

Summary: Visualizing surfaces in three dimensions

Big Picture

1. One way to understand three dimensional graphs of functions $z = f(x, y)$ is to understand slices of these graphs with different two dimensional planes. By understanding these slices, we can get a better idea of how these three dimensional shapes look. Note that this process actually works in higher dimensions too!
2. Functions are not necessarily defined for all pairs of points (x, y) . The set of points where a function is defined is called its **domain**.
3. Multivariable functions are used to describe quantities that are measured in units in the world around us. You must take special care with respect to adding constants with appropriate units to ensure that all terms in a function make sense. If you want to work more abstractly, you can identify dimensions which are fundamental to a physical quantity and independent of the units used to measure it.

Mechanics

In order to visualize the three dimensional graph of a surface $z = f(x, y)$, we can use the method of slicing.

1. Find the intersection of the xz -plane with the graph of your surface.
 - Geometrically, this is the intersection of the plane $y = 0$ with the graph of the surface $z = f(x, y)$.
 - Algebraically, this is the curve in the xz -plane defined by the function $z = f(x, 0)$.
2. Find the intersection of the yz -plane with the graph of your surface.
 - Geometrically, this is the intersection of the plane $x = 0$ with the graph of the surface $z = f(x, y)$.
 - Algebraically, this is the curve in the yz -plane defined by the function $z = f(0, y)$.
3. Find the intersection of the xy -plane with the graph of your surface.
 - Geometrically, this is the intersection of the plane $z = 0$ with the graph of the surface $z = f(x, y)$.
 - Algebraically, this is the curve on the xy -plane defined implicitly by the equation $0 = f(x, y)$.

Ask Yourself

Can you use slicing to understand higher dimensional shapes?

What are all possible dimensions of the argument of a sine function?