## Contents



## Student ID

```
ID = 316098052;
disp(ID)
```

```
316098052
```

## 1 System modeling

The system's equations are:

$$(M+m)\,\ddot{x} + MLcos\theta\ddot{\theta} - MLsin\theta\dot{\theta}^2 = -kx$$

$$gsin\theta + cos\theta\ddot{x} + L\ddot{\theta} = 0$$

approximations - small thethas and derivative :

$$(M+m)\,\ddot{x} + ML\ddot{\theta} = -kx$$

$$\ddot{x} + L\ddot{\theta} = -g\theta$$

The state variable differential matrix equation:

$$\dot{x} = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ -\frac{k}{m} & 0 & g\frac{M}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{Lm} & 0 & \frac{-g(M+m)}{Lm} & 0 \end{array}\right] \cdot x$$

## 2 System modeling 2

the system's equation are:

$$f - mg = ma$$

KVL:

$$L\dot{i} = v - iR$$

The state variable differential matrix equation:

$$\left[\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{array}\right] = \left[\begin{array}{c} x_2 \\ \frac{k}{m}\left(\frac{I_0+x_3}{X_0+x_1}\right)^2 - g \\ \frac{1}{L}\left(-Rx_3 + v\right) \end{array}\right]$$

approximation - linear near the working point:

$$Taylor\ :\quad \frac{k}{m}\left(\frac{I_0+x_3}{X_0+x_1}\right)^2 \cong \frac{k}{m}\left(-\frac{I_0^2}{X_0^3}x_1 + \frac{I_0}{X_0^2}x_3\right)$$

The linear state variable differential matrix equation:

$$\left[\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{array}\right] = \left[\begin{array}{ccc} 0 & 1 & 0 \\ -\frac{k}{m}\frac{I_0^2}{X_0^3} & 0 & \frac{k}{m}\frac{I_0}{X_0^2} \\ 0 & 0 & -\frac{R}{L} \end{array}\right] \cdot \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\right] + \left[\begin{array}{c} 0 \\ 0 \\ \frac{1}{L} \end{array}\right] \cdot v$$

The charcterized matrices and vectors of the system:

$$C = \left[\begin{array}{ccc} 1 & 0 & 0 \end{array}\right] \qquad B = \left[\begin{array}{c} 0 \\ 0 \\ \frac{1}{L} \end{array}\right] \qquad A = \left[\begin{array}{ccc} 0 & 1 & 0 \\ -\frac{k}{m}\frac{I_0^2}{X_0^3} & 0 & \frac{k}{m}\frac{I_0}{X_0^2} \\ 0 & 0 & -\frac{R}{L} \end{array}\right]$$

Transfer function at Laplace plane:

$$H(S) = \frac{X(s)}{V(s)} = C\left[sI - A\right]^{-1}B = \frac{18.19}{s^3 + 45.67s^2 + 2246s + 102600}$$

```
L = 0.508;
R = 23.2;
I_0 = 1.06;
M = 1.75;
X_0 = 4.36*10^-3;
K = 2.9*10^-4;

A = [0 1 0; (-K*I_0^2)/(M*X_0^3) 0 (K*I_0)/(M*X_0^2); 0 0 -R/L];
B = [0 0 1/L]';
C = [1 0 0];
D = 0;

sys = ss(A, B, C, D);
tf(sys)
```

```
ans =

                18.19
  -----------------------------------
  s^3 + 45.67 s^2 + 2247 s + 1.026e05

Continuous-time transfer function.
```

### 3 Comparing two systems

```
A_1 = [0 1 0;0 0 1; -4 -5 -8];
B_1 = [0 0 4]';
C_1 = [1 0 0];
D_1 = 0;

sys1 = ss(A_1, B_1, C_1, D_1);
tf_sys1 = tf(sys1)

A_2 = [0.5 0.5 0.7071;-0.5 -0.5 0.7071; -6.364 -0.7071 -8];
B_2 = [0 0 4]';
C_2 = [0.7071 1.2929 0];
D_2 = 0;

sys2 = ss(A_2, B_2, C_2, D_2);
tf_sys2 = tf(sys2)
```

```
tf_sys1 =

            4
  ---------------------
  s^3 + 8 s^2 + 5 s + 4

Continuous-time transfer function.


tf_sys2 =

     5.657 s - 1.657
  ---------------------
  s^3 + 8 s^2 + 5 s + 4

Continuous-time transfer function.
```

The two systems has the same transfer function.

### 4 State-system response

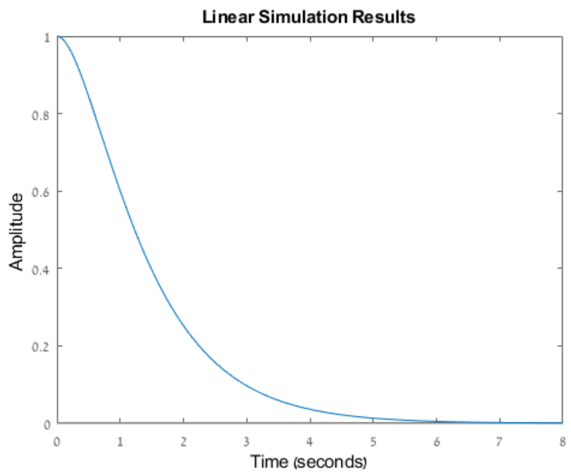
```
A_1 = [0 1 ;-2 -3];
B_1 = [0 1]';
C_1 = [1 0];
D_1 = 0;
x_0 = [1 0]';
t = 0:0.04:8;  % 201 points
u_t = t*0;
sys1 = ss(A_1, B_1, C_1, D_1);
tf_sys1 = tf(sys1)

lsim(sys1,u_t,t,x_0)
```

```
tf_sys1 =

        1
  -------------
  s^2 + 3 s + 2

Continuous-time transfer function.
```




The reponse type is ZIR, so the transfer function converge to constant value and in this case to zero, as expected.

### 5 State-system response 2

```
A_1 = [0 1 0 ;0 0 1; -3 -2 -5];
B_1 = [0 0 1]';
C_1 = [1 0 0];
D_1 = 0;
x_0 = [0 -1 1]';
t = 0:0.04:10;
u_t = t*0;
sys1 = ss(A_1, B_1, C_1, D_1);
tf_sys1 = tf(sys1)

lsim(sys1,u_t,t,x_0)

t = 10;
[V, D] = eig(A_1*t);

TM = (V*diag(exp(diag(D))))* inv(V);

x_10 = TM*x_0
```

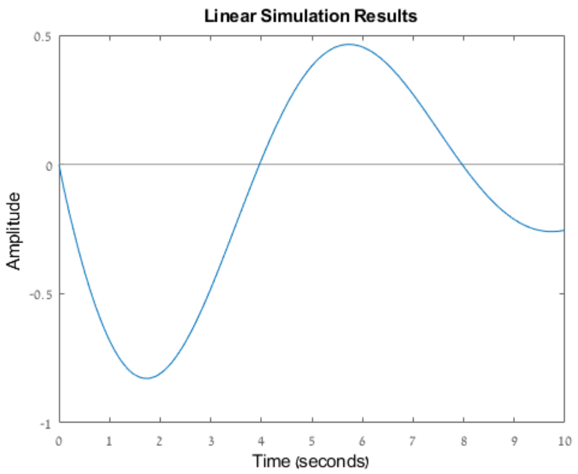
```
tf_sys1 =

            1
  ---------------------
  s^3 + 5 s^2 + 2 s + 3

Continuous-time transfer function.


x_10 =

  -0.2545 + 0.0000i
   0.0418 + 0.0000i
   0.1500 - 0.0000i
```




$x(t)$ after 10 seconds get smaller and converge to zero and it fit with the response that is shown at section 2.

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