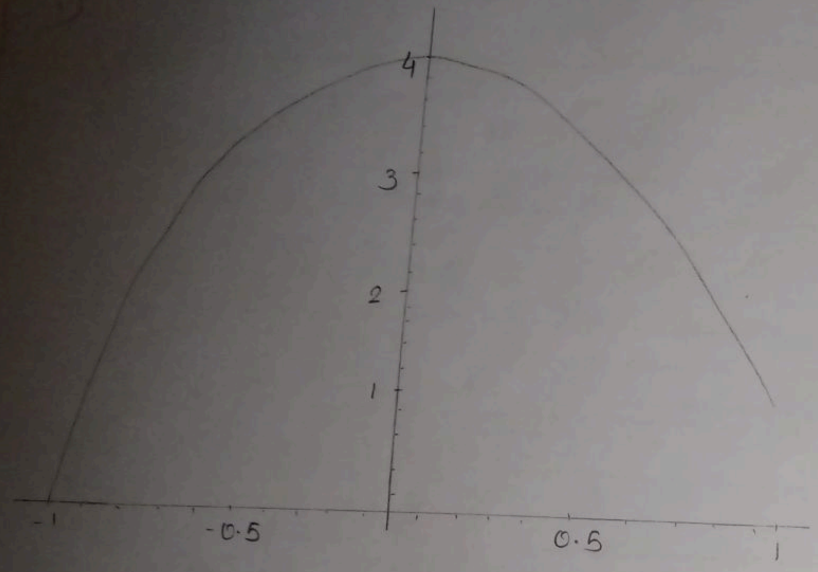


Practical - 2

Aim :- Application of Derivatives I -
a. Increasing and Decreasing functions
b. Concavity and inflection points
c. Relative Extrema
d. Absolute Extrema

1) f(x) = x^3 - 3*x^2 + 4
plot(f)

df(x) = derivative(f, x); df
Output :- x |--> 3*x^2 - 6*x

solve(df(x) == 0, x)
Output :- [x == 0, x == 2]

df(-.1)
df(.1)
Output :- 0.630000000000000
-0.570000000000000

df(1.9)
df(2.1)
Output :- -0.569999999999999
0.629999999999999

d2f(x) = derivative (f, x, 2); d2f
Output :- x |--> 6 * x - 6

solve(d2f(x) == 0, x)
Output :- [x == 1]

d2f (.9)
d2f (1.1)
Output :- -0.60000000000000
0.600000000000001

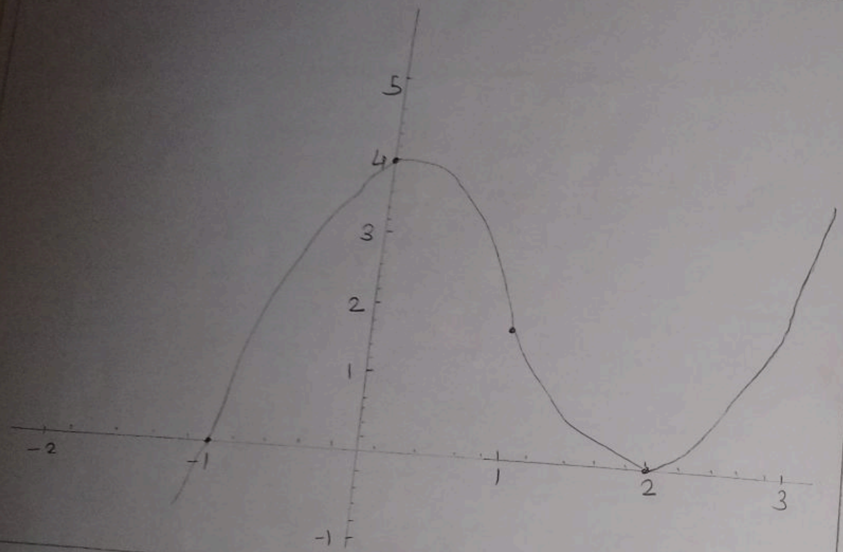
solve(f(x) == 0, x)
Output :- [x == -1, x == 2]

f(0)
Output :- 4

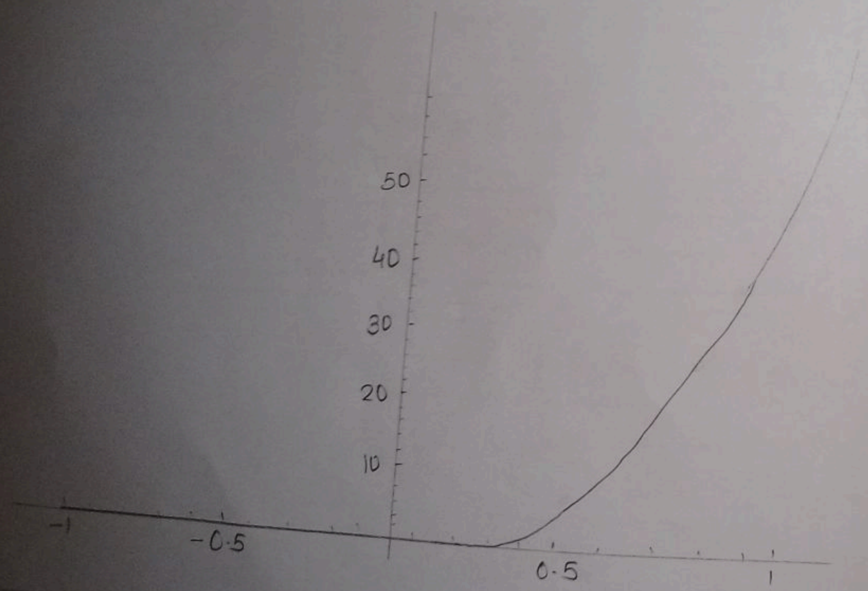
limit (f(x), x = infinity)
Output :- +Infinity

limit (f(x), x = -infinity)
Output :- -Infinity

f(0)
f(1)
f(2)
Output :- 4
2
0



2)

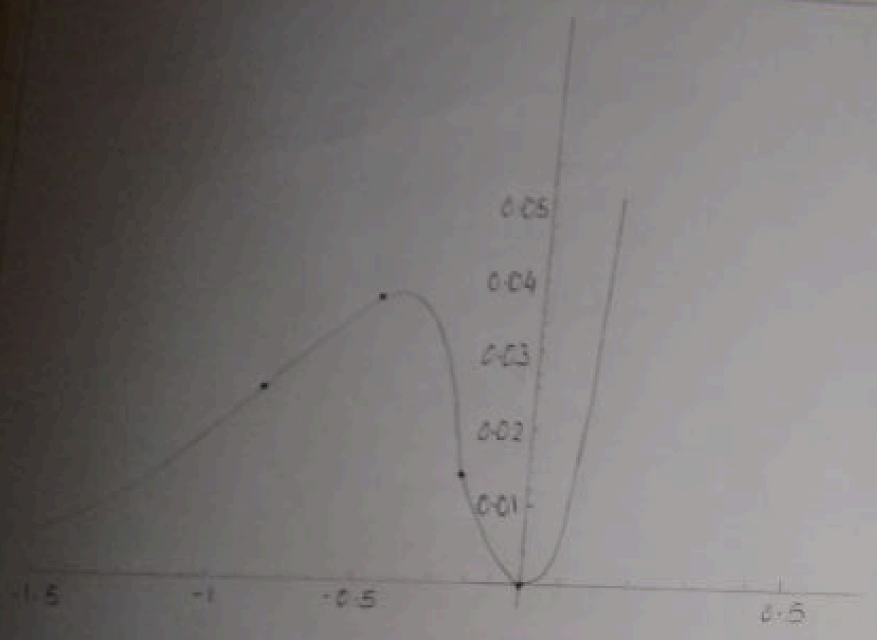



```
f(x) = x^3 - 3*x^2+4
plot(f, xmin = -2, xmax = 3, ymin = -1, ymax = 5)+
point([(-1,0), (0,4), (1,2), (2,0)], color='black',
size=20)
```

2)

```
f(x) = x^2*e^(4*x)
plot(f)

df(x) = derivative(f, x); df
Output:- x |--> 4*x^2*e^(4*x) + 2*x*e^(4*x)

solve(df(x)==0, x)
Output:- [x == (-1/2), x == 0]

df(-.6)
df(-.4)
Output:- 0.0217723087894590
         -0.0323034428791448

df(-.1)
df(.1)
Output:- -0.107251207365702
          0.358031921433905

d2f(x) = derivative(f, x, 2); d2f
Output:- x |--> 16*x^2*e^(4*x) + 16*x*e^(4*x)
         + 2*e^(4*x)
```

```
solve(d2f(x) == 0, x)
Output :- [x == -1/4*sqrt(2) - 1/2, x == 1/4*
          sqrt(2) - 1/2 ]

N(-1/4*sqrt(2) - 1/2)
N(1/4*sqrt(2) - 1/2)
Output :- -0.853553390593274
          -0.146446609406726

d2f(-.9)
d2f(-.8)
Output :- 0.0153012845704839
          -0.0228268342278850

d2f(-.2)
d2f(-.1)
Output :- -0.251624219905644
          0.375379225779958

solve(f(x) == 0, x)
Output :- [x == 0]

f(0)
Output :- 0

limit(f(x), x = infinity); limit(f(x), x = infinity)
Output :- + Infinity
          0
```



f(-0.8536)
f(-.5)
f(-0.1464)
f(0)
Output :- 0.0239692107419376
0.0338338208091532
0.0119332655442418
0

f(x) = x^2*e^(4*x)
plot(f, xmin = 1.5, xmax = .5, ymin = 0, ymax = .05)
+ point([(-.5, 0.0338), (0,0), (-0.8536, 0.0240),
(-0.1464, 0.0119)], color = 'black', size = 20)

$f(x) = x^3 - 3x^2 + 4$
$f'(x) = 3x^2 - 6x$
$f''(x) = 6x - 6$

$f'(x) = 0$
$3x^2 - 6x = 0$
$3x(x-2) = 0$
$x = 0$ or $x = 2$

Interval	(3x)(x-2)	f'(x)	Conclusion
$x < 0$	(-)(-)	+	Increasing on $(-\infty, 0]$
$0 < x < 2$	(+)(-)	-	Decreasing on $[0, 2]$
$x > 2$	(+)(+)	+	Increasing on $[2, +\infty)$

$f''(x) = 0$

$6(x-1) = 0$

$x = 1$

Interval	$6(x-1)$	$f''(x)$	Conclusion
$x < 1$	$(-)$	$(-)$	Concave down on $(-\infty, 1]$
$x > 1$	$(+)$	$(+)$	Concave up on $[1, +\infty)$

Inflection point is 1

The sign of f' changes from + to - at that point i.e $x=0$ so there is a relative maxima at that point. The sign changes from - to + at $x=2$ so there is relative minima at that point

$f(x) = x^3 - 3x^2 + 4$ —— (I)

$f'(x) = 3x^2 - 6x$

$\therefore f'(x) = 0$

$x = 0$ or $x = 2$

$\therefore$ Consider $-1, 0, 2$

Put $x = -1$ in eq (I)

$f(-1) = (-1)^3 - 3(-1)^2 + 4$

$= -1 - 3 + 4$

$= 0$

Now Put $x=0$ in eq. (I)

$f(0) = (0)^3 - 3(0)^2 + 4$

$= 4$

Put $x = 2$ in (I)

$f(2) = (2)^3 - 3(2)^2 + 4$

$= 8 - 3 \times 4 + 4$

$= 8 - 12 + 4$

$= 0$

Absolute maxima is 4 at $x=0$

Absolute minima is 0 at $x=2$ and $x=-1$

Practical - 3

Aim :- Newton's Method.

Input :-

```
f(x) = x^3+2*x-1;     # change this to whatever
                        function you like
df = diff(f, x);       # Sage will compute the
                        derivative of f
NewtonIt(x) = x-(f/df)(x);   # Newtons Iterative
                              Formula which we are
                              calling "NewtonIt"
xn = 1/2;              # initial guess
print(xn);
for i in range(10):
    xn = N(NewtonIt(xn), digits = 20)
    print(xn);
```

OUTPUT :-

```
1/2
0.45454545454545454545
0.45339833667909377689
0.45339765151664779176
0.45339765151640376764
0.45339765151640376764
0.45339765151640376764
0.45339765151640376764
0.45339765151640376764
0.45339765151640376764
0.45339765151640376764
```

$f(x) = x^3 + 2x - 1$

$f'(x) = 3x^2 + 2$

$\therefore x_{n+1} = x_n - \dfrac{(x_n^3 + 2x_n - 1)}{3x_n^2 + 2}$

$\therefore$ Put $x = 0$

$f(0) = (0)^3 + 2(0) - 1$

$= -1$

Put $x = 1$

$f(1) = (1)^3 + 2(1) - 1$

$= 1 + 2 - 1$

$= 2$

$\therefore f(0) \cdot f(1) = -2$

$\therefore x_1 = \frac{1}{2} \approx 0.5$ —— I

Put $x = 1$

$x_{1+1} = x_1 - \dfrac{x_1^3 + 2x_1 - 1}{3x_1^2 + 2}$

$x_2 = 0.5 - \dfrac{[(0.5)^3 + 2(0.5) - 1]}{3(0.5)^2 + 2}$

$= 0.5 - \dfrac{[0.125 + 1 - 1]}{0.75 + 2}$

$= 0.5 - \dfrac{0.125}{2.75}$

$= \dfrac{1.375 - 0.125}{2.75}$

$= \dfrac{1.25}{2.75}$

$x_2 \approx 0.454545$ —— II

Now Put $x = 2$

$x_{2+1} = x_2 - \dfrac{x_2^3 + 2x_2 - 1}{3x_2^2 + 2}$

$x_3 = 0.4545 - \dfrac{[(0.4545)^3 + 2(0.4545) - 1]}{3(0.4545)^2 + 2}$

$= 0.4545 - \dfrac{0.0938 + 0.909 - 1}{0.6197 + 2}$

$= 0.4545 - \dfrac{0.0028}{2.6197}$

$= \dfrac{1.1906 - 0.0028}{2.6197}$

$= \dfrac{1.1878}{2.6197}$

$x_3 \approx 0.4534$ —— III

Put $x = 3$

$x_{3+1} = x_3 - \dfrac{x_3^3 + 2x_3 - 1}{3x_3^2 + 2}$

$x_4 = 0.4534 - \dfrac{[(0.4534)^3 + 2(0.4534) - 1]}{3(0.4534)^2 + 2}$

$= 0.4534 - \dfrac{0.0932 + 0.9068 - 1}{3(0.2055) + 2}$

$$= 0.4534 - \frac{0}{3(0.2055)+2}$$

$x_4 \approx 0.4534$ ——— (IV)

Put $x = 4$

$$x_{4+1} = x_4 - \frac{x_4^3 + 2x_4 - 1}{3x_4^2 + 2}$$

$$x_5 = 0.4534 - \frac{[(0.4534)^3 + 2(0.4534) - 1]}{3(0.4534)^2 + 2}$$

$$= 0.4534 - \frac{0.0932 + 0.9068 - 1}{3(0.2055)+2}$$

$$= 0.4534 - 0$$

$x_5 \approx 0.4534$ ——— (V)

~~Put $x = 3$~~

~~$x_{3+1} = x_3$~~

Practical - 4

Aim :- Integration

1) f(x) = (x + x^2)
f. integral (x)

⇒ x |--> 1/3 * x^3 + 1/2 * x^2

2) f(x) = (3 * x^6 - 2 * x^2 + 7 * x + 1)
f. integral (x)

⇒ x |--> 3/7 * x^7 - 2/3 * x^3 + 7/2 * x^2 + x

3) f(x) = cos(x) / (sin(x))^2
f. integral (x)

⇒ x |--> -1 / sin(x)

4) f(t) = (t^2 - 2 * t^4) / t^4
f. integral (t)

⇒ t |--> -2 * t - 1/t

5) f(x) = (x^2 / (x^2+1))
f. integral (x)

⇒ x |--> x - arctan(x)

6) f(x) = (x^5+2*x^2-1) / x^4
f.integral(x)

⇒ x |--> 1/2*x^2 -1/3*(6*x^2-1) / x^3

7) f(x) = (3*sin(x) - 2*(sec(x)^2))
f.integral(x)

⇒ x |--> -3*cos(x) - 2*tan(x)

8) f(x) = (x+2)
f.integral (x, -1, 2)

⇒ 15/2

9) f(x) = (1 - 1/2*x)
f.integral (x, 0, 2)

⇒ 1

10) f(x) = (sin(x)/5)
f.integral (x, 0, pi/2)

⇒ 1/5

1) $\int (x+x^2)\,dx$

$\int x\,dx + \int x^2\,dx$

$\frac{x^{1+1}}{1+1} + \frac{x^{2+1}}{2+1}$

$\frac{x^2}{2} + \frac{x^3}{3}$

2) $\int (3x^6 - 2x^2 + 7x + 1)\,dx$

$3\int x^6\,dx - 2\int x^2\,dx + 7\int x\,dx + \int 1\,dx$

$\frac{3}{7}x^7 - \frac{2}{3}x^3 + \frac{7}{2}x^2 + x$

3) $\int \frac{\cos x}{\sin^2 x}\,dx$

Let, $t = \sin x$
diff w.r.t. 'x'
$\frac{dt}{dx} = \cos x$
$dt = \cos x\,dx$

$\therefore \int \frac{1}{t^2}\,dt$

$\frac{t^{-2+1}}{-2+1}$

$-t^{-1}$

$\frac{-1}{t}$

$\Rightarrow \frac{-1}{\sin x}$

4) $$\int \frac{t^2 - 2t^4}{t^4}\, dt$$

$$\int \frac{t^2}{t^4}\, dt - \int \frac{2t^4}{t^4}\, dt$$

$$\int \frac{1}{t^2}\, dt - 2\int 1\, dt$$

$$\frac{t^{-2+1}}{-2+1} - 2t$$

$$\frac{-1}{t} - 2t$$

5) $$\int \frac{x^2}{x^2+1}\, dx$$

$$\int \frac{x^2+1-1}{x^2+1}\, dx$$

$$\int \frac{x^2+1}{x^2+1}\, dx - \int \frac{1}{x^2+1}\, dx$$

$$\int 1\, dx - \int \frac{1}{x^2+1}\, dx$$

$$x - \tan^{-1}(x)$$

6) $$\int \frac{x^5 + 2x^2 - 1}{x^4}\, dx$$

$$\int \left(x + \frac{2}{x^2} - \frac{1}{x^4} \right) dx$$

$$\frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3}$$

$$\frac{x^2}{2} - \frac{2x^2}{x^3} + \frac{1}{3x^3}$$

$$\frac{x^2}{2} - \frac{6x^2 - 1}{3x^3}$$

7) $$\int [3 \sin x - 2 \sec^2 x]\, dx$$

$$3\int \sin x\, dx - 2\int \sec^2 x\, dx$$

$$-3 \cos x - 2 \tan x$$

8) $$\int_{-1}^{2} (x+2)\, dx$$

$$\int_{-1}^{2} x\, dx + 2\int_{-1}^{2} 1 \cdot dx$$

$$\left[\frac{x^2}{2}\right]_{-1}^{2} + 2\left[x\right]_{-1}^{2}$$

$$\left[\frac{(2)^2 - (-1)^2}{2}\right] + 2\left[2 - (-1)\right]$$

$$\frac{4-1}{2} + 2[2+1]$$

$$\frac{3}{2} + 2[3] \Rightarrow \frac{3}{2} + 6 \Rightarrow \frac{3+12}{2} \Rightarrow \frac{15}{2}$$

9) $\int_0^2 (1-\frac{1}{2}x)dx$

$\int_0^2 1\,dx - \frac{1}{2}\int_0^2 x\,dx$

$[x]_0^2 - \frac{1}{2}\left[\frac{x^2}{2}\right]_0^2$

$[2-0] - \frac{1}{2}\left[\frac{(2)^2-(0)^2}{2}\right]$

$2 - \frac{1}{2}\left[\frac{4}{2}\right]$

$2 - \frac{4}{4} \Rightarrow 2-1 \Rightarrow 1$

10) $\int_0^{\pi/2} \frac{\sin x}{5}\,dx$

$\frac{1}{5}\int_0^{\pi/2} \sin x\,dx$

$\frac{-1}{5}[\cos x]_0^{\pi/2}$

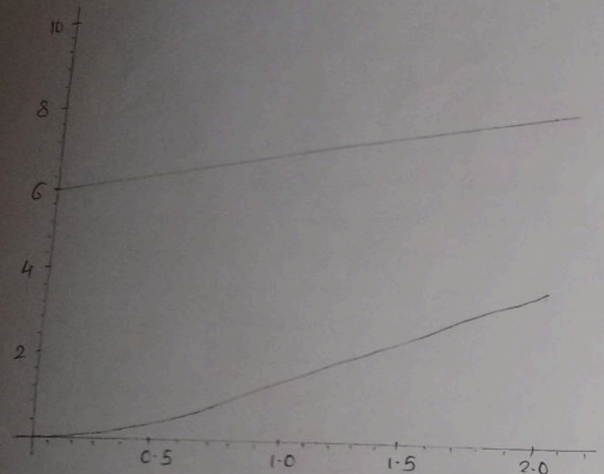
$\frac{-1}{5}[\cos(\pi/2) - \cos(0)]$

$\frac{-1}{5}[0-1]$

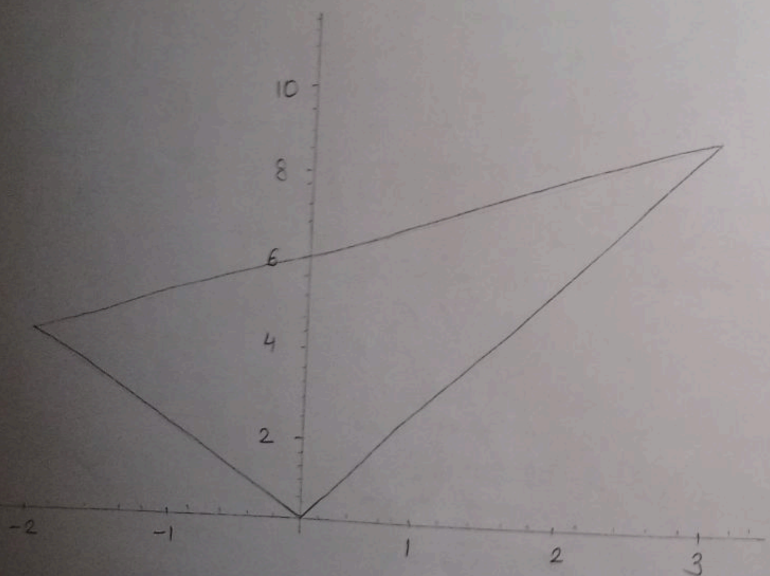
$\frac{-1}{5}(-1)$

$\frac{1}{5}$

a)



c)



Practical - 5

Aim :- Applications of Integration

a) Area between two curves

```
f(x) = x + 6
g(x) = x^2
P = plot(r(x), 0, 2, color='blue') + plot (g(x), 0, 2,
                          color = 'red')
P.show(ymin = 0, ymax = 10)
area = integral ( f(x) = g(x), x, 0, 2)
print ("area", area)
```

b) Length of a plane curve

```
f(x) = x^(3/2)
integral (sqrt (1+ derivative (f, x)^2), x, 1, 2)
```

Output :- 22/27* sqrt(22) - 13/27* sqrt(13)

c)

```
f(x) = x+6
g(x) = x^2
p.plot(f(x), -2, 3, color='blue') + plot (g(x), -2, 3,
      color = 'red')
p.show(ymin = 0, ymax = 10)
area = integral (f(x) - g(x), x, -2, 3)
print( "area", area)
```

1) $f(x) = x+6$; $g(x) = x^2$

$\therefore A = \int_0^2 [(x+6) - x^2] \cdot dx$

$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{2} \right]_0^2$

$= \frac{34}{3} - 0$

$= \frac{34}{3}$

2) $y = x^{3/2}$

Diff w.r.t. 'x',

$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$

$\therefore$ the curve extends from $x=1$, $x=2$

$L = \int_1^2 \sqrt{1+\left(\frac{3}{2} x^{1/2}\right)^2} \cdot dx$

$= \int_1^2 \sqrt{1+\frac{9}{4}x} \cdot dx$

Let, $x = 1 + \frac{9}{4} x$

Diff. w.r.t. 'x',

$\frac{dx}{dx} = \frac{9}{4}$

$dx = \frac{4dx}{9}$

$x = 1$

$x = 1+\frac{9}{4}(1) = 1+\frac{9}{4} = \frac{13}{4}$

$x = 2$

$x = 1+\frac{9}{4}(2) = 1+\frac{18}{4} = \frac{22}{4}$

$\therefore L = \frac{4}{9} \int_{13/4}^{22/4} \sqrt{x} \cdot dx$

$= \frac{4}{9} \frac{\left[x^{3/2}\right]_{13/4}^{22/4}}{\frac{3}{2}}$

$= \frac{8}{27} \left[\left(\frac{22}{4}\right)^{3/2} - \left(\frac{13}{4}\right)^{3/2} \right]$

$= \frac{8}{27} \left[\frac{22}{4} \times \frac{\sqrt{22}}{2} - \frac{13}{4} \times \frac{\sqrt{13}}{2} \right]$

$= \frac{8}{27} \left[\frac{22\sqrt{22}}{8} - \frac{13\sqrt{13}}{8} \right]$

$= \frac{8}{27} \left[\frac{22\sqrt{22} - 13\sqrt{13}}{8} \right]$

$$= \frac{22\sqrt{22} - 13\sqrt{13}}{27}$$

3) $f(x) = x + 6$; $g(x) = x^2$

$$A = \int_{-2}^{3} [f(x) - g(x)] \cdot dx$$

$$= \int_{-2}^{3} [x + 6 - x^2] \cdot dx$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3} \right]_{-2}^{3}$$

$$= \frac{27}{2} - \left(\frac{-22}{3} \right)$$

$$= \frac{125}{6}$$