Control Theory Intro: Home Assignment #4

October 28, 2021

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Introduction

The purpose of this home assignment is to base your understanding in second order systems, system response and basic control.

Your solutions should be presented in a PDF (not Word!) file. You should submit also a **.m** file. The first line should print your ID.

>> disp('ID_STUDENT_1 ID_STUDENT_2') % disp('ID_STUDENT_1') if only one student is submitting.

For clarity of the script, you can separate the different sections of the script with a **%%**. This will automatically create a block in your script. In order to run specifically this block of code press 'Ctrl+Enter'. To run the entire script press 'F5'.

1 Second-order system mesh plot

Create a 3D mesh plot, where, x axis is the value of $\zeta \in [0.2, 2]$, y axis is the value of $\omega_n t \in [2, 14]$, and the z axis is the unit step output response $y(t) \in [0, 2]$.

2 Inverted pendulum on a cart

Fig. 1 shows the inverted pendulum on a cart. We will assume that M >> m and the angle of rotation θ and $\dot{\theta}$ is small so that the equations are linearizable.

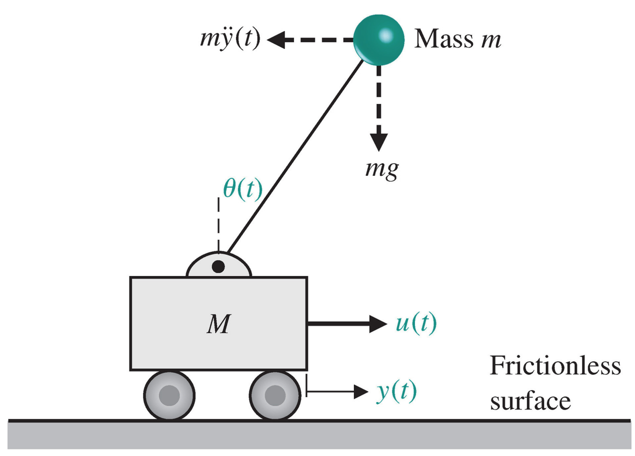

Figure 1: Inverted pendulum on a cart.

1. (optional) derive the equations of motion. If you do not wish to derive the equations you can use the following equations:

$$\begin{gathered}(M+m)\ddot{y} + ml\cos(\theta)\ddot{\theta} - ml\dot{\theta}^2\sin\theta - u(t) = 0 \\ ml\ddot{y}\cos\theta + ml\ddot{\theta} - mgl\sin\theta = 0\end{gathered} \tag{1}$$

2. Linearize the model and write the state-space equations.

3. Is the model controlable and observable? (assume $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$).

4. Is the system stable?

3 Race-car speed control

The engine, body, and tires of a racing vehicle affect the acceleration and speed attainable. The speed control of the car is represented by the model shown in Fig. 2. (a) Calculate the steady-state error of the car to a step command in speed, (b) Calculate overshoot of the speed to a step command.

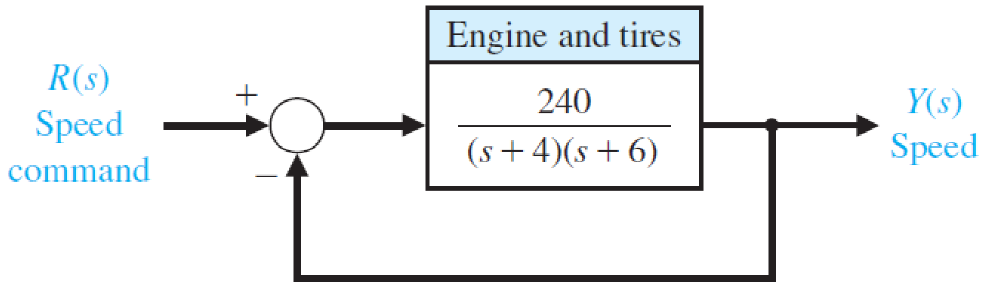

Figure 2: Racing car speed control.

4 DC motor control

The block diagram model of an armature-current-controlled DC motor is shown in Fig. 3.

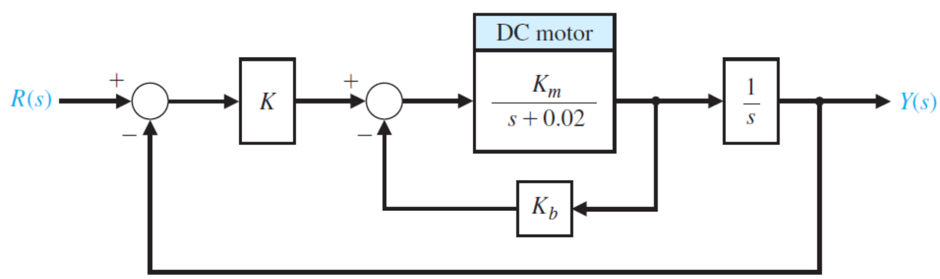

Figure 3: armature-current-controlled DC motor block diagram.

1. Determine the steady-state tracking error to a ramp input r(t) = t,$t \geq 0$, in terms of K, K_b , and K_m.

2. Let $K_m = 10$ and $K_b = 0.05$, and select K so that steady-state tracking error is equal to 1.

3. Plot the response to a unit step input and a unit ramp input for 20 seconds. Are the responses acceptable?