Chapter

6 Superposition of Waves

Quick Review

Wave motion

↓

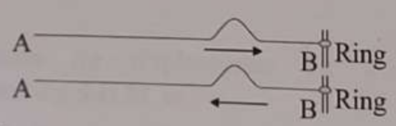
Progressive waves

Transverse

In a transverse wave, vibrations of particles are perpendicular to the direction of propagation of wave and produce crests and troughs in their medium of travel.

Reflection of transverse wave

- Reflection of a wave pulse sent as a crest from a rarer medium to a denser medium is reflected as trough and trough is reflected as crest



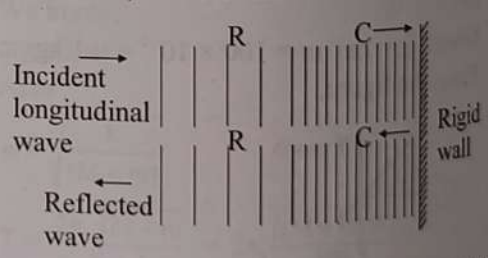
- Reflection of a wave pulse sent as a crest from a denser medium to a rarer medium is reflected as crest and trough is reflected as trough

Longitudinal

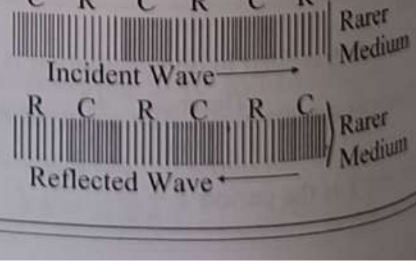
In longitudinal wave, vibrations of particles produce compressions and rarefactions along the direction of propagation of the wave.

Reflection of longitudinal wave

- Reflection of a longitudinal wave sent as a compression from a rarer medium to a denser medium is reflected as compression and rarefaction is reflected as rarefaction



- Reflection of a longitudinal wave sent as a compression from a denser medium to a rarer medium is reflected as rarefaction and rarefaction is reflected as compression

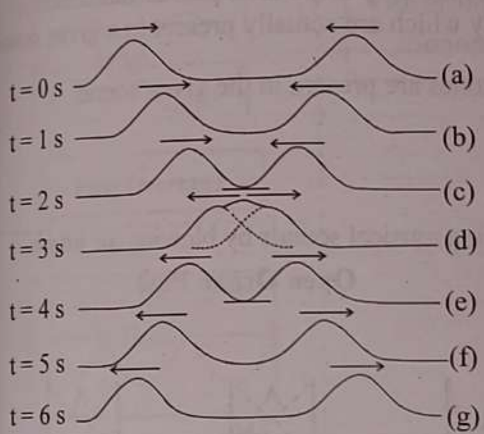


Superposition of waves:

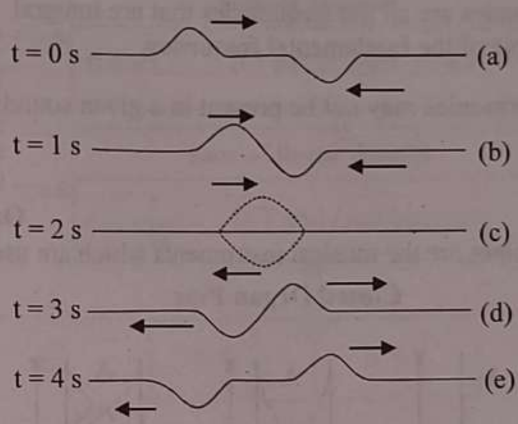
Superposition of Waves

When two or more waves arrive at a point simultaneously, then each wave produces its own displacement independent of the other. The resultant displacement at a point is a vector sum of the displacements due to the individual waves.

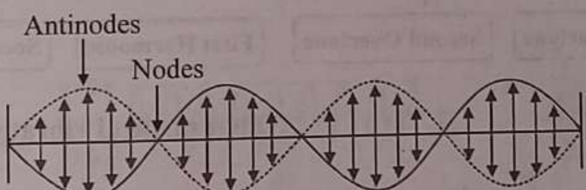
Equal Amplitude and Same Phase



Equal Amplitude and Opposite Phase



Stationary Waves



Definition: When two identical waves travelling along the same path in opposite directions interfere with each other, resultant wave is called stationary wave.

Equation of stationary wave:

$$y = 2A\cos\left(\frac{2\pi x}{\lambda}\right)\sin(2\pi nt) = R\sin(2\pi nt) \quad \left(\text{where } R = 2A\cos\frac{2\pi x}{\lambda}\right)$$

Free and Forced oscillations:

Sr. No.	Free vibrations	Forced vibrations
i.	Free vibrations are produced when a body is disturbed from its equilibrium position and released.	Forced vibrations are produced by an external periodic force of any frequency.
ii.	To start free vibrations, the force is required initially only.	Continuous external periodic force is required. If external periodic force is stopped, then forced vibrations also stop.
iii.	The frequency of free vibrations depends on the natural frequency.	The frequency of forced vibrations depends on the frequency of the external periodic force.

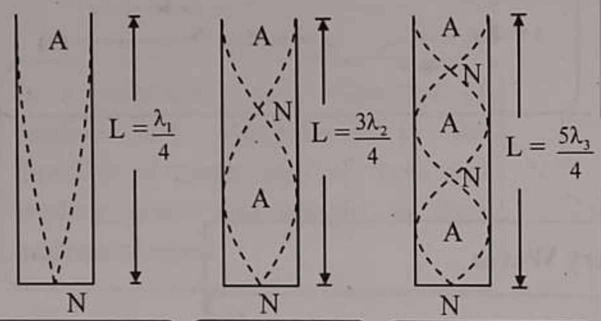
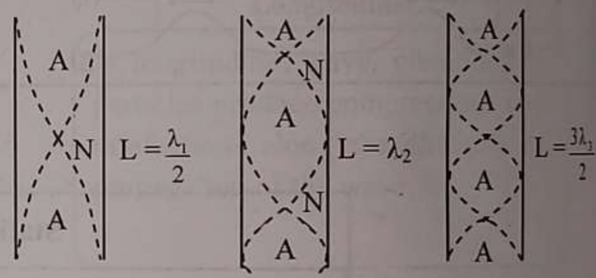
iv.	Energy of the body remains constant in the absence of friction, air resistance, etc. Due to damping forces, total energy decreases.	Energy of the ... constant by the external periodic force.
v.	Amplitude of vibrations decreases with time.	Amplitude is small but remains constant as long as external periodic force acts on it.
vi.	Vibrations stop sooner or later depending on the damping force.	Vibrations stop as soon as external periodic force is stopped.

➤ **Harmonics and Overtones:**

Harmonics	Overtone
Harmonics are all the frequencies that are integral multiple of the fundamental frequency.	Overtones are only those multiples of fundamental frequency which are actually present in a given sound.
All harmonics may not be present in a given sound.	All overtones are present in the given sound.

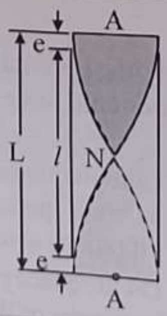
Organ Pipe

Organ pipes are the musical instruments which are used for producing musical sounds by blowing air into it.

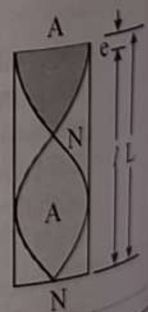
Closed Organ Pipe	Open Organ Pipe
 **Fundamental Mode** **First Overtone** **Second Overtone**	 **First Harmonic** **Second Harmonic** **Third Harmonic**
First mode of vibration: $n_1 = \frac{v}{4L}$	**First mode of vibration:** $n_1 = \frac{v}{2L}$
Second mode of vibration: $n_2 = \frac{3v}{4L} = 3n$ is called as third harmonic or first overtone.	**Second mode of vibration:** $n_2 = \frac{v}{L} = 2n_1$ is called as second harmonic or first overtone.
pth mode of vibration: $n_p = (2p-1)\frac{v}{4L}$ It is known as $(2p-1)$ harmonic or $(2p-3)$ overtone.	**pth mode of vibration:** $n_p = p\frac{v}{2L}$ It is known as pth harmonic or $(p-1)^{th}$ overtone.
It contains p nodes and p antinodes.	It contains p nodes and $(p+1)$ antinodes.
Condition: only odd harmonics are present.	**Condition:** all harmonics are present.

End Correction: The distance between the open end of the pipe and the position of antinode is called the end correction.

For Open Pipe: $L = l + 2e$

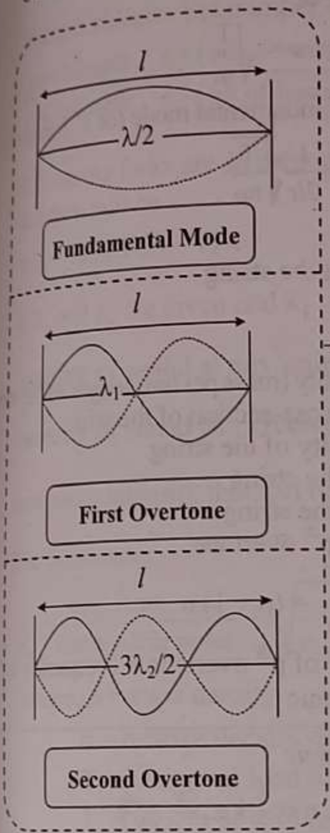


For Closed Pipe: $L = l + e$



Vibrations produced in a string:

Standing Waves are formed on a string fixed at both the ends when plucked at the centre.



Vibrating Strings (Sonometer)

Law of Length

$n \propto \frac{1}{l}$ If T and m are constant.

Law of Tension

$n \propto \sqrt{T}$ If l and m are constant.

Law of linear density

$n \propto \frac{1}{\sqrt{m}}$ If T and l are constant.

$n \propto \frac{1}{\sqrt{\rho}}$ | $n \propto \frac{1}{\sqrt{\rho}}$

Beats:

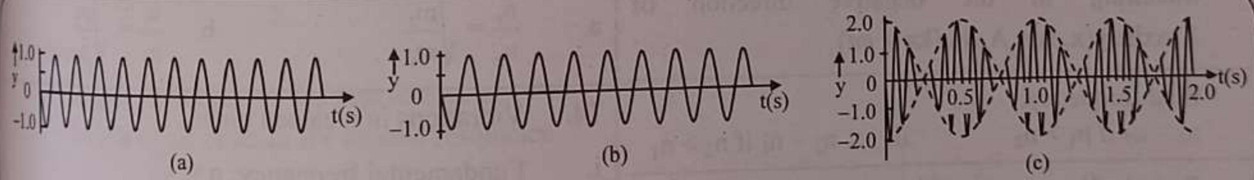


Fig. (a) and (b) are two waves having slightly different frequencies. They interfere and beats are formed.

Formation of Beats

The periodic variation of intensity of sound between maximum and minimum due to superimposition of two sound waves of same amplitude and slightly different frequencies is called as phenomenon of beats.

Waxing and Waning

Maximum intensity point is called as waxing and minimum intensity point is called as waning. They are alternatively produced.

Period and Frequency

- Period of beats:
  $T = \frac{1}{n_1 - n_2}$
- Beat frequency:
  $N = n_1 - n_2$

Formulae

1. Velocity of a wave: $v = n\lambda = \lambda/T$

2. Wave number (k) = $\dfrac{2\pi}{\lambda}$

3. Angular frequency: $\omega = \dfrac{2\pi}{T} = 2\pi n$

4. Phase difference: $\delta = \dfrac{2\pi x}{\lambda}$

5. Path difference: $x = \dfrac{\lambda}{2\pi}\delta$

6. Equation of simple harmonic progressive wave: $y(x, t) = A \sin(kx - \omega t)$

7. Different forms of equation of simple harmonic progressive wave along positive X-direction:

$$y = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

$$y = A\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$$

$$y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$y = A\sin\frac{2\pi}{\lambda}(vt - x)$$

8. Equation of simple harmonic progressive wave travelling in the negative direction of X-axis: $y(x, t) = A\sin(kx + \omega t)$

9. Beat frequency:
   i. $n_1 - n_2$ if $n_1 > n_2$ ii. $n_2 - n_1$ if $n_2 > n_1$

10. Period of beats:
    i. $T = \dfrac{1}{n_1 - n_2}$ if $n_1 > n_2$
    ii. $T = \dfrac{1}{n_2 - n_1}$ if $n_2 > n_1$

11. Equation of stationary wave:
    i. Displacement

$$x = 2A\cos\left(\frac{2\pi x}{\lambda}\right)\sin(2\pi nt) = R\sin(2\pi nt)$$

where,

$R = 2A\cos\dfrac{2\pi x}{\lambda}$ = amplitude of wave

ii. Distance between adjacent nodes and antinodes $= \dfrac{\lambda}{2}$

iii. Distance between a node and an adjacent antinode $= \dfrac{\lambda}{4}$

12. Vibrating strings:
    i. Wave velocity: $v = \sqrt{\dfrac{T}{m}}$
    ii. Frequency of fundamental mode (or 1^{st} harmonic)

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}}$$

where,
T = tension in the string
$m = \dfrac{M}{L}$ = linear density (mass per unit length of the string)
r = radius of cross-section of the wire
ρ = mass density of the string
M = mass of the string
L = length of the string

iii. Frequency of p^{th} overtone,

$$n_p = \frac{(p+1)}{2l}\sqrt{\frac{T}{m}} = (p+1)\,n$$

i.e., Frequency of p^{th} overtone = Frequency of $(p+1)^{th}$ harmonic

13. Laws of vibration:
    i. Law of length: $n \propto \dfrac{1}{l}$ i.e., $\dfrac{n_1}{n_2} = \dfrac{l_2}{l_1}$
    ii. Law of tension: $n \propto \sqrt{T_1}$ i.e., $\dfrac{n_1}{n_2} = \sqrt{\dfrac{T_1}{T_2}}$
    iii. Law of density:
    a. $\dfrac{n_1}{n_2} = \sqrt{\dfrac{m_2}{m_1}}$ b. $\dfrac{n_1}{n_2} = \sqrt{\dfrac{\rho_2}{\rho_1}}$

14. Vibrations in a closed pipe:
    i. Fundamental frequency: $n = \dfrac{v}{4L}$
    ii. Second mode = 1^{st} overtone = 3^{rd} harmonic
    $n_1 = 3n = \left(\dfrac{3v}{4L}\right)$
    iii. Frequency of k^{th} overtone $= (2k+1)n = (2k+1)\dfrac{v}{4L}$
    iv. End correction: $e = \dfrac{n_2 l_2 - n_1 l_1}{n_1 - n_2}$

15. Vibrations in an open pipe:
    i. Fundamental frequency: $n = \dfrac{v}{2L}$ (1^{st} harmonic)
    ii. 1^{st} overtone or 2^{nd} harmonic: $n_1 = 2n = 2\left(\dfrac{v}{2L}\right)$
    iii. Frequency of k^{th} overtone $= (k+1)n = (k+1)\dfrac{v}{2L}$
    iv. End correction: $e = \dfrac{n_2 l_2 - n_1 l_1}{2(n_1 - n_2)}$

Shortcuts

1. If length changes but number of beats remains same, then apply the formula,

$$\frac{\text{greater length}}{\text{smaller length}} = \frac{n + (\text{no. of beats})}{n - (\text{no. of beats})} \quad \text{(where n is unknown frequency)}$$

2. If N tuning forks are so arranged that every fork gives x beats per second with the next then the frequency of last fork will be

$$n_{Last} = n_{First} \pm (N - 1)\,x$$

3. If λ_1 and λ_2 are given and $\lambda_1 > \lambda_2$ remember $n_1 < n_2$ and the frequency of beats will be $n_2 - n_1$.

4. Intensity of sound at any point is energy emitted per second per unit area around that point and loudness is intensity of sound as perceived by human ear and the relationship $L = k \log I$ or $L = \log\left(\frac{I}{I_0}\right)$ where I_0 is minimum intensity that can be heard.

5. For Sonometer,

$$\frac{n_{load} \text{ in air}}{n_{load} \text{ immersed in water}} = \sqrt{\frac{\rho_{load}}{\rho_{load} - \rho_{water}}} = \sqrt{\frac{\sigma_{load}}{\sigma_{load} - 1}} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$$

where, ρ = mass density

σ = relative density or specific gravity of load

T_1 = weight of load in air

T_2 = weight of load in water

6. For resonance tube,

i. Velocity, $v = 4n\,(l + 0.3d) = 2n(l_2 - l_1)$

ii. End correction, $e = \frac{l_2 - 3l_1}{2}$

where,

l_1 = length of air column for first resonance

l_2 = length of air column for second resonance

7. Frequency of harmonics produced in closed organ pipe is given by $n = \frac{(2r-1)v}{4l}$ and the frequency of harmonics produced in open organ pipe is given by $n = r\frac{v}{2l}$

where r = 1, 2, 3 ……

8. For same L, $n_{open} = 2n_{closed}$

For same n (i.e. $n_{open} = n_{closed}$), $L_{open} = 2L_{closed}$

9. Whenever there is a question of masses suspended in a liquid the tension will decrease. Hence frequency will also decrease.

10. Melde's experiment:

Frequency of tuning fork for p^{th} harmonic:

i. For perpendicular position, $N = \frac{p}{2L}\sqrt{\frac{T}{m}} = n$

ii. For parallel position, $N = \frac{p}{2L}\sqrt{\frac{T}{m}} = 2n$

where, n = frequency of vibrating string