

# An Overview of Adaptive Designs

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# Introduction

- ▶ Response-adaptive designs, or response-adaptive randomization procedures, are designs that **change allocation away** from 50/50 based on responses observed so far in the trial.
- ▶ The desired allocation proportion is usually motivated by an **ethical consideration of assigning more patients to better treatments**.
- ▶ In this presentation, we discuss several classes of response-adaptive designs and the advantages and disadvantages of these designs, and the analysis of clinical trials based on adaptive designs.

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# Different Families of Response Adaptive Designs

- ▶ The early ideas of adaptive designs can be traced back to Thompson and Robbins. Since then, two main families of response-adaptive designs have been proposed :-
- ▶ **Design-driven**, where designs are driven by intuition and are **not optimal** in a formal sense.
- ▶ Few examples of these type of designs are (**Urn Models**) :-
  1. Play-the-winner rule.
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# Notations and Basic Assumptions

- ▶ Let  $p_A$  be the probability of a success on treatment  $A$  and let  $p_B$  be the success probability of treatment  $B$ , with  $q_A = 1 - p_A$  and  $q_B = 1 - p_B$ .
- ▶ In addition, let  $n$  be the total number of patients in a trial, let  $n_A$  be the number of patients assigned to treatment  $A$ , and let  $n_B$  be the number of patients assigned to  $B$ , so that  $n_A + n_B = n$ .
- ▶ We here assume that the outcome can be observed relatively quickly.
- ▶ Here we consider the simplest situation where two independent treatments “A” and “B” with binary outcomes (Success or failure) are compared in the course of a trial.

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# Urn Models

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# Play-The-Winner Rule

- ▶ This is one of the first response-adaptive designs, proposed by Zelen.
- ▶ The first patient is equally likely to receive one of the two treatments.
- ▶ The subsequent patient is assigned to the **same** treatment following a **successful** outcome, and to the **opposite** treatment following a **failure**.
- ▶ On average, the proportion of patients  $\frac{n_A}{n}$  assigned to treatment A is  $\frac{q_B}{q_A + q_B}$ .



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# Play-The-Winner Rule

- ▶ We simulate using  $R$   $10^3$  samples following Play-The-Winner Procedure where we consider  $p_A = 0.3$  and  $p_B = 0.6$ .
- ▶ Since, here we have taken  $p_A = 0.3, p_B = 0.6$ , the simulated allocation proportion for treatment A must converge to  $\frac{q_B}{q_A + q_B} = \frac{0.4}{1.1} = \frac{4}{11} \approx 0.364$ .
- ▶ We try to validate the above statements using simulation and plot the allocation proportions as a function of increasing sample size  $n$ .

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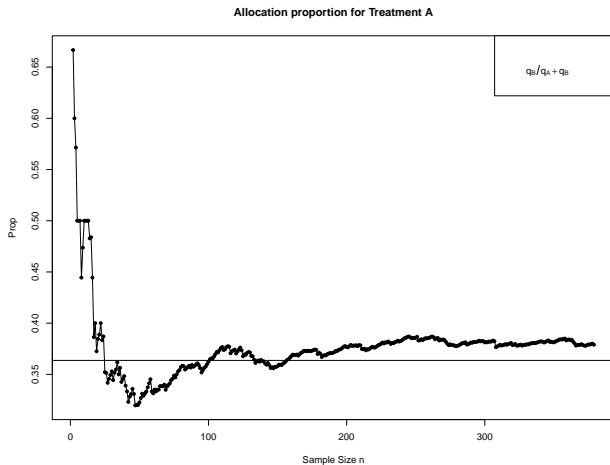


Figure: Plot of  $n_A/n$  as a function of  $n$

# Randomized play-the-winner Rule

- ▶ This rule was suggested by Wei and Durham and Wei.
- ▶ In this rule, the procedure is generally started with an urn containing one ball of each type corresponding to the two treatments.
- ▶ When a patient arrives, a ball is drawn from the urn and then replaced.
- ▶ The patient receives corresponding treatment.
- ▶ If the outcome is a **success**, **one ball of that type** is added to the urn.
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- ▶ This design has the **same limiting allocation proportion** as the play-the-winner rule i.e  $\frac{n_A}{n} \xrightarrow{P} \frac{q_B}{q_A+q_B}$ .
- ▶ We simulate for the parameter values  $p_A = 0.2, p_B = 0.4$ .
- ▶ So from theoritical perspective, the observed allocation proportion for Treatment A must get closer to  $\frac{q_B}{q_A+q_B} = \frac{0.6}{1.4} \approx 0.429$  and for Treatment B, closer to  $\frac{q_A}{q_A+q_B} \approx 0.571$ .

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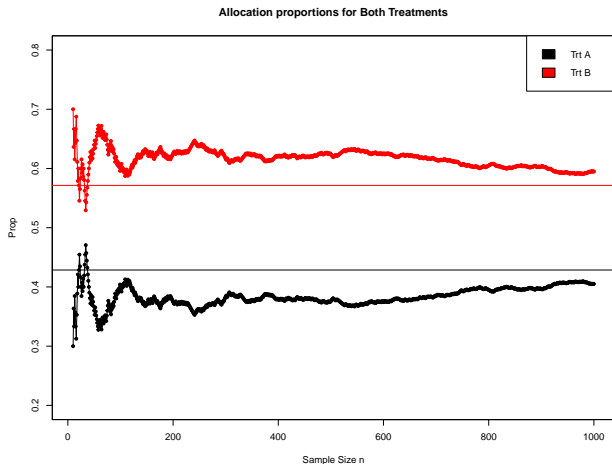


Figure:  $n_A/n$  and  $n_B/n$  as a function of  $n$

# Drop-The-Loser Rule

- ▶ According to this rule, the urn starts with **one** ball of **each treatment type** and **one** ball of the so-called **immigration type**.
- ▶ When a patient arrives to be randomized to a treatment, a ball is taken from the urn.
- ▶ If it is an immigration ball, the ball is replaced and two additional balls (one of each treatment type) are added to the urn.
- ▶ If it is an actual treatment ball, a patient is assigned to corresponding treatment and response is observed.

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- ▶ If response is a **success**, the ball is **returned** to the urn (hence the urn composition remains unchanged).
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# Advantages of Designs Based on Urn Models

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2. Because of early decision and greater flexibility it can reduce the potential cost.
3. Urn models are intuitive and easy to implement in clinical trials.
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# Neyman Allocation

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- ▶ Another important goal is to maximize the power of treatment comparison.
- ▶ These two goals usually compete with each other.
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$$\frac{n_A}{n} = \frac{\sqrt{p_A q_A}}{\sqrt{p_A q_A} + \sqrt{p_B q_B}}$$

- ▶ An interesting fact about the **Neymann Allocation** scheme is that for some values of  $p_A$  and  $p_B$  it will assign more patients to the inferior treatment. Since, then

$$\frac{n_A}{n_B} = \frac{\sqrt{p_A q_A}}{\sqrt{p_B q_B}} > 1 \Rightarrow p_A > p_B \forall p_A, p_B \in (0, 1)$$

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# Target-Based Response-Adaptive Designs

- ▶ Because optimal allocation usually **depends on** some **unknown parameters**, we **cannot implement** it in practice, unless we **estimate** the **unknown parameters sequentially**.
- ▶ In this section, we introduce two allocation procedures that use **sequential estimation of unknown parameters**.
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# Sequential Maximum Likelihood Procedure

- ▶ To start, allocate  $n_0 \geq 2$  patients to both treatments  $A$  and  $B$ .
- ▶ Assume that we have assigned  $j$  ( $j \geq 2n_0$ ) patients in the trial and their responses have been observed.
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- ▶ Then we allocate the  $(j+1)^{th}$  patient to treatment  $A$  with probability :-

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# Allocation Proportion in SML Procedure

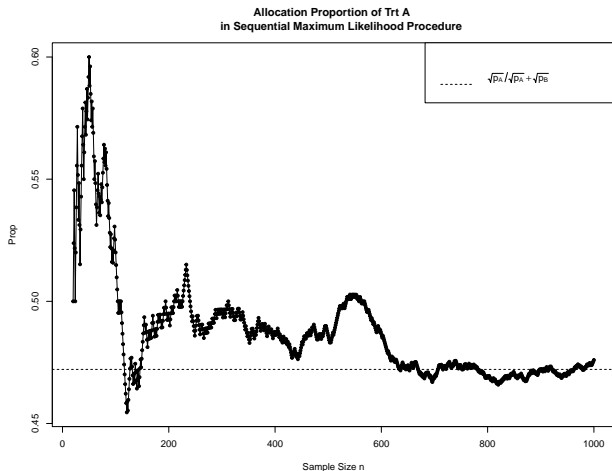


Figure:  $n_A/n$  as a function of  $n$

# Doubly Adaptive Biased Coin Design

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$$g(x, y) = \frac{y(y/x)^\gamma}{y(y/x)^\gamma + (1-y)(1-y/1-x)^\gamma} \text{ where } \gamma \geq 0$$

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At the extreme, when  $\gamma = 0$ , we have the sequential maximum likelihood procedure, which has maximum variability :-

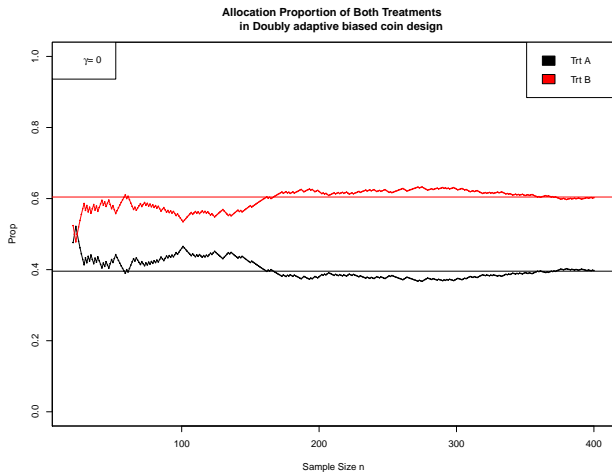


Figure:  $p_A = 0.3, p_B = 0.7, \gamma = 0$

As  $\gamma$  increases, the procedure becomes more deterministic resulting in a reduction of variance using simulation this can be shown :-

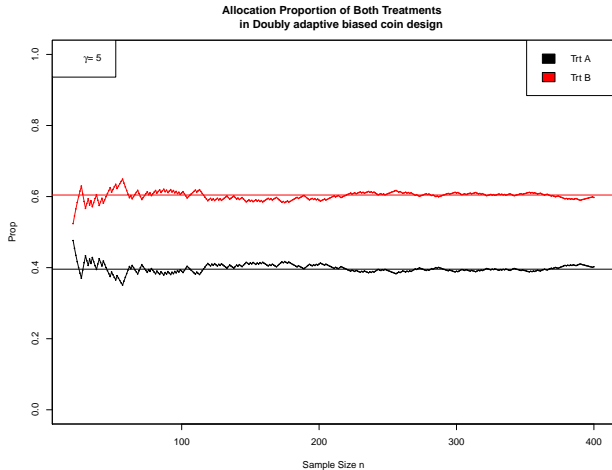


Figure:  $p_A = 0.3, p_B = 0.7, \gamma = 5$



For quite large values of  $\gamma (= 10)$ , the variability further decreases :-

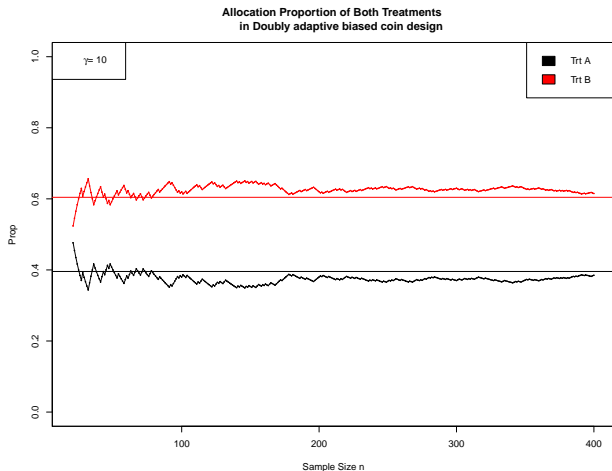


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Smaller the difference between  $p_A$  and  $p_B$  , larger is the variability of the procedure as we can notice it even doesn't seem to converge to the desired limits :-

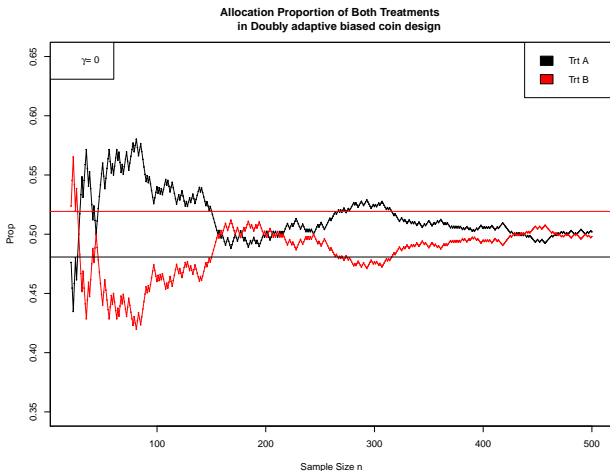


Figure:  $p_A = 0.6, p_B = 0.7, \gamma = 10$

So to stabilize the procedure, we require even larger values of  $\gamma$ , say  $\gamma = 20$  as we can see in the simulated plot :-

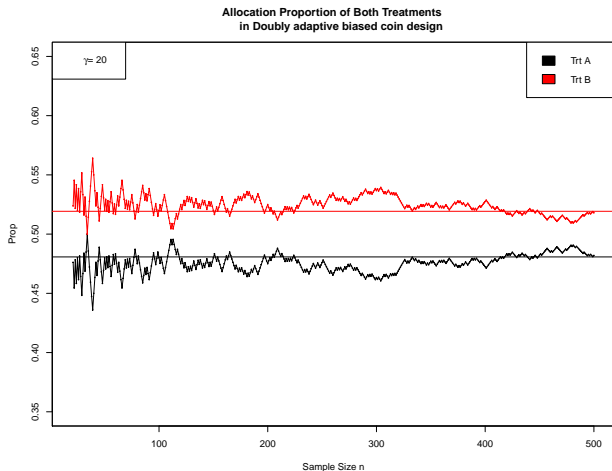


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# Variability and Power of Using Adaptive Designs

- ▶ When using response-adaptive designs in clinical trials, the number of patients assigned to each treatments ( $n_A$  &  $n_B$ ) are random variables.
- ▶ Therefore the power  $P$  (rejecting  $\mathcal{H}_0 | \mathcal{H}_0$  false) (for most testing hypotheses and alternatives) is also a random variable. Here we generally consider hypothesis like :-

$$\mathcal{H}_0 : p_A = p_B$$

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- ▶ Adaptive designs have been implemented in several clinical trials such as :-
  - ▶ A trial of crystalloid preload in hypotension for cesarean patients.
  - ▶ A trial of fluoxetine vs. placebo in depression patients.
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- ▶ Adaptive designs have also been used extensively in adaptive learning algorithms, game theory, as well as dynamical systems.

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