Summary: Level curves and partial derivatives

Big Picture

1. Multivariable functions are harder to visualize. One way to visualize such a function is to look at slices of the function at different heights. The plot looks like a hiking map and can be used to understand the graph in space more easily.

2. There is more than one notion of a derivative when we consider multivariable functions. Here we look at slicing the function with vertical planes in the x and y direction to obtain the notion of a partial derivative.

Mechanics

The **level curves** of a function $f(x, y)$ are given by $f(x, y) = k$ where k is a constant.
The **partial derivative of $f(x, y)$ with respect to x** is defined by

$$f_x(x, y) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}. \tag{1}$$

The Leibniz notation for this is $\dfrac{\partial f}{\partial x}$.
The **partial derivative of $f(x, y)$ with respect to y** is defined by

$$f_y(x, y) = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}. \tag{2}$$

The Leibniz notation for this is $\dfrac{\partial f}{\partial y}$.

Spoiler: Partial derivatives in higher dimensions
For a function in n dimensions $f(x_1, x_2, \ldots, x_n)$, the partial derivative with respect to the variable x_k is defined by

$$f_{x_k} = \lim_{\Delta x_k \to 0} \frac{f(x_1, \ldots, x_k + \Delta x_k, \ldots, x_n) - f(x_1, \ldots, x_k, \ldots, x_n)}{\Delta x_k} \tag{3}$$

for $1 \leq k \leq n$. The Leibniz notation for this is $\dfrac{\partial f}{\partial x_k}$.

Ask yourself

Is there a product rule for partial derivatives?
Is there a quotient rule or chain rule for partial derivatives?
Is a function of two variables two-dimensional or three-dimensional?

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Link	Topic
OCW 18.02: Week 4 Lecture Notes	Partial derivatives
OCW 18.02: Lecture 8 Video	Partial derivatives