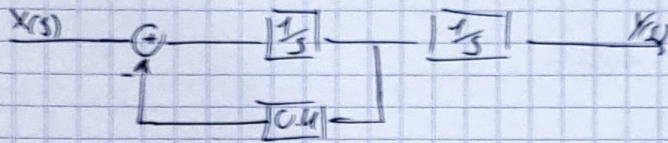
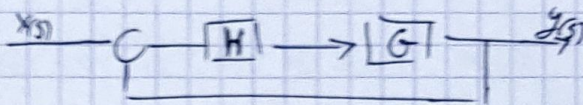


# 1) Manipulator control

$$G(s) = \frac{1}{s(s+0.4)}$$



$$\dot{X} = AX + BU$$

$$\ddot{X}(t) + 0.4\dot{X}(t) = U(t)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s(s+0.4)}$$

$$x_1 = X \quad x_2 = \dot{X}$$

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ 0 & -0.4 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$G_{cl} = \frac{h \cdot \frac{1}{s^2+0.4s}}{1 + \frac{h}{s^2+0.4s}} = \frac{h}{s^2+0.4s+h}$$

$$h = \omega_n^2, \quad \xi = \frac{0.2}{\omega_n} = \frac{0.2}{\sqrt{h}}$$

$$T_{st} \approx \frac{4}{\xi \omega_n} \leq 1.35$$

$$0.5 = 100 \exp\left[-\frac{\xi \pi}{\sqrt{1-\xi^2}}\right] \leq 5$$

$$\xi \geq 0.69$$

$$\frac{4}{0.69 \cdot 1.35} \leq \omega_n \Rightarrow \omega_n \geq 4.39$$

$$\begin{pmatrix} 0 & 1 \\ 0 & -0.4 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} (K_1, K_2) = \begin{pmatrix} 0 & 1 \\ 0 & -0.4 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ K_1 & K_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -K_1 & -0.4-K_2 \end{pmatrix}$$

$$|\lambda I - \tilde{A}| = \begin{vmatrix} \lambda & -1 \\ K_1 & \lambda + 0.4 + K_2 \end{vmatrix} = \lambda^2 + \lambda(0.4 + K_2) + K_1 = 0$$

$$K_1 = \omega_n^2 = 18.4$$

$$2\omega_n\xi = 9.92$$

$$K_2 + 0.4 = 5.92 \Rightarrow K_2 = 5.52$$



2) Pole-Placement full-state feedback control:

$$A = \begin{pmatrix} -2 & -3 & 1 \\ 5 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta_d(s) = (s+0)(s+7)(s+8) = (s^2+13s+42)(s+8) = \\ = s^3+13s^2+42s+8s^2+104s+336 = s^3+21s^2+146s+336$$

$$\tilde{A} = A - BK = \begin{pmatrix} -2 & -3 & 1 \\ 5 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} (K_1, K_2, K_3) = \begin{pmatrix} -2 & -3 & 1 \\ 5 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 5K_1 & 5K_2 & 5K_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \\ = \begin{pmatrix} -2-5K_1 & -3-5K_2 & 1-5K_3 \\ 5 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$|\lambda I - \tilde{A}| = \begin{vmatrix} \lambda + (2+5K_1) & 3+5K_2 & -1+5K_3 \\ -5 & \lambda & 0 \\ 0 & -1 & \lambda \end{vmatrix} = (\lambda + (2+5K_1))\lambda^2 + 5(\lambda(3+5K_2)) + (-1+5K_3) = \\ = \lambda^3 + \lambda^2(2+5K_1) + \lambda(5(3+5K_2)) + 5(-1+5K_3)$$

$$2+5K_1 = 21 \Rightarrow K_1 = \frac{19}{5}$$

$$5(3+5K_2) = 146 \Rightarrow K_2 = \left(\frac{146}{5} - 3\right) \frac{1}{5}$$

$$5(-1+5K_3) = 336 \Rightarrow K_3 = \frac{1}{5} \left(\frac{336}{5} + 1\right)$$



### 3) Observer design:

ers) - ממ'ית , A - ממ'ית , B - ממ'ית

$$U - 7B - 2A = 0, \quad A = \frac{0}{5}, \quad B = \frac{1}{5}A, \quad Y = 4A + B$$

$$Y = 4A + \frac{1}{5}A = \frac{4}{5}0 + \frac{1}{5}0 = 0 \left( \frac{4}{5} + \frac{1}{5} \right)$$

$$U - \frac{1}{5}7A - 2A = 0 \Rightarrow U - \frac{1}{5}7 \cdot 0 - 2 \cdot 0 = 0$$

$$U = 0 \left( 1 + \frac{7}{5} + \frac{2}{5} \right)$$

$$\frac{Y}{U} = \frac{\frac{4.5+1}{5.2}}{\frac{5.2+9.5+7}{5.2}} = \frac{4.5+1}{5.2+9.5+7}$$

$$\Delta d = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$(s - (-10 + 10j))(s - (-10 - 10j)) = s^2 - (-10 + 10j)s - (-10 + 10j)s + (-10 + 10j)(-10 - 10j)$$

$$s^2 + 20s + 200 \Rightarrow \omega_n = \sqrt{200}$$

$$2\zeta\omega_n = 20 \Rightarrow \zeta = \frac{10}{\sqrt{200}} = \frac{1}{\sqrt{2}}$$

$$\ddot{y} + 2\dot{y} + 7y = u\dot{x} + 1$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad x_1 = y, \quad x_2 = \dot{y}, \quad u = u\dot{x} + 1$$

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -7 & -2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, \quad y = \begin{bmatrix} 1 & 4 \end{bmatrix} x$$

$$\Theta = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{pmatrix} 1 & 4 \\ -28 & -7 \end{pmatrix}$$

$$\det(\Theta) = -7 + 4 \cdot 28 = 105 \neq 0 \quad - \text{ממ'ית}$$

$$\det(\lambda I - (A - LC)) = \det \left( \lambda I - \begin{pmatrix} 0 & 1 \\ -7 & -2 \end{pmatrix} - \begin{pmatrix} L_1 & uL_1 \\ L_2 & uL_2 \end{pmatrix} \right) =$$

$$= \left| \lambda I - \begin{pmatrix} -L_1 & 1-uL_1 \\ -7-L_2 & -2-uL_2 \end{pmatrix} \right| = \left| \begin{pmatrix} \lambda+L_1 & -1+uL_1 \\ 7+L_2 & \lambda+2+uL_2 \end{pmatrix} \right| =$$

$$= \lambda^2 + \lambda(2-L_1-uL_2) + 7-L_2+20L_1$$

$$-L_1 - uL_2 = 18$$

$$-L_2 + 20L_1 = 193$$

$$-L_2 + 20(-uL_2 - 18) = 193$$

$$L_2 = -6.29, \quad L_1 = 7.16$$