CHAPTER 02

Laws of Motion

Force

It is an effort in the form of push or pull causing or tending to cause motion, change in motion or deformations in a body. It is a vector quantity.

Unit of Force

- In SI system, **absolute unit** of force is **newton**.
- In CGS system, absolute unit of force is **dyne**.
- In MKS unit, **gravitational unit** of force is kilogram weight (kg-wt).

  where, 1 kg-wt = 1 kgf = 9.8 N
- In CGS system, gravitational unit of force is **gram weight** (g-wt) or **gram force** (gf).

$$1 \text{ gf} = 980 \text{ dyne}$$

Also, $1 \text{ N} = 10^5$ dyne

Fundamental Forces in Nature

All the forces in nature are classified into following four interaction that are termed as fundamental forces.

Gravitational Force

It is the force of mutual attraction between two bodies by virtue of their masses. According to Newton's law of gravitation, the gravitational attraction between two bodies of masses m_1 and m_2; and separated by distance r is given by

$$F = G\frac{m_1 m_2}{r^2}$$

Electromagnetic Force

The force which acts between charged particles and is the combination of all electrical and magnetic forces are called electromagnetic force. It can be attractive or repulsive and have infinite range, although its strength is inversely proportional to the square of the distance.

An electromagnetic force between two static charges (q_1, q_2) is called **electrostatic** or **Coulomb force** given by

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$$

where, r = distance between charges q_1 & q_2

and ε_0 = permittivity of vacuum.

Strong Nuclear Force

Nucleus consists of protons and neutrons. Protons being positively charged could repel each other and explode the nucleus.

Gravitational force is negligible comparing to electric force to overcome the repulsion of protons inside the nucleus. Thus, a new force came into existence known as strong nuclear force. The range of this force is very small, *i.e.* about 10^{-15} m. It is 100 times stronger than electromagnetic force in strength.

Weak Nuclear Force

The weak nuclear force appears only between elementary particles involved in a nuclear process of radioactively like β-decay of a nucleus. The range of weak nuclear force is very small, *i.e.* of the order of 10^{-15} m. It is 10^{25} times stronger than the gravitational force.

Inertia

It is an inherent property of all bodies, by virtue of which they cannot change by themselves their state of rest or of uniform motion along a straight line.

As inertia of a body is measured by the mass of the body. So, heavier the body, greater is the force required to change its state.

Inertia is of three types

(i) **Inertia of rest** It is the inability of a body to change its state of rest by itself.

e.g. A person standing in a train falls backward when the train suddenly starts moving forward.

(ii) **Inertia of motion** It is the inability of a body to change its state of uniform motion by itself, *i.e.* a body in uniform motion can neither accelerate nor retard on its own and comes to rest.

e.g. When a moving bus suddenly stops or apply the brake, a person standing in it falls forward.

(iii) **Inertia of direction** It is the inability of a body to change its direction of motion by itself, *i.e.* it continues to move along the straight line unless compelled by some external force to change its direction.

e.g. The rain drops falling vertically downwards cannot change their direction of motion and wet us with the umbrella on.

Momentum

Momentum of a body is the quantity of motion contained in the body. It is measured as the product of mass and velocity of a body. It is represented by p.

i.e. Momentum (p) = Mass (m) × Velocity (v)

Impulse

A large force acting for a short time to produce a finite change in momentum is called impulse.

Impulse, $J = F_{av} \times \Delta t$

$$= (p_2 - p_1) \times (t_2 - t_1) \qquad (\because \Delta t = t_2 - t_1)$$

SI unit of impulse is kg ms^{-1}.

Inertial and Non-inertial Frames of Reference

Inertial Frame of Reference In this frame of reference, Newton's first law of motion holds good. In an inertial frame of reference, if no force acts on a body, it continues to be at rest or in uniform motion.

A frame of reference moving with a constant velocity with respect to an inertial frame of reference is also an inertial frame.

Non-inertial Frame of Reference A frame of reference which is accelerating with respect to an inertial frame of reference is called non-inertial frame of reference.

Newton's laws cannot be applied to such frame of reference.

Note ***Pseudo force*** *is an imaginary force assumed to exists in a non-inertial frame of reference (to apply Newton's second law) in the direction opposite to the acceleration (**a**) of the frame. The pseudo force on a body of mass m is given by*

$$\mathbf{F}_{\text{pseudo}} = -m\mathbf{a}$$

Equilibrium of Concurrent Forces

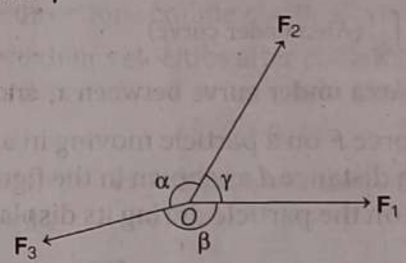
The condition for the equilibrium of a number of forces acting at the same point is that the vector sum of all these forces is equal to zero.

i.e. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + \ldots \mathbf{F}_n = 0$

Lami's Theorem

Lami's theorem states that, "if three forces acting on a particle are in equilibrium, then each force is proportional to the sine of the angle between the other two forces." If an object O is in equilibrium under three concurrent, coplanar forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ as shown in figure. Then,

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



Work

Work is said to be done when a force acts on a body such that the body is displaced through some distance in the direction of force. The work done is given by

$$W = \mathbf{F} \cdot \mathbf{s} \quad \text{or} \quad W = Fs\cos\theta$$

Work Done by a Variable Force

A force is said to be variable force, if it changes its direction or magnitude or both. The work done by a variable force can be calculated as

$$W = \int_{r_1}^{r_2} \mathbf{F} \cdot d\mathbf{r}$$

where, integration is performed along the path of particle and $d\mathbf{r}$ is the position vector of the particle.

Example 1. A position dependent force $F = 7 - 2x + 3x^2$ acts on a small body of mass 2 kg and displace it from $x = 0$ to $x = 5$ m. Calculate the work done (in joule).

a. 138J **b.** 135 J **c.** 136 J **d.** 137 J

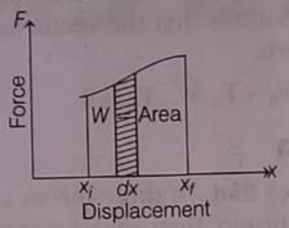
Sol (*b*) Work done, $W = \int_{x_1}^{x_2} F\,dx = \int_0^5 (7 - 2x + 3x^2)\,dx$

Here, the body changes its position from $x = 0$ to $x = 5$.

$$\therefore \quad W = \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5 = [7(5) - (5)^2 + (5)^3 - 0] = 135\text{ J}$$

Calculation of Work Done by Force-Displacement Graph

The area under force-displacement curve gives work done.



Force-displacement graph

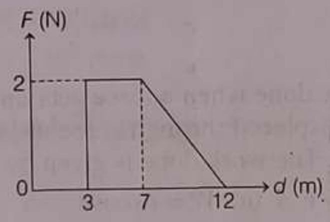
$$W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} F \cdot dx$$

where, x_i and x_f are the initial and final positions, respectively.

$$\therefore \quad W = \int_{x_i}^{x_f} \text{(Area under curve)}$$

$$\Rightarrow \quad W = \text{Area under curve between } x_i \text{ and } x_f.$$

Example 2. Force F on a particle moving in a straight line varies with distance d as shown in the figure. Find the work done on the particle during its displacement of 12 m.



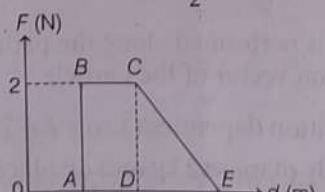
a. 15 J **b.** 16 J **c.** 13 J **d.** 12 J

Sol (c) Work done = Area under (force-displacement) graph

$= \text{Area of rectangle } ABCD + \text{Area of } \Delta DCE$

$= \text{Length} \times \text{Breadth} + \frac{1}{2} \times \text{Base} \times \text{Height}$

$= (AB \times AD) + \frac{1}{2} \times DE \times CD$



$= 2 \times (7-3) + \frac{1}{2} \times (12-7) \times 2$

$= 8 + \frac{1}{2} \times 10 = 8 + 5 = 13\text{ J}$

Conservative and Non-conservative Forces

Conservative Force

A force is said to be conservative, if work done by it in moving a body depends only on the initial and final positions of the body and not on the path travelled between initial and final positions.

e.g. Electrostatic force and gravitational force, etc.

Non-conservative Force

A force is said to be non-conservative, if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions.

e.g. Force of friction and viscous force, etc.

Energy

The capacity or ability to do work is called energy. It is calculated as the work done by a body. It is of mainly two types

Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy and it is given as $\text{KE} = \frac{1}{2}mv^2$.

Potential Energy

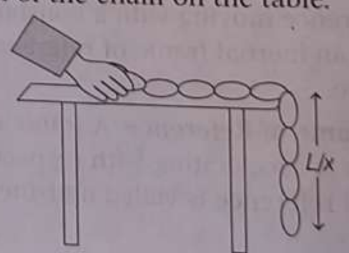
The energy possessed by a body or system by virtue of its position or configuration is known as potential energy.

Gravitational potential energy $= mgh$

Elastic potential energy of spring $= \frac{1}{2}kx^2$

Work Done in Pulling a Chain against Gravity

Consider a chain of length L and mass M is held on a frictionless table with L/x of its length hanging over the edge. We have to find the work done in pulling the hanging portion of the chain on the table.



For this, we can assume centre of mass of hanging portion of the chain at the middle.

Mass of hanging part of chain $= \frac{M}{L} \times \frac{L}{x} = \frac{M}{x}$

Now, assume hanging portion of the chain as a point mass centred at its centre of mass.

Therefore, work done to raise the centre of mass of the chain on the table,

$$W = \frac{M}{x} \times g \times \frac{L}{2x} \text{ or } W = \frac{MgL}{2x^2}$$

Collision

A collision is an isolated event in which two or more colliding bodies exert strong forces on each other for a relatively short time. For a collision to take place, the actual physical contact is not necessary.

Types of Collisions

Collision between two bodies may be classified in two ways

(i) Elastic and inelastic collision

(ii) Head-on and oblique collision

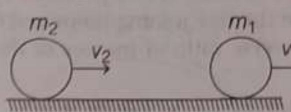
Elastic and Inelastic Collisions

A collision is said to be **elastic**, if along with linear momentum, kinetic energy also remains conserved before and after collision.

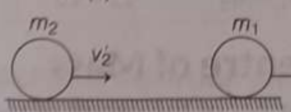
A collision is said to be **inelastic**, if only linear momentum remains conserved and not the kinetic energy. The collision is said to be **perfectly inelastic**, if approaching particles permanently stick to each other and move with common velocity.

Elastic Collision in One Dimension

Let the two balls of masses m_1 and m_2 collide each other elastically with velocities v_1 and v_2 in the directions shown in figure below. Their velocities become v_1' and v_2' after the collision along the same line.



(a) Before collision



(b) After collision

Head on elastic collision

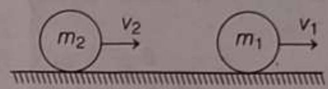
Then, final velocities are given as

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \left(\frac{2m_2}{m_1 + m_2}\right) v_2 \quad \text{...(i)}$$

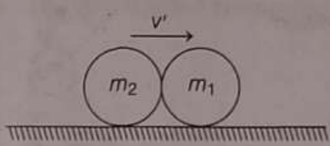
and
$$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_2 + \left(\frac{2m_1}{m_1 + m_2}\right) v_1 \quad \text{...(ii)}$$

Perfectly Inelastic Collision in One Dimension

In an inelastic collision due to permanent deformation, the kinetic energy of the particles no longer remains conserved. However, in the absence of external forces, law of conservation of linear momentum still holds good.



Before collision



After inelastic collision

Loss in KE in inelastic collision is given by

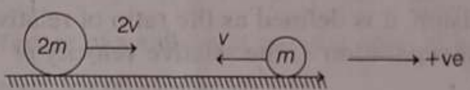
$$\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(v_1 - v_2)^2$$

Example 3. Two particles of masses m and $2m$ moving in opposite directions collide elastically with velocities v and $2v$. Find their velocities after collision.

a. $3v, 0$ b. $1, 3v$

c. $0, 1v$ d. $0, 4v$

Sol (*a*) Here, $v_1 = -v, v_2 = 2v, m_1 = m$ and $m_2 = 2m$



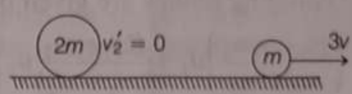
Substituting these values in Eqs. (i) and (ii), we get

$$v_1' = \left(\frac{m - 2m}{m + 2m}\right)(-v) + \left(\frac{4m}{m + 2m}\right)(2v)$$

or $$v_1' = \frac{v}{3} + \frac{8v}{3} = 3v$$

and $$v_2' = \left(\frac{2m - m}{m + 2m}\right)(2v) + \left(\frac{2m}{m + 2m}\right)(-v)$$

or $$v_2' = \frac{2}{3}v - \frac{2}{3}v = 0$$



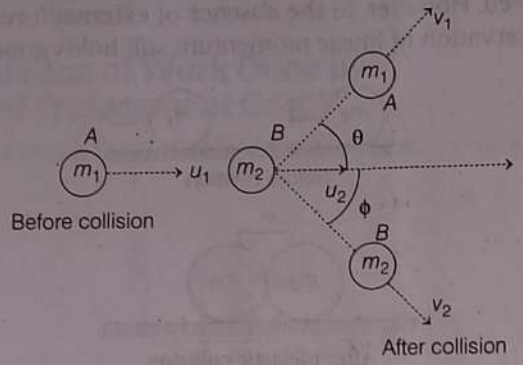
i.e. The second particle (of mass $2m$) comes to rest while the first particle (of mass m) moves with velocity $3v$ in the direction shown in figure given above.

Elastic Collision in Two Dimensions

In this type of collision, particles after collision moves in different directions. KE and momentum also remains conserved in this type of collision.

Perfectly Elastic Collision in Two Dimensions

In two dimensional (or oblique) collision between two bodies, momentum remains conserved.



$\therefore$ Along the X-axis,

$$m_1u_1 + m_2u_2 = m_1v_1\cos\theta + m_2v_2\cos\phi \quad \text{... (i)}$$

and along the Y-axis,

$$0 = m_1v_1\sin\theta - m_2v_2\sin\phi \quad \text{... (ii)}$$

As, the total kinetic energy remains unchanged.

Hence, $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...(iii)}$$

We can solve these equations provided that either the value of θ or ϕ is known to us.

Coefficient of Restitution (e)

For a collision, it is defined as the ratio of relative velocity of separation to the relative velocity of approach.

Thus, coefficient of restitution, $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

For a perfectly elastic collision, $e = 1$.

If $0 < e < 1$, then collision is said to be partially elastic. But if $e = 0$, then collision is said to be perfectly inelastic.

In a perfectly inelastic collision, $e = 0$ which means that $v_2 - v_1 = 0$ or $v_2 = v_1$.

It can be shown that for an inelastic collision, the final velocities of the colliding bodies are given by

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \frac{(1+e)m_2}{(m_1 + m_2)}u_2$$

and $$v_2 = \frac{(1+e)m_1}{(m_1+m_2)}u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2$$

If a particle of mass m, moving with velocity u, hits an identical stationary target inelastically, then final velocities of projectile and target are correlated as

$m_1 = m_2 = m$ and $u_2 = 0$; $\dfrac{v_1}{v_2} = \dfrac{1-e}{1+e}$.

System of Particles

A body of finite size is regarded as a system of particles because it is composed of a large number of particles interacting with one another.

Centre of Mass

It is defined as a point at which total mass of a finite body is supposed to be concentrated and it is given as, for a system of n particles, $r_{CM} = \sum_{i=1}^{n} \dfrac{m_i r_i}{m}$

Cartesian Coordinates of Centre of Mass

$$x = \frac{\sum_{i=1}^{N} m_i x_i}{M},\ y = \frac{\sum_{i=1}^{N} m_i y_i}{M},\ z = \frac{\sum_{i=1}^{N} m_i z_i}{M}$$

Continuous Mass Distribution

The position vector of the centre of mass is given by

$$\mathbf{r} = \frac{\int \mathbf{r}dm}{\int dm} = \frac{\int \mathbf{r}dm}{M}$$

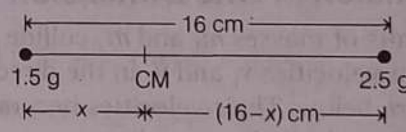
Cartesian Coordinates of Centre of Mass

$$x = \frac{\int x\,dm}{\int dm},\ y = \frac{\int y\,dm}{\int dm},\ z = \frac{\int z\,dm}{dm}$$

Example 4. Two point objects of masses 1.5 g and 2.5 g respectively are at a distance of 16 cm apart, the centre of mass is at a distance x from the object of mass 1.5 g. Find the value of x.

a. 9cm b. 8cm c. 10 cm d. 11 cm

Sol (c)



We know that, centre of mass of two particles system lies between the two particles on the line joining them and divide the distance between them in inverse ratio of masses of the particles, we can write

$$x = \frac{m_2 r}{m_1 + m_2} \Rightarrow x = \frac{2.5\times 16}{2.5 + 1.5} \Rightarrow x = \frac{2.5\times 16}{4} = 10\text{ cm}$$

Motion of Centre of Mass

The position vector $\mathbf{r}_{CM}$ of the centre of mass of n particle system is defined by

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 + ... + m_n\mathbf{r}_n}{m_1 + m_2 + m_3 + ... + m_n}$$

$$= \frac{1}{M}(m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 + ... + m_n\mathbf{r}_n)$$

$$\frac{d\mathbf{r}_{CM}}{dt} = \frac{1}{M}\left(m_1\frac{d\mathbf{r}_1}{dt} + m_2\frac{d\mathbf{r}_2}{dt} + ... + m_n\frac{d\mathbf{r}_n}{dt}\right)$$

Velocity of centre of mass,

$$\mathbf{v}_{CM} = \frac{1}{M}(m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \ldots + m_n\mathbf{v}_n)$$

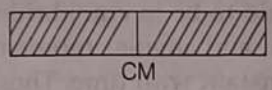
$$\mathbf{v}_{CM} = \frac{\sum_{i=1}^{n} m_i\mathbf{v}_i}{M}$$

Similarly, acceleration of centre of mass is given by

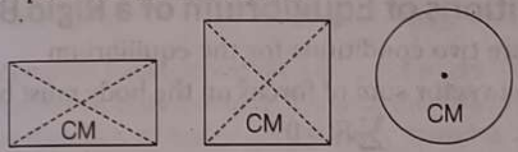
$$\mathbf{a}_{CM} = \frac{\sum_{i=1}^{n} m_i\mathbf{a}_i}{M}$$

Centre of Mass of Some Rigid Bodies

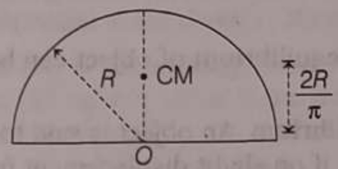
- The centre of mass of a uniform rod is located at its mid-point.



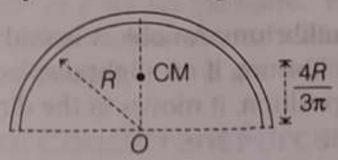
- Centre of mass of a uniform rectangular, square or circular plate lie at their centre.



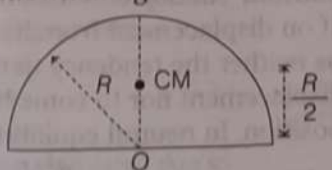
- Centre of mass of a uniform semi-circular ring lies at a distance of $h = \frac{2R}{\pi}$ from its centre, on the axis of symmetry, where R is the radius of the ring.



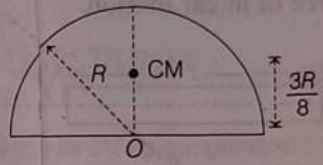
- Centre of mass of a uniform semi-circular disc of radius R lies at a distance of $h = \frac{4R}{3\pi}$ from the centre on the axis of symmetry as shown in figure.



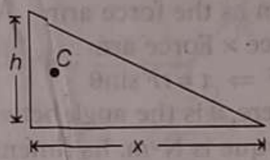
- Centre of mass of a hemispherical shell of radius R lies at a distance of $h = \frac{R}{2}$ from its centre on the axis of symmetry as shown in figure.



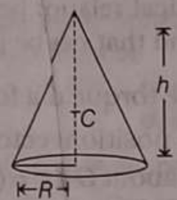
- Centre of mass of a solid hemisphere of radius R lies at a distance of $h = \frac{3R}{8}$ from its centre on the axis of symmetry.



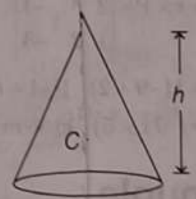
- Centre of mass of a right angled triangular plate lies at $x' = \frac{x}{3}$ and $y = \frac{h}{3}$.



- Centre of mass of a hollow right circular cone of height h lies at a distance of $y_C = \frac{h}{3}$.



- Centre of mass of a solid right circular cone of height h lies at a distance of $y_C = \frac{l}{l}$.



Centre of Gravity

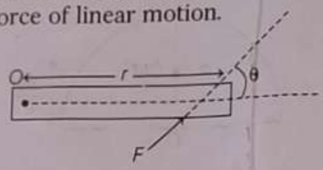
A point from which the weight of a body or system may be considered to act. In uniform gravity, it is same as centre of mass.

Rotational Motion of a Rigid Body

A body is said to possess rotational motion, if all its particles move along circles in parallel planes. The centres of these circles lie on a fixed line perpendicular to the parallel planes and this line is called the axis of rotation.

Torque or Moment of Force

Torque is a quantity which measures the capability of a force to rotate a body. Torque due to a force is also known as the moment of a force. Torque is the rotational analogue of force of linear motion.



It is defined as the product of the force and the perpendicular distance between the line of action of the force and the axis of rotation. This perpendicular distance is known as the force arm.

Torque, τ = Force × Force arm

$$\tau = \mathbf{r} \times \mathbf{F} \Rightarrow \tau = rF\sin\theta$$

(where, θ is the angle between $\mathbf{r}$ and $\mathbf{F}$)

The SI unit of torque is N-m. Its dimensions are $[ML^2T^{-2}]$.

Torque is an axial vector, *i.e.* its direction is always perpendicular to the plane containing vectors $\mathbf{r}$ and $\mathbf{F}$ in accordance with right hand screw rule.

There is a mathematical relation between torque and moment of inertia and that can be given as $\tau = I\alpha$.

Example 5. Find the torque of a force $\mathbf{F} = (\hat{i} + 2\hat{j} - 3\hat{k})$ N about a point O. The position vector of point of application of force about O is $\mathbf{r} = (2\hat{i} + 3\hat{j} - \hat{k})$ m.

a. $\tau = (-7\hat{i} + 5\hat{j} + \hat{k})$ N-m
b. $\tau = (7\hat{i} - 5\hat{j} + \hat{k})$ N-m
c. $\tau = (-7\hat{i} + 8\hat{j} + \hat{k})$ N-m
d. $\tau = (-5\hat{i} + 7\hat{j} + \hat{k})$ N-m

Sol (*a*) Torque, $\tau = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & 2 & -3 \end{vmatrix}$

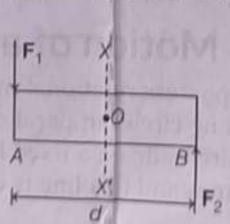
$\Rightarrow \quad \tau = \hat{i}(-9+2) + \hat{j}(-1+6) + \hat{k}(4-3)$

$\Rightarrow \quad \tau = (-7\hat{i} + 5\hat{j} + \hat{k})$ N-m

Moment of Couple

If two equal and anti-parallel forces are acting on a body, then a rotational effect produced by these two is known as moment of couple.

In given figure, F_1 and F_2 are two equal and anti-parallel forces producing a single combined rotational effect on rod AB.



Couple of forces acting on the body

Torques due to F_1 and F_2 are said to be moment of couple.

Moment of couple, τ

= (Either of the force) × (Perpendicular distance between the two parallel forces)

$$= F_1 d = F_2 d = Fd \qquad [\because |\mathbf{F}_1| = |\mathbf{F}_2| = |\mathbf{F}|]$$

Angular Momentum

It gives a measure of the turning motion of the body. SI unit of angular momentum is kgm^2s^{-1}.

Relation between torque and angular momentum is given by $\tau = \frac{dL}{dt}$.

Equilibrium of Rigid Body

A rigid body is said to be in equilibrium, if both the linear momentum and angular momentum of the rigid body remains constant with time. Thus, it is the state, where an object is not accelerating in any way.

Conditions of Equilibrium of a Rigid Body

There are two conditions for the equilibrium

(i) The vector sum of forces on the body must be zero.
i.e. $\sum F_i = 0$

(ii) The vector sum of torques on the body must be zero.
$\sum \tau_i = 0$

When force is conservative and object is in equilibrium, its potential energy is either minimum or maximum or constant.

On this basis equilibrium of object can be divided into three types

- **Stable Equilibrium** An object is said to be in stable equilibrium, if on slight displacement from equilibrium position, it has tendency to come back. Here, potential energy in equilibrium position is minimum as compared to its neighbouring points or $\frac{d^2U}{dr^2}$ = positive.
- **Unstable Equilibrium** An object is said to be in unstable equilibrium, if on slight displacement from equilibrium position, it moves in the direction of displacement.
  In unstable equilibrium, potential energy is maximum or $\frac{d^2U}{dr^2}$ = negative.
- **Neutral Equilibrium** An object is said to be in neutral equilibrium, if on displacement from its equilibrium position, it has neither the tendency to move in direction of displacement nor to come back to equilibrium position. In neutral equilibrium, potential energy of the object is constant or $\frac{d^2U}{dr^2} = 0$.