Steady 2D Diffusion

Computational Fluid Dynamics (AM5630) Assignment 2

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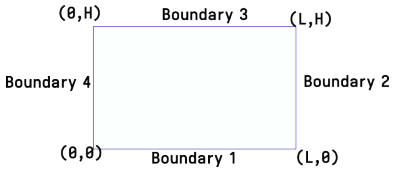
1 Assignment



$$q_x = -k\frac{\partial T}{\partial x}, \quad q_y = -k\frac{\partial T}{\partial y}$$

Case 4 is selected; the details are as follows:

2 Steps Followed

2.1 Mesh Geometry: Define Mesh Geometry

STEP 1 First create a differential 2D Control Volume with:

length along x Δ_x

length along y Δ_y

n Required number of such differential control volumes required to construct full CV

STEP 2 Compute the computational nodes for each differential control volume

Source file: `src/mesh_geometry.jl`

2.2 Computations

STEP 1 Identify the boundary nodes and apply the boundary conditions:

- Boundary 1: $T_1 = 15$
- Boundary 2: $T_2 = 10$
- Boundary 3: $T_3 = 5(1 - y/H) + 15\sin(\pi y/H)$

STEP 2 Write the equation for boundary 4

STEP 3 Write the equation for internal nodes

STEP 4 Setup the conditions for tolerance

Approach
- Pick n random nodes from grid
- Save temperature before each iteration
- Find the temperature after iteration
- $\text{diff} = T_{\text{after}} - T_{\text{before}}$
- Elementwise square each difference: diff^2
- Check: $\max(\text{diff}^2) < \text{tolerance}$

Prepare the required helper function for computation

STEP 5 Perform the computations

Source file: `./src/cfd2.jl`

3 Plots with Varying Parameters

Tolerance is set to 0.00001

Parameters:

Δ_x Length of differential CV in x direction

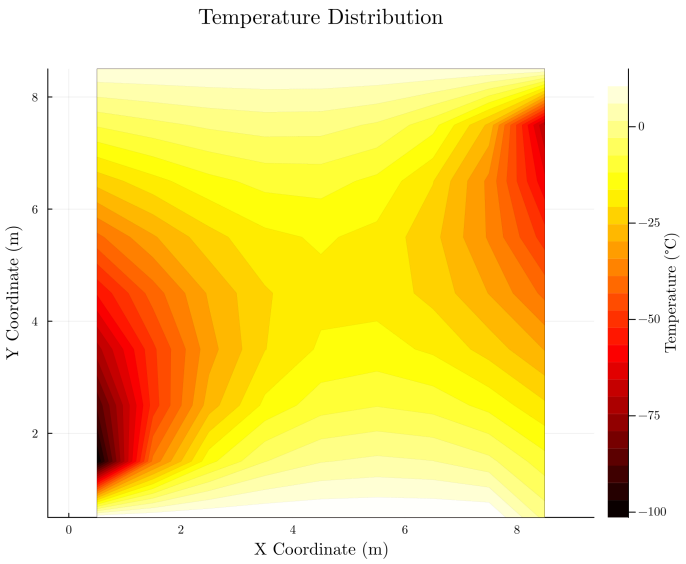
Δ_y Length of differential CV in y direction

n Number of grids

3.1 Plot with n = 10

$\Delta_x = 1.0$

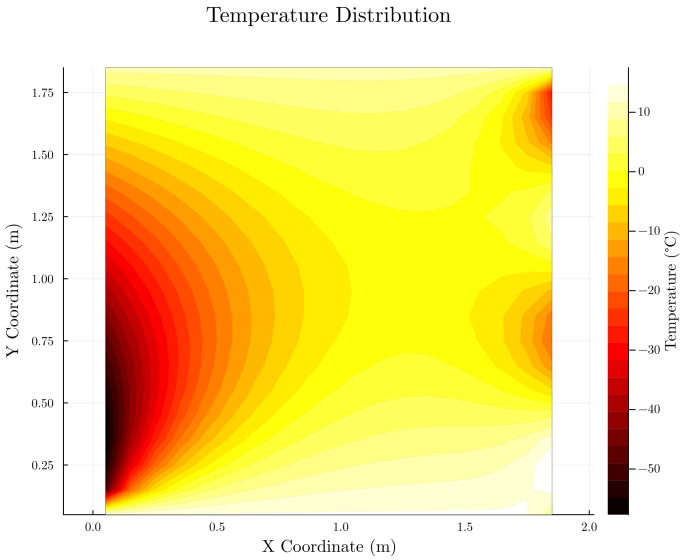
$\Delta_y = 1.0$



3.2 Plot with n = 20

$\Delta_x = 0.1$

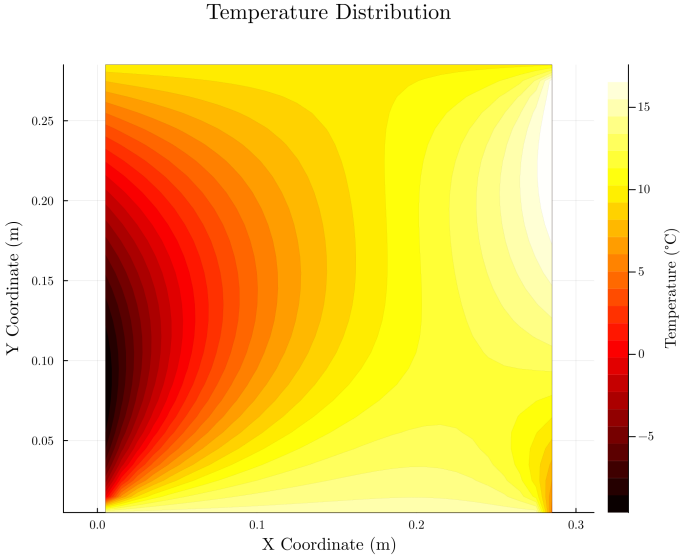
$\Delta_y = 0.1$



3.3 Plot with n = 30

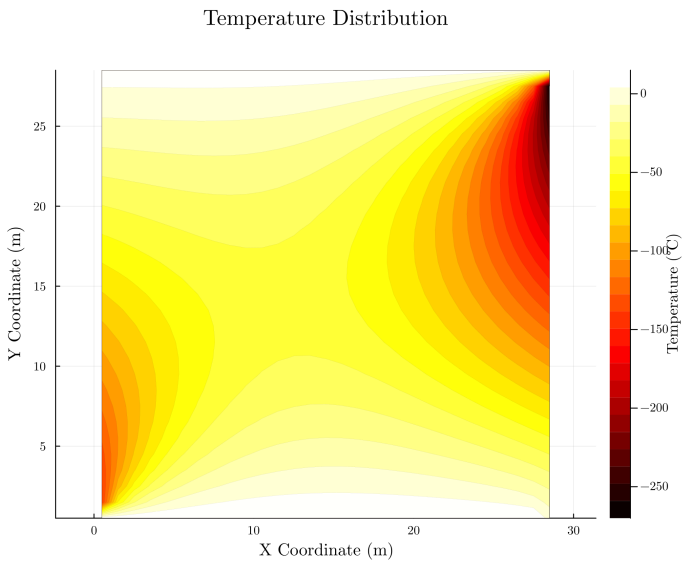
$\Delta_x = 0.01$

$\Delta_y = 0.01$



$\Delta_x = 1.0$

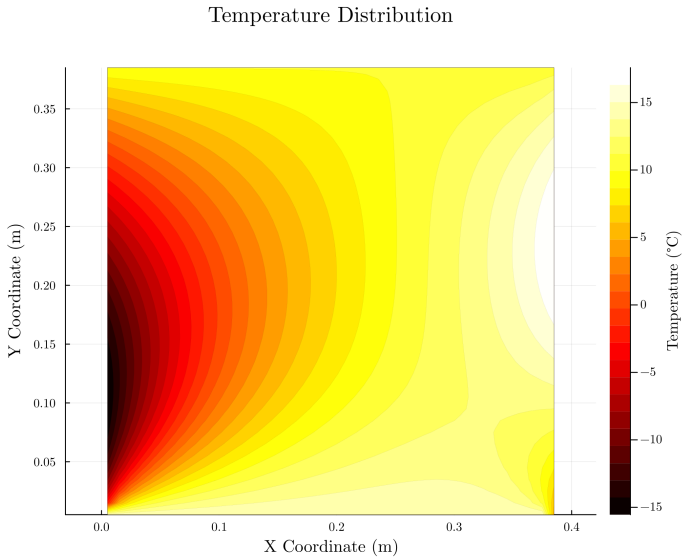
$\Delta_y = 1.0$



3.4 Plot with n = 40

$\Delta_x = 0.01$

$\Delta_y = 0.01$



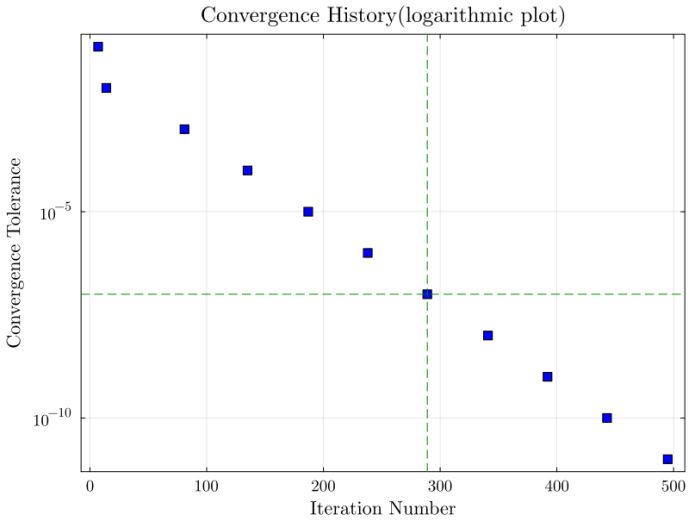
Source file: `./src/contour_plot.jl`

4 Convergence History

This section contains the number of iterations required to achieve desired convergence. The data is obtained and convergence history is plotted against logarithmic tolerance value.

For tolerance less than 10^{-7}, 289 iterations are required and is annotated in the graph.

Tolerance	Iteration Number
0.1	7
0.01	14
0.001	81
0.0001	135
10^{-5}	187
10^{-6}	238
10^{-7}	289
10^{-8}	341
10^{-9}	392
10^{-10}	443
10^{-11}	495



Source file: `./src/plot_convergence.jl`