Single Phase Circuits

Seema M

Fundamentals of AC circuits

AC – An alternating current is the current which changes periodically both in magnitude & direction.

Advantages of AC

1) The voltage in ac system can be raised or lowered with the help of a transformer. In dc system, raising & lowering of voltages is not so easy.

2) As the voltages can be raised, transmission at high voltage is possible. Because of high voltage current flowing is lesser it reduces the conducting material required & also copper losses.

3) High voltage, high speed ac generators of large capacities can be build, construction & cost of such generators are very low.

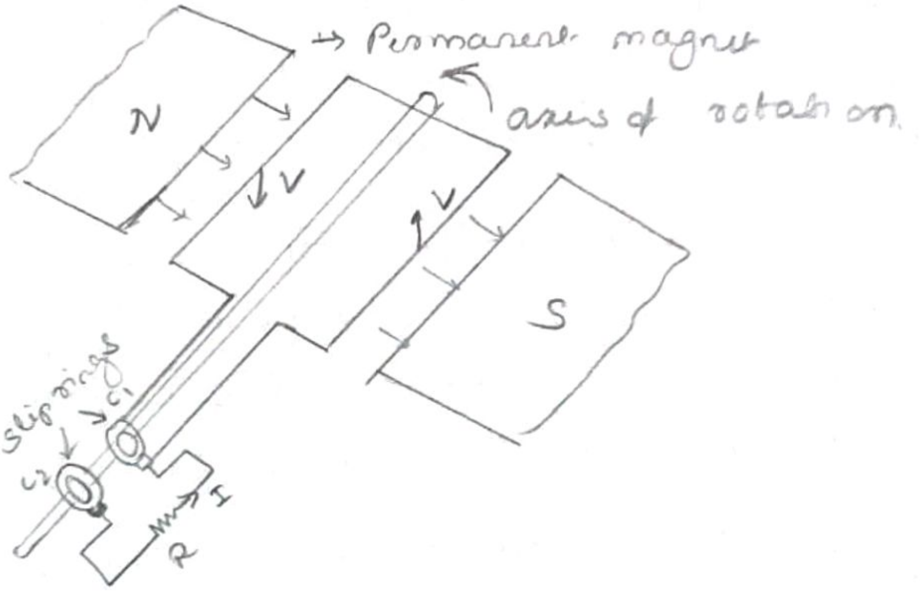
4) AC electrical motors are simple in construction, are cheaper & requires less attention from maintenance point of view.

5) Whenever it is necessary, ac supply can be easily converted to obtain dc supply, in applications like cranes, printing process, battery charging, telephone system etc. But such requirement of dc is very small compared to ac.

Generation of AC voltage

Basic principle of generation of ac voltage is electromagnetic induction. It says that whenever there is a relative motion betn the conductor & the magnetic field, an emf gets induced in the conductor.

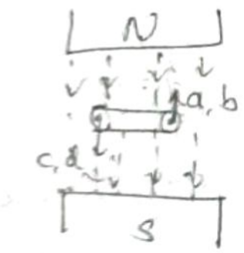
Construction



A single turn rectangular coil is kept in the vicinity of the permanent magnet. The coil is made up of 2 conductors ab & cd. The coil is so placed that it can be rotated about its own axis in clockwise or anticlockwise direction. The two ends c1 & c2 of the coil are connected to the rings mounted on the shaft called slip rings. The 2 brushes P & Q are resting on the slip rings. The slip ring & brush assembly is necessary to collect the current induced in the rotating coil & make it available to the stationary external resistance.

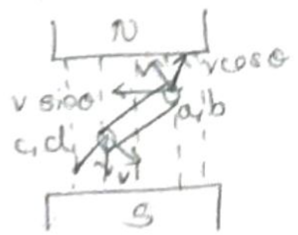
Working – The coil is rotated in anticlockwise direction. While rotating the conductors ab & cd cut the lines of flux of the permanent magnet. Due to Faraday's law of electromagnetic induction an emf gets induced in the conductors. This emf drives a current through resistance R connected across the brushes P & Q. The magnitude of the induced emf depends on the position of the coil in the magnetic field.

Instant 1 – When the plane of the coil is $\perp^{lar}$ to the direction of the magnetic field, the instantaneous component of velocity of conductors ab & cd is parallel to the magnetic field hence no cutting of the flux lines, no emf & no current.

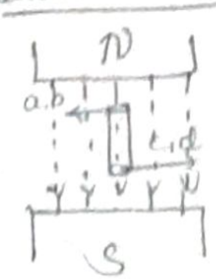


when $\theta = 0°$

Instant 2 – When the coil is rotated in anticlockwise direction through some angle θ, then the velocity will have two components $v\sin\theta$ $\perp^{lar}$ to flux lines & $v\cos\theta$ $\parallel^{lel}$ to the flux lines. Due to $v\sin\theta$ component there will be cutting of the flux & proportionally there will be induced emf in the conductors ab & cd.

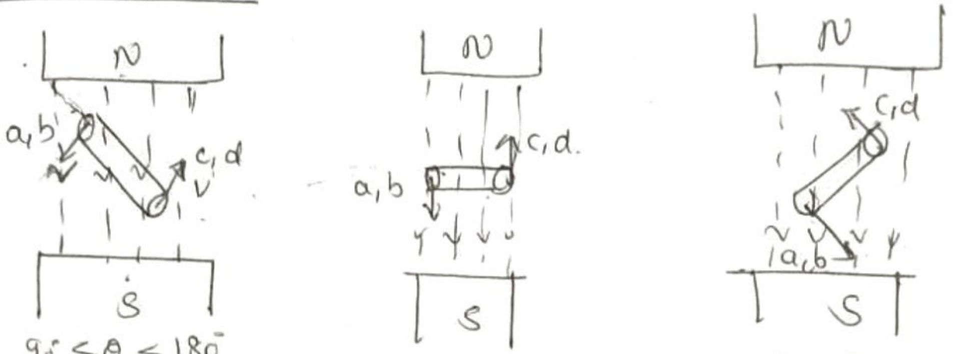


Instant 3: At $\theta = 90°$, the plane of coil is $\parallel^{lel}$ to the plane of magnetic field & velocity component is $\perp^{lar}$ to the flux lines, hence more cutting flux lines & maximum emf will induced in the circuit.



So as θ increases from 0° to 90°, emf induced in the conductors increases gradually from 0 to max. value. The current through external resistance R also varies according to the induced emf.

Instant 4 —



$90^\circ < \theta < 180^\circ$ (d) $\theta = 180^\circ$ (e) (f) $180^\circ < \theta < 270^\circ$

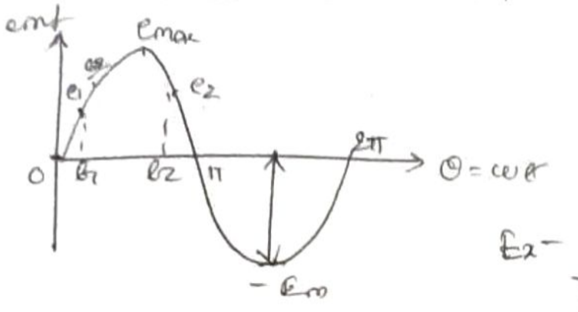
As the coil continues to rotate further from θ = 90° to 180°, the component of velocity, ⊥lar to magnetic field starts decreasing, hence gradually decreasing the magnitude of the induced emf.

Instant 5 — In this position, the velocity component is fully ∥led to the lines of flux similar to instant 1. Hence no cutting of flux lines & no emf.

Instant 6 — At θ = 270° again emf is maximum value but it is opposite in the direction. From 180° to 360° change in direction of induced emf occurs because the direction of rotation of conductors ab & cd reverses with the respect to the field.

So as θ varies from 0° to 360°, the emf in a conductor ab or cd varies in an alternating manner ie zero, increasing to achieve max in one direction, decreasing to zero, increasing to achieve max in other direction & again decreasing to zero.

Instantaneous value — The value of an alternating quantity at a particular instant is known as its instantaneous value.



Ex- e_1 at t_1, e_2 at t_2.

$\boxed{e = E_m \sin\theta}$ volts.

Waveform — The graph of instantaneous values of an alternating quantity plotted against time is called its waveform.

Cycle — Each repetition of a set of +ve & -ve instantaneous values of alternating quantity is called a cycle. Also defined as that interval of time during which a complete set of non-repeating events or wave variations occurs.

Time period (T) — Time taken by an alternating quantity to complete its one cycle. unit seconds.

Frequency (f) — The number of cycles completed by an alternating quantity / second.

f = cycles / second. Hz.

$$f = \frac{1}{T}$$

Amplitude — The maximum value attained by an alternating quantity during +ve or -ve half cycle. (E_m or I_m) → peak values.

Angular frequency (ω) — frequency expressed in electrical radians / second.

1 cycle of an alternating quantity = 2π radians,
angular frequency = $2\pi \times$ cycles / sec.

$$\boxed{\begin{aligned} \omega &= 2\pi f \ \text{rad/sec} \\ \omega &= \frac{2\pi}{T} \ \text{rad/sec} \end{aligned}}$$

Different forms of emf equation

$\theta = \omega t$ radians

$e = E_m \sin(\omega t) \longrightarrow (1)$

$e = E_m \sin(2\pi f t) \longrightarrow (2)$ $\quad \because \omega = 2\pi f$ rad/sec.

$e = E_m \sin\left(\frac{2\pi}{T} t\right) \longrightarrow (3)$ $\quad \because f = 1/T$ seconds.

$i = I_m \sin\theta$

$i = I_m \sin\omega t$, $I_m \sin(2\pi f t)$, $I_m \sin\left(\frac{2\pi}{T} t\right)$.

Ex-1) An alternating voltage of time period 0.02 seconds has maximum value of 12 V. Write the equation for its instantaneous value. Calculate the instantaneous value of the voltage after time 0.002 seconds, where reference is taken from the instant of zero voltage & is becoming +ve. Also calculate time required for the voltage to reach 4 V for the first time.

Soln — $T = 0.02$ seconds, $E_m = 12$ V.

$$e = E_m \sin\left(\frac{2\pi t}{T}\right) = 12 \sin\left(\frac{2\pi t}{0.02}\right) = 12 \sin(100\pi t) \text{ V}.$$

At $t = 0.002$ sec, $e = 12 \sin(100\pi \times 0.002)$

$e = 7.053$ V.

The instantaneous value is 4 V,

$4 = 12 \sin(100\pi t)$ — radian mode.

$t = 1.0817 \times 10^{-3}$ sec.

2) In a circuit supplied from a source 50 Hz, the voltage & current have maximum values of 500 V & 10 A respectively. At $t=0$, their respective values are 400 V & 4 A both increasing +vely.

1) write expn for their instantaneous values.

2) Find the angle betn V & I

3) I at $t = 0.015$ sec.

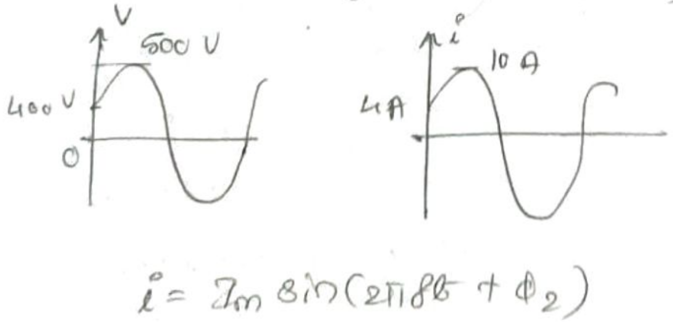
Soln – $f = 50$ Hz, $V_m = 500$ V, $I_m = 10$ A.

$v = V_m \sin(2\pi f t + \phi_1)$

$400 = 500 \sin(0 + \phi_1)$

$\phi_1 = 53.13^\circ = 0.9272$ rad.

$\therefore v = 500 \sin(100\pi t + 0.9272)$ V.



$i = I_m \sin(2\pi f t + \phi_2)$

$4 = 10 \sin(0 + \phi_2)$

$\phi_2 = 23.57^\circ = 0.4115$ rad.

$i = 10 \sin(100\pi t + 0.4115)$ A.

2) $\phi_1 = 53.13^\circ$ for $\phi_2 = 23.57^\circ$

ϕ = angle bet2 v & I $= 53.13^\circ - 23.57^\circ = 29.56^\circ$

3) At $t = 0.015$ sec

$i = 10 \sin (100\pi \times 0.015 + 0.4115)$

$i = 10 \sin (5.1238) = -9.1652$ A.

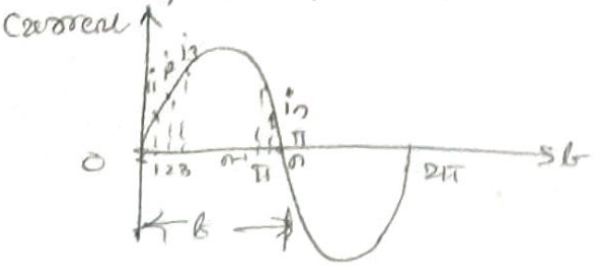
Effective value or RMS value — The effective value or rms value of an alternating current is given by that steady current (DC) which, when flowing through a given circuit for a given time, produces the same amount of heat as produced by the alternating current, which when flowing through the same circuit for the same time.

RMS value can be determined by 2 methods.

1) graphical 2) Analytical

1) **Graphical method** — Consider sinusoidally varying current. The rms value is to be obtained by comparing heat produced. (I^2R). heat produced in both +ve & -ve cycle is same.

Consider +ve half cycle, which is divided into n intervals. Width of each interval is t/n seconds & average height of each interval is assumed to be the avg instantaneous values of current ie $i_1, i_2 \ldots i_n$.



Let current be passing through resistance R ohms.

$\therefore$ Heat produced $= i^2 Rt$ joules.

$\therefore$ Heat produced due to 1st interval $= i_1^2 Rt/n$ joules

" 2nd " $= i_2^2 Rt/n$ joules

⋮

" " nth " $= i_n^2 Rt/n$ joules

$\therefore$ Total heat produced in 't' sec $= R \times t \left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right)$

Heat produced by dc I amperes passing through R for time 't' is $= I^2 Rt$ joules.

$\therefore$ For rms values, these two heats must be equal

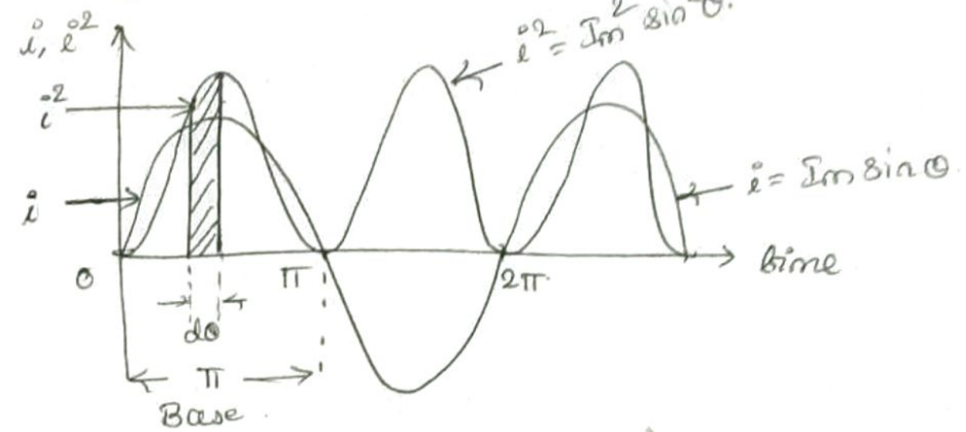
$$I^2 Rt = R \times t \times \left[\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right]$$

$$I^2 = \left[\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right]$$

$$I = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}} = I_{rms}$$

$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \ldots + v_n^2}{n}}$$

Analytical Method



The current $i = I_m \sin\theta$ $\quad i^2 = I_m^2 \sin^2\theta$

Area of curve over half a cycle can be calculated by considering an interval $d\theta$ as shown.

Area of square curve over half cycle $= \int_0^\pi i^2 d\theta$ & length of the base is π.

$\therefore$ Avg value of square of the current over half cycle =

$$= \frac{\text{area of square curve over half cycle}}{\text{length of base over half cycle}} = \frac{\int_0^\pi i^2 d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^\pi i^2 d\theta = \frac{1}{\pi}\int_0^\pi I_m^2 \sin^2\theta \, d\theta = \frac{I_m^2}{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi = \frac{I_m^2}{2\pi}[\pi] = \frac{I_m^2}{2}$$

$\therefore I_{rms} = \sqrt{\text{mean or avg of square of current}}$

$$= \sqrt{\frac{I_m^2}{2}}$$

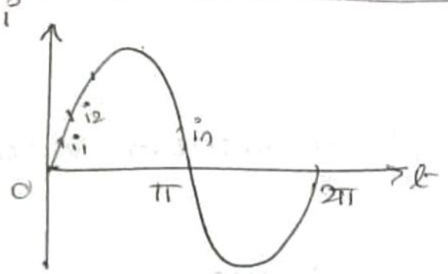
$$\therefore \boxed{I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m}$$

$\|^{ly}$ $\boxed{V_{rms} = 0.707\, V_m}$

Average Value — value which is obtained by averaging all the instantaneous values over a period of half cycle.

For a symmetrical ac, average value over a complete cycle is zero, as both +ve & −ve half cycles are exactly identical. Hence avg is defined for half cycle only.

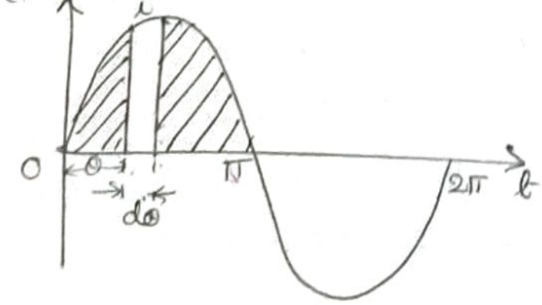
Graphical Method — Consider half cycle for n interval.



Avg of current over half cycle $= \dfrac{i_1 + i_2 + \ldots + i_n}{n}$

$$\therefore I_{av} = \frac{i_1 + i_2 + \ldots + i_n}{n} \qquad V_{av} = \frac{v_1 + v_2 + \ldots + v_n}{n}$$

Analytical Method



Consider the elementary interval of instant $d\theta$. The avg instantaneous value of current in this interval is i.

$$I_{av} = \frac{\text{area under curve for half cycle}}{\text{length of base over half cycle}}$$

$$= \frac{\int_0^{\pi} i\, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i\, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta = \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{\pi}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{I_m}{\pi}\left[1+1\right] = \frac{2\,I_m}{\pi}$$

$$I_{av} = \frac{2\,I_m}{\pi}$$

$\therefore I_{av} = 0.637\,I_m$ $\qquad$ $V_{av} = 0.637\,V_m.$

<u>Form Factor (K_f)</u> — ratio of rms value to the average value.

$$K_f = \frac{\text{rms value}}{\text{avg value}} = \frac{0.707\,I_m}{0.637\,I_m} = 1.11$$

<u>Crest or peak factor (K_p)</u> — ratio of maximum value to the rms value.

$$K_p = \frac{\text{maximum value}}{\text{rms value}} = \frac{I_m}{0.707\,I_m} = 1.414.$$

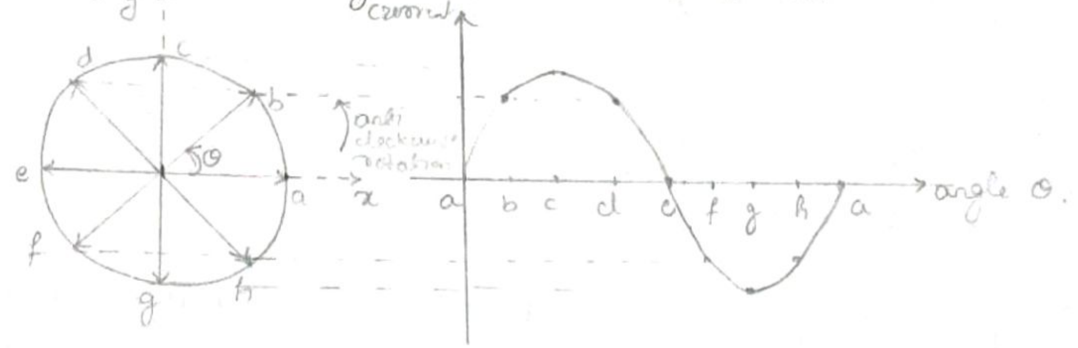
<u>Phasor Representation of an alternating quantity</u>

The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow; The length of the line represents the magnitude of the quantity & arrow indicates its direction. Such line is called phasor.

Phasors are assumed to be rotated in anti-clockwise direction.

One complete cycle of a sine wave is represented by one complete rotation of a phasor.

Consider a phasor, rotating in anticlockwise direction, with uniform angular velocity, with its starting position 'a'. If the projections of this phasor on y-axis are plotted against the angle turned through θ, we get a sine waveform.



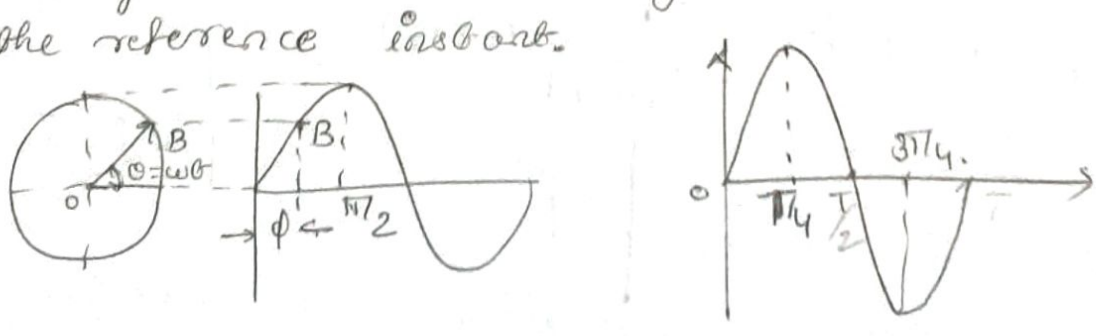
at b & d, ob & od = $I_m \sin\theta$.

at c, oc = I_m ($\because \theta = 90°$)

length of phasor — rms value of alternating quantity.
($I_m = \sqrt{2}\, I_{rms}$) → multiplied by $\sqrt{2}$ to get actual instantaneous value.

<u>Phase</u> — The phase of an alternating quantity at any instant is the angle φ (in radians or degrees) travelled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference.

or The phase of an alternating quantity at any particular instant is the fraction of the time period (T) through which the quantity is advanced from the reference instant.



phase of quantity at instant A is T/4 & at B is 3T/4.

In terms of phase the eqn of alternating quantity is

$$e = E_m \sin(\omega t \pm \phi) \qquad \phi - \text{phase.}$$

1) If $\phi = 0°$

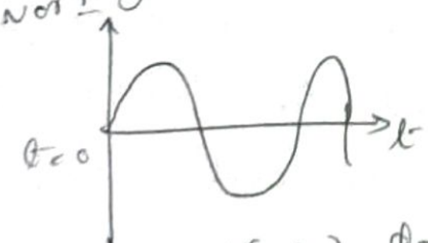
It is pure sinusoidal quantity having instantaneous value zero at $t=0$.

2) ϕ is positive phase. — when phase of an alternating quantity is +ve it means that quantity has some +ve instantaneous value at $t=0$.

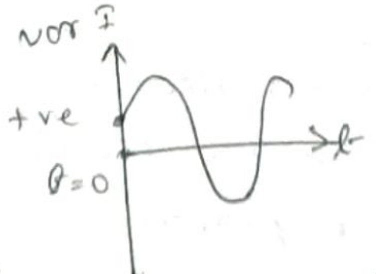
3) ϕ is -ve phase — it means that quantity has some negative instantaneous value at $t=0$.

Phase is measured with respect to reference direction ie +ve x axis direction.

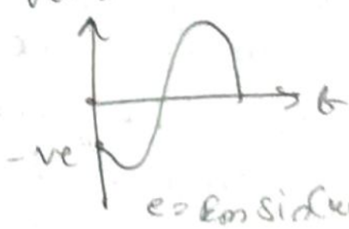
Phase measured in anticlockwise direction is +ve while the phase measured in clockwise direction is negative.



$e = E_m \sin(\omega t)$ $\phi = 0°$

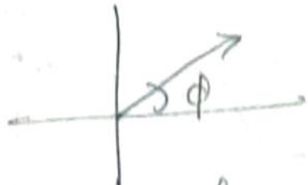


$e = E_m \sin(\omega t + \phi)$ +ve ϕ



$e = E_m \sin(\omega t - \phi)$ -ve ϕ

zero phase



+ve phase

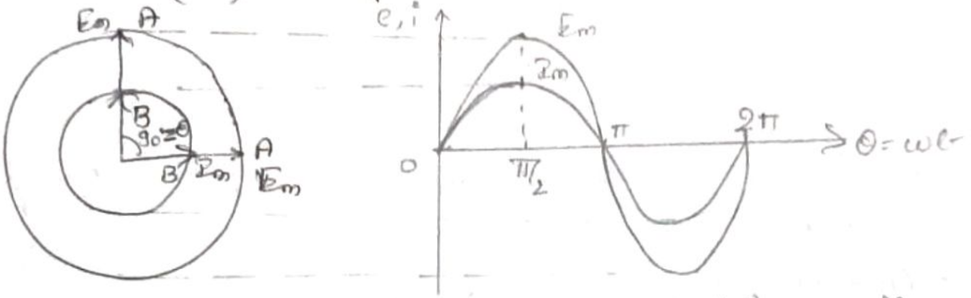
-ve phase.

Phase Difference

— The difference betⁿ the phases of two alternating quantities is called the phase difference, which is nothing but the angle difference betⁿ the two phasors representing the two alternating quantities.

Consider two alternating quantities having same frequency f Hz having different maximum value.

$e = E_m \sin(\omega t)$ & $i = I_m \sin(\omega t)$ where $E_m > I_m$.



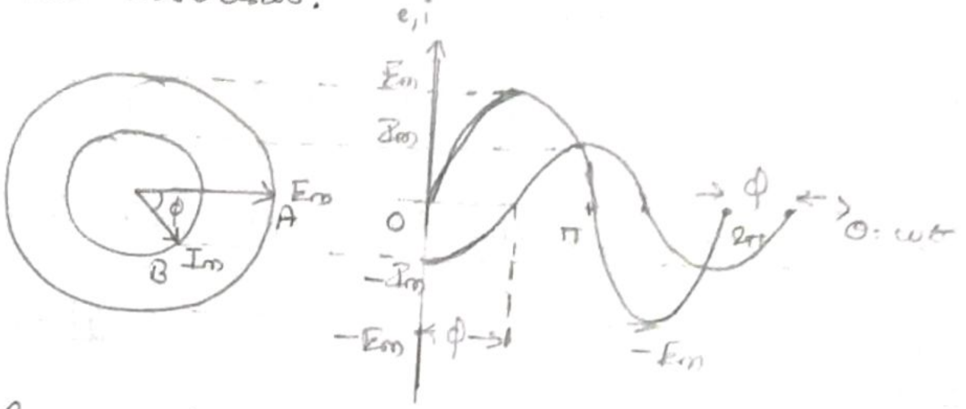
Phasors
OA = E_m
OB = I_m.

At $\theta = \pi/2$ radians, OA phasor achieves its maximum E_m while at same instant, OB phasor achieves its max I_m. As frequency of both is same, the angular velocity ω of both is also the same. So they rotate together in synchronism.

At any instant, phase of e & i will be same, thus angle travelled by both within a particular time is always the same, so phase difference will be zero. Such ~~th~~ two quantities are said to be in phase.

> The two alternating quantities having same frequency, reaching max +ve and −ve values & zero values at the same time are said to be in phase. Their amplitudes may be different.

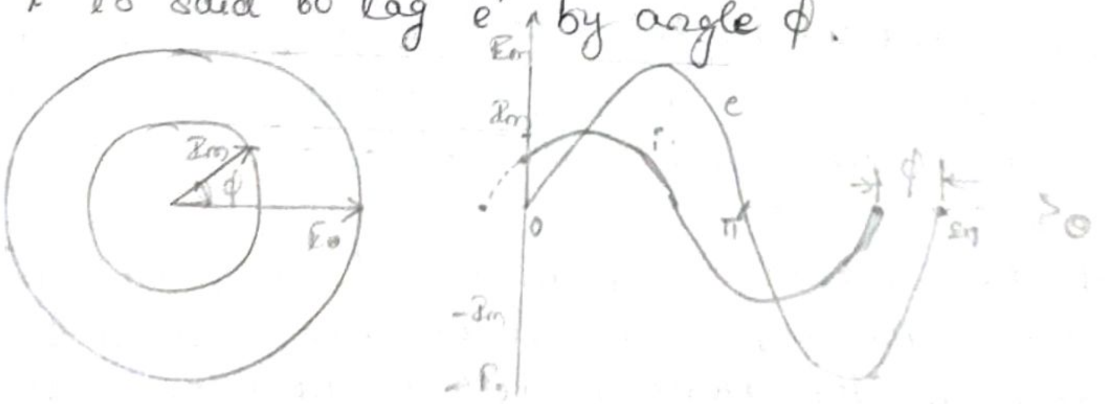
If there is difference between the phases (angles) of the two quantities, expressed in degrees or radians at any particular instant, then as both rotate with same speed, this difference remains same at all the instants.



When emf 'e' is at its zero value, the current 'i' has some -ve value. Thus there exists a phase difference ϕ betn the two phasors. As two are rotating in anticlockwise direction, the current is falling back with respect to voltage at all the times by angle ϕ. This is called lagging phase difference. The current i is said to lag the voltage e by angle ϕ.

$e = E_m \sin \omega t$ $\quad$ $i = I_m \sin(\omega t - \phi)$

'i' is said to lag 'e' by angle ϕ.

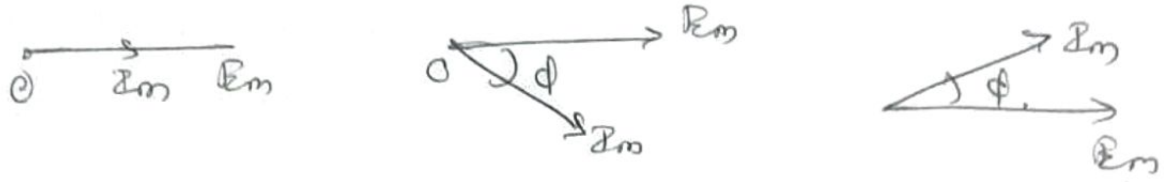


Current 'i' is ahead of voltage 'e'. Thus current is said to be leading wrt voltage & phase difference is called leading phase difference.

$e = E_m \sin\omega t \qquad i = I_m \sin(\omega t + \phi)$.

'i' is said to lead 'e' by angle 'ϕ'.

In phase   lagging   leading



3) An ac current is given by $i = 10\sin\omega t + 3\sin 3\omega t + 2\sin 5\omega t$. Find the rms value of the current.

<u>Soln</u> — Total heat is sum of heat produced by dc component & all the alternating components.

Let R – resistance of wire
t – time for which signals are flowing.

$\therefore H_{total} = H_{dc} + H_1 + H_2 + \dots$

$H_{total} = I_{rms}^2 \times R \times t$

$H_{dc} = I_{dc}^2 \times R \times t$

$H_1 = I_{rms_1}^2 Rt \qquad H_2 = I_{rms_2}^2 Rt$

$I_{rms}^2 \times Rt = I_{dc}^2 Rt + I_{rms_1}^2 Rt + I_{rms}^2 Rt + \dots$

$\therefore I_{rms} = \sqrt{I_{dc}^2 + I_{rms_1}^2 + I_{rms}^2 + \dots}$

For given ex, $I_{dc} = 0$

$I_{rms_1} = \frac{I_{m1}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07106\ A$

$I_{rms_2} = \frac{I_{m2}}{\sqrt{2}} = \frac{3}{\sqrt{2}} = 2.1213\ A$

$I_{rms_3} = I_{m3}/\sqrt{2} = 2/\sqrt{2} = 1.4142\ A$

$$\therefore \; I_{rms} = \sqrt{0+(7.07106)^2+(2.1213)^2+(1.4142)^2}$$

$$= 7.5166\,A.$$

iv) A 50 Hz sinusoidal current has peak factor 1.4 & form factor 1.1. Its avg value is 20 A. The instantaneous value of current is 15 A at $t = 0$ sec. Write the eqn of current & draw its waveform.

Soln – $f = 50\,Hz$, $K_p = 1.4$, $K_f = 1.1$ & $I_{av} = 20\,A$.

$K_p = I_m / I_{rms}$ $\qquad K_f = I_{rms}/I_{av}$.

$$K_f = \frac{(I_m / K_p)}{I_{av}} \qquad 1.1 = \frac{I_m/1.4}{20}$$

$I_m = 20 \times 1.1 \times 1.4 = 30.8\,A$.

$i = I_m \sin(\omega t + \phi) = I_m \sin(2\pi f t + \phi)$

At $t=0$ $\quad 15 = 30.8 \sin(0 + \phi)$

$$\phi = \sin^{-1}\left(\frac{15}{30.8}\right) = 29.144^\circ = \frac{29.1444\pi}{180}\,rad$$

$= 0.50866$ rad

$\therefore \; i = 30.8 \sin(100\pi t + 0.50866)$ A