FORMULA SHEET

Complex Number

- $z = x + iy$, $i = \sqrt{-1}$
- Integral Power:-

$i = \sqrt{-1}$ In general: $i^{4n} = 1$

$i^2 = -1$ $\quad i^{4n+1} = i$

$i^3 = -i$ $\quad i^{4n+2} = -1$

$i^4 = 1$ $\quad i^{4n+3} = -i$

- Equality of two complex no.s:-

$z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

iff $a_1 = a_2$ and $b_1 = b_2$

i.e. $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$

- Conjugate:-

$z = a + ib$ conjugate $= \bar{z} = a - ib$.

- Algebra of two complex no.s:-

1) $z_1 + z_2$
2) $z_1 - z_2$
3) $z_1 \cdot z_2$
4) $\frac{z_1}{z_2}$

Algebra like our simple add, sub, mul, div

- Square root of complex no.:-

$x + iy = \sqrt{a + ib}$

#Short trick:

$x^2 = \frac{\sqrt{a^2+b^2} + a}{2}$ $\quad y^2 = \frac{\sqrt{a^2+b^2} - a}{2}$

Sign in the eqn is at it is for answer also. No change in sign.

- Modulus :-

$$|z| = r = \sqrt{a^2+b^2} = \sqrt{[Re(z)]^2 + [Im(z)]^2}$$

- Argument/Amplitude :- $z \neq 0$.

$$\arg z = \theta = \tan^{-1}\left(\frac{b}{a}\right), \quad 0 \leq \theta < 2\pi$$

1^{st} Quad. : $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

2^{nd} Quad. : $\theta = \pi - \tan^{-1}\left(\frac{b}{a}\right)$

3^{rd} Quad. : $\theta = \pi + \tan^{-1}\left(\frac{b}{a}\right)$

4^{th} Quad. : $\theta = 2\pi - \tan^{-1}\left(\frac{b}{a}\right)$

- Polar Form :- $z = x + iy$

$\therefore z = r(\cos\theta + i\sin\theta)$

$\therefore r = \sqrt{x^2+y^2} \qquad \therefore |z| = \sqrt{x^2+y^2}$

$x = r\cos\theta \qquad y = r\sin\theta$

- Euler's form or Exponential Form :-

$e^{i\theta} = \cos\theta + i\sin\theta \qquad \therefore z = re^{i\theta}$

- De-Moivre's Theorem :-

1] $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$

2] $(\cos\theta - i\sin\theta)^{n} = \cos n\theta - i\sin n\theta$

3] $\dfrac{1}{(\cos\theta - i\sin\theta)^{1}} = \cos\theta - i\sin\theta$

4] $(\cos\theta - i\sin\theta)^{-n} = \cos n\theta + i\sin n\theta$.

- Cube root of unity:-

$x = 1 \Rightarrow 1$

$x = \frac{-1+\sqrt{3}i}{2} \Rightarrow \omega$

$x = \frac{-1-\sqrt{3}i}{2} \Rightarrow \omega^2$

There are 2 roots of 1.

1] $\omega^3 = 1$

2] $1+\omega+\omega^2 = 0$

3] $\omega^2 = \frac{1}{\omega}$ and $\omega = \frac{1}{\omega^2}$

4] $1+\omega = -\omega^2$, $1+\omega^2 = -\omega$, $\omega+\omega^2 = -1$

5] $\omega^{3n+1} = \omega$

6] $\omega^{3n+2} = \omega^2$

7] $\overline{\omega} = \omega^2$

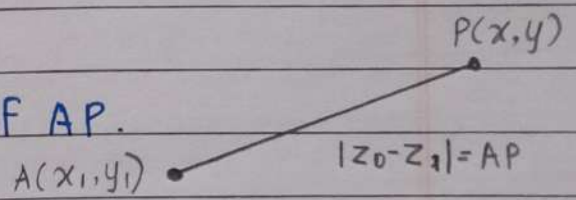
8] $(\overline{\omega})^2 = \omega$

- Set of point in complex no.s:-

$z = x+iy \Rightarrow$ Variable pt.

$z_1 = x_1+iy_1 \Rightarrow$ Fixed pt.

1] $|z-z_1|$ represent length of AP.



2] $|z-z_1| = a$ represent circle with centre $A(x_1, y_1)$ & radius a.