

# MATH1081 notes

Nira (z5417727)

September 17th, 2022

## Contents

<b>1</b>	<b>Topic 1</b>	<b>2</b>
1.1	Introduction . . . . .	2
1.2	Sets and subsets . . . . .	3
1.3	Power Sets and Stability . . . . .	4
1.4	Set Operations . . . . .	5
1.5	The Inclusion-Exclusion Principle . . . . .	6
1.6	Sets Proofs . . . . .	7
1.7	Laws of Set Algebra . . . . .	8
1.8	Generalised Set Operations . . . . .	9
1.9	Russel's Paradox . . . . .	10
1.10	Cartesian Product . . . . .	11

# **1 Topic 1**

## **1.1 Introduction**

1. addition, multiplication, division and subtraction
2. Mainly dealing with finite sets

## 1.2 Sets and subsets

A set is a well defined collection of distinct objects

Example:  $S = \{1, a, 3\}, A = \{\Pi, 1\}$ .

1.  $e \notin A$ ; it is not in A
2. For example, if A is a set of all integers;  $\{\text{all even integers}\} = \{n \in \mathbb{R} | n \text{ is even}\}$ .
3. We can remove superfluous items (elements that occur more than one).  
 $A = \{1, 2, 3, 3\}$  where 3 can be removed.

Example:

$A = \{1, 2, 3\}, B = \{2, 3, 1\}, C = \{1, 2, 3, 3\}, D = \{1, 3\}$ .

Here, D is a proper subset of A, B, C; A, B, C are supersets of D.

$\subseteq$ : Subset (proper subset),  $\supseteq$ : Superset.

1. To prove if a set is a proper subset; do the following:

For example, if  $D \in A$ , then check if  $e \in D$

If  $e \in D$ , then  $e \in A$ . Thus, it would be a proper subset (here, e is just an element).

2. To prove that two sets are equal;

For example, if  $A = B$ , prove:

- i)  $A \subseteq B$ ; if an element is in A, then the element is in B.
- ii)  $B \subseteq A$ ; if an element is in B, then the element is in A.

### 1.3 Power Sets and Stability

Subsets of  $A = \{1, 2, 3\}$ :

1. Could throw everything out to get empty set  $\Phi$ ,
2. One element each:  $\{1\}, \{2\}, \{3\}$ ,
3. Two elements:  $\{1, 2\}, \{2, 3\}, \{1, 3\}$ ,
4. Set itself:  $A$ .

The set containing 1, 2, 3, 4 is called the powerset of A.

Given  $A = \{1, 2, 3\}, B = \{1, 2, 3, 3\}, C = \{1, 3\}, D = \{1, 3\}$ , where  $A = B, C \subseteq A, B$  and  $D \not\subseteq A, B, C$ .

1. size of A = 3, B = 3, C = 2, D = 2.

[Exercise with A = 0, 1, 0, 1, B done in word].

## 1.4 Set Operations

Boolean Operators ("not" operation in programming):

1. Complement:

Let there be a set  $A$  in  $U$  ( $A$ : all of the people in the video,  $U$ : universal set of everyone in the world,  $A^c$  = complement of  $A$ ).

$$A^c = \{x \in U | x \notin A\}.$$

2. Intersecting ("and" operation in programming):

If there is  $A, B$ , intersecting,

$$A \cap B = \{x \in A | x \in B\}.$$

3. Union ("or" operation in programming): If there is  $A, B$ ,  $A$  or  $B$  is:

$$A \cup B = \{x \in U | x \in A \text{ or } x \in B\}.$$

4. Difference: If there is  $A, B$ , intersecting,

$$A - B = \{x \in A | x \notin B\}.$$

[examples in word doc]

## 1.5 The Inclusion-Exclusion Principle

[example in Word]

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

For three elements,

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

[example in word]

## 1.6 Sets Proofs

[proof question in word]

Hints for proofs:

1. To prove that  $S \subseteq T$ , we can assume that  $x \in S$  and show that  $x \in T$ .
2. To prove that  $S = T$ , we can show that  $S \subseteq T$  and  $T \subseteq S$ .

Scaffold:

Proof: Suppose that ..... (proof) we see that/ it follows ... (conclusion) (end with shaded box to indicate end of proof)
---

Note that the "Suppose that" part of the proof is usually whatever the if statement mentions.

For example, if the question is "Prove that if  $A \cap B = A$ , then  $A \cup B = B$ ", then the proof starts like this:

<u>Proof</u> : Suppose that $A \cap B = A$ .
--

For questions like "is this statement true", there are two ways to approach the question:

1. If the statement is true (if you think it is true), then prove it.
2. If the statement is false, then give a counter-example that proves it false.

[examples in word]

## 1.7 Laws of Set Algebra

### Laws of Set Algebra

1.  $A \cap B = B \cap A$  : Commutative Law.
2.  $A \cap (B \cap C) = (A \cap B) \cap C$  : Associative Law.
3.  $A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$  : Distributive Law.
4.  $A \cap (A \cup B) = A$  : Absorption Law.
5.  $A \cap U = U \cap A = A$  : Identity Law.
6.  $A \cap A = A$  : Idempotent Law.
7.  $(A^c)^c = A$  : Double Complement Law.
8.  $A \cap \emptyset = \emptyset \cap A = \emptyset$  : Domination Law.
9.  $A \cap A^c = \emptyset$  : Intersection with Complement Law.
10.  $(A \cup B)^c = A^c \cap B^c$  : De Moirve's Law.

The intersection can be swapped with the union to form another law (like,  $A \cup B = B \cup A$  swapped as  $A \cap B = B \cap A$ ). Similarly,  $U$  should be swapped with  $\emptyset$  and vice versa.

[examples in word]



## 1.8 Generalised Set Operations

Unions and Intersections; A saga:

$$1. \cup_{i=1}^n A_i = A_1 \cup A_2 \cup \cdots \cup A_n,$$

$$2. \cap_{i=1}^n A_i = A_1 \cap A_2 \cap \cdots \cap A_n.$$

Example:

$$\begin{aligned} A_k &= k, k+1; \\ \cup_{i=1}^3 A_k &= A_1(\{1, 2\}) \cup A_2(\{2, 3\}) \cup A_3(\{3, 4\}), \\ &= \{1, 2, 3, 4\}. \end{aligned}$$

[example in word]

## 1.9 Russel's Paradox

A set may contain another set as one of its elements.

This raises the possibility that a set may contain itself as an element.

**Problem:** Try to let  $S$  be the set of all sets that are not elements of themselves, i.e.,  $S = \{A \mid A \text{ is a set and } A \notin A\}$ .

**Is  $S$  an element of itself?**

i) If  $S \in S$ , then the definition of  $S$  implies that  $S \notin S$ , a contradiction.

ii) If  $S \notin S$ , then the definition of  $S$  implies that  $S \in S$ , also a contradiction.

Hence neither  $S \in S$  nor  $S \notin S$ . This is Russell's paradox.

## 1.10 Cartesian Product

[example in word]

The Cartesian product of two sets A and B, denoted by  $A \times B$ , is the set of all ordered pairs from A to B:

$$A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$$

If  $|A| = m$  and  $|B| = n$ , then we have  $|A \times B| = mn$ .

Sets with more than 2 elements:

**Example:**  $A = \{a, b\}, B = \{1, 2, 3\}$ .

Cartesian Product  $(A \times B) = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

(all of the ordered pairs – combinations)

[example in word]

When X and Y are small finite sets, we can use an arrow diagram to represent a subset S of  $X \times Y$  : we list the elements of X and the elements of Y , and then we draw an arrow from x to y for each pair  $(x, y) \in S$ .