TECHNION
Israel Institute
of Technology

Kinematics, dynamics and control of robots

Homework project 4

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1) PD + Inverse Dynamics

$$\tau = C(q, \dot{q})\dot{q} + G(q) + H[\ddot{q}_d - K_P(q - q_d) - K_D(\dot{q} - \dot{q}_d)]$$

In this approach, we augment the PD tracking controller with the required torques and forces to overcome the system's dynamics. To implement this controller, we assume that the motion equations of the system are known in their entirety. Additionally, we aim to select the matrix values of K_p and K_d in such a way that the control efforts are as low as possible, while ensuring good tracking performance. As an initial guess (to grasp insight into error dynamics and control behavior), we substitute the controller into the motion equations:

$$H\ddot{q} + C\dot{q} + G = \tau = C\dot{q} + G + H[\ddot{q_d} - K_p(q - q_d) - K_d(\dot{q} - \dot{q_d})]$$
$$\Rightarrow \; H(\ddot{q} - \ddot{q_d}) + HK_d(\dot{q} - \dot{q_d}) + HK_p(q - q_d) = 0 \;\backslash H^{-1}\{\, symetric \,\&P.D\}$$
$$(\ddot{q} - \ddot{q_d}) + K_d(\dot{q} - \dot{q_d}) + K_p(q - q_d) = 0 \quad \{e = q - q_d\}$$
$$\ddot{e} + K_d\dot{e} + K_p e = 0$$

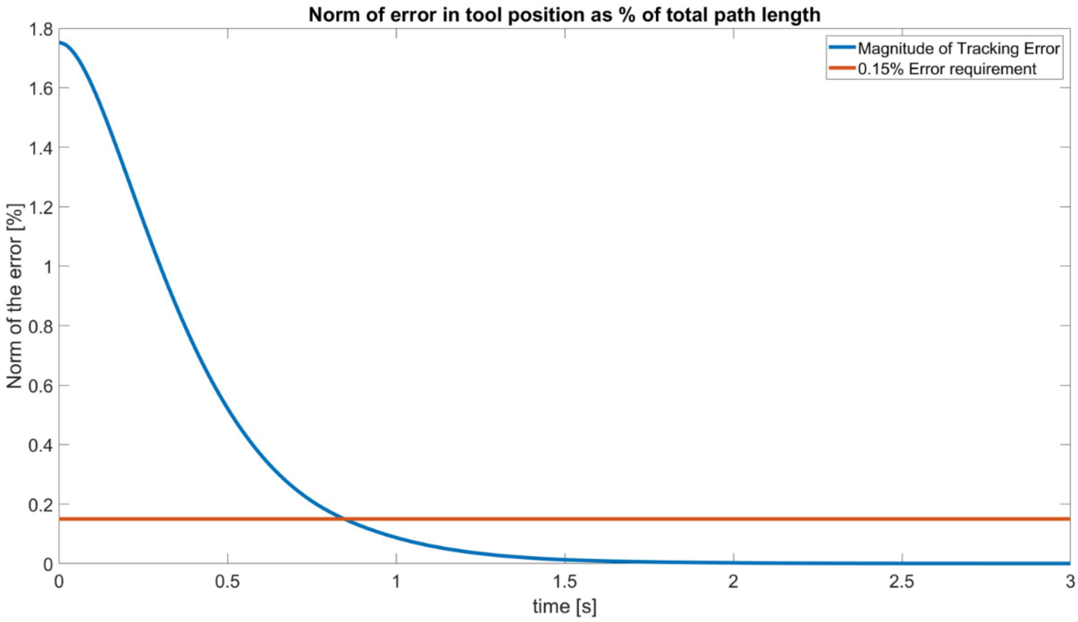
We would like to use the characterize equation responds for the second order equation. We perform a calculation to approximate the convergence percentage for the error magnitude and the total path length for the gripper's position along the z-axis.
The coefficient matrices of the PD components were chosen to be diagonal matrices (to prevent coupling between the joints) and strictly positive definite.
Control values alignment

$$s^2 + 2\zeta\omega_n + \omega^2 \Rightarrow \begin{cases} K_d = 2\zeta\omega_n \\ K_p = \omega_n^2 \end{cases}$$

$$e_{0z} = 0.01[m], |Z_B - Z_A| = |0.7 - 0.5| = 0.2$$
$$e_{settling} = 0.15\% \cdot |Z_B - Z_A| = 0.2 \cdot 0.0015 = 3 \cdot 10^{-4}$$
$$e_{ration} = \frac{e_{settling}}{e_{0z}} = 0.0015$$
$$t_s = -\frac{\ln(0.002)}{\sigma} = 1.5[s] \rightarrow \sigma = 4.335 \rightarrow \zeta\omega_n = 4.335$$
$$\Rightarrow K_d = 2\zeta\omega_n = 8.67$$
$$K_p = 2\zeta\omega_n = 23.2$$

$$K_P = \begin{bmatrix} 23.2 & 0 & 0 \\ 0 & 23.2 & 0 \\ 0 & 0 & 23.2 \end{bmatrix}, K_D = \begin{bmatrix} 8.67 & 0 & 0 \\ 0 & 8.67 & 0 \\ 0 & 0 & 8.67 \end{bmatrix}$$

Pluging these coefficients showed good results and the magnitude tracking error in end effector's position converges to 0.15% of the total path length in less than 0.8 seconds:



We adjusted the gains to minimize force and torque while meeting requirements. The values were found through trial and error. Control values for the first joint had minor impact; KP and KD were set at 1. Increasing KP reduced early error but increased later error. KD increased in early error but decreased later error. Values were increased gradually to meet requirements with minimal control efforts. Control values converged the error to 0.15% just before 1.5 seconds. The resulting controller is as follows:

$$K_P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 5 \end{bmatrix}, K_D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Simulation Result:

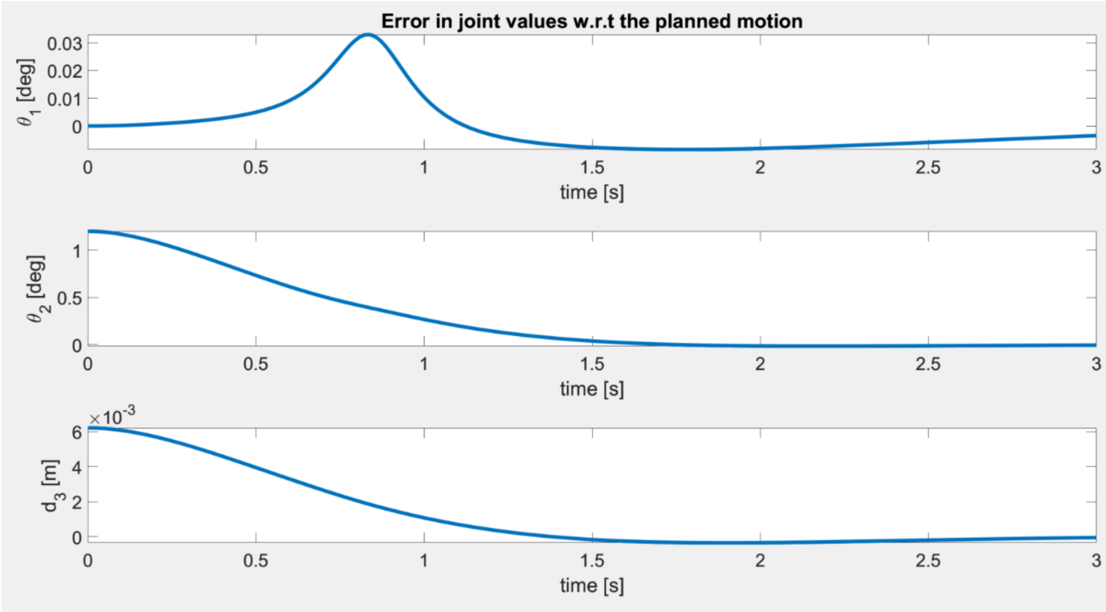


Figure 1 presents the error in the joints relative to the desired motion.

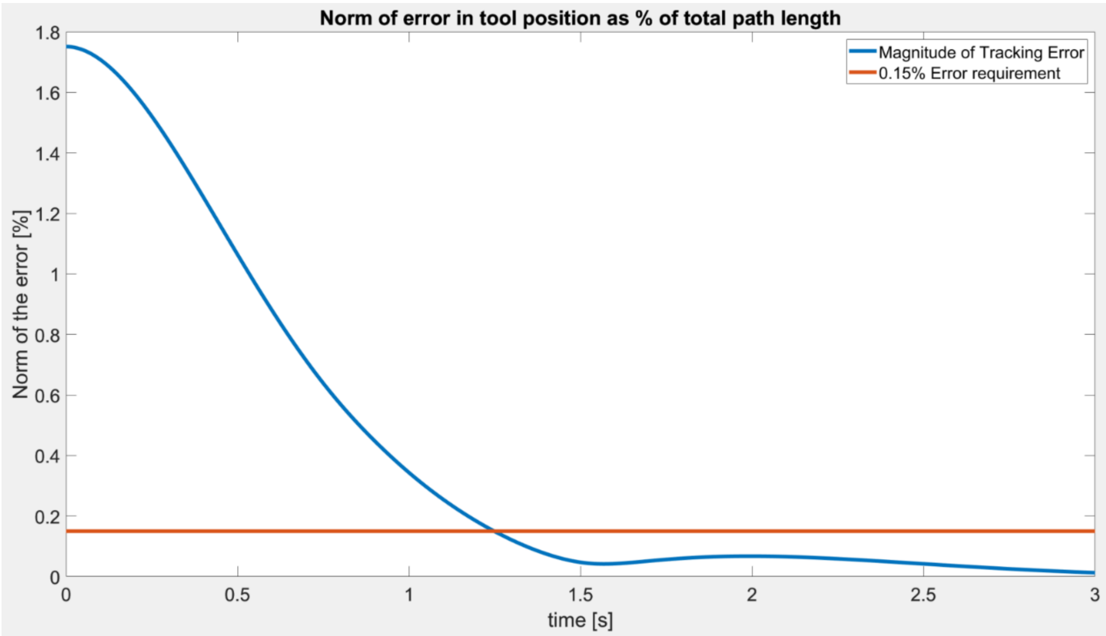


Figure 2 illustrates the error norm of the tool position relative to the path length.

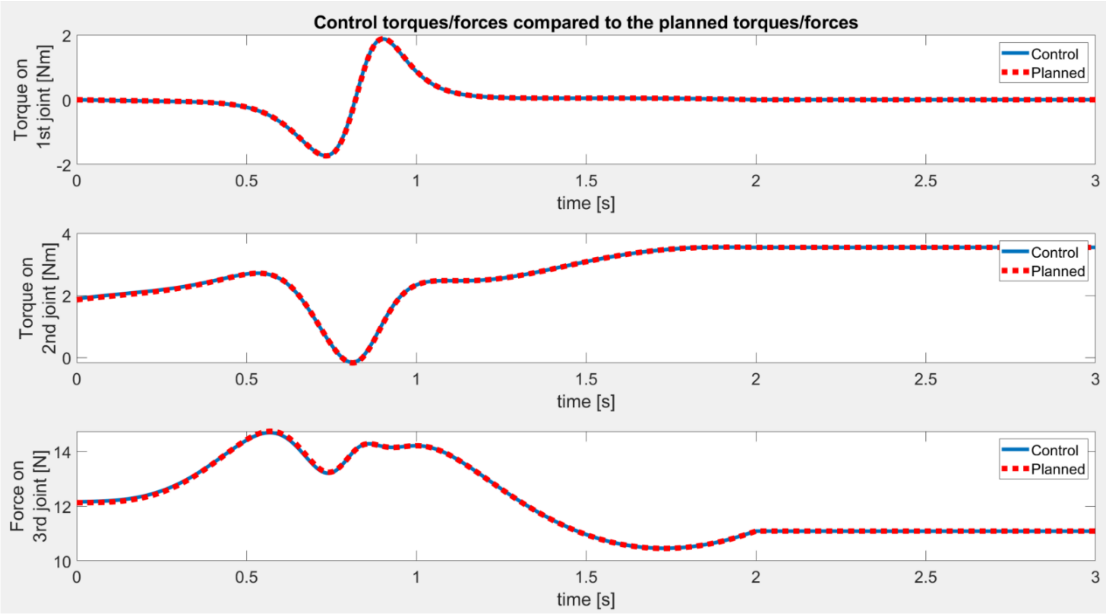


Figure 3 displays the torques and forces exerted by the controller in relation to the desired forces and torques.

Simulation without load mass at the end of the gripper:

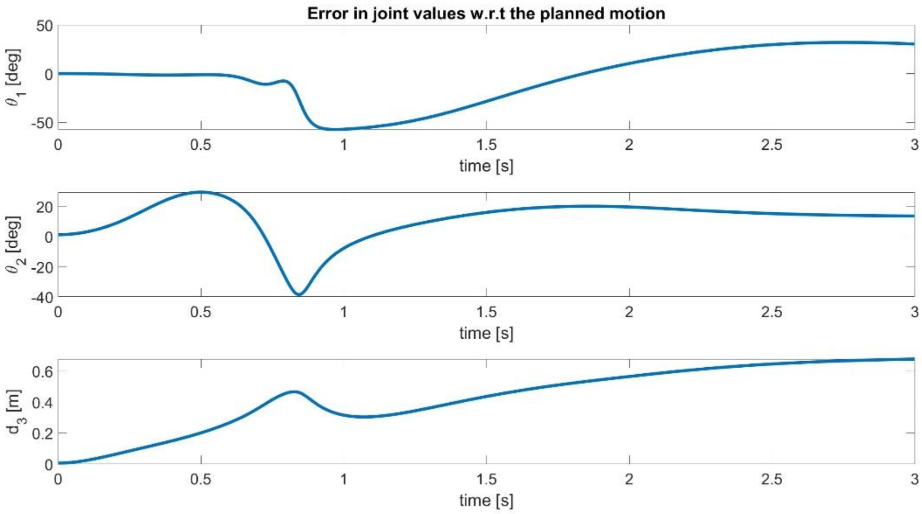


Figure 4 presents the error in the joints relative to the intended motion.

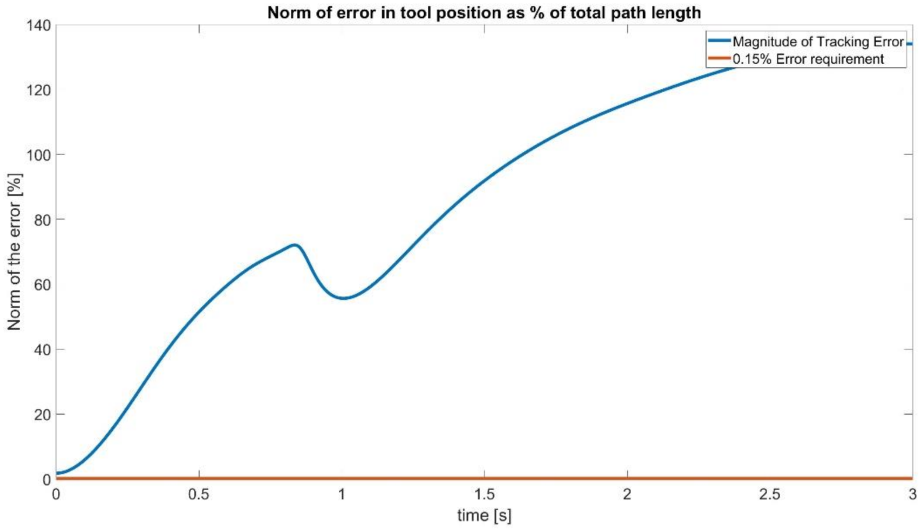


Figure 5 illustrates the error norm of the tool's position relative to the path length.

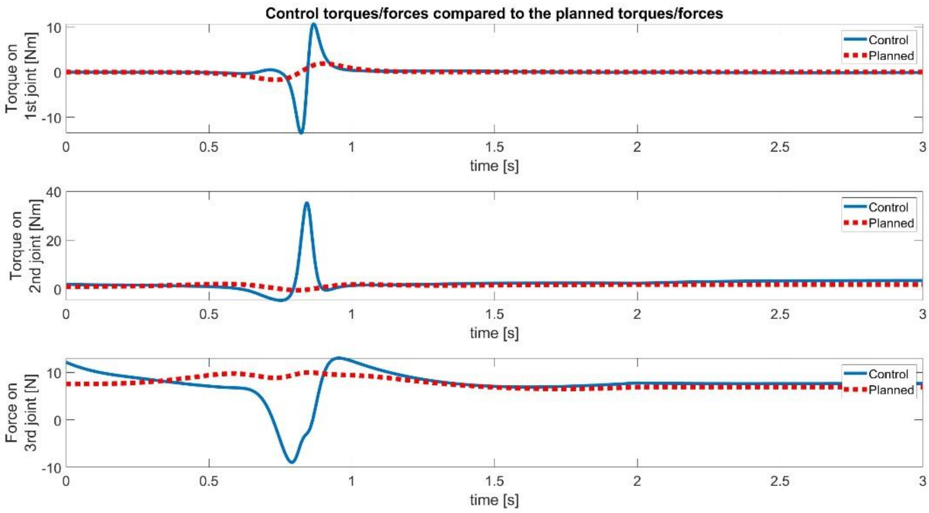


Figure 6 displays the torques and forces in the joints exerted by the controller in relation to the desired forces and torques.

Explanation of the Results:

By removing the mass at the gripper's end, it can be observed that the error increases to a considerable percentage. The reason for this lies in the fact that the computed torques and forces take into account a specific mass (0.5 kg). When there is no mass at all, additional forces are required to move the robot. When external forces are applied to a massless tool, its position diverges and moves further away. This can also be seen in the analysis of joint positions: the error compared to the dynamically computed values is significant, causing the error to increase rapidly, preventing the system from reaching its destination point (point B).

This control loop heavily depends on the robot's parameters through the inertia matrix H, velocity matrix C, and gravity compensation G. All of them involve the masses and inertial moments of the system components. Thus, a change in mass disrupts the system's stability, rendering this loop unsuitable for cases with uncertainties in the constant values.

Summary of Inverse Dynamics + PD Control:
The complexity of this control is not trivial due to the necessity to compute matrices H, C, G at each time step, making it unsuitable for real-time requirements. If matrices H, C, G are precisely known, the control loop will converge to the desired error. However, if any parameter constituting these matrices is imprecise, such as mass or inertia of a controlled body, the control will not converge to the desired value. The required forces and torques are not significantly greater than the intended values. Since the control relies on knowledge of these matrices, it lacks robustness and is highly sensitive to errors.

2) PD+G Control

Finding the Controller Constant

$$\tau = G(q) - K_P(q - q_d) - K_D(\dot{q} - \dot{q}_d)$$

This controller combines gravity compensation and PD control. Just like the control method used for inverse dynamics and PD (which follows the same criteria from Section 1), this controller also needs 2 control settings. We select control matrices in a similar way to how we did for the inverse dynamics and PD control (ensuring they are symmetric and positively defined).
Tuning this controller is like the previous one, where we try things out and adjust until it works best. Initially, we set up the controller with diagonal control settings and a value of 1 on the diagonal. Then, we gradually increase the control settings by a factor of 10 until they meet our requirements. We fine-tune the values to find the best performance. Here are the final controller settings:

$$K_P = \begin{bmatrix} 825 & 0 & 0 \\ 0 & 1200 & 0 \\ 0 & 0 & 900 \end{bmatrix}, K_D = \begin{bmatrix} 750 & 0 & 0 \\ 0 & 750 & 0 \\ 0 & 0 & 450 \end{bmatrix}$$

Simulation Result:

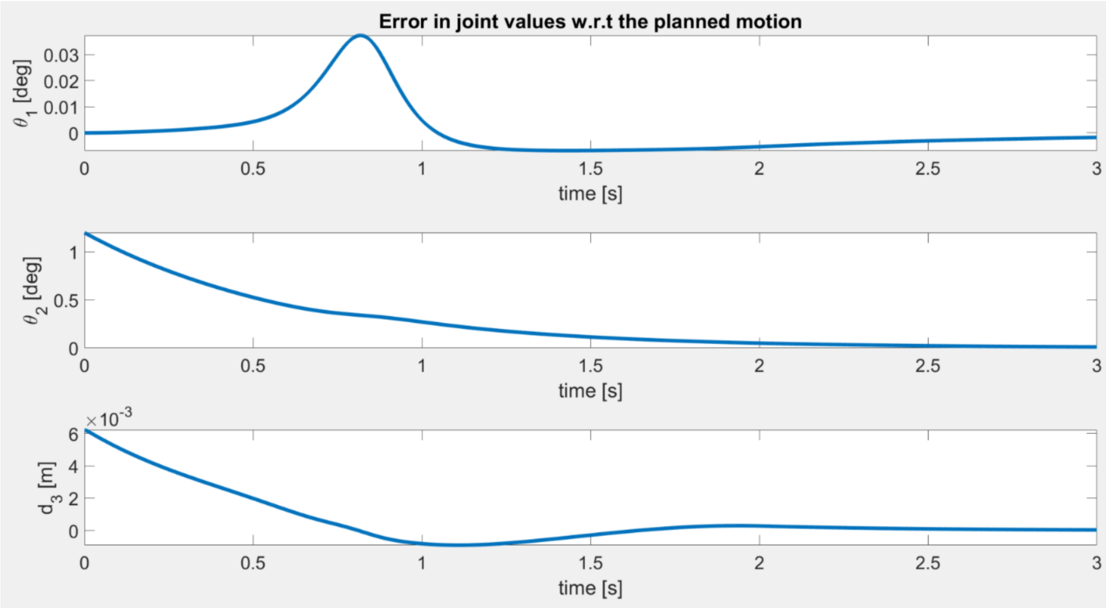


Figure 7 depicts the error in the joints relative to the intended motion.

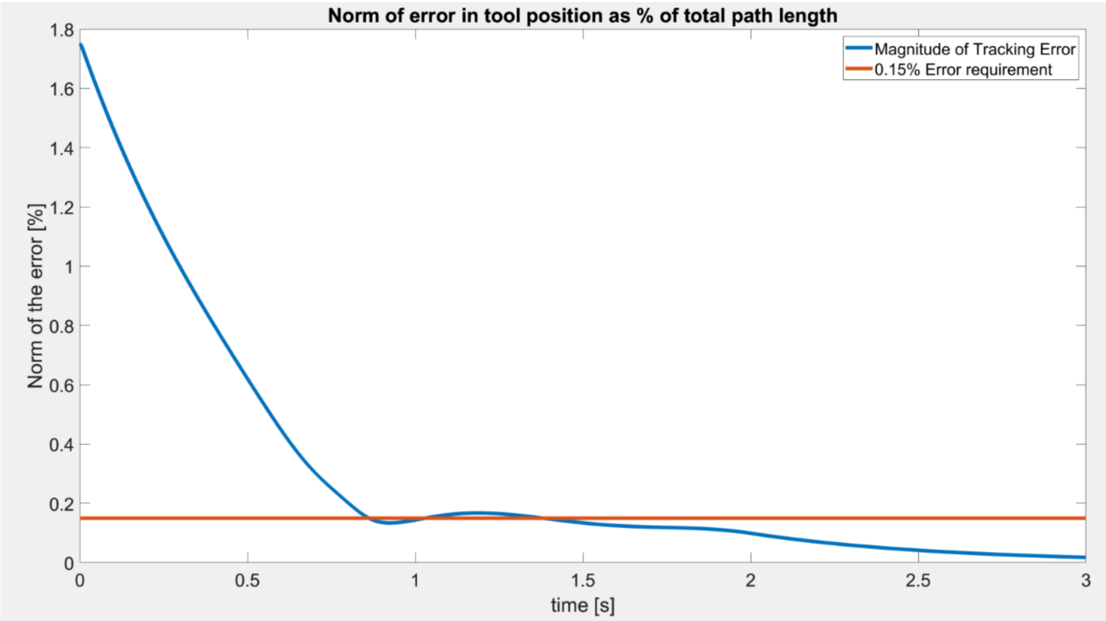


Figure 8 illustrates the error norm of the tool's position relative to the path length.

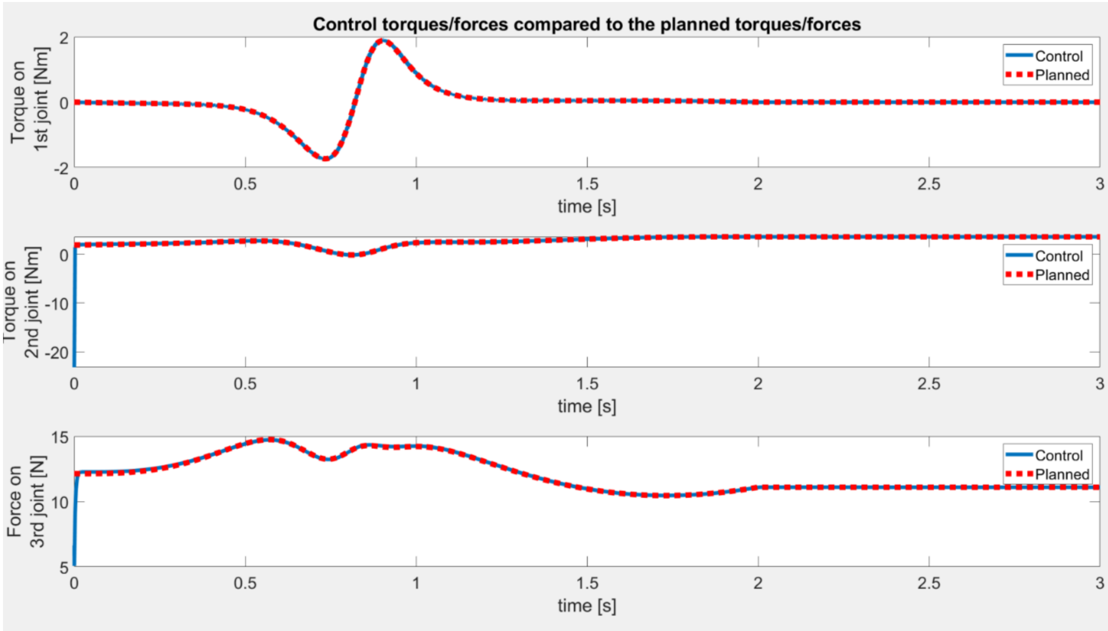


Figure 9 displays the torques and forces in the joints exerted by the controller in relation to the desired forces and torques

Simulation without load mass at gripper end:

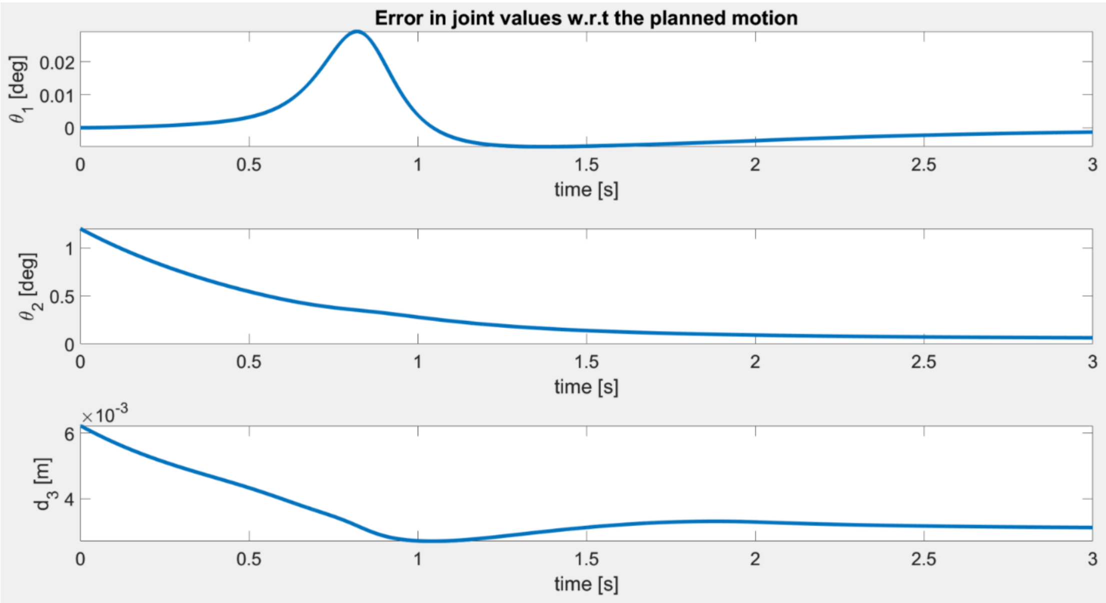


Figure 10 presents the error in the joints relative to the intended motion.

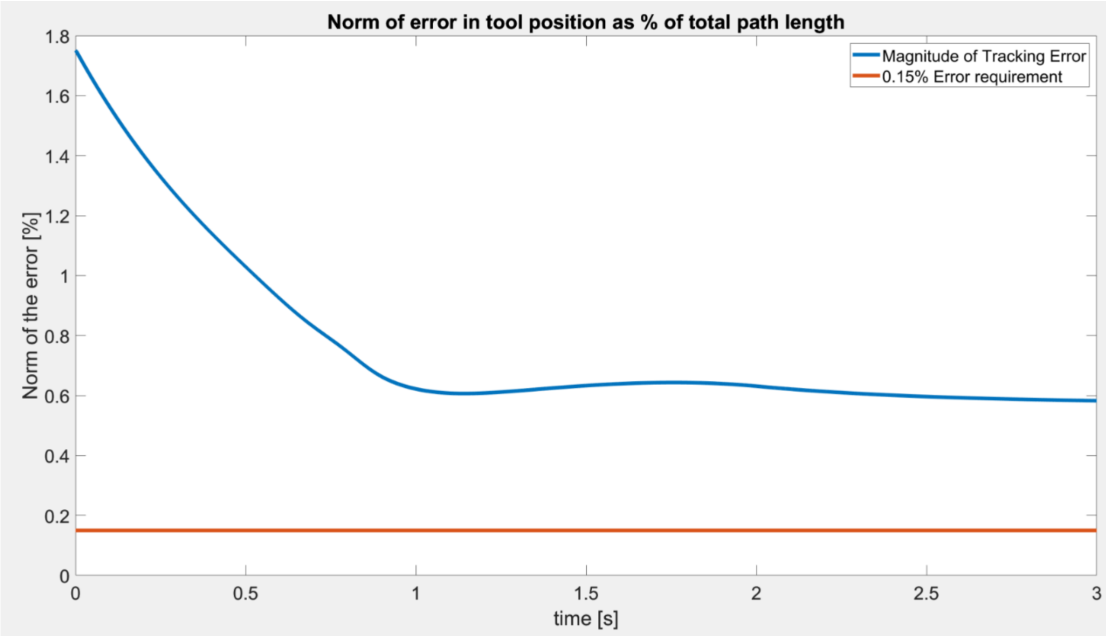


Figure 11 illustrates the error norm of the tool's position relative to the path length.

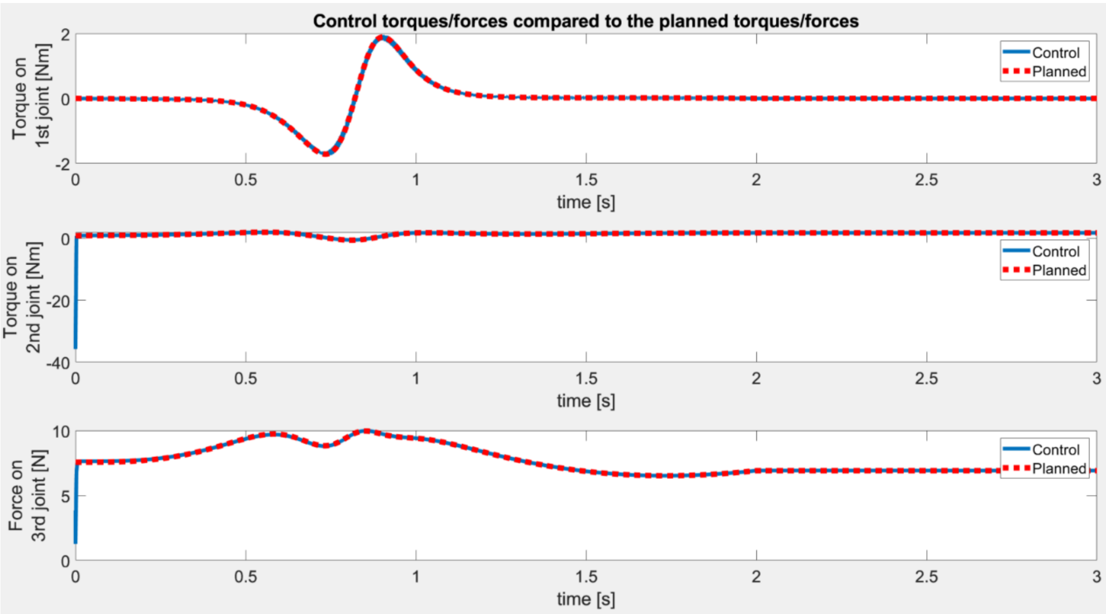


Figure 12 displays the torques and forces in the joints exerted by the controller in relation to the desired forces and torques.

Explanation of Results:

Neglecting compensation for most dynamics except G results in higher control gains, suboptimal performance, and increased error oscillations. While this controller exhibits slight improvements in robustness compared to the first one with full dynamics, it remains insufficient to fully meet the requirements and cannot be considered a robust solution. Removing the load mass has minimal impact on position, resulting in a small error increase. The controller's insensitivity to changes in manipulator mass is due to its reliance on the gravity compensation term, G.

Summary of PD Controller :

This controller has low complexity. Only the gravity vector G needs to be calculated, making it suitable for real-time requirements. When the matrix H is accurately known, the control loop converges to the desired error. The required forces and torques are not significantly higher than the intended values, but the variations are greater compared to the PD+ID controller. This controller is not robust and is sensitive to changes in masses resulting from gravity compensation, but it is more effective than the inverse dynamics controller.

3) PID Control

$$\tau = -K_P\left(q - q_d\right) - K_I \int \left(q - q_d\right) dt - K_D\left(\dot{q} - \dot{q}_d\right)$$

For this controller, we initially tried the PD+G controller constants, starting with low gains for the derivative term (KD) and gradually increasing them. We observed their impact on the system's response. Increasing these gains led to an excessive response and oscillations, which helped mitigate the steady-state error. The summation of the error amplifies control signals when the controller struggles to converge due to constant forces. However, this summation creates excessive response and subsequent oscillations as the controller initially tries to reach the desired path values.

To reduce oscillations and excessive response, we introduced values for Kd, which adds damping to the system. This allowed us to individually examine the error in each response. Finally, the following parameters were determined:

Here are the final controller coefficients:

$$K_P = \begin{bmatrix} 1500 & 0 & 0 \\ 0 & 4000 & 0 \\ 0 & 0 & 1600 \end{bmatrix}, K_I = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 2000 & 0 \\ 0 & 0 & 4000 \end{bmatrix},$$

$$K_D = \begin{bmatrix} 1750 & 0 & 0 \\ 0 & 420 & 0 \\ 0 & 0 & 700 \end{bmatrix}$$

Simulation Result-with load mass at gripper:

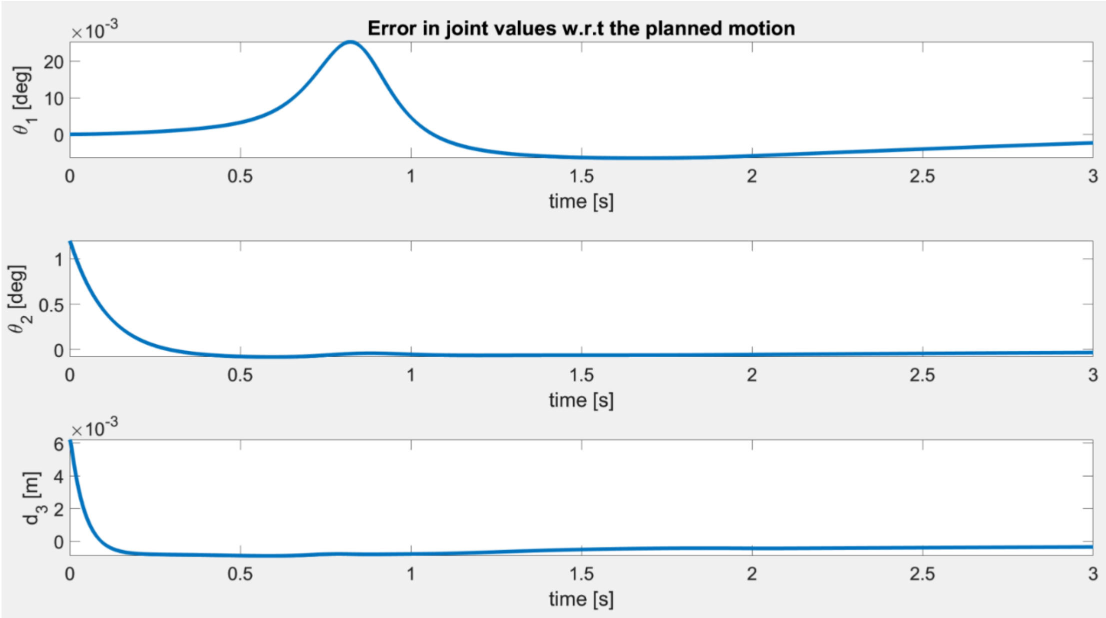


Figure 13 depicts the error in the joints relative to the intended motion.

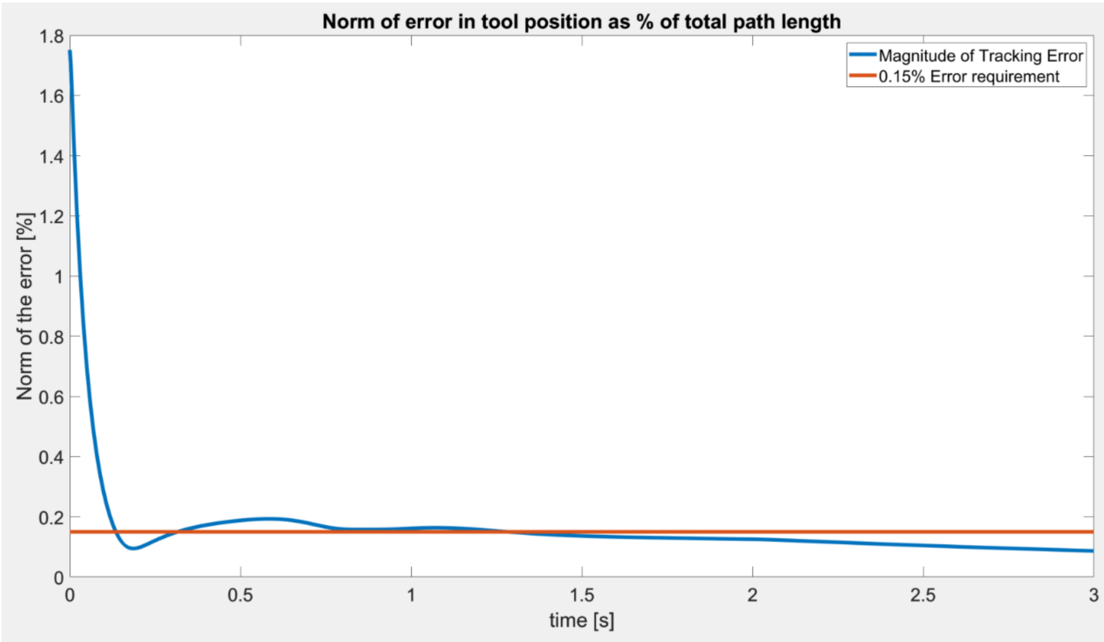


Figure 14 illustrates the error norm of the tool's position relative to the path length.

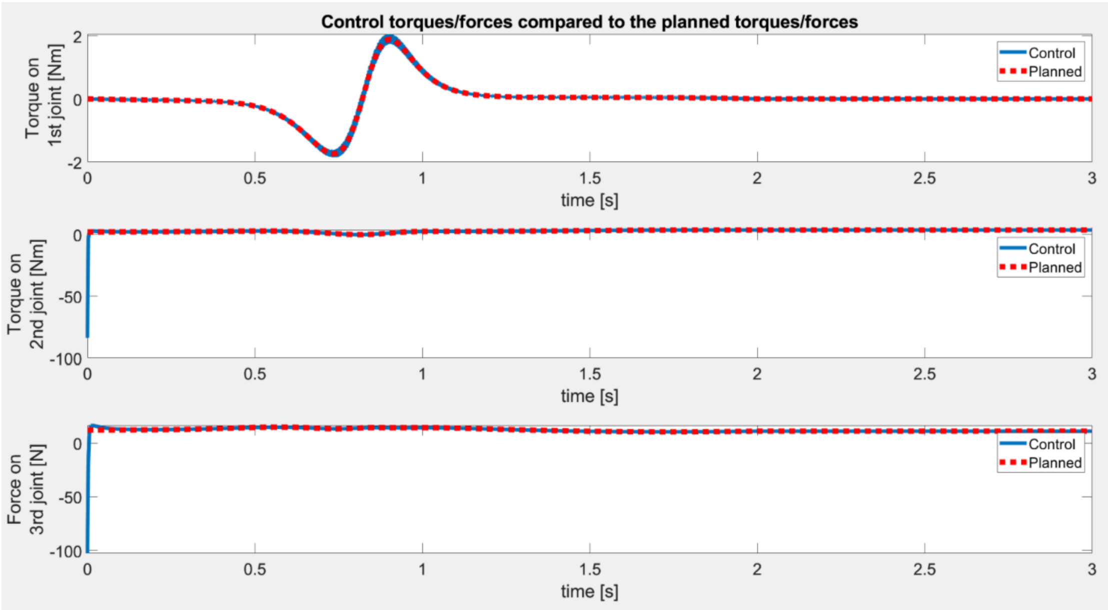


Figure 15 displays the torques and forces in the joints exerted by the controller in relation to the desired forces and torques

We can see that even though the control constants needed are higher initially than PD+G, the control efforts become stable very quickly and work well for accurate tracking, similar to a feed-forward controller. Even after 2 seconds, the integral part of the controller doesn't fully reset, showing that it helps balance out forces when the system is at rest, and the effect of the PD parts is small.This indicates that since the control effort required when the input is 1 is zero in the static state according to the forward-feed calculations, the integration of the error resets once the robot reaches the desired endpoint.

Simulation without load mass at gripper

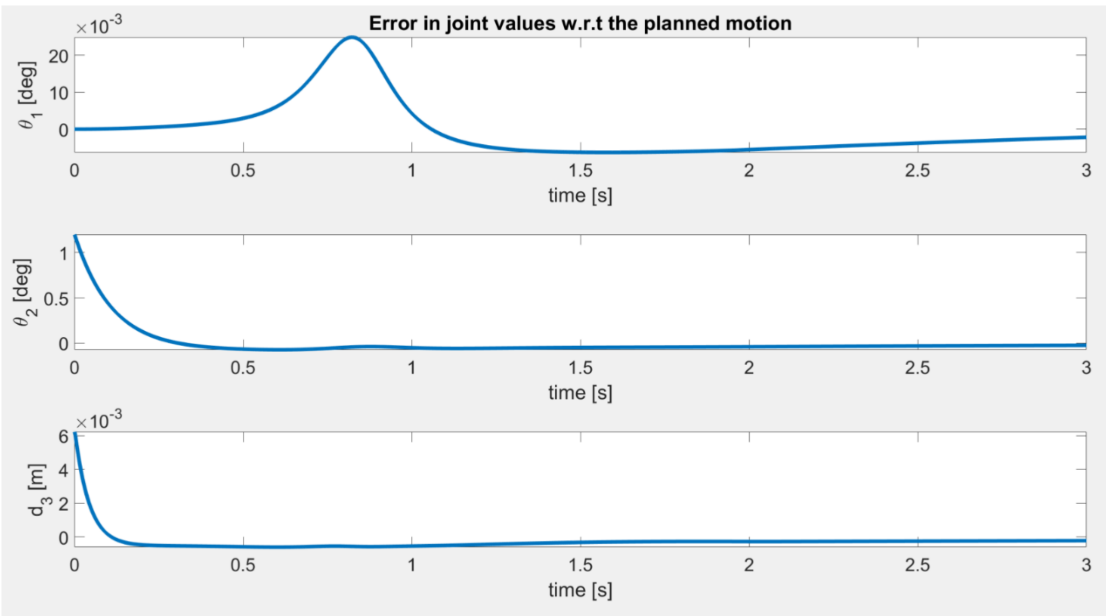


Figure 16 depicts the error in the joints relative to the intended motion.

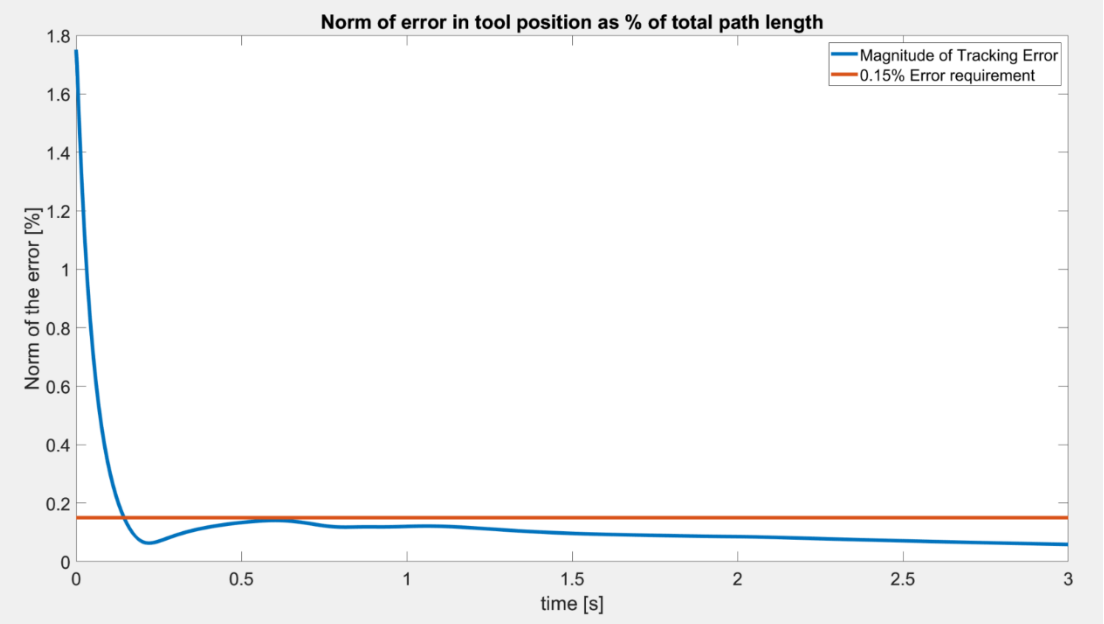


Figure 17 illustrates the error norm of the tool's position relative to the path length.

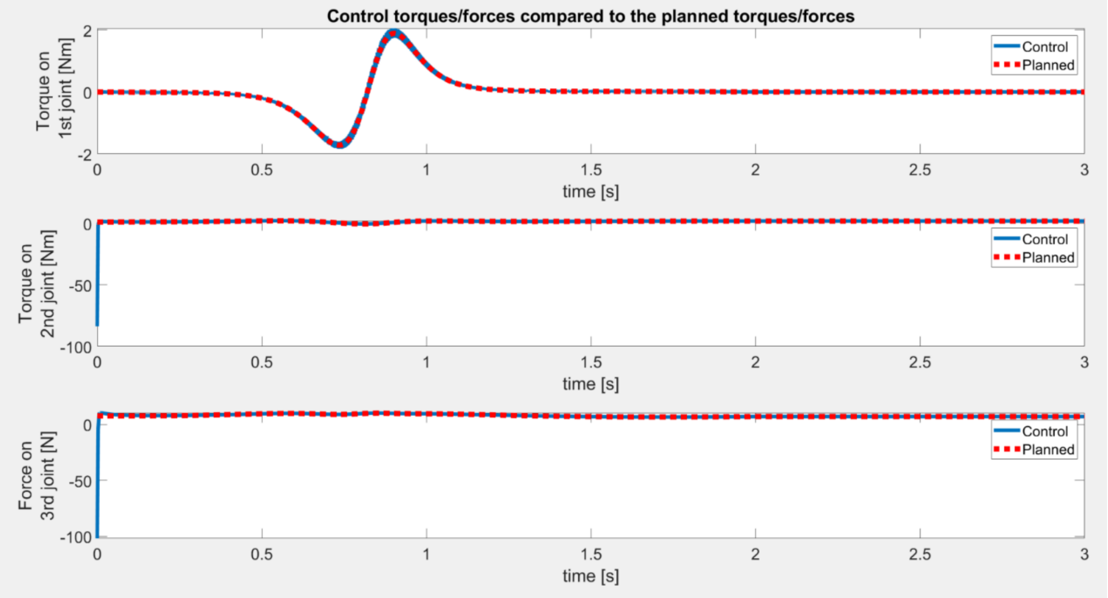


Figure 18 displays the torques and forces in the joints exerted by the controller in relation to the desired forces and torques

We can see that not only did the controller successfully accomplish the task, but its performance even improved because it didn't have to deal with the forces and moments generated when the robot holds a load. This suggests that this controller, by accumulating error signals, can effectively handle disturbances and uncertainties, making this controller robust.

4) MINMAX Control

To design the controller and define its criteria for reducing errors to 0, we aim to create a potential Lyapunov function and analyze the circumstances where its derivative is consistently negative. Initially, we introduce the following variable:

$$e = q - q_d \text{ ,} x = [x_1 \;\; x_2]^T = [e \;\; \dot{e}]^T$$

$$s = \dot{e} + ke = x_2 = kx_1 \text{ , } k = k^T\{k \; is \; positive \; definet\}$$

$$\dot{q}_r = \dot{q}_d - ke \;\; \Rightarrow s = \dot{q} - \dot{q}_r \text{ , } \dot{s} = \ddot{q} - \ddot{q}_r$$

When k (P.D matrix), selectable and determines the error vector e and the dynamic correction $\dot{q}_r$. We define a candidate Lyapunov function as follows:

$$V(x) = \frac{1}{2}(S^T Hs + e^T Pe)$$

When matrix P is P.D matrix, which is selectable, it can be shown that taking the derivative of this function leads to the expression:

$$\dot{V}(x) = W(x) = s^T H\dot{s} + s^T\dot{H}s + \dot{x}_1^T Px_1 = \cdots = s^T(u - \eta) - ek^T Pe$$

$$\eta = -H\ddot{q}_r - C\dot{q}_r - G + Pe$$

Given bounded uncertainty in the held mass at the gripper's end, we separate it into a nominal mass value (known) and an uncertain value as follows:

$$M = M_0 + \Delta M, 0 < M < 1[kg]$$

$$M_0 = 0.5[kg]$$

$$-0.5 < \Delta M < 0.5, |\Delta M| < 0.5$$

Where '0' represents the known part and Δ represents the uncertain part.
Since there is a linear dependency between the dynamic matrices and the mass at the gripper's end, we can separate the dynamic matrices into the known part and the uncertain part as follows:

$H(q) + \Delta H(q)$

$H_0 = H(M = M_0 = 0.5), \Delta H = H(M = \Delta M, m_i = 0)$

$C(\dot{q}, q) = C_0(\dot{q}, q) + \Delta C(\dot{q}, q)$

$C_0 = C(M = M_0 = 0.5), \Delta C = C(M = \Delta M, m_i = 0)$

$G(\dot{q}, q) = G_0(q) + \Delta G(q)$

$G_0 = G(M = M_0 = 0.5), \Delta G = C(M = \Delta M, m_i = 0)$

Therefore, disturbance can be defined as follows:

$$\eta = \eta_0 + \Delta\eta$$

$$\eta_0 = -G_0 - H_0\ddot{q}_r - C_0\dot{q}_r + Pe$$

$$\Delta\eta = -\Delta G - \Delta H\ddot{q}_r - \Delta C\dot{q}_r$$

When analyzing the $\dot{V}(x)$, we know that P, k are P.D matrices, and their product is positive. Therefore, the second term in $\dot{V}(x)$ is negative (for all $x \neq 0$). The first term depends on the input control vectors and the disturbance vector.

We can define the unknown disturbance at its worst case (acting in the direction of "s" in its maximum):

$\Delta\eta = \rho_\Delta \frac{s}{||s||}, ||\Delta\eta|| < \rho_\Delta$

$$||\Delta\eta|| = || - \Delta G - \Delta H\ddot{q}_r - \Delta C\dot{q}_r||$$

$$||\Delta\eta|| = ||-\Delta G - \Delta H\ddot{q}_r - \Delta C\dot{q}_r||$$

$$\Delta G = \Delta M \cdot \Delta G(\Delta M = 1)$$

$$\Delta H = \Delta M \cdot \Delta H(\Delta M = 1)$$

$$\Delta C = \Delta M \cdot \Delta C(\Delta M = 1)$$

$$\rho_\Delta = ||\Delta G(\Delta M = 0.5) + \Delta H(\Delta M = 0.5)\ddot{q}_r + \Delta C(\Delta M = 0.5)\dot{q}_r||$$

As can be seen, ρ_Δ is well-defined and can be computed at any given moment (assuming that the position, velocities, and joint angles can be measured). In order to ensure that the control input acts against s, we define the controller:

$u = -\rho_u \frac{s}{||s||+\delta}, 0 < \delta \ll 1$

For $\delta = 0$ we can see that the first element of $\dot{V}(x)$ is bounded by:

$$s^T\Delta\eta \leq s^T \cdot ||\Delta\eta|| \frac{s}{||s||} \leq s^T \cdot \rho_\Delta \frac{s}{||s||}$$

Thus:

$$s^T(u + \eta_0 + \Delta\eta) < -\frac{s^T s}{||S||}\rho_u + s^T\eta_0 + \frac{s^T s}{||S||}\rho_\Delta = (\rho_\Delta - \rho_u)||s|| + s^T\eta_0$$

$$= (\rho_\Delta + \frac{s^T}{||S||}\eta_0 - \rho_u)$$

Now, if we choose $\rho_\Delta + \frac{s^T}{||s||}\eta_0 \leq \rho_u$ we can ensure that the Lyapunov function will be negative for $||s|| = ||x_2 + kx_1|| \neq 0$.

In this situation, the Lyapunov derivative is entirely negative since the second term in the derivative becomes zero only when $x_1 = 0$ and the first term becomes zero only when both x_1 and $x_2 = 0$. In other words, the Lyapunov derivative becomes zero only when x = 0 and is negative in all other states (entirely negative).
Observe that when $\left(\rho_\Delta + \frac{s^T}{||s||}\eta_0\right) \leq 0$, the Lyapunov derivative has a negative value without requiring control action. This means that in this situation, we can choose $\rho_u = 0$ to minimize control effort. Therefore, to reduce control effort, we set:

$$\rho_u = \max\left(0, \rho_\Delta + \frac{s^T}{||S||}\eta_0\right)$$

For implementation purposes, we aim to avoid situations where calculations become undefined. Notice that $\rho_\Delta + \frac{s^T}{||s||}\eta_0 = \frac{\rho_\Delta\, ||s|| + s^T\, ||s||}{||s||}\eta_0$ and since the norm is always positive, the sign (positive/negative) of the expression is determined by the numerator
The controller implementation is carried out as follows:

$$\begin{cases} u = -\left[\beta\rho_\Delta + \frac{s^T}{||s||}\eta\right]\frac{s}{||s|| + \delta} = -\frac{\beta\rho_\Delta \cdot s + \eta_0||s||}{||s|| + \delta} & , \rho_\Delta|s|| + s^T\eta_0 > 0 \\ u = 0 & , \quad else \end{cases}$$

When $\beta \geq 1$, that allows increasing the weight of the constraint size for handling uncertainty disturbance in the control signal. Since adding β to the control signal can only increase it, the stability properties we discussed earlier are preserved.

Finding the controller coefficient

The controller coefficients were determined through a trial-and-error process until meeting the requirements.
The matrices K and P were chosen to be diagonal. Here are the values after tuning:

$$K = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

β was chosen to be 1.05

δ was chosen to be 0.001

The result of the simulation:

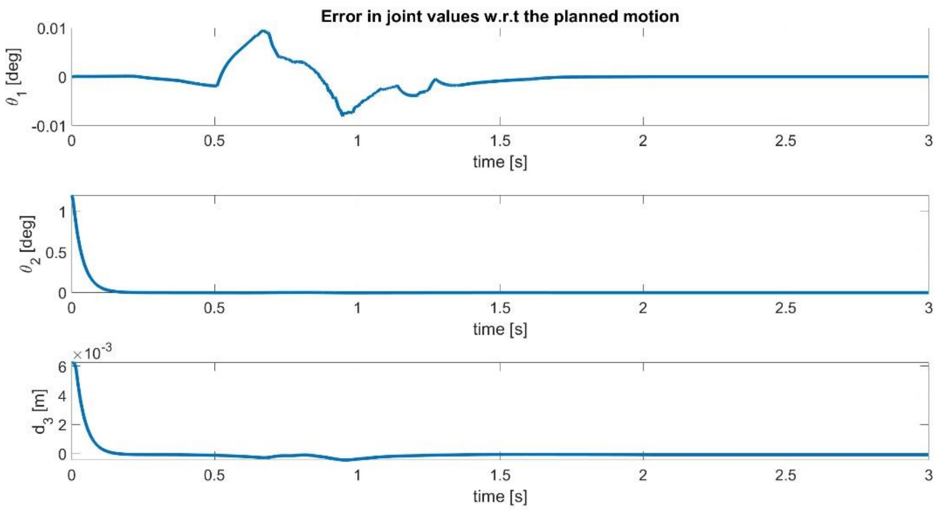


Figure 19 presents the error in the joints relative to the intended motion

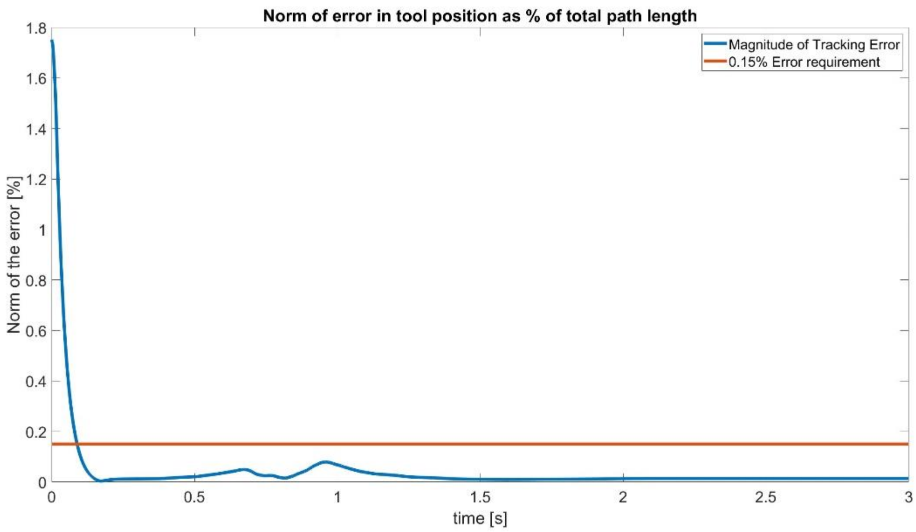


Figure 20 illustrates the error norm of the tool's position relative to the path length

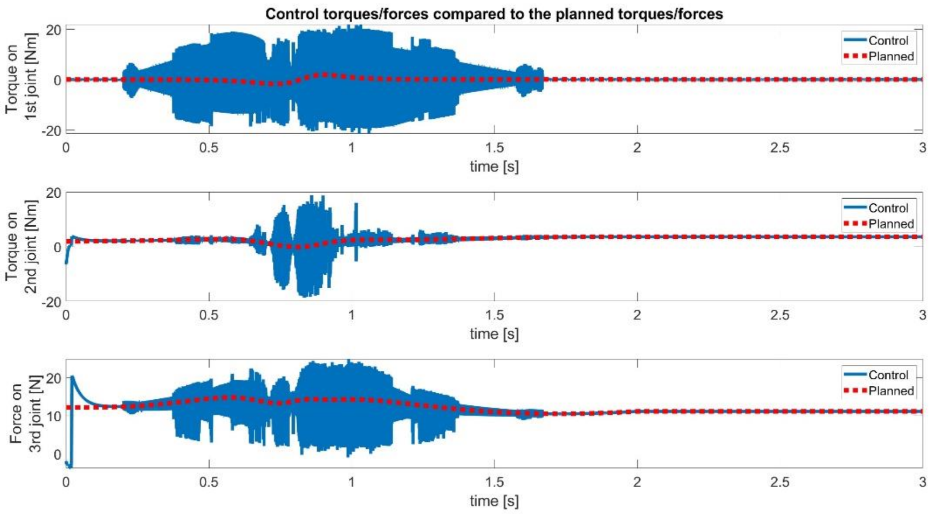


Figure 21 displays the torques and forces in the joints exerted by the controller in relation to the desired forces and torques.

Simulation without load mass at gripper:

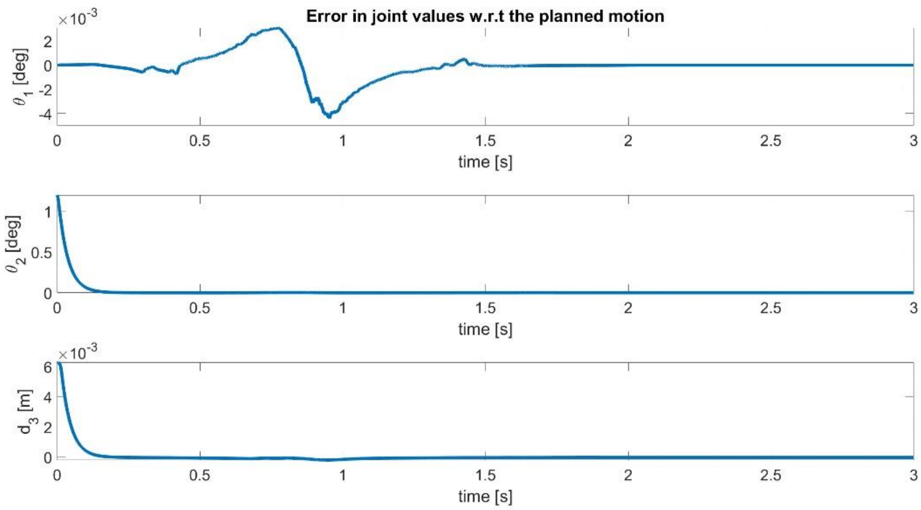


Figure 22 presents the error in the joints relative to the intended motion

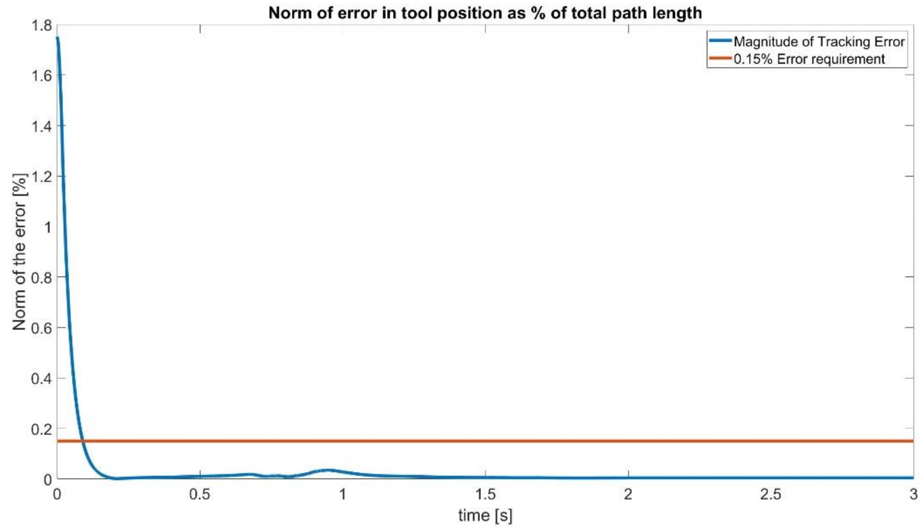


Figure 23 illustrates the error norm of the tool's position relative to the path length

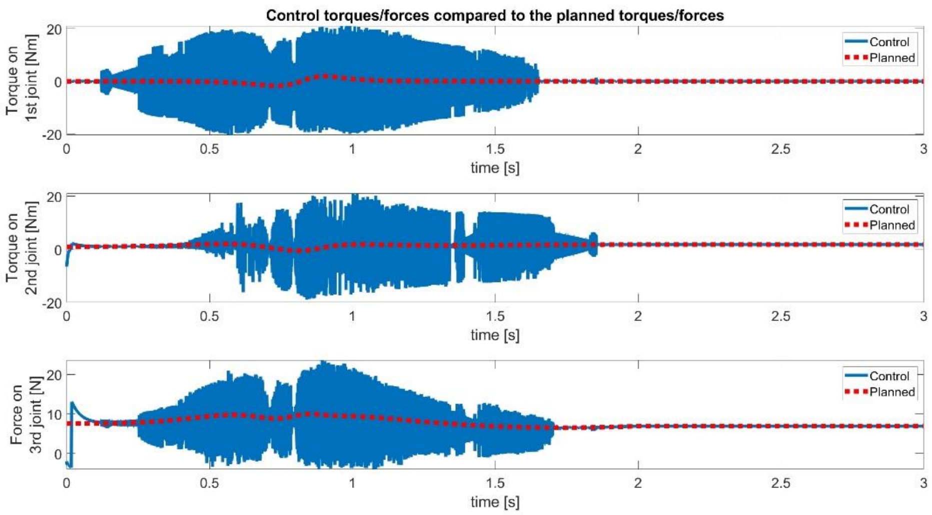


Figure 24 displays the torques and forces in the joints exerted by the controller in relation to the desired forces and torques.

Explanation and summary of Results:

This controller quickly corrects position errors, considering the unknown mass M through the definition of the threshold. As a result, the controller successfully converges for both the original mass and when the mass is removed.

Regarding forces and torques, there's observed oscillation due to error correction: whenever an error occurs, the controller reacts forcefully in the opposite direction of the 'weighted' error.

The controller's complexity is relatively low but higher than a PID controller. Complexity depends on the upper threshold determination method. By defining a feasible maximum error and limits on positions, velocities, and accelerations of the components, the upper threshold can be set as a constant, lowering complexity. However, this may raise the threshold value. In this scenario, the controller suits real-time requirements without precise system parameter knowledge. The error remains consistent despite load changes, indicating its robustness. While forces are significant yet notably smaller than those in a PID controller, their dynamics can strain the manipulator's mechanical components. Forces converge relatively quickly (around 1.7 seconds) to forces resembling the planned forces.

5) Adaptive Control

$$H\ddot{q} + C\dot{q} + G = H\ddot{q} + h(q,\dot{q}) = H_0\ddot{q} + H'\ddot{q} + h_0(q,\dot{q}) + h'(q,\dot{q})$$

When H_0, h_0 are known and H', h' are unknown.

$$H_0 = H(M=0)$$
$$h_0 = C(M=0)\dot{q} + G(M=0)$$
$$H' = H - H_0$$
$$h' = C\dot{q} - C_0\dot{q} + G - G_0$$

We are interested in finding and P that satisfy the following relationship:

$$H'\ddot{q} + h' = Y \cdot P$$

We choose:

$$P_{1\times1} = M$$
$$Y_{3\times1} = \frac{1}{M}(H'\ddot{q} + h')$$

control constants:

$$K_P = \begin{bmatrix} K_{P11} & 0 & 0 \\ 0 & K_{P22} & 0 \\ 0 & 0 & K_{P33} \end{bmatrix}, K_D = \begin{bmatrix} K_{D11} & 0 & 0 \\ 0 & K_{D22} & 0 \\ 0 & 0 & K_{D33} \end{bmatrix}$$

$$Q = \begin{bmatrix} \tilde{Q} & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{Q} & 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{Q} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{Q} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{Q} & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{Q} \end{bmatrix}, \Gamma_{1\times1} = \left[\tilde{\Gamma}\right]$$

F is calculated by Solving the Lyapunov equation: $A^T F + FA = -Q$

When:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -K_{P_{11}} & 0 & 0 & -K_{D_{11}} & 0 & 0 \\ 0 & -K_{P_{22}} & 0 & 0 & -K_{D_{22}} & 0 \\ 0 & 0 & -K_{P_{33}} & 0 & 0 & -K_{D_{33}} \end{bmatrix}$$

An initial guess must be added for an estimated P: $\hat{P}(0)$

The augmented state vector is updated as: $\begin{bmatrix} q & \dot{q} & \hat{P} \end{bmatrix}^T$

Estimation law: $\dot{\hat{P}} = -\Gamma^{-1} Y^T \hat{H}^{-T} B^T F^T \begin{bmatrix} e \\ \dot{e} \end{bmatrix}$

This value must be placed in M and then calculate the estimated matrices: $\hat{H} \quad \hat{h}$

When:

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The control law:

$$u = \hat{H}(q)\left[\ddot{q}_d - K_D \dot{e} - K_P e\right] + \hat{h}(q, \dot{q})$$

The controller in this section responds to the uncertainty of the mass parameter at the end of the gripper. The mass is unknown, but its limits are known: 0<M<0.5 kg.
The controller is required to meet the same criteria listed in section 1.

Finding the controller constants:

The controller constants were found by trial and error until the requirements were met. KP and KD matrices were chosen diagonal, below are the values after tuning.
As demonstrated below, in order to fulfill the requirements, this method demanded the highest constant gains compared to all other methods.

$$K_P = \begin{bmatrix} 27000 & 0 & 0 \\ 0 & 27000 & 0 \\ 0 & 0 & 9000 \end{bmatrix}, K_D = \begin{bmatrix} 3000 & 0 & 0 \\ 0 & 9000 & 0 \\ 0 & 0 & 3000 \end{bmatrix}$$

The matrix Q:

The parameter $Q = 20 I_{6\times 6}$

The parameter Γ was chosen as 0.02.

The result of the simulation

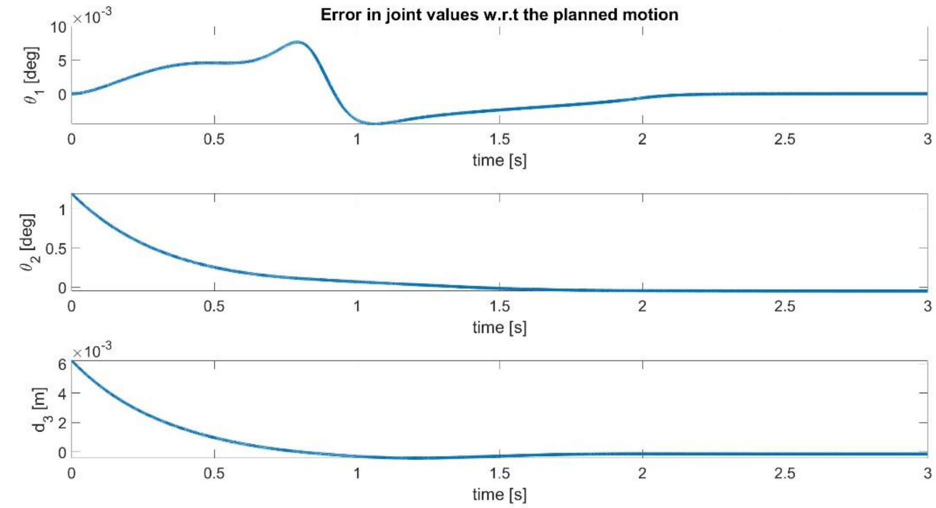


Figure 25 presents the error in the joints relative to the intended motion

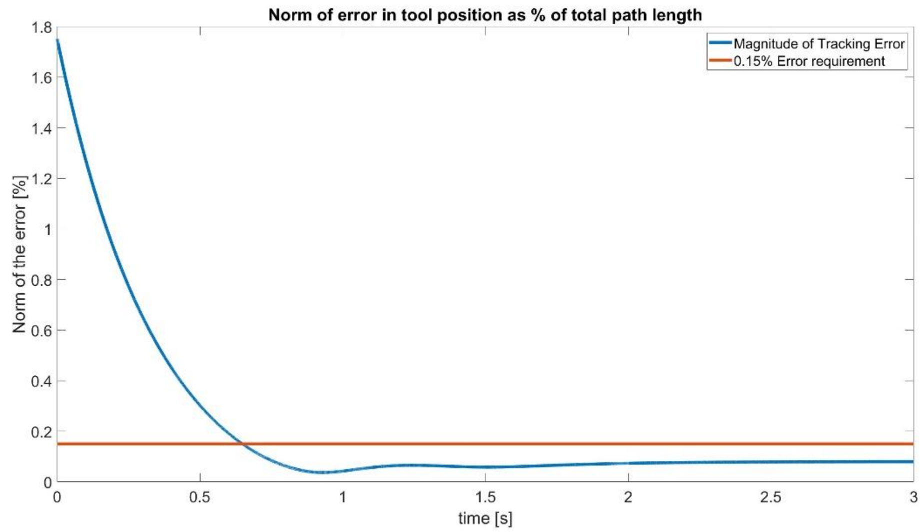


Figure 26 illustrates the error norm of the tool's position relative to the path length

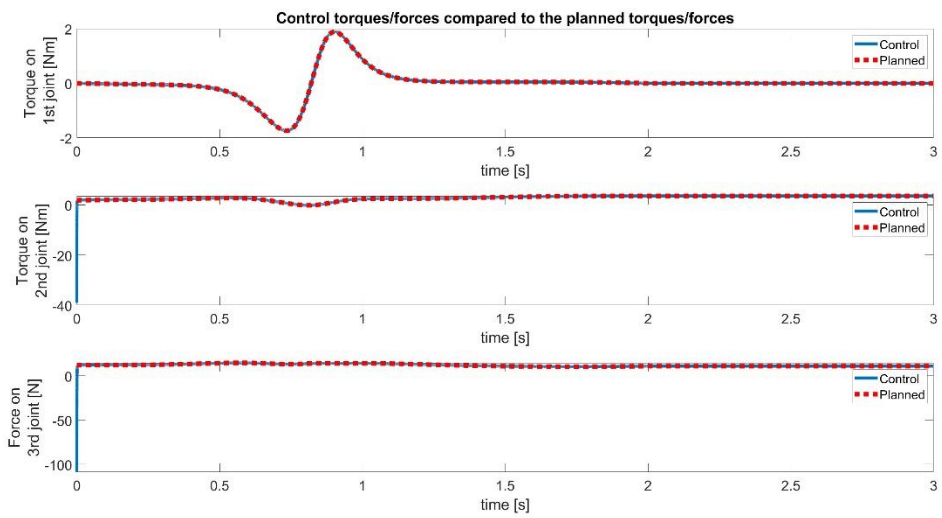


Figure 27 displays the torques and forces in the joints exerted by the controller in relation to the desired forces and torques.

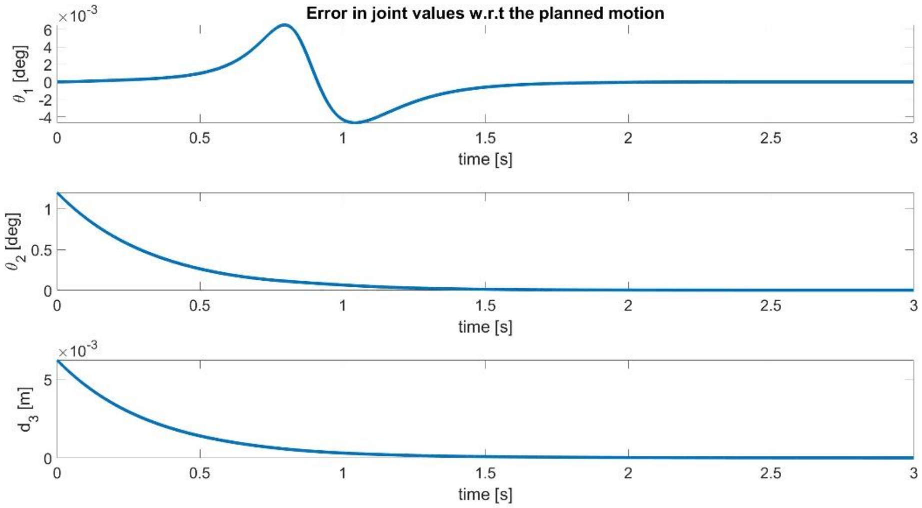


Figure 28 presents the error in the joints relative to the intended motion

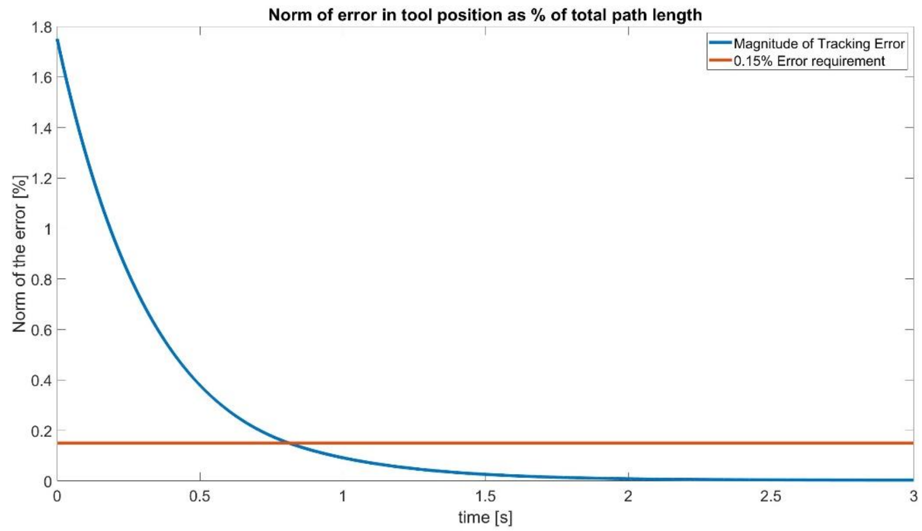


Figure 29 illustrates the error norm of the tool's position relative to the path length

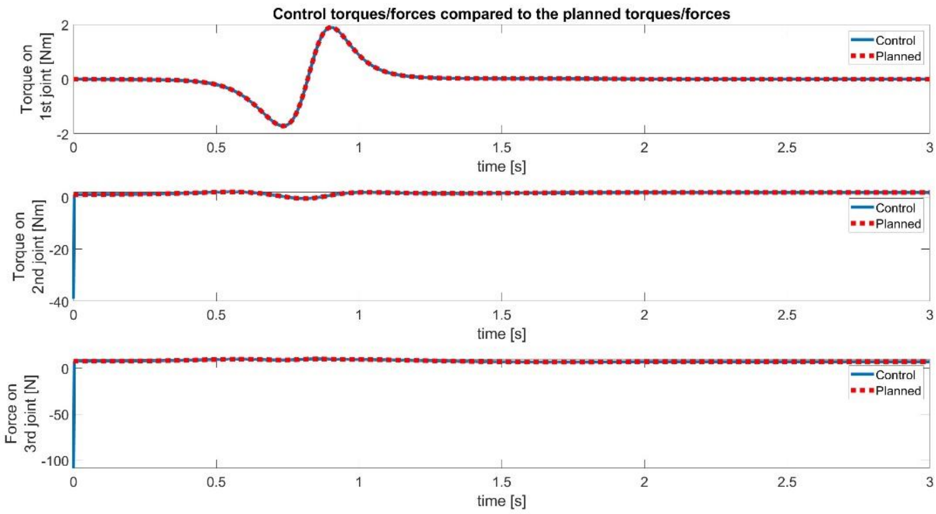


Figure 30 displays the torques and forces in the joints exerted by the controller in relation to the desired forces and torques.

Summary of AC Controller:

The adaptive controller, after calibration of its values, exhibits consistent convergence without oscillations. From the error graph, it can be observed that the system starts with an error of 2.3% due to the robot's initial position being 1 cm higher than the calculated required path point (true for all controllers).

The design of the controller takes into consideration the fact that the mass M is not known in advance and calculates an estimated value at each moment. Consequently, it is evident that the controller manages to converge for both the original mass and even better when the mass is removed. After mass removal, the error does increase, albeit in a relatively minor manner, while still adhering to requirements.

Forces and Torques: It can be seen that there is very minor fluctuation in force calculations. This arises from variations in the estimated mass value, leading to updates in the dynamic matrices.

Unlike other controllers, this controller requires acceleration measurement in order to calculate matrix Y, which is used to estimate unknown parameters.

Complexity: The controller's complexity is high due to the need for matrix decomposition, necessary for computing their inverses. Additionally, numerous operations are required due to matrix computations and multiplication.

Sensitivity: The controller can overcome initial condition errors to a certain extent, although it is influenced by them to a low degree.

Forces and Torques: Forces and torques initially are higher than those of the MINMAX controller but lower than the PID controller. There is oscillation in forces, yet with a very low amplitude, and forces converge rapidly to the desired forces.

Robustness: The controller is robust since parameters can be set to satisfy requirements even when there are changes in system parameters, although a slight change in errors remains due to uncertainty.