# MATH1081 notes

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September 17th, 2022

## Contents

# 1 Topic 1

## 1.1 Introduction

1. addition, multiplication, division and subtraction
2. Mainly dealing with finite sets

## 1.2 Sets and subsets

A set is a well defined collection of distinct objects

Example: $S = \{1, a, 3\}, A = \{\Pi, 1\}$.
1. $e \notin A$; it is not in A
2. For example, if A is a set of all integers; $\{$all even integers$\} = \{n \in \mathbb{R} |$n is even$\}$.
3. We can remove superfluos items (elements that occur more than one). $A = \{1, 2, 3, 3\}$ where 3 can be removed.

Example:
$A = \{1, 2, 3\}, B = \{2, 3, 1\}, C = \{1, 2, 3, 3\}, D = \{1, 3\}$.
Here, D is a proper subset of A, B, C; A, B, C are supersets of D.
$\subseteq$: Subset (proper subset),$\supseteq$: Superset.

1. To prove if a set is a proper subset; do the following:
For example, if $D \in A$, then check if $e \in D$
If $e \in D$, then $e \in A$. Thus, it would be a proper subset (here, e is just an element).

2. To prove that two sets are equal;
For example, if A = B, prove:
i) $A \subseteq B$; if an element is in A, then the element is in B.
ii) $B \subseteq A$; if an element is in B, then the element is in A.

### 1.3 Power Sets and Stability

Subsets of $A = \{1, 2, 3\}$:
1. Could throw everything out to get empty set $\Phi$,
2. One element each: $\{1\}, \{2\}, \{3\}$,
3. Two elements: $\{1, 2\}, \{2, 3\}, \{1, 3\}$,
4. Set itself: $A$.
The set containing 1, 2, 3, 4 is called the powerset of A.

Given $A = \{1, 2, 3\}, B = \{1, 2, 3, 3\}, C = \{1, 3\}, D = \{1, 3\}$, where $A = B$, $C \subseteq A, B$ and $D \nsubseteq A, B, C$.
1. size of A = 3, B = 3, C = 2, D = 2.
[Exercise with A = 0, 1, 0, 1, B done in word].

## 1.4 Set Operations

Boolean Operators (”not” operation in programming):
1. Complement:
Let there be a set A in U (A: all of the people in the video, U: universal set of everyone in the world, $A^c$ = complement of A).

$$\boxed{A^c = \{x \in U | x \notin A\}.}$$

2. Intersecting (”and” operation in programming):
If there is $A, B$, intersecting,

$$\boxed{A \cap B = \{x \in A | x \in B\}.}$$

3. Union (”or” operation in programming): If there is $A, B$, A or B is:

$$\boxed{A \cup B = \{x \in U | x \in A \text{ or } x \in B\}.}$$

4. Difference: If there is $A, B$, intersecting,

$$\boxed{A - B = \{x \in A | x \in B\}.}$$

[examples in word doc]

## 1.5 The Inclusion-Exclusion Principle

[example in Word]

$\boxed{|A \cup B| = |A| + |B| - |A \cap B|.}$

For three elements,

$$\boxed{|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.}$$

[example in word]

### 1.6 Sets Proofs

[proof question in word]
Hints for proofs:
1. To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
2. To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Scaffold:

| Proof: Suppose that ...... (proof)we see that/ it follows ... (conclusion) (end with shaded box to indicat |
|---|

Note that the "Suppose that" part of the proof is usually whatever the if statment mentions.

For example, if the question is "Prove that if $A \cap B = A$, then $A \cup B = B$, then the proof starts like this:

$\boxed{\underline{Proof}:\ \text{Suppose that } A \cap B = A.}$

For questions like "is this statement true", there are two ways to approach the question:
1. If the statement is true (if you think it is true), then prove it.
2. If the statement is false, then give a counter-example that proves it false.
[examples in word]