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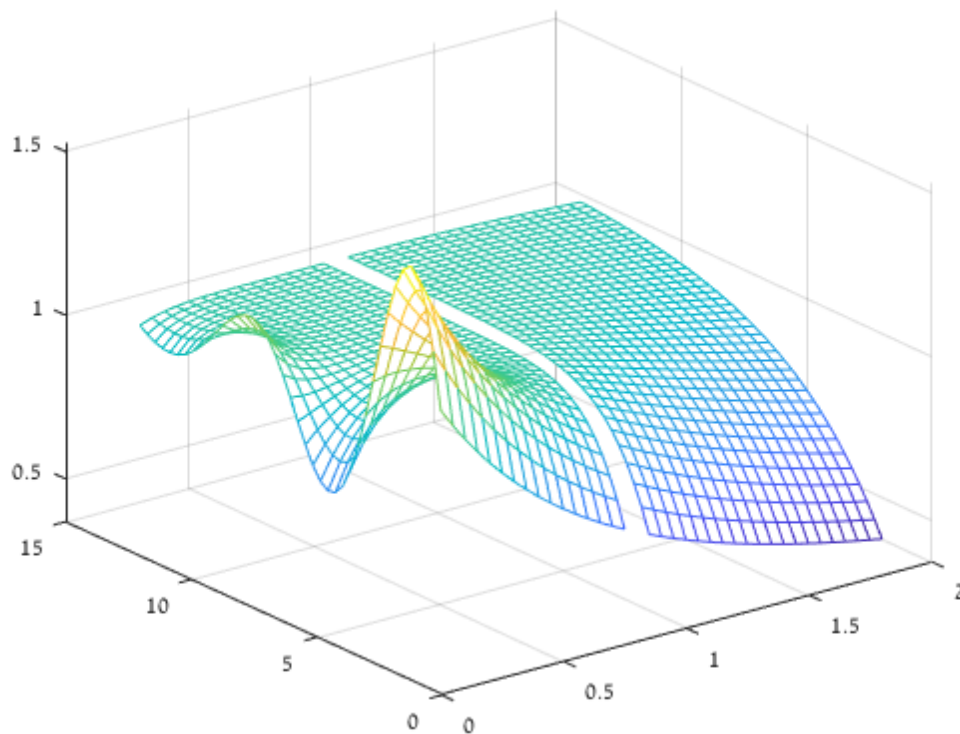
Student ID

```
ID = 316098052;  
disp(ID)
```

316098052

1 Second-order system mesh plot

```
zeta = 0.2:0.05:2;  
omega_t = 2:0.3:14;  
[ZETA,OMEGA_T] = meshgrid(zeta,omega_t);  
  
beta = (1-ZETA.^2).^(1/2);  
tetha = acos(ZETA);  
Y_T = 1-(1./beta).*exp(-1.*ZETA.*OMEGA_T).*sin(OMEGA_T.*beta+tetha);  
  
mesh(ZETA,OMEGA_T,Y_T)
```



2 Inverted pendulum on a cart

2. State-space equations.

The linearized model of the system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{Mg}{Ml} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{-Ml} \end{bmatrix} \cdot u(t)$$

3. Controlable and observable.

The matrix cont is full rank and therefore the system is controllable.

The matrix obs is full rank and therefore the system is observable.

```
syms m g l M
C = [1 0 0 0; 0 1 0 0];
A = [0 1 0 0; 0 0 (-m*g)/M 0; 0 0 0 1; 0 0 (g/l) 0];
B = [0; 1/M; 0; -1/(M*l)];

cont = [B A*B A^2*B A^3*B]
rank(cont)
obs = [C ; C*A ; C*(A^2) ; C*(A^3)]
rank(obs)
```

```
cont =

[ 0, 1/M, 0, (g*m)/(M^2*l)]
[ 1/M, 0, (g*m)/(M^2*l), 0]
[ 0, -1/(M*l), 0, -g/(M*l^2)]
[ -1/(M*l), 0, -g/(M*l^2), 0]
```

ans =

4

obs =

```
[ 1, 0, 0, 0]
[ 0, 1, 0, 0]
[ 0, 1, 0, 0]
[ 0, 0, -(g*m)/M, 0]
[ 0, 0, -(g*m)/M, 0]
[ 0, 0, 0, -(g*m)/M]
[ 0, 0, 0, -(g*m)/M]
[ 0, 0, -(g^2*m)/(M*l), 0]
```

ans =

4

4 DC motor control

1. Steady state error:

The system's error function in laplace domain:

$$E(s) = \frac{s(s + 0.02 + k_b k_m)}{(k k_m + s k_b + s(s + 0.02))}$$

The error's steady state response to a ramp unit using the final value theorem:

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s) \frac{1}{s^2} = \frac{0.02 + k_b k_m}{k k_m}$$

2. Required K calculation:

For $k_m = 10$ and $k_b = 0.05$.

$$k = 10^{-3}$$

```
syms k k_m k_b

k = 10^-3;
k_m = 10;
k_b = 0.02;
s = tf('s')

H = (k*k_m+s*k_b*(1-k_m))/(k*k_m+s*k_b+s*(s+0.02))

ramp = H*(1/s);

figure(1)
step(H,20)
figure(2)
step(ramp,20)
```

s =

s

Continuous-time transfer function.

H =

$$\frac{-0.18 s + 0.01}{s^2 + 0.04 s + 0.01}$$

Continuous-time transfer function.

The two plots are acceptable by the derivative link between them.

```
function [zeros,poles] = pzplot2(a,b)

poles = complex(roots(b));
max_pole = max(real(poles));
zeros = complex(roots(a));
axe = max([max(abs(zeros)) max(abs(poles))]);
```

```

figure(1)

plot(zeros, 'o', 'MarkerEdgeColor', 'red', 'MarkerSize', 10)
hold on
plot(poles, 'x', 'MarkerEdgeColor', 'blue', 'MarkerSize', 10)
grid, axis([-axe-1 axe+1 -axe-1 axe+1])
hold on
xL = xlim;
yL = ylim;
line([0 0], yL); %x-axis
line(xL, [0 0]); %y-axis
hold on
patch_x = [max_pole; max_pole; axe+1; axe+1];
patch_y = [-axe-1; axe+1; axe+1; -axe-1];
patch(patch_x, patch_y, [0, 0.8, 0.8], 'edgeAlpha', 0.1);
alpha(0.3)
hold off

title('Pole-Zero map')
xlabel('Real Part (\sigma)')
ylabel('Imaginary Part (j\omega)')

```

end