

# BASIC ELECTRICAL ENGINEERING

Prepared by  
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Sub Code:	<b>Basic Electrical Engineering</b>	C	L	T	P	CH
Duration: 14 Weeks		3	2	1	0	4

### **Course Objectives:**

1. To establish a broad concept of various types of generation of electricity.
2. To make students understand the basics of representation of electrical quantities and relationship among them.
3. To provide an overview of various types of electrical apparatus.
4. To introduce the concept of domestic wiring and importance of safety and sensing devices.
5. To provide an insight into various sources of power generation.

### **Course Contents:**

#### **UNIT-I: Introduction to Electrical Parameters**

**[11hrs]**

Concept of Alternating Voltage and Current, Sinusoidal functions-specifications, Phasor representation, concept of impedance, admittance, conductance and susceptance –series and parallel circuits of RLC. Concept of power and power factor. Kirchoff's laws and network solutions. Electromagnetic induction-laws, direction & magnitude of induced emf, mmf, permeability, reluctance and comparison of electric and magnetic circuits. Self and mutual inductance of a coil, coupling coefficients. Concept of energy storage in L & C, resonance between L &C. Generation of three phase voltages, star-Wye configurations, relation between line and phase quantities and expression for power.

#### **UNIT-II: Electrical Apparatus**

**[11hrs]**

DC generator, DC motor- concept of force, torque and mechanical work. Single and three phase induction motors, shaded pole motor, universal motor, stepper motor: Basic construction, principle of operation and applications. Single and three-phase transformers: Principle, emf equation.

#### **UNIT-III: Generation & Distribution:**

**[10hrs]**

Block diagram representation of generation, transmission and distribution. Current generation and transmission scenario, need for transmission at high voltage. Block diagram representation of thermal, hydel, nuclear, diesel and renewable power plants. Concept of smart-grid and role of ICT in smart-grid.

## **UNIT-IV: Tariff, Protective Devices and Sensors**

**[10hrs]**

Tariff schemes, basic concepts of domestic wiring and types, earthing, protective fuses, MCB. Sensors: pressure sensor, strain gage, proximity sensor, displacement sensor, rotary encoder and ultrasonic sensors (applications in relevant disciplines- ref to 8 and 9)

### **References:**

1. Theodore Wildi, “Electrical Machines, Drives, and Power Systems”, Pearson Education, 5<sup>th</sup> Edition, 2007
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3. Kulshreshtha C, “Basic Electrical Engineering” Tata McGraw Hill, 2<sup>nd</sup> Edition, 2011
4. Mittle V.N. and A. Mittal, “Basic Electrical Engineering” Tata McGraw Hill, 2<sup>nd</sup> Edition, 2005
5. Kothari D.P., L.J. Nagrath “Basic Electrical Engineering”, Tata McGraw Hill, 2009
6. Robert L. Boylestad and Louis Nashelsky, “Introduction to Electricity, Electronics and Electromagnetics” Prentice Hall, 5<sup>th</sup> edition, 2001
7. Introduction to smart grid:  
[http://www.occc.ohio.gov/publications/electric/Smart\\_Grid\\_An\\_Introduction.pdf](http://www.occc.ohio.gov/publications/electric/Smart_Grid_An_Introduction.pdf)
8. Role of ICT in smart grid:  
<http://users.atlantis.ugent.be/cdvelder/papers/2010/develder2010sgc.pdf>
9. Sensors: [http://www.omron-ap.co.in/technical\\_guide/](http://www.omron-ap.co.in/technical_guide/)
10. Strain gage with bridge circuit:  
<http://www.facstaff.bucknell.edu/mastascu/elessonshtml/Sensors/StrainGage.htm#SensorsInVoltageDividerCircuits>

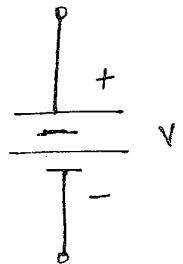
### **Course Outcomes:**

On completion of this course the students will be able to:

1. Describe the operation and control of various types of generation of electricity
2. Describe the principle of operation of electrical apparatus
3. Differentiate between single and three phase systems
4. Solve simple mathematical relationships related to electrical apparatus.
5. Relate the applications of electronic devices and sensors in practical life.

UNIT - 1INTRODUCTION TO ELECTRICAL PARAMETERS :-Independent Voltage Source :-

If the voltage of a source is constant and will not depend on any parameters of the circuit, is called an independent voltage source.

Resistance & Resistivity :-

The property of a material due to which it opposes (or restricts) the flow of current through it is called resistance.

The resistance of a conductor is directly proportional to its length and inversely proportional to its area of cross-section. i.e.,

$$R \propto \frac{l}{A}$$

$$(or) R = \rho \frac{l}{A}$$

$\rho$  (Rho) is a constant & is called resistivity or specific resistance of the conductor material.

Unit for resistance is  $\Omega$  (ohm)

Unit for resistivity is  $\Omega \cdot m$  (ohm-meter).

## The Electric Current (I) :-

The rate at which the electric charge is transferred across a point in a conductor is known as the current flowing through the conductor.

i.e.

$$I = \frac{dq}{dt} = \frac{q}{t}$$

The unit of current is A (Ampere).

## The Ampere (A) :-

One ampere of current is defined as that current which, when flowing through a resistance of one ohm, causes a potential difference of one volt across it.

## Electric Potential :-

The electric potential at any point in a charged conductor is defined as the work done to bring a unit positive charge from infinity to that point.

The unit of electric potential is volt.

## Potential Difference :-

The potential difference between any two points of a charged conductor is the amount of work that has to be done to bring a unit positive charge from the point of lower potential to the point of higher potential.

The unit of potential difference is Volt.

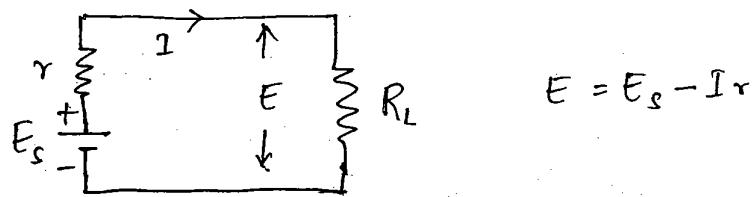
## Volt (V) :-

One Volt is defined as the potential difference across a resistance of one ohm, through which a current of one ampere is flowing.

## E.M.F (Electromotive Force) of a Source (E) :-

The E.M.F of a source is the voltage available across its terminals.

The voltage available across the terminals of a voltage source is slightly less than the ~~theoretical~~ internal voltage  $E_s$  because of the small voltage drop across its internal resistance ' $r$ '.

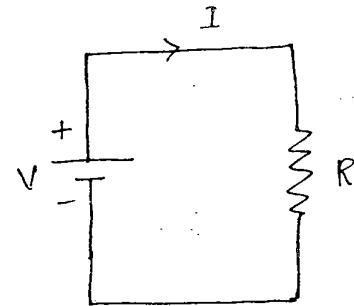


## OHM'S LAW :-

The ohm's law states that, temperature remaining constant, the current through a passive element is directly proportional to the voltage across the element.

$$I \propto V$$

$$\text{i.e., } \frac{V}{I} = \text{Constant} = R$$

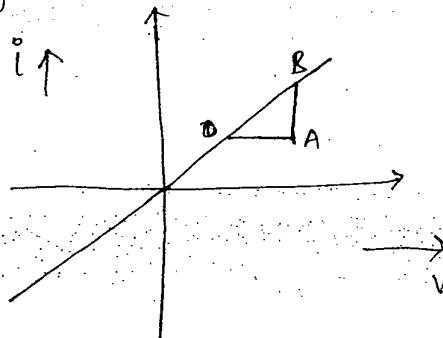


Where, 'R' is known as the resistance of the element.

$$\therefore I = \frac{V}{R} \quad (\text{or}) \quad V = IR. \quad (\text{or}) \quad I = GV$$

Graphical representation of ohm's law:

$$\text{Slope} = \frac{1}{R}$$



## Limitations of ohm's law :-

- (i) It does not hold true for non-linear devices.  
Such as Semiconductors & Zener diodes.

(ii) It is not applicable to non-metallic conductors, such as Silicon Carbide, where the following relation is applicable:

$$V = K I^m$$

Where,  $K$  &  $m$  are Constants.

(iii) Ohm's law cannot be applied to arc-lamps.

(iv) It does not hold good where temperature rise is rapid in some metals.

### SERIES CIRCUIT:-

The circuit in which resistances are connected end-to-end, so that there is only one path for current flow, is called as Series Circuit.

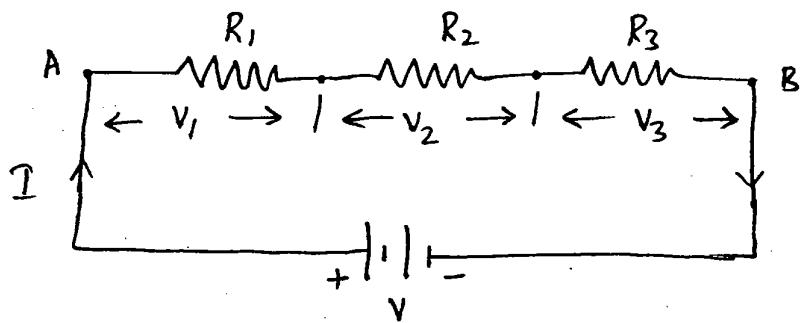
In a Series Circuit:-

(i) The same current flows through all the resistances.

(ii) There will be a voltage drop across each resistance, according to Ohm's law.

(iii) The sum of the voltage drops is equal to the applied voltage.

Fig. below shows a Series Circuit, where resistors  $R_1$ ,  $R_2$  and  $R_3$  are connected in Series and a voltage of 'V' Volts is applied at the extreme ends A & B, to cause a current of 'I' amperes to flow through all these resistors.



Let,  $V_1$ ,  $V_2$  and  $V_3$  be the Voltage drops across resistors  $R_1$ ,  $R_2$  and  $R_3$  respectively.

$$\begin{aligned} \text{Now, } V &= V_1 + V_2 + V_3 \\ &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \end{aligned}$$

$$\therefore \frac{V}{I} = R_1 + R_2 + R_3$$

A/c to Ohm's law,  $\frac{V}{I}$  is the total circuit Resistance, ' $R_s$ '.

$$\therefore R_s = R_1 + R_2 + R_3$$

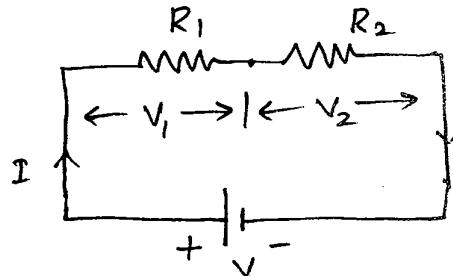
i.e., Total Resistance = Sum of Individual resistances.

## VOLTAGE DIVISION :-

Let us consider the circuit shown in fig (1). which deals with the division of voltage between only two resistors connected in series.

The total resistance of the circuit is,

$$R_s = R_1 + R_2$$



Hence, the current in the circuit is,

$$I = \frac{V}{R_s} = \frac{V}{R_1 + R_2}$$

The voltage drop across the resistor  $R_1$ , is given by,

$$V_1 = I R_1 = \frac{V}{R_1 + R_2} \times R_1$$

$$\therefore V_1 = \frac{V R_1}{R_1 + R_2}$$

Similarly,

$$V_2 = \frac{V R_2}{R_1 + R_2}$$

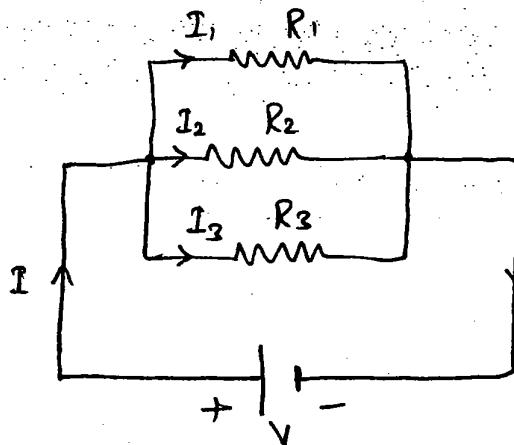
## PARALLEL CIRCUIT :-

When a number of resistors are connected in such a way that one end of each of them is joined to a common point, & the other end of each of them is joined to another common point, then the resistors are said to be connected in parallel.

In a parallel circuit :-

- (i) The voltage across all the resistors are same.
- (ii) The current divides into as many paths as the number of resistances.
- (iii) The sum of all the branch currents is equal to the total current of the circuit.

Fig below shows, the resistors  $R_1$ ,  $R_2$  &  $R_3$  connected in parallel across a voltage ' $V$ ' & the total current flowing is ' $I$ ' amperes.



Let,  $I_1$ ,  $I_2$  &  $I_3$  be the currents flowing in the resistors  $R_1$ ,  $R_2$  &  $R_3$  respectively.

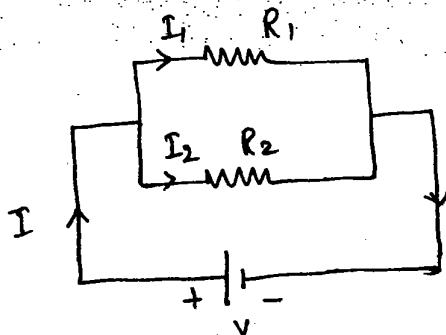
$$\begin{aligned}
 \text{Now, } I &= I_1 + I_2 + I_3 \\
 &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad (\text{by ohm's law}) \\
 &= V \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \\
 \therefore \frac{I}{V} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\
 \frac{1}{R_p} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\
 \therefore R_p &= \frac{V}{I} \\
 \Rightarrow \frac{I}{V} &= \frac{1}{R_p}
 \end{aligned}$$

i.e. The reciprocal of total resistance is equal to the sum of the reciprocals of the individual resistances.

### CURRENT DIVISION :-

If two resistors  $R_1$  &  $R_2$  are connected across a potential difference of 'V' Volts, As per ohm's law,

The Current flowing through  $R_1$  &  $R_2$ ,



$$I_1 = \frac{V}{R_1} \rightarrow (i)$$

$$I_2 = \frac{V}{R_2} \rightarrow (ii)$$

Dividing expression (i) by (ii)

$$\frac{I_1}{I_2} = \frac{V/R_1}{V/R_2} = \frac{R_2}{R_1} \rightarrow (\text{iii})$$

Also,  $I = I_1 + I_2$

$$\Rightarrow I_2 = I - I_1$$

from eqn (iii)

$$I_1 = \frac{I_2 \cdot R_2}{R_1} \quad \cancel{\text{as}} = \frac{(I - I_1) R_2}{R_1}$$

$$I_1 R_1 = I R_2 - I_1 R_2$$

$$I_1 R_1 + I_1 R_2 = I R_2$$

$$I_1 (R_1 + R_2) = I R_2$$

$$\therefore I_1 = \underline{\underline{\frac{I R_2}{(R_1 + R_2)}}}$$

Similarly,  $I_2 = \underline{\underline{\frac{I R_1}{(R_1 + R_2)}}}$

## KIRCHHOFF'S LAWS :-

Kirchhoff's laws are very useful in solving circuits which cannot be easily solved by using Ohm's law.

Kirchhoff's first law (KCL) :-  
(Kirchhoff's Current Law)

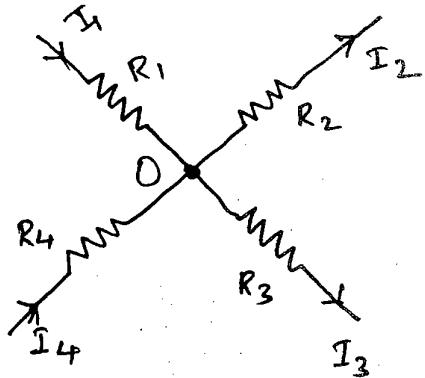
It states that, the algebraic sum of currents meeting at a junction in a circuit is zero.

(or)

The sum of incoming currents towards any point is equal to the sum of outgoing currents away from that point.

Sign Convention :-

- \*> The current entering the node/junction is treated as positive (+ve).
- \*> The current leaving the node/junction is treated as Negative (-ve).



Fig(1):-

(12)

Applying KCL at node 'O' for the fig (1):-

$$(+I_1) + (-I_2) + (-I_3) + (+I_4) = 0$$

$$I_1 - I_2 - I_3 + I_4 = 0$$

$$(08) \quad \underline{I_1 + I_4 = I_2 + I_3}$$

Kirchhoff's Second law :- (KVL)

(Kirchhoff's Voltage Law)

It states that, at any instant the algebraic sum of voltages around a closed loop (or) circuit is zero.

(08)

In any closed circuit/mesh, the algebraic sum of products of currents & resistances (i.e. voltage drops) plus the algebraic sum of all emf's in that circuit is zero.

Sign Convention :-

→ Sign of emf's :- A rise in potential should be taken as '+ve' & a fall in potential should be taken as '-ve'.

(13)

\* Signs of voltage drops :- If we go <sup>with</sup> the direction of current, the voltage drop should be considered to be '-ve', if we go against the direction of current, the voltage drop should be considered to be '+ve'.

Illustration :-

Assuming the directions of the loop currents  $I_1$  &  $I_2$  as shown in fig (2).

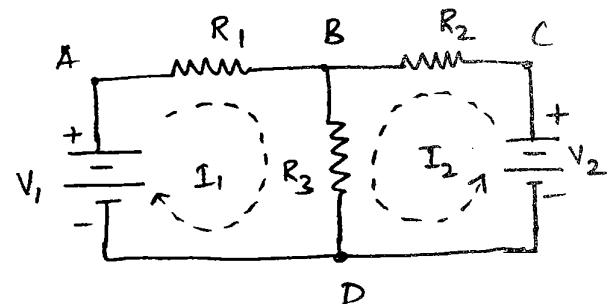


fig (2) :-

Applying KVL to loop ABDA :-

$$V_1 - I_1 R_1 - R_3 (I_1 + I_2) = 0$$

$$\text{or } V_1 = I_1 R_1 + R_3 (I_1 + I_2) \rightarrow (i)$$

Applying KVL to loop CBDC :-

$$V_2 - R_2 I_2 - R_3 (I_2 + I_1) = 0$$

$$\text{or } V_2 = R_2 I_2 + R_3 (I_2 + I_1) \rightarrow (ii)$$

By solving eqn's (i) & (ii) simultaneously, the loop currents  $I_1$  &  $I_2$  can be found.

## ELECTRICAL WORK :- (W) :-

Electrical work is done when there is a transfer of charge.

The unit is joule.

One joule of electrical work done is that work done in moving a charge of 1 Coulomb through a potential difference of 1 volt.

$$\therefore W = V \times Q \text{ joules}$$

$$I = \frac{Q}{t}$$

$$\therefore \underline{W = V I t \text{ joules}}$$

|   
 t = time in  
 seconds.

## ELECTRICAL POWER (P) :-

The rate at which electrical work is done in an electric circuit is called Electrical power.

$$\therefore P = \frac{W}{t} = \frac{V I t}{t}$$

$$\therefore \underline{\underline{P = V I \text{ joules/sec (or) Watts}}}$$

The unit of power is 'watt'.

## ELECTRICAL ENERGY (E) :-

Electrical energy is the total amount of electrical work done in an electrical circuit.

$$E = P \times t$$

$$E = V I \times t$$

$$\therefore E = \underline{VIt} \text{ watt-secs (or) Joules.}$$

## POWER & ENERGY Using Ohm's Law :-

w.k.t.,  $P = VI$  watts

f.  $V = IR$  by ohm's law.

$$\therefore P = IR \cdot I = \underline{\underline{I^2 R}} \text{ watts}$$

$$(or) P = V \cdot \underline{\underline{\frac{V}{R}}} = \underline{\underline{\frac{V^2}{R}}} \text{ watts}$$

Energy,  $E = VIt$  joules

$$\therefore E = I^2 R t = \underline{\underline{\frac{V^2}{R} t}} \text{ joules (or) Watt-secs}$$

## ELECTROMAGNETISM :-

Materials such as iron, cobalt, nickel and their alloys are magnetic materials. When bars of such materials are wound with a coil and current is passed through them, they become "electromagnets".

The strength of such an electromagnet depends on the number of turns in the coil & the magnitude of the current passing through it. Hence, by increasing the no. of coils & the current passing through them, an electromagnet of any strength may be obtained.

### Magnetic field :-

The space around the poles of a magnet is called the magnetic field & is represented by magnetic lines of force.

The magnetic field due to a magnet is strongest near the poles & goes on decreasing as we move away from the poles.

### Magnetic lines of force:-

The direction of each line of force is from the N-pole to the S-pole outside the magnet, but

from S-pole to the N-pole inside the magnet.

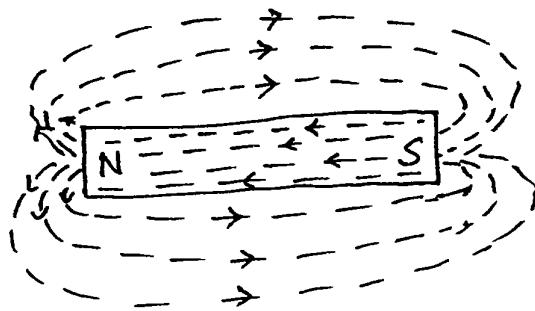


fig :- Magnetic lines of force.

Magnetic Flux ( $\phi$ ) :-

The total number of magnetic lines of force in a magnetic field is called magnetic flux. It is represented by  $\phi$ . & the unit is Weber (wb).

Magnetic Flux Density (B) :-

The magnetic flux density at any point is given by the flux passing per unit area at that point, through a plane that is at right angles to the flux.

$$\text{Flux density, } B = \frac{\phi}{A} \text{ wb/m}^2$$

Magnetic Circuit :-

It is defined as the path which is followed by the magnetic flux.

## Magnetomotive Force (M.M.F) :-

M.M.F is defined as the magnetic force, which creates magnetic flux in a magnetic material.

The unit of M.M.F is 'Ampere-turns' (AT)

$$\text{M.M.F} = N \cdot I$$

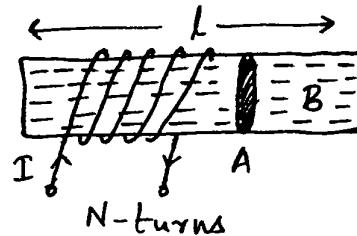


fig :-

|  $N = \text{No. of turns}$

|  $I = \text{Current}$

Also,  $\text{M.M.F} = \text{Flux} \times \text{Reluctance}$

$$\therefore N \cdot I = \phi \times S$$

$$(or) \phi = \frac{NI}{S}$$

## Reluctance (S) :-

Reluctance is the property of a <sup>magnetic</sup> ~~material~~ material by virtue of which it opposes the creation of magnetic flux in it.

The unit is Ampere-turn per weber (AT/web).

The reluctance of a magnetic material is directly proportional to the length of the magnetic material and inversely proportional to the area of cross-section.

(19)

$$S \propto \frac{l}{A} = \frac{1}{4} \cdot \frac{l}{A} = \frac{l}{\mu_0 \mu_r A} \rightarrow (i)$$

Where,  $\mu$  = a constant known as the absolute permeability of the magnetic material.

$$\mu = \mu_0 \mu_r$$

Where,  $\mu_0$  = Permeability of free space (or) air

$$\mu_0 = 4\pi \times 10^{-7}$$

&  $\mu_r$  = Relative permeability of the magnetic material

$$W.K.T, \quad \phi = \frac{NI}{S} \rightarrow (ii)$$

Eqn (i) in (ii)

$$\phi = \frac{NI}{\frac{l}{\mu_0 \mu_r A}} \text{ Hb}$$

### Permeability :-

It is defined as the ability of a material to conduct magnetic flux through it.

$$\mu = \frac{B}{H}, \quad \mu_r = \frac{B}{B_0}$$

$\mu_r = 1$ , for free space (or) Air.

## Magnetising Force (or) Magnetic field Intensity (H) :-

Magnetising force at any point is defined as the force experienced by a unit north pole, when placed at that point.

$$\therefore H = \frac{m}{4\pi \mu_0 \mu_r d^2} \quad \text{Newtons/weber}$$

Where,  $m \rightarrow$  pole strength  
 $d \rightarrow$  distance.

Magnetising force may also be defined as the number of ampere turns produced per unit length of the magnetic path.

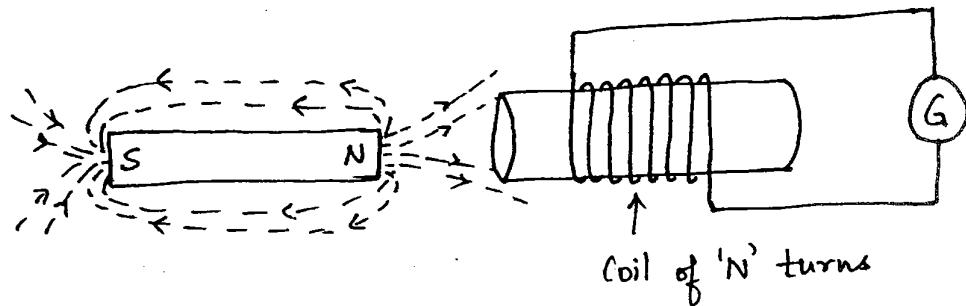
$$\therefore H = \frac{NI}{l} \quad \text{AT/m.}$$

## ELECTROMAGNETIC INDUCTION:-

The process of ~~producing~~ <sup>producing</sup> an e.m.f in a conductor by changing the magnetic flux linking with it, is known as Electromagnetic Induction (EMI).

The emf ~~so~~ produced is called induced emf and the ~~current~~ resulting current is called induced current.

Consider a coil wound with a large number of turns be connected to a galvanometer as shown in fig below



- \*> If a permanent magnet NS is moved towards the Coil, the pointer of the galvanometer will deflect, indicating that there must be an emf induced in the Coil.

Whenever the flux linking a conductor changes, an emf is induced in it. This emf persists as long as the flux linking the conductor is changing.

- \*> If the permanent magnet NS is now moved away from the Coil, the deflection in the galvanometer pointer is in the reverse direction  
 $\therefore$  The direction of the induced emf depends upon the direction of magnetic flux and upon the direction in which the flux moves relative to the conductor.

SGN

\*> Also, the magnitude of the Induced emf, depends upon the rate of change of flux linking with the conductor. (i.e., The speed at which the magnet is moved).

### FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION:

First Law:- Whenever the flux linking a coil or circuit changes, an emf is induced in it.

Second Law:- The magnitude of the induced emf in a coil is directly proportional to the rate of change of flux linkages.

$$\text{Flux linkages} = \text{Flux} \times \text{No of turns}$$

$$\therefore \text{Rate of change of flux linkages} = \frac{N\phi_2 - N\phi_1}{t}$$

As per faraday's second law of EMI,  
the emf 'e' is given by,

$$e \propto \frac{N\phi_2 - N\phi_1}{t} \text{ volts}$$

$$e = K \frac{N(\phi_2 - \phi_1)}{t}$$

| ∵ K is unity

$$e = N \frac{d\phi}{dt} \text{ volts} \rightarrow (i)$$

Lenz's law :- The direction of the induced current in the coil is such that it opposes the very cause producing it.

$$(i) \Rightarrow e = -N \frac{d\phi}{dt} \text{ volts}$$

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### Fleming's Right-Hand Rule :-

This rule is to find out the direction of induced emf & current in a coil.

The Thumb indicates

The direction of

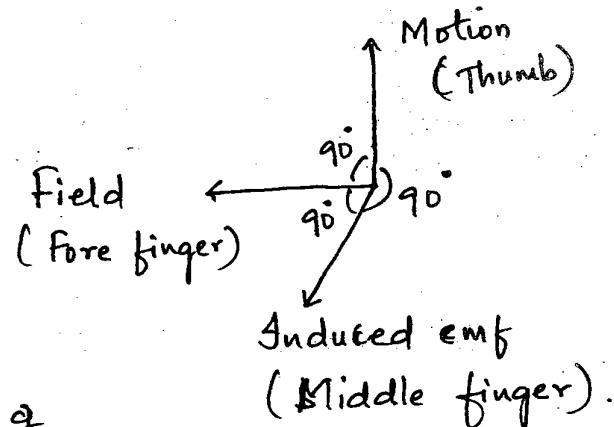
Motion of the

coil, the fore finger

Indicates the direction of

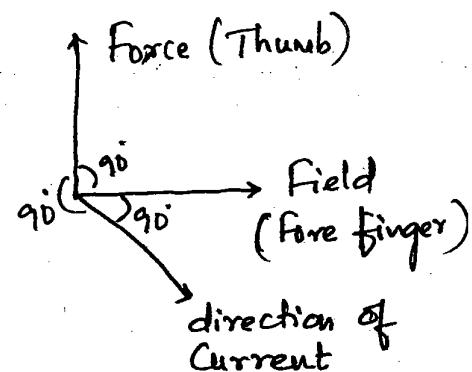
field & the middle finger

Indicates the direction of Induced emf.



### Fleming's Left-Hand Rule :-

This rule is to find out the direction of force experienced by the conductor.



Application of "FRH Rule"  $\rightarrow$  To find the direction of Induced current in the "Generator."

Application of "FLH Rule"  $\rightarrow$  To find the direction of force experienced by the Conductor in the "motor."

### Methods of producing Induced EMF :-

There are three requirements to produce an Induced emf in a Conductor,

- (i) The presence of a conductor.
- (ii) The presence of a magnetic field.
- (iii) The linking of the magnetic flux by the Conductor (flux linkages).

Following are the three methods by which this can be achieved.

- 1) By using stationary Conductor, a stationary electromagnet & varying the magnetic flux by supplying ac source to the electromagnet.

This ~~method~~ is emf Induced by this method is termed as "statically Induced emf".

- 2) By using a stationary conductor & a moving permanent magnet (or electro magnet).
- 3) By using a stationary permanent magnet (or an electromagnet, fed by a dc source) & a moving conductor.

The emf induced by the methods (2) & (3) is termed as "dynamically induced emf".

### Statically Induced emf :-

- a) Self - Induced emf (or)
- b) Mutually - Induced emf .

#### a) Self - Induced emf :-

The emf induced in a coil due to the change of its own flux linked with it, is called self-induced emf.

This property of the coil is known as Self-Inductance.

The self-induced emf is directly proportional to the rate of change of current.

$$e \propto \frac{di}{dt} \quad (\text{or}) \quad e = L \frac{di}{dt}$$

Where, 'L' is called the Co-eff of Self Inductance

(or) Simply Inductance.

The unit is Henry (H).

w.k.t. webers turns per ampere are  $\frac{N\phi}{I}$

$$\therefore L = \frac{N\phi}{I} \text{ henry.} \rightarrow (i)$$

---


$$\text{Also, } \phi = \frac{NI}{L/M_0\mu_r A} \text{ wb.}$$

$$\frac{\phi}{I} = \frac{N}{L/M_0\mu_r A}$$

$$(i) \Rightarrow L = N \left( \frac{1}{L/M_0\mu_r A} \right)$$

$$\therefore S = \frac{L}{M_0\mu_r A}$$

$$L = \frac{N^2}{S} \text{ henry} \rightarrow (ii)$$

---


$$L = \frac{N\phi}{I} \Rightarrow L I = N\phi$$

$$\frac{d}{dt}(LI) = \frac{d}{dt}(N\phi) \Rightarrow L \frac{dI}{dt} = N \frac{d\phi}{dt}$$

$$\text{w.k.t, } e = N \frac{d\phi}{dt} \quad \therefore \boxed{e = L \frac{dI}{dt}}$$

## Mutually Induced emf :-

Consider two coils A & B.

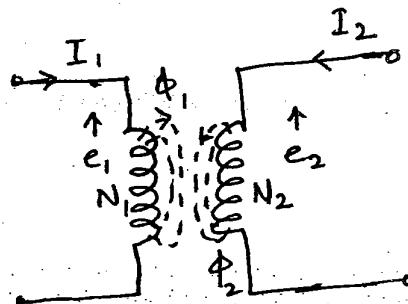
The coils A & B are said to be mutually coupled, if there is an interchange of energy between the two coils A & B.

$\therefore$  The emf induced in one coil due to the change of flux linkages from the other coil, when the current flows through the other coil.

The Co-eff of mutual inductance between two coils is defined as the weber turns in one coil due to one ampere current in the other coil.

$$\therefore M = \frac{N_2 \phi_1}{I_1}$$

$$(or) \quad M = \frac{N_1 \phi_2}{I_2}$$



## Co-efficient of Coupling (k) :-

Let us consider two magnetically coupled coils A & B, which have  $N_1$  &  $N_2$  turns respectively.

The Self Inductances of the coils can be written as,

$$\text{Coil A}, \quad L_1 = \frac{N_1^2}{S} \quad (\text{or}) \quad L_1 = \frac{N_1^2}{l/\mu_0 \mu_r A}$$

$$\& \text{ Coil B}, \quad L_2 = \frac{N_2^2}{S} \quad (\text{or}) \quad L_2 = \frac{N_2^2}{l/\mu_0 \mu_r A}$$

When Current  $I_1$  amperes flows in coil A,  
The flux produced is,

$$\phi_1 = \frac{N_1 I_1}{S} = \frac{N_1 I_1}{l/\mu_0 \mu_r A}$$

~~If~~ if a fraction  $k_1$  of  $\phi_1$ , i.e.,  $k_1 \phi_1$  is linked with coil B,

$$\text{then, } M = \frac{k_1 \phi_1 N_2}{I_1} \quad k_1 \leq 1$$

$\phi_1$  in M

$$\therefore M = k_1 \frac{N_1 N_2}{S} \rightarrow (i)$$

Similarly, when a current of  $I_2$  amperes flows in Coil B, the flux produced is,

$$\phi_2 = \frac{N_2 I_2}{s} = \frac{N_2 I_2}{4\pi M_0 A}$$

Let a fraction  $k_2$  of  $\phi_2$ , i.e.,  $k_2 \phi_2$  be linked with coil A.

Then,  $M = \frac{k_2 \phi_2 N_1}{I_2} \quad | k_2 \leq 1$

$\phi_2$  in M

$$\therefore M = k_2 \frac{N_1 N_2}{s} \rightarrow (ii)$$

Multiplying eqn's (i) & (ii),

$$M^2 = k_1 k_2 \frac{N_1^2 N_2^2}{s^2} = k_1 k_2 \frac{N_1^2}{s} \cdot \frac{N_2^2}{s}$$

Substituting for,  $\frac{N_1^2}{s}$  &  $\frac{N_2^2}{s}$

$$M^2 = k_1 k_2 L_1 L_2$$

$$M = \sqrt{k_1 k_2} \cdot \sqrt{L_1 L_2}$$

$$\therefore M = k \sqrt{L_1 L_2}$$

$$(r) \quad k = \frac{M}{\sqrt{L_1 L_2}}$$

$k$  is known as Co-eff of Coupling, which gives the relation between Self & Mutual Inductance.

## Energy stored in the magnetic field of an Inductor:-

Let us consider an electric circuit with an ideal inductor of Inductance 'L'. When the current increases from zero to a final steady state value after a definite time lag, the self inductance opposes the growth of current. Some work has to be performed in increasing the current from zero to a max value against the induced emf. This work done by the current is stored up in the magnetic field as magnetic energy.

The applied voltage is equal to the induced emf.

$$\therefore e = L \frac{di}{dt} \quad \& \quad V = e$$

$$\therefore V = L \frac{di}{dt}$$

The work done in small time interval  $dt$  seconds is given by,

$$dw = V \cdot i \cdot dt \text{ joules}$$

$$dw = \left( L \frac{di}{dt} \right) \cdot i \cdot dt$$

$$\therefore dw = L \cdot i \cdot di \text{ joules}$$

From the instant the current is zero to its final steady state value of  $I$  amperes, the total work (or energy stored) is the summation of work done over the interval 0 to  $I$ .

$$\therefore E = \int_0^t dw = \int_0^I L \cdot i \cdot di$$

$$\underline{E = \frac{1}{2} L I^2 \text{ joules}}$$

### Analogy between Electric and Magnetic circuits:-

- \*> Energy must be continuously supplied to an electric circuit to maintain current in it, whereas the magnetic flux once set up, does not need any further supply of energy.
- \*> The magnetic circuit stores energy in its field, whereas the electric circuit immediately releases its energy as heat.
- \*> In magnetic circuits involving ferromagnetic materials, on increasing the magnetic field strength the flux density increases only till the state of saturation is reached. But in electric circuits, there is no phenomenon as saturation.

## Similarities between Electric and Magnetic Circuits:-

### Electric Circuit

### Magnetic Circuit

- 1) The closed path for electric current is called Electric circuit. The closed path for magnetic flux is called magnetic circuit.
- 2) The cause of current flow is emf. The cause of generation of flux is mmf.
- 3) Opposition to flow of current is offered by resistance ( $R$ ). Opposition to flow/creation of flux is offered by reluctance ( $S$ )
- 4) Current,  $I = \frac{V}{R}$  Amperes Flux,  $\phi = \frac{\text{MMF}}{S}$  wb.
- 5)  $R = \rho \frac{l}{A}$   $\Omega$   $S = \frac{1}{4} \frac{l}{A}$  AT/wb
- 6) Current density, Flux density,  
 $J = \frac{I}{A}$  A/m<sup>2</sup>  $B = \frac{\phi}{A}$  wb/m<sup>2</sup>
- 7) Conductance, Permeance,  
 $G = \frac{1}{R}$  Siemens  $\mathcal{G} = \frac{1}{S}$  wb/A

### Numerical on Electromagnetism

Q.1) A conductor of length 1m moves at right angles to a uniform magnetic field of flux density 1.5Wb/m<sup>2</sup> with a velocity of 50m/s. Calculate the emf induced in it. Find also the value of induced, when the conductor moves at an angle of 30° to the direction of the field & when it moves parallel to the field.

**Soln:**

Given Data: Length (l) = 1m, B= 1.5Wb/m<sup>2</sup>, v= 50m/s.

Emf induced (e) = Blv Sine

(a ) θ = 90°                    (Right angle to the field)

$$e = 1.5 \times 1 \times 50 \times \sin 90 = 75V$$

(b) θ = 30°

$$e = 1.5 \times 1 \times 50 \times \sin 30 = 37.5V$$

(c ) θ = 0°                    (Parallel to the field)

$$e = 1.5 \times 1 \times 50 \times \sin 0 = 0V$$

Q.2) An iron-cored torriodal coil has 100 turns, a cross sectional area 10cm<sup>2</sup> & a mean length of 314cm. If the relative permeability of iron is 1000. Calculate the inductance of the coil.

**Soln:**

Given data: N= 100 turns, a= 10cm<sup>2</sup> = 10 \*10<sup>-4</sup> m<sup>2</sup>, l= 314cm = 314 \*10<sup>-2</sup>m, μ<sub>r</sub> = 1000,

$$\mu_0 = 4\pi \times 10^{-7}H/m$$

$$L = N^2 / S \quad \text{where } S \text{ is reluctance} = l / \mu_r \mu_0 a$$

$$\text{Therefore, } L = N^2 \mu_r \mu_0 a / l$$

$$= [ (100)^2 * 1000 * 4\pi \times 10^{-7} * 10 * 10^{-4} ] / (314 * 10^{-2})$$

$$L = 4 * 10^{-3} H = 4mH.$$

Q.3) A coil consist of 750 turns, a current of 10A in the coil gives rise to a magnetic flux of 1200μWb. Determine the inductance of the coil & average emf induced in the coil, when the current is reversed in 0.01 sec.

**Soln:**      Given Data:N= 750 turns, I=10A,

$$\Phi = 1200\mu Wb = 1200 * 10^{-6} Wb, \quad t = 0.01sec.$$

$$e = L \frac{di}{dt} \quad \text{-----(eqn 1)}$$

$$\text{Inductance (L)} = \frac{N\Phi}{I} = \frac{750 * 0.0012}{10} = 0.09H = 90mH$$

$$\frac{di}{dt} = \frac{10 - (-10)}{0.01} = 2000 A/s$$

**Sub: values in eqn (1)**

Average emf induced in the coil (e) = 0.09 \* 2000 = 180V

Q.4) Two coils with 800 & 2000 turns are mutually coupled. A current of 10A in coil 2 produces a flux of 1mWb in coil 2 & 0.6mWb links with coil 1. Calculate L<sub>1</sub>, L<sub>2</sub>, M & K.

**Soln:**      Given Data:N<sub>1</sub>= 800 turns, N<sub>2</sub>= 2000 turns I<sub>2</sub> =10A,

$$\Phi_2 = 1mWb = 1 * 10^{-3}Wb, \quad \Phi_{21} = 0.6mWb = 0.6 * 10^{-3}Wb$$

$$K = \Phi_{21} / \Phi_2 = 0.0006 / 0.001 = 0.6$$

$$L_2 = N_2 \Phi_2 / I_2 = (2000 * 10^{-3}) / 10 = 0.2 H$$

$$\text{i.e, } L = N^2 \mu_r \mu_0 a / l$$

$$L_1 = N_1^2 \mu_r \mu_0 a / l, \quad \& \quad L_2 = N_2^2 \mu_r \mu_0 a / l$$

$$L_1 / L_2 = N_1^2 / N_2^2 \quad \text{i.e, } L_1 = (800/2000)^2 * 0.2 = 0.032H$$

$$M = K \sqrt{L_1 * L_2} = 0.6 \sqrt{0.2 * 0.032} = 0.048 H$$

Q.5) Two coils having 150 & 250 Turns are wound on an iron circuit with μ<sub>r</sub> = 2000, l=250cm & area of X-section 140cm<sup>2</sup>. Find (i) inductance of each coil, (ii) Mutual inductance & (iii) Emf induced in first coil, if the current through the second coil changes from 0 to 5A in 0.025sec, K= 1.

**Soln:** Given Data:N<sub>1</sub>= 150 turns, N<sub>2</sub>= 250 turns I =5A, μ<sub>r</sub> = 2000, l=250cm=2.5m, a= 140cm<sup>2</sup> = 140 \* 10<sup>-4</sup> m<sup>2</sup>, μ<sub>0</sub> = 4π x 10<sup>-7</sup>H/m

**(i) Inductance of each coil**

$$L_1 = N_1^2 \mu_r \mu_0 a / l = (150)^2 * 2000 * 4\pi * 10^{-7} * 140 * 10^{-4} / 2.5 = 0.316 \text{ H}$$

$$L_2 = N_2^2 \mu_r \mu_0 a / l = (250)^2 * 2000 * 4\pi * 10^{-7} * 140 * 10^{-4} / 2.5 = 0.879 \text{ H}$$

**(ii) Mutual inductance**

$$M = K\sqrt{L_1 * L_2} = \sqrt{0.316 * 0.879} = 0.527 \text{ H}$$

**(iii) Emf induced in the first coil due to second coil ,  $e_M = M * \frac{di}{dt}$**

$$= 0.527 * \frac{(5-0)}{0.025}$$

$$= 105.4 \text{ V}$$

## SINGLE-PHASE CIRCUITS:-

### Advantages of Sinusoidal waveforms :-

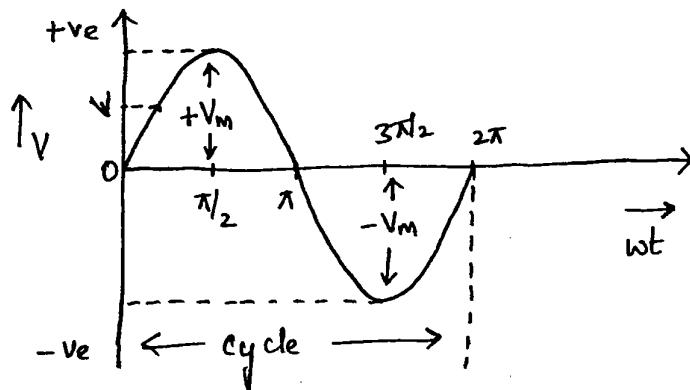
- \*> Many phenomena occurring in nature are of sinusoidal type. ex:- The motion of pendulum, Swing.
- \*> The derivative & integral of a sinusoidal functions are also sinusoidal in nature.
- \*> When a 3 $\phi$  sinusoidal voltage is applied to the windings of a motor, It produces a RMF, which has the capacity to do work.
- \*> When the Current in a Capacitor (or) an Inductor is sinusoidal, The voltage across them is also sinusoidal. This is not true for any other waveform.

### Important Definitions :-

- 1) Alternating Quantity :- An alternating quantity is one which acts in alternate positive and negative directions.
- 2) Waveform :- The graph between an alternating quantity (Voltage or Current) and time is called waveform.

Fig(1). Shows the waveform of a Sinusoidal Voltage.

3) Instantaneous value :- The value of an alternating quantity at any instant is called Instantaneous value.



4) Cycle :- One complete set of +ve & -ve values of an alternating quantity is called a cycle.

The values of sine wave repeat after every  $2\pi$  radians.

5) Time Period & Frequency :-

The time duration required for the quantity to complete one cycle is known as Time period ( $T$ ).

The no of cycles completed per second by an alternating quantity is known as Frequency ( $f$ ).

$$f = \frac{1}{T}$$

The unit of frequency is Hertz.

$$\text{If, } f = 50 \text{ Hz}, \quad T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ secs}$$

(\* Motors & transformers are smaller & lighter at higher frequency).

6) Amplitude :- The maximum value, +ve or -ve, which an alternating quantity attains during one complete cycle is called amplitude. (or) peak value (or) maximum value.

7) Angular frequency :- It is the no of radians ~~covered~~ covered in one second.  
It is denoted as ' $\omega$ '.

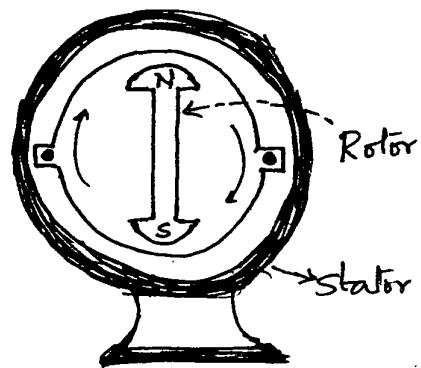
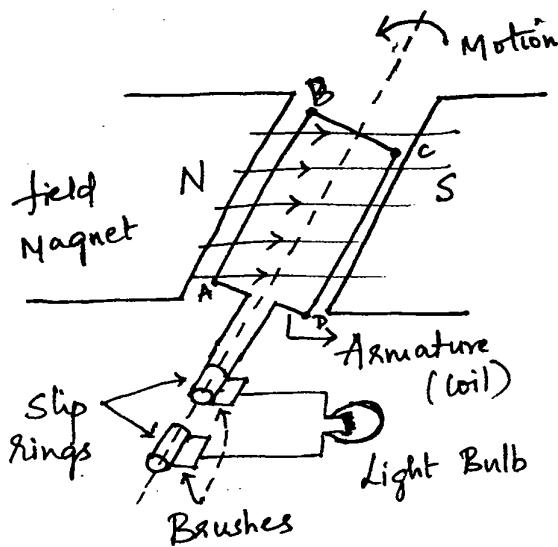
$$\omega = 2\pi f \quad (\text{or}) \quad \omega = \frac{2\pi}{T}$$

Generation of Sinusoidal AC Voltage :-

Alternating voltage may be generated :-

a) By rotating a coil in a magnetic field as shown in fig(a).

b) By rotating a magnetic field within a stationary coil as shown in fig(b).



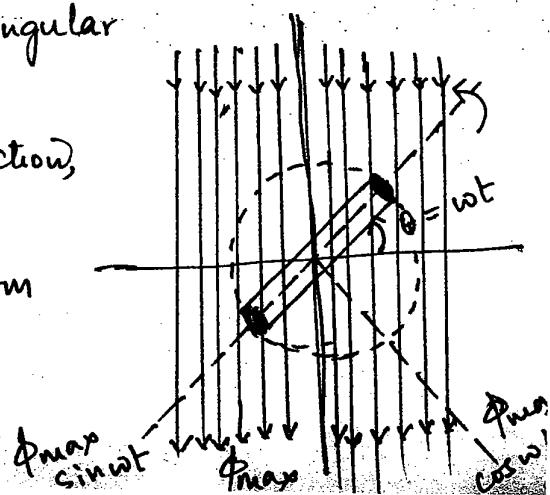
fig(b) :-

fig (a) :-

- The value of the alternating voltage depends upon the no. of turns in the coil, the strength of the field & the speed at which the coil (or) magnetic field rotates.
- The alternating voltage generated has regular changes in magnitude & direction.

### Equation of Alternating EMF :-

Let us consider a rectangular coil of 'N' turns rotating in the anti-clockwise direction, with an angular velocity of  $\omega$  rad/sec in a uniform magnetic field as shown in fig.



Let the time be measured from the instant of coincidence of the plane of the coil with x-axis. At this instant max flux ( $\phi_{max}$ ) links with the coil. As the coil rotates, the flux linking with it changes & hence emf is induced in it. Let the coil turn through an angle  $\theta$  in time 't' seconds. & let it assume the position as shown in fig. Obviously  $\theta = \omega t$ .

When the coil is in this position, the  $\phi_{max}$  acting vertically downwards can be resolved into two components, each perpendicular to the other, namely :-

a) Component  $\phi_{max} \sin \omega t$ ,  $\parallel$  to the plane of the coil.

This component doesn't induce emf as it is  $\parallel$  to the plane of the coil.

b) Component  $\phi_{max} \cos \omega t$ ,  $\perp$  to the plane of the coil.

This component induces emf in the coil.

$\therefore$  flux linkages of the coil at that instant

(at  $\theta$ ) is

$$= N \text{ of turns} \times \text{flux linking}$$

$$= N \phi_{max} \cos \omega t$$

=

As per faraday's laws of EMI, the emf induced in a coil is equal to the rate of change of flux linkages of the coil. So instantaneous emf 'e' induced in the coil at this instant is :-

$$\begin{aligned}
 e &= -\frac{d}{dt} (\text{flux linkages}) \\
 &= -\frac{d}{dt} (N \phi_{\max} \cos \omega t) \\
 &= -N \phi_{\max} \frac{d}{dt} (\cos \omega t) \\
 &= -N \phi_{\max} \omega (-\sin \omega t) \\
 \therefore e &= -N \omega \phi_{\max} \sin \omega t \quad \text{Volts} \quad \rightarrow (i)
 \end{aligned}$$

It is apparent from eqn (i), that the value of 'e' will be maximum ( $E_m$ ), when the coil has rotated through  $90^\circ$  ( $\because \sin 90^\circ = 1$ ).

$$\text{thus, } E_m = N \omega \phi_{\max} \text{ Volts.} \rightarrow (ii)$$

Substituting the value of  $N \omega \phi_{\max}$  in (i).

$$\underline{e = E_m \sin \omega t \quad \text{Volts}} \rightarrow (iii)$$

$$\theta = \omega t, \quad e = E_m \sin \omega t$$

$$\omega = 2\pi f, \quad e = E_m \sin 2\pi f t$$

$$f = \frac{1}{T}, \quad e = E_m \sin \left( \frac{2\pi}{T} \right) t$$

### Equation of Alternating Current:

When an alternating voltage,  $e = E_m \sin \omega t$  is applied across a load, an alternating current flows through the circuit which will also have a sinusoidal variation.

The expression for the alternating current is,  $i = I_m \sin \omega t$ .

### RMS Value :-

The rms (or) effective value of an alternating current is defined as that steady current which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current, when flowing through the same resistance for the same time.

Let us consider an elementary strip of thickness  $d\theta$  in the first half cycle of the squared wave, as shown in fig:-

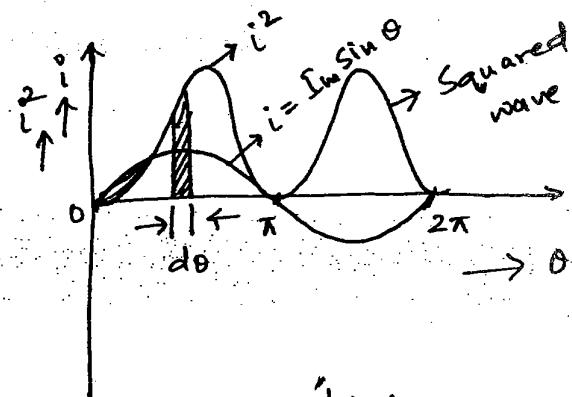


fig:-

$$\text{Area of the strip} = i^2 d\theta$$

$$\text{Area of first half-cycle of Squared wave} = \int_0^{\pi} i^2 d\theta$$

$$\begin{aligned}
 &= \int_0^{\pi} (I_m \sin \theta)^2 d\theta = \int_0^{\pi} I_m^2 \sin^2 \theta \cdot d\theta \\
 &= I_m^2 \int_0^{\pi} \frac{1 - \cos 2\theta}{2} \cdot d\theta \quad \left| \because \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \right. \\
 &= \frac{I_m^2}{2} \int_0^{\pi} (1 - \cos 2\theta) \cdot d\theta \\
 &= \frac{I_m^2}{2} \left[ \theta - \frac{\sin 2\theta}{2} \right]_0^{\pi} \\
 &= \frac{I_m^2}{2} \cdot \pi
 \end{aligned}$$

$$\begin{aligned}
 \therefore I &= \sqrt{\frac{\text{Area of first half-cycle of Sq-wave}}{\text{base}}} \\
 &= \sqrt{\frac{\frac{I_m^2}{2} \cdot \pi}{\pi}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}
 \end{aligned}$$

$$\therefore I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

R.M.S Value of Current =  $0.707 \times$  maximum value of current

$$\text{Similarly, } E = 0.707 E_m$$

OR

Power in 'R' due to DC ~~Current~~ (I) is  
 $I^2 R$ .

$$\therefore P_{avg} = \frac{I_m^2 R}{2}$$

$$\therefore I^2 R = \frac{I_m^2 R}{2} \quad (\text{or}) \quad I^2 = \frac{I_m^2}{2}$$

Taking square root, we get the effective value of ac sinusoidal current as,

$$I_{eff} \quad (\text{or}) \quad I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

### Average Value :-

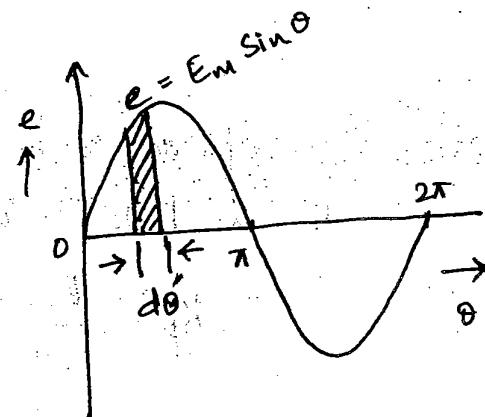
The arithmetic average of all <sup>the</sup> values of an alternating quantity over one cycle is called Average value.

The average value over the entire cycle is zero. Hence, the average value is obtained by considering only half - cycle.

$$E_{avg} = \frac{\text{Area under } \frac{1}{2} \text{ cycle}}{\text{Length of } \frac{1}{2} \text{ cycle.}}$$

$$= \int_0^{\pi} e \cdot d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi} E_m \sin \theta \cdot d\theta$$



(42)

$$= \frac{E_m}{\pi} \int_0^{\pi} \sin \theta \cdot d\theta$$

$$= \frac{E_m}{\pi} \left[ -\cos \theta \right]_0^{\pi} = \frac{E_m}{\pi} (2)$$

$$\therefore E_{avg} = \underline{\frac{2 E_m}{\pi}} = 0.637 E_m$$

Similarly,  $I_{avg} = \underline{\frac{2 I_m}{\pi}} = 0.637 I_m$

$\therefore$  Average Value of Current =  $0.637 \times$  maximum value of current.

### FORM FACTOR & PEAK FACTOR :-

Form factor ( $k_f$ ) :- The ratio of the RMS value to the average value.

$$k_f = \frac{E_{rms}}{E_{avg}} = \frac{0.707 E_m}{0.637 E_m} = \underline{\underline{1.11}}$$

Peak factor ( $k_p$ ) :- The ratio of peak value to its RMS value.

$$k_p = \frac{E_m}{0.707 E_m} = \underline{\underline{1.414}}$$

### PHASE :-

An alternating Voltage or Current changes its magnitude & direction at every instant. So it is necessary to know the condition of the alternating quantity at a particular instant. The location of the condition of the alternating quantity at any particular instant is called its phase.

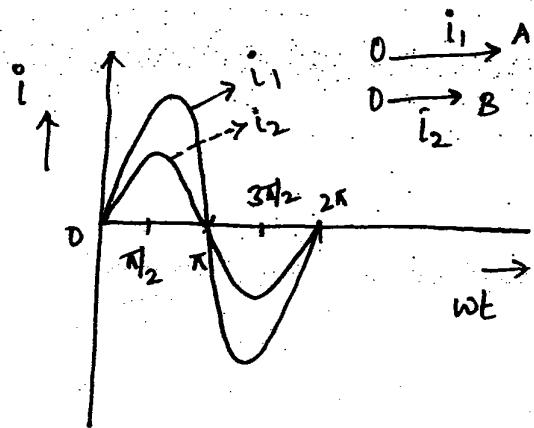
"The phase of an alternating quantity at any particular instant is the fractional part of a period (or) cycle through which the quantity has advanced from the selected origin".

### PHASE DIFFERENCE :- (Leading (or) Lagging)

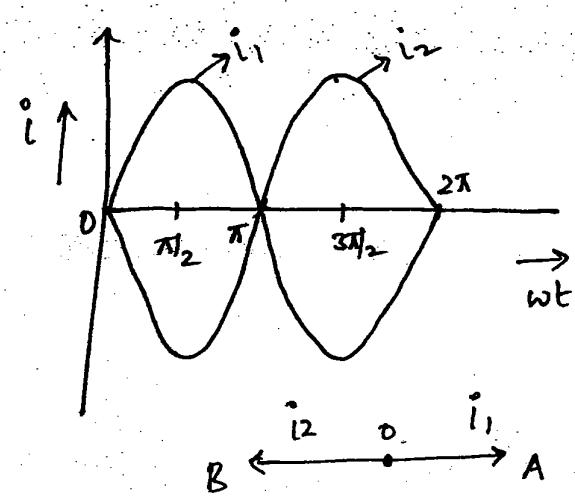
When two alternating quantities are considered simultaneously, the frequency being the same, they may not pass through a particular point at the same instant. One may pass through its maximum value at the instant when the other passes through a value other than its maximum one. These two quantities are said to have a phase difference. Phase difference is specified either in degrees (or) in radians.

Two quantities are said to be in phase with each other if they pass through zero values at the same instant & rise in the same direction,

as shown in fig (1), However if the two quantities pass through zero values at the same instant but rise in opposite directions, as shown in fig (2), they are said to be in phase opposition. i.e., the phase difference is  $180^\circ$ .



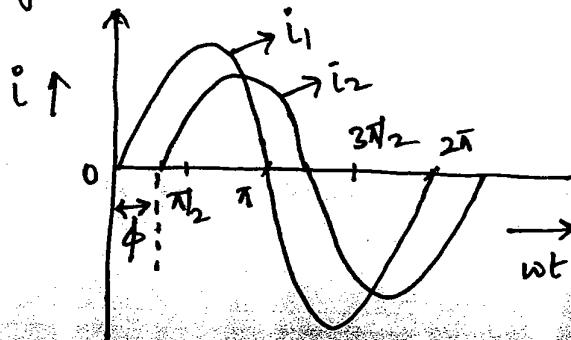
fig(1):-



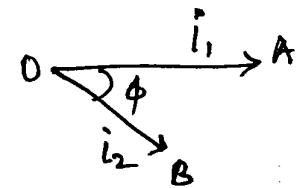
fig(2):-

The quantity ahead in phase is said to lead the other quantity, whereas the second quantity is said to lag the first one.

$i_1$  leads  $i_2$  by an angle  $\phi$  (or)  $i_2$  lags  $i_1$  by an angle  $\phi$ . as shown in fig (3).

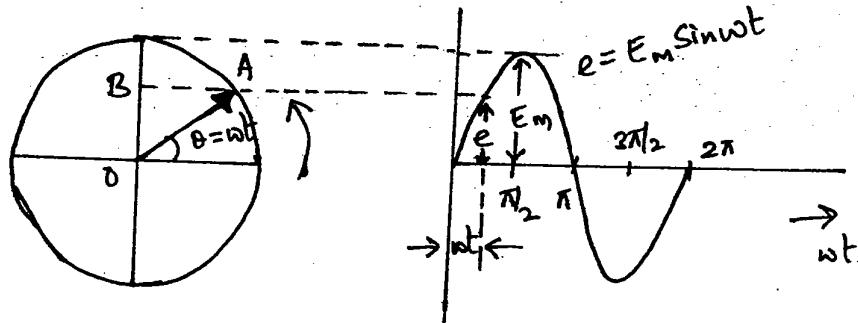


fig(3):-



## Phasor representation of Alternating quantities :-

We know that, an alternating quantity (Voltage or Current) has Sine waveform, & generators are designed to give emf's with the sine waveforms. The method of representing alternating quantities continuously by equations giving instantaneous values ( $e = E_m \sin \omega t$ ) is quite tedious. So it is more convenient to represent a sinusoidal quantity by a phasor rotating in an anticlockwise direction.



While representing an alternating quantity by a phasor (Vector).:-

- \*> The length of the phasor should be equal to the maximum value of the alternating quantity.
- \*> The phasor should be in the horizontal position at the instant the alternating quantity is zero. & is increasing in the positive direction.

- \*> The inclination of the line w.r.t some axis of reference gives the direction of that quantity & an arrow-head placed at one end indicates the direction in which that quantity acts.
- \*> the angular velocity in an anti-clockwise direction of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

$$\therefore OB = OA \sin \omega t$$

$$e = E_m \underline{\sin \omega t}$$

## AC Circuit Containing pure Ohmic Resistance

When an alternating Voltage is applied across a pure ohmic resistance, electrons (current) flow in one direction during the first half cycle & in the opposite direction during the next half-cycle, thus constituting alternating current in the circuit.

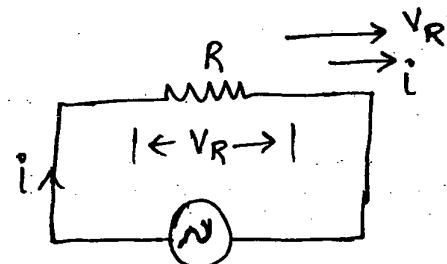
Let us consider an ac circuit with just a pure-resistance  $R$ . as shown in the fig (1):-

Let the applied voltage be given by the equation

$$V = V_m \sin \theta = V_m \sin \omega t \rightarrow (i). \quad V = V_m \sin \omega t$$

As a result of this alternating Voltage, an alternating current ( $i$ ) will flow through the circuit.

fig (1):-



The applied voltage has to supply the drop in the resistance, i.e.,

$$V = iR$$

$$V_m \sin \omega t = iR$$

$$(or) \quad i = \frac{V_m}{R} \sin \omega t \rightarrow (ii)$$

(48)

The value of the alternating current 'i' is maximum when,  $\sin \omega t = 1$

$$\text{i.e., } I_m = \frac{V_m}{R}$$

$\therefore$  (ii) becomes,

$$i = I_m \sin \omega t \rightarrow (\text{iii})$$

from eqn's (i) & (iii), it is apparent that voltage & current are in phase with each other. This is indicated by the wave & vector diagram as shown in fig (2):-

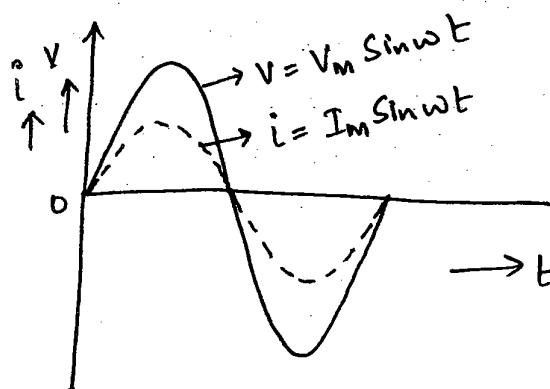


fig (2):-

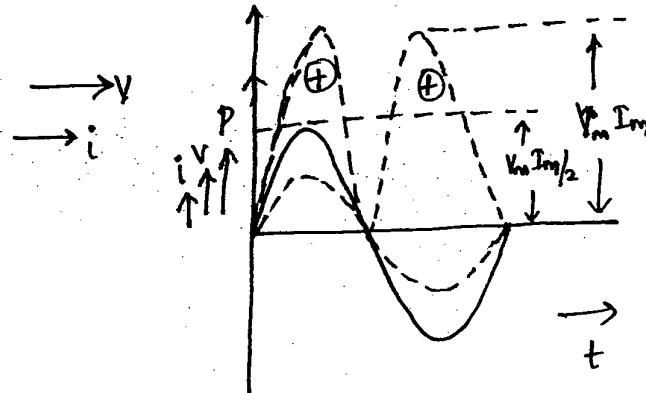


fig (3):-

power :- The voltage & current are changing at every instant.

$\therefore$  Instantaneous power,  $p = V_m \sin \omega t \cdot I_m \sin \omega t$

$$= V_m I_m \sin^2 \omega t$$

$$= V_m I_m \left( \frac{1 - \cos 2\omega t}{2} \right)$$

(49)

$$P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

thus, Instantaneous power consists of a  
Constant part  $\frac{V_m I_m}{2}$  & a fluctuating part

$\frac{V_m I_m}{2} \cos 2\omega t$  of frequency double that of  
the voltage & current waves.

The average value of  $\frac{V_m I_m \cos 2\omega t}{2}$  over a  
complete cycle is zero.

$$\therefore P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$P = VI \text{ watts}$$

V = r.m.s value of applied voltage.

I = r.m.s value of current.

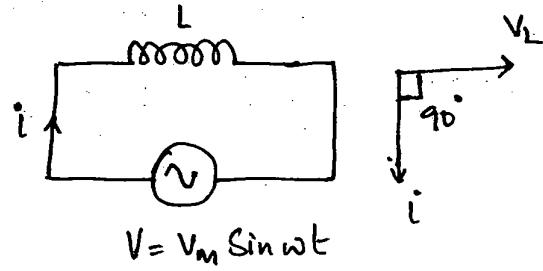
power curve for a purely resistive circuit is  
as shown in fig (3). This power is dissipated  
as heat.

### AC Circuit Containing Pure Inductance:

An Inductive Coil is a coil with or without an Iron Core & has negligible resistance (of laminated Iron Core).

In the application of an alternating voltage to a circuit containing a pure Inductance, a back emf is produced due to the self-inductance of the coil. This emf opposes the rise or fall of current, at every stage. Because of the absence of voltage drop, the applied voltage has to overcome this self-induced emf only.

Let the applied voltage be,  $V = V_m \sin \omega t$ , & the self-inductance of the coil =  $L$  Henry.



Self Induced emf in the coil,  $e_L = -L \frac{di}{dt}$

Applied voltage is equal & opposite to  $e_L$

$$\therefore V = -e_L$$

$$V_m \sin \omega t = -\left(-L \frac{di}{dt}\right)$$

$$di = \frac{V_m}{L} \sin \omega t \cdot dt$$

$$i = \frac{V_m}{L} \int \sin \omega t \cdot dt$$

$$i = \frac{V_m}{\omega L} (-\cos \omega t)$$

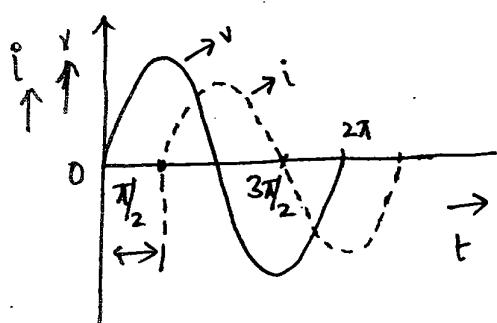
$$(or) i = \frac{V_m}{\omega L} \sin(\omega t - \pi/2)$$

'i' will be maximum when,  $\sin(\omega t - \pi/2) = 1$ .

$$I_m = \frac{V_m}{\omega L}$$

$$\therefore i = I_m \sin(\omega t - \pi/2).$$

from the expressions of applied voltage & current flowing through a purely Inductive coil, it is clear that the current lags behind the voltage by  $\pi/2$  as shown in fig (4).



fig(4):-

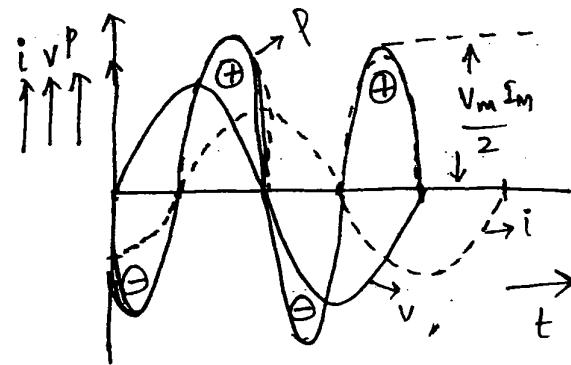


fig (5):-

' $\omega L$ ' is known as Inductive reactance & is denoted as ' $X_L$ '.

Power :- Instantaneous power,  $P = V I$

$$\begin{aligned}
 P &= V_m \sin \omega t \cdot I_m \sin (\omega t - \pi/2) \\
 &= -V_m I_m \sin \omega t \cos \omega t \\
 \therefore P &= -\frac{V_m I_m}{2} \sin 2\omega t
 \end{aligned}$$

power for the whole cycle,  $P = -\frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = 0$

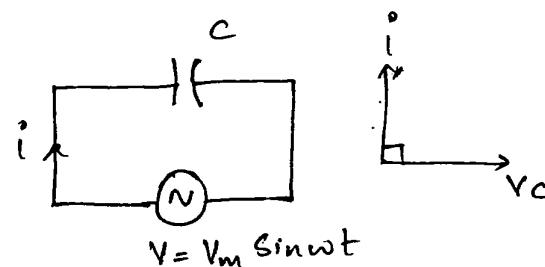
$\therefore$  The power absorbed in a pure Inductive Circuit is zero.

The power Curve is as shown in fig (5) :-

### AC Circuit Containing Pure Capacitance :-

When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction & then in the opposite direction as the voltage reverses.

Let,  $V = V_m \sin \omega t$ , be applied across a capacitor of capacitance 'C' farads.



Instantaneous charge,

$$q = CV = C V_m \sin \omega t$$

Capacitor Current is equal to the rate of Change of charge.

$$i = \frac{dq}{dt} = \frac{d(C V_m \sin \omega t)}{dt}$$

$$i = \omega C V_m \cos \omega t$$

$$i = \frac{V_m}{\omega C} \cos \omega t$$

$$(or) i = \frac{V_m}{\omega C} \sin(\omega t + \pi/2)$$

'i' is maximum, when  $\sin(\omega t + \pi/2)$  is 1.

$$I_m = \frac{V_m}{\omega C}$$

$$\therefore i = I_m \underline{\sin(\omega t + \pi/2)}$$

From the expressions of applied voltage & current it is clear that the current leads the voltage by  $\pi/2$  as shown in fig (6):

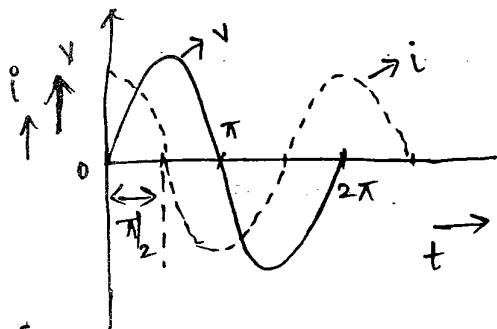


fig (6) :-

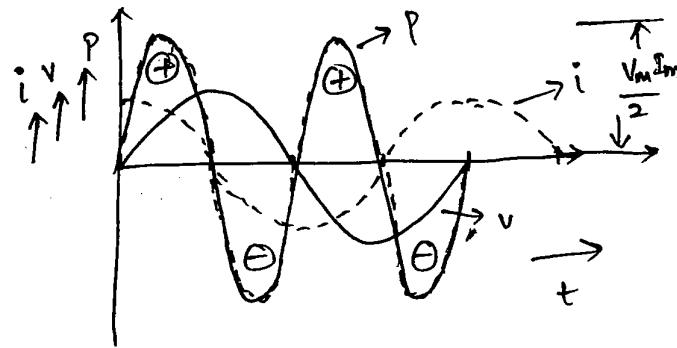


fig (7) :-

' $\frac{1}{\omega C}$ ' is known as capacitive reactance & is denoted as ' $X_c$ '.

Power :- Instantaneous power,  $P = V i$

$$P = V_m \sin \omega t \cdot I_m \sin(\omega t + \pi/2)$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

power for the complete cycle :-

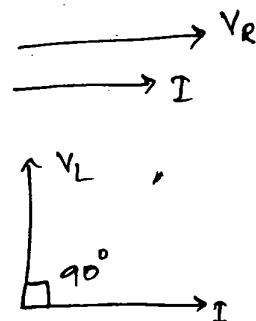
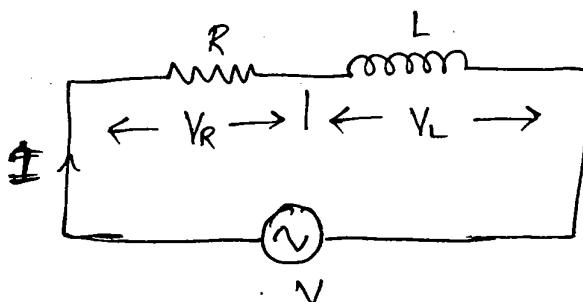
$$P = \frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \cdot dt = 0$$

thus, the ~~average~~ power ~~consumed~~ by a pure capacitor is zero.

The power curve is as shown in fig (7):-

### Series R-L Circuit :-

Let us consider an ac circuit containing pure resistance  $R$  & a pure Inductance  $L$  as shown in fig (8):-



fig(8):-

Let,  $V$  = rms value of the applied Voltage.

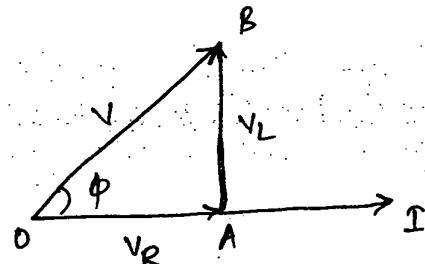
$I$  = rms value of the Current.

Voltage drop across  $R$ ,  $V_R = IR$  (in phase with  $I$ ).

Voltage drop across  $L$ ,  $V_L = IX_L$  (leading  $I$  by  $90^\circ$ ).

as shown in fig (9) :-

$$\begin{aligned} \therefore V &= \sqrt{V_R^2 + V_L^2} \\ &= \sqrt{(IR)^2 + (IX_L)^2} \\ &= I \sqrt{R^2 + X_L^2} \end{aligned}$$



fig(9) :-

$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

The term  $\sqrt{R^2 + X_L^2}$  offers opposition to current flow & is called the Impedance ( $Z$ ) of the circuit.

$$\therefore I = \frac{V}{Z}$$

Referring to the Impedance  $\triangle ABC$ , fig (10) :-

$$Z^2 = R^2 + X_L^2$$

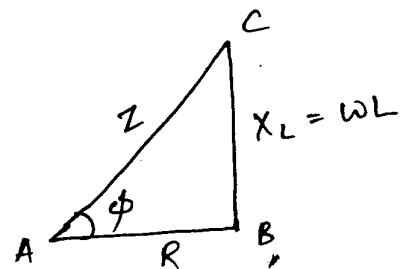


fig (10) :-

$$\& \tan \phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$\therefore \phi = \tan^{-1} \left( \frac{X_L}{R} \right)$$

$\therefore$  We observe that the Circuit Current lags behind applied voltage by an angle  $\phi$ .

$$\therefore i = I_m \sin(\omega t - \phi) \quad | \text{As shown in fig (11)}$$

$$I_m = \frac{V_m}{Z}$$

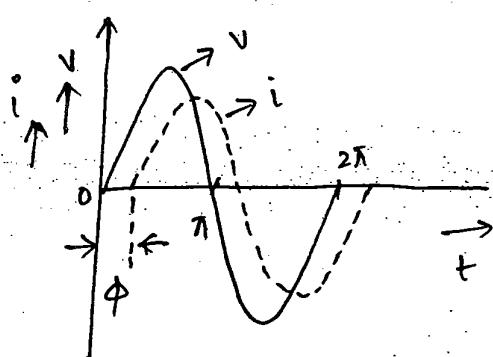


fig (11) :-

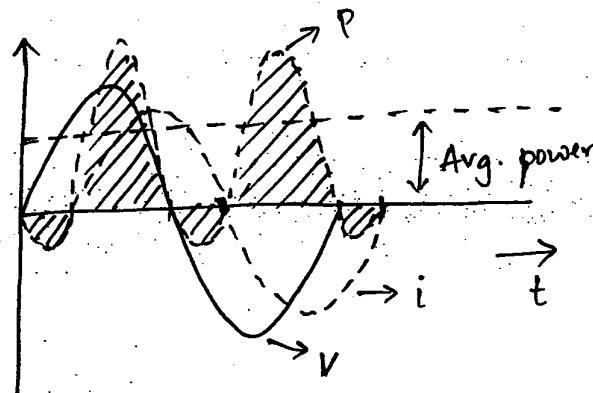


fig (12) :-

Power:- Instantaneous power,  $P = V i$

$$P = V_m \sin \omega t \cdot I_m \sin (\omega t - \phi)$$

$$= V_m I_m \sin \omega t \cdot \sin (\omega t - \phi)$$

$$= \frac{1}{2} V_m I_m [\cos \phi - \cos (2\omega t - \phi)]$$

$$\therefore P = \frac{1}{2} V_m I_m \cos \phi - \frac{1}{2} V_m I_m \cos (2\omega t - \phi)$$

(i) Constant part,  $\frac{1}{2} V_m I_m \cos \phi$ , which contributes to real power.

(ii) pulsating component,  $\frac{1}{2} V_m I_m \cos (2\omega t - \phi)$ , whose average value is zero over one complete cycle.

$$\therefore \text{Average power}, P = \frac{1}{2} V_m I_m \cos \phi$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi$$

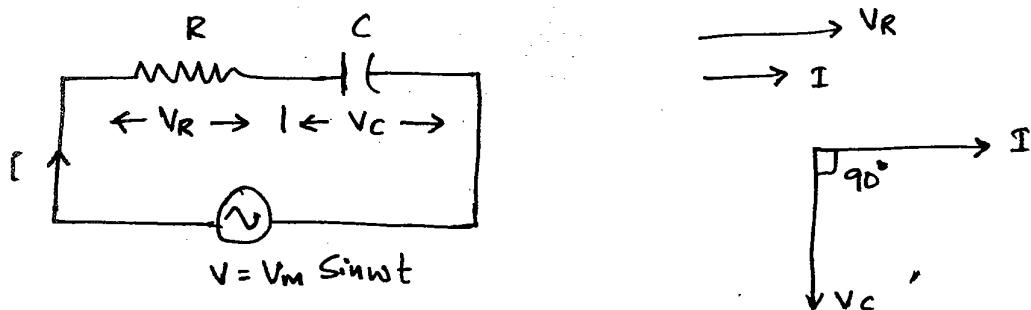
$$\therefore P = \underline{V I \cos \phi}$$

~~losses~~

The net power in the R-L series circuit over the cycle is positive & is as shown in fig(12).

### SERIES R-C CIRCUIT :-

Consider an a.c. circuit containing resistance 'R' ohms & Capacitance 'C' Farads, as shown in fig (13).



fig(13):-

Let,  $V$  = rms value of voltage

$I$  = rms value of current

$\therefore$  Voltage drop across  $R$ ,  $V_R = iR$

(in phase with  $I$ )

& Voltage drop across  $C$ ;  $V_C = I X_C$

as shown in fig (14). (Lagging  $I$  by  $90^\circ$ )

The Capacitive reactance is negative, so  $V_C$  is in the negative direction of Y-axis.

$$V = \sqrt{V_R^2 + (-V_C)^2}$$

$$= I \sqrt{R^2 + X_C^2}$$

$$(or) I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

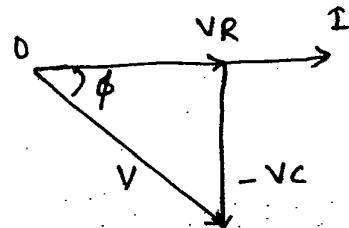


fig (14):-

$Z = \sqrt{R^2 + X_C^2}$ , is the Impedance of the circuit.

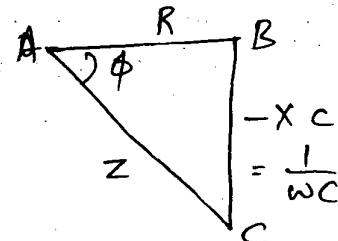
Referring to the Impedance  $\Delta$  ABC, fig(15):-

$$\phi = \tan^{-1} \left( -\frac{X_C}{R} \right)$$

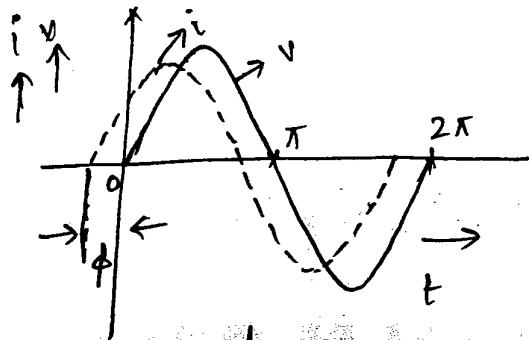
∴ We observe that the circuit current leads the applied voltage by an angle  $\phi$ .

$$\therefore i = I_m \sin(\omega t + \phi)$$

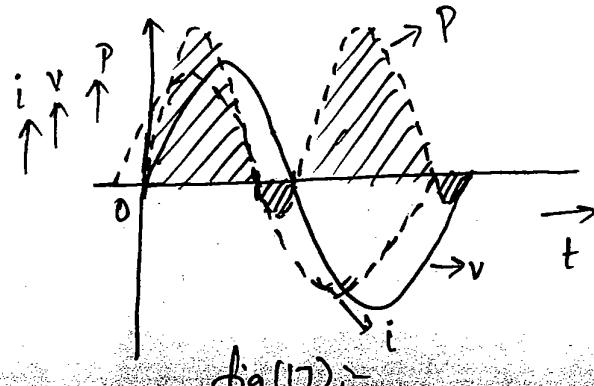
$$I_m = \frac{V_m}{Z}$$



fig(15):-



fig(16):-



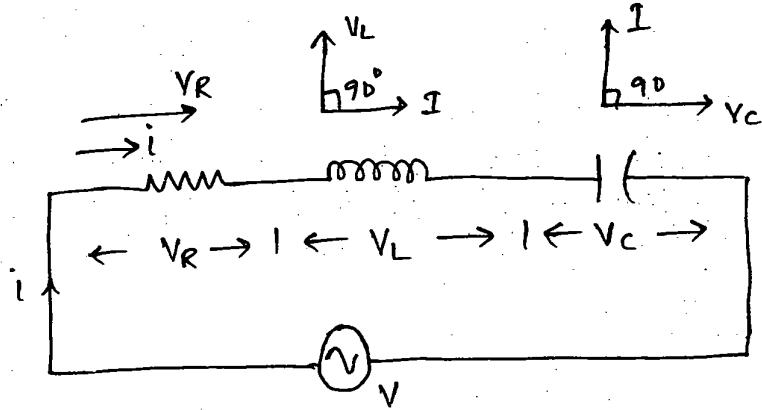
fig(17):-

$$\text{Power} := P = V I \cos \phi.$$

The waveforms for,  $P$ ,  $V$  &  $i$  are as shown in fig(16) & fig(17):-

### SERIES R-L-C CIRCUIT:-

Consider an a.c. circuit containing resistance ' $R$ ' ohms, Inductance ' $L$ ' henries & Capacitance ' $C$ ' farads, connected in series as shown in fig(18):-



fig(18):-

Let,  $V$  = rms value of applied voltage;

$I$  = rms value of current.

$\therefore$  Voltage drop across  $R$ ,  $V_R = IR$  (in phase with  $I$ )

$\Rightarrow$   $V_L = I \cdot X_L$  (Leading  $I$  by  $90^\circ$ )

$\Rightarrow$   $V_C = I \cdot X_C$  (Lagging  $I$  by  $90^\circ$ )

Referring to the Voltage  $\Delta^{\text{de}}$ , as shown in fig(19).~

60

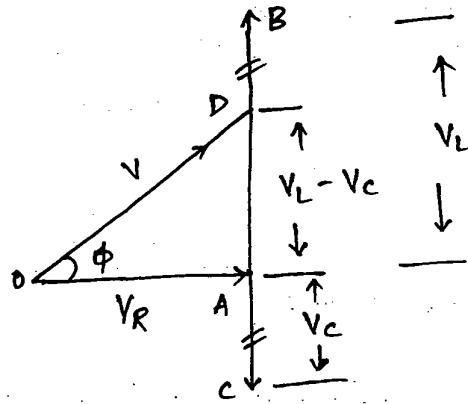
DA represents  $V_R$ ,

AB & AC represent  $V_L$  &  $V_C$

we observe that,

$V_L$  &  $V_C$  are  $180^\circ$

out of phase.



∴ The net reactive drop  
across the combination is

fig (19) :-

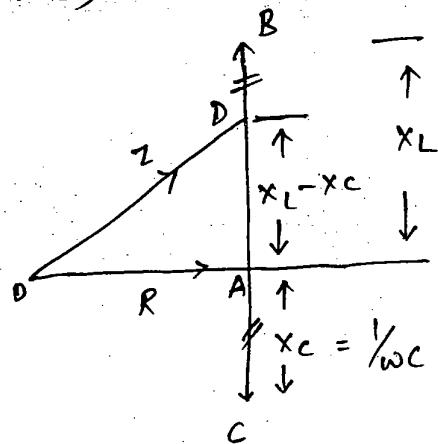
$$\begin{aligned} AD &= AB - AC \\ &= AB - BD \quad (\because BD = AC) \end{aligned}$$

$$= V_L - V_C$$

$$AD = I(X_L - X_C)$$

$$\therefore V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= I \sqrt{R^2 + (X_L - X_C)^2}$$



$$(08) \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

fig (20) :-

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ , is the Impedance of the Circuit.

Referring to the Impedance  $Z$ , fig (20) :-

$$\phi = \tan^{-1} \frac{(X_L - X_C)}{R} = \frac{x}{R}$$

$$\text{Powerfactor, } \cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + x^2}}$$

$$\underline{P = V I \cos \phi}$$

The Current expression in an R-L-C circuit  
is given as,

$$i = I_m \sin(\omega t \pm \phi)$$

Case (i) :- If  $X_L = X_C$ ,  $Z = R$

$$\therefore i = I_m \sin \omega t$$

Case (ii) :- If  $X_L > X_C$ ,

$$\text{then, } i = I_m \sin(\omega t - \phi).$$

Case (iii) :- If  $X_C > X_L$

$$\text{then, } i = I_m \sin(\omega t + \phi).$$

### PARALLEL AC CIRCUITS:-

In a parallel AC circuit, the voltage across each branch of the circuit is the same whereas current in each branch depends upon the branch impedances. Since, alternating currents are vector quantities, total line current is the vector sum of branch currents.

The following are the three methods of solving parallel a.c. circuits:-

- (i) Vector method
- (ii) Admittance method.
- (iii) Symbolic (or)  $j$ -method

## ADMITTANCE METHOD :-

The reciprocal of impedance of a circuit is called its "admittance". It is represented by "Y".

$$Y = \frac{1}{Z}$$

Its unit is Siemens (S) or mho ( $\omega$ ).

Let us consider a parallel circuit with three branches as shown in fig (a).

We can determine the conductance by just adding the conductances of three branches. & the susceptance is determined by the algebraic addition of the susceptances of all the three branches.

Total Conductance,

$$G = g_1 + g_2 + g_3$$

Total Susceptance,

$$B = (-b_1) + (-b_2) + b_3$$

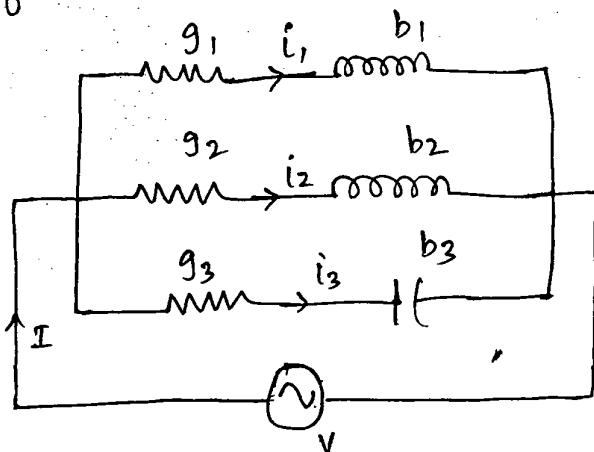


fig (a) :-

∴ Total Admittance,  $Y = \sqrt{G^2 + B^2}$

Total Current,  $I = YV$ .

Power factor,  $\cos \phi = \frac{G}{Y}$

NOTE :-

$G$  is always '+ve'.

$b$  is '+ve' if  $X_C$

$b$  is '-ve' if  $X_L$

### THREE-PHASE CIRCUITS:-

Comparison of three-phase system with single-phase system and Advantages of three-phase systems :-

- \*> A 3φ apparatus is more efficient than a 1φ apparatus.
- \*> For the same Capacity, a 3φ apparatus costs less than a 1φ apparatus.
- \*> The size of a 3φ apparatus is smaller than the size of a 1φ apparatus for the same Capacity & hence, requires less material for construction.
- \*> The amount of conductor material required is less in the case of a 3-φ system.
- \*> 3φ motors produce uniform torque, whereas the torque produced by the 1φ motors is pulsating.
- \*> Harmonics are produced when 1φ generators are connected in parallel, whereas 3φ generators can be conveniently connected in parallel without giving rise to harmonics.
- \*> In case of a 3φ Star ( $\lambda$ ) connected system, two different voltages can be obtained, one voltage between the lines & the other between the line & phase, whereas only one voltage can be obtained in a 1φ system.
- \*> 3φ Motors are self starting, whereas 1φ motors are not self starting.

### 3-phase E.M.F. Generation :-

In the three-phase system, there are three equal voltages of the same frequency but displaced from one another by  $120^\circ$  electrical. These voltages are produced by a 3-phase generator which has three identical windings or phases displaced by  $120^\circ$  electrical apart. When these windings are rotated in a magnetic field, emf is induced in each winding or phase. These emf's are of same magnitude & frequency but are displaced from one another by  $120^\circ$  electrical.

Consider 3 electrical coils  $a_1, a_2, b_1, b_2$  &  $c_1, c_2$  mounted on the same axis but displaced from each other by  $120^\circ$  electrical. Let the 3 coils be rotated in an anticlockwise direction in a bipolar magnetic field with an angular velocity of ' $w$ ' rad/sec, as shown in fig (a). Here  $a_1, b_1$  &  $c_1$  are the start terminals &  $a_2, b_2$  &  $c_2$  are the end terminals of the coils.

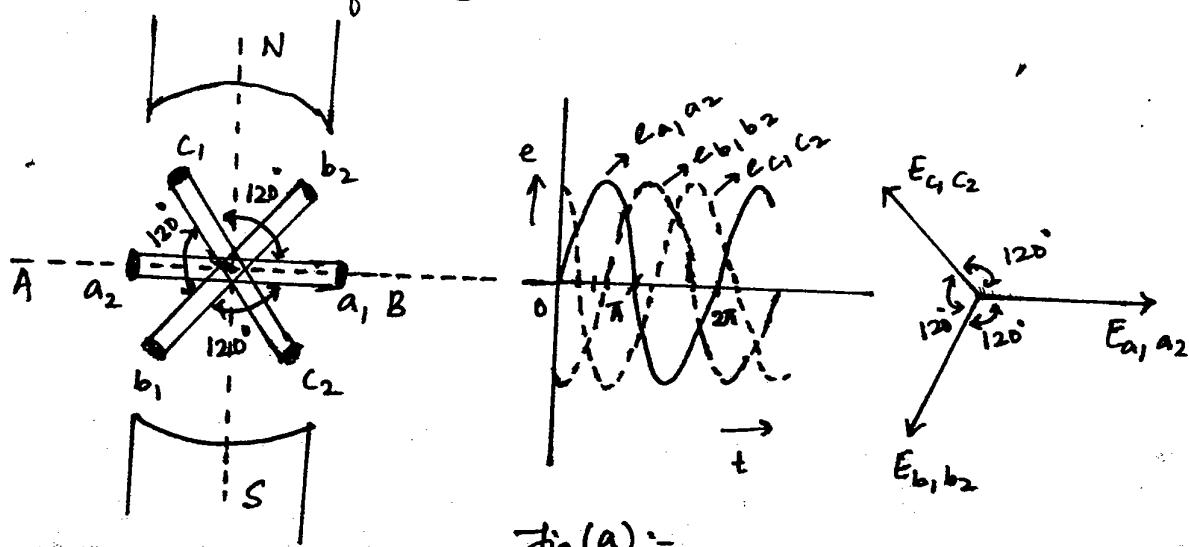


Fig (a) :-

When the coil is in position AB as shown in fig(a), the magnitude & direction of the emf's induced in the various coils is as follows :-

- (i) EMF induced in coil  $a_1, a_2$  is zero & is increasing in the positive direction. This is indicated by  $e_{a_1, a_2}$  wave in the fig (a).
- (ii) The coil  $b_1, b_2$  is  $120^\circ$  electrically behind  $a_1, a_2$ . The emf induced in this coil is negative & is approaching a maximum negative value. This is shown by the  $e_{b_1, b_2}$  wave.
- (iii) The coil  $c_1, c_2$  is  $240^\circ$  electrically behind  $a_1, a_2$  or  $120^\circ$  electrically behind the coil  $b_1, b_2$ . The emf induced in this coil is positive & is decreasing. This is indicated by wave  $e_{c_1, c_2}$ .

Thus, it is apparent that the emf's induced in the three coils are of the same magnitude & frequency but displaced  $120^\circ$  electrical from each other.

Vector Diagram :- The rms values of the three phase voltages are shown vectorially in fig(a):-

Equations :- The eqn's for three Voltages are:-

$$e_{a_1, a_2} = E_m \sin \omega t$$

$$e_{b_1, b_2} = E_m \sin(\omega t - 2\pi/3) \quad (\text{or}) \quad e_{b_1, b_2} = E_m \sin(\omega t - 120^\circ)$$

$$e_{c_1, c_2} = E_m \sin(\omega t - 4\pi/3) \quad (\text{or}) \quad e_{c_1, c_2} = E_m \sin(\omega t - 240^\circ)$$

### Phase Sequence :-

The order in which the voltages in the phases reach their maximum positive values is called the phase sequence.

For the coils  $a_1, a_2, b_1, b_2$  &  $c_1, c_2$ , the emf's in the 3-φ's attaining their maximum positive values is abc. Hence, the phase sequence is a, b, c.

### Naming the Phases :-

The 3 phases may be numbered (1, 2, 3) or lettered (a, b, c) or specified Colours (R Y B). By normal convention, Sequence R Y B is considered positive and RBY is negative.

### Double-Subscript Notation :-

It is necessary to employ some systematic notation for the solution of a.c. Circuits & Systems containing a no of emf's acting & currents flowing so that the process of solution is simplified & less prone to errors.

If emf is expressed as  $E_{ab}$ , it indicates that, emf acts from a to b;

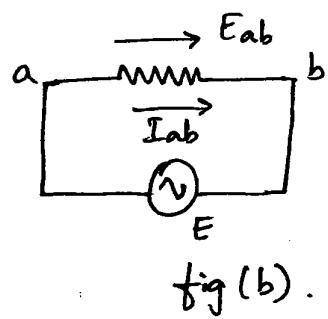


fig (b).

If it is expressed as  $E_{ba}$ , then the emf acts in a direction opposite to that in which  $E_{ab}$  acts.

$$\text{i.e., } E_{ba} = -E_{ab}$$

Similarly,  $I_{ab}$  indicates that current flows in the direction from a to b, but  $I_{ba}$  indicates that current flows in the direction from b to a.

$$\text{i.e., } I_{ba} = -I_{ab}$$

### Three-phase Balanced Supply and Load ( $\lambda$ & $\Delta$ ):-

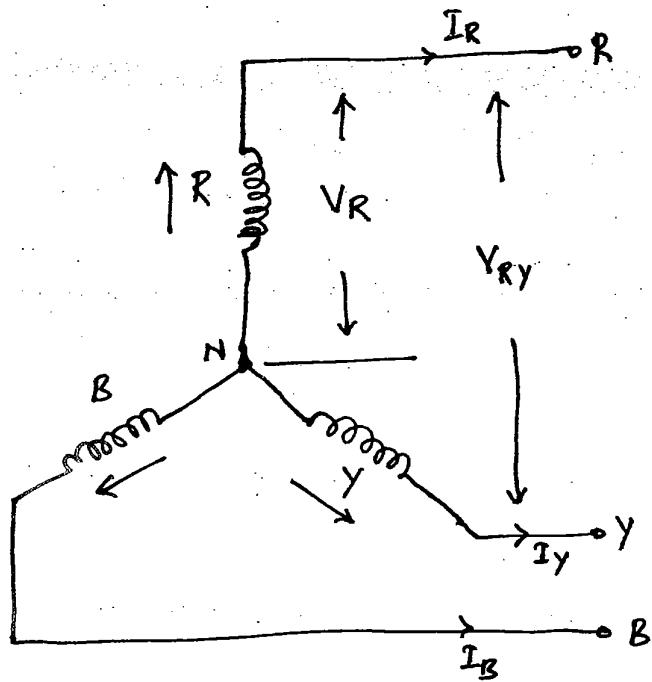
When a balanced generating supply, where the 3- $\phi$  voltages are equal, & the phase difference is  $120^\circ$ , supplies balanced equipment load, where the impedances of the 3- $\phi$  or 3 circuit loads are equal, then the current flowing through these 3- $\phi$ 's will also be equal in magnitude, & will also have a phase difference of  $120^\circ$  with one another. Such an arrangement is called a balanced load.

#### \* Significance of phase Sequence:-

- If the phase sequence is changed, then both the magnitudes & phases of the currents flowing in the lines & phases of the load will change.
- If the load is a 3 $\phi$  Inductor motor, with the change in phase sequence, the direction of the rotation of the motor also changes.

Relationship between Line & phase values &  
Expression for power in a balanced Star connection :

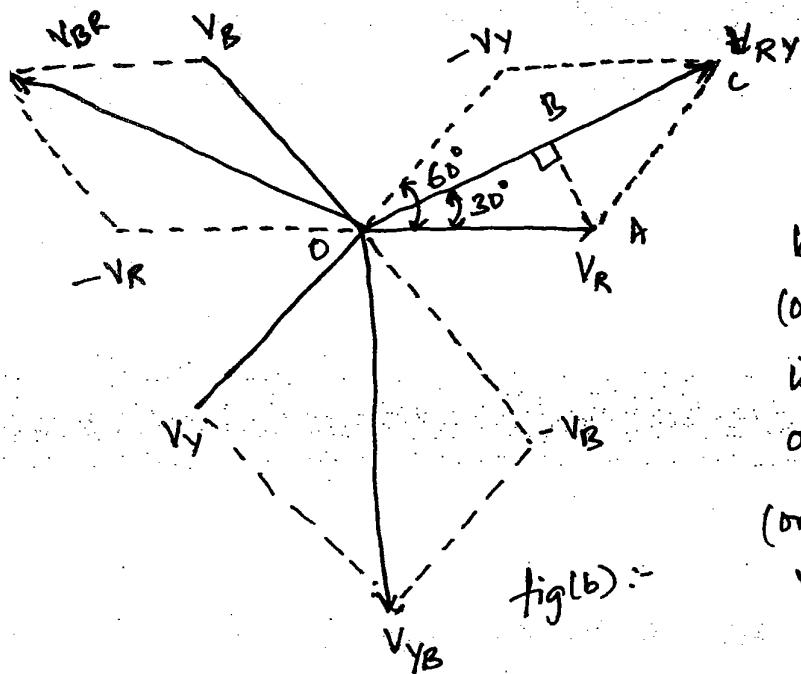
This system is obtained by joining together similar ends, either the start or the finish ends; the other ends are connected to the line wires, as shown in fig(a). The common point 'N' at which similar (start or finish) ends are connected is called the neutral or star point.



fig(a).

The voltage between any line & the neutral point, i.e., the Voltage across the phase winding is called the "phase Voltage". While the Voltage between any two others is called the "Line Voltage". Usually, the neutral point is connected to earth.

In fig(b), the emf's induced in the three phases are shown vectorially. In star-connection there are two windings between each pair of others & due to joining of similar ends together, the emf's induced in them are in opposition.



The potential difference between neutrals R & Y (or) Line voltage  $V_{RY}$ , is the vector difference of phase voltages  $V_R$  &  $V_Y$  (or) Vector sum of phase voltages  $V_R$  &  $(-V_Y)$

$$\text{i.e., } V_{RY} = V_R - V_Y \quad (\text{vector difference})$$

$$(\text{or}) \quad V_{RY} = V_R + (-V_Y) \quad (\text{vector sum})$$

As the phase angle between vectors  $V_R$  &  $(-V_Y)$  is  $60^\circ$ , from fig (b).

$$V_{RY} = \cos 30^\circ = \frac{OB}{OA}$$

$$\frac{\sqrt{3}}{2} = \frac{OC/2}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{V_{RY}/2}{V_R}$$

$$\Rightarrow V_R \frac{\sqrt{3}}{2} = \frac{V_{RY}}{2}$$

$$\therefore \underline{V_{RY}} = \sqrt{3} \underline{V_R}$$

i.e., line Voltage =  $\sqrt{3}$  phase Voltage.

$$\underline{V_L} = \sqrt{3} \underline{V_{ph}}$$

Since, in a star-connected system, each line conductor is connected to a separate phase.

Line current, ( $I_L$ ) = Phase current, ( $I_{ph}$ ).

If the phase current has a phase difference  $\phi$  with the phase voltage.

$$\text{power of P per phase} = \cancel{V_{ph} I_{ph}} \cos \phi$$

$$\text{Total power of P, } P = 3 V_{ph} I_{ph} \cos \phi$$

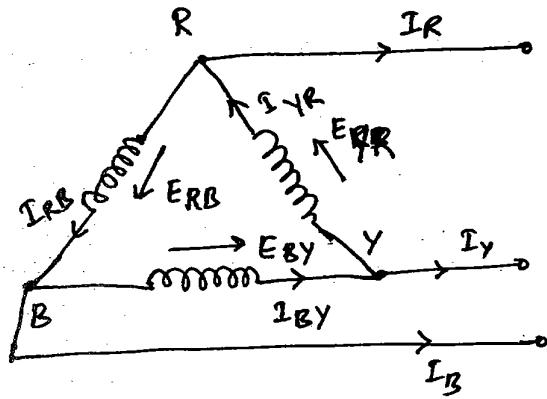
$$= 3 \cdot \frac{V_L}{\sqrt{3}} I_L \cos \phi$$

$$\therefore P = \underline{\underline{\sqrt{3} V_L I_L \cos \phi}}$$

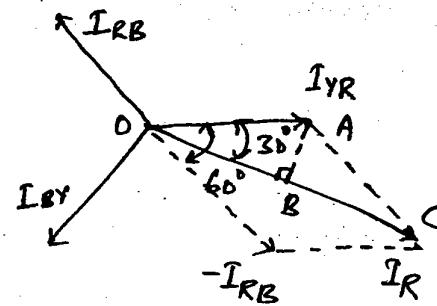
i.e., Power =  $\sqrt{3} \times \text{line voltage} \times \text{line current} \times \text{P.F.}$

Relationship between Line and Phase Values,  
and Expression for Power in a balanced Delta  
Connection:-

When starting end of one coil is connected to the finishing end of another coil, as shown in fig (i), delta or mesh connection is obtained. The direction of the emf's is as shown in the diagram.



fig(i)



fig(ii).

From fig it is clear that line Current is the Vector difference of phase currents of the two phases concerned.

$$\text{Line Current, } I_R = I_{YR} - I_{RB} \quad (\text{Vector difference})$$

$$(\text{or}) \quad I_R = I_{YR} + (-I_{RB}) \quad (\text{Vector sum})$$

from fig(ii)

$$\cos 30^\circ = \frac{OB}{OA} = \frac{OC/2}{OA}$$

$$\sqrt{3}/2 = \frac{I_R/2}{I_{YR}}$$

$$\frac{\sqrt{3} \cdot I_{YR}}{2} = \frac{I_R}{2}$$

$\therefore I_R = \sqrt{3} \cdot I_{YR}$   
 i.e., line current =  $\sqrt{3}$  . phase current.

$$\underline{I_L = \sqrt{3} \cdot I_{ph}}$$

In a delta connection, there is only one phase between any pair of line outs, so the potential difference b/w the line outs, called line voltage is equal to the phase voltage

$$\text{i.e. } \underline{\underline{V_L = V_{ph}}}$$

$$\text{Power per phase} = V_{ph} \cdot I_{ph} \cdot \cos \phi$$

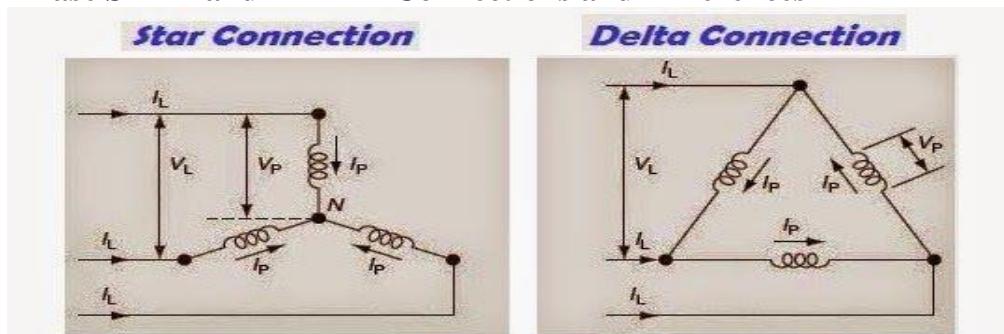
$$\text{Total power o/p, } P = 3 V_{ph} \cdot I_{ph} \cdot \cos \phi$$

$$= 3 \cdot V_L \cdot \frac{I_L}{\sqrt{3}} \cdot \cos \phi$$

$$\therefore P = \sqrt{3} V_L I_L \cos \phi$$

i.e., power =  $\sqrt{3} \times \text{Line Voltage} \times \text{Line Current} \times \text{P.F.}$

### 3 Phase STAR and DELTA Connections and Differences



Star (Y) Connection	Delta ( $\Delta$ ) Connection
In STAR connection, the starting or finishing ends (Similar ends) of three coils are connected together to form the neutral point. A common wire is taken out from the neutral point which is called Neutral.	In DELTA connection, the opposite ends of three coils are connected together. In other words, the end of each coil is connected with the start of another coil, and three wires are taken out from the coil joints
There is a <b>Neutral or Star Point</b>	No Neutral Point in Delta Connection
Three phase four wire system is derived from Star Connections ( <b>3-Phase, 4 Wires System</b> ) We may Also derived 3 Phase 3 Wire System from Star Connection	Three phase three wire system is derived from Delta Connections ( <b>3-Phase, 3 Wires System</b> )
Line Current = Phase Current $I_L = I_{PH}$	Line Voltage = Phase Voltage $V_L = V_{PH}$
Line Voltage is $\sqrt{3}$ times of Phase Voltage. i.e. $V_L = \sqrt{3} V_{PH}$	Line Current is $\sqrt{3}$ times of Phase Current. i.e. $I_L = \sqrt{3} I_{PH}$
The Total Power of three phases could be found by $P = \sqrt{3} \times V_L \times I_L \times \cos\Phi \dots \text{Or}$ $P = 3 \times V_{PH} \times I_{PH} \times \cos\Phi$	The Total Power of three phases could be found by $P = \sqrt{3} \times V_L \times I_L \times \cos\Phi \dots \text{or}$ $P = 3 \times V_{PH} \times I_{PH} \times \cos\Phi$
The speeds of Star connected motors are slow as they receive $1/\sqrt{3}$ voltage.	The speeds of Delta connected motors are high because each phase gets the total of line voltage
In Star Connection, the phase voltage is low as $1/\sqrt{3}$ of the line voltage, so, it needs low number of turns, hence, saving in copper.	In Delta connection, The phase voltage is equal to the line voltage, hence, it needs more number of turns.
Low insulation required as phase voltage is low	Heavy insulation required as Phase voltage = Line Voltage.
In Power Transmission, Star Connection system is general and typical to be used.	In Power Distribution and industries, Delta Connection is general and typical to be used.

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# Unit II

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**Contents** Working principle of D.C.Machine as generator and motor, constructional features, EMF equation of generator and simple problems, back emf and torque equation of DC motors, simple problems, types of DC motors.

A machine which works on direct current is defined as a D.C.Machine.

D.C.Machines are of two types. (i) D.C.Generator and (ii) D.C.Motor.

Table 2.1: Differences between DC Generator & DC Motor

D.C. Generator	D.C.Motor
<b>Definition:</b> A generator is a rotating machine which converts mechanical energy into electrical energy	<b>Definition:</b> A motor is a machine which converts electrical energy into mechanical energy
<b>Principle:</b> Whenever a coil is rotated in a magnetic field an e.m.f. will be induced in this coil and is given by $e=Blv\sin\theta$ volts/coil side where, B=The flux density in Tesla, l=the active length of the coil side in meters, v=the velocity with which the coil is moved in meters/sec and $\theta$ is the angle between the direction of the flux and the direction of rotation of the coil side.	<b>Principle:</b> Whenever a current coil is placed under a magnetic field the coil experiences a mechanical force, and is given by $F=BIl\sin\theta$ Newtons/coil side. Where, I is the current through the coil in ampere.
The direction of the emf induced is fixed by applying the Fleming's right hand rule	The direction of the force acting is fixed by applying the Fleming's left hand rule.

## 2.1 Construction of D.C.Machine

Salient parts of a D.C.machine are:

- Field system (poles)
- Coil arrangement (armature)
- Commutator
- Brushes
- Yoke

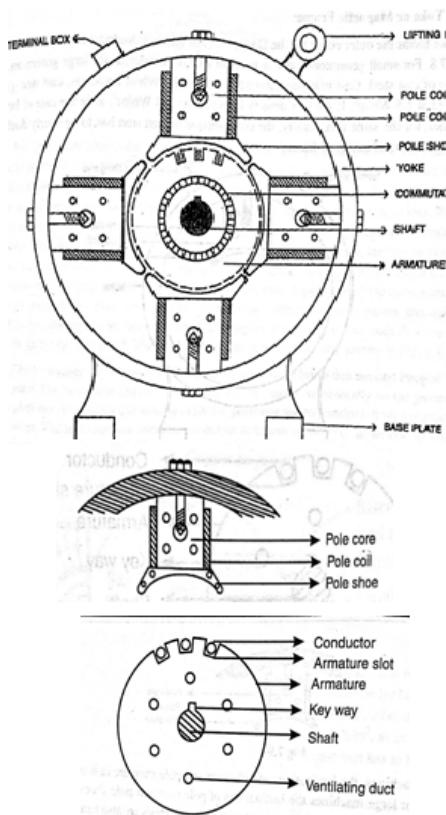


Figure 2.1: Construction of DC Machine

**Field system:** This is made of electromagnets, wherein a iron laminated core is wound with well insulated enameled copper wire. The core is laminated to minimize the eddy current loss. Each lamination is dipped in varnish and dried. A pole shoe is attached to the pole face to direct the flux to concentrate radially on to the armature thereby reducing the leakage and fringing flux. Poles are fixed to the yoke by means of bolts.

**Armature:** This is the rotating part of the machine made of laminated iron core cylindrical in structure with slots on its periphery. Insulated copper coils are laid in these slots, and these coils are connected for lap or wave connection. The core laminations are firmly mounted on a shaft fitted with smooth bearings on either side for smooth rotation.

**Commutator:** As the induced e.m.f. in the armature is alternating commutator converts alternating e.m.f. into unidirectional e.m.f. This is cylindrical in structure made of copper segments with mica insulation between them and is firmly fixed on to the shaft carrying the armature and the armature coil free ends are brazed to the commutator segments.

**Brushes:** These are current collecting devices placed on the body of the commutator with a holder. Brushes are made of carbon, copper or graphite, to have good conductor and low coefficient of friction to reduce the electrical loss and excessive wear, respectively.

**Yoke:** This is the outer most part of the machine made of cast steel which is the mechanical enclosure for the machine to protect it from dust and moisture and also provides the return path for the magnetic flux and carries half the flux per pole.

### 2.1.1 Comparison of lap and wave windings

LAP	WAVE
Number of armature parallel paths is equal to the number of poles.(A=P)	Number of parallel paths is always equal to two.(A=2)
Preferred when large current at lesser voltage is the requirement.	Preferred when large voltage with lesser current is the requirement.

## 2.2 DC Generator

### 2.2.1 E.M.F. Equation of DC Generator

Let the D.C. machine has P number of poles, Z number of armature conductors arranged in A number of parallel paths. Let  $\phi$  be the flux per pole and N is the speed of rotation in revolutions per minute.

Consider one North Pole of the machine under which a group of armature conductors all being connected in series. Let x be the spacing between any two neighboring conductors and t be the time taken to move through this distance of x.

The total flux per pole  $\phi$  is made of several lines and one line of flux is cut by one conductor when it moves through a distance of x in t seconds.

Therefore the induced emf in the 1st conductor when cut by the flux of  $\phi_1$  is  $e_1 = \frac{\phi_1}{t}$  volts

Similarly in the 2nd conductor  $e_2 = \frac{\phi_1}{t}$  volts, and so on.

Therefore the total emf induced in all the conductors under one pole is the sum of all these emf's.

$$E = e_1 + e_2 + e_3 + e_4 + \dots$$

$$E = \frac{\phi_1}{t} + \frac{\phi_2}{t} + \frac{\phi_3}{t} + \frac{\phi_4}{t} + \dots$$

$$E = \frac{\phi}{t} \text{ volts/pole.}$$

$$\text{For all the } P \text{ number of poles } E = \frac{P\phi}{t} \text{ volts}$$

The speed is defined as N revolutions per minute,

N revolutions in one minute or 60 seconds.

1 revolutions will be in time of  $60/N$  seconds, and as one revolution corresponds to all the Z number of conductors the time t for a travel of distance x can be written as  $t = 60/NZ$  seconds.

$$\text{Therefore the induced EMF } E = \frac{P\phi}{t} = \frac{P\phi}{60/NZ} = \frac{PZN\phi}{60A}$$

As the Z number of conductors are arranged in A number of parallel paths,

$$\text{The induced e.m.f per parallel path is } E = \frac{PZN\phi}{60A} \text{ volts.}$$

As P, Z, A are fixed the induced e.m.f is mainly dependent on the flux and the speed, and hence we write that the induced e.m.f E is proportional to the product of the speed N and the flux  $\phi$ .

**Types of D.C. Generators:** D.C. Generators are classified on the basis of the method of exciting the field coils as (i) Separately excited generators and (ii) Self excited generators.

## 2.3 D.C. MOTOR

**Principle:** Whenever a current coil is placed in a magnetic field the coil experiences a mechanical force , and is given by

$$F = BIL \sin\theta \text{ newtons.}$$

Where B is the flux density in Tesla

I is the current through the coil

$L$  is the active length of the coil side

$\theta$  is the angle between the movement of the coil and the direction of the flux

The direction of the force acting can be decided by applying Fleming' s left hand rule.

The construction of a D.C.Motor is same as the construction of a D.C.generator.

**Types of D.C.Motors:** Depending on the interconnection between the armature and the field circuit D.C.Motors are classified as (i) Shunt Motor, (ii) Series Motor and (iii) Compound motors just like D.C.Generators.

**Back EMF:** Whenever a current coil is placed under a magnetic field the coil experiences a mechanical force due to which the coil starts rotating. This rotating coil again cuts the magnetic lines of force resulting an EMF induced in it whose direction is to oppose the applied EMF (as per Fleming' s right hand rule), and hence the name BACK EMF or Counter Emf.

**Significance of Back EMF:** Back EMF is a must in a motor which helps to regulate the armature current and also the real cause for the production of torque.

Expression for the back Emf is given by  $E=V-I_a R_a$ ,

Where  $E$  is the back emf,  $V$  is the applied emf,  $I_a$  is the armature current and  $R_a$  is the armature circuit resistance. And also  $E = \frac{PZN\phi}{60A}$  volts, from the machine parameters.

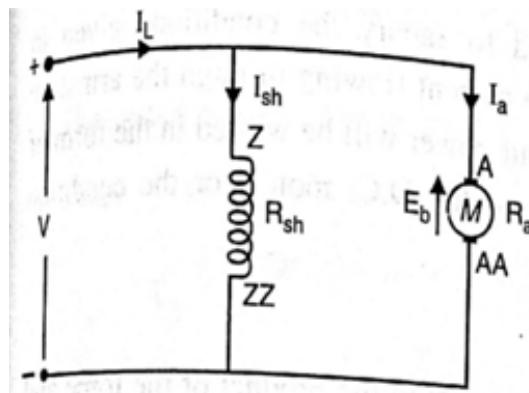


Figure 2.2: Eq Circuit Diagram of DC Motor

**Production of torque in a D.C. Motor:** A DC machine is loaded either as a motor or as generator. The field of a machine is excited and a potential difference is impressed upon the machine terminals. The rotor conductors carry current. These conductors are the surface of the rotor at a common radius from its center. The torque is produced around the circumference of the rotor and the rotor starts rotating.

The meaning of torque is turning or twisting of a force about an axis. The torque is measured by the product of the force and radius at which this force acts.

In case of generators, the machine operates at constant speeds, and the torque is equal to and opposite to that provided by the prime mover.

In case of motors, the torque is transferred to the shaft of the rotor and drives the mechanical load. The torque expression for the generator and the motor is same.

**Torque Equation:** Let P be the total number of poles, Z be the total number of armature conductors arranged in A number of parallel paths. Let  $\phi$  be the flux per pole, N be the speed of rotation in rpm, and T be the torque in Nm.

From equivalent circuit the back emf , $E=V-I_a R_a$

Multiplying the above equation by  $I_a$  on both sides

We get  $E I_a = V I_a - I_a^2 R_a$

Where  $V I_a$  represents the Power input to the armature, $I_a^2 R_a$  represents the armature copper loss and  $E I_a$  represents the Total power output of the armature which is the electrical power converted into mechanical power called the electro-mechanical power in watts. The equivalent mechanical power is given by  $\frac{2\pi NT}{60A}$  watts.

Therefore,  $E I_a = \frac{2\pi NT}{60A}$  watts

But  $E = \frac{PZN\phi}{60A}$ , therefore the torque  $T = \frac{PZN\phi}{60A}$  Nm.

From the above equation it can be seen that the torque is directly proportional to the product of the flux and the armature current.

**Speed of a D.C.Motor:** We know that for a motor in general the back emf e is given by

$$E = V - I_a R_a = \frac{PZN\phi}{60A}$$

From which we write,

$$N = (V - I_a R_a) / \frac{PZN\phi}{60A},$$

and the speed N is proportional to  $\frac{(V - I_a R_a)}{\phi}$

From the above equation we write the speed is directly proportional to the applied voltage V, and the armature current  $I_a$  and inversely proportional to the flux  $\phi$ .

## 2.4 Transformer

Transformer is a static device which transfer electric energy from one electric circuit to another at any desired voltage with out any change in frequency.

**Principle:-** A transformer works on the principle of mutual induction. “Whenever a change in current takes place in a coil there will be an induced emf in the other coil wound over the same magnetic core”. This is the principle of mutual induction by which the two coils are said to be coupled with each other. The fig1 shows the general arrangement of a transformer. C is the iron core made of laminated sheets of about 0.35mm thick insulated from one another by varnish or thin paper. The purpose of laminating the core is to reduce the power loss due to eddy currents induced by the alternating magnetic flux. The vertical portions of the core are called limbs and the top and bottom portions are called the yokes. Coils P and S are wound on the limbs. Coil P is connected to the supply and therefore called as the primary, coil S is connected to the load and is called as the secondary.

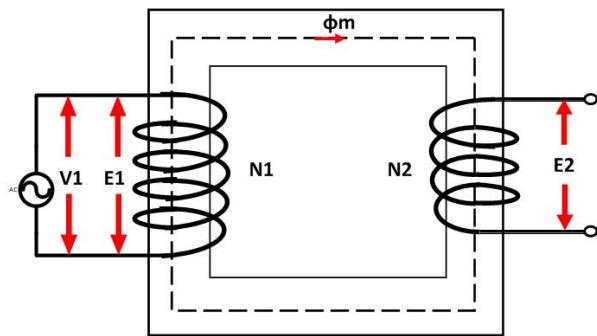


Figure 2.3: Single Phase Transformer  
The diagram illustrates a single-phase transformer. It features a central iron core with two vertical limbs labeled N1 and N2. A dashed line labeled  $\phi_m$  represents the magnetic flux. Two coils, labeled E1 and E2, are wound around these limbs. The primary coil E1 is connected to an AC voltage source V1. The secondary coil E2 is connected to a load. The core is shown with its yoke and laminated structure.