

# Introduction to Modern Communications

361-1-3221

## Computer Assignment 1

Spring 2022

### General Instructions

- Electronic submission via the course website by **10.05.2022, 23:59**
  - All figures must be labeled correctly on both the x and y axes.
  - In case a figure includes more than one plot, a legend should be added. Use markers to facilitate readability in the case of black and white printing.
- The assignment has three questions: In Question 1 you will simulate an analog communication system. This question is intended to be carried out using MATLAB. Questions 2 and 3 are analytical questions and need to be done **without** the aid of a computer.
  - Analytic expressions should be simplified and explicit as much as possible.
- All solutions and graphs should be submitted as a PDF file. MATLAB code should be attached to the end of the file.
- **Submission should be done in pairs.**
- **Only one person should upload the assignment to the moodle, and the pdf file should contain the names and IDs of both students.**

# 1 AM and FM Modulation - MATLAB (50%)

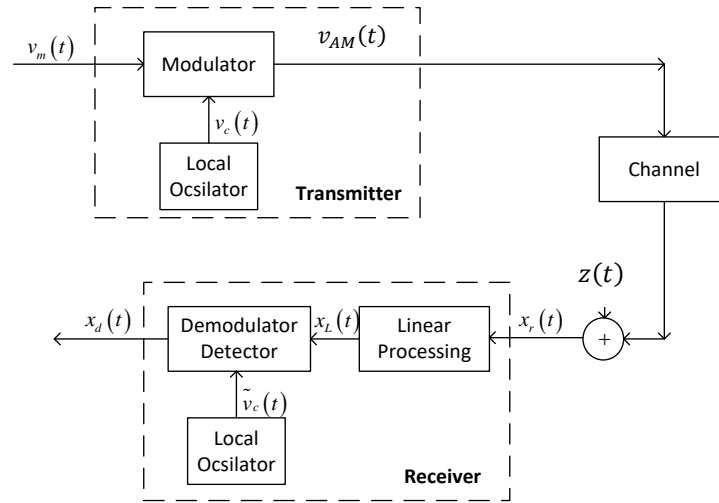


Figure 1-Analog Communication System

## 1.1 Data Generation (5%)

To generate the information signal  $v_m(t)$ , consider the following steps:

- Download the wav file “in-the-air.wav” from Moodle.
- Load the file in MATLAB using the command:

“`[v_m,fs] = audioread('in-the-air.wav')`”

The MATLAB vector ‘v\_m’ is of dimension  $107968 \times 2$ , where the two columns of the vector are referred to the right and left channel. The parameter ‘ $f_s$ ’ is the sample rate for the data signal ‘v\_m’.

- To make things simpler, we will work with a one dimensional signal ( $107968 \times 1$ ), i.e., extract only the left channel by using the command: “`v_m = v_m(:,1)`”.
- You can listen to the information signal by using the command: “`sound(v_m,fs)`”.
- Define the sampling interval  $T_s = 1/f_s$ , and define the number of sample points  $N$  as the length of the vector v\_m (i.e., use the MATLAB command “`N=length(v_m)`”).
- create the time vector “t”, and the frequency vector “f”, using the commands:

“`t = 0: Ts: (N-1)*Ts`”

“`f = linspace(-fs/2,fs/2,N)`”

From now on, when we refer to the time  $t$  and the frequency  $f$ , we will refer to the discrete-time vectors created above.

- Finally, to evaluate the Fourier transform of  $v_m(t)$ ,  $V_m(f) = \mathcal{F}\{v_m(t)\}$ , use the “fft” command: “`V_m = fftshift(fft(v_m)) / sqrt(N)`”

**1.1.1. Plot** on the same figure (using “subplot”) the data signal  $v_m(t)$  and its Fourier transform  $V_m(f)$ .

**1.1.2. Evaluate** the bandwidth of the information signal.

## 1.2 Modulator (10%)

In this section, we will simulate the AM with Carrier modulation learned in class, for the information signal  $v_m(t)$ . The modulated signal will be denoted by  $v_{AM}(t)$ .

Choose the carrier frequency  $f_c = 15 \cdot 10^3 [Hz]$  and the modulation index  $k_{AM} = 0.02$ .

**1.2.1.** Create the signal  $v_{AM}(t)$ . You can use the MATLAB function “ammod”.

**1.2.2.** Compute in MATLAB the Fourier transform of  $v_{AM}(t)$ , denoted  $V_{AM}(f)$ , by using the “fft” command (as explained above), and **plot  $V_{AM}(f)$** . **Explain.**

**1.2.3.** Listen to the modulated signal  $v_{AM}(t)$  (using the “sound” command). Do you hear something? **Explain!**

## 1.3 Channel (10%)

In this section, we will generate the noise sequence  $z(t)$ .

In order to simulate the continuous-time noise in MATLAB, you will generate a discrete time sequence  $z(t)$  (again, remember that  $t$  is a discrete-time vector). The sequence is of independent and identically distributed random variables, each drawn from a zero-mean normal distribution with variance  $N_0/2$ , i.e.,  $z(t) \sim N(0, N_0/2), \forall t$ .  $z(t)$  is further generated independently of the data signal.

**1.3.1.** Generate in MATLAB a real white Gaussian noise sequence  $z(t)$  as described above.

- MATLAB instructions: To generate the noise in MATLAB, use the function “randn” in

the following command:  $z = \sqrt{\frac{N_0}{2}} * \text{randn}(1, N)$ .

Make sure that the noise vector has the same size as the signal vector.

Choose  $\sqrt{N_0/2} = 0.02$ .

**1.3.2.** Generate the received signal at the channel output:  $x_r(t) = v_{AM}(t) + z(t)$ .

Compute and **plot** in MATLAB the Fourier transform of  $x_r(t)$ ,  $X_r(f)$ . **Explain.**

#### 1.4 Demodulator (15%)

In this section, you will demodulate the channel output obtained in the previous section. We first filter the received noisy signal with a bandpass filter.

- 1.4.1.** Generate the signal  $x_L(t)$  by passing the received signal  $x_r(t)$  through a bandpass filter. You can use the MATLAB function “bandpass”. Note that you need to choose the passband frequency range of the filter appropriately.

Compute and **plot** in MATLAB the Fourier transform of  $x_L(t)$ ,  $X_L(f)$ . **Explain.**

- 1.4.2.** Demodulate the received signal  $x_L(t)$ . You can use the MATLAB function “amdemod”. Let  $x_d(t)$  denote the decoded signal at the receiver. **Plot** the decoded signal, together with  $v_m(t)$  on the same graph.

**Plot** the decoded Fourier transform of the signals  $X_d(f)$  and  $V_m(f)$  on the same graph.

**NOTE: You may need to lowpass filter the received signal  $x_d(t)$ . You can use the MATLAB function “lowpass” if needed.**

- 1.4.3.** Listen to the signal  $x_d(t)$ . How does it sound?
- 1.4.4.** Finally, use the MATLAB function “xcorr” to determine the correlation between the original signal  $v_m(t)$  and the decoded signal  $x_d(t)$ . You can use the following command: “xcorr(x\_d, v\_m, 0, 'coeff’)”. **Write the result.**

#### 1.5. Now with different noise and modulations (10%)

- 1.5.1.** Repeat items 1.3 and 1.4, this time choosing  $\sqrt{N_0/2} = 0.1$ .
- 1.5.2.** Repeat items 1.3-1.5.1, this time for FM modulation. Choose the frequency deviation  $\Delta f_d = 10 \cdot 10^3 [Hz]$ . You can use the MATLAB functions “fmmod” and “fmdemod”.
- 1.5.3.** Conclude by writing the four correlation coefficients from item 1.4.4. Explain the differences.

## 2 PM Modulation (25%)

Consider the information signal (a periodic signal with period  $T_m = 1/f_m$ ):

$$v_m(t) = \begin{cases} 1, & |t| \leq \frac{T_m}{4} \\ 0, & \frac{T_m}{4} < |t| < \frac{T_m}{2} \end{cases}$$

Assume that  $v_M(t)$  is a PM modulation of the signal  $v_m(t)$ , with maximal phase deviation

$\Delta\varphi_{\max}$ .

**2.1 (10%)** Show that the low pass equivalent of  $v_M(t)$  is given by:

$$v_{LPE}(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{j2\pi f_m n \cdot t},$$

and find the coefficients  $\{C_n\}_{n=-\infty}^{\infty}$  explicitly.

**2.2 (5%)** Show that:

$$|C_n| = \begin{cases} \left| \frac{1}{2} \cos\left(\frac{\Delta\varphi_{\max}}{2}\right) \right|, & n = 0 \\ \left| \frac{1}{\pi n} \sin\left(\frac{\Delta\varphi_{\max}}{2}\right) \right|, & n \text{ odd} \\ 0, & n \text{ even}, n \neq 0 \end{cases}$$

**2.3 (10%)** We define the cut-off index:  $n_c(\varepsilon) \triangleq \min\{n: |C_n| \leq \varepsilon\}$ .

Find  $n_c(\varepsilon)$  (as a function of  $\varepsilon$ ) for:

**2.3.1**  $\Delta\varphi_{\max} = \frac{\pi}{2}$

**2.3.2**  $\Delta\varphi_{\max} \ll 1$  (you can use the approximation  $\sin(\alpha) \approx \alpha$  for  $\alpha \ll 1$ ).

### 3 AM and FM Modulations (25%)

Consider the information signal  $v_m(t) = \cos(2\pi f_m t)$ .

The signal  $v_m(t)$  is modulated to the signal  $z(t)$  using amplitude modulation, around a carrier frequency  $\bar{f} = 2f_m$ , where the carrier signal is  $v_c(t) = \cos(2\pi \bar{f} \cdot t)$ .

Then, the signal  $z(t)$  is modulated to the signal  $x(t)$  using frequency modulation, around a carrier frequency  $f_c \gg f_m$ :

$$x(t) = \cos\left(2\pi f_c t + 2\pi \Delta f_d \int_{\tau=-\infty}^t z(\tau) d\tau\right)$$

where  $\Delta f_d$  is the maximum deviation frequency.

**3.1 (10%)** Assume that  $z(t)$  is an **SSB modulation** of  $v_m(t)$ .

Find the bandwidth of the signal  $x(t)$  by using Carson's rule. Your answer should depend on the parameters  $f_m$ ,  $\Delta f_d$ . Consider the two methods of SSB modulation: USSB and LSSB (i.e., you should provide two different answers according to each method).

**3.2 (15%)** Assume that  $z(t)$  is a **DSB modulation** of  $v_m(t)$ .

Derive an expression for the spectrum of the transmitted signal,  $X(f)$ . Your answer should depend on the parameters  $f_m$ ,  $\Delta f_d$  and the Bessel function, defined by:

$$J_n(\beta) \triangleq \frac{1}{2\pi} \int_{\alpha=-\pi}^{\pi} e^{j(\beta \sin(\alpha) - n \cdot \alpha)} d\alpha$$