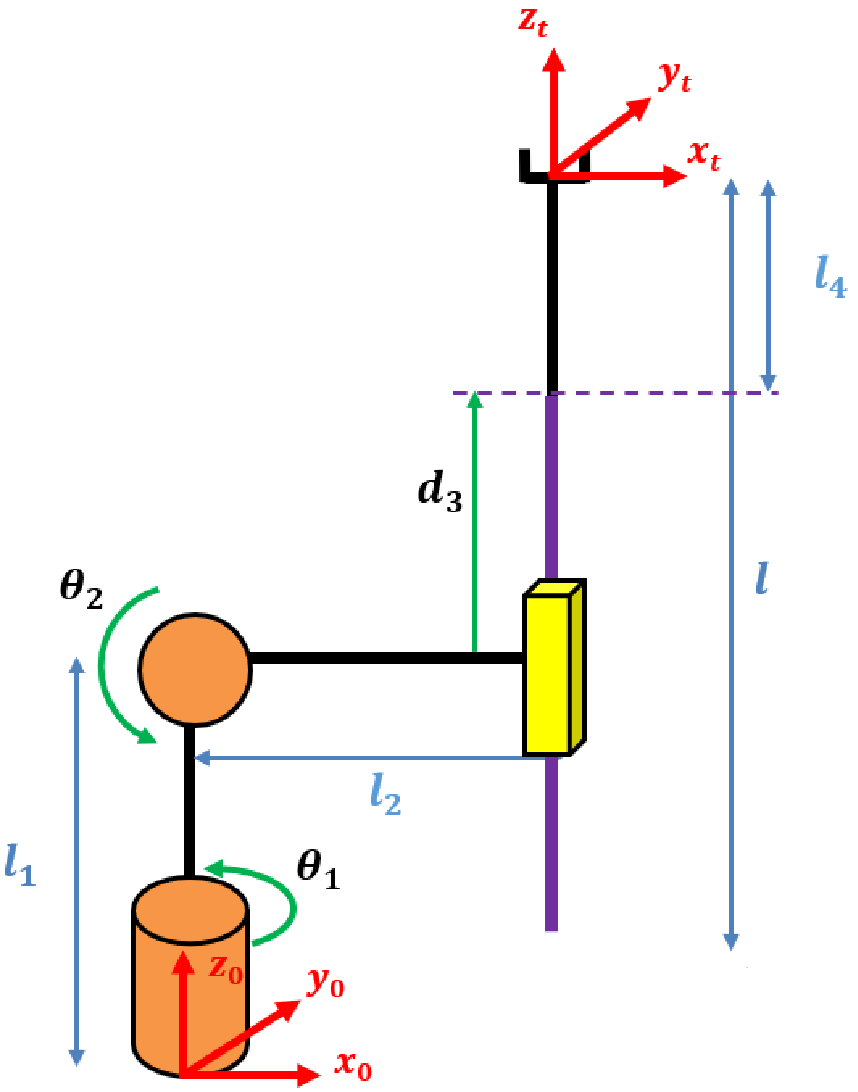
Kinematics, dynamics and control of robots - Homework project 3

Remark: In case you had mistakes in project 1, you must correct them and update the functions for inverse kinematics and trajectory planning. **Use the analytic calculation for velocities and accelerations** (using the Jacobian and its time derivative).

In this project we consider a simplified case of the serial robot from project 1,
where $\boldsymbol{\theta_4, \theta_5, \theta_6 = 0}$ and $\boldsymbol{l_5, l_6 = 0}$:



1. Formulate the dynamic equations of motion for a simplified case of the serial robot in project 1:
   Under vector of joint variables $\mathbf{q} = [\theta_1, \theta_2, d_3]^T$
   You must take into account the following:
   - Links' masses m_i and lengths l_i, position of center of mass and the tensor of inertia for each link.
   - Gravitation in the $-\hat{z}_0$ direction.
   - A point mass M is held by the gripper.
   - Joint torque\force vector $\boldsymbol{\tau}$.
   - The option of some external force $\boldsymbol{F}_e$ acting on the end effector.

a. Formulate the kinetic and potential energy for each link, find the Lagrange equations by differentiation of the energies, and find the H and G matrices.

b. Formulate the dynamic equations of motion using the partial Jacobian matrices $\mathbf{J}_{Li}$ and $\mathbf{J}_{Ai}$ matrices.

Write the equations in the form:

$$\mathbf{H}(\mathbf{q})\ddot{q} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{q} + G(\mathbf{q}) = \tau + \mathbf{J}^T F_e$$

where, $\mathbf{q} = [\theta_1, \theta_2, d_3]^T$ is the vector of joint variables.

Write explicitly the elements of H, C, G as functions of $\mathbf{q}, \dot{\mathbf{q}}$ and the parameters l_i, m_i, M, g.

All calculations should be done parametrically, numerical values are only to be substituted for the simulations and creating figures!

2. "Inverse dynamics": Plan the vector of control torques\forces τ needed to create the motion from point A to B planned in project 1, according to the polynomial profile. The end effector of this simplified robot is the point ***P*** of the robot from project 1, with a load of $M = 0.5[kg]$. Plan the control value for discrete times with $\Delta T = 0.001$ [sec].
   Draw and submit figures showing the control actuation forces/torques at the joints, as a function of time.

3. Calculate the reaction **force** acting on joint 2 in the direction of the joint axis, as a function of the state, velocities and accelerations. Write explicit expression as a function of $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ and constant parameters.
   Plot a graph of this force as a function of time for the trajectory in section 2.

4. "Forward dynamics": perform a dynamic simulation of the robot motion from the initial position A, zero initial velocities, and the control torques\force calculated in section 2 are applied at the joints. Integration step should be at least 10 times smaller than ΔT. (Use Matlab command **ode45**, Or Python scipy **odeint** function)

5. Repeat section 4 with the initial position of the end-effector moved 1cm above point A $(+\hat{z}_0)$, and the control vector $\boldsymbol{\tau}(t)$ is the same one planned in section 2.

6. Repeat section 4 with initial conditions at point A but the load mass is changed to $M = 0.6[Kg]$, while the control vector $\boldsymbol{\tau}(t)$ is the same one planned in section 2, without knowing about the change in load mass.

For sections 4,5,6, present the following results:

- Figures of joint values as a function of time compared to the planned values.
- Figure of the error norm for the position of the end-effector $\sqrt{(x - x_d)^2 + (y - y_d)^2 + (z - z_d)^2}$ as a function of time, expressed as a percentage of the full path length.

7. Write a short paragraph summarizing the project. Give your conclusion referring to your results.

Mass and inertia values (simplified model):

Assume, for simplicity, that the links can be approximated as thin rods with cross-section diameter of $d = 0.015[m]$ made of steel with uniform density $7800[kg/m^3]$. Assume the center of mass for each link is at half its length.

When calculating the mass and moment of inertia for d_3, consider a constant length of $l = 0.6[m]$. The center of mass of the link is at $l/2$ before the end effector.

Reminder: the moment of inertia (about the center of mass) of a thin rod of mass m and length L is negligible with respect to the longitudinal axis and is equal to $\frac{mL^2}{12}$ in the perpendicular directions.

Link lengths: $l_1 = 0.4[m],\ l_2 = 0.15[m], l_4 = 0.1[m]$

Initial and final positions (in meters):

$$x_A = 0.2, \quad y_A = 0, \quad z_A = 0.5$$
$$x_B = -0.3, \quad y_B = -0.2, \quad z_B = 0.7$$

Mechanical limitations:

$$-\pi \leq \theta_1 \leq \pi, -\pi \leq \theta_2 \leq \pi, \quad |d_3| < 0.7[m]$$

Functions that need to be programmed and submitted:

1. Calculate matrices for dynamics [H,C,G]=dynamics_mat(q,qdot)
   Input – joint values and velocities q,qdot.
   Output – matrices\vectors H,C,G for the dynamic equations of motion.

2. Calculate vector of required joint torque/force tau=tau_plan(prof,t)
   Input – velocity profile 'prof' and time 't'.
   Output – Vector of required joint torque/force $\tau(t)$.

3. Dynamic equation of motion Xdot=state_eq(t,X)
   Input – state vector $\mathbf{X} = (\mathbf{q}, \dot{\mathbf{q}})$.
   Output – velocity vector $\dot{\mathbf{X}}(t)$.

4. A short program for running the dynamic simulation with some initial condition and processing the results.

There should be no symbolic calculations or variables in any of these functions!
(You may use command **matlabFunction**)

Submission in pairs by 13.06.2023 on the course moodle page.

Good luck!