LINE AND PLANE

Remember This: Line

- The vector equation of the line passing through $A(\bar{a})$ and parallel to vector $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$
- The vector equation of the line passing through $A(\bar{a})$ and $B(\bar{b})$ is $\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$.
- The Cartesian equations of the line passing through $A(x_1, y_1, z_1)$ and having direction ratios a, b, c are $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$
- The distance of point $P(\bar{\alpha})$ from the line $\bar{r} = \bar{a} + \lambda\bar{b}$ is given by $\sqrt{|\bar{\alpha} - \bar{a}|^2 - \left[\frac{(\bar{\alpha} - \bar{a}) \cdot \bar{b}}{|\bar{b}|}\right]^2}$
- The shortest distance between lines $\bar{r} = \bar{a}_1 + \lambda_1\bar{b}_1$ and $\bar{r} = \bar{a}_2 + \lambda_2\bar{b}_2$ is given by $d = \left|\frac{(\bar{a}_2 - \bar{a}_1) \cdot (\bar{b}_1 \times \bar{b}_2)}{|\bar{b}_1 \times \bar{b}_2|}\right|$
- Lines $\bar{r} = \bar{a}_1 + \lambda_1\bar{b}_1$ and $\bar{r} = \bar{a}_2 + \lambda_2\bar{b}_2$ intersect each other if and only if $(\bar{a}_2 - \bar{a}_1) \cdot (\bar{b}_1 \times \bar{b}_2) = 0$
- Lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ intersect each other if and only if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$
- The distance between parallel lines $\bar{r} = \bar{a}_1 + \lambda\bar{b}$ and $\bar{r} = \bar{a}_2 + \lambda\bar{b}$ is given by $d = |(\bar{a}_2 - \bar{a}_1) \times \hat{b}|$

Plane

- The vector equation of the plane passing through $A(\bar{a})$ and normal to vector $\bar{n}$ *is* $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$
- Equation $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$ is called the **vector equation of plane in scalar product form.**
- If $\bar{a} \cdot \bar{n} = d$ then equation $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$ takes the form $\bar{r} \cdot \bar{n} = d$
- The equation of the plane passing through $A(x_1, y_1, z_1)$ and normal to the line having direction ratios a,b,c is $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$.

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- The vector equation of the plane passing through point $A(\bar{a})$ and parallel to $\bar{b}$ and $\bar{c}$ is $\bar{r}\cdot(\bar{b}\times\bar{c}) = a\cdot(\bar{b}\times\bar{c})$
- Equation $\bar{r} = \bar{a} + \lambda\bar{b} + \mu\bar{c}$ is called the vector equation of plane in parametric form.
- The vector equation of the plane passing through non-collinear points $A(\bar{a})$, $B(\bar{b})$ and $C(\bar{c})$ is $(\bar{r}-\bar{a})\cdot(\bar{b}-\bar{a})\times(\bar{c}-\bar{a}) = 0$
- Cartesian from of the above equation is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$
- The equation of the plane at distance p unit from the origin and to which unit vector $\hat{n}$ is normal is $\bar{r}\cdot\hat{n} = p$
- If l, m, n are direction cosines of the normal to a plane which is at distance p unit from the origin then its equation is $lx + my + nz = p$.
- If N is the foot of the perpendicular drawn from the origin to a plane and $ON = p$ then the co-ordinates of N are (pl, pm, pn).
- If planes $(\bar{r}\cdot\bar{n}_1 - d_1) = 0$ and $\bar{r}\cdot\bar{n}_2 - d_2 = 0$ intersect each other, then for every real value of λ,equation $\bar{r}\cdot(\bar{n}_1 + \lambda\bar{n}_2) - (d_1 + \lambda d_2) = 0$ represents a plane passing through the line of their intersection.
- If planes and $a_1x+b_1y+c_1z+d=0$ and $a_2x+b_2y + c_2z + d_2 = 0$ intersect each other, then for every real value of λ, equation $(a_1x+b_1y + c_1z + d_1) + \lambda\ (a_2x+b_2y\ c_2z + d_2) = 0$ represents a plane passing through the line of their intersection.
- The angle between the planes $\bar{r}\cdot\bar{n}_1 = d_1$ and $\bar{r}\cdot\bar{n}_2 = d_2$ is given by $\cos\theta = \left|\dfrac{\bar{n}_1\cdot\bar{n}_2}{|\bar{n}_1||\bar{n}_2|}\right|$
- The acute angle between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r}\cdot\bar{n} = d$ is given by $\sin\theta = \left|\dfrac{\bar{b}\cdot\bar{n}}{|\bar{b}_1||\bar{n}|}\right|$
- Lines $\bar{r} = a_1 + \lambda_1 b_1$ and $\bar{r} = \bar{a}_2 + \lambda_2\bar{b}_2$ are coplanar if and only if $(\bar{a}_2 - \bar{a}_1)\cdot(\bar{b}_1\times\bar{b}_2) = 0$ and the quation of the plane determined by them is $(\bar{r} - \bar{a}_1)\cdot(\bar{b}_1\times\bar{b}_2) = 0$
- Lines $\dfrac{x-x_1}{a_1} = \dfrac{y-y_1}{b_1} = \dfrac{z-z_1}{c_1}$ and $\dfrac{x-x_2}{a_2} = \dfrac{y-y_2}{b_2} = \dfrac{z-z_2}{c_2}$.

are coplanar if and only if $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$, and the equation of the plane determined

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by them is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

- The distance of the point $A(\bar{a})$ from the plane $\bar{r}\cdot\hat{n} = p$ is given by $\left|p - \left|\bar{a}\cdot\hat{n}\right|\right|$