An Overview of PPAS Sampling Scheme

Rajdeep Datta
Niranjan Dey

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Contents

Introduction

- Firstly, we have n samples $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ of the Paired variable (X, Y) from a Population of Size N.
- The ratio estimator is :

$$\widehat{R} = \frac{\overline{y}}{\overline{x}}$$

where, $\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

- Now, Ratio estimate of population total Y is :

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where $X = \sum_{i=1}^{N} X_i$=Population Total of X-values.

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Why PPAS?

- We are here to **find a Probability allocation** by which we can Unbaisedly Estimate $R = \frac{Y}{X} = \frac{\sum_{i=1}^{N} Y_i}{\sum_{i=1}^{N} X_i}$ by $\widehat{R}$.
- So, we will change P =Probability allocation in the Sampling Design (U, S, P), where U =Population, S =Sample Space.
- **Methods of Probability allocation** we already know is:
- **SRSWOR:** This is the most simple method by which we allocate equal probabilities to all the samples i.e. equally likely allotment.

$$Cov(\widehat{R}, \bar{x}) = E(\widehat{R}\bar{x}) - E(\widehat{R})E(\bar{x})$$
$$= E(\frac{\bar{y}}{\bar{x}}\bar{x}) - E(\widehat{R})\bar{X} = \bar{Y} - E(\widehat{R})\bar{X}$$

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- So, we have

$$E(\widehat{R}) = \frac{\overline{Y}}{\overline{X}} - \frac{Cov(\widehat{R}, \bar{x})}{\overline{X}}$$
$$= R - \frac{Cov(\widehat{R}, \bar{x})}{\overline{X}}$$

- which is not unbiased. To make it unbaised we have Hartly-Ross estimator:

$$\widehat{R}_{UE} = \overline{r} - \widehat{B(\overline{r})} = \overline{r} + \frac{\hat{Cov}(r_j, x_j)}{\overline{X}}$$
$$= \overline{r} + \frac{n}{n-1}(N-1)(\overline{y} - \overline{rx})$$

where $\overline{r} = \frac{1}{n}\sum_{i=1}^{n} r_i = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{x_i}, r_j = \frac{y_j}{x_j}$

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Why PPAS?

- **PPS:** In this method we give more probability to the unit that has greater size. So it is called Probability Proportional to Size (PPS) sampling scheme.
  For PPSWR the expressions for $V(\widehat{X}), V(\widehat{Y}), cov(\widehat{X}, \widehat{Y})$are:
  $$V(\widehat{X}) = \frac{1}{n}(\sum_{i=1}^{N} \frac{X_i^2}{P_i} - X^2), V(\widehat{Y}) = \frac{1}{n}(\sum_{i=1}^{N} \frac{Y_i^2}{P_i} - Y^2)$$
  $$cov(\widehat{X}, \widehat{Y}) = \frac{1}{n}(\sum_{i=1}^{N} \frac{X_i Y_i}{P_i} - XY)$$
  Where, P_i =Inclution probability of ith population unit.

Why PPAS?

- Which is again not unbaised. Expression of bias of $\widehat{R}$ is:

$$B(\widehat{R}) = \frac{1}{\overline{\overline{X}}}(RV(\widehat{X}) - Cov(\widehat{X}, \widehat{Y}))$$
$$= \frac{1}{nX^2}(\sum_{i=1}^{N} \frac{X_i}{P_i}(RX_i - Y_i))$$

- **PPAS**: Now we shall see in Probability Proportional to Aggregative Size (PPAS) Sampling the probability of selecting a sample will be proportional to the aggregative size (total size). This method will make $\widehat{R} = \frac{\overline{y}}{\overline{x}}$ to be unbaised for $R = \frac{Y}{X}$ i.e. $E_{PPAS}(\widehat{R}) = R$

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Method of PPAS Sampling

- Now, we will see what is the method PPAS Sampling.
- Let us denote the probability alloted to a sample 's' to be $P(s)$, where $s \in S$ is the sample space.
- Then the sampling design is given by $(s, P(s), s \in S)$
- Now, the ratio estimator is given by $\widehat{Y_R} = \widehat{R_S}X$,where $\hat{R}_S = \frac{\hat{Y}_s}{\hat{X}_s}$.
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Expression of $\hat{X}_s$

- *E*xpression of $\hat{X}_s$ (sample estimate of aggregate size):

$$E(\hat{X}_s) = X = \sum_{i=1}^{N} X_i$$

$$= N\overline{X} = N.E(\overline{x})$$

$$\Rightarrow \widehat{X_s} = N\overline{x} = N(\frac{1}{n}\sum_{i=1}^{n} x_i)$$

- Simillarly for $\widehat{Y_s}$.

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Method of PPAS Sampling

- Now, $E_P(\hat{R}_S) \neq R$ i.e. it is a biased estimator.
- To get an unbiased estimator, we modify the sampling design as (s, P(s), s∈S) where $P'(s) \propto P(s)\hat{X}_s$
- This sampling design is called Probability proportional to aggregative size (PPAS) sampling design.
- Now, $\sum_{s \in S} P'(s) = 1$
  $\implies \sum_{s \in S} kP(s)\hat{X}_s = 1$,$k$ being the constant of proportionality
  $\implies k\sum_{s \in S} P(s)\hat{X}_s = 1$
  $\implies kE_P(\hat{X}) = 1$
  $\implies kX = 1$
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P'(s) &= \frac{1}{X} P(s)\hat{X}_s \\
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&= \frac{\sum_{i=1}^{n} x_i}{X.\binom{N-1}{n-1}}
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- This is the changed inclusion probability of the sample s in sample space S under **PPAS** sampling scheme.

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Method of PPAS Sampling

- Under this new sampling design,
  $E_{P'}(\hat{R}_s)$
  $= \sum_{s \in S} \hat{R}_s P'(s)$
  $= \sum_{s \in S} \frac{\hat{Y}_s}{\hat{X}_s} . \frac{1}{X} . P(s) . \hat{X}(s)$
  $= \frac{1}{X} . Y$
  $= R$ for all R
- Thus, $\hat{R}_s$becomes unbiased for R under P'.
- So, finally we have an method of probability allocation by which $\hat{R}_s$becomes unbiased for R.

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\begin{aligned}
E_{P'}(\hat{Y}_R) &= E_{P'}(\hat{R}_S X) \\
&= \sum_{s \in S} X \frac{\sum y_i}{\sum x_i} \cdot \frac{\sum x_i}{X.\binom{N-1}{n-1}} \\
&= RX \\
&= Y \text{ for all } Y
\end{aligned}
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- Hence, $\hat{Y}_R$beomes unbiased for Y under P'.

Sampling Method of PPAS Samples

- In PPAS, first unit is selected using PPS, and the remaining $n-1$ units are selected using simple random sampling without replacement from the remaining $N-1$ population units.
- Now, we will see how to perform PPAS Sampling.
- **Midzuno's Scheme:**
  - Draw one unit from N units of the population U such that the initial probability is $P_i = \frac{X_i}{X}$ i.e. using PPS sampling.
  - Select remaining $n-1$ units from the remaining $N-1$ units by SRSWOR scheme.
- **Lahiri's Method:**
  - Take M $\geq$ max$\{\sum_{j \in s} x_j : s \in S\}$ (sum of the n largest values of x)
  - Select a sample of size n by SRSWOR. Let s is the selected sample.
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Sampling Method of PPAS Samples

- In PPAS, first unit is selected using PPS, and the remaining $n-1$ units are selected using simple random sampling without replacement from the remaining $N-1$ population units.
- Now, we will see how to perform PPAS Sampling.
- **Midzuno's Scheme:**
  - Draw one unit from N units of the population U such that the initial probability is $P_i = \frac{X_i}{X}$ i.e. using PPS sampling.
  - Select remaining $n-1$ units from the remaining $N-1$ units by SRSWOR scheme.
- **Lahiri's Method:**
  - Take M $\geq$ max$\{\sum_{j \in s} x_j : s \in S\}$ (sum of the n largest values of x)
  - Select a sample of size n by SRSWOR. Let s is the selected sample.
  - Now, draw a random number R from 1 to M.
    - If $R \leq \sum_{j \in s} x_j$,then select the sample.
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