Summary: Gradients

The partial derivatives of a function together form a vector quantity. This vector quantity gives us information about the shape and slope of the graph of the function.

Mechanics

Gradients

Definition

The vector $\langle f_x, f_y \rangle$ is called the **gradient** of f.
The abbreviation for the gradient of f is ∇f.

Vector fields

Definition

A **vector field** on the plane is a function that attaches a vector to each point (x, y) in the plane.
Equivalent definitions:
A vector field is a function $\mathbf{F}$ that maps points in the plane to vectors:

$$\mathbf{F}(x, y) = \langle F_1(x, y), F_2(x, y) \rangle.$$

A vector field is sometimes called a vector-valued function.

The magnitude and direction of the gradient

Theorem

1. At any point (x_0, y_0), the vector $\langle f_x(x_0, y_0), f_y(x_0, y_0) \rangle$ is perpendicular to the level curve of f through (x_0, y_0).
2. ∇f points in the direction of steepest increase.
3. $|\nabla f|$ is the slope of that increase.

Ask yourself

Ask Yourself

Why is the gradient useful?
What is the direction of steepest decrease?