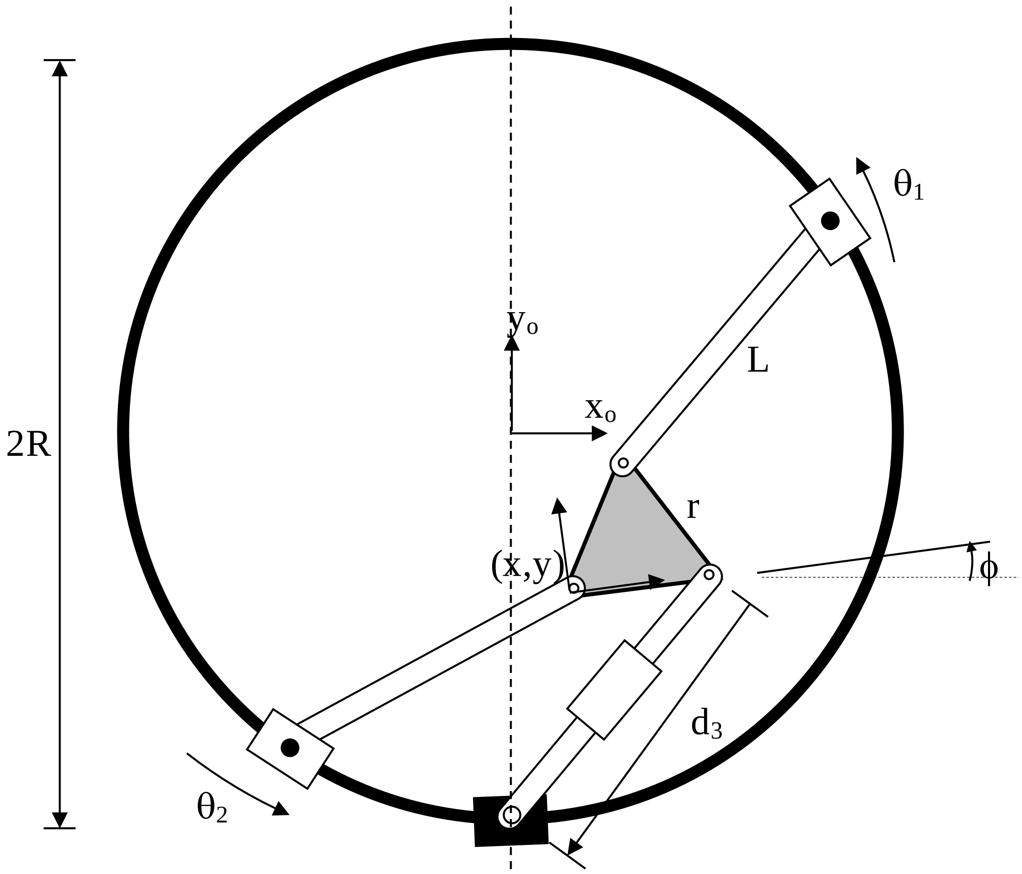
Kinematics, dynamics and control of robots - Homework project 2

The drawing depicts a 3 DOF parallel robot. The central plate is an equilateral triangle with a side length of r. Two of the plate's vertices are connected via revolute joints to identical links of length L, whose other end is connected via revolute joints to sliders. The sliders move along a circular track of radius R. The third vertex of the plate is connected via revolute joint to a link of varying length $d_3 \geq 0$, whose other end is connected via revolute joint to a fixed point on the circular track, along $-y_0$ direction of the frame located at the circle's center. All revolute joints are passive. The robot's actuated degrees of freedom are the two angular positions of the sliders along the track and the varying length of the prismatic joint d_3 so that $\boldsymbol{q} = [\theta_1, \theta_2, d_3]^T$. The task vector is the position and orientation of the moving frame attached to the plate $\boldsymbol{x} = [x, y, \phi]^T$.



1. Solve the inverse kinematics of the robot, that is, find the values of the joints $\boldsymbol{q}$ as a function of the robot's position and orientation $\boldsymbol{x}$.
   Write a code for function of the robot's inverse kinematics (in Matlab or python. The input of the function is the robot's position and orientation $\boldsymbol{x}$, and the output is a matrix whose columns are all the possible solutions of the joints $\boldsymbol{q}$. In addition, the function needs to draw the robot in all the possible solutions in separate figures (use equal axis in the plots).
   Submit the output of the function, values and draws, for the task $[x, y, \phi] = [-3, -1, 45°]$.

2. Solve the forward kinematics of the robot, that is, find the position and orientation of the end effector $\boldsymbol{x}$ as a function of the joint values $\boldsymbol{q}$.
   Directions:
   - By manipulating the inverse kinematics equations, eliminate all $\{x^2, y^2\}$ terms to obtain a linear system in $\{x, y\}$ whose coefficients depend nonlinearly on ϕ.
   - Substitute the solution for $\{x, y\}$ into one of the original equations.
   - Substitute:
   $$\cos\phi = \frac{1-t^2}{1+t^2}, \qquad \sin\phi = \frac{2t}{1+t^2}, \qquad \tan\frac{\phi}{2} = t$$
   - Solve the high degree polynomial in t numerically.

   Detail all the steps of the solution. There is no need to write the coefficients of the polynomial.

   Write a code for function of the robot's forward kinematics (in Matlab or python). The input of the function is the joints' values $\boldsymbol{q}$, and the output is all matrix whose columns are all the possible solutions of the robot's position and orientation $\boldsymbol{x}$. In addition, the function needs to draw the robot in all the possible solutions in separate figures (use equal axis in the plots).
   Check one of your solutions for question 1 using this function, add the output, and explain it.
   Verify each solution for $\boldsymbol{x}$ by recalculating the inverse kinematics $\boldsymbol{q} = f(\boldsymbol{x})$. What is the magnitude of the error obtained? Explain.

3. Plan the motion of the plate, $\boldsymbol{x}$ from point $\boldsymbol{x}_a = [-3, -2, 45°]^T$ to point $\boldsymbol{x}_b = [-2, 0, 0°]^T$ along a straight line in constant linear and angular velocity over 2 seconds, under restriction that the links cannot cross each other. Present analytical expression for $\boldsymbol{x}(t)$, and plots of the joint variables position as a function of time. In addition, draw the robot's platform and the 3 links at $t = (0, 0.5, 1, 1.5, 2)[sec]$ in x-y plane, overlaid on the same plot.

4. (*) Calculate parametrically the Jacobian matrices $J_q\dot{\boldsymbol{q}} = J_x\dot{\boldsymbol{x}}$ by differentiating the geometric constraints $F(\boldsymbol{x}, \boldsymbol{q}) = 0$. Express the Jacobian matrices as a function of $\boldsymbol{x}, \boldsymbol{q}$.

5. (*) For $\phi = 10°, y = 0$, and the range $0 < x < 2$, find and draw all, qualitatively different, singular positions. For each position, calculate the direction of free motion where $\mathrm{J}_x \cdot \dot{\boldsymbol{x}} = 0$ holds.

6. Write a short paragraph summarizing the assignment, refer to the results and your conclusions.

General comment: It is recommended to use symbolic programs such as Matlab/Maple/Python, but the printed results do not replace a detailed explanation of all the steps to the solution.
Exercises with (*) can be solved after lecture and tutorial 4 (18.04.2023)

Submission in pairs by 9.5.2023 on the course site.

Robot parameters: $r = 2, L = 3.5, R = 4$ (all sections should be solved parametrically. Substitute values only for final numeric solutions.)

<u>Useful Matlab commands:</u>

Spring semester 2023

Numeric calculation of polynomial roots by its coefficients	sol=roots([a3 a2 a1 a0])
Calculate eigenvalues and eigenvectors	[M, D]=eig(A)
Draw a triangle	patch([x1 x2 x3], [y1 y2 y3], 'green', 'edgecolor', 'red')
Allow drawing additional line on figure	hold on
Draw a line	plot(xvec, yvec, 'blue', 'linewidth', 2)
Use the same length for the data units along each axis	axis equal
Set figure limits	axis([-1 5 0 6])
Set symbolic variables	syms x y
Differentiate symbolic expression by x	diff(f, x)
Substitute values in symbolic expression	subs(f, [x, y], [5, z^2])
Calculate Jacobian of symbolic vector	J=jacobian([Px, Py, Pz], [q1, q2, q3])
Symbolic solution of system of equations	solve('a*x+b*y=1', 'c*x+d*y=0', 'x, y')
Simplify, reorder symbolic expression	collect, simplify, factor
converts a symbolic expression to a numeric	matlab function