CHAPTER 15

Binomial Distribution

Bernoulli Trial

In a random experiment, if there are only two outcomes success and failure and the sum of the probabilities of these two outcomes is one, then any trial of such experiment is known as a Bernoulli trial.

Generally, the trials of an experiment are called Bernoulli trials, if they satisfy the following requirements:

(i) There should be a finite number of trials.
(ii) The trials should be independent.
(iii) Each trial has exactly two outcomes, success or failure.
(iv) The probability of success remains the same in each trial.

Binomial Distribution

The probability distribution of number of successes in an experiment consisting n-Bernoulli trials obtained by the binomial expansion of $(q+p)^n$ is called binomial distribution.

Where p denote the probability of success and q denote the probability of failure.

In this distribution, the number of success X can be written as

X	0	1	2	r	n
$P(X)$	${}^nC_0\, p^0\, q^n$	${}^nC_1 p^1 q^{n-1}$	${}^nC_2 p^2\, q^{n-2}$	${}^nC_r\, p^r\, q^{n-r}$	${}^nC_n\, p^n \cdot q^0$

This probability distribution is known as binomial distribution with parameters n and p, because for given values of n and p, we can find the complete probability distribution.

Here, $P(X=r) = {}^nC_r p^r\, q^{n-r}$ is called the **probability function** of the binomial distribution.

where, p = probability of success,
q = probability of failure
and n = number of trials.

Also, $p+q=1$ such that $0<p, q<1$

A binomial distribution with n Bernoulli trials and probability of success in each trial as p is denoted by $X \sim B(n, p)$.

In a binomial distribution, an outcome having highest probability is called **most likely outcome**.

Mean and Variance of Binomial Distribution

Let $X \sim B(n,p)$, then $P(X=r) = {}^nC_r p^r q^{n-r}$

where, $r = 0, 1, 2, \dots, n$ and $p+q=1$

$\therefore$ Mean, $\overline{X} = E(X) = np$

and variance, $\text{var}(X) = \sigma_x^2 = npq$

Standard deviation, $\text{SD}(x) = \sigma_x = \sqrt{npq}$