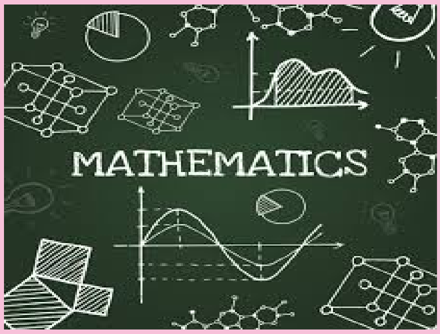
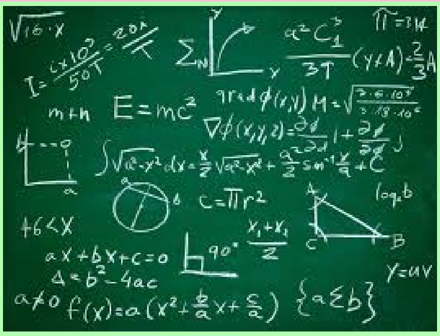
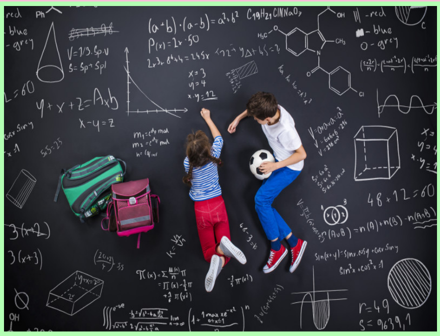
SSC MATHS HELP GUIDE

- with CAS Approach

[First Edition, 2020]



A B M Shahadat Hossain, PhD
Kamrun Nahar Putul, BS

MDA
Publication

Contents

1 Real Numbers **3**
- 1.1 Algebraic Expressions . . . 3
  - 1.1.1 Algebraic Formulae . . . 4
  - 1.1.2 Extension of formula for square . . . 6
- 1.2 Formula of Cubes . . . 13
- 1.3 Resolution into Factors . . . 19
- 1.4 Middle term factorization: . . . 19
- 1.5 Formulae of addition or subtraction of two cubes: . . . 20
- 1.6 Factors of the expression with fractional coefficients: . . . 21
- 1.7 Remainder Theorem . . . 22
- 1.8 Forming and applying algebraic formulae in solving real life problems 25

2 Sets and Functions **36**
- 2.1 Algebraic Expressions . . . 36
  - 2.1.1 Algebraic Formulae . . . 37
  - 2.1.2 Extension of formula for square . . . 39
- 2.2 Formula of Cubes . . . 46
- 2.3 Resolution into Factors . . . 52
- 2.4 Middle term factorization: . . . 52
- 2.5 Formulae of addition or subtraction of two cubes: . . . 53
- 2.6 Factors of the expression with fractional coefficients: . . . 54
- 2.7 Remainder Theorem . . . 55
- 2.8 Forming and applying algebraic formulae in solving real life problems 58

3 Algebraic Expressions **69**
- 3.1 Algebraic Expressions . . . 69
  - 3.1.1 Algebraic Formulae . . . 70
  - 3.1.2 Extension of formula for square . . . 72
- 3.2 Formula of Cubes . . . 81
- 3.3 Resolution into Factors . . . 87
- 3.4 Middle term factorization: . . . 87
- 3.5 Formulae of addition or subtraction of two cubes: . . . 88

3.6 Factors of the expression with fractional coefficients: 89
3.7 Remainder Theorem 90
3.8 Forming and applying algebraic formulae in solving real life problems 93

Chapter 3

Algebraic Expressions

Introduction

⊙ Algebraic formulae are used to solve many algebraic problems.

⊙ Moreover,many algebraic expressions are presented by resolving them into factors. That is why the problem solved by algebraic formulae and the contents of resolving expressions into factors by making suitable for the students.

⊙ Moreover,different types of mathematical problems can be solved by resolving into factors with the help of algebraic formulae.

⊙ In the previous class, algebraic formulae and their related corollaries have been discussed elaborately.

⊙ In this chapter, those are reiterated and some of their applications are presented through examples.Besides, extension of the formulae of square and cube, resolution into factors using remainder theorem and formation of algebraic formulae and their applications in solving practical problems have been discussed here in detail.

At the end of the chapter , the students will be able to

⇒ expand the formulae of square and cube by applying algebraic formulae.

⇒ explain the remainder theorem and resolve into factors by applying the theorem.

⇒ form algebraic formulae for solving real life problems and solve the problems by applying the formulae.

3.1 Algebraic Expressions

⊙ Meaningful organization of operational signs and numerical letter symbol is called **Algebraic Expressions**. Such as, $2a + 3b - 4c$ is an algebraic expression.

⊙ In algebraic expression, different types of information are expressed through the letters $a, b, c, p, q, r, m, n, x, y, z \dots$ etc.

⊙ These alphabet are used to solve different types of problems related to algebraic expressions.In arithmetic, only positive numbers are used,where as, in algebra, both positive and negative numbers including zero are used.

⊙ **Algebra is the generalization of arithmetic.**

⊙ The numbers used in algebraic expressions are **constants**, their values are fixed.

⊙ The letter symbols used in algebraic expressions are **variables**, their values are not fixed, they can be of any value.

3.1.1 Algebraic Formulae

⊙ Any general rule or resolution expressed eby algebraic symbols is called **Algebraic Formula**.

⊙ In class VII and VIII,algebraic formulae and related corollaries have been discussed.

⊙ In this chapter,some applications are presented on the basis of that discussion.

♠ **Formula 1.** $(a+b)^2 = a^2 + 2ab + b^2$

♠ **Formula 2.** $(a-b)^2 = a^2 - 2ab + b^2$

Remark 3.1.1.

⇒ *It is seen from* **Formula 1** *and* **Formula 2** *that, adding* $2ab$ *or* $-2ab$ *to* $a^2 + b^2$ *, we get a perfect square, i.e. we get* $(a+b)^2$ *or* $(a-b)^2$.

⇒ *Substituting* $-b$ *instead of* b *in* **Formula 1** *we get* **Formula 2:** $\{a+(-b)\}^2 = a^2 + 2a(-b) + (-b)^2$. *That is,*$(a-b)^2 = a^2 - 2ab + b^2$

♣ Corollary 1. $a^2 + b^2 = (a+b)^2 - 2ab$

♣ Corollary 2. $a^2 + b^2 = (a-b)^2 + 2ab$

♣ Corollary 3. $(a+b)^2 = (a-b)^2 + 4ab$

Proof.

$(a+b)^2 = a^2 + 2ab + b^2 = a^2 - 2ab + b^2 + 4ab = (a-b)^2 + 4ab$ □

♣ Corollary 4. $(a-b)^2 = (a+b)^2 - 4ab$

Proof.

$(a-b)^2 = a^2 - 2ab + b^2 = a^2 + 2ab + b^2 - 4ab = (a+b)^2 - 4ab$ □

♣ Corollary 5. $a^2+b^2=\dfrac{(a+b)^2+(a-b)^2}{2}$

Proof.

From **Formula 1** and **Formula 2**,

$(a+b)^2 = a^2+2ab+b^2$
$(a-b)^2 = a^2-2ab+b^2$

$2a^2+2b^2=(a+b)^2+(a-b)^2$(Adding)

or,$2(a^2+b^2)=(a+b)^2+(a-b)^2$

Hence, $a^2+b^2=\dfrac{(a+b)^2+(a-b)^2}{2}$ □

♣ Corollary 6. $ab=\left(\dfrac{a+b}{2}\right)^2-\left(\dfrac{a-b}{2}\right)^2$

Proof: From **Formula 1** and **Formula 2**,

$a^2+2ab+b^2=(a+b)^2$
$a^2-2ab+b^2=(a-b)^2$

$4ab=(a+b)^2-(a-b)^2$ (Subtracting)

or,$ab=\dfrac{(a+b)^2}{4}-\dfrac{(a-b)^2}{4}$

Hence,$ab=\left(\dfrac{a+b}{2}\right)^2-\left(\dfrac{a-b}{2}\right)^2$

Remark 3.1.2.

⇒ *By applying the* **Corollary 6**, *product of any two quantities can be expressed as the difference of two squares.*

♠ **Formula 3.** $\boxed{a^2-b^2=(a+b)(a-b)}$

Therefore,the difference of the squares of two expressions = sum of two expressions × difference of two expressions.

♠ **Formula 4.** $\boxed{(x+a)(x+b)=x^2+(a+b)x+ab}$

Therefore, $(x+a)(x+b)=x^2+$(algebraic sum of a and b)x +(the product of a and b)

3.1.2 Extension of formula for square

⊙ There are three terms in the expression $a+b+c$. It can be considered the sum of two terms $(a+b)$ and c.

⊙ Therefore,by applying **Formula 1**,the square of the expression is,

$$\begin{aligned}(a+b+c)^2 =&\{(a+b)+c\}^2 = (a+b)^2 + 2(a+b)c + c^2\\ =&a^2 + 2ab + b^2 + 2ac + 2bc + c^2\\ =&a^2 + b^2 + c^2 + 2ab = 2bc + 2ac\end{aligned}$$

♠ **Formula 5.** $\boxed{(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab = 2bc + 2ac}$

♣ Corollary 7. $a^2 + b^2 + c^2 = (a+b+c)^2 - 2(ab+bc+ac)$

♣ Corollary 8. $2(ab+bc+ac) = (a = b+c)^2 - (a^2 = b^2 + c^2)$

⊙ Note: extension of **Formula 5**

(1) $$\begin{aligned}(a+b-c)^2 =&\{a+b+(-c)\}^2\\ =&a^2 + b^2 + (-c)^2 + 2ab + 2b(-c) + 2a(-c)\\ =&a^2 + b^2 + c^2 + 2ab - 2bc - 2ac\end{aligned}$$

(2) $$\begin{aligned}(a-b+c)^2 =&\{a+(-b)+c\}^2\\ =&a^2 + (-b)^2 + c^2 + 2a(-b) + 2(-b)c + 2ac)\\ =&a^2 + b^2 + c^2 - 2ab - 2bc + 2ac\end{aligned}$$

(3) $$\begin{aligned}(a-b-c)^2 =&\{a+(-b)+(-c)\}^2\\ =&a^2 + (-b)^2 + (-c)^2 + 2a(-b) + 2(-b)(-c) + 2a(-c)\\ =&a^2 + b^2 + c^2 - 2ab + 2bc - 2ac\end{aligned}$$

Example 3.1.1.

What is the square of $(4x+5y)$?

Solution:

$(4x+5y)^2 = (4x)^2 + 2 \times (4x) \times (5y) + (5y)^2 = 16x^2 + 40xy + 25y^2$

Input

(1) To clear all saved values for variables:

```
Clear["`*"]
```

(2) To get the desired expansion:

```
Expand[(4 x + 5 y)^2]
```

Output

```
16 x^2 + 40 x y + 25 y^2
```

Example 3.1.2.

What is the square of $(3a - 7b)$?

Solution:

$(3a - 7b)^2 = (3a)^2 - 2 \times (3a) \times (7b) + (7b)^2 = 9a^2 - 42ab + 49b^2$

Input

To get the desired expansion:

```
Expand[(3 a - 7 b)^2]
```

Output

```
9 a^2 - 42 a b + 49 b^2
```

Example 3.1.3.

Find the square of 996 *by applying the formula of square.*

Solution:

$$\begin{aligned}(996)^2 =& (1000 - 4)^2\\ =& (1000)^2 - 2 \times 1000 \times 4 + 4^2\\ =& 1000000 - 8000 + 16 = 1000016 - 80000 = 992016\end{aligned}$$

Input

```
(996)^2
```

Output

```
992016
```

Example 3.1.4.

What is the square of $a+b+c+d$

Solution:

$$\begin{aligned}(a+b+c+d)^2 =& \{(a+b)+(c+d)\}^2\\ =&(a+b)^2+2\times(a+b)\times(c+d)+(c+d)^2\\ =&a^2+2ab+b^2+2(ac+ad+bc+bd)+c^2+2cd+d^2\\ =&a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc+2bd+2cd\end{aligned}$$

Input

```
Expand[(a + b + c + d)^2]
```

Output

```
a^2 + 2 a b + b^2 + 2 a c + 2 b c + c^2 + 2 a d + 2 b d + 2 c d + d^2
```

Work:

Find the square with the help of the formulae:

(i) $3xy+2ax$ **(ii)** $4x-3y$ **(iii)** $x-5y+2x$

Solution:

Example 3.1.5.

Simplify $(5x+7y+3z)^2+2(7x-7y-3z)(5x+7y+3z)+(7x-7y-3z)^2$

Solution:

Let, $5x + 7y + 3z = a$ and $7x - 7y - 3z = b$

$\therefore$ Given expression is:

$$
\begin{aligned}
&= a^2 + 2.b.a + b^2 = a^2 + 2ab + b^2 \\
&= (a+b)^2 \\
&= \{(5x + 7y + 3z) + (7x - 7y - 3z)\}^2 \\
&= (5x + 7y + 3z + 7x - 7y - 3z)^2 \\
&= (12x)^2 \\
&= 144x^2
\end{aligned}
$$

Example 3.1.6.

If $x - y = 2$ and $xy = 24$, What is the value of $x + y$?

Solution:

$$
\begin{aligned}
& (x+y)^2 = (x-y)^2 + 4xy = (2)^2 + 4(24_{=}4 + 96 = 100 \\
\therefore \quad & x + y = \pm\sqrt{100} = \pm 10
\end{aligned}
$$

Input

```
soln = Solve[{x - y == 2, x y == 24}, {x, y}]
res1=soln[[1, 1, 2]] + soln[[1, 2, 2]]
res2=soln[[2, 1, 2]] + soln[[2, 2, 2]]
```

Output

```
{{x -> -4, y -> -6}, {x -> 6, y -> 4}}
-10
10
```

Example 3.1.7.

If $a^4 + a^2b^2 + b^4 = 3$ and $a^2 + ab + b^2 = 3$, what is the value of $a^2 + b^2$?

Solution:

Given that

$$\begin{aligned}
a^4 + a^2b^2 + b^4 &= (a^2)^2 + 2a^2b^2 + (b)^2 - a^2b^2 \\
&= (a^2+b^2)^2 - (ab)^2 \\
&= (a^2+b^2+ab)(a^2+b^2-ab) \\
&= (a^2+ab+b^2)(a^2-ab+b^2)
\end{aligned}$$

$$\Rightarrow \qquad 3 = 3(a^2-ab+b^2)[\text{Substituting the values}]$$

$$\Rightarrow a^2-ab+b^2 = \frac{3}{3} = 1$$

Now adding, $a^2+ab+b^2=3$ and $a^2-ab+b^2=1$, we get,

$$2(a^2+b^2) = 4 \quad \Rightarrow \quad a^2+b^2 = \frac{4}{2} = 2$$

Input

```
In[11]:= res1 = Factor[a^4 + a^2 b^2 + b^4]
Out[11]= (a^2 - a b + b^2) (a^2 + a b + b^2)

In[23]:= res2 = res1 /. (a^2 + a b + b^2) -> 3
Out[23]= 3 (a^2 - a b + b^2)

         (*equn: 3 (a^2-a b+b^2)==3, let x=a^2-ab+b^2)

In[20]:= Solve[3 x == 3, x]
Out[20]= {{x → 1}}

         (*i.e. a^2-ab+b^2=1)
         (* given a^2+ab+b^2=3)

In[25]:= Simplify[(a^2 - a b + b^2) + (a^2 + a b + b^2)]
Out[25]= 2 (a^2 + b^2)

         (*let y=a^2+b^2)

In[26]:= Solve[2 y == 1 + 3, y]
Out[26]= {{y → 2}}
```

Example 3.1.8.

Prove that: $(a+b)^4 - (a-b)^4 = 8ab(a^2+b^2)$

Solution:

Given that

$$\begin{aligned}(a+b)^4-(a-b)^4 &= \{(a+b)^2\}^2-\{(a-b)\}^2\\ &= \{(a+b)^2+(a-b)^2\}\{(a+b)^2-(a-b)^2\}\\ &= 2(a^2+b^2)\times 4ab \quad \text{[Applying Corollary 5 and Corollary 6]}\\ &= 8ab(a^2+b^2)\end{aligned}$$

Example 3.1.9.

If $a+b+c=15$ and $a^2+b^2+c^2=83$, what is the value of $ab+bc+ac$?

Solution:

We know that

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ac)$$

$$\Rightarrow 2(ab+bc+ac) = (a+b+c)^2-(a^2+b^2+c^2) = (15)^2-83 = 142$$

$$\therefore \quad ab+bc+ac = \frac{142}{2} = 71$$

Input

```
Expand[(a + b + c)^2]
```

Output

```
a^2 + 2 a b + b^2 + 2 a c + 2 b c + c^2
```

Example 3.1.10.

If $a+b+c=1$ and $ab+bc+ac=1$, what is the value of $(a+b)^2+(b+c)^2+(c+a)^2$?

Solution:

Given that

$$\begin{aligned}(a+b)^2+(b+c)^2+(c+a)^2 &= a^2+2ab+b^2+b^2+2bc+c^2 = c^2+2ca+a^2\\ &= (a^2+b^2+c^2+2ab = 2bc+2ca)+(a^2+b^2+c^2)\\ &= (a=b+c)^2+(a+b+c)^2-2(ab+bc+ca)\\ &= (2)^2+(2)^2-2\times 1 = 4+4-2 = 8-2 = 6\end{aligned}$$

Example 3.1.11.

Express $(2x+3y)(4x-5y)$ *as the difference of two squares.*

Solution:

Let, $2x+3y=a$ and $4x-5y=b$. Then we have,

$$\begin{aligned}
ab &= \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 \\
&= \left(\frac{2x+3y+4x-5y}{2}\right)^2 - \left(\frac{2x+3y-4x+5y}{2}\right)^2 \\
&= \left(\frac{6x-2y}{2}\right)^2 - \left(\frac{8y-2x}{2}\right)^2 \\
&= \left\{\frac{2(3x-y)}{2}\right\}^2 - \left\{\frac{2(4y-x)}{2}\right\}^2 \\
&= (3x-y)^2 - (4y-x)^2
\end{aligned}$$

Work:

(a) Simplify: $(4x+3y)^2+2(4x+3y)(4x-3y)+(4x-3y)^2$

(b) If $x+y+z=12$ and $x^2+y^2+z^2=50$, find the value of

$$(x-y)^2+(y-z)^2+(z-x)^2.$$

Exercise 3.1

Exercise 41 (1).

Find the square with the help of the formulae:

1) $2a+3b$	*2)* $x^2+\dfrac{2}{y^2}$	*3)* $4y-5x$
4) $5x^2-y$	*5)* $3b-5c-2a$	*6)* $ax-by-cz$
7) $2a+3x-2y-5z$	*8)* 1007	

Exercise 42 (2).

Simplify:

1) $(7p+3q-5r)^2 - 2(7p+3q-5r)(8p-4q-5r) + (8p-4q-5r)^2$

2) $(2m+3n-p)^2 + (2m-3n+p)^2 - 2(2m+3n-p)(2m-3n+p)$

3) $6.35 \times 6.35 + 2 \times 6.35 \times 3.65 + 3.65 \times 3.65$

4) $\dfrac{2345 \times 2345 - 759 \times 759}{2345 - 759}$

Exercise 43 (3-8).

(i) *If* $a-b=4$ *and* $ab=60$*, what is the value of* $a+b$*?*

(ii) *If* $a+b=9m$ *and* $ab=18m^2$*, what is the value of* $a-b$*?*

(iii) *If* $x-\dfrac{1}{x}=4$*, Prove that,* $x^4+\dfrac{1}{x^4}=322$

(iv) *If* $2x+\dfrac{2}{x}=3$*, what is the value of* $x^2+\dfrac{1}{x^2}$*?*

(v) *If* $a+\dfrac{1}{a}=2$*, Show that,* $a^2+\dfrac{1}{a^2}=a^4+\dfrac{1}{a^4}$

(vi) *If* $a+b=\sqrt{7}$ *and* $a-b=\sqrt{5}$*, Prove that,* $8ab(a^2+b^2)=24$

Exercise 44 (9-12).

(a) *If* $a+b+c=9$ *and* $ab+bc+ca=31$*, what is the value of* $a^2+b^2+c^2$*?*

(b) *If* $a^2+b^2+c^2=9$ *and* $ab+bc+ca=8$*, what is the value of* $(a+b+c)^2$*?*

(c) *If* $a+b+c=6$ *and* $ab+bc+ca=14$*, what is the value of* $(a-b)^2+(b-c)^2+(c-a)^2$*?*

(d) *If* $x=3, y=4$ *and* $z=5$*, what is the value of* $9x^2+16y^2+4z^2-24xy-16yz+12zx$*?*

Exercise 45 (13-15).

(a) *Express* $(a+2b)(3a+2c)$ *as the difference of two squares.*

(b) *Express* $x^2+10x+24$ *as the difference of two squares.*

(c) *If* $a^4+a^2b^2+b^4=8$ *and* $a^2+ab+b^2=4$ *find the value of,*

  (i) a^2+b^2

  (ii) ab