Summary: Linear approximations and tangent planes

Big Picture

1. When you zoom in on the level curves of any function near a point where the function is differentiable, the level curves begin to look like parallel lines.
2. The level curves of a plane are parallel lines.
3. Close enough to a point, a function can be well approximated by its tangent plane.

Mechanics

Equations of lines and planes

1 variable	$y = ax + b$	line
2 variables	$z = ax + by + c$	plane

Given a function $f(x, y)$, the **linear approximation** of f near (x_0, y_0) is the **tangent plane** given by the equation

$$f(x_0 + \Delta x, y_0 + \Delta y) \approx f(x_0, y_0) + f_x(x_0, y_0)\Delta x + f_y(x_0, y_0)\Delta y$$

Ask Yourself

Can we do linear approximation on the equation for a plane?

Do you need to graph a 2 variable function to find its tangent plane approximation?

Extensions to higher dimensions

Given a function $f(x_1, \ldots, x_i, \ldots, x_n)$, the linear approximation of f near $(\tilde{x}_1, \ldots, \tilde{x}_i, \ldots, \tilde{x}_n)$ is an n dimensional hyperplane defined by the equation

$$\begin{aligned} f(\tilde{x}_1 + \Delta x_1, \ldots, \tilde{x}_i + \Delta x_i, \ldots, \tilde{x}_n + \Delta x_n) \approx\ & f(\tilde{x}_1, \ldots, \tilde{x}_i, \ldots, \tilde{x}_n) + f_{x_1}(\tilde{x}_1, \ldots, \tilde{x}_i, \ldots, \tilde{x}_n)\Delta x_1 \\ & + \cdots + f_{x_i}(\tilde{x}_1, \ldots, \tilde{x}_i, \ldots, \tilde{x}_n)\Delta x_i \\ & + \cdots + f_{x_n}(\tilde{x}_1, \ldots, \tilde{x}_i, \ldots, \tilde{x}_n)\Delta x_n \end{aligned}$$

where f_{x_i} is the ith partial derivative of f.