Digital communication

Why digital?

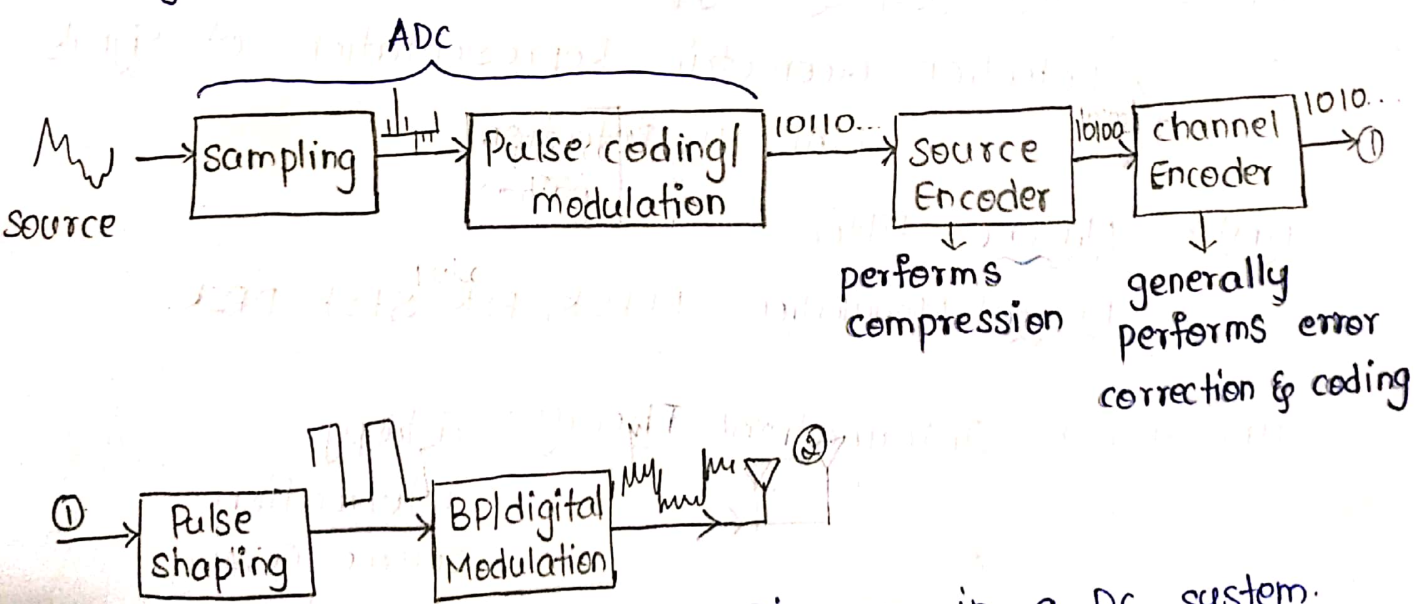
Telegraph → 1^{st} communication system.

* '—', '.' & ' ' → 3 symbols used to encode the text
* Example for digital communication system.

Advantages of Digital communication over Analog:

1) can recover the original signal from the noisy one.
2) Error correction and coding is possible.
3) compression } can be performed.
4) Encryption. }
5) Processor (or) Algorithm instead of components (or) circuits and hence easy to modify (or) upgrade the communication system
6) common format (or) protocol for storage (or) communication of different types of signals.
ex: Audio, Video, Image etc...

* The source generates a signal that is typically analog in nature.



Transmitter Block Diagram in a DC system.

* pulse shaping associates the physical signal (manife-station) to the logical entities (bits)

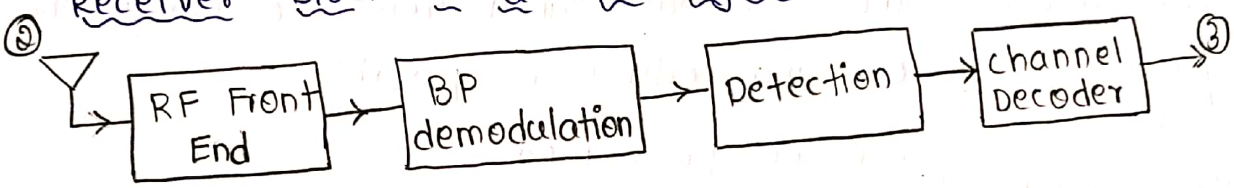
ex: represent 1 →

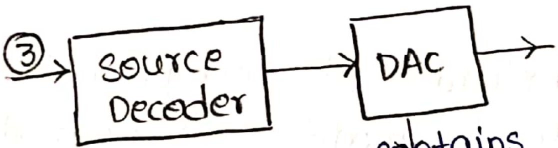
0 →

↓

this representation is not used generally because the above signals ventures into the neighbouring band.

Receiver Block in a DC system:





contains Reconstruction Filter.

Receiver Block in a DC system

Unit 1: Pulse Coding: PCM – Pulse coded Modulation

DPCM – Differential PCM

DM – Delta Modulation.

Pulse Shaping: unipolar, polar, bipolar, ...

Unit 2: Pulse Shaping: Nyquist criterion

*. Detection: Geometric Representation of Signals.

MAP, ML Detection.

Unit 3: Matched Filter:

Digital Modulation: BPFSK, FSK, QFSK, DFSK.

unit 4 & 5: Information Theory: Entropy

Information

Source Coding.

channel capacity

* Textbooks

1) "Digital communications"
 - by Simon Haykin.

2) "Elements of Information Theory"
 - by TM cover and JA Thomas.

why digital ?

Lec 2

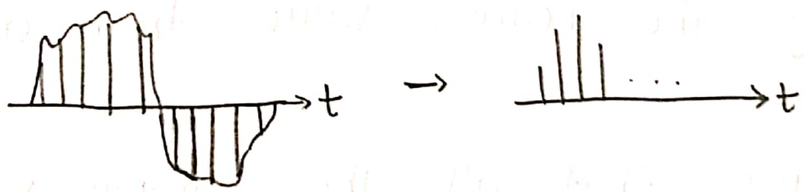
1) unlike analog communication the original signal can be recovered from its distorted version in digital communication.

2) compression, Error correction & Encryption can be performed.

3) A common format can be used for storage/processing/communication of different kinds of data like audio, image, video etc...

4) Processor/Algorithms are used instead of circuits this helps in modifying and upgrading the system.

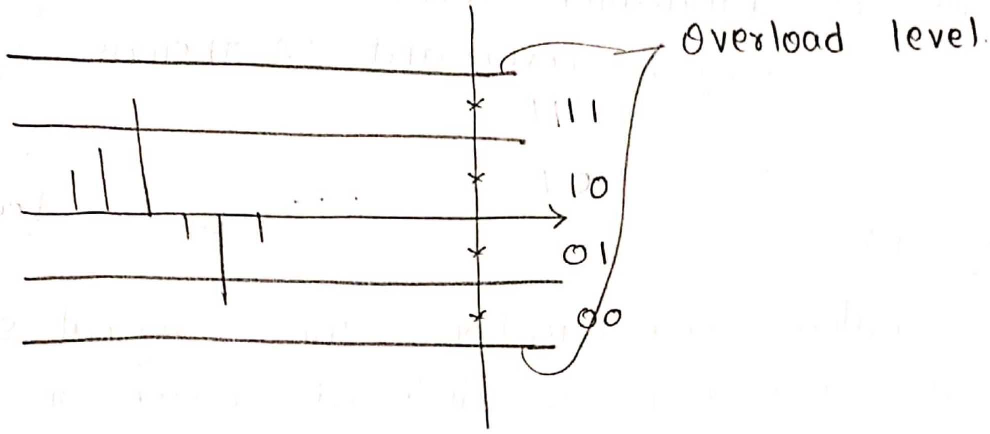
Sampling:



* "Sampling" converts a "continuous random process" to a "Discrete-Time Random process".

* Discrete-Time Random process" → Random variable at discrete time (i.e.. sequence of continuous random variables).

* Sampling results in a sequence of continuous Random variables. Each sample has an amplitude taken from a continuous range of values.



PCM : Pulse Coded Modulation :

PCM is the process of representing the sequence of samples by a sequence of bits. Here each sample is represented by a bit pattern.

PCM involves 2 steps :-

1) Quantization
2) Bit Encoding.

*Quantization :-

Quantization is the process of approximating each sample by the nearest value from a discrete set of value.

We assume that all the sample values are within the overload level represented by ±A Volt.

The Range [-A,+A] is divided into "L" levels each sample is approximated by the mid-point of the level it belongs to.

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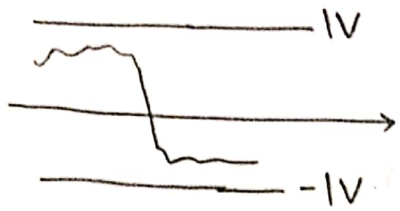
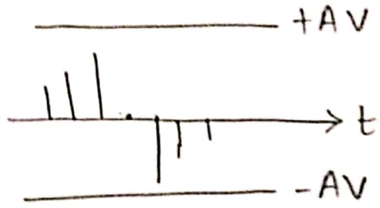
Ex:

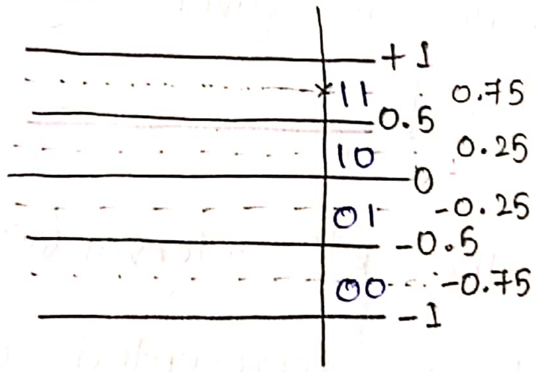




* let the sequence of samples be 0.34
0.51
0.65
0.28
-0.06
-0.43.

* let each sample be represented by 2 bits.

$\Rightarrow$ L = 4 levels

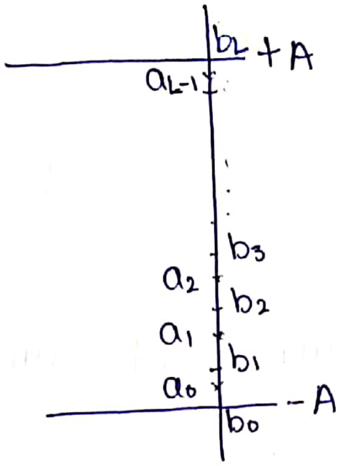


Quantization:

x(n)	y(n)	Bit Encoding
0.35	0.25	10
0.51	0.75	11
0.65	0.75	11
0.28	0.25	10
-0.06	-0.25	01
-0.43	-0.25	01
-0.71	-0.75	00

* Quantization converts a sequence of continuous random variables to a sequence of discrete random variables.

$$x(n) \longrightarrow \boxed{\text{Quantizer}} \longrightarrow y(n)$$



* The range [-A, +A], is divided into L levels. let $b_0, b_1, \ldots b_L$ indicate the Level boundary and $a_0, a_1, \ldots a_{L-1}$ indicate the mid-point of these level. The

* The Quantization Rule is given by.

$$\boxed{y(n) = a_k \quad \text{if} \quad b_k \leq x(n) < b_{k+1}}$$

(or)

$x(n) \in I_k$ where I_k is the k^{th} interval (or) k^{th} level

* Suppose each sample is represented using N bits then we have 2^N levels.

$\therefore$ $\boxed{L = 2^N \text{ levels.}}$

* If the range [-A, +A] is divided into $L = 2^N$ levels of equal width then it is called uniform quantization.

* Each level has the step with $\boxed{\Delta = \frac{2A}{2^N}}$

(in case of uniform quantization).

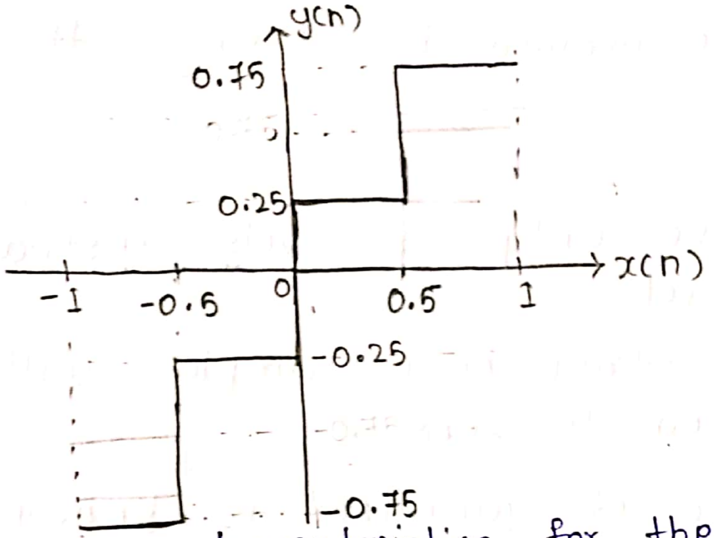
* The quantization error is given by

$$\boxed{q(n) = y(n) - x(n)}$$

④

* Uniform Quantizer can be of two types:

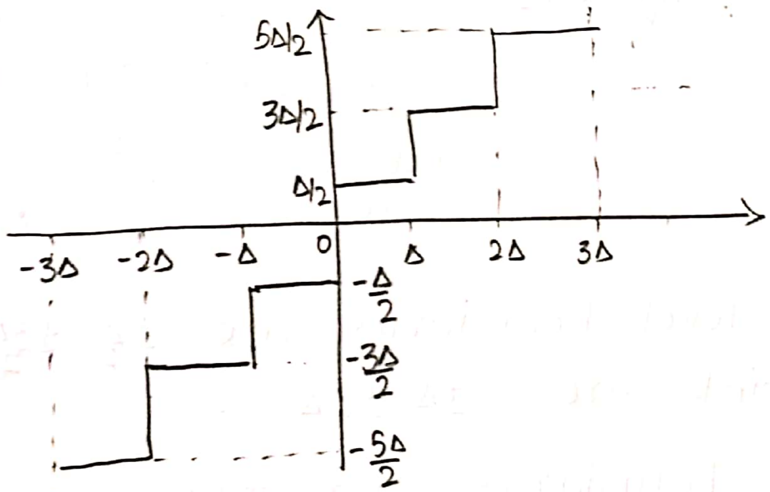
1) Mid- Riser
2) Mid- Thread



i/p - o/p characteristics for the previous example

In this case the level boundaries are $0, \pm\Delta, \pm 2\Delta, \ldots$

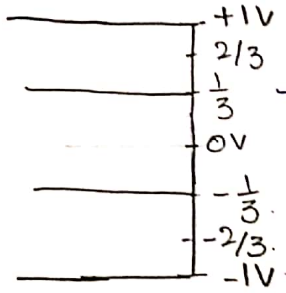
the mid-points are at $\pm\frac{\Delta}{2}, \pm\frac{3\Delta}{2}, \ldots$



* Note: Quantization introduces an error $q(n)$. that cannot be removed.

* Due to the stray noise samples we will have a stream of bits even though there is no signal (i.e.. i/p signal is zero).

* This can be avoided by taking off zero as a boundary.

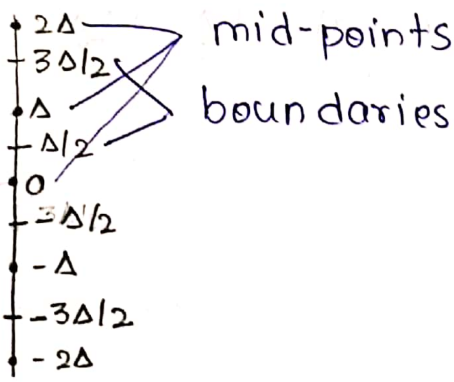


→ Have only 3 levels instead of 4 levels.

→ so stray noise samples will be mapped to zero.

* This type of quantizer is known as Mid-Tread Quantizer.

Mid-Tread Quantizer:

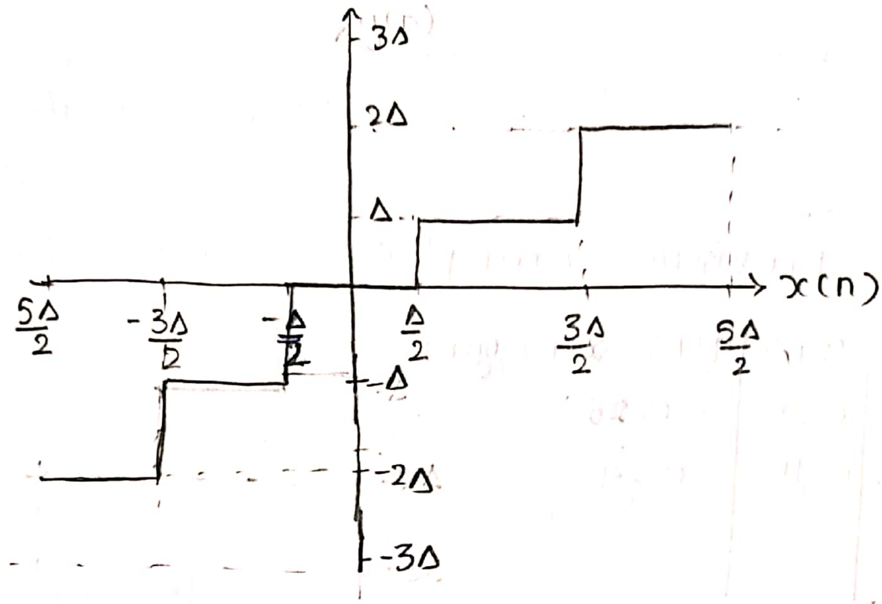


* Here the level boundaries are $\pm\frac{\Delta}{2}, \pm\frac{3\Delta}{2}, \ldots$ and the mid-points are $0, \pm\Delta, \pm2\Delta, \ldots$

* Here the boundaries and mid-points are flipped when compared to the Mid-Riser Quantizer.

* Here the no. of Levels $\boxed{L = 2^N - 1}$

$\therefore$ $\boxed{\Delta = \frac{2A}{2^N - 1}}$ → step width of each level.



i/p o/p characteristics of a Mid- Tread Quantizer.

Mid - Riser v/s Mid-Tread :-

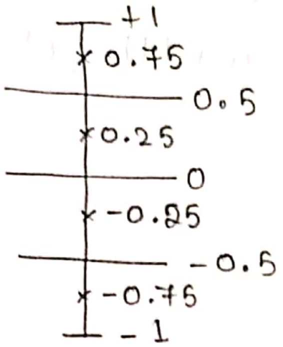
* In Mid- Riser quantizer when the signal is not present the output alternates between $+\frac{\Delta}{2}$ & $-\frac{\Delta}{2}$ due to noise samples resulting in a spurious bit pattern. This problem is avoided in Mid- Tread quantizer.

* The number of levels is 2^N in Mid - Riser quantizer and $2^N - 1$ in Mid - Tread quantizer.

for large values of N

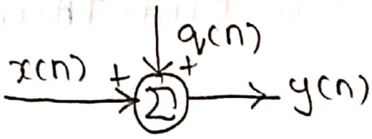
$\Delta = 2^N$ (for both the quantizer).

Quantization Error | Noise :-



consider the previous example :

$x(n)$	$y(n)$	$q(n) = y(n) - x(n)$
0.38	0.25	-0.13
0.51	0.75	0.24
⋮	⋮	⋮



w.k.t. $y(n) = a_k$ if $x(n) \in \Delta_k$.

$\therefore$ $q(n) = y(n) - x(n)$

$\boxed{q(n) = a_k - x(n)}$ if $x(n) \in \Delta_k$.

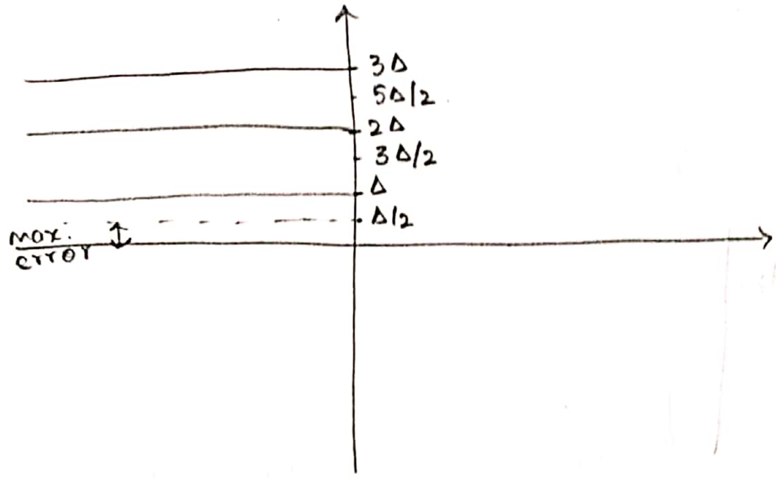
* $q(n)$ is called the quantization error | noise.

[q(n) = approx. - actual value]

* quantization error cannot be removed.
* since $x(n)$ is random $q(n)$ is also random.

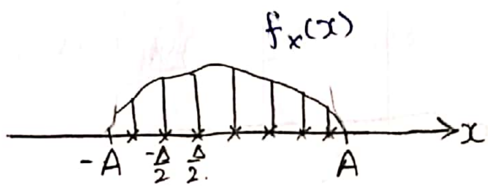
6

*



-x. $\therefore$ q(n) takes values in the range $-\frac{\Delta}{2}$ to $\frac{\Delta}{2}$.

★ pdf of x(n) & q(n)



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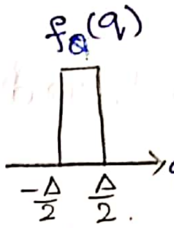
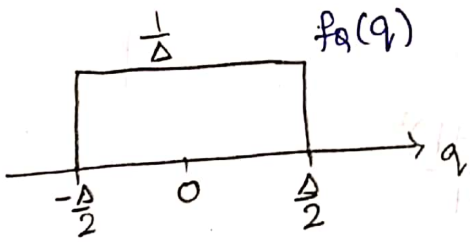


pdf of q(n) for large values of N.

pdf of x(n)

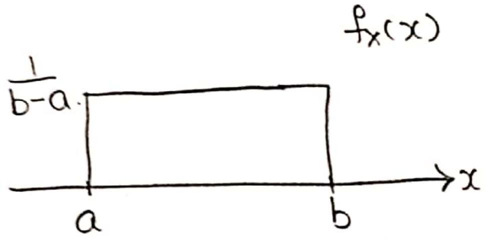
* if N is large then $\boxed{\Delta = \frac{2A}{2^N}}$ is small hence we can assume that the pdf of the quantization error is uniform over the range $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$



pdf of q(n) for large values of N.

-x. * $\boxed{E[Q] = 0}$ (i.e... mean of the random variable Q = 0).

* x(n) & q(n) are the realizations of the random variable X & Q rptly.



$$\boxed{\begin{aligned} E[X] &= \frac{a+b}{2} = \mu_X \\ \sigma_X^2 &= \frac{(b-a)^2}{12} \end{aligned}}$$

Q) S.T. the variance of X is $\frac{(b-a)^2}{12}$ for the given uniform pdf.

Sln w.k.t. $\sigma_X^2 = E[X^2] - \mu_X^2$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$= \int_a^b x^2 \cdot \frac{1}{(b-a)}\,dx$$

$$= \left.\frac{x^3}{3(b-a)}\right|_a^b$$

$$E[X^2] = \frac{b^3-a^3}{3(b-a)}$$

$$\sigma_X^2 = \frac{b^2+ab+a^2}{3} - \frac{(a+b)^2}{4}$$

$$= \frac{4b^2+4ab+4a^2-3a^2-6ab-3b^2}{12}$$

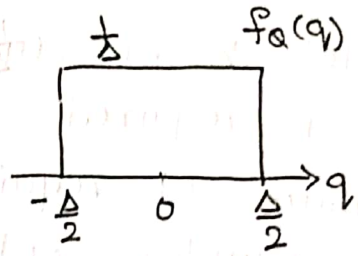
$$= \frac{b^2-2ab+a^2}{12}$$

$$\therefore \quad \boxed{\sigma_X^2 = \frac{(b-a)^2}{12}}$$

(7)

* Hence, we get $\boxed{\sigma_Q^2 = \frac{\Delta^2}{12}}$

as $(b-a) = \Delta$.



* ∴ The Average Power of the Quantization is. $E[Q^2] = \sigma_Q^2 + \mu_Q^2$.

as $\mu_Q^2 = 0$

∴ $E[Q^2] = \sigma_Q^2$

* $\boxed{E[Q^2] = \frac{\Delta^2}{12}} \rightarrow$ Average power for the zero mean Random variable.

* * A measure of the performance of the Quantizer is the signal to noise ratio defined as

$$\boxed{SNR = \frac{\text{Average signal power}}{(\text{Average power of the quantization noise})}}$$

$$\boxed{\text{Average Signal Power} = E[X^2]}$$

* * The signal is usually assumed to have zero mean hence we have average signal power as

$\boxed{E[X^2] = \sigma_X^2}$

$$\therefore \boxed{SNR = \frac{\sigma_X^2}{\sigma_Q^2} = \frac{\sigma_X^2}{\Delta^2/12}}$$

ex: let X be uniform over the range −10 to 10. If it is required that $\sigma_Q^2 < 0.2$ what is the minimum N required. (by default we take mid-riser quantizer only).

Sln

Given: $\sigma_Q^2 < 0.2$

$$\frac{\Delta^2}{12} < 0.2$$

$$\Delta < \sqrt{2.4}$$

$$\Delta < 1.549.$$

$$\Delta = \frac{2A}{2^N} < 1.549.$$

$$\frac{2 \times (10)}{2^N} < 1.549$$

$$2^N > \frac{20}{1.549}.$$

$$N > \log_2\left(\frac{20}{1.549}\right)$$

$$N > 3.69$$

$$\boxed{N \geq 4.}$$

∴ minimum value of N required is 4.

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ex: let X be uniform over the range [-A, A]. Find the SNR for N bit quantization. (assume N is large).

Sn.

$$SNR = \frac{\frac{(2A)^2}{12}}{\Delta^2/12} \qquad \left(SNR = \frac{\sigma_x^2}{\sigma_Q^2} \right).$$

$$SNR = \frac{4A^2}{\Delta^2}.$$

w.k.t. $\Delta = \frac{2A}{2^N}$.

$$\therefore SNR = \frac{4A^2}{4A^2/2^{2N}}$$

$$\boxed{SNR = 2^{2N}}$$

In dB we have

$$SNR_{dB} = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_Q^2}\right).$$

$$= 10\log_{10}(2^{2N}).$$

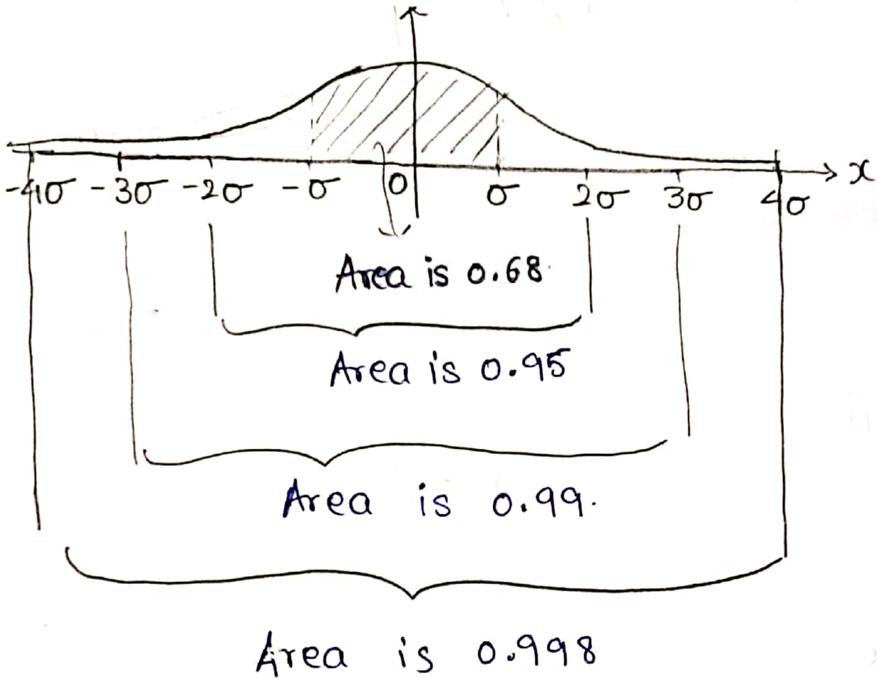
$$SNR_{dB} = 20N\log_{10}(2).$$

$$\boxed{SNR_{dB} = 6.02N \approx 6N}$$

ex) let $X \sim N(0, \sigma^2)$. ($N \to$ Normal distribution (Gaussian)).

let X be Gaussian with mean 0 and varia-nce σ^2. Find SNR for N-bit Quantization.

let $A = 4\sigma$. ($\because$ peak for Gaussian distribution is at ∞).



* Hence for all practi-cal purposes the value of $x(n)$ can be take between -4σ & 4σ.

s/n

w.k.t. for Gaussian distribution

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

here $\mu = 0$.

$$\therefore f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$\cancel{\Rightarrow \sigma_x^2 = E[X^2] = \int_{-4\sigma}^{4\sigma} \frac{1}{\sqrt{2\pi\sigma^2}} x^2 e^{-x^2/2\sigma^2} dx}$$

$$\sigma_Q^2 = \frac{\Delta^2}{12} = \frac{(2 \times 4\sigma_x)^2}{12 \times 2^{2N}} = \frac{64\,\sigma_x^2}{12 \times 2^{2N}} = \frac{16}{3}\frac{\sigma_x^2}{2^{2N}}$$

$$SNR = \frac{\sigma_x^2}{\sigma_Q^2} = \frac{\sigma_x^2}{\frac{16}{3}\frac{\sigma_x^2}{2^{2N}}}$$

⑨

-X- $\boxed{SNR = \frac{3}{16} 2^{2N}}$

$\therefore SNR_{dB} = 10\log_{10}\left(\frac{3}{16} 2^{2N}\right)$

-X- $\boxed{SNR_{dB} = 6N - 7.269}$

ex: let $x(n) = A\cos\omega_0 n$. find SNR. in terms of N.

s/n -X- since $x(n) = A\cos\omega_0 n$. i.e.. $x(n)$ is deterministic

-X- $\boxed{SNR = \frac{\text{Avg. Power of Input signal}}{\sigma_Q^2} = \frac{P_x}{\sigma_Q^2}}$

w.k.t $P_x = \frac{A^2}{2}$. when $x(n) = A\cos\omega_0 n$.

$SNR = \frac{A^2}{2} / (\Delta^2/12)$.

$= \frac{\cancel{A^2}}{2} / \left(\frac{4\cancel{A^2}}{\cancel{12}_3 \times 2^{N}}\right)$

-X- $\boxed{SNR = \frac{3}{2} 2^{N}}$

$\boxed{SNR_{dB} = 6N + 1.76}$

$\boxed{P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2} \to ①$

*when $x(n)$ is periodic

$\boxed{P_x = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2} \to ②$

use ① has we don't know weather $x(n)$ is periodic (or) not & express $x(n) = A\cos\omega_0 n$ as $\frac{A}{2}\left(e^{j\omega_0 n} + e^{-j\omega_0 n}\right)$.

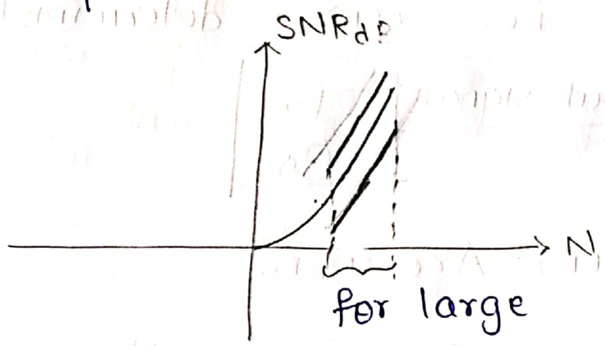
* -X- From these results we note the following:

1) SNR depends on the input signal's pdf.

2) $\boxed{SNR_{dB} = 6N + c.}$ is an "incrementally linear" function of N with a slope of 6dB/bit

3) for every additional bit we get an improveme-nt of 6dB in SNR. (from ② Result).

*. So whenever an additional bit is added the number of levels doubles and the width size will reduces by half hence we will be having a finer quantization as the error in quantization reduces.



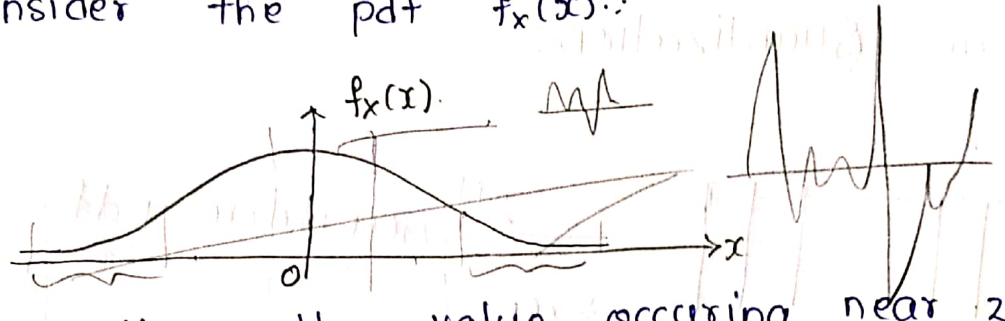
for large values of N.

4). If the number of bits is increased by 1 σ_Q^2 decreases by a factor of 4.

*. The difference in performance between different signals with same peak to peak values is due to the change in signal power variation. (i.e. σ_x^2).

i.e. SNR depends on pdf of $x(n)$ i.e. the input signal.

* consider the pdf $f_x(x)$:-



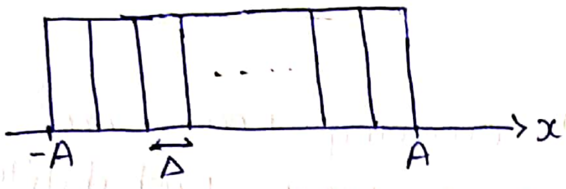
* Here the value occuring near zero is more probable. so there is more probability of values being close to zero than towards the extremes.

* So if we look at the instantaneous power it varies depending upon the amplitude. In Gaussian the probability of occurrance of low power signal is more whereas occurrance of high power parts is less hence its more of a low power signal. Therefore, the signal power is low and hence the SNR is also low.

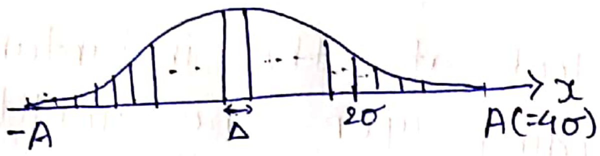
* Thus the performance of a signal is affected by σ_x^2 & not the σ_Q^2.

* So the differentiating factor for different SNRs is σ_x^2 and not the σ_Q^2.

Non-Uniform Quantization:



2^N uniform pdf

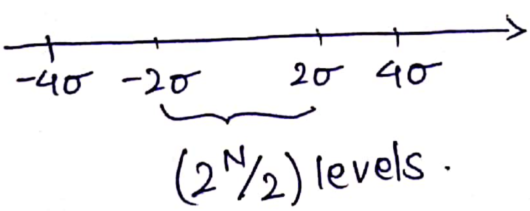


Gaussian pdf.

* In uniform pdf the probability of the i/p signal being in any level remains the same as the area of each interval remains constant.

* In Gaussian pdf the probability of occurrance of one level is higher than the other levels (i.e... the probability of occurrance of each level is different from one another).

* Between -2σ to 2σ the probability of occurance is $\boxed{P\{-2\sigma < x < 2\sigma\} = 0.95}$



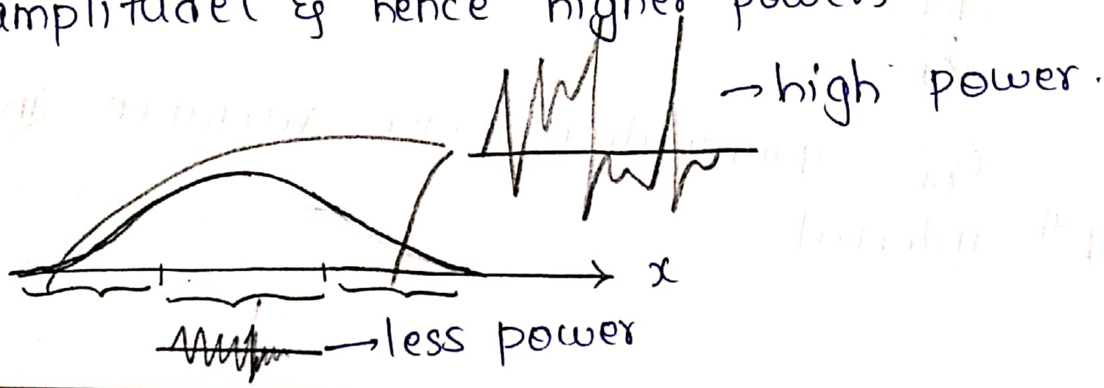
** half of the total number of levels ($\frac{2^N}{2}$) lies between the interval $(-2\sigma, 2\sigma)$.

11

Motivation:

* With uniform quantization for i/p signal with uniform all the levels are equally likely to occur. But for the Gaussian i/p the levels around the mean are much more likely to occur than the levels at the extremes. Since $\boxed{P\{-2\sigma < x < 2\sigma\} = 0.95}$ the levels b/w -2σ & 2σ occur 95% of the time whereas the levels b/w 2σ & 4σ (also b/w -2σ & -4σ) occur only 5% of the time. Therefore, half the levels are almost wasted. To improve performance we need to have more number of levels around the mean (with smaller step size) and less number of levels at the extreme ends (with larger step size). This leads to Non-Uniform Quantization.

* Speech signal can be considered to have a Gaussian pdf. Normal speech has lesser amplitude (& hence lesser power) and is more likely to occur than the loud speech which has higher amplitude (& hence higher power).



Motivation 2

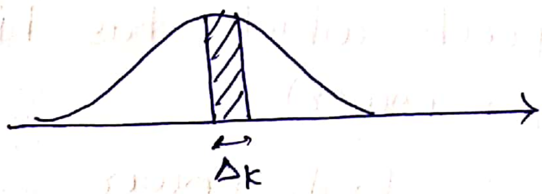
* Perceptual consideration require that the SNR be almost constant a/c the different i/p power levels. This means we need to provide more levels (with smaller step size) when the signal power is low & provide less no. of levels (with larger step size) when signal power is high. such a quantization scheme in which the SNR is almost same a/c the different i/p power levels is called "Robust Quantization".

* In Non-Uniform Quantization. we have the same number of levels $L = 2^N$ but each level can have different width. Let Δ_k be the width of the k^{th} level. It can be shown that the quantization Noise variance is given by

$$\boxed{\sigma_Q^2 = \sum_{k=0}^{L-1} p_k \sigma_{Q_k}^2}$$

where, $p_k \rightarrow$ the probability of occurance of the k^{th} level and is given by.

$$\boxed{p_k = \int_{b_k}^{b_{k+1}} f_X(x)\,dx}$$



$\sigma_{Q_k}^2 \Rightarrow$ quantization noise variance in the k^{th} interval.

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(12)

when N is large we have

$$\boxed{\sigma_Q^2 = \sum_{k=0}^{L-1} p_k \frac{\Delta_k^2}{12}}$$

↳ (to prove this we need conditional pdf concepts). →

↳ here we are taking the avearge of the variances across the different levels.

* Consider the expression

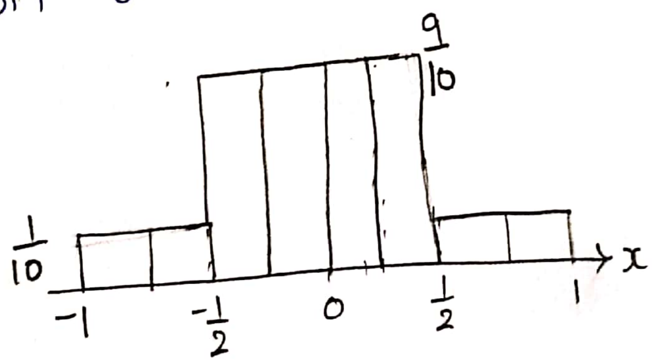
$$\sigma_Q^2 = \sum_{k=0}^{L-1} p_k \frac{\Delta_k^2}{12}$$

In case of uniform Quantization $\frac{\Delta_k^2}{12} = \frac{\Delta^2}{12}$

$$\Rightarrow \boxed{\sigma_Q^2 = \frac{\Delta^2}{12} \sum_{k=0}^{L-1} p_k = \frac{\Delta^2}{12}}$$

→ (as seen in case of uniform quanti-zation.

Q: For the following pdf find the SNR for 3 bit uniform quantization.



$$\Delta = \frac{2A}{2^N} = \frac{2}{8} = 0.25$$

$$\sigma_x^2 = \int_{-1/2}^{1/2} x^2\left(\frac{9}{10}\right)dx + 2\int_{1/2}^{1} x^2 \frac{1}{10}\,dx$$

$$= 2 \cdot \frac{9}{10} \left.\frac{x^3}{3}\right|_0^{1/2} + \frac{2}{10}\left.\frac{x^3}{3}\right|_{1/2}^{1}$$

$$= \frac{3}{40} + \frac{7}{120}$$

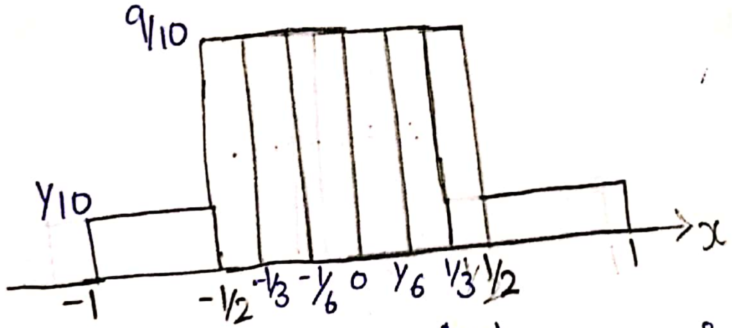
$$\sigma_x^2 = \frac{2}{15}$$

$$SNR = \frac{\sigma_x^2}{\sigma_Q^2} = \frac{\frac{2}{15}}{\Delta^2/12} = \frac{2/15}{0.25^2/12}$$

$\therefore$ $\boxed{SNR = 25.6}$

$\therefore$ $\boxed{SNR_{dB} = 14.08\,dB}$

* For the same pdf find the SNR for the following non-uniform quantization.



Sol. w.k.t $\sigma_Q^2 = \sum_{k=0}^{L-1} p_k^2 \frac{\Delta_k^2}{12}$

(13)

$$\sigma_Q^2 = \sum_{k=0}^{7} p_k^2 \frac{\Delta_k^2}{12}.$$

for $k = 1$ to 6. $\quad p_k = \frac{9}{10} \times \frac{1}{6} = \frac{3}{20}$.

$$\Delta_k = \frac{1}{6}.$$

for $k=0$ & $k=7$ $\quad p_k = \frac{1}{20}$

$$\Delta_k = 0.5$$

$$\therefore \sigma_Q^2 = 6 \times \frac{3}{20} \times \frac{1}{36} \times \frac{1}{12} + 2 \times \frac{1}{20} \times 0.25 \times \frac{1}{12}$$

$$\sigma_Q^2 = \frac{1}{240}.$$

$$\therefore SNR = \frac{\sigma_x^2}{\sigma_Q^2} = \frac{2/15}{1/240}$$

$$\therefore \boxed{SNR = 32}$$

$$\therefore \boxed{SNR_{dB} = 15.05}$$

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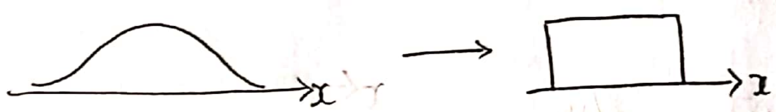
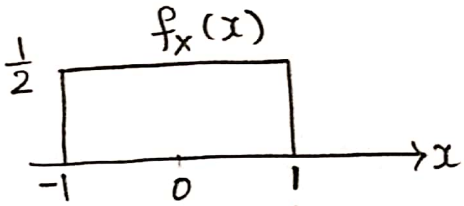
* **Robust quantization** → Is one type of non uniform quantization where we select the step sizes so as to make the SNR almost same across all the different power levels.

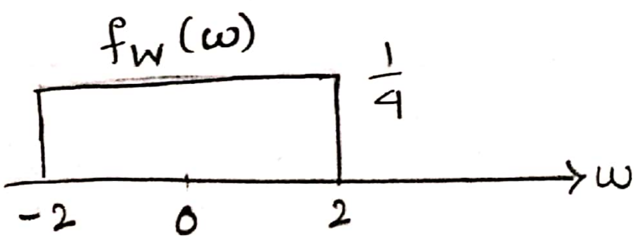
* In practice we first perform a non-linear transformation of the input signal and then apply a uniform quantizer. This transformation is called "compression". At the receiver we perform the inverse transformation called "Expansion". Together the process is called "companding".



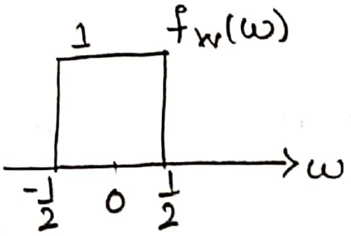
*



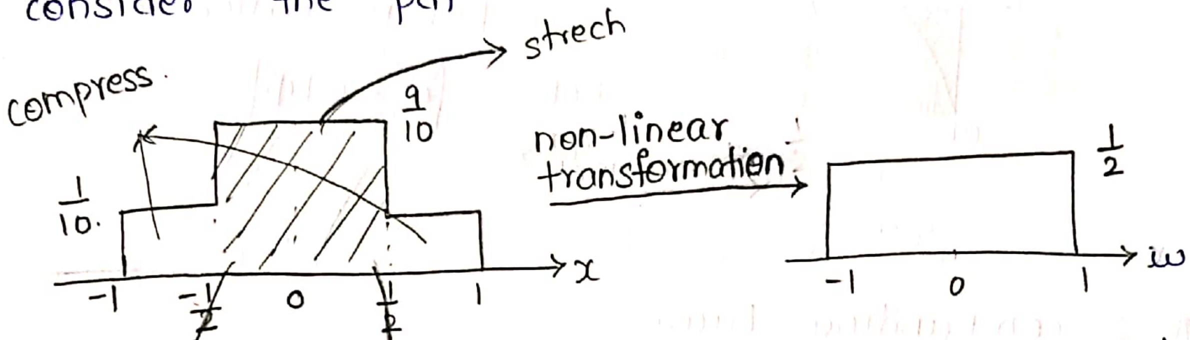
* if $W = 2X$ → linear transformation
  ↳ Here w is also a Random variable.



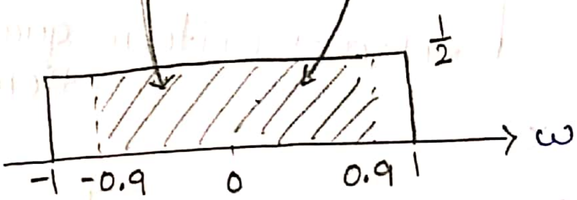
* if $W = \frac{X}{2}$. → linear transformation.

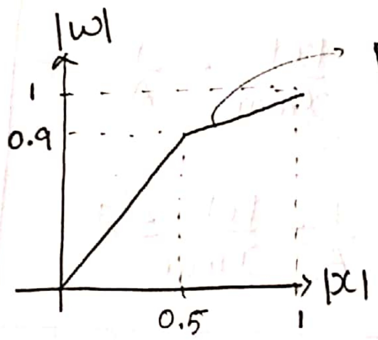


* consider the pdf.



* In the compression operation the overload level should remain the same

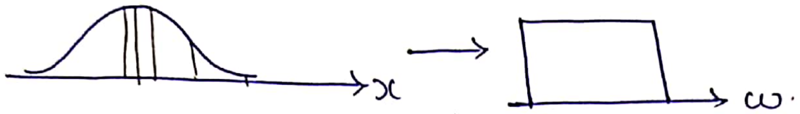




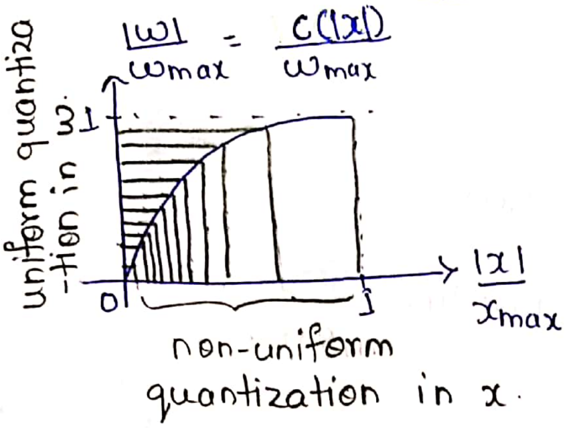
piecewise-linear function (its a non-linear function)

Ex: write the expression for the given transforma-tion.

$$|\omega| = \begin{cases} \frac{9}{5}|x| & , 0 < |x| < \frac{1}{2} \\ \frac{1}{5}|x| + 0.8, & \frac{1}{2} < |x| < 1 \end{cases}$$



$$\frac{|\omega|}{\omega_{max}} = \frac{c(|x|)}{\omega_{max}}$$



the compression function $|\omega|$ is called $c(|x|)$.

$$\boxed{\omega = c(|x|)}$$

⁂ 2 companding Laws:

1) μ-Law (US, Japan):

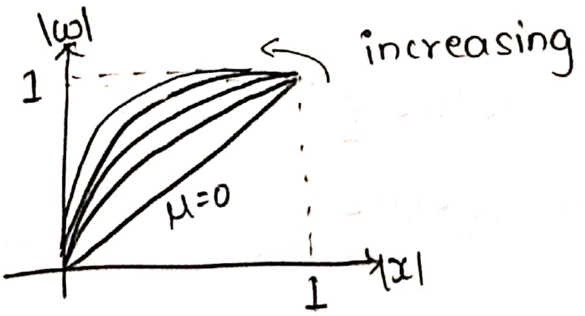
⁂ $$\boxed{\frac{c(|x|)}{x_{max}} = \frac{\ln\left(1+\mu\frac{|x|}{x_{max}}\right)}{\ln(1+\mu)}}, \quad 0 \le \frac{|x|}{x_{max}} \le 1.$$

⁂ $\mu = 0 \Rightarrow$ uniform quantization

2) A-Law (India, Europe):

⁂ $$\boxed{\frac{c(|x|)}{x_{max}} = \begin{cases} \dfrac{A\frac{|x|}{x_{max}}}{1+\ln A} & 0 \le \frac{|x|}{x_{max}} \le \frac{1}{A} \\[2ex] \dfrac{1+\ln\left(A\frac{|x|}{x_{max}}\right)}{1+\ln(A)}, & \frac{1}{A} \le \frac{|x|}{x_{max}} \le 1. \end{cases}}$$

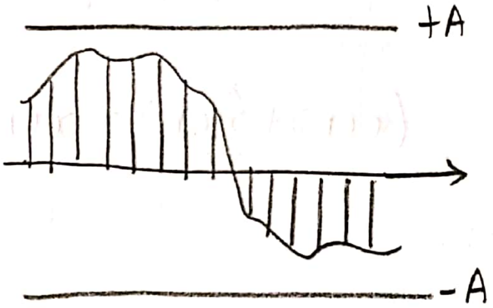
⁂ $A = 1 \Rightarrow$ uniform quantization

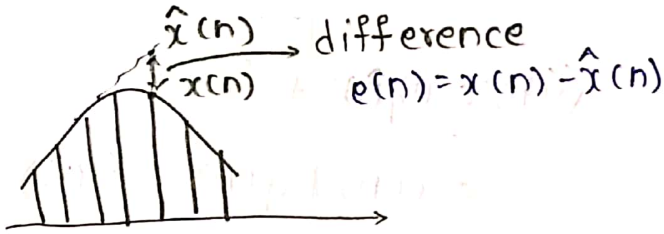


* $\mu = 255$ is practically used.
  $\mu = 0$ gives uniform quantization.
* $A = 1$ gives uniform quantization
  $A = 87.56$ is practically used.

Differential PCM (DPCM):

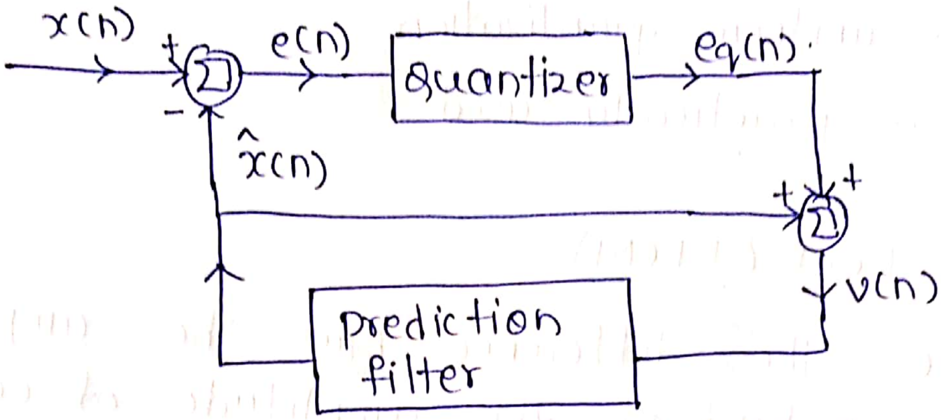
↳ Quantize the difference b/w the amplitudes of samples instead of the amplitude of each sample. so $\Delta = \frac{2A}{2^N}$ decreases as A decreases and hence σ_Q^2 decreases ⇒ SNR increases.





* To improve the quantization performance one approach is to quantize the signal $\boxed{e(n) = x(n) - \hat{x}(n)}$ where $\hat{x}(n)$ is the predicted (estimated) value of $x(n)$ based on the previous samples. Since there is good amount of correlation across the samples in practical signals we can obtain a good estimate of $x(n)$ resulting in a small value for the error signal $e(n)$. Therefore the SNR performance for quantizing $e(n)$ will be much better than the performance w.r.t. $x(n)$.

* Let $x_q(n)$ and $e_q(n)$ denote the quantized versions of $x(n)$ & $e(n)$ respectively



* $$\boxed{\begin{aligned} x_q(n) &= x(n) + q(n) \\ e_q(n) &= e(n) + q(n). \end{aligned}}$$ where, $q(n) \to$ quantization noise.

$$v(n) = e_q(n) + \hat{x}(n)$$

$$= e(n) + q(n) + \hat{x}(n). \qquad (e(n) + \hat{x}(n) = x(n))$$

$$= x(n) - \hat{x}(n) + q(n) + \hat{x}(n).$$

$$= x(n) + q(n).$$

* $\boxed{v(n) = x_q(n)}$

* If the prediction filter is just a delay then $\hat{x}(n) = x_q(n-1)$. $\Rightarrow$ we are using the previous quantized sample as an estimate.

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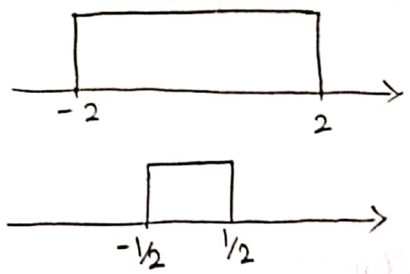
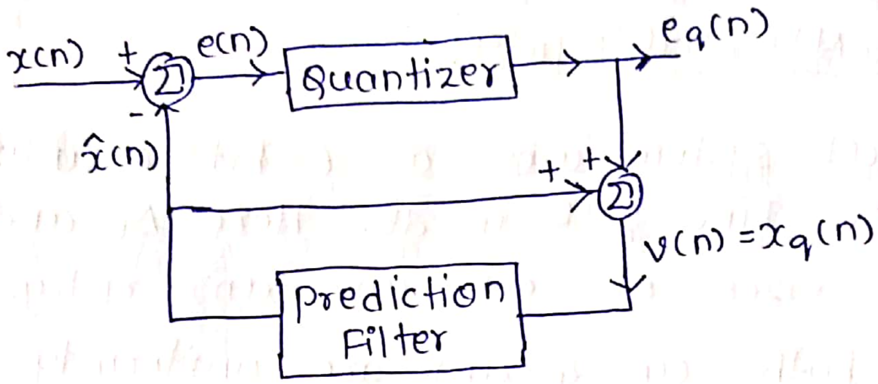
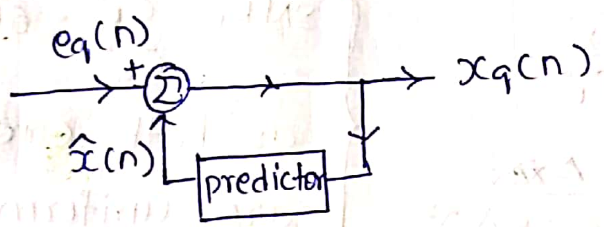




* we cannot obtain $x(n)$ at the receiver but we can get only $x_q(n)$ at the receiver.



* same feed back loop can be used at both transmitter and receiver.

$$SNR = \frac{\sigma_x^2}{\sigma_Q^2}$$

$$\boxed{SNR = \underbrace{\frac{\sigma_x^2}{\sigma_E^2}}_{\text{Gain due to the DPCM operation.}} \underbrace{\frac{\sigma_E^2}{\sigma_Q^2}}_{\text{SNR of the PCM system.}}}$$

let → $\boxed{\frac{\sigma_E^2}{\sigma_Q^2} = SNR_P}$ (SNR of the PCM system)

$\hookrightarrow$ $\boxed{\dfrac{\sigma_x^2}{\sigma_E^2} = G_p \text{ (Predictor Gain)}}$ > 1 (always)

$\therefore$ $\boxed{SNR = SNR_p G_p}$

$SNR_{dB} = 10 \log_{10}(SNR_p) + 10\log_{10} G_p.$

$\boxed{SNR_{dB} = (6N + C) + 10\log_{10} G_p}$

Example: A DPCM system uses a 6-bit quantizer. If $\Delta_E = 0.25\Delta_x$. Find SNR in dB. Here Δ_E and Δ_x are the step sizes of $e(n)$ and $x(n)$ rptly.

Sln. Assume both $e(n)$ & $x(n)$ are uniformly distributed.

w.k.t. * $SNR = SNR_p G_p$

$$= \frac{\sigma_E^2}{\sigma_Q^2} \cdot \frac{\sigma_x^2}{\sigma_E^2}$$

$$= 2^{2N} \times \frac{\Delta_x^2}{(0.25\Delta_x)^2}$$

$SNR_p = \dfrac{\sigma_E^2}{\sigma_Q^2} = 2^{2N}$ as both $e(n)$ & $q(n)$ are uniform

$\therefore$ $\boxed{SNR = 2^{12} \times 16 \doteq 65536}$

* $SNR_{dB} = 6N + 10\log_{10} 16.$

$SNR_{dB} = 6 \times 6 + 12.04$

$\therefore$ $\boxed{SNR_{dB} \approx 48.04}$

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Example: $x(n)$ is a WSS process with zero mean and $R_x(k) = 0.9^{|k|}$. It is input to a DPCM system. If the SNR is 8 find G_p. The predictor is just a delay.

sln

w.k.t. $SNR = \frac{\sigma_x^2}{\sigma_q^2}$.

$\sigma_x^2 = E[X^2]$ ($\because x(n)$ has zero mean).

$\sigma_x^2 = R_x(0) = 1$.

$\therefore \frac{1}{\sigma_q^2} = 8$.

$\boxed{\sigma_q^2 = \frac{1}{8}}$

since the predictor is just a delay $\hat{x}(n) = x_q(n-1)$

w.k.t. $e(n) = x(n) - \hat{x}(n)$.

$e(n) = x(n) - x_q(n-1)$

$e(n) = x(n) - x(n-1) + q(n-1)$

w.k.t. $var(X+Y+Z) = \sigma_X^2 + \sigma_Y^2 + \sigma_Z^2 + 2C_{XY} + 2C_{YZ} + 2C_{XZ}$

$var(E) = \sigma_x^2 + \sigma_x^2 + \sigma_q^2 - 2R_x(1) + 0$.

* since $q(n)$ (noise) is independent of $x(n)$ and zero mean. C_{YZ} & $C_{XZ} = 0$

$\sigma_E^2 = 1 + 1 + \frac{1}{8} - 2 \times 0.9$

$\sigma_E^2 = 0.325$

$\therefore \boxed{G_p = \frac{\sigma_x^2}{\sigma_E^2} = \frac{1}{0.325} = 3.07}$

* variance for $x(n)$ & $x(n-1)$ is same.

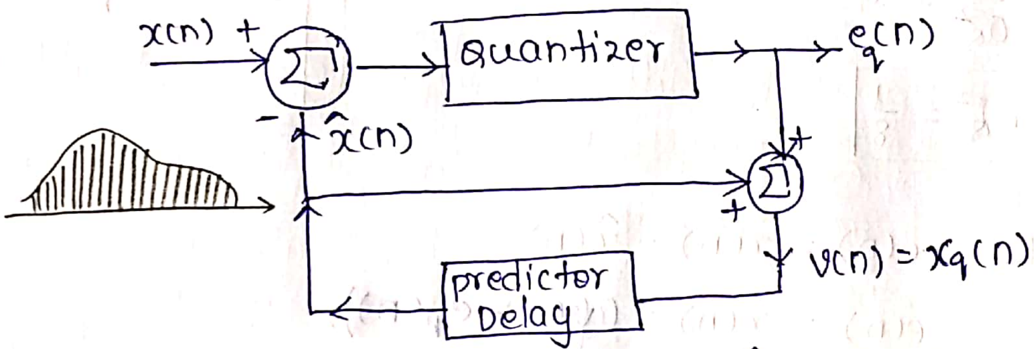
* C_{XY} for $x(n)$ & $x(n-1)$ is $R_x(1) = E[X(n)X(n-1)]$

* zero mean
⇒ co-relation = covariance.

* Noise is independent of signal

Delta Modulation :-

※ Delta Modulation is a special case of DPCM that uses an one-bit quantizer. The samp-ling rate is choosen to be much higher than the Nyquist rate so that the successive samples display a high degree of correlation. The difference b/w successive samples is quantized into two levels $\pm\Delta$. In this case the predictor is just a delay.



w.k.t. $v(n) = x_q(n) = e_q(n) + \hat{x}(n)$. (from block diagram).

$$= e_q(n) + x_q(n-1).$$

$$= e_q(n) + e_q(n-1) + \hat{x}(n-1).$$

$$= e_q(n) + e_q(n-1) + x_q(n-2)$$

$$= e_q(n) + e_q(n-1) + e_q(n-2) + \hat{x}(n-2).$$

$$\vdots$$

let $\hat{x}(0) = 0$. (i.e. set the initial estimate as '0')

then $\boxed{x_q(n) = \sum_{k=1}^{n} e_q(k)}$ — Accumulation.

※ The quantized sample is obtained as an accumulation of the quantized error samples.

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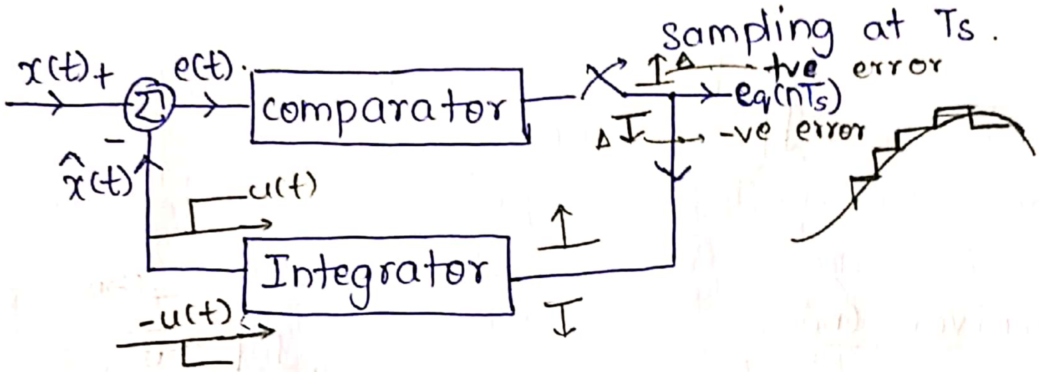
*: $x_q(n)$ is the accumulation of the quantized error samples.

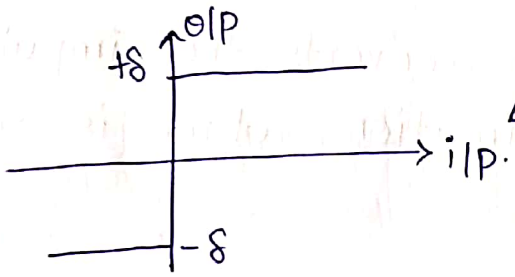
Practical Implementation:

In delta modulation we perform both samp-ling and quantization together & it will be like flat top samples.



Lec-8

Transfer characteristics of Deltamodulation

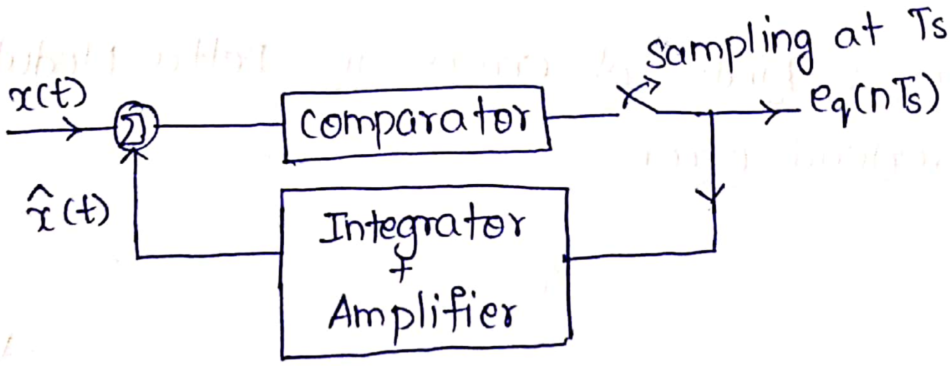


$\Delta = 2\delta$ (step size).

$$x_q(n) = e_q(n) + e_q(n-1) + \dots + e_q(1) + \hat{x}(0)$$

if $\hat{x}(0) = 0$ then

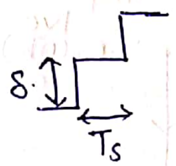
*: $$\boxed{x_q(n) = \sum_{k=1}^{n} e_q(k)}$$



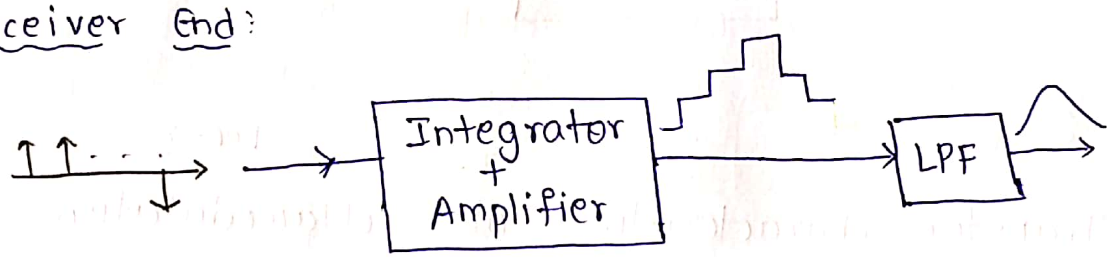
* Here Amplifier is used to obtain the required step size.

* Here we get the stair case approximation of the i/p signal $x(t)$.

* Delta Modulation results in a stair case approximation of $x(t)$.



* **Receiver End:**



* * The Integrator converts the impulses to stair-case approximation which is smoothened by the LPF.

* * The advantages of DM are:
  1) one-bit quantization avoids the requirement for word formation
  2) Simple structure for the transmitter and the receiver.

* * We have 2 kinds of errors in Delta Modulation

* 1) slope-overload error:

P.T.O.

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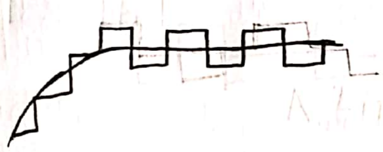
·X· **Slope overload Error**: when the slope of the signal $x(t)$ is too high then $\hat{x}(t)$ is unable to follow $x(t)$ & deviates from it. This is called slope overload.

To avoid slope overload we require that the slope of $x(t)$ at any point of time should be less than (or) equal to the slope of the stair case approximation.

·X· $\therefore$ $\boxed{\dfrac{\delta}{T_s} \geq \max\left|\dfrac{dx(t)}{dt}\right|}$

·X· 2). **Granular Noise.**



·X· Granular Noise occur when $x(t)$ is almost constant.

This noise can be reduced by decreasing δ.

*·X· To avoid slope overload we need to either
i) increase δ (but this will increase the granular nois he
ii) decrease T_s – but this will increase the bitrate and the BW required for transmission.

Example : The signal $x(t) = A\cos 2\pi f_0 t$ undergoes DM with parameters δ & f_s. Find the condition on A to avoid slope overload. Also find the condition on the power of $x(t)$ to avoid slope overload.

s/n

$$x(t) = A\cos 2\pi f_0 t.$$

$$\frac{dx(t)}{dt} = A\, 2\pi f_0 \sin 2\pi f_0 t.$$

$$\max\left|\frac{dx(t)}{dt}\right| = 2\pi f_0 A.$$

$\therefore$ to avoid slope overload.

$$\frac{\delta}{T_s} = \delta f_s \geq \max\left|\frac{dx(t)}{dt}\right|$$

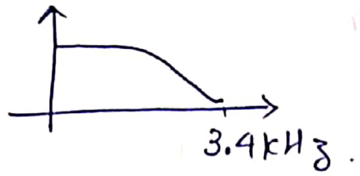
$$\delta f_s \geq 2\pi f_0 A$$

$$\boxed{A \leq \frac{\delta}{2\pi}\,\frac{f_s}{f_0}}$$

$$\boxed{P \leq \frac{\delta^2}{8\pi^2}\,\frac{f_s^2}{f_0^2}} \qquad \left(\because P = \frac{A^2}{2}\right)$$

$\therefore$ The max. possible Amplitude is inversely proportional to the frequency of the sinusoid. But this is not a problem for speech signal as f_{max} (speech signal) will be around 3.4 kHz.

* DM is widely used for speech signal since the Amplitude spectrum of speech signal decays with increasing frequency and hence the slope overload is not a concern.



Speech Signal.

* The Integrator in the Receiver can result in noise enhancement to avoid this the Integrator can be moved to the transmitter itself. This is called Sigma-Delta Modulation. & it is widely used in practice.

Example : A message signal has a BW of W Hz and is sampled at Nyquist Rate. The samples are quantized using an 8-bit PCM system. If the resulting bit rate is 40Mbps. Find W.

Sln : Bit rate is 40 Mbps.

No. of words/samples per second $= \frac{40 \times 10^6}{8}$

$= 5 \times 10^6$.

$\therefore f_s = 5\,MHz$

* Hence $\boxed{W = \frac{f_s}{2} = 2.5\,MHz.}$ (Since $f_s = 2W$ i.e.. Nyquist Rate Sampling)

Example : A signal $x(t) = 5\cos 10^5 \pi t$ is sampled at twice the Nyquist Rate. It is to be quantized so that the SNR is atleast 43dB. Find the minimum possible bitrate & the correspond step size for the quantizer.

$SNR_{dB} \geq 43dB$.

$6N + 1.76 \geq 43dB$.

$N \geq 6.87$

$N \geq 7$

$f_s = 4f_o$.

$f_o = 5 \times 10^4$ (50kHz).

$f_s = 200kHz$.

$\therefore$ Minimum possible bitrate is 7×200 kHz

= 1.4Mbps.

Example : Repeat the problem for $x(t) = 5\cos 10^5\pi t + 10\cos 10^4\pi t$

$SNR_{dB} \geq 43dB$.

Signal power $(P_x) = \frac{5^2}{2} + \frac{10^2}{2} = \frac{125}{2}$

since the signals are orthogonal
$\because f_1 = 50kHz$; $f_2 = 5kHz$
signals are orthogonal

$\Delta = \frac{2 \times 15}{2^N} = \frac{30}{2^N}$.

$\sigma_q^2 = \frac{\Delta^2}{12} = \frac{900}{12 \times 2^{2N}}$.

$\therefore$ $SNR = \frac{P_x}{\sigma_q^2} = \frac{62.5}{900} \times 12 \times 2^{2N}$.

$SNR = \frac{5}{6} \times 2^{2N}$

$SNR_{dB} = 6N - 0.079$.

$SNR_{dB} \geq 43dB$.

$6N - 0.079 \geq 43$.

$6N \geq 43.079$

$N \geq 7.179$

$\underline{N \geq 8}$.

$f_s = 4f_{max}$

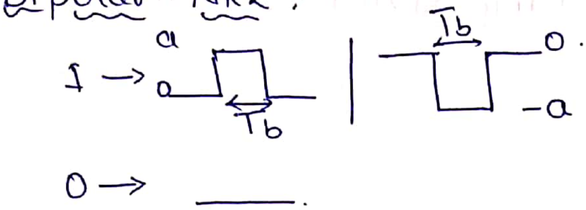
$f_s = 4 \times 50kHz$.

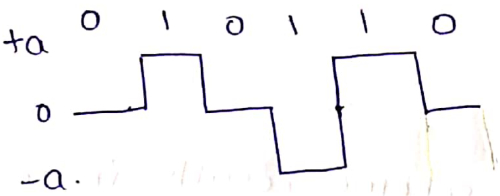
$\boxed{f_s = 200kHz.}$

$\therefore$ Minimum possible bitrate is $8 \times 200kHz$.

$= \underline{1.6}$ Mbps

3) Bipolar NRZ :



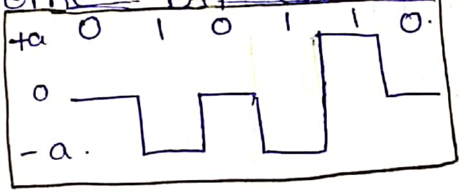


* In this scheme alternate 1s are represented by +ve & -ve pulses rptly. zero is represented by no pulse.

A typical Bipolar NRZ signal.

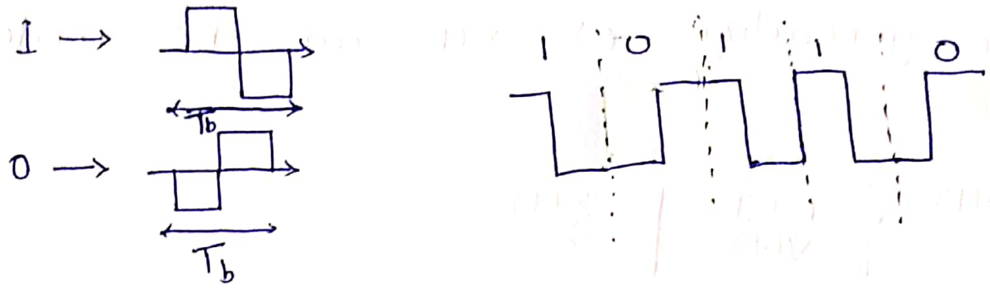
* In this scheme DC content is zero. Polarity inversion is not a problem.

* Some bit errors can be detected.



* In these 3 schemes a long string of 1s (or) 0s (for bipolar scheme → long string of 0s). can result in loss of synchronization b/w the clks at the transmitter & the receiver. At the receiver the transitions in the incoming waveform are used for clock recovery hence absence of transitions for a long period results in considerable drift in the clk. To avoid this we consider Manchester coding.

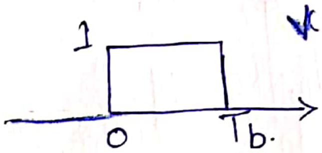
Manchester Coding

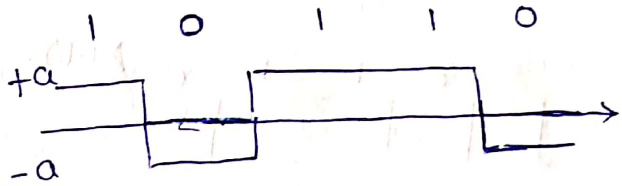


✻ In this scheme there is a transition in the middle of every bit duration and hence clock recovery becomes easier.

✻ Cost → Band width gets doubled.

Power Spectrum of Discrete PAM Signals:





✻ $\boxed{x(t) = \sum_{k=-\infty}^{\infty} A_k\, v(t-kT_b)}$

✻✻ let $v(t)$ denote the basic pulse of duration T_b sec. The discrete PAM signal is a random process given by

✻ $$\boxed{x(t) = \sum_{k=-\infty}^{\infty} A_k\, v(t-kT_b)}$$

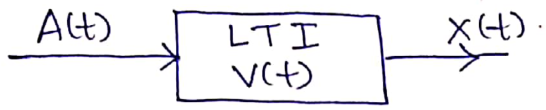
where, $A_k \rightarrow$ is a discrete random variable whose value depends on the k^{th} bit.

$$x(t) = \sum_{k=-\infty}^{\infty} A_k \left\{ \delta(t-kT_b) * v(t) \right\}$$

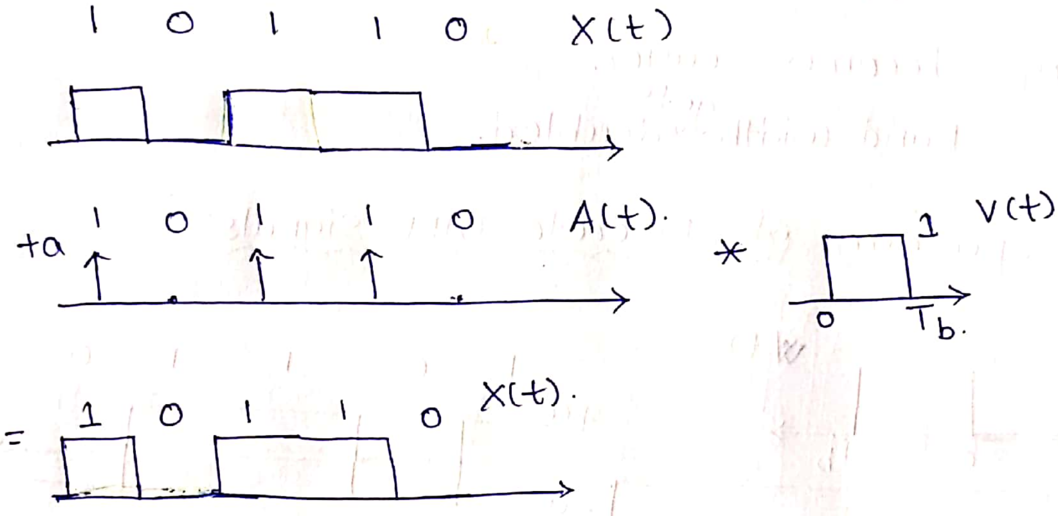
✻ $$\boxed{x(t) = \left\{ \sum_{k=-\infty}^{\infty} A_k\, \delta(t-kT_b) \right\} * v(t)}$$

※ $\boxed{x(t) = A(t) * v(t)}$

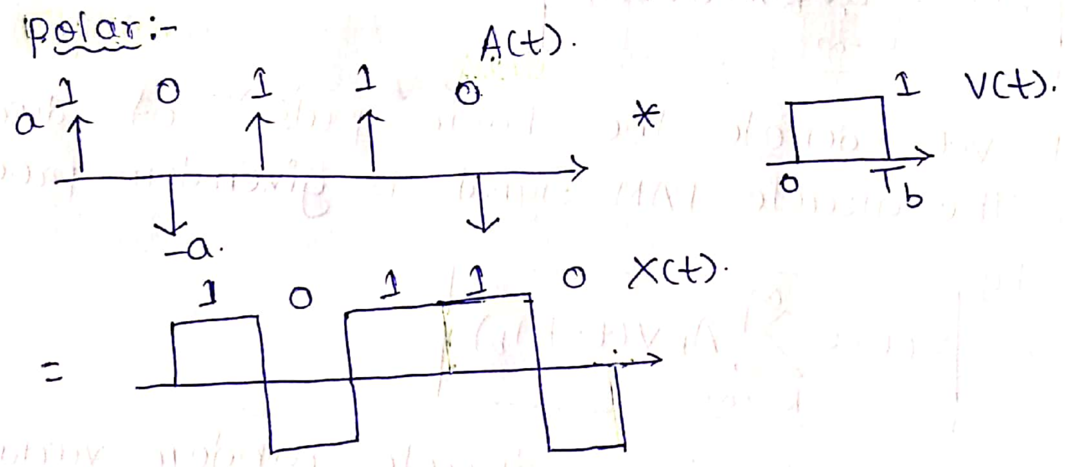
※ The generation of x(t) can be modelled as



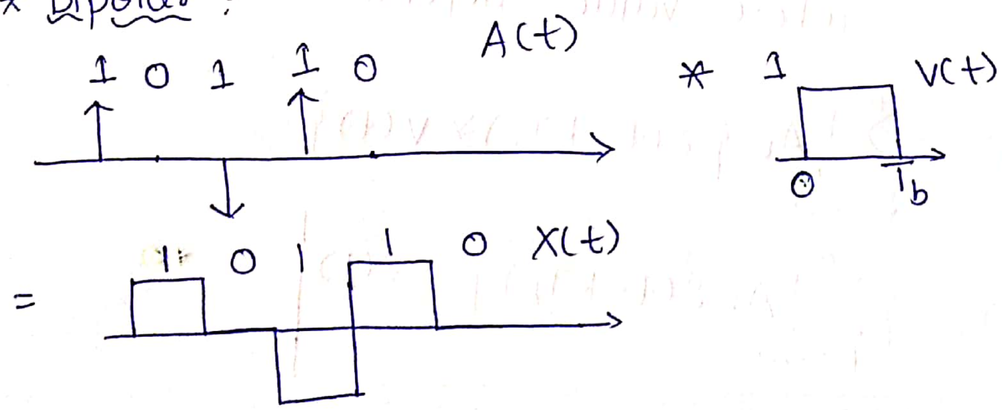
* Unipolar



* Polar:-



* Bipolar :



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Power Spectrum of PAM signals:

$$w(t) \longrightarrow \boxed{h(t)} \longrightarrow v(t)$$

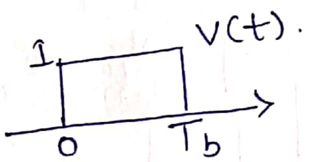
- When a WSS process is input to an LTI System the output is a WSS process whose power spectrum is given as follows.

$$\boxed{S_v(f) = |H(f)|^2 S_w(f)}$$

- Here the discrete PAM signal is a cyclo Stationary process with period T_b. Hence its power spectrum is given by

$$\boxed{S_x(f) = \frac{|V(f)|^2}{T_b} S_A(f)}$$

(cyclo-stationary process ÷ is a stationary process with a periodic component).



$$V(f) = \int_0^{T_b} v(t) e^{-j2\pi f t} \, dt.$$

$$= \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right|_0^{T_b}.$$

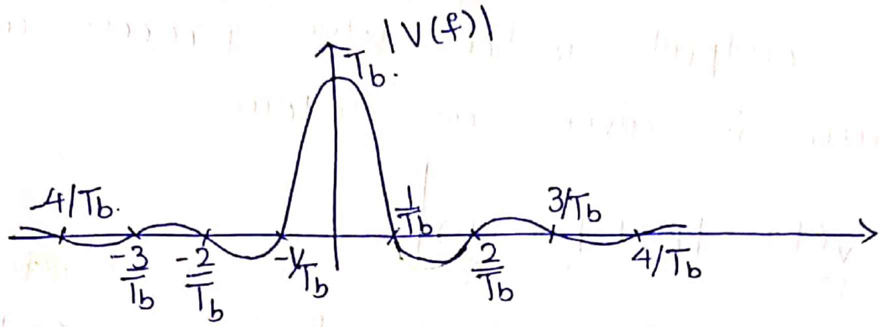
$$= \frac{1 - e^{-j2\pi f T_b}}{j2\pi f}$$

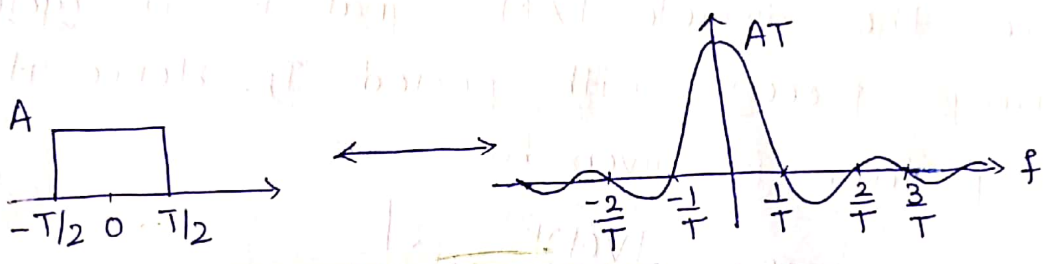
$$= \frac{e^{-j\pi f T_b}}{\pi f} \cdot \sin(\pi f T_b).$$

$$\boxed{V(f) = T_b . e^{-j\pi f T_b} \operatorname{sinc}(f T_b)}$$

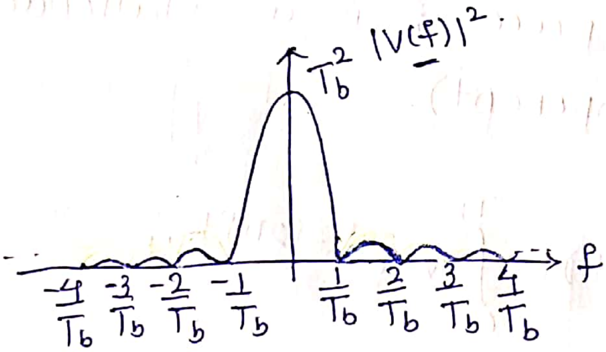
$$|V(f)| = T_b \frac{\sin \pi f T_b}{\pi f T_b}.$$

$$\boxed{|V(f)| = T_b \, \text{sinc}(f T_b).}$$





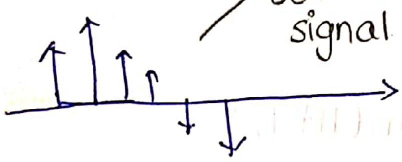
$$\boxed{|V(f)|^2 = T_b^2 \, \text{sinc}^2(f T_b).}$$



$x(n)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}.$$

both are same signal



※ For Random processes we don't talk about the magnitude spectrum we talk about the power spectrum.

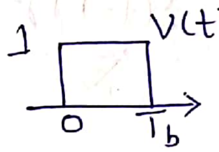
$$x(t),\, X(t) \rightarrow \boxed{h(t)} \rightarrow y(t),\, Y(t)$$

$S_Y(f) = |H(f)|^2 S_X(f)$. → Stationary process.

$S_Y(f) = \frac{|H(f)|^2}{T_b} S_X(f)$ → cyclo-stationary process
↓
for discrete PAM signal.

$$A(t) \rightarrow \boxed{V(t)} \rightarrow X(t).$$

※ $\boxed{S_X(f) = \frac{|V(f)|^2}{T_b} S_A(f).}$

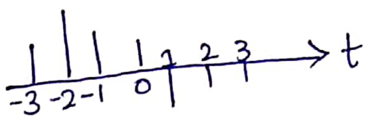


$\boxed{S_X(f) = T_b \operatorname{sinc}^2(fT_b)\, S_A(f)}$ $\left(\because |V(f)| = T_b \operatorname{sinc}(fT_b)\right)$

※ w.k.t. the power spectrum is the Fourier Transform of the auto-correlation function.

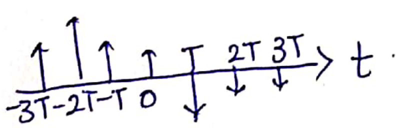
* consider the sequence of samples x_k we can represent the sequence as either the discrete time sequence $x(n)$ (or) as a continuous time signal $x(t)$.

$$x(n) = \sum_{k=-\infty}^{\infty} x_k \delta(n-k).$$



$$x(t) = \sum_{k=-\infty}^{\infty} x_k \delta(t-kT).$$

Two different viewpoints of the same sequence of samples.



* $x(t)$ contains the information about the sampling period T which is absent in $x(n)$.

$$x(n) = \sum_{k=-\infty}^{\infty} x_k \delta(n-k) \xleftrightarrow{F} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$F\left\{ \sum_{k=-\infty}^{\infty} x_k \delta(t-kT) \right\} = \sum_{k=-\infty}^{\infty} F\{ x_k \delta(t-kT)\}.$$

$$= \sum_{k=-\infty}^{\infty} x_k F\{\delta(t-kT)\}.$$

$$\delta(t) \xleftrightarrow{F} 1.$$

$$\delta(t-kT) \xleftrightarrow{F} e^{-jkT2\pi f} = e^{-j2\pi f kT}$$

$$\therefore \boxed{\sum_{k=-\infty}^{\infty} x_k F\{\delta(t-kT)\} = \sum_{k=-\infty}^{\infty} x_k e^{-j2\pi f kT}.}$$

* $\therefore$ The power spectrum of $A(t)$ is

$$* \quad \boxed{S_A(f) = \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}.}$$

where, $R_A(n) = E[A_k A_{k-n}]$

(26)

Unipolar NRZ:

b_k	A_k	P_k
0	0	1/2
1	a	1/2

※ $\boxed{E[X] = \sum_k x_k p(x - x_k)}$

※ We assume that 0 & 1 occur with equal probability & the sequence of bits is independent.

$$R_A(0) = E[A_k^2] = 0 \cdot \frac{1}{2} + a^2 \cdot \frac{1}{2} = \frac{a^2}{2}$$

※ $\boxed{R_A(0) = \frac{a^2}{2}}$

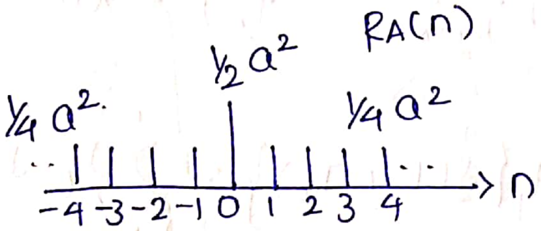
$$R_A(1) = E[A_k A_{k-1}]$$

※ $\boxed{R_A(1) = \frac{a^2}{4}}$

b_k	b_{k-1}	A_k	A_{k-1}	P_k	$A_k A_{k-1}$
0	0	0	0	1/4	0
0	1	0	a	1/4	0
1	0	a	0	1/4	0
1	1	a	a	1/4	a^2

* We can see that $R_A(n) = \frac{1}{4}a^2,\ n \neq 0$.

$$\therefore\ R_A(n) = \begin{cases} \frac{1}{4}a^2, & n \neq 0 \\ \frac{1}{2}a^2, & n = 0. \end{cases}$$



※ $\therefore\ \boxed{R_A(n) = \frac{a^2}{4}(1 + \delta(n))}$

$$\therefore \; S_x(f) = \frac{|V(f)|^2}{T_b} \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}$$

$$= T_b \operatorname{sinc}^2(fT_b) \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}$$

$$= T_b \operatorname{sinc}^2(fT_b) \sum_{n=-\infty}^{\infty} \left\{ \frac{a^2}{4}(1+\delta(n)) \right\} e^{-j2\pi f n T_b}$$

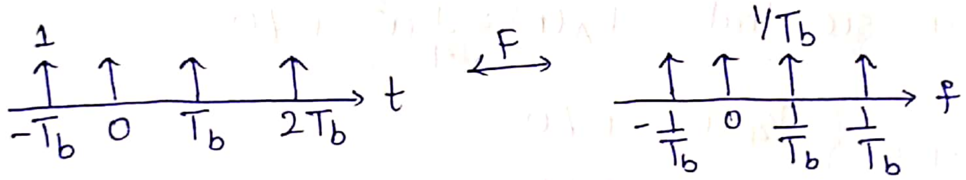
$$= \frac{a^2 T_b}{4} \operatorname{sinc}^2 fT_b + T_b \operatorname{sinc}^2(fT_b) \sum_{n=-\infty}^{\infty} \frac{a^2}{4} e^{-j2\pi f n T_b}$$

$$\therefore \; \boxed{S_x(f) = \frac{a^2 T_b}{4} \operatorname{sinc}^2 fT_b + \frac{a^2}{4} T_b \operatorname{sinc}^2 fT_b \sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b}}$$

$$\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} = 1 + \cos 2\pi f T_b + \cos 4\pi f T_b + \cdots$$

$\downarrow$

we will get a sequence of impulses.



$$\sum_{k=-\infty}^{\infty} \delta(t - kT_b) \qquad\qquad \frac{1}{T_b}\sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T_b}\right) \rightarrow (1)$$

$$\therefore \; F\left\{ \sum_{k=-\infty}^{\infty} \delta(t - nT_b) \right\} = \sum_{k=-\infty}^{\infty} F\{\delta(t - nT_b)\} = \sum_{k=-\infty}^{\infty} e^{-j2\pi f n T_b} \rightarrow (2)$$

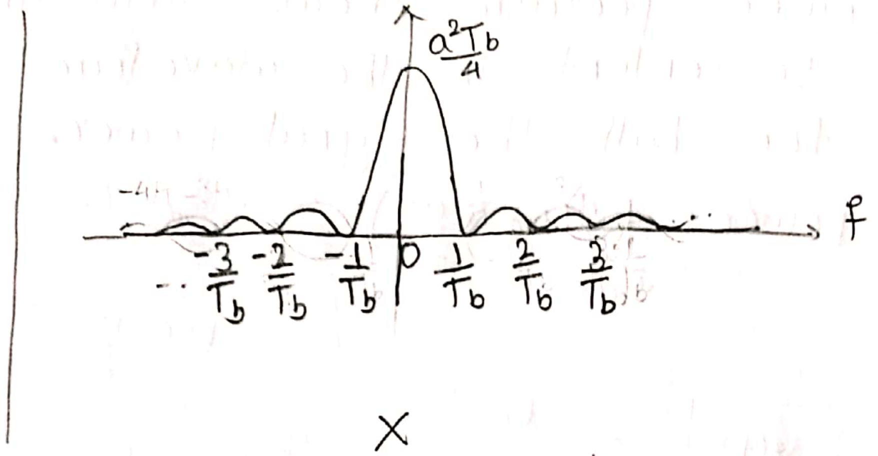
$\therefore$ from (1) & (2) we get.

$$\boxed{\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{k}{T_b}\right)}$$

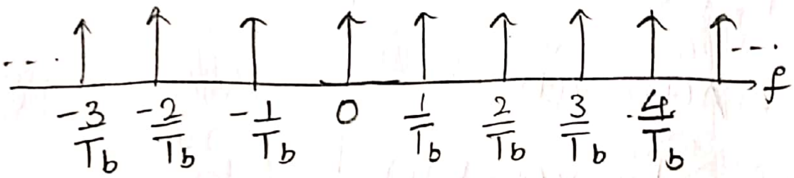
27

$\therefore$ $S_x(f)$ becomes.

$$S_x(f) = \frac{a^2}{4} T_b \operatorname{sinc}^2 f T_b + \frac{a^2}{4} T_b \operatorname{sinc}^2 f T_b \cdot \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)$$



X



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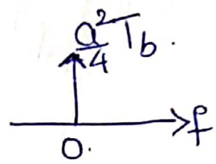
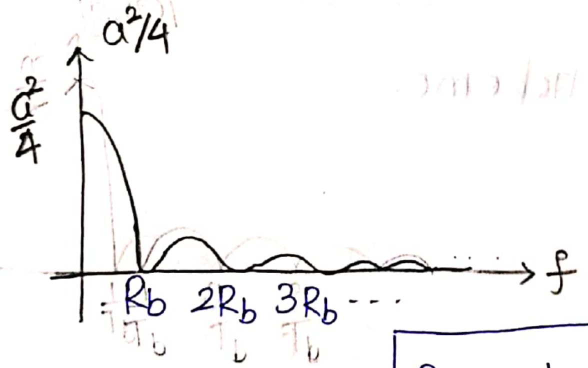


※. All the impulses except at $T=0$ with the zeros of the sinc^2 function. hence we have.

$$S_x(f) = \frac{a^2}{4} T_b \operatorname{sinc}^2 f T_b + \frac{a^2}{4} \delta(f).$$

※ $$\boxed{S_x(f) = \frac{a^2}{4}\left(T_b \operatorname{sinc}^2 f T_b + \delta(f)\right)}$$

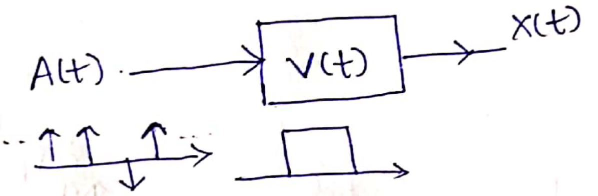


$$\boxed{R_B = \frac{1}{T_b} = \text{Bitrate}}$$

※ By considering the 1st non-DC null as a Band-width we see that unipolar NRZ has a Bandwidth of R_B Hz.

※ The power spectrum shows that there is non-zero DC content in the waveform that accounts for half the signal power. (signal power $= R_x(0) = \frac{a^2}{2}$).

Lec-12



$$x(t) = A(t) * v(t).$$

$$x(t) = \sum_{k=-\infty}^{\infty} A_k \, v(t - kT_b).$$

$$S_x(f) = \frac{|V(f)|^2}{T_b} S_A(f).$$

※ $S_A(f) = \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}.$

$$S_x(f) = \frac{a^2}{4} T_b \, \text{sinc}^2 f T_b + \frac{a^2}{4} \delta(f).$$

※ $\boxed{S_x(f) = \frac{a^2}{4} \left(T_b \, \text{sinc}^2 f T_b + \delta(f) \right)}$

for unipolar NRZ

(28)

Polar NRZ:

b_k	A_k	P_k
0	$-a$	0.5
1	a	0.5

* 1^{st} find $R_A(n)$.

$R_A(0) = E[A_k^2]$

$= \frac{a^2}{2} + \frac{a^2}{2}$

$R_A(0) = a^2$

$R_A(1) = E[A_k A_{k-1}]$

b_k	b_{k-1}	A_k	A_{k-1}	$A_k A_{k-1}$	P_k
0	0	$-a$	$-a$	a^2	1/4
0	1	$-a$	a	$-a^2$	1/4
1	0	a	$-a$	$-a^2$	1/4
1	1	a^2	a	a^2	1/4

$R_A(1) = \frac{a^2}{4} - \frac{a^2}{4} - \frac{a^2}{4} + \frac{a^2}{4}$

$R_A(1) = 0$.

III^{ly} $R_A(2) = 0$

similar analysis for other values of n leads to the result

$$\boxed{R_A(n) = \begin{cases} a^2, & n = 0 \\ 0 & n \neq 0 \end{cases}}$$

$\therefore \boxed{R_A(n) = a^2\delta(n)}$

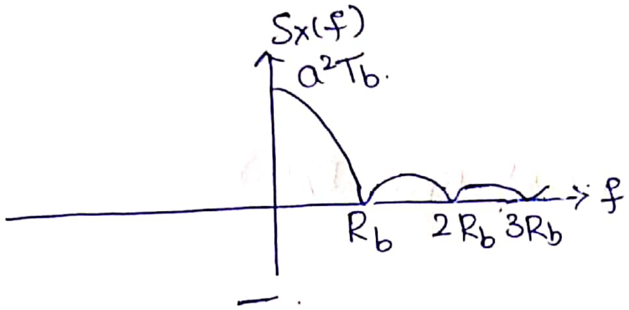
$\therefore S_X(f) = \frac{|V(f)|^2}{T_b} S_A(f)$

$= T_b \operatorname{sinc}^2 f T_b \cdot \sum_{n=-\infty}^{\infty} a^2\delta(n) e^{-j2\pi f n T_b}$.

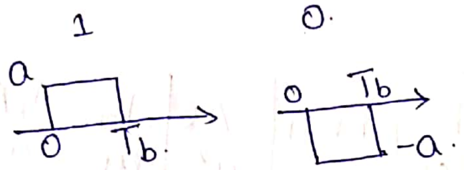
$$\boxed{S_X(f) = a^2 T_b \operatorname{sinc}^2 f T_b}$$

↳ for polar NRZ.

The Band width is R_b Hz.



* The average power of the polar PAM signal.



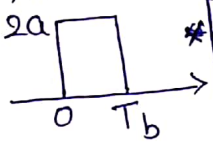
* The energy per bit is
  * $\boxed{E_b = a^2T_b}$ for both 0 & 1

* $\boxed{\text{Avg power} = \frac{E_b}{T_b} = a^2.}$

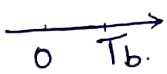
* For the same error performance the unipolar scheme has to be.

* For the same error performance we need the same gap of $2a$ b/w the representation values for 0 & 1.

$1 \rightarrow$



$0 \rightarrow$



* The average power of the unipolar PAM scheme having the same error performance.
  * the Energy per bit is.

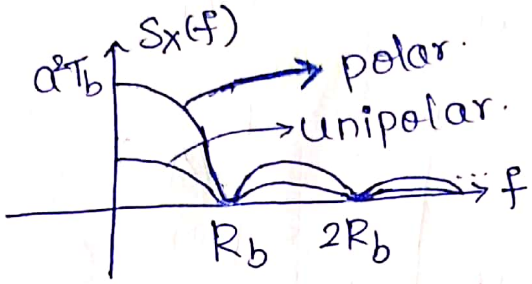
* $$\boxed{E_b = \begin{cases} 4a^2T_b & \text{for } 1 \\ 0 & \text{for } 0. \end{cases}}$$

* Average power $= 0.5\,\frac{4a^2T_b}{T_b} + 0.5\,\frac{0}{T_b}$

$$\boxed{\text{Average Power} = 2a^2}$$

※ ∴ for the same error performance the unipolar scheme requires twice the average power of the polar scheme.

Reason: Half the power is lost as DC in unipolar scheme.



only ac power is considered.

Bipolar:

* $R_A(0) = E[A_k^2]$

b_k	A_k	p_x
0	0	0.5
1	a	0.25
	-a	0.25

$R_A(0) = 0.(0.5) + a^2(0.25) + a^2(0.25)$

※ $\boxed{R_A(0) = \frac{a^2}{2}}$

* $R_A(1) = E[A_k . A_{k-1}]$

b_k	b_{k-1}	A_k	A_{k-1}	A_kA_{k-1}	p_k
0	0	0	0	0	1/4
0	1	0	a -a	0 0	1/4
1	0	a -a	0	0 0	1/4
1	1	a -a	-a a	$-a^2$	1/4

$R_A(1) = -a^2 . \frac{1}{8} - a^2 . \frac{1}{8}$

※ $\boxed{R_A(1) = -a^2/4}$

* $R_A(2) = E[A_k A_{k-2}]$

b_k	b_{k-2}	A_k	A_{k-2}	$A_k A_{k-2}$	P_k
0	0	0	0	0	1/4
0	1	0	a $-a$	0	1/4
1	0	a $-a$	0	0	1/4
1	1	a a $-a$ $-a$	a $-a$ $-a$ a	a^2 $-a^2$ $-a^2$ a^2	1/4 (1/16 for each outcome)

$R_A(2) = (a^2 - a^2 - a^2 + a^2) \times \frac{1}{16} = 0.$

$$\boxed{R_A(2) = 0}$$

* Therefore, we have $R_A(n) = \begin{cases} a^2/2 & , n=0 \\ -a^2/4 & , n = 1, -1. \\ 0 & \text{elsewhere.} \end{cases}$

& auto correlation is symmetric

$\therefore\ R_A(n) = \frac{a^2}{2}\delta(n) - \frac{a^2}{4}\left(\delta(n-1) + \delta(n+1)\right).$

$\therefore\ S_A(f) = \sum_{n=-\infty}^{\infty}\left[\frac{a^2}{2}\delta(n) - \frac{a^2}{4}\left(\delta(n-1) + \delta(n+1)\right)\right]$

$S_A(f) = \frac{a^2}{2} - \frac{a^2}{4}\left(e^{-j2\pi f T_b} + e^{j2\pi f T_b}\right).$

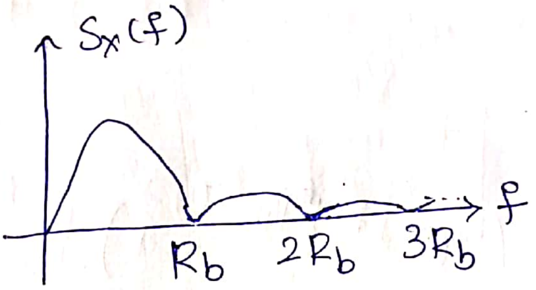
w.k.t. $S_X(f) = \dfrac{|V(f)|^2}{T_b} S_A(f)$.

$$S_X(f) = T_b \operatorname{sinc}^2 fT_b \left[\frac{a^2}{2} - \frac{a^2}{4}\left(e^{-j2\pi fT_b} + e^{j2\pi fT_b}\right)\right]$$

$$= T_b \operatorname{sinc}^2 fT_b \left[\frac{a^2}{2} - \frac{a^2}{2}\cos 2\pi fT_b \right]$$

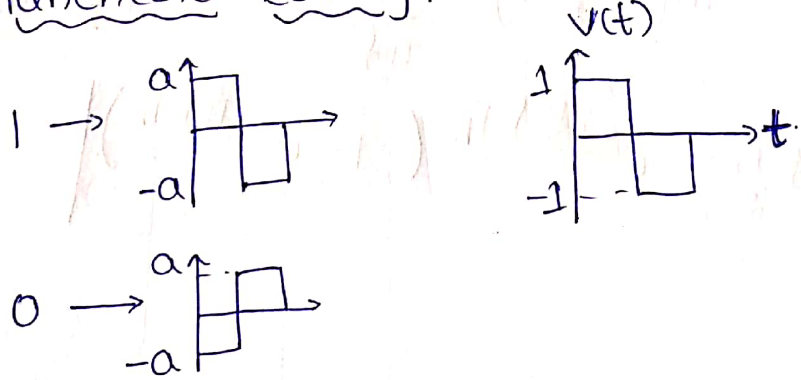
$$= a^2 T_b \sin^2 \pi fT_b \operatorname{sinc}^2 fT_b .$$

※ $\boxed{S_X(f) = a^2 T_b \sin^2(\pi f T_b) . \operatorname{sinc}^2 \pi f T_b}$ → for bipolar NRZ scheme.



※ The power spectrum has a null at f=0.
The BW is R_b Hz.

Manchester Coding :-



※ for Manchester coding the i/p A(t), hence $R_A(n)$ is same as that for the polar case. only v(t) is different.

$$V(f) = \int_0^{T_b/2} e^{-j2\pi ft}\,dt + \int_{T_b/2}^{T_b} -e^{-j2\pi ft}\,dt$$

$$= \frac{1}{-j2\pi f}\left(e^{-j2\pi ft}\Big|_0^{T_b/2}\right) + \frac{1}{j2\pi f}\left(e^{-j2\pi ft}\Big|_{\frac{T_b}{2}}^{T_b}\right)$$

$$= \frac{e^{-j\pi fT_b} - 1}{-j2\pi f} + \frac{e^{-j2\pi fT_b} - e^{-j\pi fT_b}}{j2\pi f}$$

$$= \left\{\begin{array}{l} \dfrac{e^{-j\frac{\pi}{2}fT_b}}{-j2\pi f}\left(e^{-j\frac{\pi}{2}fT_b} - e^{j\frac{\pi}{2}fT_b}\right) \\ + \\ \dfrac{e^{-j\pi fT_b}}{j2\pi f}\left(e^{-j\pi fT_b} - 1\right). \end{array}\right.$$

$$= \frac{e^{-j\frac{\pi}{2}fT_b}}{\pi f}\sin\left(\frac{\pi}{2}fT_b\right) - \frac{e^{-j\frac{3\pi}{2}fT_b}}{\pi f}\sin\left(\frac{\pi}{2}fT_b\right)$$

$$\boxed{V(f) = \frac{T_b}{2}\operatorname{sinc}\left(f\frac{T_b}{2}\right)\cdot e^{-j\frac{\pi}{2}fT_b}\left(1 - e^{-j\pi fT_b}\right)}$$

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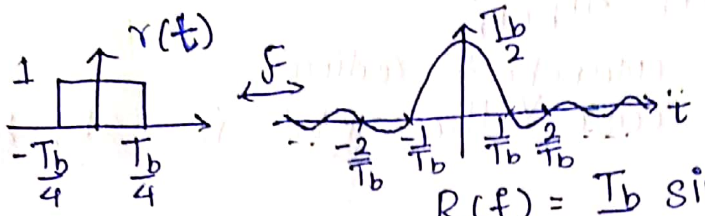
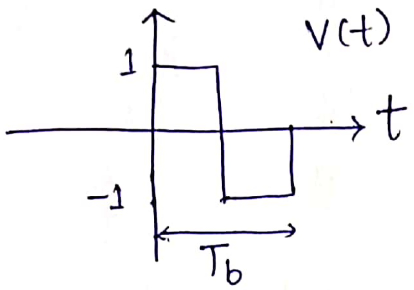
A(t) is the same as that for the polar scheme and hence $S_A(f) = a^2$



$$R(f) = \frac{T_b}{2}\,\text{sinc}\left(\frac{fT_b}{2}\right)$$

$$v(t) = r\left(t - \frac{T_b}{4}\right) - r\left(t - \frac{3T_b}{4}\right)$$



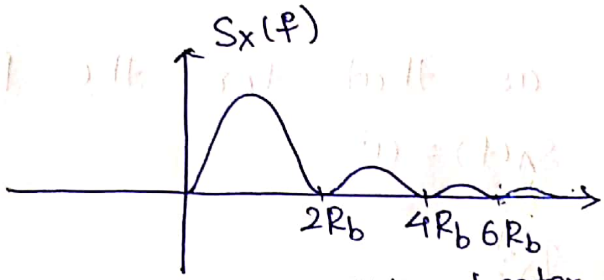
$$\therefore\ V(f) = e^{-j2\pi f \cdot \frac{T_b}{4}} R(f) - e^{-j2\pi f \cdot \frac{3T_b}{4}} R(f)$$

$$V(f) = \frac{T_b}{2}\,\text{sinc}\left(\frac{fT_b}{2}\right) e^{-j2\pi f \frac{T_b}{2}} \left(e^{j2\pi f \frac{T_b}{4}} - e^{-j2\pi f \frac{T_b}{4}}\right)$$

$$\boxed{V(f) = T_b j\, e^{-j2\pi f \frac{T_b}{2}}\ \text{sinc}\left(\frac{fT_b}{2}\right) \sin\left(\frac{\pi f T_b}{2}\right)}$$

$$* \quad \boxed{\frac{|V(f)|^2}{T_b} = T_b\,\text{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right)}$$

$$\boxed{S_X(f) = \frac{|V(f)|^2}{T_b} S_A(f) = T_b a^2\,\text{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right)}$$



(Manchester coding)

* In this case the BW is $2R_b$ Hz.

Scanned by CamScanner

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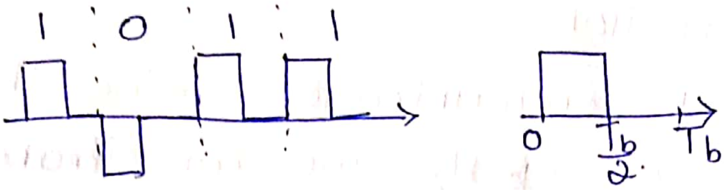
Polar RZ waveform

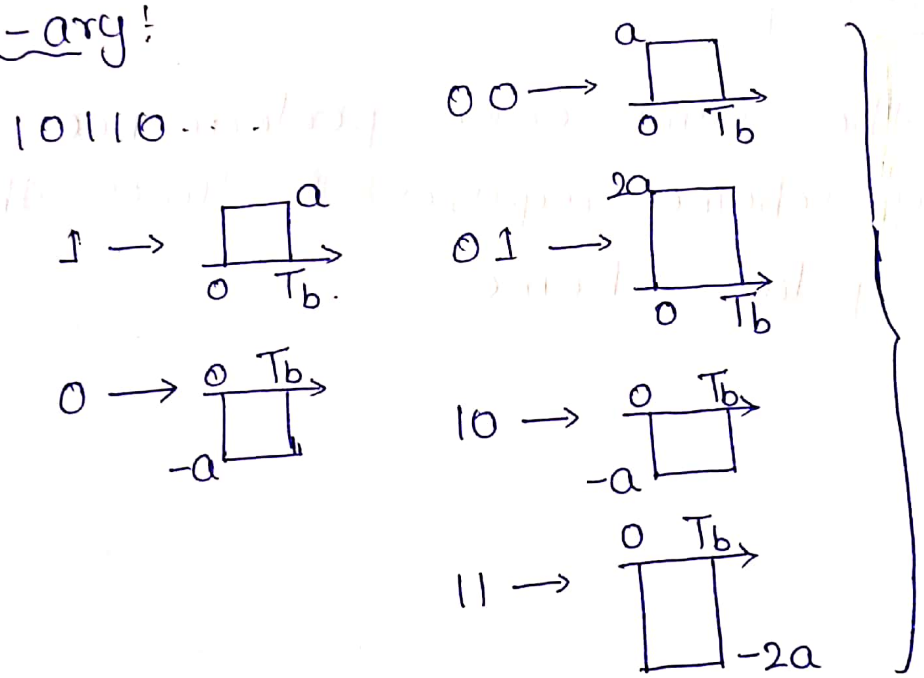


BW ⟷ $\frac{2}{T_b} = 2R_b$

※ Gaining in power (less avg. power) by spending in BW (BW increases).

** For the RZ scheme A(t) is the same but v(t) has lesser width and hence the BW increases correspondingly. But RZ schemes require lesser Avg. power than the NRZ schemes.

M-ary:



Quaternary Scheme.

※ Instead of coding the individual bits we can consider L bits at a time this results in $M = 2^L$ possible combinations. When a basic pulse is modulated to represent these M combinations it is known as M-ary communication.

* With M-ary communication using the same band width of R_b Hz we can increase the bit-rate to LR_b bits/sec.

* Average power for Quaternary scheme:

$$P_{avg} = \frac{1}{4}[2 \times 9a^2 + 2 \times a^2]$$

$$\boxed{P_{avg} = 5a^2}$$

* for polar NRZ scheme.

$$\boxed{P_{avg} = a^2}$$

* For the same error performance the Quaternary scheme requires 5 times the power of polar scheme.