Summary: Graphing and Critical Points

Definition of Critical Points

The critical points of a function $f(x)$ to be all points x in the domain of $f(x)$ such that

- $f'(x) = 0$, or
- $f'(x)$ does not exist.

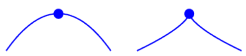
The First Derivative Test

Finding Local Maxima and Minima

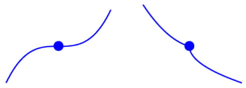
Suppose the function $f(x)$ is continuous at $x = a$ and has a critical point at $x = a$.

f has a local minimum at $x = a$ if $f'(x) < 0$ just to the left of a and $f'(x) > 0$ just to the right of a.

f has a local maximum at $x = a$ if $f'(x) > 0$ just to the left of a and $f'(x) < 0$ just to the right of a.



The point $x = a$ is neither a local minimum nor a local maximum of f if $f'(x)$ has the same sign just to the left of a and just to the right of a.



Just to the left or right

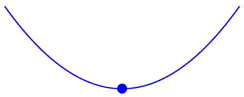
When we use the phrase " $f'(x) > 0$ just to the left of a," we mean that there is some open interval $(a-c, a)$ of positive width c on which f' is positive. This interval does not have to be very big, as long as it has some size!

Similarly, " $f'(x) > 0$ just to the right of a" means that there is some open interval $(a, a+d)$ of positive width d on which f' is positive.

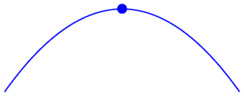
The Second Derivative Test

Suppose that $x = a$ is a critical point of f, with $f'(a) = 0$.

If $f''(a) > 0$, then f has a local minimum at $x = a$.



If $f''(a) < 0$, then f has a local maximum at $x = a$.



If $f''(a) = 0$, or does not exist, then the test is inconclusive — there might be a local maximum, or a local minimum, or neither.

The First Derivative Test vs. the Second Derivative Test

We've developed two tests for determining whether critical points are local minima or maxima. Each has its pros and cons.

- The Second Derivative Test requires just one value of f'', but it is sometimes inconclusive.
- The First Derivative Test requires a bit more data, but is often able to provide more information as a result.

It is important to be comfortable using both tests!

Definition of Inflection Point

An **inflection point** is a point where the concavity of the function changes. That is the second derivative $f''(x)$ changes sign— $f''(x) > 0$ just to the left of x and $f''(x) < 0$ just to the right of x (or vice versa).