$T(n) = $ The no of additions required to compute $F(n)$

$T(n) \in O(2^n)$

$T(2) = 1.$

$F(0)$

$F(1)$

$T(n) = T(n-1) + T(n-2) + 1$

$T(n-1) = T(n-2) + T(n-3) + 1$

$\geq T(n-2) + 1$

$n - k = 2$

$k = n - 2$

$\leq T(n-1) + T(n-1)$

$= 2T(n-1) \leq 2 \cdot 2T(n-2)$

$\leq 2 \cdot 2 \cdot 2T(n-3)$

$\leq 2^k T(n-k) \leq 2^{n-2}$

$T(100) \leq 1/4 \, 2^{100}$

$\leq 1/4 \times 10^{30}$

$$T(n) = T(n-1) + T(n-2) + 1$$

$$T(2) = 1$$

$$> T(n-1) + T(n-2)$$

$$n - 2k = 2$$

$$> T(n-2) + T(n-2) = 2T(n-2)$$

$$2k = n-2$$

$$k = \frac{n-2}{2} = \frac{n}{2} - 1$$

$$> 2 \cdot 2 \cdot T(n-4)$$

$$> 2 \cdot 2 \cdot 2 \cdot T(n-6)$$

$$> 2^k T(n-2k) > \frac{1}{2} 2^{n/2}$$

$$T(100) > \frac{1}{2} 2^{50}$$

$$> \frac{1}{2} 10^{15}$$

$$\frac{1}{2} \boxed{10^5 \text{ sec}}$$

$$\frac{1}{2} 2^{n/2} < T(n) < \frac{1}{4} 2^{n}$$

$T(n)$ is $O(2^n)$

$T(n)$ is $\Omega(2^{n/2})$

$S(n)$ = Amount of space required to compute $F(n)$

$F(n-1)$ $F(n-2)$

$$S(n) = S(n-1) + 1$$
$$= S(n-2) + 1 + 1$$
$$= S(n-k) + k$$
$$= n + S(0)$$

$$S(n) = \Theta(n).$$

$(a,b) = (c,d)$

$a = b$

$b = d$

$$\underline{0 \le F(n) \le 9}.$$

$10$

$(a,b)$

$|A| = 10$

$10 \quad 10$

$F(0), F(1)$

$F(1), F(2)$

$F(2), F(3) \to i$

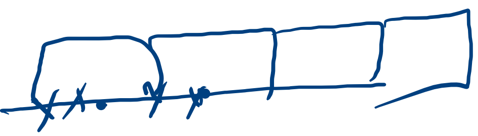
$A \times A = \{(a,b) \mid a, b \in A\}$

$F(0)$

$F(1)$

$\vdots$

$F(10)$



$\to j$

$|A \times A| = 10 \times 10$

$(F(i), F(i+1)) = (F(j), F(j+1))$

$F(i) = F(j)$

$F(i+1) = F(j+1)$

$F(100), F(101)$

Space $(\log n)$

Time.

$\underline{\theta(m)} + \log n + \theta(m)$ q

$\theta(m+\log n)$

$\theta(q(m+\log n))$

$\underline{\theta(m + \log n) \text{ space.}}$

$\underline{m = 10^9}$.

$m \le 10^6$

$F(n) = F(n-1) + F(n-2) \;\%\; m$

$n_1$

$n_2$

$\vdots$

$n_q$

Space $\underline{\theta(m+\log n)}$ ✓

Time. $\underline{\theta(m+q\log n)}$ ✓

$q = 10^4$

$\log n = 100$

$m = 10^9$

$F(0) \ldots F(1)$

$$\boxed{\begin{array}{c}\theta(\log n) \\ \theta(\log^2 n)\end{array}}$$

$p < 6m$

$F(n) = F(n-1) + F(n-2)\ \%\ m$

$F(n) = F(n-1) + 2F(n-3)\ \%\ m$

$n = 2K+1$

$x^n$

$n/2 = K$

$y = x$

$(x^2)^K$

$= x^{2K}$

i → 0 to n

$y = y \times x$

$O(n)$

$n = 2K$ $\quad x^n = x^{2K}$

$= (x^2)^K$

$O(\log n)$.

Power(x, n)

if (n == 0) return 1

else if (n % 2 == 0)

→ return Power($x^2$, $n/2$)

else return

→ $x \times$ Power($x^2$, $n/2$)

$T(n)$ = the number of multiplications done by pow function to compute $x^n$

$T(n)$ is $\Theta(\log n)$.

$$T(n) \geq 1 + T(n/2)$$
$$\geq 1 + 1 + T(n/4)$$
$$\geq k + T(n/2^k)$$
$$\geq \log n$$

$$T(n) \leq 2 + T(n/2)$$
$$T(n) \leq 2k + T(n/2^k)$$
$$\leq 2\log n$$

$$\log n \leq T(n) \leq 2\log n$$