PPAS

① $P' = \{ P'(s) : s \in S \}$,

where $P'(s) \propto P(s) . \hat{X}_s$, $\hat{X}_s$ = an u.e. of X based on the sample s

② $P'(s) = \frac{1}{X} . P(s) \hat{X}_s$ $\quad = \left(N . \frac{1}{n} \sum_{j \in s} x_j\right)$

③ $\hat{Y}_R = \hat{R}_s X = \frac{\hat{Y}_s}{\hat{X}_s} . X \Rightarrow$ u.e. of Y, under P',

$\hat{Y}_s = N . \frac{1}{n} \sum_{j \in s} y_j$

④ ~~For SRSWOR,~~

$$P'(s) = \frac{1}{{}^{(N-1)}C_{(n-1)}} . \frac{\sum_{j \in s} x_j}{X}$$

(under SRSWOR)

⑤ Variance of $\hat{R}_s$ under PPAS

$$\left[= \left(\sum_{j \in s} y_j \Big/ \sum_{j \in s} x_j \right) \right]$$

⑥ $V_{P'}(\hat{R}_s) = E_{P'}(\hat{R}_s^2) - R^2$, where

$$E_{P'}(\hat{R}_s^2) = \frac{1}{X \binom{N-1}{n-1}} \sum_{s \in S} \left(\frac{\left(\sum_{j \in s} y_j\right)^2}{\sum_{j \in s} x_j} \right)$$

⑦ ① Midzuno's scheme :-

ⓐ Draw one unit from N units of the popl^n U $\ni$ the initial prob. $P_i = \frac{x_i}{X}$

ⓑ Select the remaining $(n-1)$ units from $(N-1)$ units by SRS WOR scheme.

$$P(\text{the sample is selected}) = \frac{1}{X\binom{N-1}{n-1}} \sum_{j\in s} x_j = p'(s)$$

③ Lahiri's method:

ⓐ Define: $M =$ a no. $\geq \max\left\{\sum_{j\in s} x_j : s \in S\right\}$

ⓑ Select a sample of size n by SRS WOR (provisionally)

ⓒ Draw R from 1 to M

- ⓘ If $R \leq \sum_{j\in s} x_j \Rightarrow$ select
- ⓘⓘ o.w. $\Rightarrow$ repeat

⑨ P(a trial results in selection of a particular sample s)

$$= \frac{1}{\binom{N}{n}} \cdot \frac{\sum_{j\in s} x_j}{M} = a, \quad M = \text{a no.} \geq \max\left\{\sum_{j\in s} x_j : s \in S\right\}$$

ⓐ P(a trial results in no. selection)

$$= \left(1 - \frac{1}{M}\sum_{s\in S} \frac{1}{\binom{N}{n}} \cdot \frac{N}{n}\sum_{j\in s} x_j \cdot \frac{n}{N}\right)$$

$$= \left(1 - \frac{1}{M}\cdot\frac{n}{N}\sum_{i=1}^{N} x_i\right) = \left(1 - \frac{nX}{MN}\right) = b$$

⑩ P(a specified sample s is selected)

$$= \sum_{j\in s} x_j \Big/ X\binom{N-1}{n-1} = p'(s)$$

(10) $$V_{pr}(\hat{Y}_R) = \frac{X}{\binom{N-1}{n-1}} \sum_{s \in S} \left(\frac{\left(\sum_{j \in s} y_j\right)^2}{\left(\sum_{j \in s} x_j\right)} \right) - Y^2$$

V_{PPAS} $\nearrow$

(11) an u.e. of Y^2 $\Rightarrow$

$$Y^2 \hat{=} \frac{\left[\left(\sum_{j \in s} y_j^2\right) \Big/ \binom{N-1}{n-1} + 2\left(\sum_{\substack{i \neq j \\ i \in s \\ j \in s}} y_i y_j\right) \Big/ \binom{N-2}{n-2}\right]}{p'(s)} \quad (= G)$$

(12) an u.e. of $V(\hat{Y})$

$\Downarrow$

$$V(\hat{Y}) \hat{=} \hat{Y}^2 - G$$