# Practical - 6

Aim :- Differencial Equations

a) Solve differential equation.

```
x = var ('x')
y = function ('y') (x)
de = diff (y,x) ==(x*(1-x^2))/(y*(2-y))
sol = desolve(de, y)
show (sol)
```

OUTPUT :-

$$\frac{1}{3} y(x)^3 - y(x)^2 = \frac{1}{4} x^4 - \frac{1}{2} x^2 + C$$

b) Solve differential equation using initial value problem.

```
x = var('x')
y = function ('y')(x)
de = diff(y,x) == (x*(1-x^2)) / (y*(2-y))
sol = desolve (de, y)
show (sol)
sol2 = desolve(de, y, ics = [0,1])
show(sol2)
```

OUTPUT :-

$$\frac{1}{3} y(x)^3 - y(x)^2 = \frac{1}{4} x^4 - \frac{1}{2} x^2 + C$$

$$\frac{1}{3} y(x)^3 - y(x)^2 = \frac{1}{4} x^4 - \frac{1}{2} x^2 - \frac{2}{3}$$

c) Numerical solution of first-order equations using Euler's method

Code:

```
def euler(f, x0, y0, h, x1):
    n = int((1.0)*(x1 - x0)/h)
    xv, yv = x0, y0
    soln = [[x0, y0]]
    for i in range(1, n+1):
        xv = x0 + (h*i)
        yv = yv + (h* f(xv, yv))
        soln.append([xv, yv])
    return soln
var('x, y')
f(x, y) = (x*(1-x^2)) / ((y*(y-2))
x0 = 0
y0 = 1
h = 0.1
x1 = 1
euler(f, x0, y0, h, x1)
```

OUTPUT :-

```
[[0, 1],
[0.100000000000000, 0.990100000000000]
[0.200000000000000, 0.970898118023547],
[0.300000000000000, 0.943574977522111],
[0.400000000000000, 0.909867660734499],
[0.500000000000000, 0.872060521639262],
[0.600000000000000, 0.833021511201714],
[0.700000000000000, 0.796297581357226],
[0.800000000000000, 0.766250799915668]]
```

1) $\frac{dy}{dx} = \frac{x(1-x^2)}{y(2-y)}$

$y(2-y)\,dy = x(1-x^2)\,dx$
$y(y-2)\,dy = x(x^2-1)\,dx$
$(y^2-2y)\,dy = (x^3-x)\,dx$

Integrate on b.t.s.

$\int (y^2-2y)\,dy = \int (x^3-x)\,dx$

$\int y^2 dy - \int 2y dy = \int x^3 dx - \int x \, dx$

$\frac{y^3}{3} - 2\int \frac{y^2}{2} = \frac{x^4}{4} - \frac{x^2}{2} + C$

$\frac{y^3}{3} - y^2 = \frac{x^4}{4} - \frac{x^2}{2} + C$

2) $(x, y) = (0, 1)$

$\therefore \frac{y^3}{3} - y^2 = \frac{x^4}{4} - \frac{x^2}{2} + C$

$\therefore \frac{(1)^3}{3} - (1)^2 = \frac{(10)^4}{4} - \frac{(10)^2}{2} + C$

$\frac{1}{3} - 1 = C$

$c = \frac{1-3}{3}$

$c = \frac{-2}{3}$

$\therefore \frac{y^3}{3} - y^2 = \frac{x^4}{4} - \frac{x^2}{2} - \frac{2}{3}$

3) $f(x, y) = \frac{x(1-x^2)}{y(y-2)}$ ; $x_0 = 0, y_0 = 1, \Delta 2 = 0.1$

| i | $x_i$ | $y_i$ |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0.1 | 0.9901 |
| 2 | 0.2 | 0.9709 |
| 3 | 0.3 | 0.9438 |

Put, $x = 0$

$y_{0+1} = y_0 + f(x_0, y_0)\Delta x$

$y_1 = 1 + f(x_0, y_0)(0 \cdot 1)$

$\cancel{\therefore f(x_0, y_0) = \frac{x_0(1-x_0^2)}{y_0(y_0-2)}}$

$\therefore f(x_0, y_0) = \frac{x_0(1-x_0^2)}{y_0(y_0-2)}$

$= \frac{0(1-0^2)}{1(1-2)}$

$=$

$\therefore y_1 = 1 + (0)(0.1)$

$= 1$

Put, $x = 1$

$y_{1+1} = y_1 + f(x_1, y_1)\Delta x$

$y_2 = 1 + f(x_1, y_1)(0.1)\cancel{-1)}$

$\therefore f(x_1, y_1) = \frac{x_1(1-x_1^2)}{y_1(y_1-2)}$

$$= \frac{(0.1)(1-(0.1)^2)}{1(1-2)}$$

$$= \frac{(0.1)\ (1-0.01)}{-1}$$

$$= -(0.1)\ (0.99)$$

$$= -0.099$$

$\therefore y_2 = 1 - 0.099\ (0.1)$

$= 1 - 0.0099$

$= 0.9901$

Put $x = 2$

$y_{2+1} = y_2 + f(x_2, y_2)\ \Delta x$

$y_3 = 0.9901 + f(x_2, y_2)\ (0.1)$

$$\because f(x_2, y_2) = \frac{x_2(1-x_2^2)}{y_2(y_2-2)} = \frac{(0.2)(1-(0.2)^2)}{(0.99)(0.99-2)}$$

$$= \frac{-0.192}{0.9999} \approx -0.192$$

$y_3 = 0.9901 - 0.192\ (0.1)$

$= 0.9901 - 0.0192$

$= 0.9709$

Put $x = 3$

$y_{3+1} = y_3 + f(x_3, y_3)\ \Delta x$

$y_4 = 0.9709 + f(x_3, y_3)\ (0.1)$

$$\because f(x_3, y_3) = \frac{x_3\ (1-x_3^2)}{y_3\ (y_3-2)}$$

$$= \frac{0.3\ (1-(0.3)^2)}{0.9709\ (0.9709-2)}$$

$$= \frac{-0.273}{0.999}$$

$$\approx -0.2732$$

$\therefore y_4 = 0.9709 - 0.2732(0.1)$

$= 0.9709 - 0.02732$

$= 0.94358$

# Practical - 7

Aim :- First order partial order

1)
```
var('x,y')
f(x,y) = (x*y) / (x+y)
fx = f.diff(x)
show(fx(x,y))
fy = f.diff(y)
show(fy(x,y))
```

OUTPUT :-

$$\frac{y}{x+y} - \frac{xy}{(x+y)^2}$$

$$\frac{x}{x+y} - \frac{xy}{(x+y)^2}$$

Aim :- Second order partial derivative

1)
```
var('x,y')
f(x,y) = (x*y) / (x+y)
fx = f.diff(x)
show(fx(x,y))
fy = f.diff(y)
show(fy(x,y))
fxx = f.diff(x,2)
show(fxx(x,y))
```

fyy = f.diff( y, 2)
show (fyy (x, y))

OUTPUT :-

$$\frac{y}{x+y} - \frac{xy}{(x+y)^2}$$

$$\frac{x}{x+y} - \frac{xy}{(x+y)^2}$$

$$-\frac{2y}{(x+y)^2} + \frac{2xy}{(x+y)^3}$$

First order partial derivative

$$f(x) = \frac{(xy)}{(x+y)}$$

Diff. w.r.t. 'x'

$$f'(x) = \frac{(x+y)\ y(1) - xy\ (1+0)}{(x-y)^2}$$

$$= \frac{(x+y)y}{(x+y)^2} - \frac{xy}{(x+y)^2}$$

$$= \frac{y}{(x+y)} - \frac{xy}{(x+y)^2}$$

$$f(y) = \frac{xy}{x+y}$$

Diff. w.r.t. 'y'

$$f'(y) = \frac{(x+y) \quad x(1) - xy \quad (0+1)}{(x+y)^2}$$

$$f'(y) = \frac{(x+y) \quad x - xy}{(x+y)^2}$$

$$f'(y) = \frac{(x+y)(xy)}{(x+y)^2} - \frac{xy}{(x+y)^2}$$

$$f'(y) = \frac{x}{(x+y)} - \frac{xy}{(x+y)^2}$$

Second order partial derivative

$$f'(x) = \frac{y}{(x+y)} - \frac{xy}{(x+y)^2}$$

$$f''(x) = y\left(\frac{-1}{(x+y)^2}\right) - \left[\frac{(x+y)^2 \, y(1) - xy\,(2(x+y)(1+0)}{((x+y)^2)^2}\right]$$

$$= \frac{-y}{(x+y)^2} - \left[\frac{(x+y)^2 y - 2xy\,(x+y)}{(x+y)^4}\right]$$

$$= \frac{-y}{(x+y)^2} - \left[x+y\left(\frac{(x+y)y - 2xy}{(x+y)^4}\right)\right]$$

$$= \frac{-y}{(x+y)^2} - \left[\frac{(x+y) \quad y - 2xy}{(x+y)^3}\right]$$

$$= \frac{-y}{(x+y)^2} - \left[\frac{(x+y) \quad y}{(x+y)^3} + \frac{2xy}{(x+y)^3}\right.$$

$$= \frac{-y}{(x+y)^2} - \frac{y}{(x+y)^2} + \frac{2xy}{(x+y)^3}$$

$$= \frac{-2y}{(x+y)^2} + \frac{2xy}{(x+y)^3}$$

$$f''(y) = \frac{x}{(x+y)} - \frac{xy}{(x+y)^2}$$

$$f''(y) = x\left(\frac{-1}{(x+y)^2}\right) - \left[\frac{(x-y)^2 x(1) - xy(2(x+y)[0+1])}{((x+y)^2)^2}\right]$$

$$= \frac{x}{(x+y)^2} - \left[\frac{(x+y)^2 x - 2xy(x+y)}{(x+y)^2}\right]$$

$$= \frac{-x}{(x+y)^2} - \left[\frac{(x+y)[(x+y)x - 2xy}{(x+y)^4}\right]$$

$$= \frac{-x}{(x+y)^2} - \left[\frac{(x+y)x}{(x+y)^3} - \frac{2xy}{(x+y)^3}\right]$$

$$= \frac{-x}{(x+y)^2} - \frac{x}{(x+y)^2} + \frac{2xy}{(x+y)^3}$$

$$= \frac{-x-x}{(x+y)^2} + \frac{2xy}{(x+y)^3}$$

$$= \frac{-2x}{(x+y)^2} + \frac{2xy}{(x+y)^3}$$

# Practical - 8

Aim :- Directional derivatives

1) $D_u f(x_0, y_0) = f_x(x_0, y_0) u_1 + f_y(x_0, y_0) u_2$

```
var('x y')
f(x,y) = 4*x*y*exp(-x^2-y^2)
(a,b) = 1,1
(u1, u2) = (1/sqrt(2), 1/sqrt(2))
f.diff(x)(x=a, y=b)*u1 + f.diff(y)(x=a, y=b)*u2
```

OUTPUT :-

-4*sqrt(2)*e^(-2)

2) Gradient

```
var('x, y')
f(x, y) = x^2*e^y
(a, b) = -2, 0
v = f.gradient() (x=a, y=b)
show(v)
```

OUTPUT :-

(-4, 4)

$= \frac{1}{\sqrt{2}}$ ; $x_2 = \frac{1}{\sqrt{2}}$ ; $x_0 = 1$ ; $y_0 = 1$

$f(x, y) = 4xy\, e^{(-x^2-y^2)}$

→ $f_x(x_0, y_0) = 4y[x\, e^{(-x^2-y^2)}(-2x) + e^{(-x^2-y^2)}(1)]$

$$f_x(1,1) = 4(1)[(1).e^{(\quad)}(-2) + e^{(-(1)^2-(1)^2)}]$$
$$= 4[e^{-2}(-2) + e^{-2}]$$
$$= e^{-2}\,4(-2+1)$$
$$= -4e^{-2} \quad \text{———} \quad (i)$$

$$f_y(x_0,y_0) = 4x[y e^{(-x^2-y^2)}(-2y) + e^{(-x^2-y^2)}(1)]$$
$$f_y(1,1) = 4(1)[(1)e^{(-(1)^2-(1)^2)}(-2(1)) + e^{(-(1)^2-(1)^2)}]$$
$$= 4[e^{-2}(-2) + e^{-2}]$$
$$= 4e^{-2}[-2+1]$$
$$= -4e^{-2} \quad \text{———} \quad (ii)$$

By substituting in formula,

$$u\,f(x_0,y_0) = f_x(x_0,y_0)\,x_1 + f_y(x_0,y_0)\,x_2$$

$$= (-4e^{-2})\frac{1}{\sqrt{2}} + (-4e^{-2})\frac{1}{\sqrt{2}} \quad \text{———} \text{(from (i) \& (ii))}$$

$$= \frac{-4e^{-2} + (-4e^{-2})}{\sqrt{2}}$$

$$= \frac{-8e^{-2}}{\sqrt{2}}$$

$$= -4 \times \sqrt{2} \times e^{-2}$$

# Practical - 9

Aim :- Tangent Planes and Normal Vectors for functions of two or three variables.

1)
```
var('x, y, z')
f(x,y) = x^2 + 4*y^2 + z^2
(a,b,c) = (1,2,1)
fx = f.diff(x)(x=a, y=b, z=c)
fy = f.diff(y)(x=a, y=b, z=c)
fz = f.diff(z)(x=a, y=b, z=c)
T(x,y) = fx*(x-a) + fy*(y-b) + fz*(z-c)
show(T(x,y))
n = (fx, fy, fz)
show(n)
```

OUTPUT :-

$2x + 16y + 2z - 36$

$(2, 16, 2)$

$x^2 + 4y^2 + z^2 ; (x, y, z) \equiv (1, 2, 1)$

$f_x(x^2 + 4y^2 + z^2) = 2x$

$f_y(x^2 + 4y^2 + z^2) = 8y$

$f_z(x^2 + 4y^2 + z^2) = 2z$

$\nabla f(x, y, z) = [f_x(x, y, z), f_y(x, y, z), f_z(x, y, z)]$

$\nabla f(1, 2, 1) = [2x, 8y, 2z]$

$= [2(1), 8(2), 2(1)]$

$= [2, 16, 2]$

eqⁿ of tangent plane,

$$f_x(x,y,z)(x-x_0) + f_y(x,y,z)(y-y_0) + f_z(x,y,z)(z-z_0) = 0$$

$$2(x-1) + 16(y-2) + 2(z-1) = 0$$

$$2x - 2 + 16y - 32 + 2z - 2 = 0$$

$$2x + 16y + 2z - 36 = 0$$