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On Combination of Ratio and PPS Estimators

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Abstract

Agarwal and Kumar (1980) proposed an estimator, combining ratio and *pps* estimators of population mean and proved that the proposed estimator would always be better (in minimum mean square error sense) than the *pps* estimator or the ratio estimator under *pps* sampling scheme for optimum value of constant k (parameter). The optimum value of k is rarely known in practice, hence the alternative is to replace k from the sample-values. In this paper, an estimator depending on estimated optimum value of k based on sample-values, under *pps* sampling scheme is proposed and studied.

Key words: Ratio estimator, PPS estimator.

1. Introduction

For the ith unit ($i=1, 2, \dots, N$) of the population of size N, let y_i, x_{1i} and x_{2i} be the values of the characters y (under study), x_1 and x_2 respectively; y be the population total of y, X_1 and X_2 be the population totals of the two auxiliary characters x_1 and x_2 respectively. Also let

$$u_i=\frac{y_i}{Np_{1i}}; \quad v_i=\frac{x_{2i}}{Np_{1i}}; \quad p_{1i}=\frac{x_{1i}}{X_1}$$

$$\bar{u}_n=n^{-1}\sum_{i=1}^{n} u_i=\bar{y}_{1pps}$$

$$\bar{v}_n=n^{-1}\sum_{i=1}^{n} v_i=\bar{x}_{2pps}$$

$$\sigma_u^2=\sum_{i=1}^{N} p_{1i}(u_i-\bar{Y})^2$$

$$\sigma_v^2=\sum_{i=1}^{N} p_{1i}(v_i-\bar{X}_2)^2$$

$$\bar{y}_R=\frac{\bar{u}_n}{\bar{v}_n}\bar{X}_2; \quad C_u=\frac{\sigma u}{\bar{Y}}; \quad C_v=\frac{\sigma v}{\bar{X}_2}$$

$$\text{MSE}\,(\bar{y}_R)=n^{-1}\bar{Y}^2\,(C_u^2+C_v^2-2\varphi_{uv}C_uC_v)$$

$$\varphi_{uv}=\sum_{i=1}^{N}\frac{p_{1i}\,(u_i-\bar{Y})\,(v_i-\bar{X}_2)}{\sigma_u\sigma_v}$$

Using information on two auxiliary variables, AGARWAL and KUMAR (1980) proposed the following estimator combining ratio and pps estimators for estimating the population mean ($\bar{Y}$):

$$T_0=k\bar{y}_R+(1-k)\,\bar{y}_{1pps} \tag{1.1}$$

where, k is a constant to be determined so that mean square error of T_0 is minimum.

The value of k which minimises mean square error (MSE) of T_0 is

$$k_{\text{opt.}}=\varphi_{uv}\frac{C_u}{C_v} \tag{1.2}$$

and for this value of k, the MSE of T_0, to the first degree of approximation, is

$$\text{MSE}\,(T_0)=n^{-1}\sigma_u^2\,(1-\varphi_{uv}^2) \tag{1.3}$$

$k_{\text{opt.}}$ involves certain population parameters. Therefore, in practical application, T_0 lacks its utility.

In this paper, we have suggested an estimate for $k_{\text{opt.}}$. Even substituting this estimate of $k_{\text{opt.}}$ in T_0 the mean square error of the resulting estimator $\hat{T}_0$ remains the same.

The proposed estimator of $k_{\text{opt.}}$ is

$$\hat{k}_{\text{opt.}}=\frac{s_{uv}}{s_v^2}\,\frac{\bar{X}_2}{\bar{u}_n} \tag{1.4}$$

where

$$s_v^2=\frac{1}{n}\sum_{i=1}^{n}v_i^2-\bar{X}_2^2$$

$$s_{uv}=\frac{1}{n}\sum_{i=1}^{n}u_iv_i-\bar{u}_n\bar{X}_2$$

with

$$\text{E}(s_v^2)=\sigma_v^2 \quad\text{and}\quad \text{E}(s_{uv})=\sigma_{uv}\,.$$

so that $\hat{T}_0$ becomes

$$\begin{aligned}\hat{T}_0&=\hat{k}_{\text{opt.}}\bar{y}_R+(1-\hat{k}_{\text{opt.}})\,\bar{y}_{1pps}\\&=\hat{k}_{\text{opt.}}\,\frac{\bar{u}_n}{\bar{v}_n}\,\bar{X}_2+(1-\hat{k}_{\text{opt.}})\,\bar{u}_n\\&=\bar{u}_n\left[1-\hat{k}_{\text{opt.}}\left(1-\frac{\bar{X}_2}{\bar{v}_n}\right)\right]\end{aligned}$$

$$=\bar{u}_n-\frac{\hat{k}_{\text{opt.}}\bar{u}_n}{\bar{v}_n}(\bar{v}_n-\bar{X}_2)$$

$$=\bar{u}_n-\frac{s_{uv}}{s_u^2}\frac{\bar{X}_2(\bar{v}_n-\bar{X}_2)}{\bar{v}_n}$$

It will be shown that MSE of $\hat{T}_0$ (to the first degree of approximation) comes out to be equal to MSE of T_0 with $k_{\text{opt.}}$

2. Mean Square Error of $\hat{T}_0$

To evaluate mean square error of $\hat{T}_0$, let

$$\bar{u}_n=\bar{Y}(1+e_1)$$

$$\bar{v}_n=\bar{X}_2(1+e_2)$$

$$s_{uv}=\sigma_{uv}(1+e_3)$$

$$s_v^2=\sigma_v^2(1+e_4)$$

with

$$E(e_i)=0 \quad \text{for} \quad i=1,2,3,4$$

Then

$$\hat{T}_0=\bar{Y}(1+e_1)-\frac{\sigma_{uv}(1+e_3)}{\sigma_v^2(1+e_4)}\frac{\bar{X}_2e_2}{\bar{X}_2(1+e_2)}\bar{X}_2 \tag{2.1}$$

$$=\bar{Y}(1+e_1)-\frac{\sigma_{uv}}{\sigma_v^2}[\bar{X}_2(1+e_3)e_2(1+e_4)^{-1}(1+e_2)^{-1}]$$

or

$$\hat{T}-\bar{Y}=\bar{Y}e_1-\frac{\varphi_{uv}\sigma_u\sigma_v}{\sigma_v^2}\bar{X}_2[(e_2+e_2e_3)(1-e_4+e_4^2-\ldots)\cdot(1-e_2+e_2^2-\ldots)].$$

From (2.1) mean square error of $\hat{T}_0$, to the first degree of approximation, is

$$\text{MSE}(\hat{T}_0)=\text{E}(\hat{T}_0-\bar{Y})^2 \tag{2.2}$$

$$=\text{E}\left[\bar{Y}e_1-\varphi_{uv}\frac{C_u}{C_v}\bar{Y}e_2\right]^2$$

$$=\bar{Y}^2\left[E(e_1^2)+\left(\varphi_{ur}\frac{C_u}{C_v}\right)^2\text{E}(e_2^2)-2\left(\varphi_{uv}\frac{C_u}{C_v}\right)\text{E}(e_1e_2)\right]$$

$$=\frac{\bar{Y}^2}{n}\left[C_u^2+\left(\varphi_{uv}\frac{C_u}{C_v}\right)^2\cdot C_v^2-2\varphi_{uv}\frac{C_u}{C_v}\varphi_{uv}C_uC_v\right]$$

$$=\frac{\bar{Y}^2}{n}[C_u^2-\varphi_{uv}^2C_u^2]$$

$$=\frac{\bar{Y}^2}{n}C_u^2(1-\varphi_{uv}^2)$$

$$=n^{-1}\sigma_u^2(1-\varphi_{uv}^2)$$

3. Concluding Remarks

From (1.1) to (1.3), the estimator T_0 by AGARWAL & KUMAR (1980) involves the parameter $k=\varphi_{uv}(C_u/C_v)$ which is unknown in practice, hence the alternative is to replace $k=\varphi_{uv}C_u/C_v$ by its estimated value. When $k=\varphi_{uv}(C_u/C_v)$ is replaced by its estimated value $\hat{k}_{\text{opt.}}$ given by (1.4), the resulting proposed estimator $\hat{T}_0$, from (2.2), attains the minimum mean square error given by (1.3). Thus in the light of practical applications and minimum mean squared error sense the proposed estimator $\hat{T}_0$ may be prefered to the estimator T_0.

References

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