Control Theory Intro: Home Assignment #3

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Introduction

The purpose of this home assignment is to base your understanding in system modeling with state-space.

Your solutions should be presented in a PDF (not Word!) file. You should submit also a **.m** file. The first line should print your ID.

>> disp('ID_STUDENT_1 ID_STUDENT_2') % disp('ID_STUDENT_1') if only one student is submitting.

For clarity of the script, you can separate the different sections of the script with a **%%**. This will automatically create a block in your script. In order to run specifically this block of code press 'Ctrl+Enter'. To run the entire script press 'F5'.

1 System modeling

Fig. 1 shows a mass M suspended from another mass in by means of a light rod of length L. Obtain the state variable differential matrix equation using a linear model assuming a small angle for θ.

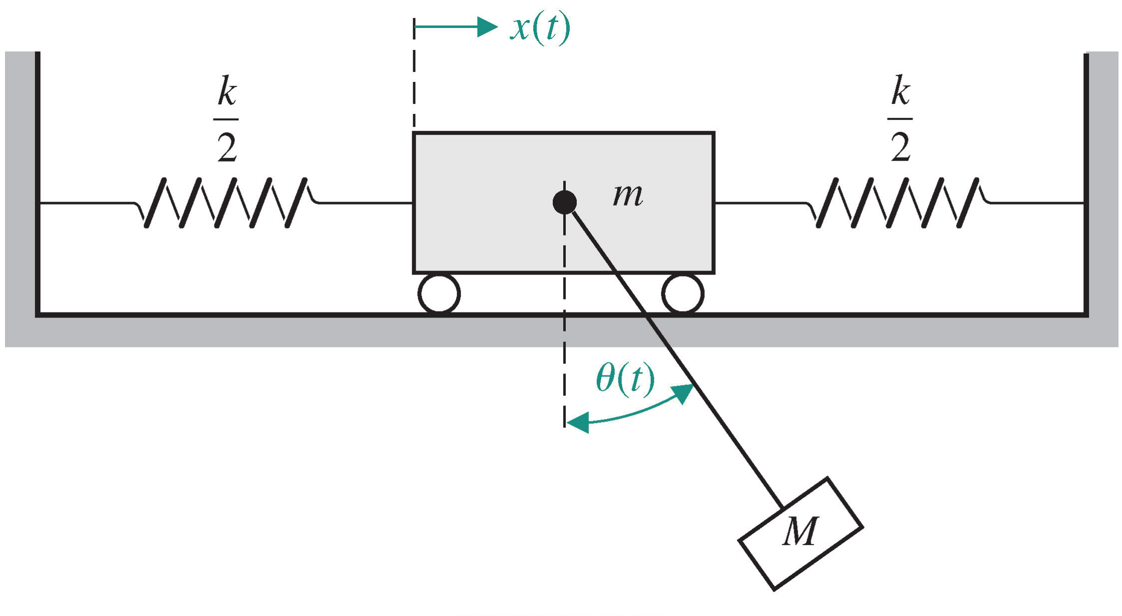

Figure 1: Mass suspended from cart.

2 System modeling 2

Consider the electromagnetic suspension system shown in Fig. 2. An electromagnet is located at the upper part of the experimental system. Using the electromagnetic force f, we want to suspend the iron ball. Note that this simple electromagnetic suspension system is essentially unworkable. Hence feedback control is

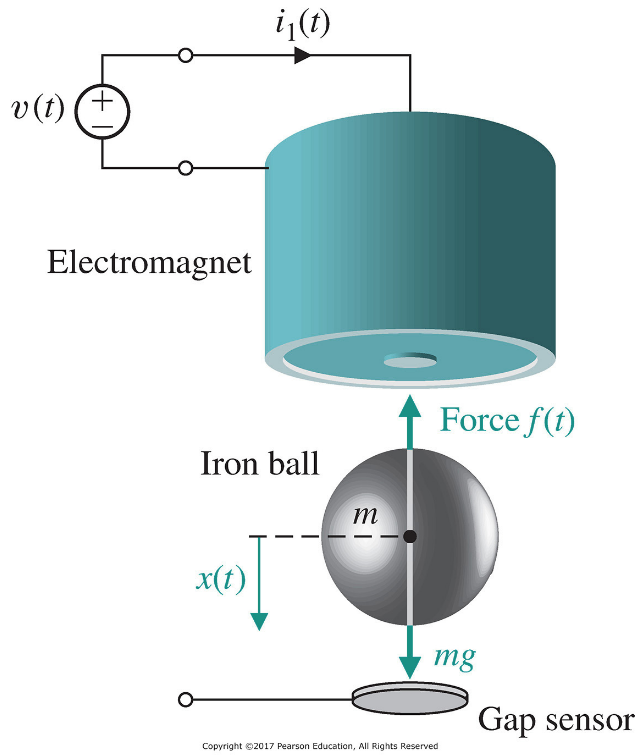

Figure 2: Electromagnetic suspension system.

indispensable. As a gap sensor, a standard induction probe of the type of eddy current is placed below the ball.

Assume that the state variables are $x_1 = x$, $x_2 = \frac{dx}{dt}$, and $x_3 = i$. The electromagnet has an inductance L $\approx$ 0.508 H and a resistance $R \approx 23.2\Omega$. Use a **Taylor series approximation** for the electromagnetic force. The current is $i_1 = I_0 + i$, where $I_0 = 1.06A$ is the operating point and i is the variable. The mass m is equal to 1.75 kg. The gap is $x_g = X_0 + x$, where $X_0 = 4.36mm$ is the operating point and x is the variable. The electromagnetic force is $f = k(i_1/x_g)^2$, where $k = 2.9X10^{-4}Nm^2/A^2$. Determine the matrix differential equation and the equivalent transfer function $X(s)/V(s)$.

Assistance - $L\frac{di}{dt} = v - Ri$

3 Comparing two systems

Consider the two systems

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -5 & -8 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u \qquad (1)$$
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_1$$

and,

$$\dot{x}_2 = \begin{bmatrix} 0.5 & 0.5 & 0.7071 \\ -0.5 & -0.5 & 0.7071 \\ -6.364 & -0.7071 & -8.000 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u \qquad (2)$$
$$y = \begin{bmatrix} 0.7071 & -0.7071 & 0 \end{bmatrix} x_2$$

1. Using the `tf` function, determine the transfer function $Y(s)/U(s)$ for system (1).
2. Using the `tf` function, determine the transfer function $Y(s)/U(s)$ for system (2).
3. Compare the results in parts 1 and 2.

4 State-system response

Consider the following system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

with,

$$x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Using the `lsim` function obtain and plot the system response (for $x_1(t)$ and $x_2(t)$) when $u(t) = 0$.

5 State-system response 2

Consider the svstem

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -5 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_1$$

1. Using the `tf` function, determine the transfer function $Y(s)/U(s)$.
2. Plot the response of the system to the initial condition $x(0) = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix}^T$ for $0 \leq t \leq 10$.
3. Compute the state transition matrix, and determine $x(t)$ at $t = 10$ for the initial condition given in part 2. Compare the result with the system response obtained in part 2.