

Maths Assignment - Applied Mathematics Flavour

z5417727

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1 Question 1

1.1 a)

Prove that in modulo 9, it is not possible for a perfect square to be congruent to 2, 3, 4, 6 or 8. Proof: For any integer $n \in \mathbb{Z}$, we say that $n^2 \equiv 0, 1, 4, 7 \pmod{9}$.

This can be deduced by finding the squares of 0, 1, 2, 3, 4 respectively and applying the \equiv theorem for numbers up to 9.

$$0^2 \equiv 0 \pmod{9},$$

$$1^2 \equiv 1 \pmod{9},$$

$$2^2 \equiv 4 \pmod{9},$$

$$3^2 \equiv 0 \pmod{9},$$

$$4^2 \equiv 7 \pmod{9}.$$

Through \equiv , we find the similar rule applied to 5 through 8 (since $9^2 \equiv 0 \pmod{9}$).

$$5^2 \equiv (-4)^2 \equiv 7 \pmod{9},$$

$$6^2 \equiv (-3)^2 \equiv 0 \pmod{9},$$

$$7^2 \equiv (-2)^2 \equiv 4 \pmod{9},$$

$$8^2 \equiv (-1)^2 \equiv 1 \pmod{9}.$$

Here, we see that the perfect squares always end with the digits 0, 1, 4 and 7. Thus, it can be proved that in modulo 9, it is not possible for a perfect square to be congruent to 2, 3, 4, 6, or 8.

1.2 b)

Hence (and not otherwise) prove that there do not exist three consecutive integer values of n for which $41n + 39$ is a perfect square.

2 Question 2

A certain relation \star is defined on the set \mathbb{Z}^+ by:

$x \star y$ if and only if every factor of x is a factor of y .

For each of the questions below, be sure to provide a proof supporting your answer.

2.1 a)

Is \star reflexive?

2.2 b)

Is \star symmetric?

2.3 c)

Is \star anti-symmetric?

2.4 d)

Is \star transitive?

2.5 e)

Is \star an equivalence relation, a partial order, both or neither?

3 Question 3

Consider the two functions $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ for non-empty sets X, Y, Z . Decide whether each of the following statements is true or false, and prove each claim.

3.1 a)

If $g \circ f$ is injective, then g is injective.

3.2 b)

If $g \circ f$ is injective, then f is injective.

3.3 c)

If $g \circ f$ is injective and f is surjective, then g is injective