Summary: Lagrange Multiplier

A differentiable function $f(x, y)$ of two variables on a closed bounded region R attains an absolute maximum (and absolute minimum) on R.

- The absolute maximum (or minimum) occurs at a critical point, or
- the absolute maximum (or minimum) occurs on the boundary of R.

Key point 1 : Along the boundary, the maximum occurs when the gradient ∇f is normal (perpendicular) to the boundary.

Key point 2 : If the boundary of the region R is described as the level curve $g(x, y) = k$. Then the maximum occurs where ∇f and ∇g point in the same (or opposite) direction:

$$\nabla f = \lambda \nabla g.$$

Mechanics

The method of **Lagrange multipliers** is used to optimize a function $f(x, y)$ (find the max or min) along a curve C described as a level curve $g(x, y) = k$ for some function $g(x, y)$. The curve C is called the **constraint**. A summary of the steps is given below.

1. Solve the following system of equations

$$f_x(x, y) = \lambda g_x(x, y) \tag{1}$$
$$f_y(x, y) = \lambda g_y(x, y) \tag{2}$$
$$g(x, y) = k \tag{3}$$

   for x and y. (The scalar λ is called the **Lagrange multiplier**.)

2. Compute the value of $f(x, y)$ at each point found in Step 1.

3. Identify which points give the maxima and minima of $f(x, y)$.

Ask Yourself

How do you determine which function plays which role?

If a function is only defined along a curve and has no meaning otherwise, do you still check critical points?