Formula sheet

Indefinite Integration

- Basic integration formula:-

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq 1$.
2. $\int \frac{1}{x} dx = \log|x| + c, x \neq 0$
3. $\int e^x dx = e^x + c$.
4. $\int a^x dx = \frac{a^x}{\log_e a} + c$
5. $\int \sin x dx = -\cos x + c$
6. $\int \cos x dx = \sin x + c$
7. $\int \sec^2 x \, dx = \tan x + c$
8. $\int \operatorname{cosec}^2 x dx = -\cot x + c$
9. $\int \sec x \tan x dx = \sec x + c$
10. $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$
11. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$ or $-\cos^{-1} x + c$
12. $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$ or $-\cot^{-1} x + c$
13. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$ or $\operatorname{cosec}^{-1} x + c$
14. $\int 1 \, dx = x + c$.
15. $\int \underset{\downarrow(ax+b)}{F(x)} dx = \frac{F(ax+b)}{a} + c$

- Important Results:-

1] $\int \tan^2 x dx = \tan x - x + c$

2] $\int \sqrt{1+\sin 2x} = -\cos x + \sin x + c$

3] $\int \sqrt{x} \, dx = \frac{2}{3} x^{3/2} + c$

4] $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$

5] $\int \frac{F'(x)}{F(x)} dx = \log(F(x)) + c$

6] $\int \cot x . dx = \log|\sin x| + c$

7] $\int \tan x . dx = \log|\sec x| + c$
$= -\log|\cos x| + c$

8] $\int \sec x \cdot dx = \log|\sec x + \tan x| + c$
$= \log\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right] + c$

9] $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c$
$= \log\left|\tan\left(\frac{x}{2}\right)\right| + c.$

10] $\int (f(x))^n \cdot f'(x) \cdot dx = \frac{[f(x)]^{n+1}}{n+1} + c \quad , n \neq -1$

11] $\int \frac{f'(x)}{\sqrt{f(x)}} \cdot dx = 2\sqrt{f(x)} + c.$

12] $\int \sin a \cdot dx = \sin a \int dx = (\sin a)x.$

- Integrals of Form:

$\int \frac{p\sin x + q\cos x}{a\sin x + b\cos x} dx$, $\int \frac{p\sin x \text{ or } \cos x}{a\sin x + b\cos x} dx$ and $\int \frac{ae^x + b}{ce^x + d} dx$

$$Nr = A(Dr) + B\frac{d}{dx}(Dr)$$

Denominator of function.

Short trick: $Ax \pm B\,|\log(f(x))| + c$

Depends on value of B is +ve or −ve

- Some special integrals:-

1] $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

2] $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

3] $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$

4] $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

5] $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

6] $\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left|x+\sqrt{x^2+a^2}\right| + c$

7] $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x+\sqrt{x^2-a^2}\right| + c$

- Special integrals with substitution:

1] $\sqrt{a^2-x^2}$	$x = a.\sin\theta$ or $x = a.\cos\theta$
2] $\sqrt{a^2+x^2}$	$x = a.\tan\theta$
3] $\sqrt{x^2-a^2}$	$x = a.\sec\theta$
4] $\sqrt{\frac{a-x}{a+x}}$	$x = a.\cos 2\theta$

- Types of Integrals:

1] $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

Reduced to, $\int \frac{dx}{\sqrt{A^2-x^2}}$, $\int \frac{dx}{\sqrt{x^2-A^2}}$, $\int \frac{dx}{\sqrt{x^2+A^2}}$

formula for 3^{rd} term $= \left(\frac{1}{2} \times \text{coeff. } x\right)^2$

- Integrals of form:

$\int \frac{1}{a+b\sin^2 x}\,dx$, $\int \frac{1}{a+b\cos^2 x}\,dx$, $\int \frac{1}{a\sin^2 x + b\cos^2 x + c}\,dx$

1) divide Nr. and Dr. by $\cos^2\theta$
2) Replace $\sec^2 x$, by $1+\tan^2 x$ (In dr.)
3) Put $\tan x = t$.

- Types of integrals:

1] $\int \frac{1}{a+b\cos x} dx$, $\int \frac{1}{a+b\sin x} dx$, $\int \frac{1}{a\sin x + b\cos x + c} dx$

To evaluate: Put $\tan\frac{x}{2} = t$ we get, $dx = \frac{2dt}{1+t^2}$

Also put, $\sin x = \frac{2t}{1+t^2}$ $\cos x = \frac{1-t^2}{1+t^2}$

2] $\int \frac{1}{a+b\cos 2x} dx$, $\int \frac{1}{a+b\sin 2x} dx$, $\int \frac{1}{a\cos 2x + b\sin 2x} dx$

and $\int \frac{1}{a\sin 2x + b\cos 2x} dx$

$\therefore$ Put $\tan x = t$ $\therefore$ we get, $dx = \frac{dt}{1+t^2}$

Also put, $\sin 2x = \frac{2t}{1+t^2}$ $\cos 2x = \frac{1-t^2}{1+t^2}$

3] $\int \frac{Px+q}{ax^2+bx+c} dx$, $\int \frac{Px+q}{\sqrt{ax^2+bx+c}} dx$

Here, $Px+q \longrightarrow$ Linear term.

Nr can be expressed as,

$Nr = A\left[\frac{d}{dx}(Dr)\right] + B$

- Integration by Parts.

$\int u.v\, dx = u\int v.dx - \int\left[\frac{d}{dx}u \cdot \int v\,dx\right]dx$

How to decide:-

L — Logarithmic ($\log x$ ...)
I — Inverse ($\tan^{-1}x$ ...)
A — Algebric (x^2+2x+1)
T — Trigo ($\sin x$ ...)
E — Exponential (e^x ...)

- Short tricks:-

1] $\int x^n \cdot f(x)\,dx$ → Only for this type.

Ex. $\int x^2 \cdot \sin 3x$ ∴ Here $u = x^2$
$v = \sin 3x$

Take, Diff. (Derivative) of u   Integration of v

Diff. of u	Integration of v
$+\ x^2$	$\sin 3x$
$-\ 2x$	$-\cos 3x/3$
$+\ 2$	$-\sin 3x/9$
$-\ 0$	$\cos 3x/27$

So, $I = \frac{-x^2}{3}\cos 3x + \frac{2x}{9}\sin 3x + \frac{2}{27}\cos 3x + c.$

2] $\int \log x \cdot x^n\,dx$ → Only for this type.

Ex. $\int \log x \cdot x^2$ ∴ Derivative (u)   Integration (v)

Derivative (u)	Integration (v)
$\log x$	x^2
$\frac{1}{x}$	$\frac{x^3}{3}$

Take Multiplication of U and V

∴ $\frac{1}{x} \times \frac{x^3}{3} = \frac{x^3}{3}$

If multiplication is integrable the stop and then

$\log x \to x^2$
$1/x \quad x^3/3$

∴ $I = \frac{x^3}{3}\log x - \int \frac{x^2}{3}$

Take ~~m~~ integration of multiplication.   ∴ $I = \frac{x^3}{3}\log x - \frac{x^3}{9} + c.$

- Important Results.

1] ~~$\int e^{ax} \cdot \sin(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2} \cdot [a\cos(bx+c)+b\sin(bx+c)] + k$~~

1] $\int e^{ax} \cdot \cos(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2} \cdot [a\cos(bx+c)+b\sin(bx+c)]+k$

2] $\int e^{ax} \cdot \sin(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2} \cdot [a\sin(bx+c)-b\cos(bx+c)]+k$

3] To evaluate the integrals of type $\int \sin^{-1}x\,dx$; $\int \tan^{-1}dx$; $\int \sec^{-1}dx$; $\int \log x \cdot dx$ take second function (v) to be 1

4] $\int \left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx = x\log(\log x)-\frac{x}{\log x}+c$

5] $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

6] $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c$

7] $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$

- Note: $\int (px+q)\sqrt{ax^2+bx+c}\,dx$

To evaluate,

$$px+q = A\left[\frac{d}{dx}(ax^2+bx+c)\right]+B$$

- Integral of type: $\int e^x[F(x)+F'(x)]dx = e^x F(x)+c$

- Integration by partial fraction:

1] $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} \Rightarrow \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$ → Non repeated Linear Factor.

2] $\frac{px^2+qx+r}{(x-a)^2(x-b)} \Rightarrow \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$ → Repeated Linear Factor.

3] $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} \Rightarrow \frac{A}{(x-a)} + \frac{B}{x^2+bx+c}$ → Product of Linear and Quadratic.

Partial fraction applicable only for proper rational function.

- Long division method:

  Dividend = (Divisior × Quotient) + Remainder.

- In Repeated Linear:

  Take/Put 1 time value by self. (eg: $x=0$)

- In Product:

  Take/Put 2 time value by self. (eg: $x=0$, $x=1$)