Chapter

# 8 Electrostatics



*[Note: Concepts of electric flux and Gauss' law discussed as introduction in this chapter are recollection of the concepts studied in chapter 10, Electrostatics of std. XI. To avoid repetition, questions on introduction are not included here since they are thoroughly covered in chapter 10.]*

## Quick Review

**Electrostatics**: The study of electricity/electric charges at rest

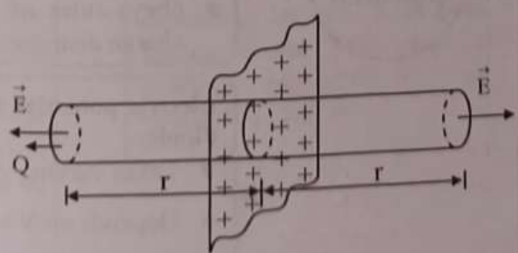
→ **Gauss' theorem**: The flux of the net electric field through a closed surface equals the net charge enclosed by the surface divided by $\varepsilon_0$.

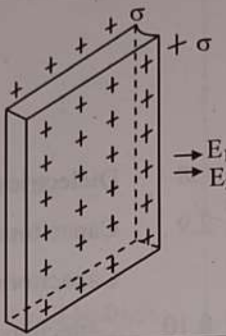
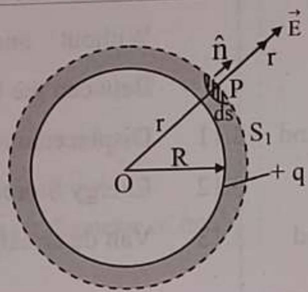
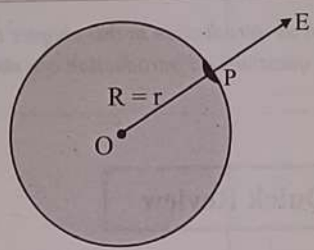
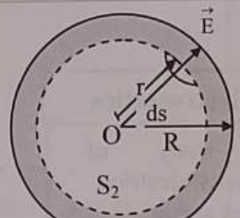
→ **Van de Graaff generator**

Charge gives rise to:

- **Electric field**
- **Electrostatic force**
- **Electric potential**
- **Electric potential energy**

### Applications of Gauss' Theorem:

| Application | Diagram | Formula |
|---|---|---|
| i. Electric field due to uniformly charged infinite plane sheet. |  | i. Total electric flux over entire surface of cylinder,<br>$\phi = 2\text{ E ds}$<br>ii. Total charge enclosed by cylinder<br>$q = \sigma\text{ ds}$<br>iii. From Gauss' theorem,<br>$\phi = 2\text{ E ds} = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$ |

| | | |
|---|---|---|
| ii. Electric field due to uniformly charged infinite plane sheet having uniform thickness. |  | According to Gauss' theorem<br>$E = E_1 + E_2$<br>$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$<br>$E = \frac{\sigma}{\varepsilon_0}$ |
| iii. Electric field due to uniform charged spherical shape [for field outside shell $(r > R)$] |  | According to Gauss' theorem,<br>$\oint \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$<br>$E \cdot (4\pi r^2) = \frac{q}{\varepsilon_0}$<br>$E = \frac{q}{4\pi r^2 \varepsilon_0}$ |
| iv. Electric field due to uniform charged spherical shape $(r = R)$ |  | At a point on the surface of the shell$(r = R)$:<br>$E = \frac{q}{4\pi\varepsilon_0 r^2}$<br>For surface charge density σ,<br>$q = 4\pi R^2 \cdot \sigma$<br>$\therefore\ E = \frac{4\pi R^2 \sigma}{4\pi R^2 \varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \text{constant}$ |
| v. Electric field due to uniform charged spherical shape (for $r < R$) |  | According to Gauss' theorem,<br>$E \times 4\pi r^2 = \frac{q}{\varepsilon_0} = 0$<br>$E = 0$<br>Thus, the field due to a uniformly charged spherical shell is zero at all points inside shell. |

**Electric potential**

**Electric potential** at any point in an electric field is defined as work done in bringing a unit charge from infinity to that point against the direction of electric field intensity.

**Potential difference:**
$\Delta V = V_B - V_A$
$\Delta V = \frac{W_{AB}}{q_0}$

**Relation between electric field intensity and pontential difference:**
$E = -\frac{dV}{dx}$

**Electrostatic potential due to a point charge:**
- +ve charge produces +ve potential
- –ve charge produces –ve potential
- Spherically symmetric

**Electric potential due to system of charges:**
- Obeys superposition principle for a system of charges
- obeys rules of integration for continuous charge distribution

**Electric potential at any point due to electric dipole:**
- Takes varying values
- Depends on $V \propto \frac{1}{r^2}$ and $V \propto \cos\theta$
- $V_{max}$ at axial point, $V_{equator} = 0$

## Electric potential energy

Electrostatic potential energy is the work done against the electrostatic forces to achieve a certain configuration of charges in a given system.

**1. Electric potential energy of a point charge:**

i. Varies directly with $qq_0$

ii. Varies inversely with r

**2. Electric potential energy of a system of two point charges:**

The work done in bringing the two charges to their respective locations is stored as the potential energy of the configuration of two charges.

$$\therefore \ U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$$

**3. Electric potential energy of system of charge particles:**

i. Obeys superposition principle for a system of charges

ii. obeys rules of integration for continuous charge distribution.in addition to the corner particles.

**4. Electric potential energy of an electric dipole in uniform electric field:**

$$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$$

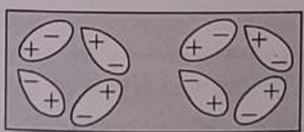
## Dielectrics

*Dielectrics are non-conducting substances which cannot transmit electric charge through them.*

### Polar dielectrics

Polar dielectrics has permanent electric dipole moment even if the electric field is absent.

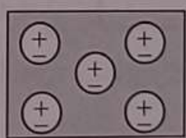
Example: HCl, water, alcohol, $NH_3$



### Non polar dielectrics

Non polar dielectrics: Every molecule has zero dipole moment in its normal state.

Examples: $H_2$, $N_2$, $O_2$, $CO_2$, benzene, methane



## Capacitors

Capacitor is a system consisting of two conductors having equal and opposite charges separated by an insulator or dielectric.

### Capacitance

i. The ability of a conductor to store charge is called capacity of conductor.

ii. Capacitance of capacitor depends on the size, shape and separation of the system of two conductors.

### Energy stored

The work done, while charging a capacitor, is stored in the form of electrostatic energy between the plates. This energy can later be recovered by discharging the capacitor.

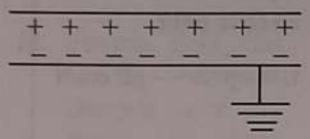
## Capacitors

### Types of capacitor

i. Parallel plate capacitor:

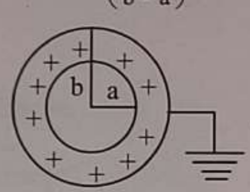
a. $C = \frac{k\varepsilon_0 A}{d}$

b. $C_m = k\, C_{air}$



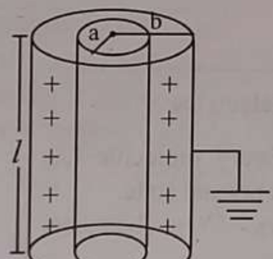
ii. Spherical capacitor:

$C = 4\pi k\varepsilon_0 \left(\frac{ab}{b-a}\right)$



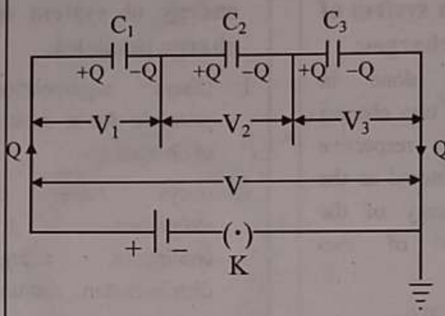

iii. Cylindrical capacitor:

$C = \frac{2\pi k\varepsilon_0 l}{2.303 \log\left(\frac{b}{a}\right)}$




### Combinations

**i. Capacitors in series:**




a. Potential difference across each capacitor is different.

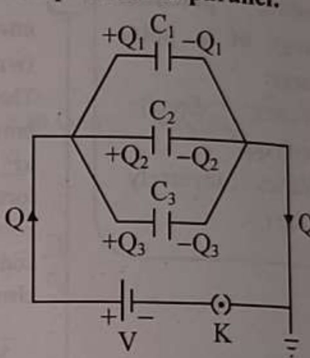
b. Charge on each capacitor is same.

c. Stands high voltage and divides high voltage

d. Used when a high voltage is to be divided on capacitors. Capacitor with minimum capacitance has the maximum potential difference between the plates.

e. Cannot store large number of charges.

**ii. Capacitors in parallel:**




a. Potential difference across each capacitor is same.

b. Charge on each capacitor is different.

c. Stands low voltage.

d. Capacitors are combined in parallel when a large capacitance at small potentials is required.

e. Can store large number of charges.

## Formulae

**1.** Charge per unit length (Linear charge density):

$\lambda = \frac{q}{l}$

**2.** Charge per unit surface area (Surface charge density): $\sigma = \frac{q}{A}$

**3.** Electric flux:

i. $\phi = \int \vec{E}\cdot\vec{ds} = Es\cos\theta$

ii. $\phi = \frac{q}{\varepsilon_0}$

iii. $\phi = \frac{q}{k\varepsilon_0}$

**4.** Dielectric constant of a medium: $k = \frac{\varepsilon}{\varepsilon_0}$

**5.** Electric intensity: $E = \frac{1}{4\pi k\varepsilon_0}\frac{q}{r^2}$

**6.** Electric intensity at a point outside a charged spherical conductor:

i. $E_{medium} = \frac{q}{4\pi k\varepsilon_0 r^2} = \frac{\sigma R^2}{k\varepsilon_0 r^2}$ ....(r > R)

ii. $E_{vacuum} = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2}$ ....(r > R)

where, $\sigma = \frac{q}{4\pi R^2}$

iii. $E_{inside} = 0$ ....(r < R)

7. Electric intensity at a point outside a charged cylindrical conductor:

i. Cylinder in any medium,

$$E = \frac{\lambda}{2\pi\varepsilon r} = \frac{\lambda}{2\pi k \varepsilon_0 r} = \frac{\sigma R}{k\varepsilon_0 r} \quad \ldots(r > R)$$

ii. Cylinder in free space or vacuum,

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r} \quad \ldots(r > R)$$

iii. $E_{inside} = 0 \quad \ldots(r < R)$

8. Electric intensity at short distance from a charged conductor of any shape:

i. $E = \dfrac{\sigma}{k\varepsilon_0}$

ii. Conductor in free space or air or vacuum,

$$E_0 = \frac{\sigma}{\varepsilon_0} = kE$$

9. Electric intensity at a point outside a uniformly charged infinite plane sheet: $E = \dfrac{\sigma}{2\varepsilon}$

10. Relationship between electric field and potential:

i. $E = -\dfrac{dV}{dx}$ ii. $V = -\int E\, dx$

11. Electric potential due to an electric dipole:

i. $V_C = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p\cos\theta}{r^2}$

ii. $V_{axial} = \pm\dfrac{1}{4\pi\varepsilon_0}\dfrac{p\cos\theta}{r^2}$

iii. $V_{equator} = 0$

12. Electrostatic potential due to system of charges:

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

13. Potential energy of single charge:

P.E. = $q\,V(\vec{r})$, where $\vec{r}$ is position vector of charge q

14. Potential energy for a system of two charges separated by a distance r:

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

15. Potential energy for a system of two charges placed in an external electric field:

$$U = q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

16. Potential energy for a system of 'N' point charges:

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}} \quad \text{where, } j \neq k$$

17. Work done:

i. $W = qV$ ii. $W = q(V_B - V_A)$

18. Torque on a dipole:

i. $\vec{\tau} = \vec{p} \times \vec{E}$ ii. $\tau = pE\sin\theta$

iii. For $\theta = 90°$, $\tau_{max} = pE$ iv. For $\theta = 0$, $\tau_{min} = 0$

19. Work done by the external torque on dipole:

$$W = \int_{\theta_0}^{\theta}\tau_{ext}(\theta)\,d\theta = \int_{\theta_0}^{\theta} pE\sin\theta\, d\theta$$

$$= pE[\cos\theta_0 - \cos\theta]$$

20. Potential energy of electric dipole in external electric field:

$$U(\theta) - U(\theta_0) = pE(\cos\theta_0 - \cos\theta)$$

21. Capacity of condenser: $C = \dfrac{Q}{V}$

22. Capacitance of capacitor with dielectric:

$$C_d = C_0\frac{E_0}{E_d}$$

where, $C_0$ is original capacitance

$E_0$ is original electric field

$E_d$ is electric field with dielectric

23. Parallel plate capacitor with dielectric medium between the plates:

i. Capacitance of a parallel plate capacitor with a dielectric slab between the plate:

$$C = \frac{A\varepsilon_0}{d - t + \dfrac{t}{k}}$$

ii. If the dielectric fills up the entire space then t = d

$$C = \frac{A\varepsilon_0 k}{d} = kC_0$$

iii. If the capacitor is filled with 'n' dielectric slabs of thickness $t_1, t_2, \ldots, t_n$

$$C = \frac{A\varepsilon_0}{\dfrac{t_1}{k_1} + \dfrac{t_2}{k_2} \ldots + \dfrac{t_n}{k_n}}$$

iv. If the arrangement consists of 'n' capacitors in parallel with plate area $A_1, A_2, \ldots, A_n$

$$C = \frac{\varepsilon_0}{d}(A_1k_1 + A_2k_2 \ldots A_nk_n)$$

v. If the capacitor is filled with conducting slab

$$C = \left(\frac{d}{d - t}\right)C_0$$

**24.** Series combination of 'n' condensers:

i. $V = V_1 + V_2 + V_3 \ldots\ldots + V_n$

ii. $Q = Q_1 = Q_2 = Q_3 = \ldots\ldots = Q_n$

iii. $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots + \frac{1}{C_n}$

**25.** Parallel combination of 'n' condensers:

i. $Q = Q_1 + Q_2 + \ldots\ldots + Q_n$

ii. $V = V_1 = V_2 = V_3 = \ldots\ldots = V_n$

iii. $C = C_1 + C_2 + C_3 + \ldots\ldots + C_n$

**26.** Parallel plate condenser:

i. Intensity between the plates,

$$E = \frac{\sigma}{\varepsilon} = \frac{Q}{A\varepsilon} = \frac{\sigma}{k\varepsilon_0} = \frac{Q}{Ak\varepsilon_0}$$

ii. Potential difference between the plates, $V = Ed$

iii. Capacity between the plates, $C = \frac{A\varepsilon}{d} = k\,C_0$

iv. Capacity of vacuum, $C_0 = \frac{A\varepsilon_0}{d}$

**27.** Energy stored in a charged capacitor:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

## Shortcuts

1. If two capacitors are connected, then the resultant potential after joining them will be given by $\frac{C_1V_1 + C_2V_2}{C_{eff}}$, where, $C_1$ and $C_2$ are their capacities, $V_1$ and $V_2$ are their potentials and $C_{eff}$ is resultant capacity after joining. Therefore, $C_{eff}$ will be $C_1 + C_2$ in parallel and $\frac{C_1C_2}{C_1 + C_2}$ in series.

2. The capacity of a parallel plate capacitor is given by, $C = \frac{A\varepsilon_0}{d}$ where, A is the area of the plate and d is the distance between two plates.

i. If a good conductor of thickness t is inserted between the plates, replace 'd' by '$d - t$'

ii. If an insulator of dielectric constant k is inserted between the plates, replace 'd' by '$d - t\left[1 - \frac{1}{k}\right]$'.

iii. If the whole space between the plates is filled with insulator of dielectric constant k, capacity will be Ck.

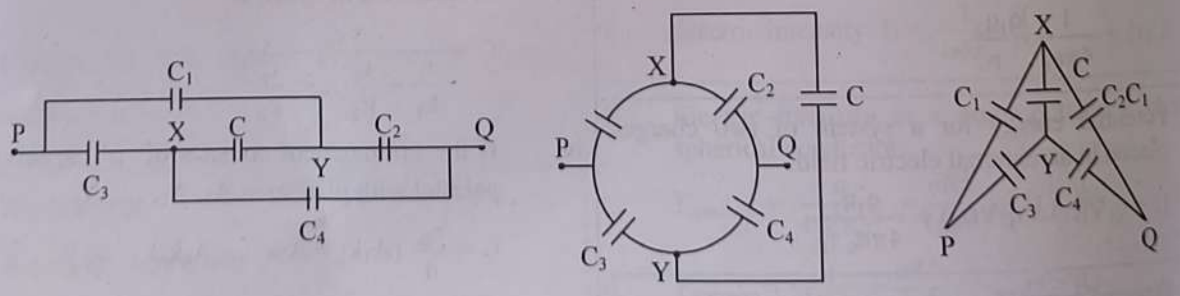
3. If n charged droplets, each of capacity C, are charged to the potential V with charge q, then

i. total charge = nq  ii. total capacity = $n^{1/3}$ C  iii. potential = $n^{2/3}$V

4. The relationship between inducing charge q and induced charge q′ is

$q' = -q\left[1 - \frac{1}{k}\right]$ where, k is dielectric constant of the medium.

5. All the following circuits represent Wheatstone's bridge of capacitors.




If $\frac{C_1}{C_2} = \frac{C_3}{C_4}$, then the bridge is said to be balanced. For the balanced bridge, while calculating effective capacitance between PQ, value of C can be ignored.