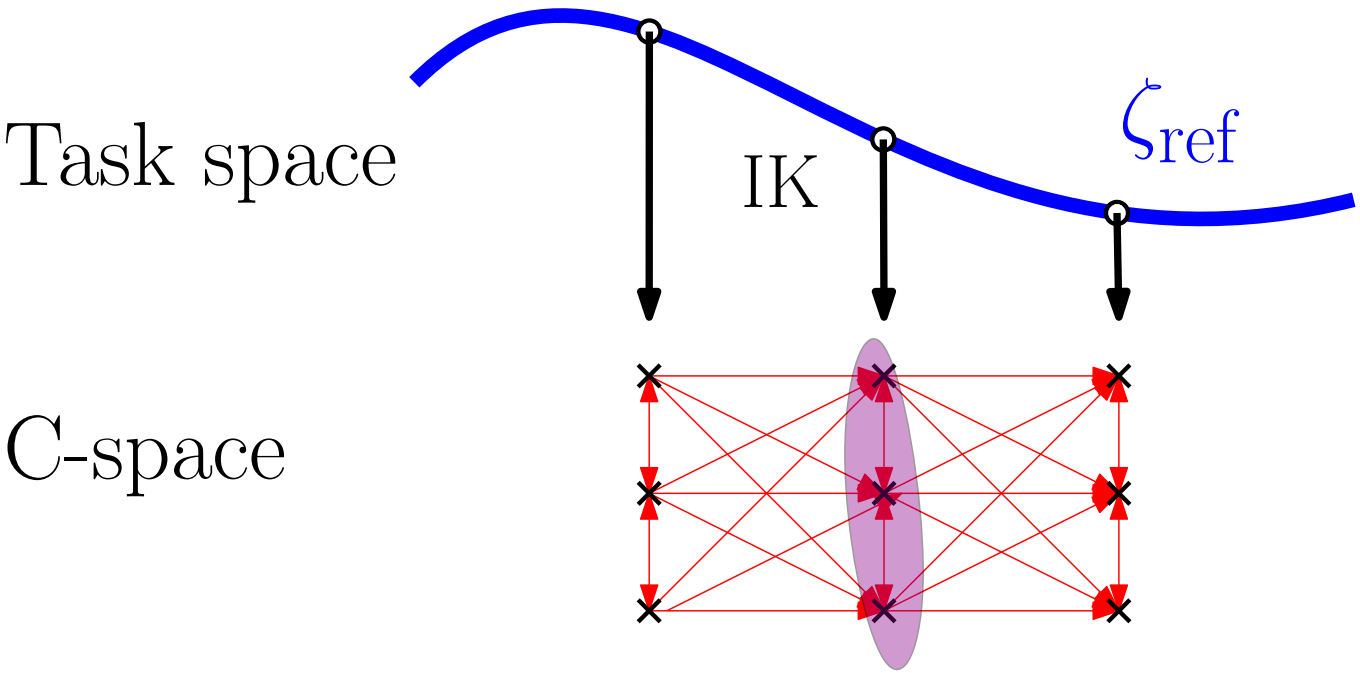

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Approach—from layered graph to NN graph

Planning, take (3) [Niyaz, Kuntz, S., Alterovitz, Srinivasa18]

- Sample along ζ_{ref}
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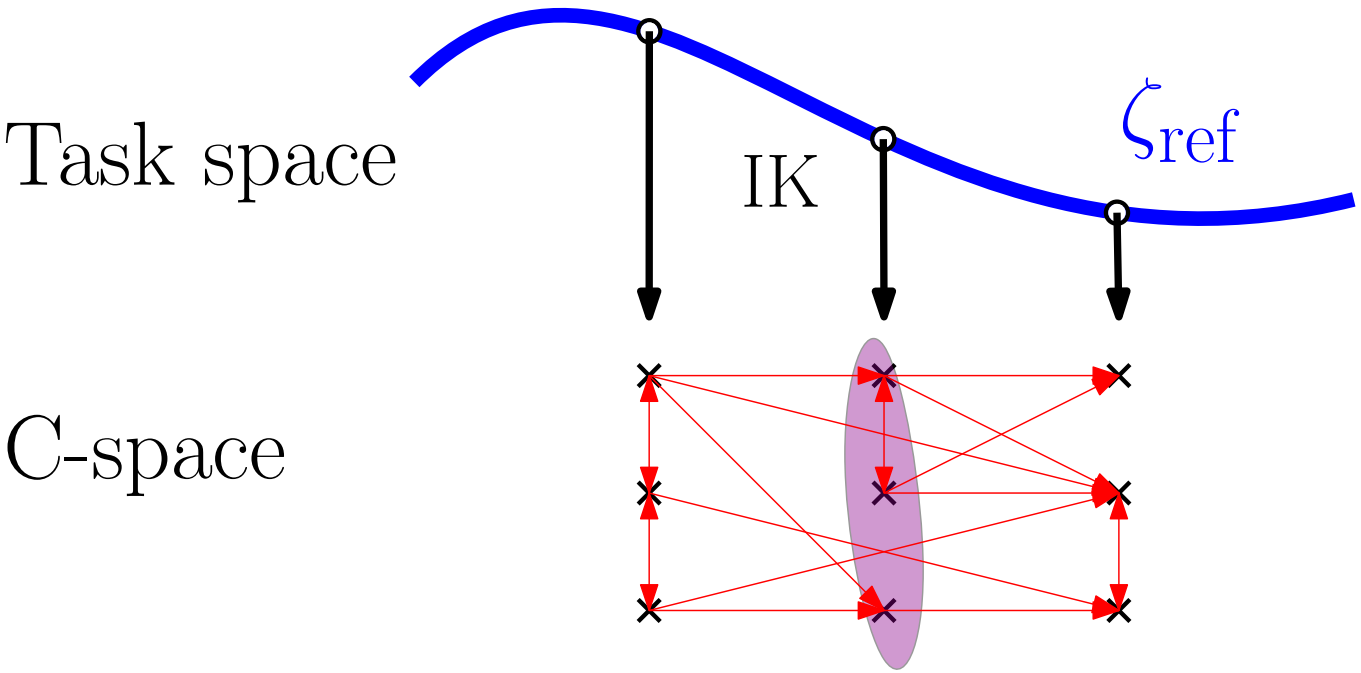
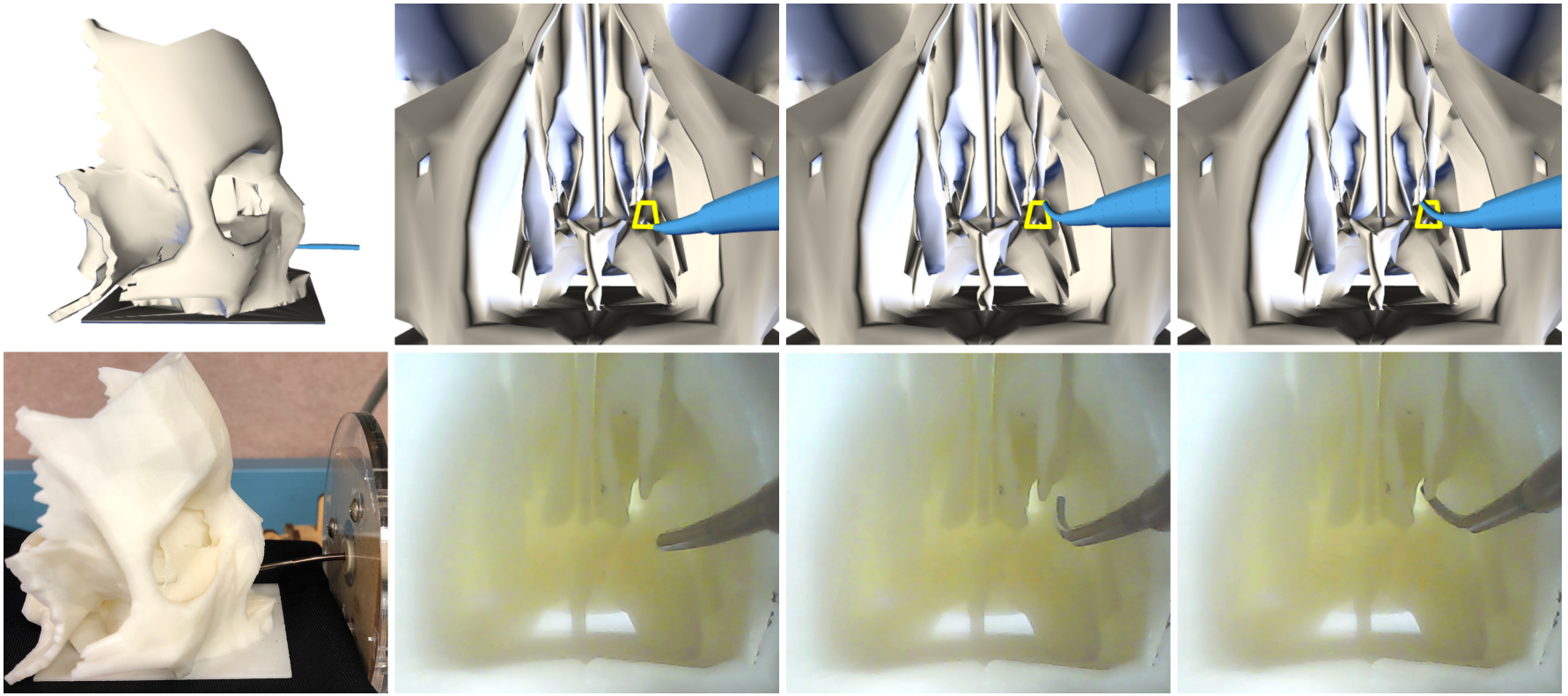
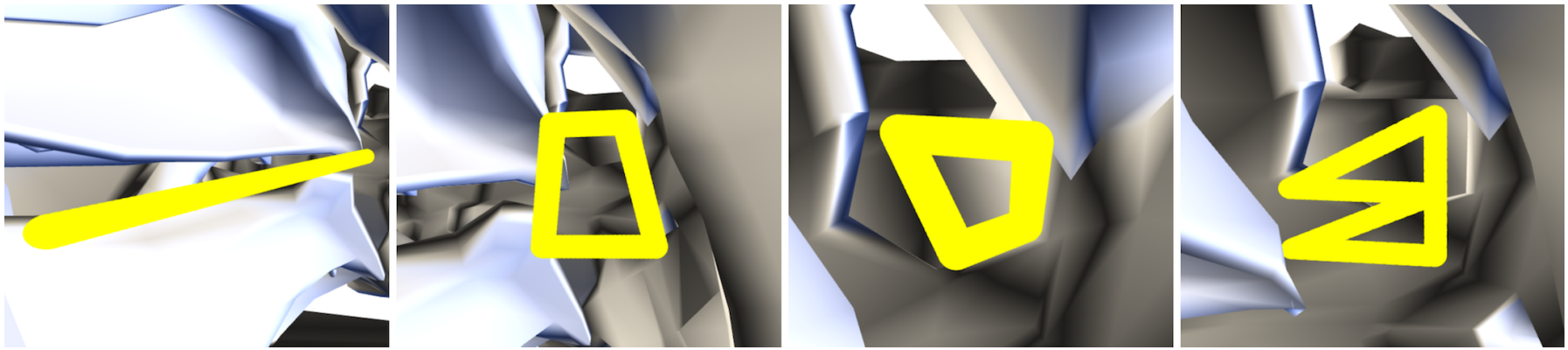
- Construct a ~~layered~~ nearest-neighbor graph N
- Find minimal-cost path in N



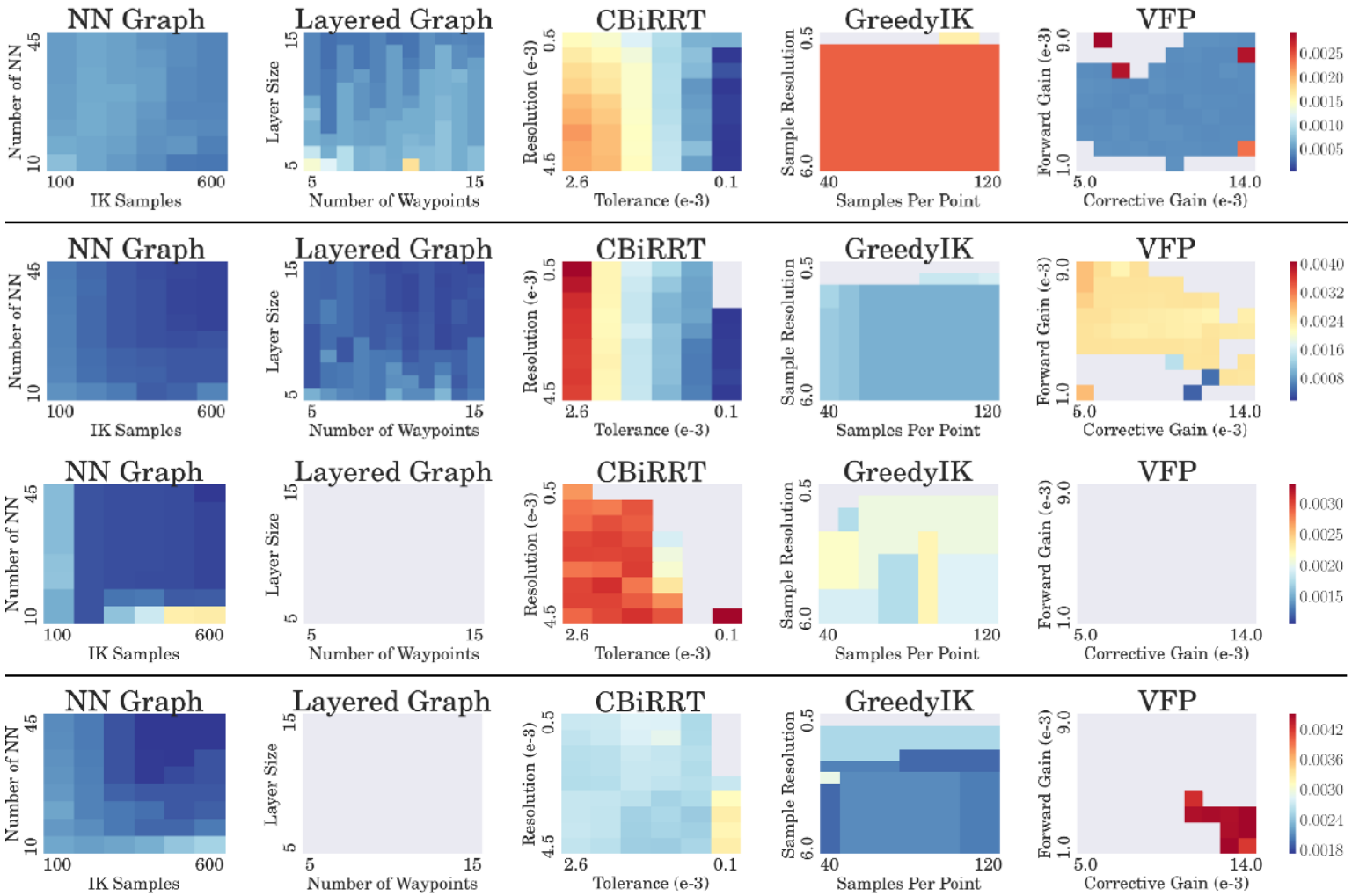
Experimental results—Simulation



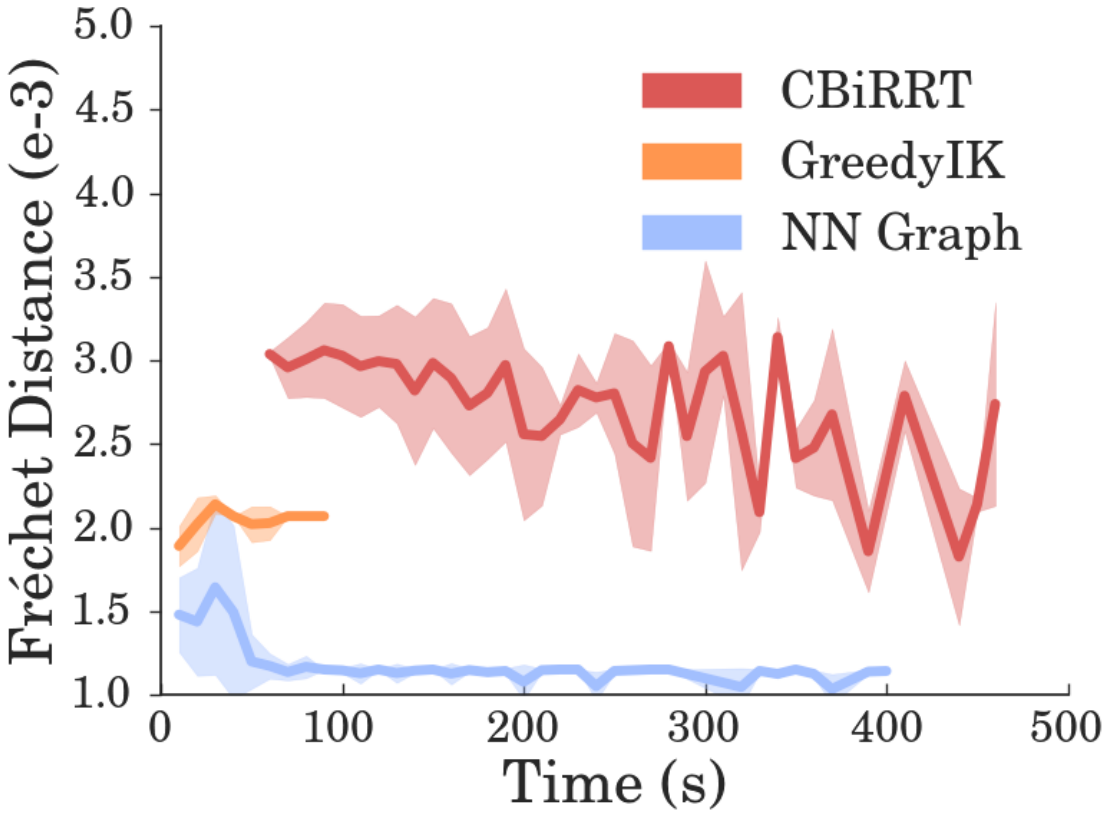
Experimental results—Simulation



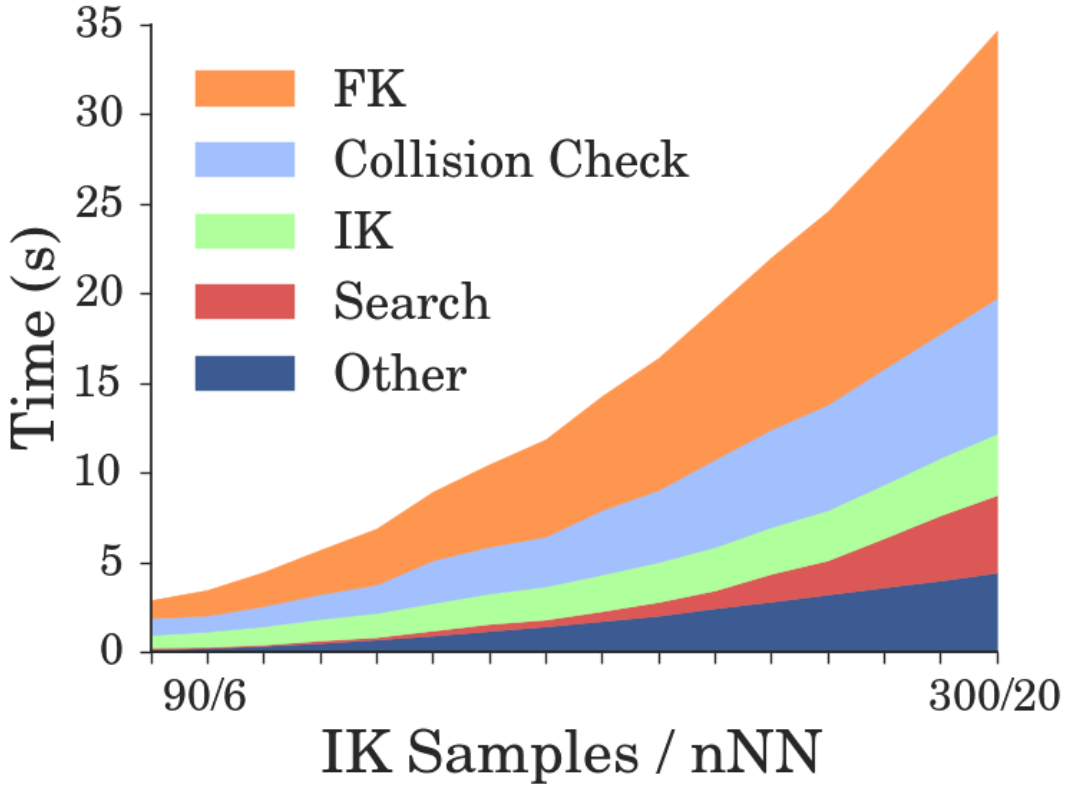
Experimental results—Simulation



Experimental results—Simulation



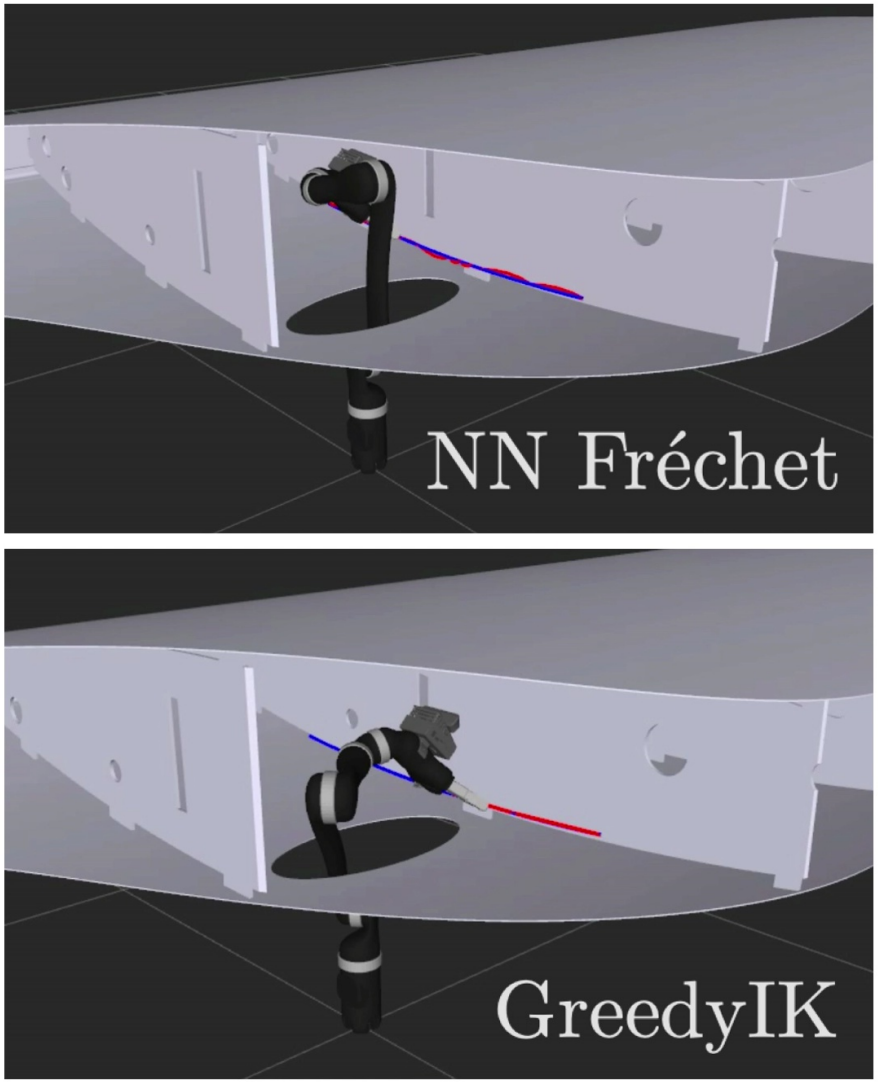
Experimental results—Simulation



Experimental results—Execution on CTR

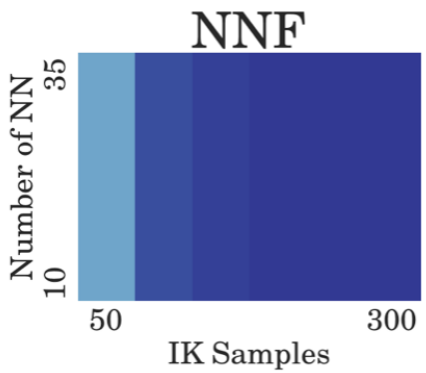
Frechet error: 8mm (real robot) vs. 2mm (simulation) due (primarily) to inaccuracies in kinematic model.

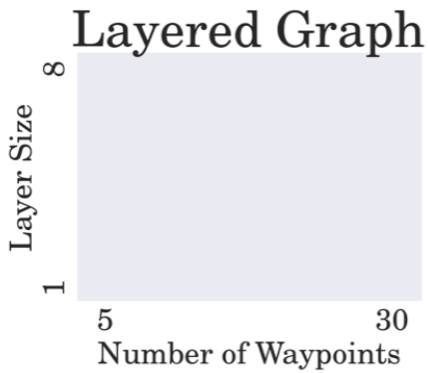
Experimental results—Aircraft wing inspection

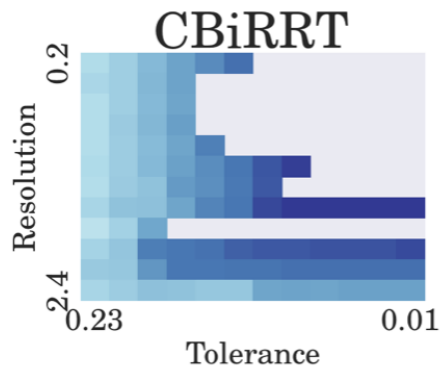


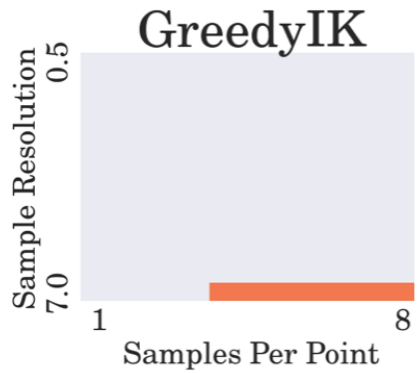
Experimental results—Aircraft wing inspection

Path M-1

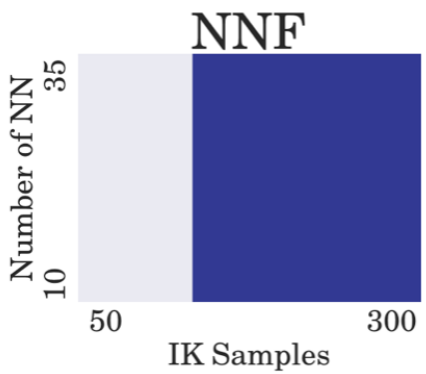


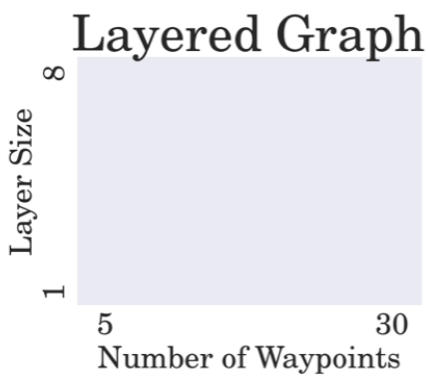


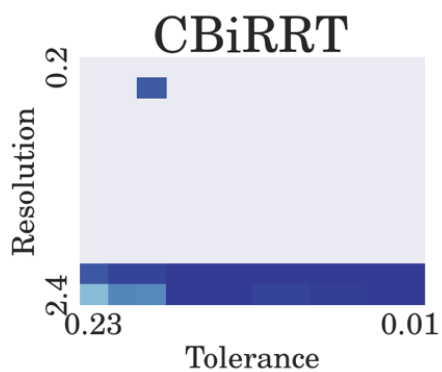


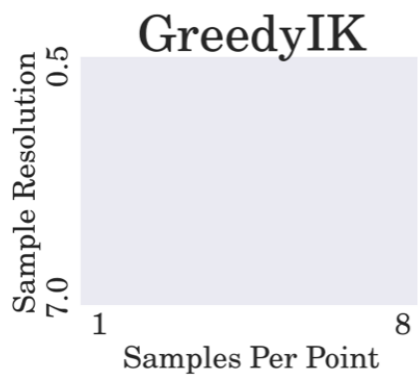


Path M-2

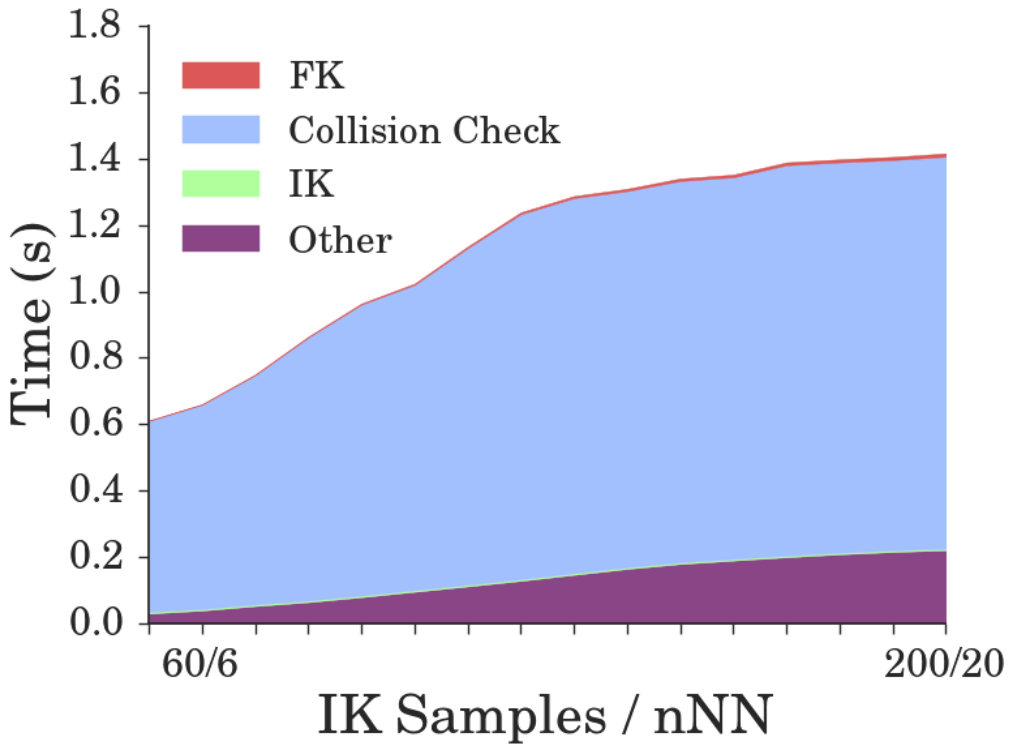




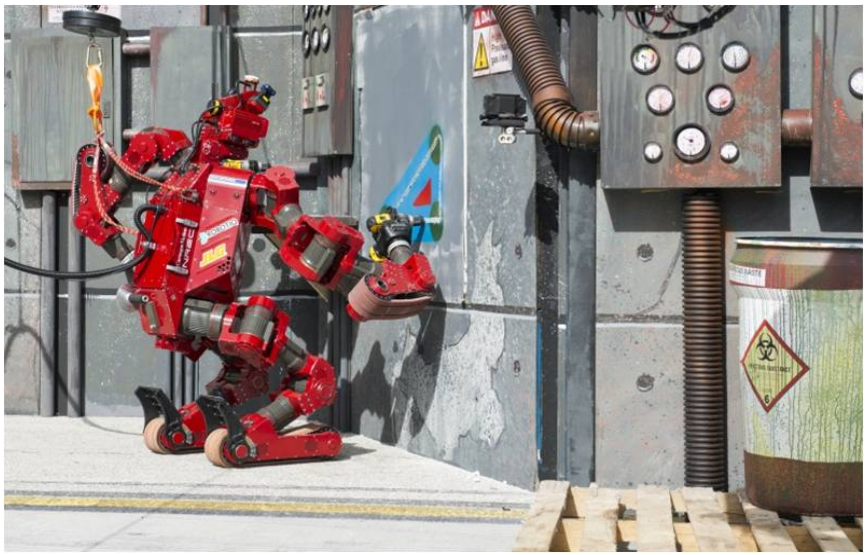


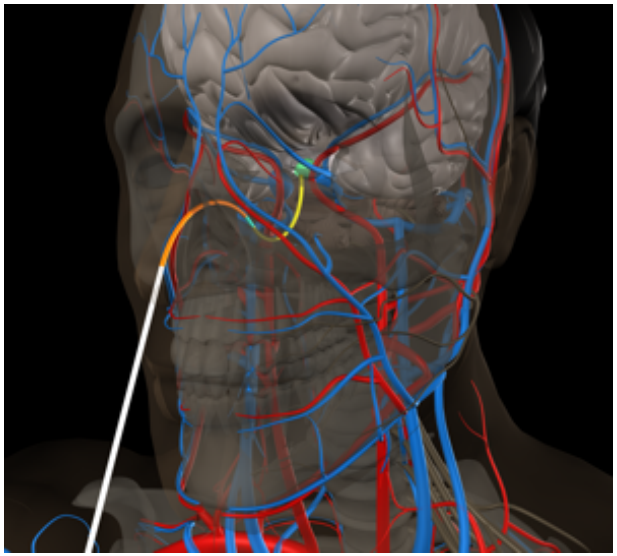


Experimental results—Aircraft wing inspection



Following Paths in Task Space

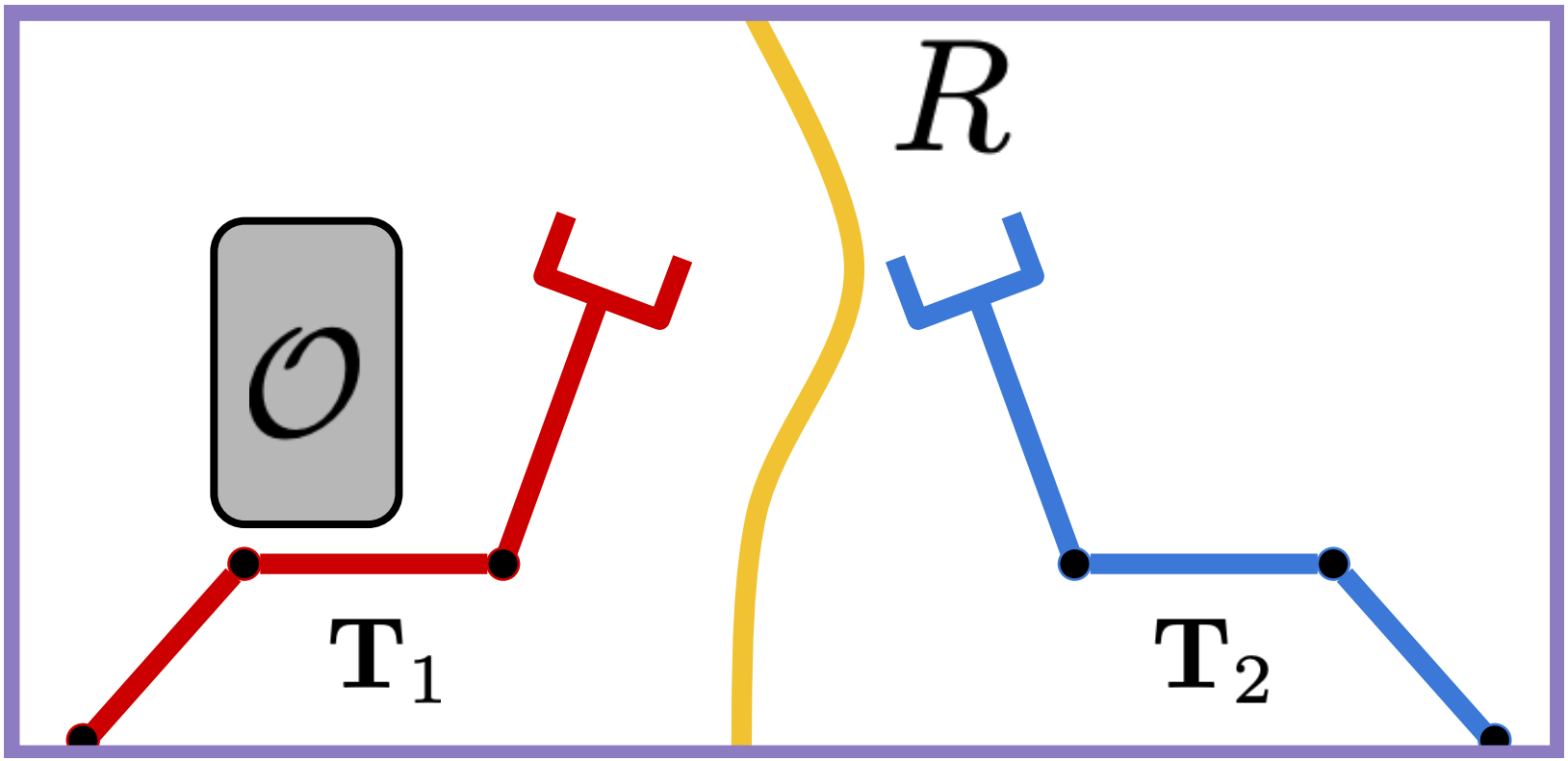




Can we do better?

Following Paths in Task Space

Robot placement matters!



Following Paths in Task Space

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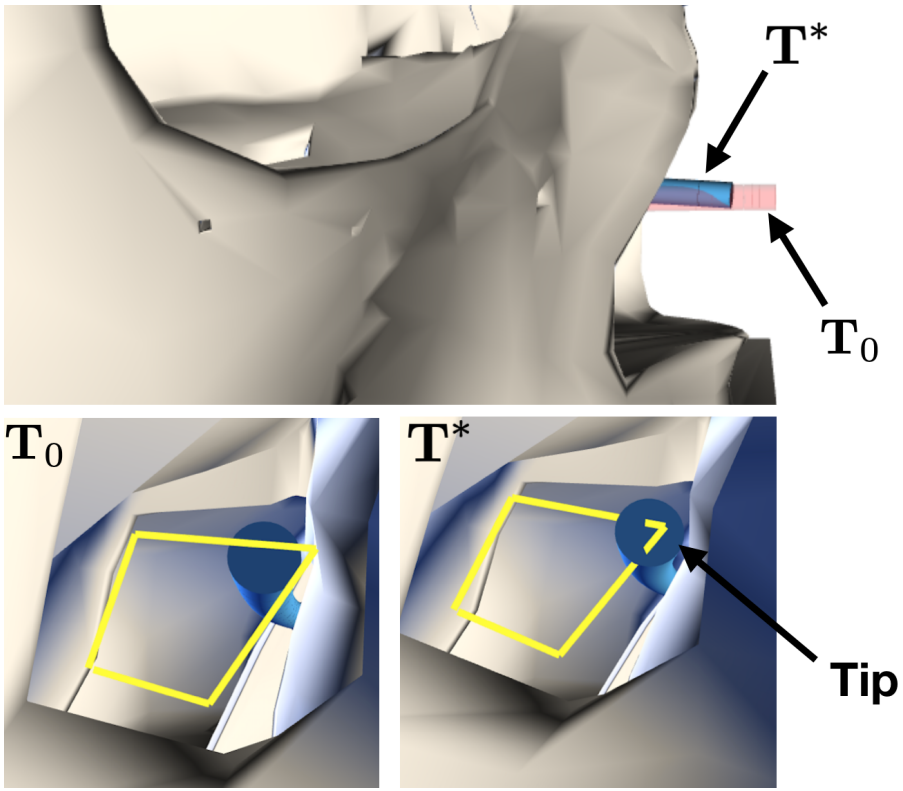

Following reference paths—Extending the optimization function

Let $T \in SE(3)$ be a placement of the robot base

$$\zeta^* = \underbrace{\arg\min_{\zeta \in \mathcal{C}_{\text{free}}}}_{\text{How do we plan ?}} \underbrace{||\text{FK}(\zeta), \zeta_{\text{ref}}||}_{\text{How do we measure distances ?}}$$

NN Graph

dFrećhet distance

Following reference paths—Extending the optimization function

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Design or setup optimization

What we are actually trying to do is to optimize the setups of motionp-lanning problems to maximize the utility of a given task

(informal) definition of setup

The setup of a planning problem is the set of parameters we have control over that must be set before the planner is invoked

Design optimization via reachability maps

- Reachability maps can be used to approximate the cost of motions required by the robot as a function of placement [Vahrenkamp et al. 16; Makhal, Goins 17]
- They fail to account for actual motions required by the robot, especially in highly constrained tasks

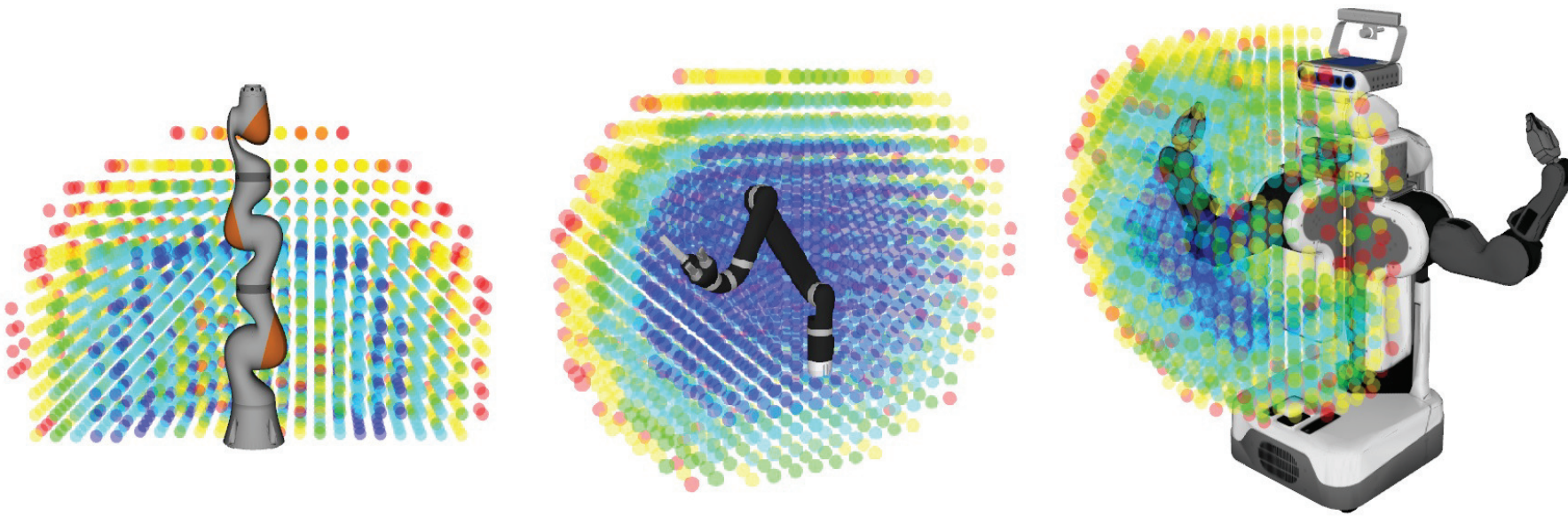
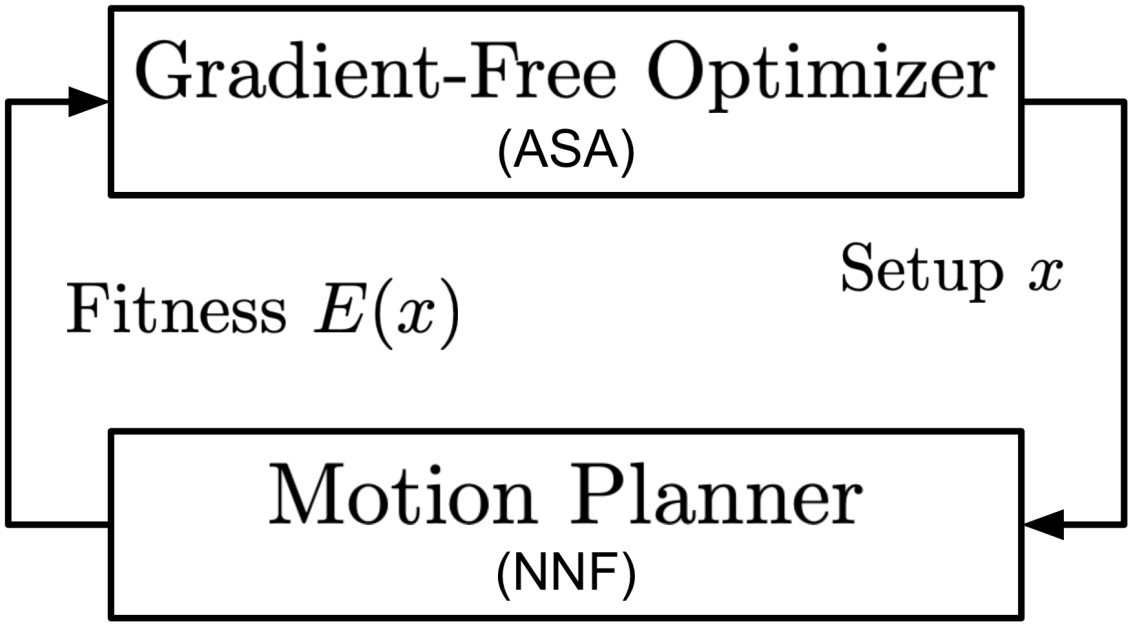


Figure adapted from [Makhal, Goins 17]

Design optimization via global optimization

We want to use the motion planner to asses the quality of a design



- Combines gradient-free optimization over design (insertion pose) with a motion planner used to evaluate cost of each one

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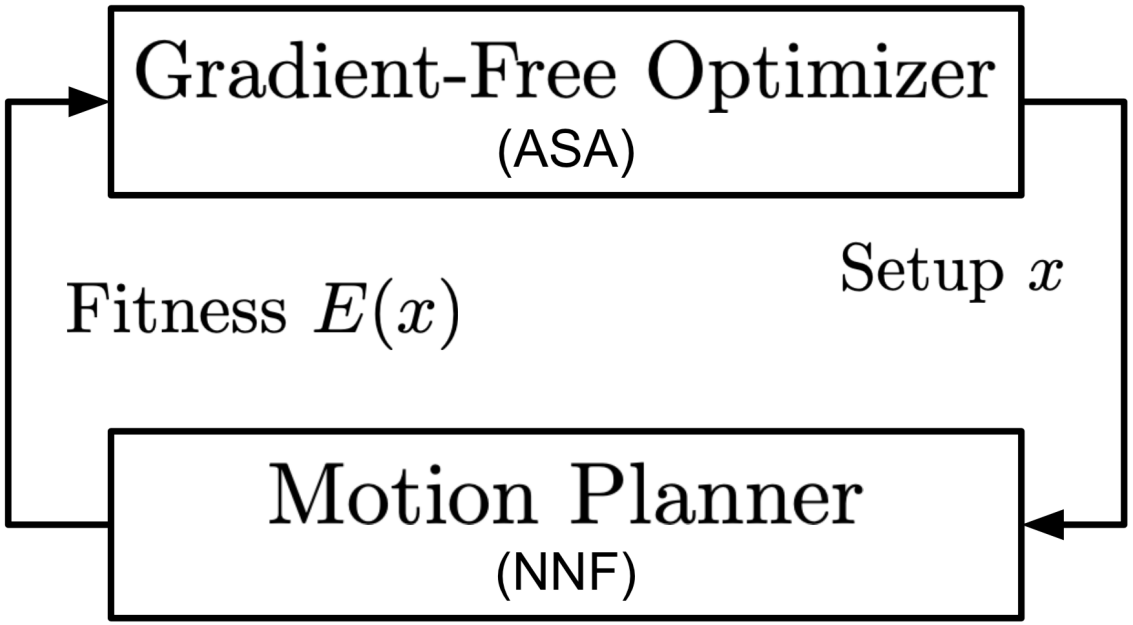

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Adaptive Simulated Annealing (ASA) [Locatelli 02]

- Annealing is a thermal process for obtaining low energy states of a solid in a heat bath
- Two-step process
  - Increase temperature of heat bath to a maximum value at which the solid melts
  - Carefully decrease temperature of heat bath until particles arrange themselves in solid's ground state
- Ground state is a minimum energy state of the solid
- The ground state is obtained only if the maximum temperature is high enough and the cooling is done slowly

Adaptive Simulated Annealing (ASA) [Locatelli 02]

- Analogy with thermodynamics
- Incorporate a temperature parameter into the minimization procedure
- At high temperatures, explore parameter space
- At lower temperatures, restrict exploration

Adaptive Simulated Annealing (ASA) [Locatelli 02]

- Consider decreasing series of temperatures
- For each temperature, iterate these steps:
  - Propose an update and evaluate function
  - Accept updates that improve solution
  - Accept some updates that don't improve solution
    - Acceptance probability depends on "temperature" parameter
- If cooling is sufficiently slow, the global minimum will be reached

Adaptive Simulated Annealing (ASA) [Locatelli 02]

Each iteration,

- sample some $u \in [0, 1]$ uniformly at random
- sample a new setup T "close" to the old setup
- accept T, if:

$$exp\left(\frac{-E(x)-e'}{K}\right) > u$$

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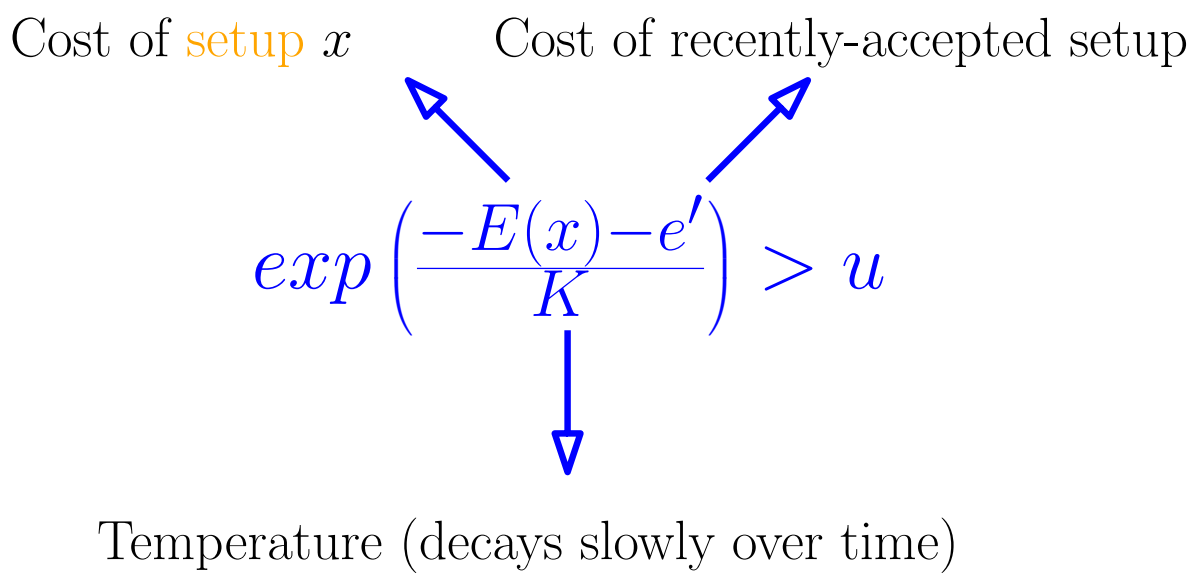
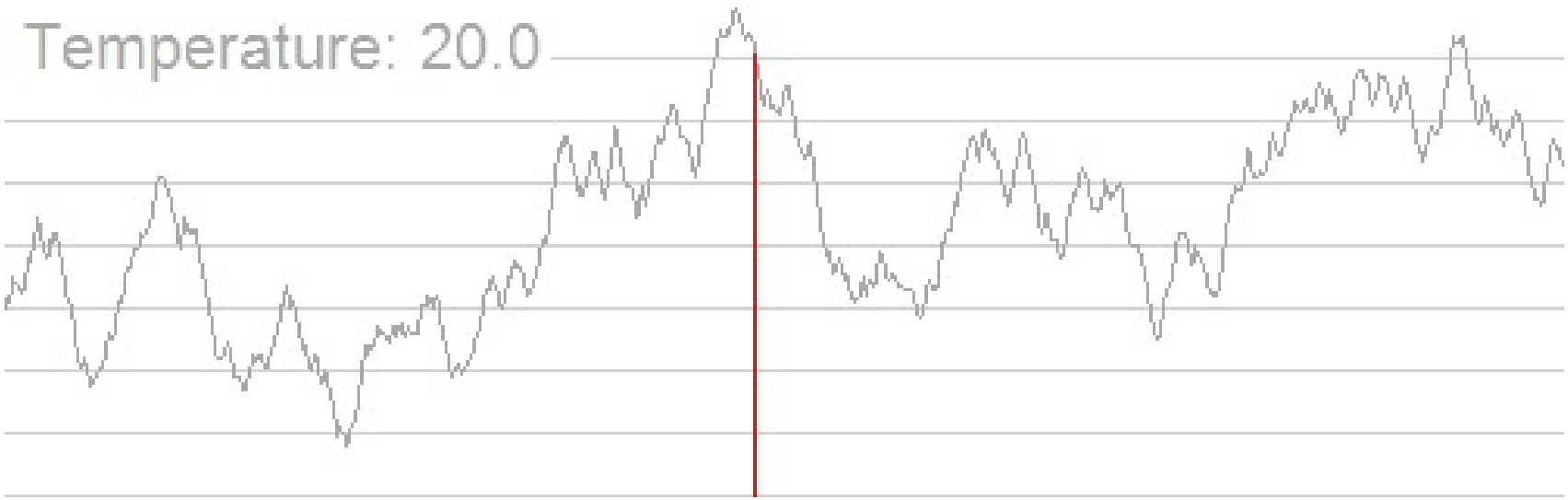


Adaptive Simulated Annealing (ASA) [Locatelli 02]



Adapted from https://en.wikipedia.org/wiki/Simulated_annealing

Recap - ASA for design optimization

- ASA was used to optimize the kinematic design of surgical robots [Baykal, Bowen, Alterovitz 18; Kuntz et al. 18]
- These methods treat the planner as a black box
- However, the planner is computationally expensive

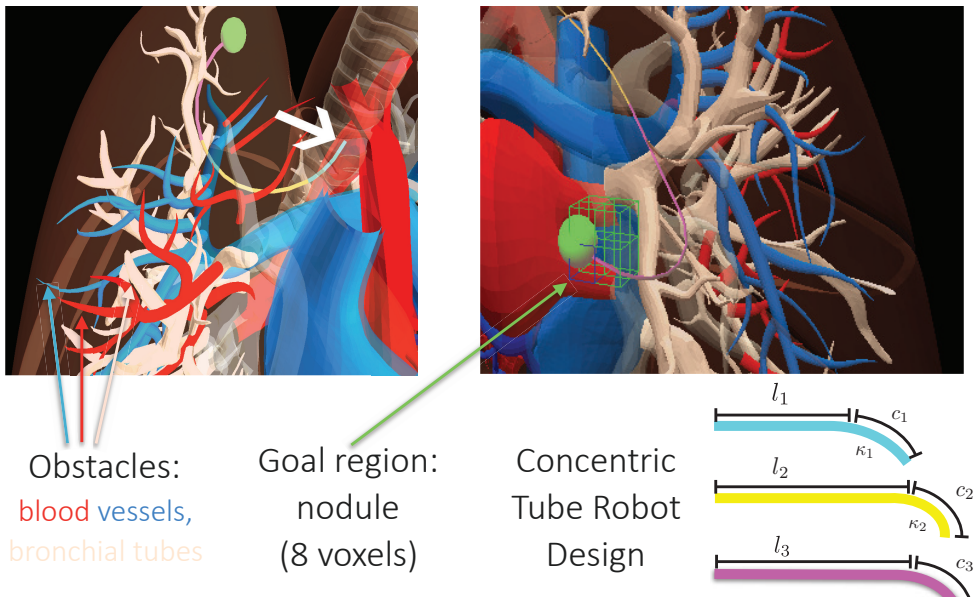


Figure adapted from [Baykal, Alterovitz 17]

ASA for efficient design optimization

- Key insight: open the black box optimizer
- We can show that a setup T cannot be accepted if

$$E(x) \geq \mathcal{B} \text{ s.t. } \mathcal{B} = e' - (K \cdot \ln(u))$$

- Useful!—Many motion planners are able to lower-bound the cost of their final solution as it is being computed
  - E.g. priority of node popped from priority queue in Dijkstra

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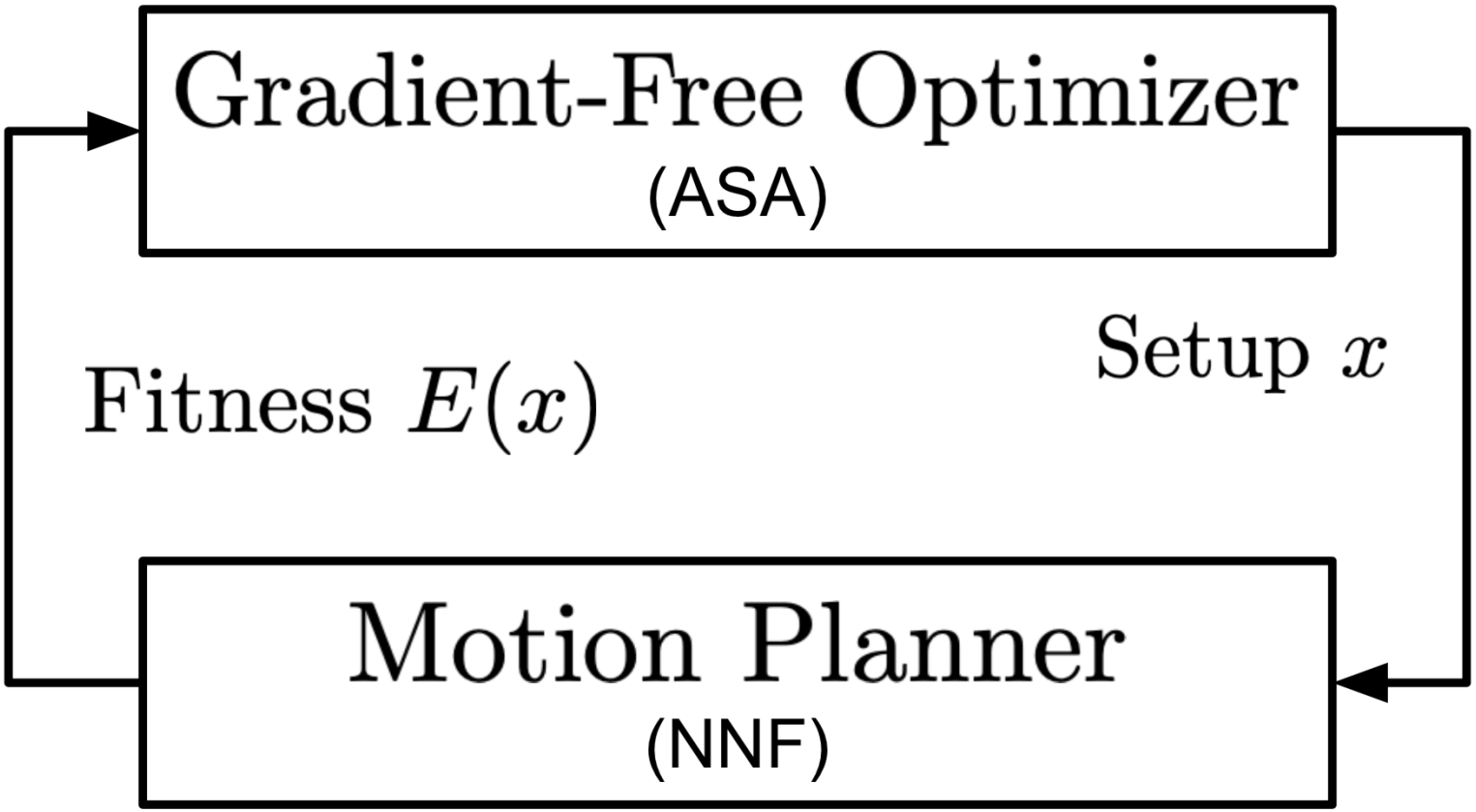

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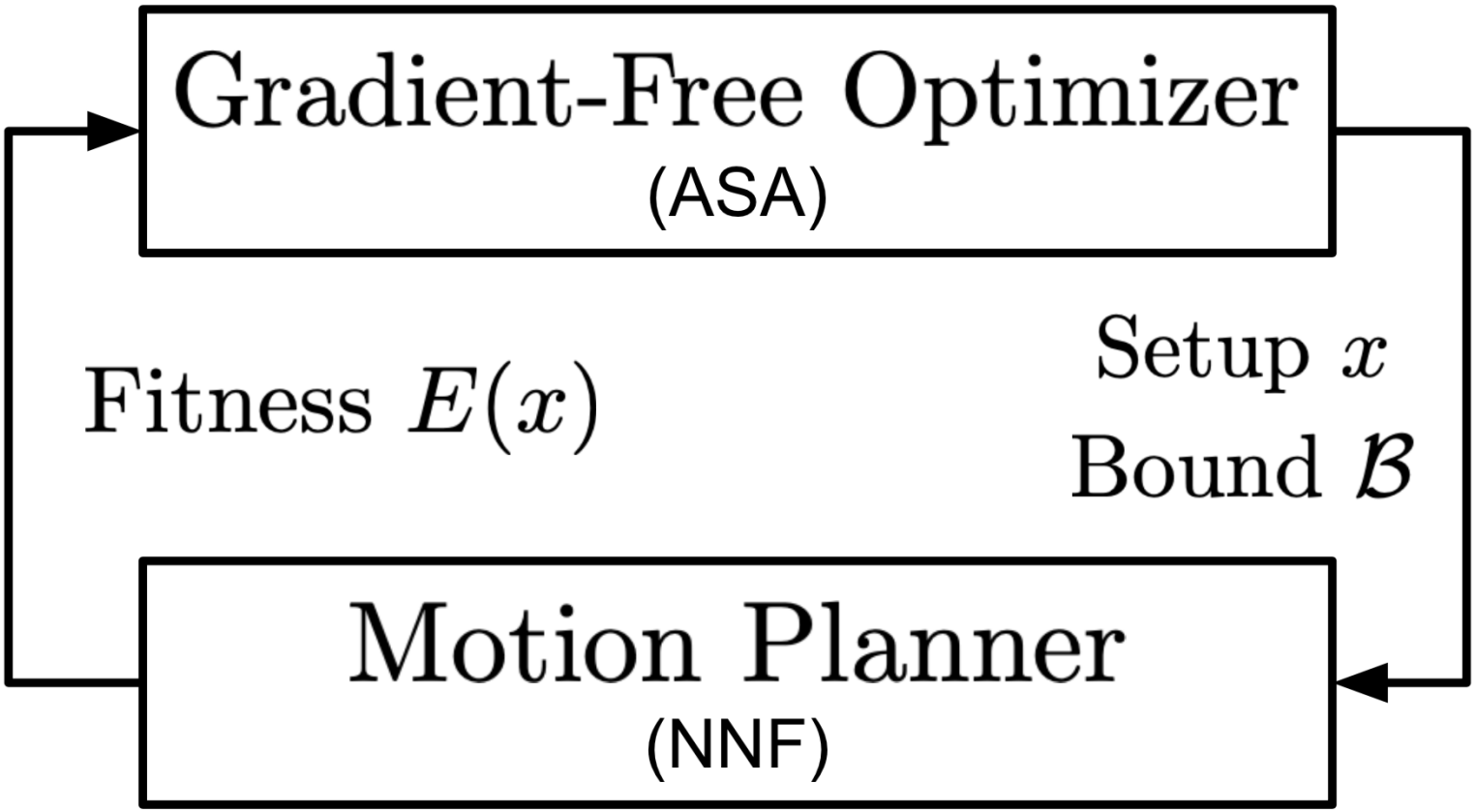
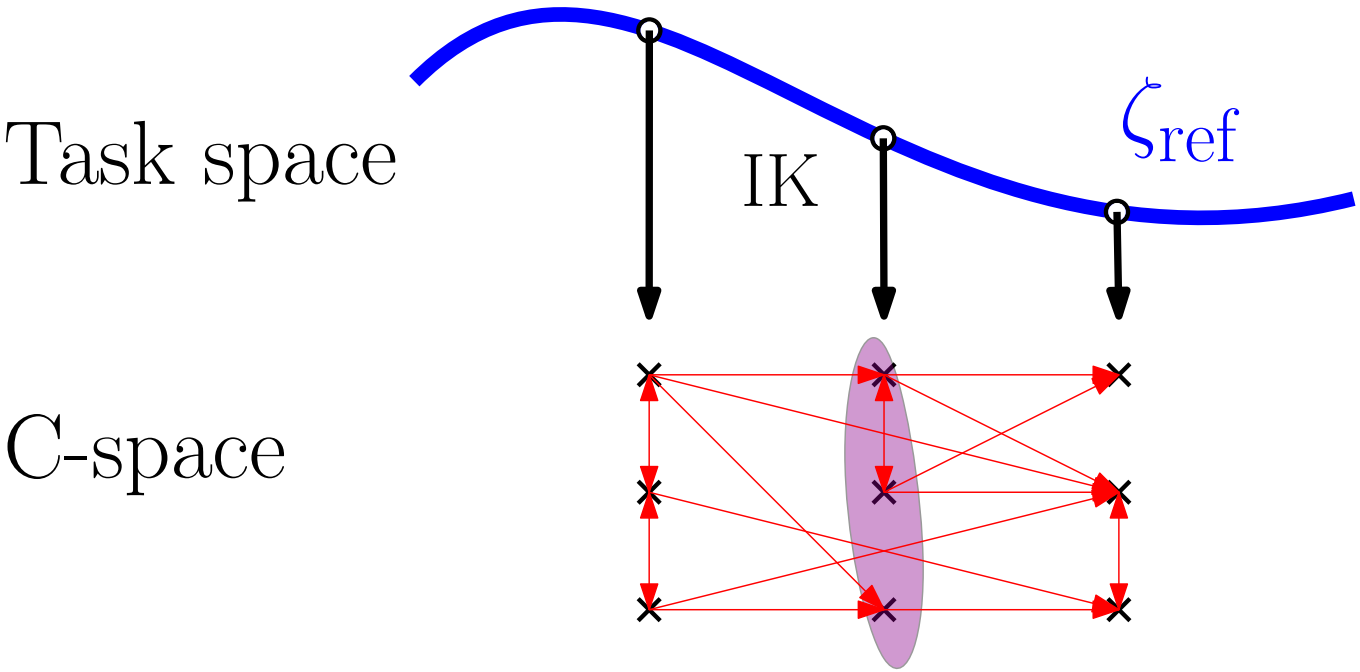
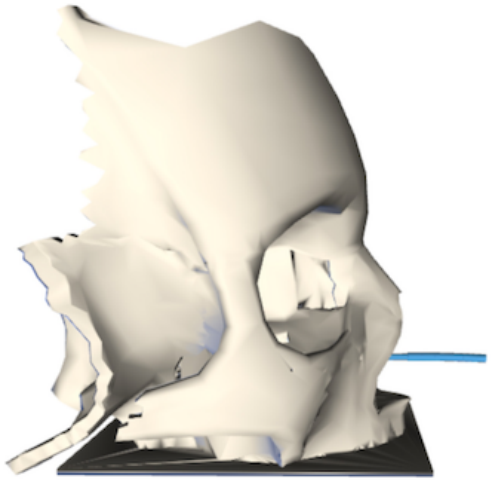


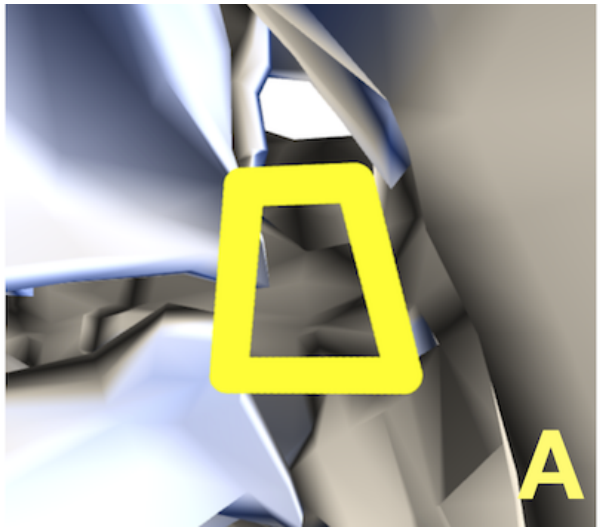
Opening the black box

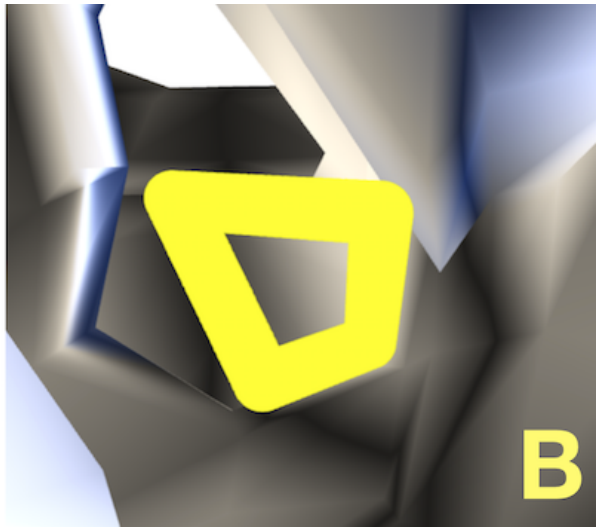


- For every node n expanded by the search algorithm, compare its priority $\text{key}(n)$ to $B = e' - (K \cdot \ln(u))$
- Abort if $\text{key}(n) > B$ (further computation is useless to the optimizer)

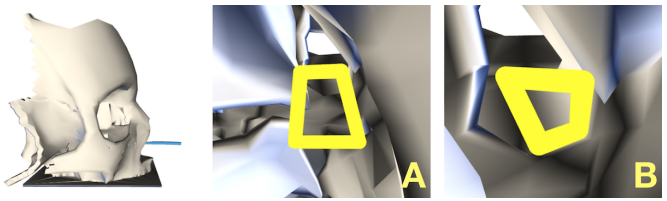
Evaluation

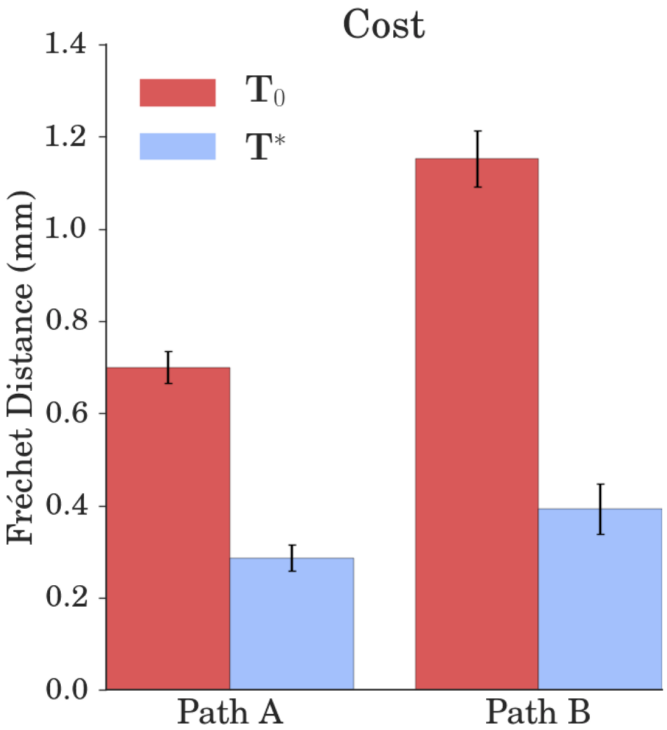


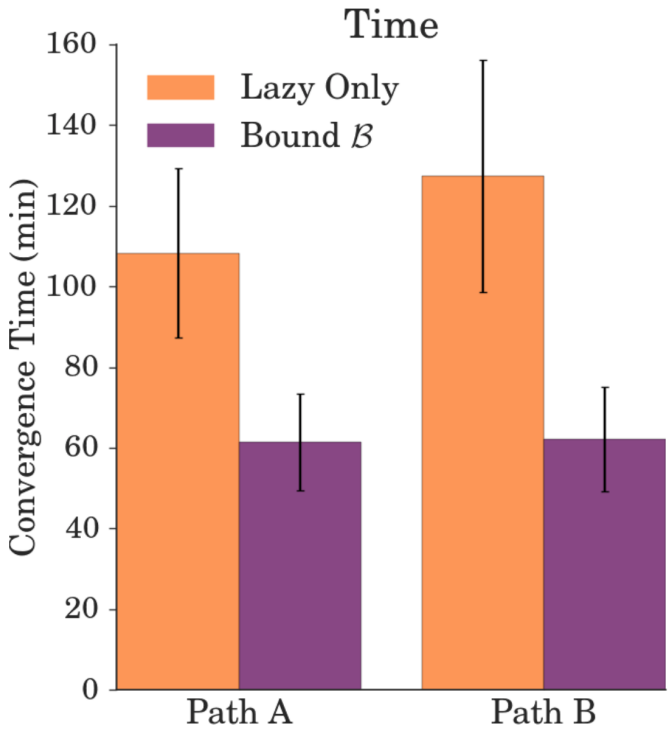




Evaluation


A
B


Cost
T_0
T^*
Fréchet Distance (mm)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Path A
Path B


Time
Lazy Only
Bound $\mathcal{B}$
Convergence Time (min)
160
140
120
100
80
60
40
20
0
Path A
Path B

Evaluation