

TRANSIENT CONDUCTION IN SQUARE SPECIMEN $([0, 1] \times [0, 1])$

Given Transient Heat conduction in square domain having dimension $[0, 1] \times [0, 1]$ with temperature distribution $T(x,y,t=0)$: $20^\circ C$ if $(x - 0.5)^2 + (y - 0.5)^2 < 0.2$, $40^\circ C$ otherwise

Method: [Finite Difference Method](#)

Discretization of Heat Equation : [The Explicit Method](#)

The heat equation for 2^{nd} dimension is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{\partial T}{\alpha \times \partial t}$$

discretizing the problem in time and using forward difference approximations to the time derivative we obtain

$$\frac{\partial T}{\partial t}_{m,n} = \frac{T_{m,n}^{p+1} - T_{m,n}^p}{\Delta T}$$

where $t = p \times \Delta t$ and the m, n subscripts may be used to designate the x - *locations* and y - *locations* of discrete nodal points. substituting and solving for the nodal temperature at the new $(p + 1)$ time and assuming that $\Delta x = \Delta y$ it follows that

$$T_{m,n}^{p+1} = F_0(T_{m+1,n}^p + T_{m-1,n}^p + T_{m,n+1}^p + T_{m,n-1}^p) + (1 - 4F_0)T_{m,n}^p$$

where F_0 is finite difference number given as

$$F_0 = \frac{\alpha \Delta t}{(\Delta x)^2}$$

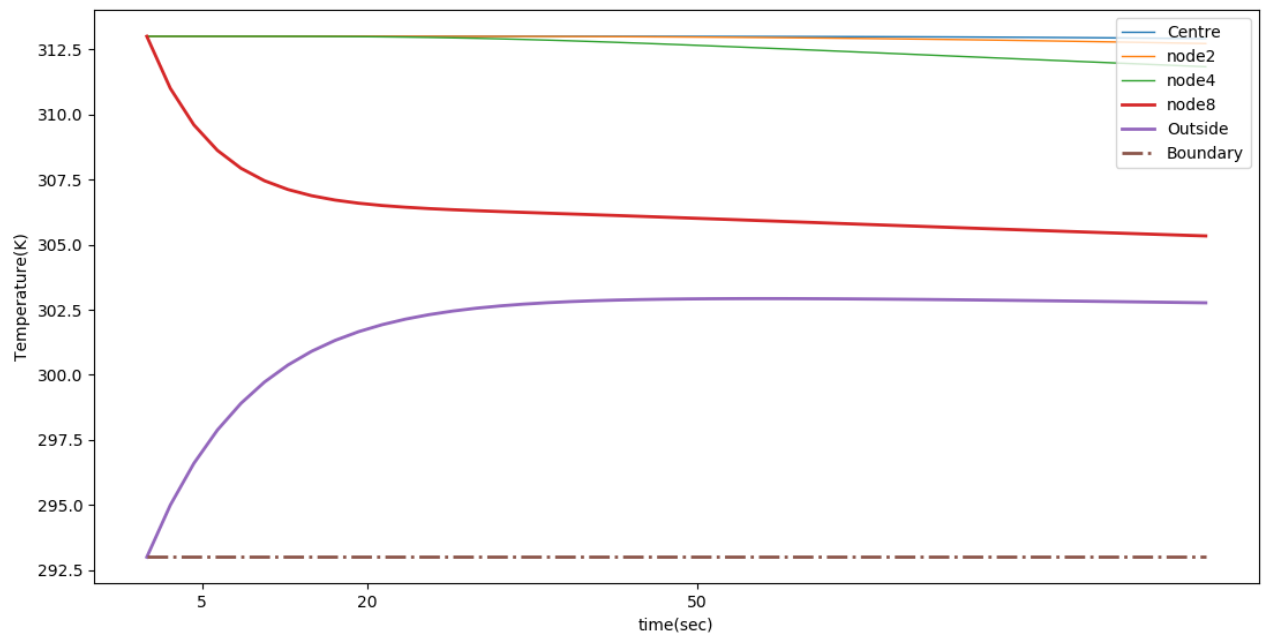
Assumed : dimensions $[0, 1] \times [0, 1]$ is in meter and the test specimen is of copper with thermal diffusivity $\alpha = 117 \times 10^{-6}$.

Selecting the 11 nodes where as node 0 is at centre of circle made by the equation $(x - 0.5)^2 + (y - 0.5)^2 = 0.2$ and corresponding nodes 1, 2, 3, 410 are at scale of $\Delta x = 0.05m$ from origin to edge of square. since the radii of circle is $\sqrt{0.2}$. the node 9 lies outside of circle and node 10 lies at the boundry of square and the remaining nodes lies inside of the circe.

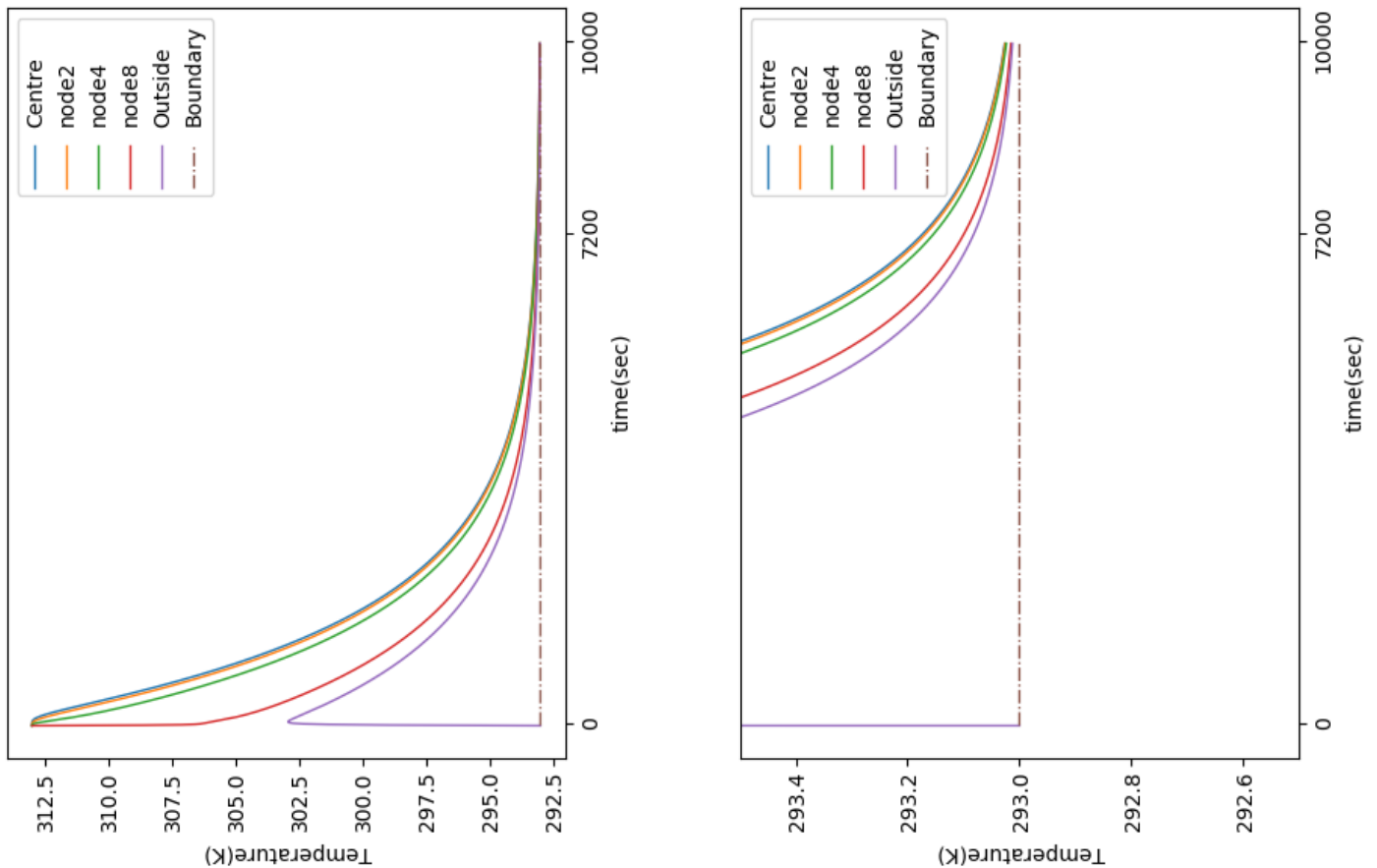
selecting $F_0 = 0.1$ this gives $\Delta t = 2.13$ by symmetry the transfer of heat takes place radially in 1 dimension .thus for 1-Dimensional conduction in x . we have following equation for interior node

$$T_m^{p+1} = F_0(T_{m+1}^p + T_{m-1}^p) + (1 - 2 * F_0)T_m^p$$

Calcluating and plotting response for 5 , 20 and 50 Seconds at node 0(Centre),node 10(Boundary), node 2(at distance 2×0.05 from orgin) ,node 4(at distance 4×0.05)similarly for node 8 and node 9(lie outside of circle)



Now plotting response for 7200 Seconds



link for Github : <https://github.com/Niraj-apr4/Assignment-Report>