DEFINITE INTEGRALS AND APPLICATION OF INTEGRALS

CONCEPT MAP

Class XII

DEFINITE INTEGRALS

For any two values a and b, we have $\int_a^b f(x)\,dx = [F(x)+c]_a^b = F(b) - F(a)$

Limit of Sum

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h[f(a) + f(a+h) + \ldots + f(a+(n-1)h)],$$

where $h = \dfrac{b-a}{n} \to 0$ as $n \to \infty$

Fundamental Theorem of Calculus

- **First Fundamental Theorem :** Let $f(x)$ be a continuous function in the closed interval $[a, b]$ and let $A(x)$ be the area function. Then $A'(x) = f(x)$, for all $x \in [a, b]$.
- **Second Fundamental Theorem :** Let $f(x)$ be a continuous function in the closed interval $[a, b]$ and $F(x)$ be an integral of $f(x)$, then $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

Solving by Substitution

When definite integral is to be found by substitution, change the lower and upper limits of integration. If substitution is $t = f(x)$ and lower limit of integration is a and upper limit is b, then new lower and upper limits will be $f(a)$ and $f(b)$ respectively.

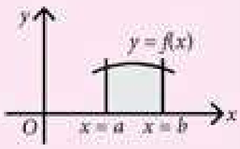
Properties

- $\int_a^b f(x)dx = \int_a^b f(t)dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$. In particular $\int_a^a f(x)dx = 0$
- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ where $a < c < b$
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
- $\int_{-a}^{a} f(x)dx = \begin{cases} 0 & \text{, if } f(-x) = -f(x) \\ 2\int_0^a f(x)dx & \text{, if } f(-x) = f(x) \end{cases}$
- $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
- $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$

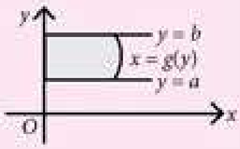
APPLICATION OF INTEGRALS

Area Under Simple Curves

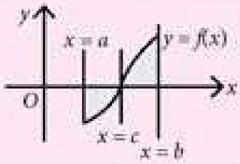
- Area $= \int_a^b y\,dx$ $= \int_a^b f(x)dx$ (where $b > a$)



- Area $= \int_a^b x\,dy$ $= \int_a^b g(y)dy$ (where $b > a$)

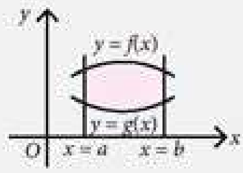


- Area $= \left|\int_a^c f(x)dx\right| + \int_c^b f(x)dx$

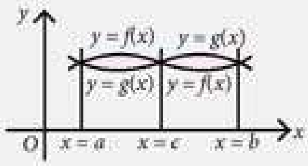


Area Between Two Curves

- Area $= \int_a^b [f(x) - g(x)]dx$, $f(x) \geq g(x)$ in $[a, b]$



- Area $= \int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$



where $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b]$

- Area $= \int_a^c f(x)dx + \int_c^b g(x)dx$