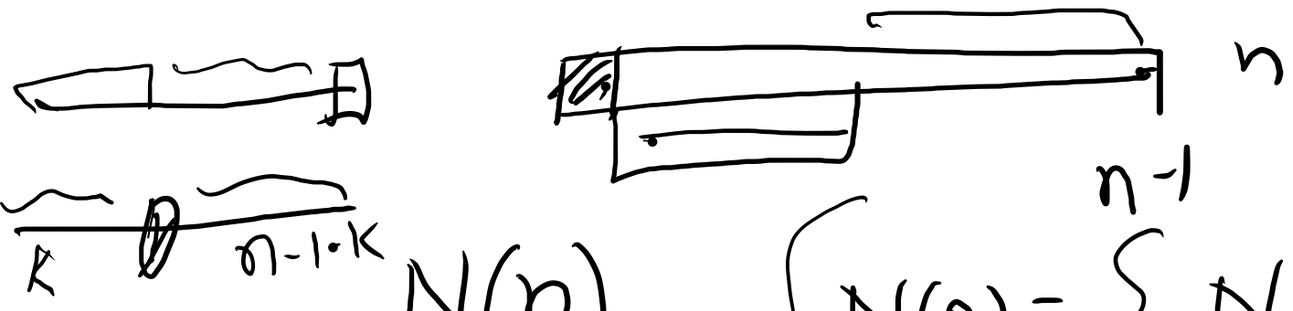


$N(n)$

$N(0) = 1$

$N(1) = 1$

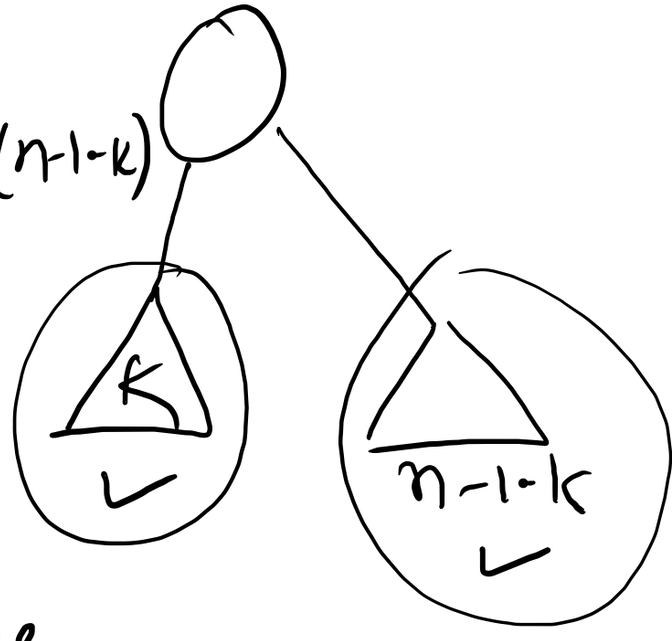
$N(2) = 2$

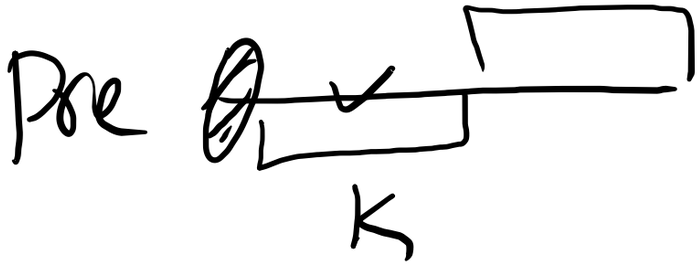
$N(3) = 5$

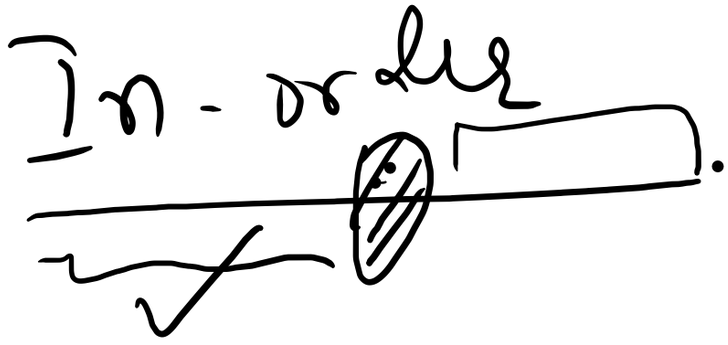
$$N(n) = \sum_{k=0}^{n-1} N(k) \times N(n-1-k)$$

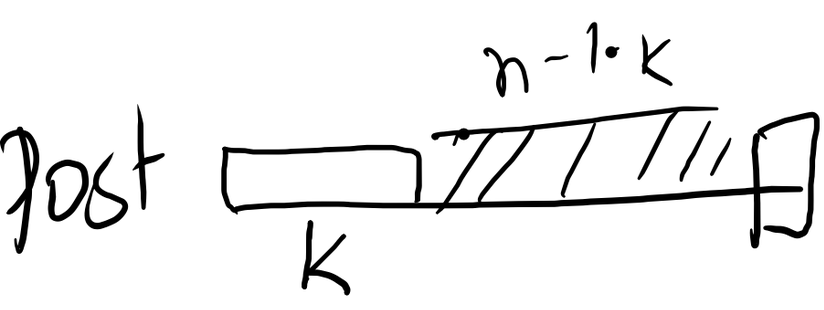
$$= \frac{1}{n+1}\binom{2n}{n}$$

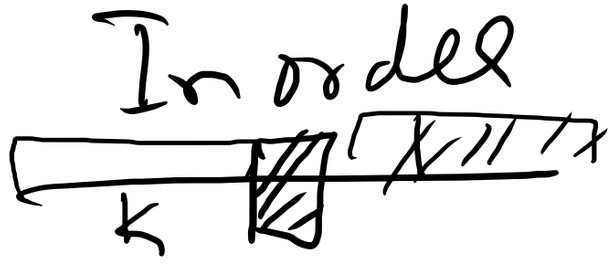
(Catalan numbers.

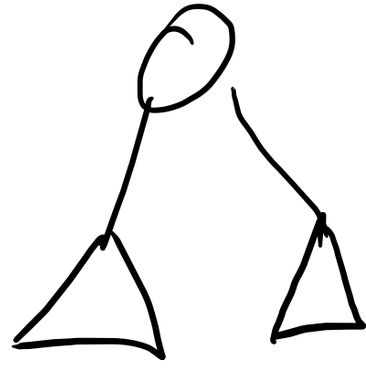



Pre
k


In-order


n-1-k
Post
k

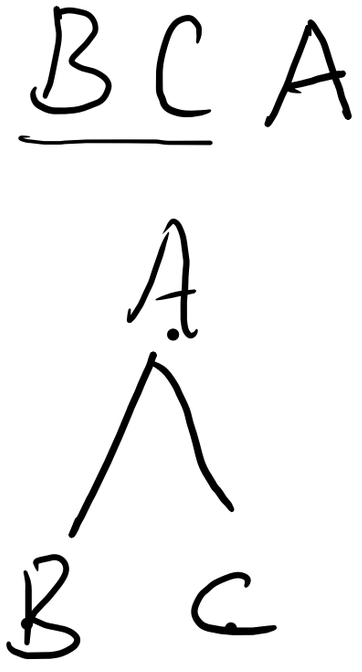

Inorder
k




Pre

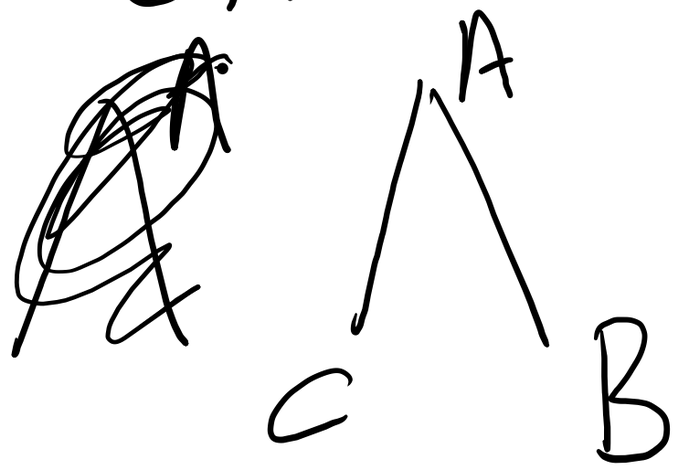

Post.

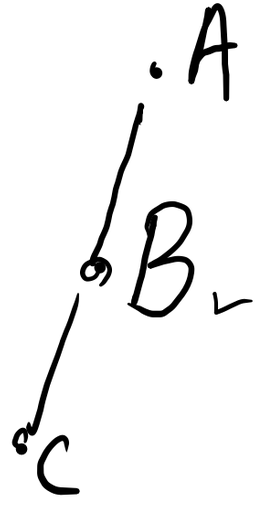
Рес.

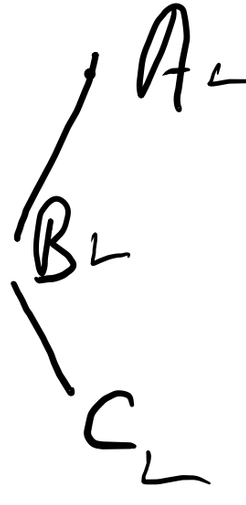


A B C

C B A







$h+1 \le n \le 2^{h+1}-1$

$\log(n+1)-1 \le h \le n-1$

fix n

max h

$h \le n-1$

n h

fix h

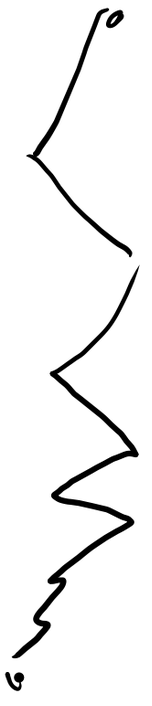
max n.

$h \le n-1$

$n+1 \le 2^{h+1}$

$n+1 = 2^{h+1}$

degree

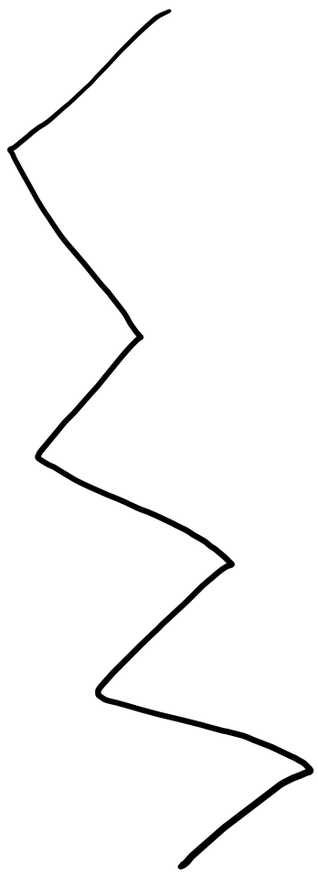


$$n+1 = 2^{h+1}$$

h is $\Theta(\log n)$.

Balanced BT

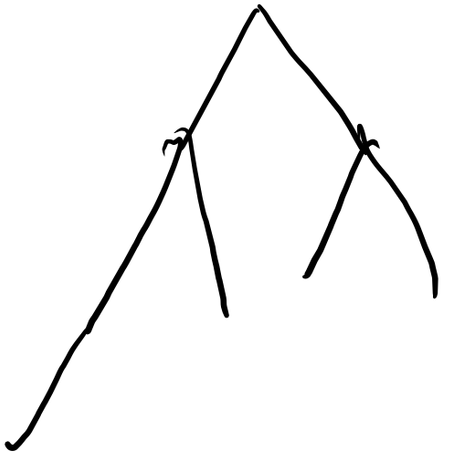
CBT BBT



h

n

C BT



$$n \geq \sum_{i=0}^{h-1} 2^i = 2^h - 1 + 1 = 2^h$$

$$h < \log n$$

$$h > (\log n + 1) - 1$$

$$h \text{ is } \theta(\log n)$$

$$N(n) = \sum_{k=0}^{n-1} N(k) \times N(n-1-k)$$

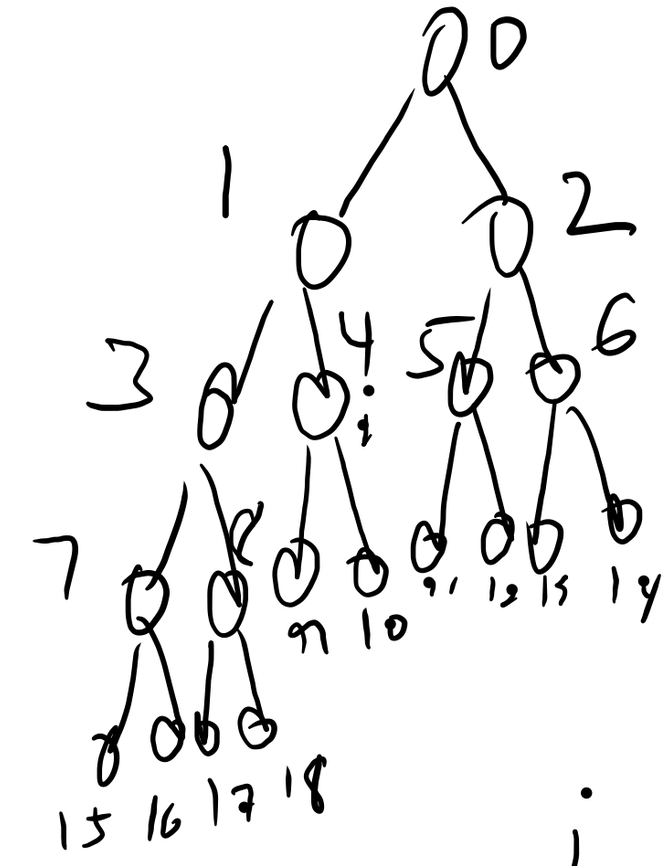
$$> N(0)N(n-1) + N(n-1) \times N(0)$$

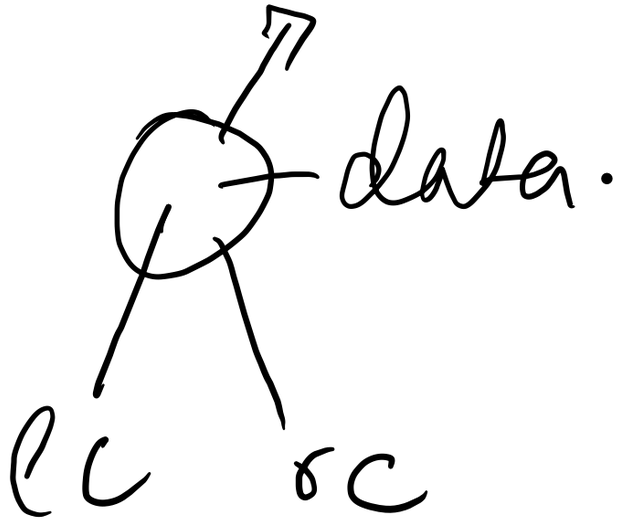
$$> 2N(n-1)$$

$$> 2^2 N(n-2)$$

$$> 2^k N(n-k) > 2^{n-1} N(1) > 2^{n-1}$$

$n \geq 2$

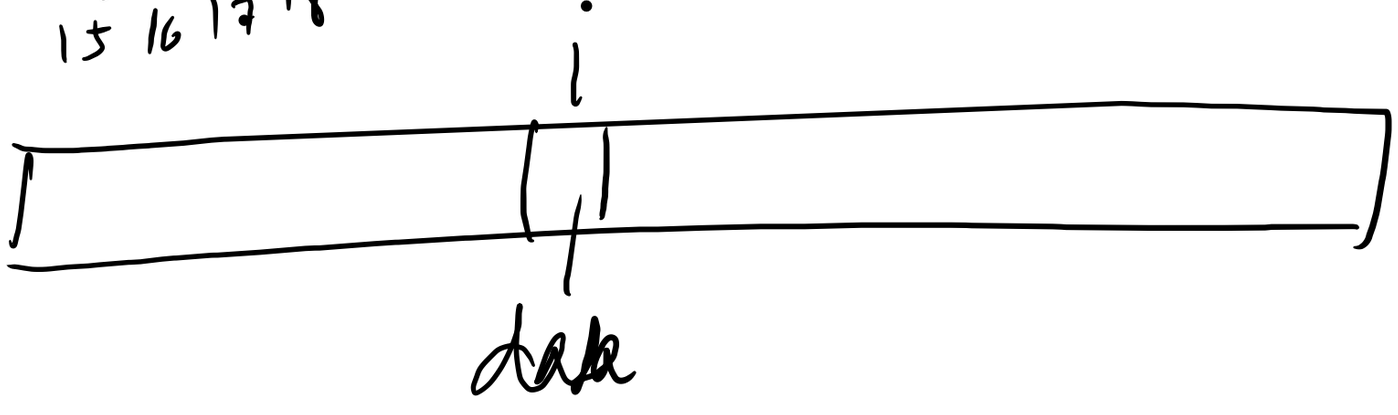




$$L(i) = 2i+1$$

$$R(i) = 2i+2$$

$$P(i) = \frac{i-1}{2}$$

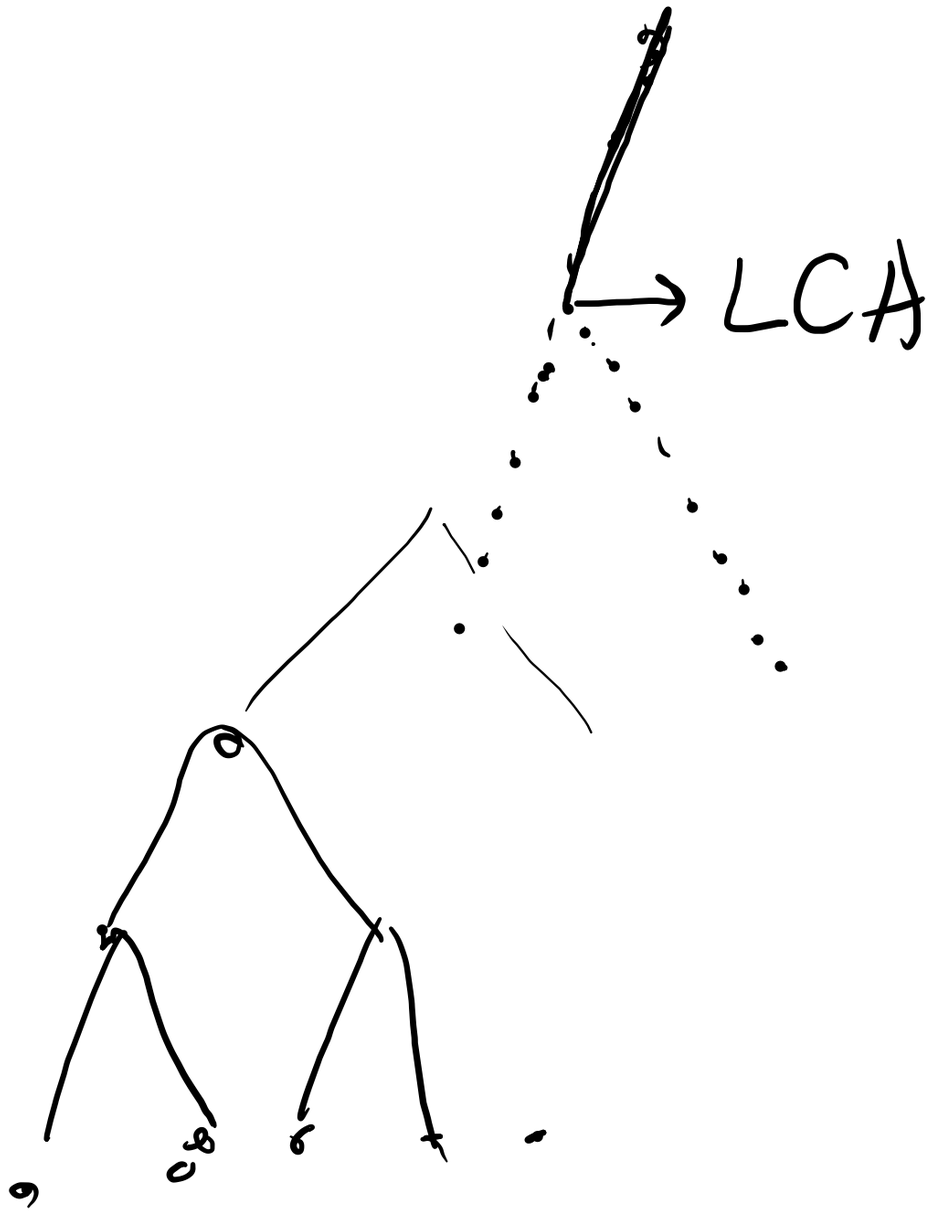


$n = 10^{18}$. CBI

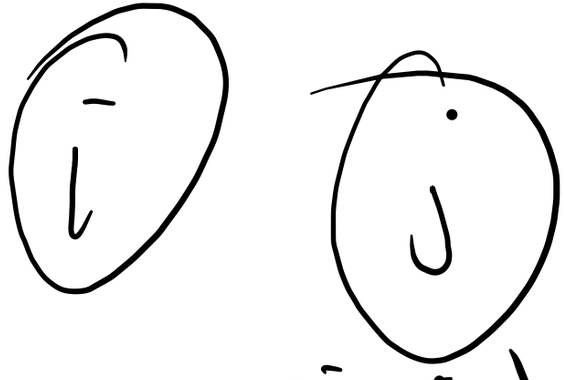
Level (i)

$\theta(\log n)$.

```
l = 0
while (i > 0)
    l++;
    i = i - 1
          2
```



CBT → n



LCA(i, j)

$\theta(\log n)$ $\theta(1)$

Space