# Maths Assignment - Applied Mathematics Flavour

z5417727

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# 1 Question 1

## 1.1 a)

**Prove that in modulo 9, it is not possible for a perfect square to be congruent to $2, 3, 4, 6$ or $8$.** **Proof:** For any integer $n \in \mathbb{Z}$, we say that $n^2 \equiv 0, 1, 4, 7(\mod 9)$.

This can be deduced by finding the squares of $0, 1, 2, 3, 4$ respectively and applying the [] theorem for numbers up to 9.

$$0^2 \equiv 0(\text{mod } 9),$$
$$1^2 \equiv 1(\text{mod } 9),$$
$$2^2 \equiv 4(\text{mod } 9),$$
$$3^2 \equiv 0(\text{mod } 9),$$
$$4^2 \equiv 7(\text{mod } 9).$$

Through [], we find the similar rule applied to 5 through 8 (since $9^2 \equiv 0(\mod 9)$).

$$5^2 \equiv (-4)^2 \equiv 7(\text{mod } 9),$$
$$6^2 \equiv (-3)^2 \equiv 0(\text{mod } 9),$$
$$7^2 \equiv (-2)^2 \equiv 4(\text{mod } 9),$$
$$8^2 \equiv (-1)^2 \equiv 1(\text{mod } 9).$$

Here, we see that the modulo of perfect squares always end with the digits $0, 1, 4$ and 7. Thus, it can be proved that in modulo 9, it is not possible for a perfect square to be congruent to $2, 3, 4, 6,$ or 8.

### 1.2 b)

ff **Hence (and not otherwise) prove that there do not exist three consecutive integer values of $n$ for which $41n + 39$ is a perfect square.** Consider a number $n$, $n + 1$ and $n + 2$ for $n \in \mathbb{R}$.

## 2 Question 2

**A certain relation $\star$ is defined on the set $\mathbb{Z}^+$ by:**

$x \star y$ **if and only if every factor of** $x$ **is a factor of** $y$.

**For each of the questions below, be sure to provide a proof supporting your answer.**

### 2.1 a)

**Is $\star$ reflexive?**

## 2.2 b)

**Is $\star$ symmetric?**

### 2.3 c)

**Is $\star$ anti-symmetric?**

## 2.4 d)

**Is $\star$ transitive?**

## 2.5 e)

**Is $\star$ an equivalence relation, a partial order, both or neither?**

# 3 Question 3

**Consider the two functions $f : X \to Y$ and $g : Y \to Z$ for non-empty sets $X, Y, Z$. Decide whether each of the following statements is true or false, and prove each claim.**

## 3.1 a)

**If $g \circ f$ is injective, then $g$ is injective.**

### 3.2 b)

**If $g \circ f$ is injective, then $f$ is injective.**

### 3.3 c)

**If $g \circ f$ is injective and $f$ is surjective, then $g$ is injective**