04. Oscillations

4.0 Introduction

In SHM, the particle performs the same set of movements again and again after equal intervals of time. One set of movements of the particle is called oscillation.

4.1 Periodic motion

1. It is the motion that repeats itself after a fixed interval of time about the mean position is called periodic motion. The periodic motion may have constant speed or may not have constant speed.
2. The time interval after which the motion is repeated is called period of motion.

Ex.:

i) The revolution of earth around the sun, its period of revolution is one year.
ii) The rotation of earth about its polar axis, its period of revolution is 24 hours.
iii) The motion of the needle of sewing machine.
iv) Oscillatory motion of the particle about mean position.

Non periodic motion:

It is that motion which is not identically repeated after a fixed interval of time.

Oscillatory motion or vibratory motion:

1. It is the periodic motion in which a body moves to and fro or back and forth or up and down about a fixed point (called mean position), in a definite interval of time.
2. In such a motion, the body is confined or remains constant within well defined limits (called extreme positions) on either side of mean position.

Ex.:

i) The motion of the pendulum clock is oscillatory motion.
ii) The motion of a loaded spring when the load attached to the spring is pulled once a little from its mean position and left to itself.
iii) The vibration of the tuning fork.
iv) Vibration of a wire of Sitar.
v) Vibration of the drum of a Tabala.
vi) Motion of the balance wheel of a watch.
vii) Motion of an object along the tunnel dug to the diameter of the earth or along the chord: A tunnel is dug to any diameter 'or a chord of the earth. If a mass 'm' is droped into the tunnel (the friction is assumed to be zero), then, the mass of the object executes SHM about the centre of the earth with a time period T. The gravitational force exerted on the body by the dotted portion of the earth at a distance y from the centre of the earth is,

$$F = \frac{GM'm}{y^2} \qquad \text{..... (i)}$$

The mass of the earth interms of density is,

$$M' = \rho\ V'$$

$$M = \rho V$$

$$\frac{M'}{M} = \frac{V'}{V} = \frac{\frac{4\pi}{3}y^3}{\frac{4\pi}{3}R^3}$$

$$M' = \frac{My^3}{R^3}$$

Put M' in equation (i) we have,

$$F = -\frac{GMmy^3}{y^2R^3} = -\frac{GMmy}{R^3}$$

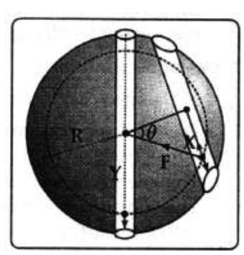
But $F = m a$

$$\therefore \quad m a = -\frac{GMmy}{R^3}$$

$$a = -\frac{GM y}{R^3}$$

Equating with standard equation of acceleration

$$a = -\omega^2 y$$



$$\therefore \quad \omega^2 = \frac{GM}{R^3}$$

$$\omega = \sqrt{\frac{GM}{R^3}}$$

But $T = \frac{2\pi}{\omega}$

$$\therefore \quad T = 2\pi\sqrt{\frac{R}{g}}$$

$$= 84.6 \text{ min}$$

Thus, the period of SHM about the centre of earth is 84.6 min.

The motion of the mass along the chord can be obtained by resolving F into its components is,

$$F' = F \sin\theta \quad \left(\because \sin\theta = x/y \text{ from geometry of figure}\right)$$

$$F' = -\left(\frac{GMmy}{R^3}\right) \times \frac{x}{y}$$

$$= -\left(\frac{GMm}{R^3}\right) \times$$

$$\therefore \quad \text{Acceleration} = - -\left(\frac{GM}{R^3}\right) \times$$

As acceleration is directly proportional to the displacement and both are oppositely directed. Thus, the object performs linear SHM.

$$\therefore \quad \omega^2 = \frac{GM}{R^3}$$

$$\therefore \quad \omega = \sqrt{\frac{GM}{R^3}}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

$$= 84.6 \text{ min}$$

Thus the period of the chord is same as that of the diameter of the earth.

3. All the oscillatory motions are periodic motions but all the periodic motions are not oscillatory motions.

Ex.:

i) Motion of the needle of sewing machine is periodic and oscillatory.

ii) The motion of a particle along the circumference of the circle is periodic but not oscillatory, since its path is not oscillatory path.

Types of oscillations:

1) Damped oscillation and
2) Undamped oscillation.

1. **Damped oscillations:** A body oscillates with its amplitude gradually decreasing with time. The energy of the oscillation decreases with time. The amplitude of oscillation decreases due to presence of frictional and air resistance of atmosphere. In damped oscillations, the frequency of oscillations remains constant. The amplitude of oscillation is given by $A = A_0 e^{-bt}$, where A_0 = maximum amplitude and A = instantanous amplitude, b = damping factor related to velocity of the oscillating body.
2. **Undamped oscillations:** A body oscillates with a constant amplitude is called undamped oscillations. The frequency of oscillation remains same. The total energy of the oscillating body remains constant.

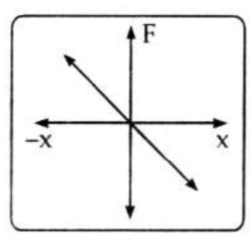
4.2 Simple Harmonic Motion

1. It is a periodic motion in which a particle moves to and fro repeatedly about a mean position under a restoring force which is always directed towards the mean position and whose magnitude at any position is directly proportional to the displacement of the particle from the mean position.

$$F \propto -x$$
$$F = -kx$$

where k is constant of proportionality known as force constant. Negative sign shows that the restoring force (F) is always directed towards the mean position and displacement is away from the mean position.

2. The displacement of the particle executing SHM at any instant can be expressed in terms of single harmonic function (i.e. sine or cosine). Hence it is called simple harmonic motion.
3. If the graph is plotted between force and displacement of a particle performing SHM, the nature of graph is straight line passing through second and fourth quadrant.



4. The slope of the graph is negative force constant.

5. The same graph is obtained between acceleration and displacement with slope is $-\omega^2$.
6. The unit of force constant is Newton per meter (N/m).
7. The dimensions of force constant are $[L^0 M^1 T^{-2}]$.
8. The value of force constant is $K = m\omega^2 = m4\pi^2 n^2$.

Geometrical interpretation of SHM :

1. SHM is defined as the projection of a uniform circular motion on any diameter of a reference circle.
2. SHMs can be linear and angular SHM.
3. The linear SHM is always along a straight line about a fixed point, whereas the angular SHM is always along an arc of a circle about a fixed point, on the arc.
4. The linear SHM is controlled by force law, $F = -kx$, where k is the restoring force constant.
5. The angular SHM is controlled by torque law, $\tau = -C\theta$ where C is the restoring torque constant.
6. The unit of C is Nm/rad.
   The dimension are $[L^2 M^1 T^{-2}]$

Periodic function :

1. The functions used to represent periodic motions.
2. A function f (t) is said to be periodic if $f(t) = f(t+T) = f(t+2T)$ where T is called the period of periodic function. $\sin\theta$ and $\cos\theta$ are the examples of periodic functions with period equal to 2π second, becaus $\sin\theta = \sin(\theta + 2\pi) = \sin(\theta + 4\pi)$

   and $\cos\theta = \cos(\theta + 2\pi) = \cos(\theta + 4\pi)$

   Angular frequency (ω) of a body executing periodic motion is equal to the prod uct of frequency of the body with a factor 2π. $\omega = n \times 2\pi$. Its unit is hertz or s^{-1}.

4.3 Projection of UCM as SHM

1. Projection of UCM as SHM about any diameter of the circle.
2. When two particles performing SHM have same period frequency and amplitude but change of phase of $\pi/2$, then resultant motion is circle i.e. $x = A\sin\omega t$ and $y = A\cos\omega t$. Squaring and adding above two equations, we have,

   $$x^2 + y^2 = A^2$$

   It is the standard equation of circle.
3. When two particles performing SHM have same amplitude. One particle have frequency ω and other particle have twice the frequency of first but their phase difference is $\pi/2$, then resultant motion followed by the particle is parabola.
4. When particle starts motion from mean position and reach to midway, then time taken by the particle is $t = T/12$.
5. When particle starts motion from midway and reach to extreme position, then minimum time taken by the particle is $t = T/6$.
6. The minimum time required for the particle to move from one midway to other midway is $t = T/6$.

The differential equation of linear SHM :

1. Linear differential equation of SHM is,

   $$\frac{d^2x}{dt^2} + \omega^2 x = 0, \text{ where } \omega^2 = k/m$$

2. The differential equation of angular SHM is,

   $$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \text{ where } \omega^2 = C/I$$

   Here I is the moment of inertia of body executing angular SHM.
3. It is the second order differential equation of SHM.

4.4 Displacement, velocity and acceleration of a particle performing in SHM

1. The displacement of a particle executing linear SHM at an instant is defined as the distance of the particle from the mean position is,

   $$x = A\sin(\omega t \pm \phi)$$

2. The displacement of a particle executing linear SHM at an instant is defined as the distance of the particle from the extreme position is,

   $$x = A\cos(\omega t \pm \phi)$$

3. The maximum displacement either sides of mean position is called amplitude of motion.
4. The direction of displacement is always away from the mean position.
   The direction of displacement varies twice in a period of oscillation.
5. The average value of displacement over a period of SHM is zero.

6. The distance covered by the particle over a period of SHM is 4 A.
7. The displacement of the particle always measure from mean position, whether the particle starts motion either from mean or extreme position.

Velocity in SHM :

1. The time rate of change of displacement of the a particle with respect to time.
2. Velocity of a particle performing SHM is,

$$v = \frac{dx}{dt} = \frac{d}{dt}(A \sin \omega t) = A \omega \cos \omega t$$

$$= A \omega \sqrt{1 - \sin^2 \omega t} = \omega\sqrt{A^2 - x^2}$$

3. Velocity of the particle performing SHM is maximum at mean potion ($A\omega$) and it is zero at extreme position.
4. The maximum value of velocity is called velocity amplitude of a particle performs SHM i.e. $v_m = A\omega$.
5. The direction of velocity of a particle in SHM is either towards or away from the mean position.
6. In SHM, the velocity varies simple harmonically with time.
7. The velocity–displacement (v – x) graph of a particle performs simple harmonic motion is an ellipse shown in figure (4.2 a).

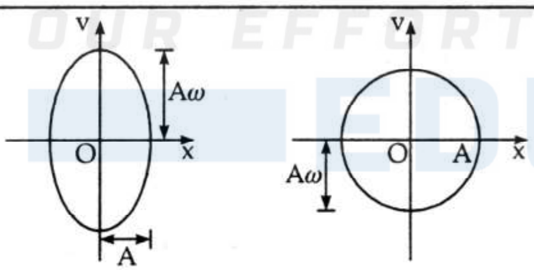


Fig. 4.2 (a) Fig. 4.2 (b)

8. If $\omega = 1$ or $T = 2\pi$, then it is represented by a circle shown in figure (4.2 b).

The velocity of a particle performing SMH is,

$$v = \omega\sqrt{A^2 - x^2}$$

$$v^2 = \omega^2\left(A^2 - x^2\right)$$

$$\therefore \quad \frac{y^2}{\omega^2} = \left(A^2 - x^2\right)$$

$$\frac{y^2}{\omega^2} + x^2 = A^2$$

$$\omega^2 \frac{v^2}{A^2\omega^2} + \frac{x^2}{A^2} = 1$$

This is standard equation of ellipse.

9. If we plot a graph between velocity against the displacement, the nature of graph is ellipse shown in figure (4.2 a).
10. The velocity amplitude is angular velocity times the displacement amplitude.
11. The average value of velocity of a particle performing SHM over a period is zero.
12. The average value of speed of a particle performing SHM over a period is,

$$v_{av} = \frac{\text{Average distance over one oscillation}}{\text{Time}}$$

$$= \frac{4A}{T} = 4\,A\,n$$

$$\therefore \quad v_{av} = \frac{2A\omega}{\pi}$$

13. The motion of oscillating particle is faster initially and later on beomes slower.
14. When the particle is at a distance half of the amplitude, its velocity is 86.67 % v_{max}

i.e. $v = \omega\sqrt{A^2 - \frac{A^2}{4}} = \frac{\sqrt{3}}{2} A\omega = 8667\%\ A\omega$

15. If $v = \frac{v_{max}}{2} = \frac{A\omega}{2}$ Then, $v = \omega\sqrt{A^2 - x^2}$

$$\therefore \quad \frac{A\omega}{2} = \omega\sqrt{A^2 - x^2} \quad \therefore \quad x = 86.6\%\ A$$

A particle starting from mean position with maximum velocity, it will lose only 50% of its velocity even after travelling 86.67% of the maximum possible distance in one direction.

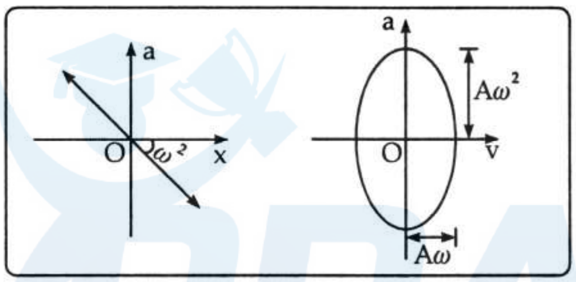
Acceleration in SHM :

1. The time rate of change of the velocity of the particle at the given instant.

$$a = \frac{dv}{dt} = \frac{d}{dt}(A\,\omega \cos \omega t) = -\omega^2 A \sin \omega t = -\omega^2 x$$

2. The acceleration of the particle executing SHM is minimum at the mean position and is maximum at the extreme position.

3. The maximum value of acceleration is called acceleration amplitude in SHM i.e. $a_m = A\omega^2$
4. The acceleration of a particle executing SHM is always directed towards the mean position.
5. In SHM the acceleration varies simple harmonically with the time.
6. The acceleration amplitude is the product of square of the angular velocity (ω^2)and the displacement amplitude.
7. The average acceleration of particle performing SHM over a period is zero.
8. The force or acceleration in SHM varies periodic in direction i.e. half the period of oscillating particle. But magnitude varies instantaneously
9. The velocity acceleration (v – a) graph in SHM, is an ellipse.



We know that,

$$v = \omega\sqrt{A^2 - x^2}$$

$$\frac{v}{\omega} = \sqrt{A^2 - x^2}$$

$$\frac{v^2}{\omega^2} = A^2 - x^2$$

$$\frac{v^2}{\omega^2} + x^2 = A^2$$

$$\frac{v^2}{A^2\omega^2} + \frac{x^2}{A^2} = 1 \qquad \text{...(i)}$$

But $a = \omega^2 x$

$$\therefore \quad x = \frac{a}{\omega^2}$$

$$x^2 = \frac{a^2}{\omega^2}$$

Substute x^2 in equaiton (i), we have,

$$\frac{v^2}{A^2\omega^2} + \frac{a^2}{A^2\omega^2} = 1$$

which is an equation of ellipse.

Phase relationship between displacement, velocity acceleration and restoring force SHM :

1. The displacement (x) of a particle executing SHM at an instant is,

$$x = A\sin(\omega t + \phi)$$

2. Velocity, $v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$

$$= A\omega\sin[(\omega t + \phi) + \pi/2]$$

3. Acceleration, $a = \frac{dv}{dt} = -A\omega^2\sin(\omega t + \phi)$

$$= \omega^2 A\sin[(\omega t + \phi) + \pi]$$

4. Restoring force, $F = -A\omega^2\sin(\omega t + \pi)$

$$= -A\omega^2\sin[(\omega t + \phi) + \pi]$$

From above we note that,

phase of displacement $\omega t + \phi$

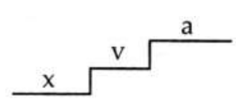
phase of velocity $(\omega t + \phi + \pi/2)$

phase of acceleration $(\omega t + \phi + \pi)$

phase of restoring force = $(\omega t + \phi + n)$

We conclude that,

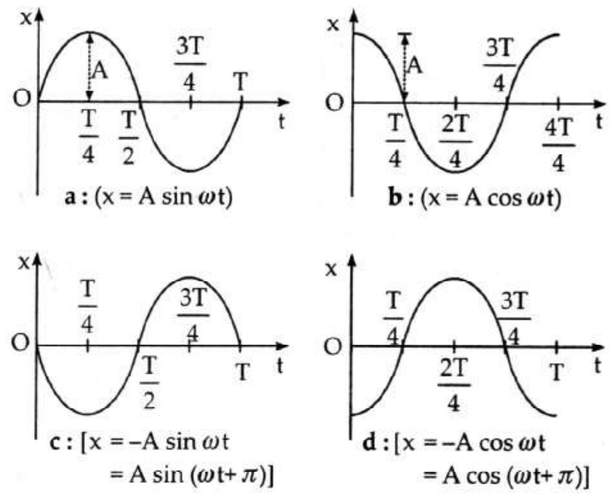
5. The velocity in SHM is leading the displacement by a phase $\pi/2$ radian.
6. The acceleration in SHM is leading the displacement by a phase π radian.
7. The acceleration in SHM is leading the velocity by a phase $\pi/2$ radian.
8. Restoring force in SHM is leading the velocity by a phase $\pi/2$ radian.
9. To remember phase of lead or lags, we follow the figure.



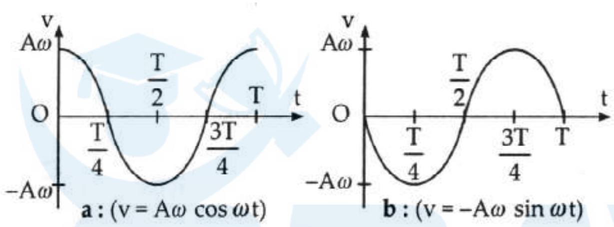
4.5 Graphical Representation of SHM

Displacement, velocity, acceleration, restoring force and time in SHM :

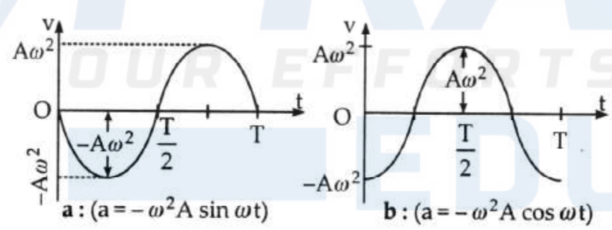
1. The displacement–time (x – t) graphs for different cases of SHM are shown in figure.



2. The velocity–time graph (v – t) for different simple harmonic motions is shown in figure (a, b).



3. The acceleration–time (a – t) graph for different simple harmonic motions is shown in figure.



4.6 Phase, period and frequency of SHM

1. If the displacement of the particle executing a vibratory motion is represented by $x = A\sin(\omega t + \phi)$, then the angle $(\omega t + \phi)$ is called phase of the displacement. A particle performs oscillatory motion and ϕ is called the initial phase or epoch or starting phase angle of the particle.
2. Phase can be expressed in terms of time period or frequency.

   Phase angle $= (\omega t + \phi) = \left(\frac{2\pi t}{T} + \phi\right) = (2\pi n t + \phi)$.
3. The phase angle depends upon time 't', displacement and direction.
4. Initial phase angle is independent of amplitude and time.
5. Phase of a vibrating particle at any instant is a physical quantity which completely expresses the position and direction of motion of particle at that instant with respect to its mean position.,
6. It is measured either in terms of fraction of time period or fraction of 2π angle, which has elapsed, since the vibrating particle has crossed its mean position in the positive direction.
7. In oscillatory motion, the phase of a vibrating particle is the arrangement of sine or cosine function involved to represent the generalised equation of motion of the vibrating particle.

Phase difference between two vibrating particles:

1. It tells the lack of harmony in the vibrating states of the two particles at a given instant.
2. The displacement at any instant of the two particles executing a vibratory motion be given by

   $x_1 = A_1 \sin(\omega_1 t + \phi_1)$ and

   $x_2 = A_2 \sin(\omega_2 t + \phi_2)$

   Then the instantaneous phase difference between two motions $\Delta\phi = (\omega_1 t + \phi_1) - (\omega_2 t + \phi_2)$

   i) If $\Delta\phi$ is positive, then the disp lacement of first particle is more than that of second particle i.e. the first particle is leading the second particle by a phase $\Delta\phi$.

   ii) If $\Delta\phi$ is negative, then the displacement of the first particle is less than that of second particle i.e. the second particle is leading the first particle by a phase $\Delta\phi$.
3. If $\Delta\phi = 2n\pi$, where n = 0, 1, 2, 3, ..., then the two vibrating particles are said to be in the same phase.
4. If $\Delta\phi = (2n + 1)\pi$, where n = 0, 1, 2, ..., then two vibrating particles are said to be in opposite phase.

Time period and frequency:

1. When a particle performs SHM, its displacement, velocity and acceleration are periodic. Time period and frequency are the same for these three.

2. When a particle performing SHM and it is at $x = A/2$ from mean position. The time required is $t = T/12$.
3. When the particle starts motion from the extreme position and reach to the midway, the time taken is T/6 s.
4. The shortest time for a particle to move from one midway to other midway is $\left(\frac{T}{12}+\frac{T}{12}\right) = \frac{T}{6}$.
5. Time period in SHM is given by

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} \text{ or}$$

$$T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

6. Frequency of vibration in SHM,

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}}$$

$$\text{or} \quad n = \frac{1}{2\pi}\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

7. For a simple harmonic motion, time period and frequency is independent of the amplitude of motion. This type of motion is called Isochronous motion.
8. If T_1 and T_2 are the time periods of a body oscillating under the restoring forces F_1 and F_2, then the time period of the body under the influence of resultant force $\vec{F}$. When the forces are in the same direction.

$$T = \frac{T_1T_2}{\sqrt{T_1^2+T_2^2}}$$

when the forces are in the opposite direction. Then the period is,

$$T = \frac{T_1T_2}{\sqrt{T_1^2+T_2^2}}$$

9. The force constant k of a stiffer spring is higher than that of a soft spring. So the time period of a stiffer is less than that of a soft spring.
10. Amplitude of an oscillating particle

$$A = \sqrt{\frac{(v_1x_2)^2-(v_2x_1)^2}{v_1^2-v_2^2}}$$

11. Period of an oscillating particle

$$T = 2\pi\sqrt{\frac{x_2^2-x_1^2}{v_1^2-v_2^2}}$$

12. Frequency of oscillation of a particle is,

$$n = \frac{1}{2\pi}\sqrt{\frac{v_1^2-v_2^2}{x_2^2-x_1^2}}$$

4.7 Energy in SHM

Energy of a particle performing SHM is,

$$E = P.E. + K.E. = E_P + E_K = \frac{1}{2}m\omega^2A^2 = \text{constant}.$$

1. The total energy of a particle in SHM is same at all instants and at all displacements.
2. It depends upon (a) mass, (b) amplitude and (c) frequency of vibration of the particle executing SHM.

t	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T
$\theta = \omega t$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
K.E.	$\frac{1}{2}m\omega^2A^2$	0	$\frac{1}{2}m\omega^2A^2$	0	$\frac{1}{2}m\omega^2A^2$
P.E.	0	$\frac{1}{2}m\omega^2A^2$	0	$\frac{1}{2}m\omega^2A^2$	0
T.E.	$\frac{1}{2}m\omega^2A^2$	$\frac{1}{2}m\omega^2A^2$	$\frac{1}{2}m\omega^2A^2$	$\frac{1}{2}m\omega^2A^2$	$\frac{1}{2}m\omega^2A^2$

3. Variation of K.E. P.E. and total energy E in SHM with respect to t or $\theta = \omega t$.

If $x = A\sin\omega t$, then potential energy in SHM is,

$$P.E. = \frac{1}{2}m\omega^2A^2\sin^2\omega t$$

$$= \frac{1}{4}m\omega^2A^2(1-\cos 2\omega t)$$

$$\left(\because \sin^2\omega t = \frac{1-\cos 2\omega t}{2}\right)$$

Kinetic energy in SHM is,

$$K.E. = \frac{1}{2}m\omega^2A^2\sin^2\omega t$$

$$= \frac{1}{4}m\omega^2A^2(1+\cos 2\omega t)$$

$$(\because \cos^2 \omega t = \frac{1+\cos 2\omega t}{2})$$

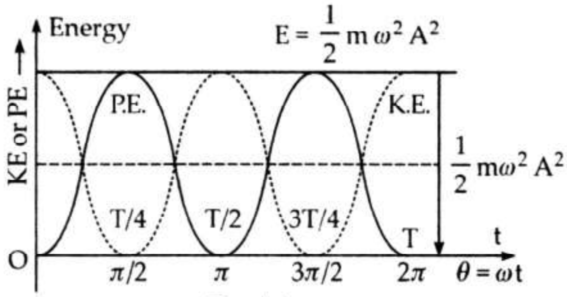


Fig. (a)

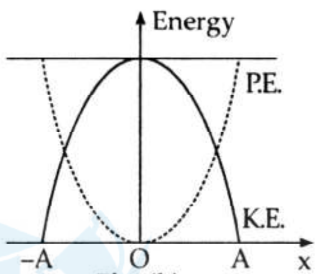


Fig. (b)

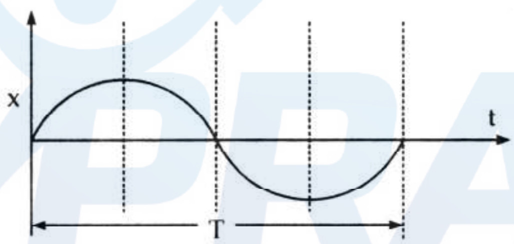


Fig. (c)

4. Both potential energy and kinetic energy vary periodically but the variation is not simple harmonic.
5. The frequency of oscillation of potential energy or kinetic energy is twice frequency of displacement or velocity or acceleration of a particle executing SHM.
6. Graphical representation of potential energy, kinetic energy and total energy with time (t) and displacement (x) is shown in figure (a) and (b).
7. Average value of P.E. or K.E. over time period is,

$$\langle K.E. \rangle = \langle P.E. \rangle = \frac{1}{4} m \omega^2 A^2$$

8. At displacement, $x = \frac{A}{2}$, P.E. = $\frac{1}{4}$ E = 25% of E and K.E. = $\frac{3}{4}$ E = 75% E. The ratio of K.E. to P.E. at this position is 3 : 1.
9. At distance $x = \frac{A}{\sqrt{2}} = 0.707$ A, P. E. = $\frac{1}{2}$ E and K.E.= $\frac{1}{2}$ E.

   The ratio of K.E. to P.E. at this position is 1 : 1.
10. The frequency of P.Eor K.E. is double the frequency of particle performs of SHM i.e. n' = 2n.
11. The period of K.E. or P.E. is half of the period of a particle performing SHM i.e T' = T/2.
12. In a period of SHM, the kinetic energy and potential energy are equal in four times.
13. In a period of kinetic energy or potential energy, the K.E. and P.E. are equal in twice.
14. The work done by a restoring force in one complete oscillation is zero.
15. Kinetic energy and potential energy of a particle peforming SHM are non–conservative.
16. The total energy of a particle peforming SHM is conservative.
17. Total energy is independent of the displacement of the particle.
18. Potential energy lags in phase with the kinetic energy by (π/2).
19. The average value of total energy over one oscillation is, $\frac{1}{2}$ m $\omega^2 A^2$.

4.8 Composition of SHM

The displacement equations of the two particles starts from different positions have different amplitudes and initial phase angles, but same frequency are

x_1 = At sin ($\omega t + \alpha_1$) and $x_2 = A_2 \sin(\omega t + \alpha_2)$

By principle of superposition,

$x = x_1 + x_2$

$= A_1 \sin(\omega t + \alpha_1) + A_2 \sin(\omega t + \alpha_2)$

After solving the above equation, we have,

$x = R \sin(\omega t + \alpha)$

where R is the amplitude of resultant SHM and it is given by

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha_1 - \alpha_2)}$$

Initial phase angle of resultant SHM is,

$$\delta = \tan^{-1}\left[\frac{A_1 \sin\alpha_1 + A_2 \sin\alpha_2}{A_1 \cos\alpha_1 + A_2 \cos\alpha_2}\right]$$

From the above equation we conclude that,

1. The amplitude of resultant motion not only depends upon the amplitude of individual, but it also upon the initial phase angles of individuals.
2. **When the amplitude of individual displacements is same, but the phase changes as,**

i) If the phase difference is $\pi/2$ i.e. a $\alpha_1 - \alpha_2 = \pi/2$, then the amplitude of resultant motion is root two times amplitude of individual i.e. $R = \sqrt{2}$ A.

ii) If the phase difference is π (i.e. $\alpha_1 - \alpha_2 = \pi$), then the amplitude of resultant motion is zero.

iii) If $\alpha_1 - \alpha_2 = 0$, then the amplitude of the resultant motion is twice the amplitude of individual.

3. If the amplitude of individual wave is same but the initial phase angles are zero and $\pi/2$, then the phase angle of the resultant motion is $\pi/4$.
4. Composition of two SHM is a SHM, whose, amplitude and initial phase angle are different from individuals.
5. If two particles performing SHM have the same initial phase angles and amplitudes combine to form a single SHM, then the resultant SHM have the intial phase angle as that of the individual or average of the individual phase angles.
6. Two simple harmonic motions are combined perpendicular to each other,

i) the resultant path is straight line, if the phase difference is zero between those two

$$x = A_1 \sin \omega t \text{ and } y = A_2 \sin \omega t$$

$$\therefore \quad y = \frac{A_2}{A_1}x$$

which is equation of a straight line.

ii) The resultant path is a straight line, if phase difference is π

$$x = A_1 \sin \omega t \text{ and } y = A_2 \sin(\omega t + \pi)$$

$$y = \frac{-A_2}{A_1}x$$

which is equation of a straight line.

Here resultant motion is simple harmonic with amplitude $R = \sqrt{A_1^2 + A_2^2}$.

iii) The resultant path is an ellipse, if the phase difference is $\pi/2$.

$$x = A_1 \sin \omega t \text{ and } y = A_2 \sin(\omega t + \pi/2)$$

$$\therefore \quad \frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$$

which is equation of ellipse.

iv) The resultant path is circle, if phase difference is $\pi/2$ and amplitudes are same.

$$x = A \sin \omega t \text{ and } y = A \sin(\omega t + \pi/2)$$

$\therefore \quad x^2 + y^2 = A^2$ which is equation of a circle.

4.9 Simple pendulum

Ideal simple pendulum:

The heavy point mass suspended by means of weightless in extenssible string from a perfectly rigid support.

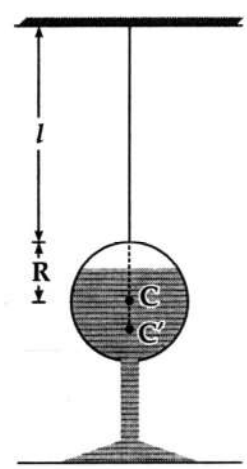
Practical simple pendulum:

It consists of heavy metalic sphere suspended by means of light weight slightly extenssible string from a rigid support.

1. For simple pendulum angular displacement is at the most of 5° to 10°.
2. Time period, $T = 2\pi\sqrt{l/g}$ where l is the length of simple pendulum, g is the acceleration due to gravity. This relation is valid when $l << R_e$ (radius of earth).
3. If l is comparable with the radius of the earth, then the time period of simple pendulum is given by,

$$T = \sqrt{\frac{R_e}{(1 + R_e/l)g}}$$

4. Period of simple pendulum is
   i) independent of amplitude,
   ii) depends on length of pendulum,
   iii) independent of mass and
   iv) depends.upon acceleration due to gravity.



5. A pendulum whose time period is two second is called second's pendulum and its frequency is 0.5 Hz.
6. Clock pendulum whose time period is one second.
7. The tension in the string is maximum when the pendulum is at mean position and minimum when the pendulum is at extreme position.
8. The frequency of the clock pendulum is one hertz.
9. If the spherical bob of the pendulum is filled with sand/mercury/liquid and the bob is provided with a fine hole at the bottom, then the filled material falls down continuously. The centre of mass, will first go below the centre, increasing the length of the simple pendulum, and when the entire material falls down, the CM will again come to point C (centre of the bob). Therefore, the effective length of the pendulum first increases from $l + R$ value, and then decreases back to $l - R$ value. The time period of such a leaky pendulum, therefore, first increases and then decreases. Later on remains same.
10. If the liquid is highly viscous, then due to high resistance of the medium, the amplitude of simple pendulum decreases exponentially and $T = \infty$, It means the pendulum does not oscillate.
11. If the bob of simple pendulum is negatively charged and a positively charged plate is placed below it, then the effective acceleration on bob increases and consequently time period decreases.

$$T = 2\pi\sqrt{\frac{l}{(g + qE/m)}}$$

12. If the bob of simple pendulum is negatively charged and is made to oscillate above the negatively charged plate, then the effective acceleration on bob decreases and the time period increases.

$$2\pi\sqrt{\frac{l}{(g - qE/m)}}$$

13. Time period is independent of the mass of bob and the angular amplitude θ, if it is small.
14. If no resistive forces are active, the work done by a simple pendulum, in one complete oscillation is zero.
15. Work done in producing angular displacement 8 to the pendulum, from its mean position is,

$$\omega = E_p = mgl\,(1 - \cos\theta).$$

16. If a lift is accelerated upwards with acceleration a, then time period of a simple pendulum in the lift is,

$$T' = 2\pi\sqrt{\frac{l}{g + a}}\,.$$

17. If a lift is accelerated downwards with acceleration a, then time period of a simple pendulum in the lift is,

$$T' = 2\pi\sqrt{\frac{l}{g - a}}$$

18. If a lift is moving upwards or downwards with a constant velocity v, then a = 0

$$\therefore \quad T' = 2\pi\sqrt{\frac{l}{g}}$$

19. When a lift is freely falling with acceleration g. then

$$T' = 2\pi\sqrt{\frac{l}{g - g}} = \infty$$

20. When a vehicle (car, bus or train) is moving with an acceleration a in the horizontal direction, then time period of simple pendulum on vehicles is,

$$T' = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$$

21. If a car is moving in a horizontal curve road then the period of the pendulum is,

$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}}}$$

where r is the radius of curvature of the road.
22. If a simple pendulum bob whose density is ρ, made to oscillate in a liquid of density ρ_0, then its time period of vibration in liquid will increase and is,

$$T = 2\pi\sqrt{\frac{l}{\left(1 - \frac{\rho_0}{\rho}\right)g}} \quad \text{where, } \rho > \rho_0$$

23. The percentage change in time period of simple pendulum when its length changes is,

$$\frac{\Delta T}{T}\times 100=\frac{1}{2}\left(\frac{\Delta l}{l}\right)\times 100\%$$

It means if l is increased by x% then T is increased by (x/2) %.

24. The percentage change in time period of simple pendulum when g changes but l remains constant is,

$$\frac{\Delta T}{T}\times 100=-\frac{1}{2}\left(\frac{\Delta g}{g}\right)\times 100\%$$

If g is decreased by x%, then T will be increased by (x/2) %.

25. The % change in time period of simple pendulum when both l and g change is,

$$\frac{\Delta T}{T}\times 100=-\frac{1}{2}\left(\frac{\Delta l}{l}-\frac{\Delta g}{g}\right)\times 100\%$$

26. The time period of a simple pendulum whose length L is not negligible in comparison to radius of earth R, is

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{R}+\frac{1}{L}\right)}}$$

27. The time period of a simple pendulum of infinite length is,

$$T = 2\pi\sqrt{R/g} = 86.4 \text{ minutes}$$

28. The time period of oscillation of a simple pendulum of

i) Infinite length is,

$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ minutes}$$

ii) Length of the pendulum is equal to radius of the earth its period is 59.40 min.

29. Work done in producing an angular displacement θ to the bob of simple pendulum of mass m from its mean position is,

$$W = mgh$$
$$= m\,g\,l\,(1-\cos\theta) \because h = l\,(1-\cos\theta)$$

where l is the length of simple pendulum. This work done appears as potential energy of the pendulum at height h from mean position.

Kinetic energy of pendulum at the angular displacement θ is given by

$$K_E = m\,g\,l\cos\theta.$$

30. In case of oscillations of a liquid in U–tube, time period where h is the height of undisturbed liquid in each limb of U–tube.

31. Time period of simple pendulum is also independent of mass of the bob. This is why

i) If the solid bob is replaced by a hollow sphere of same radius but different mass, time period remains unchanged.

ii) If a girl is swinging a swing and another sits with her, the time period remains unchanged.

32. Time period depends on L as T $\propto$ $\sqrt{L}$. Here L is the distance between point of suspension and centre of mass of the bob and is called' effective length'.

This is why

i) When a sitting girl on a swinging swing stands up, her centre of mass will go up and so L and hence T will decrease.

ii) If a hole is made in the bottom of a hollow sphere full of water and water comes out slowly through the hole and time period is recorded till the sphere is empty, initially and finally the centre of mass will be at the centre of the sphere. However, as water drains off the sphere, the centre of mass of the system will first move down and then will come up. Due to this L and hence T first increase, reach maximum and then decrease till become equal to its initial value.

iii) If the bob is suspended by a wire, due to change in temperature, length L will change and so the time period. If $\Delta\theta$ is the increase in temperature then as $L = L_0\,(1+\alpha\,\Delta\theta)$

$$\frac{T}{T_0}=\sqrt{\frac{L}{L_0}}=(1+\alpha\Delta\theta)^{1/2}\cong\left[1+\frac{1}{2}\alpha\Delta\theta\right]$$

By using Binomial theorem

or $\frac{T}{T_0}=1\cong\frac{1}{2}\alpha\Delta\theta$ i.e. $\frac{\Delta T}{T}\cong\frac{1}{2}\alpha\Delta\theta$

iv) If a mass M is suspended from a wire of natural length L and the wire stretches by ΔL due to elasticity its period increases.

33. Time period of a simple pendulum also depends on acceleration due to gravity and as T $\propto$ $(1/\sqrt{g})$, with increase in g, T will decrease and

vice versa. This is why?

Ex.: If a clock based on simple pendulum is taken to moon or a hill, g will decrease and so T will increase. So the clock will take longer time to complete a given number of oscillations and hence will become slow or lose time.

Laws of simple pendulum:

a) **Law of lengths:** Time period of a simple pendulum is directly proportional to square root of its length

i.e. $T \propto \sqrt{l}$ or $\frac{1}{T^2}$ is constant (at a given place)

$$\frac{l_1}{T_1^2} = \frac{l_2}{T_2^2}$$

1. If length of the pendulum is increased to P times, time period increases to $\sqrt{p}$ times.
2. If fractional change in length of a pendulum is dl/l (which is small) then fractional change in its time period is $\frac{dT}{T} = \frac{1}{2}\left(\frac{dl}{l}\right)$
3. For percentage change

$$\left(\frac{dT}{T}\right)\times 100 = \frac{1}{2}\left(\frac{dl}{l}\right)\times 100.$$

b) **Law of gravity:** Time period of simple pendulum is inversely proportional to square root of acceleration due to gravity at that place.

1. $T \propto \frac{1}{\sqrt{g}}$ or $T^2 g$ = constant (for same length)

$T_1^2 g_1 = T_2^2 g^2$.

2. If fractional change in acceleration due to gravity is $\frac{dg}{g}$ (which is very small) fractional change in time period is $\frac{dT}{T} = \frac{1}{2}\left(\frac{dg}{g}\right)$.
3. Percentage change $\frac{dT}{T}\times 100 = \frac{-1}{2}\left(\frac{dg}{g}\right)\times 100$

Here percentage change

$$\frac{dT}{T}\times 100 = \frac{-1}{2}\left(\frac{dg}{g}\right)\times 100.$$

4. For the time period of simple pendulum to remain constant at different places, $l \propto g$ (T is constant)

$$\frac{l_1}{l_2} = \frac{g_1}{g_2} = \frac{l}{g} = \text{constant.}$$

c) **Law of isochronism:** Time period of a simple pendulum is independent of amplitude, provided it is small.

d) **Law of mass:** Time period of simple pendulum is independent of size, shape, material and mass of the bob (effective length should be constant).

Special characteristics of pendulum:

1. The time period of a simple pendulum is independent of size, shape, material and mass of the bob.
2. If the time period of pendulum used in a clock increases, clock takes longer time to. complete a given number of oscillations. So, clock loses time and runs slow. If the time period of pendulum decreases, clock gains time and runs fast.
3. If a simple pendulum makes n_1 oscillation at one place and n_2 oscillation at another place in a given time (say t seconds),

then $\frac{g_1}{n_1^2} = \frac{g_2}{n_2^2}$ or $\frac{n_1}{n_2} = \sqrt{\frac{g_1}{g_2}}$

where g_1 and g_2 are accelerations due to gravity at those two places.

4. At a given place two simple pendulums of different lengths l_1 and l_2 make n_1 and n_2 oscillations respectively in a given time. Then $n_1^2 l_1 = n_2^2 l_2$.
5. Two pendulums of lengths l_1 and l_2 ($l_2 > l_1$) start vibrating from the mean position in the same phase. They will be again in the same phase at the mean position after longer pendulum completes n oscillations and shorter pendulum completes (n + 1) oscillations such that

$n_1\sqrt{l_1} = n_2\sqrt{l_2}$ where $n_2 = n$ and $n_1 = (n + 1)$

$$\frac{n}{n+1} = \sqrt{\frac{l_1}{l_2}}$$

6. **Graph:**

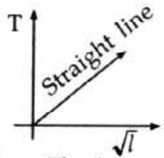


Fig. (a)

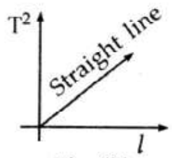


Fig. (b)

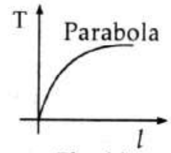


Fig. (c)

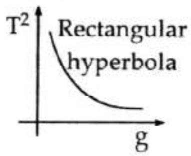


Fig. (d)

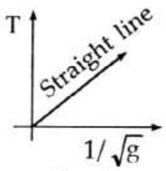


Fig. (e)

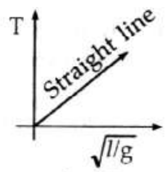


Fig. (f)

i) A graph between T and $\sqrt{l}$ is a straight line shown in figure (a) and the graph between T_2 and l is also a straight line shown in figure (b).

ii) The graph between T and l is a parabola shown in figure (c).

iii) The graph between T^2 and g is a rectangular hyperbola as shown in figure (d).

iv) The graph between T and $1/\sqrt{g}$ is a straight line shown in figure (e).

v) The graph between T and $\sqrt{l/g}$ is a straight line shown in figure (f).

7. Pendulum lose or gain period per day:

i) When the pendulum loses time per day when it is at height 'h' above the surface of earth which is,

$$T' = 86400 \times \frac{h}{R}.$$

ii) The pendulum loses time per day 'when it is at depth 'd' below the surface of earth which is,

$$T' = 86400 \times \frac{d}{2R}.$$

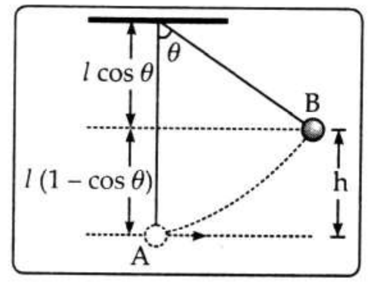
iii) When the length of pendulum increases by n %, then the pendulum loses time per day will be

$T' = 432 \times n.$

iv) When the length of pendulum decreases by n %, then the pendulum gain time per day will be

$T' = 432 \times n.$

8. If θ is the angular amplitude of pendulum, then height rises by the ball of pendulum is, $h = l(1 - \cos\theta)$ Velocity at mean position when it is released at height 'h' is,



$$v = \sqrt{2gl(1-\cos\theta)}$$

Work done in a displacement

$W = U = mgl\,(1 - \cos\theta)$

Kinetic energy at mean position

$KE = mgl\,(1 - \cos\theta)$

Tension in the string of pendulum at mean position

$$T_{max} = mg + \frac{mv^2}{l}$$

$$= 3\,mg - 2\,mg\cos\theta$$

$$= mg\,(3 - 2\cos\theta)$$

At extreme position

$T = mg\cos\theta.$

Conical pendulum:

It is an arrangement in which a point mass suspended from an elastic thread clamped at one end, and revolved in a horizontal circle with constant speed.

$\sin\theta = r/L,$

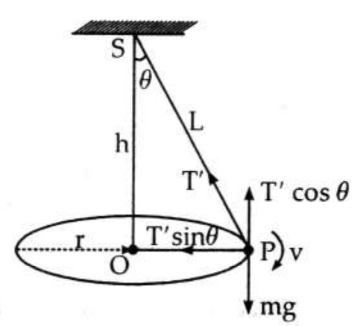
$T'\cos\theta = mg$ and

$T'\sin\theta = m\,v^2/r.$

If SO = h

then $h = L\cos\theta$

and $r = L\sin\theta$.



Time period,

$$T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

Angular frequency $\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L\cos\theta}}$

4.10 Oscillation of a loaded spring

1. Time period,

$$T = 2\pi\sqrt{m/K}$$

where m is the mass of body attached at the free end of spring and K is the force constant of spring.

2. If a body of mass m is suspended from one end of a spring whose mass is ms and spring constant K, time period of vibration is given by

$$T = 2\pi\sqrt{\frac{m+m_s}{K}}$$

3. If n identical springs are connected in series each of spring constant k, then effective spring constant k' = k/n and effective time period,

$$T' = \sqrt{n}\,T$$

where T is the time period of vibration of one spring and is given by $T = 2\pi\sqrt{m/K}$.

4. If n identical springs are connected in parallel each of spring constant or force constant k, then effective spring constant, k' = n k and effective time period of vibration,

$$T' = T/\sqrt{n} \qquad \text{where } T = 2\pi\sqrt{\frac{m}{k}}.$$

5. In a two particle system as shown in figure (a), if k is the force constant of the spring, then time period of vibration of the system is given by

$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass.

Fig. (a)

6. In a system of two particles if only one mass m_1 is oscillating, figure (b), then time period,

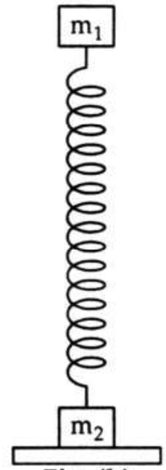


Fig. (b)

$$T = 2\pi\sqrt{\frac{m_1}{k}}$$

where $k = (m_1 + m_2)/l$.

Here l is the length of spring.

7. If a mass is suspended from a spring and made to oscillate in a liquid, its time period remains unchanged.

Spring constant (K) of a spring:

It is defined as force per unit extension or compression of the spring i.e. K = F/x.

Its SI units is N m^{-1} and its dimensional formula is $[L^0 M^1 T^{-2}]$.

1. The spring constant of the combination of two springs in series is,

$$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2} \text{ or } K = \quad K = \frac{k_1 k_2}{k_1 + k_2}$$

2. The spring constant of the combination of two springs in parallel is $K = k_1 + k_2$.
3. If the spring is cut into n equal parts then the force constant for each part of the spring is k_1 = nk. Where k is force constant of original length of the spring.
4. If the spring is cut into n unequal parts then the force constant for the longer length of the spring is $k_1 = \frac{k}{n}(n+1)$.
5. The spring constant for smaller length of the spring is $k_2 = k(n + 1)$. Where k is spring constant of original spring.

Important techniques for problems

For a complete oscillation :

Oscillation	Time required from mean position	Phase angle
$\left(\frac{1}{8}\right)^{th}$	$\frac{T}{12}$	30° or $\frac{\pi}{6}$ rad
$\left(\frac{1}{4}\right)^{th}$	$\frac{T}{4}$	90° or $\frac{\pi}{2}$ rad
$\left(\frac{3}{8}\right)^{th}$	$\frac{5T}{12}$	150° or $\frac{5\pi}{6}$ rad
$\frac{1}{2}$ or half	$\frac{T}{2}$	180° or π rad
$\left(\frac{5}{8}\right)^{th}$	$\frac{7T}{12}$	210° or $\frac{7\pi}{6}$ rad
$\left(\frac{3}{4}\right)^{th}$	$\frac{3T}{4}$	270° or $\frac{3\pi}{2}$ rad
$\left(\frac{7}{8}\right)^{th}$	$\frac{11T}{12}$	330° or $\frac{11\pi}{6}$ rad

❍❍❍

MULTIPLE CHOICE QUESTIONS

4.1 Periodic and Simple harmonic Motion

1. A particle is moving in a circle with a constant speed. Its moving is
   a) Periodic b) Oscillatory
   c) Simple harmonic d) both 'a' and 'b'
2. A particle moves in a circular path with a continuously increasing speed. The motion of its projection on a diameter of the circle is
   a) nonperiodic b) Oscillatory
   c) Simple harmonic d) UCM
3. A system executing SHM must have
   a) inertia only
   b) restoring force only
   c) both restoring force and inertia
   d) only external force
4. The distance (from initial position) of a particle executing SHM with amplitude A in air at any periodic time is always
   a) Zero b) A
   c) A/2 d) any thing from 0 to 2A
5. The displacement of a particle executing SHM with amplitude A in a period is
   a) Zero b) A
   c) 2A d) 4A
6. The distance covered by a particle executing SHM with amplitude A in a period is
   a) 4A b) 2A
   c) A d) Zero
7. The motion of a particle in SHM is of
   a) uniform speed
   b) uniform acceleration
   c) uniform velocity
   d) non uniform speed
8. Simple harmonic oscillations are
   a) one dimensional b) three dimensional
   c) two dimensional d) four dimensional
9. A body is said to be in simple linear harmonic motion (S.H.M.) about a fixed point, if
   A) it moves along a straight line
   B) it's acceleration is directed towards a fixed point
   C) the restoring force acting on it, is directly proportional to its displacement and both are oppositely directed.
   D) the direction of force and displacement varies periodically
   The correct statements is / are
   a) A and B only b) B and C only
   c) A, B and C d) A, B, C and D
10. In a simple harmonic motion, the restoring force or restoring acceleration is directly proportional to
    a) the displacement and both are in the same direction
    b) the displacement and both are oppositely directed
    c) the angular displacement and both are in the same direction
    d) the angular displacement and both are in the opposite direction
11. The motion of a body which repeats itself after equal intervals of time is
    a) non oscillatory motion
    b) non periodic motion
    c) periodic motion
    d) periodic and oscillatory motion
12. The oscillatory motion is simple harmonic motion since
    a) its path is straight line
    b) its displacement, velocity and acceleration are represented by trigonometric function sine and cosine
    c) its displacement, velocity and acceleration are represented by trigonometric function sine, cosine and tangent
    d) both 'a' and 'b'
13. In ideal simple harmonic motion, the constant' quantity is
    a) amplitude b) kinetic energy
    c) potential energy d) force
14. All oscillatory motions are necessarily periodic motions but
    a) all periodic motions are not oscillatory
    b) all periodic motions are oscillatory

c) all oscillatory motions are not periodic motion

d) all periodic motions are non harmonic

15. A particle is moving in a circle with uniform speed. Its motion is

a) periodic and simple harmonic

b) a periodic

c) periodic but not simple harmonic

d) non periodic but simple harmonic

16. In simple harmonic motion, the quantities are not constant

a) amplitude and frequency

b) potential energy and kinetic energy

c) total energy and propagation constant

d) path length.

17. The necessary and sufficient condition for S.H.M. is

a) constant period and inertial property

b) constant acceleration and elasticity property

c) proportionality between restoring force and displacement from equilibrium position in opposite direction

d) periodic and harmonic

18. Periodic motion is called harmonic motion, since

a) the expressions of displacement, velocity, and acceleration containing sine and cosine function and it oscillates with unique frequency

b) the expressions of displacement, velocity, and acceleration containing cosine and tangent function

c) it performs motion with unique frequency

d) straight line, to and fro motion

19. When the amplitude of a particle executing S.H.M. is increased slightly its period

a) increases

b) remains unchanged

c) decreases

d) may increase or decrease

20. The position at which the net force on the oscillating particle is zero, the position is

a) mean position

b) equilibrium position

c) extreme position

d) both' a' or 'b'

21. The constant of proportionality of a particle performing S.H.M. depends upon

a) elastic properties of string

b) the dimensions of displacement

c) the dimensions of force

d) the dimensions of displacement and force

22. The condition for oscillations of the body is

a) inertial property

b) applied force

c) elastic property

d) inertial and elasticity property

23. A particle performing S.H.M. with the initial phase angle is $\pi/2$. Then the particle is at

a) maximum displacement position

b) minimum. energy position

c) maximum velocity position

d) minimum acceleration

24. A package is on a platform vibrates vertical in S.H.M. with a period of 0.5 s. The package can lose contact with the platform

a) if a mass exceeds a certain limit

b) at the highest point of its motion

c) at the lowest point of its motion

d) any position of the package

25. A person is standing on a platform which executing vertical S.H.M. his weight be largest at

a) the highest position

b) atthe equilibrium position

c) at the lowest position

d) midway between highest point and mean position

26. The distance covered by a particle executing S.H.M. in one complete oscillation is

a) A b) A/2

c) 2 A d) 4A

27. Which of the following is not essential for S.H.M.?

a) inertia b) restoring force

c) material medium d) gravity

28. The quantity which does not vary periodically in SHM is

a) displacement b) acceleration

c) total energy d) velocity

29. If a hole is drilled along the diameter of the earth

and you leave a coin in the hole, then

a) it falls 'off and leaves the earth

b) it falls off and finally stops at the centre of the earth

c) it falls off but does not leave the earth

d) it falls and comes back to you

30. If– hole is bored along the diameter of the earth and a stone is dropped into the hole then the stone

a) reaches the centre of the earth and stops

b) reaches the other side of the earth and stops

c) executes simple harmonic motion about the centre of the earth

d) reaches the other side of the earth and escapes into space

31. The dimensions of force constant 'k' are

a) $[L^1M^1\ T^{-2}]$ b) $[L^0M^1T^{-2}]$

c) $[L^1M^0T^{-2}]$ d) $[L^1M^1T^{-1}]$

32. A particle performing linear S.H.M., the kinematical equations of motion are not applied to solve the problems, of S.H.M. since

a) velocity of the particle in linear S.H.M. is not uniform

b) acceleration of the particle in linear S.H.M. is not uniform

c) displacement of the particle in linear S.H.M. is proportional to force

d) force acting on the particle is not uniform

33. When a mass undergoes simple harmonic motion, there is always a constant ratio between its displacement and

a) period b) mass

c) acceleration d) velocity

34. The force and acceleration in S.H.M. changes in direction over a period

a) periodically with quarter period

b) twice in a period

c) periodically with period half of the motion

d) both 'b' and 'c'

35. Which of the following equation does not represent a simple harmonic motion?

a) $x = A \sin \omega t$

b) $x = A \cos \omega t$

c) $x = A \sin \omega t + B \cos \omega t$

d) $x = A \tan \omega t$

36. Which one of the following is a simple harmonic motion?

a) Wave moving through a string fixed at both ends.

b) Earth spinning about its own axis.

c) Ball bouncing between two rigid vertical wall.

d) Particle moving in a circle with uniform speed.

37. When the displacement of a particle in SHM from the mean position is 4 cm, the force acting on the particle is 6 N. Then the force acting on it when its displacement is 6 cm from the mean position is

a) 3 N b) 16 N

c) 8 N d) 9 N

38. Displacement 'x' of a simple harmonic oscillator varies with time, according to the differential equation $(d^2x/dt^2) + 4x = 0$. Then its time period is

a) $\pi/2$s b) π s

c) 2π s d) 4π s

39. If a body of mass 1 gm execute linear S.H.M. with a frequency of 5 oscillations per second and an amplitude 1cm, then the magnitude of force at the extreme position will be

a) zero b) 987 dyne

c) 103 dyne d) 102 dyne

40. The time period of a particle executing S.H.M. is 1 s. If the particle starts motion from the mean position, then the time during which it will be at mid way between mean and extreme position will be

a) 1/6 s b) 1/4 s

c) 1/12 s d) 3/2 s

41. The equation $(d^2x/dt^2) + a\, x = 0$ for a particle performing S.H.M. Then the time period of the motion will be

a) $2\pi\alpha$ b) $2\pi\sqrt{\alpha}$

c) $\frac{2\pi}{\alpha}$ d) $\frac{2\pi}{\sqrt{\alpha}}$

42. If a small body of mass 0.1 kg is executing S.H.M. of amplitude 1m and period 0.2 s, then the maximum force acting on it will be ($\pi^2 = 9.87$)

a) 98.7 N b) 985.96 N

c) 100.2 N d) 76.23 N

43. One kg weight is suspend to a massless spring and it has a period T. If now 4 kg weight is suspended from the same spring the new period will be
a) T　b) T/2
c) 2T　d) 4T

4.2 Projection of U.C.M. along a diameter as S.H.M.

44. Projection of U.C.M. along its any diameter is
a) linear S.H.M.
b) angular S.H.M.
c) neither linear S.H.M. nor angular S.H.M.
d) complex oscillatory motion.

45. The phase angle between the two projections of uniform circular motion on any two mutually perpendicular diameter is
a) zero　b) $\pi/2$
c) $3\pi/4$　d) π

46. When two particles performing SHM of same amplitude and frequency arriving at a point of medium simultaneously with phase difference of $\pi/2$, then the resultant path is
a) parabolic　b) elliptical
c) circle　d) straight line

4.3 Differential equation of linear S.H.M. and acceleration, velocity and displacement

47. The velocity of a particle performing S.H.M. at mean position is
a) maximum　b) half of the maximum
c) minimum　d) zero

48. The velocity of a particle performing S.H.M. at extreme position is
a) minimum
b) constant
c) maximum
d) half of the maximum velocity

49. The acceleration of a particle performing S.H.M. at extreme position is
a) minimum
b) constant
c) maximum
d) in between maximum and minimum

50. The acceleration of a particle performing S.H.M. at mean position is
a) minimum or zero　b) constant
c) maximum　d) half of the maximum

51. The particle performing S.H.M., about mean position it has
a) maximum acceleration and maximum velocity
b) minimum acceleration and maximum velocity
c) maximum acceleration and minimum velocity
d) minimum acceleration and minimum velocity

52. The graph between instantaneous velocity and acceleration of a particle performing S.H.M. is
a) parabola　b) straight line
c) ellipse　d) circle

53. The graph between instantaneous velocity and acceleration of a particle performing S.H.M. with a period of 6.28 s is
a) parabola　b) straight line
c) ellipse　d) circle

54. The graph between instantaneous velocity and displacement of a particle performing S.H.M. is
a) parabola　b) straight line
c) ellipse　d) circle

55. The graph between instantaneous velocity and displacement of a particle performing S.H.M. with period 2π sec or $\omega = 1$ is
a) parabola　b) straight line
c) ellipse　d) circle

56. The graph between instantaneous velocity and angular displacement of a particle performing S.H.M.is
a) parabola　b) straight line
c) sinusoidal　d) circle

57. The graph between instantaneous acceleration and angular displacement of a particle performing S.H.M. is
a) parabola　b) straight line
c) sinusoidal　d) circle

58. A particle performing S.H.M., its velocity when the particle moves from mean to extreme position is
a) slower initially and faster laterally
b) uniformly moves
c) faster initially and momentraily falls to zero
d) fast moves and stop at extreme position

59. The equation of a S.H.M. of amplitude 'A' and angular frequency ro in which all distances are measured from one extreme position and time is taken to be zero, at the other extreme position is

a) $x = A \sin \omega t$ b) $x = A - A \sin \omega t$

c) $x = A \cos \omega t$ d) $x = A - A \cos \omega t$

60. A particle performing S.H.M. about equilibrium position. Then the velocity of the particle is

a) slower from mean to extreme position

b) faster from mean to extreme position

c) slower initial from mean position and faster lateral and stop at the end

d) faster initial from mean position and later on falls off, suddenly

61. Acceleration amplitude of a particle performing S.H.M. is the product of

a) amplitude and velocity

b) amplitude and acceleration

c) amplitude and square of angular velocity

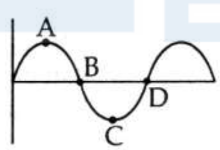
d) square of amplitude and angular velocity

62. The ratio of the maximum velocity and maximum displacement of a particle executing simple harmonic motion is equal to

a) ω b) T

c) g d) n

63. The figure gives the displacement versus time graph of a simple harmonic oscillator. The position with maximum speed directed down wards is at



a) A b) B

c) C d) D

64. The differential equation of angular S.H.M. is in the order of

a) 2 b) 0

c) 3 d) 1

65. A particle performing S.H.M. with amplitude 'A' and period T. The average values of magnitude of distance over a half period is

a) depends upon periodic time

b) depends upon amplitude of motion

c) independents upon path of the particle

d) 'b' and' c'

66. The frequency of oscillation of a particle executing SHM with amplitude A and having velocity 'v' at the mean position is

a) $\frac{v}{2\pi A}$ b) $\frac{v}{A}$

c) $\frac{A}{v}$ d) $\frac{2\pi A}{v}$

67. A particle executing linear S.H.M. performs 30 oscillations per minute. It's velocity when passing through the middle of its path is 0.157 mfs. The length of the path is

a) 0.2 m b) 0.5 m

c) 0.1 m d) 500 cm

68. A ball attached to a string travels in uniform circular motion in a horizontal circle of 50 cm radius in 1 s. Sun light shining on the ball throws its shadow on a wall. The velocity of the shadow at the centre of the path is

a) π m/s b) 0.5 π m/s

c) 0.5 m/s d) 1 m/s

69. A particle starts simple harmonic motion from the mean position. If It's amplitude is A and time period T, then at a certain instant, its speed is half of its maximum speed at this instant, the displacement is

a) $\frac{\sqrt{2}A}{3}$ b) $\frac{\sqrt{3}A}{2}$

c) $\frac{2A}{\sqrt{3}}$ d) $\frac{3A}{\sqrt{2}}$

70. The initial phase of a simple harmonic oscillator is zero. At what fraction of the period, the velocity is half of its maximum value?

a) 1 b) 1/2

c) (2/3) d) 1/6

71. If a particle is performing simple harmonic motion along X–axis with amplitude 4cm and time period 1.2 s. Then the minimum time taken by the particle to move from 2 cm to 4 cm and again back will be

a) 0.6 s b) 0.4 s

c) 0.3 s d) 0.2 s

72. A particle starts S.H.M. from mean position along straight line and comes to rest momentraily at x = 1 m and t = 1 s. If the motion is simple harmonic, then the maximum acceleration will be ($\pi^2 = 10$)

a) 1 m/s^2 b) 4 m/s^2

c) 2.5 m/s^2 d) 7 m/s^2

73. The time period of S.H.M. is 16 seconds and it starts motion from the equilibrium position, after two seconds the velocity is π m/s. Then the displacement amplitude is

a) $\sqrt{2}$ m b) $2\sqrt{2}$ m

c) $4\sqrt{2}$ m d) $8\sqrt{2}$ m

74. The time taken by a particle executing simple harmonic motion of period T, to move from the mean position to half the maximum displacement is

a) $\frac{T}{2}$s b) $\frac{T}{4}$s

c) $\frac{T}{6}$s d) $\frac{T}{12}$s

75. If a particle performing S.H.M. with a period of T s, then the time required to reach from midway to extreme position will be

a) $\frac{T}{2}$s b) $\frac{T}{4}$s

c) $\frac{T}{6}$s d) $\frac{T}{12}$s

76. The ratio of maximum acceleration to the maximum velocity of a particle performing S.H.M. is equal to

a) amplitude

b) angular velocity

c) square of amplitude

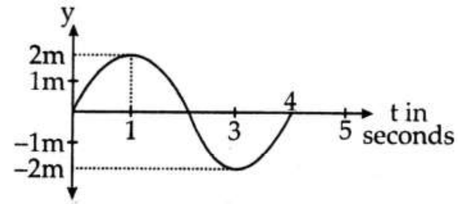
d) square of angular velocity

77. A particle executes SHM with an amplitude of 0.2 m. Its displacement when its phase is 90° is

a) 0.1 m b) 0.2 m

c) 0.4 m d) $0.1/\sqrt{2}$ m

78. The displacement – time graph of a particle executing SHM is as shown in the figure. The maximum velocity of the particle is



a) π ms^{-1} b) 2π ms^{-1}

c) 4π ms^{-1} d) 2 ms^{-1}

79. A simple harmonic oscillator has a period of 0.01 s and an amplitude of 0.2m. The magnitude of the velocity in m/s at the centre of oscillation, is

a) π b) 10π

c) 0.1π d) 40π

80. The 'particle performing S.H.M. about mean position, displacement and acceleration have initial phase difference of

a) $\pi/2$ rad b) $3\pi/2$ rad

c) π rad d) 2π rad

81. A body is oscillating vertical about its mean position with period $\sqrt{3}$ s. If its maximum and minimum heights above the surface of the ground are 2 m and 1.5 m, then the speed at midway between mean and extreme position will be

a) 5 m/s b) $\frac{\pi}{4}$ m/s

c) 10 m/s d) $\frac{\pi}{\sqrt{3}}$ m/s

82. If the maximum acceleration of a particle performing S.H.M. is numerically equal to twice the maximum velocity then the period will be

a) 1.57 s b) 3.142 s

c) 6.28 s d) 2 s

83. The velocity of the particle midway between mean and extreme position, performing S.H.M. is

a) $A\omega$ b) $\frac{\sqrt{3}}{2}A\omega$

c) $\frac{2}{\sqrt{2}}A\omega$ d) $\sqrt{3}\,A\omega$

84. The displacement x of a particle in simple harmonic motion is related to time is,

$x = 0.01 \cos[\pi t + (\pi/4)]$m.

The frequency of the motion is

a) 0.5 Hz b) 1 Hz
c) $\pi/2$Hz d) π Hz

85. A mass m attached to a light spring oscillates with a period of 2 s. If the mass is increased by 2 kg, the period increases by 1 s. Then the value of m is
a) 1 kg b) 1.6 kg
c) 2 kg d) 2.4 kg

86. A simple harmonic motion of amplitude A has a time period T. The acceleration of the oscillator when its displacement is half the amplitude is
a) $\frac{4\pi^2A}{T^2}$ b) $\frac{2\pi^2A}{T^2}$
c) $-\frac{4\pi^2A}{T^2}$ d) $-\frac{2\pi^2A}{T^2}$

87. A body oscillates harmonically with amplitude 0.05 m, at a certain instant of time. If its displacement is 0,01 m and acceleration is 1 m/s^2, then the period of oscillation will be
a) 0.1 s b) 0.2 s
c) $\pi/10$s d) $\pi/5$s

88. The maximum velocity of a particle in the above problem is
a) 0.25 m/s b) 0.5 m/s
c) 0.75 m/s d) 1 m/s

89. The displacement x is in centimeter of an oscillating particle varies with time t in seconds as $x = 2\cos[0.05\pi t + (\pi/3)]$. Then the magnitude of the maximum acceleration of the particle will be
a) $\frac{\pi}{2}$ cm/s^2 b) $\frac{\pi}{4}$ cm/s^2
c) $\frac{\pi^2}{200}$ cm/s^2 d) $\frac{\pi^2}{4}$ cm/s^2

90. A particle executes S.H.M. of amplitude 25 cm and time period 3 s. What is the minimum time required for the particle to move between two points located at 12.5 cm on either side of the mean position?
a) 0.25 s b) 0.5 s
c) 0.75 s d) 1 s

91. A body executing linear simple harmonic motion has a velocity of 3 cm/s when its displacement is 4 cm and a velocity of 4 cm/s when its displacement is 3 cm. Then amplitude of oscillation will be
a) 5 cm b) 7.5 cm
c) 10 cm d) 12.5 cm

92. The period of oscillation of the particle in the above problem is
a) 3 : 142 s b) 6.28 s
c) 12.56 s d) 9.426 s

93. The average velocity of a particle executing SHM with an amplitude A and angular frequency ω, during one oscillation is
a) ωA b) $\frac{\omega A}{2}$
c) $2\omega A/\pi$ d) zero

94. The speed of the particle on the reference circle of radius R is v. The time period of oscillation of the projection on a diameter of the circle is
a) $\frac{R}{v}$ b) $\frac{2\pi R}{v}$
c) $\frac{2\pi}{v}$ d) vR

95. A body is oscillating vertical about its mean position. If its maximum and minimum heights above the surface of the ground are 2 m and 1.5 m, then the speed at the mean position will be (g = 10 m/s^2)
a) 5 m/s b) $\sqrt{2.5}$ m/s
c) 10 m/s d) $\sqrt{10}$ m/s

96. A body of mass 0.5 kg executes S.H.M. of frequency 4 Hz. If the amplitude of S.H.M. is 10 m, then the maximum restoring force will be ($\pi^2 = 10$)
a) 0.32 N b) 3.2×10^3 N
c) 32 N d) 320 N

97. A particle executes SHM of amplitude 4 cm and period 4 s. The time taken by it to move from positive extreme position to half the amplitude is
a) 0.5 s b) 2/3 s
c) 0.75 s d) 1 s

98. A particle is executing S.H.M. with amplitude 'A' and maximum velocity V_m. The displacement at which its velocity is half of the maximum velocity

is

a) 86.6 % A b) 13.4% A

c) 70.7 % A d) 36.72 % A

99. A particle is executing S.H.M. with amplitude 'N and maximum velocity v_m. Its velocity at displacement A/2 is

a) 13.4 % vm b) 86.6 % vm

c) 37.5 % vm d) 66.67 % vm

100. The displacement of a particle executing periodic motion is given by

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin 1000\,t$$

The independent constituent of simple harmonic motion is / are

a) 1 b) 2

c) 3 d) 4

101. A body of mass 5 gm is executing S.H.M. about a point with amplitude 10 cm. If its maximum velocity is 100 cm/s, then its velocity will be 50 cm/s at a distance of

a) 5 cm b) $5.\sqrt{3}$ cm

c) $5\sqrt{2}$ cm d) $10\sqrt{2}$ cm

102. If particle executing S.H.M. has an amplitude of 6 cm, its acceleration at a distance 2 cm from mean position is 8 cm/s^2, then the maximum speed of the particle will be

a) 8 cm/s b) 12 cm/s

c) 16 cm/s d) 24 cm/s

103. A horizontal platform with an object placed on it is executing S.H.M. in the vertical direction. The amplitude of oscillation is 2.5 cm. If the object is not detached from the platform, then the least period of these oscillations will be, (g = 10 m/s^2)

a) $0.1\,\pi$ s b) π s

c) $0.5\ \pi$ s d) $2\ \pi$ s

104. The velocity and acceleration of a particle executing S.H.M. have a steady phase relationship. The acceleration leads velocity in phase by

a) π b) $-\pi/2$

c) $\pi/2$ d) $-\ \pi$

105. The maximum velocity of a particle executing S.H.M. is u. If the amplitude is doubled and the time period of oscillation decreases to (1/3) of its original value, then the maximum velocity will be

a) 18 u b) 6 u

c) 12 u d) 3 u

106. In S.H.M., the velocity of the particle at the mean position is 1 m/s and acceleration at the extremity is 2 m/s^2, the period of motion is

a) 2 s b) 0.5 s

c) 1 s d) 3.142 s

107. A particle executes S.H.M. of period 1.2 sand amplitude 8 cm. What is the time taken to travel 2.344 cm from the positive extremity?

a) 0.17 s b) 1 s

c) 0.15 s d) 0.7 s

108. A ball of mass 5 kg hanging from a spring oscillates wi th a time period of 2 π s. At any instant the ball is at equilibrium position, now the ball' is removed, then spring shortens by

a) 2 π m b) 9/2 m

c) 9.8 m d) 2 m

109. A block on a horizontal slab is moving horizontally with a simple harmonic motion of frequency two oscillations per second. If the coefficient of static friction between block and slab is 0.5, then the amplitude of the oscillation will be (if the block does not slip along the slab)

a) 3.3 cm b) 3.5 cm

c) 3.15 cm d) 7 cm

110. The velocity of a particle performing S.H.M. at any position

a) leads in phase by ($\pi/2$) than the displacement

b) lags in phase by ($\pi/2$) than the displacement

c) leads in phase by ($\pi/2$) than the acceleration

d) 'a' and 'b'

111. The velocity at mean position and acceleration at the extreme position have the phase difference of

a) $\pi/2$ rad b) π rad

c) $\pi/4$ rad d) $3\pi/4$ rad

112. A simple pendulum performs simple harmonic motion about X = 0 with an amplitude A and period T. The speed of the pendulum at midway betw,een mean and extreme position is

a) $\frac{\pi A\sqrt{3}}{T}$ b) $\frac{\pi A}{T}$

c) $\frac{\pi A\sqrt{3}}{2T}$ d) $\frac{3\pi^2 A}{T}$

113. A body is executing simple harmonic motion with an angular frequency 2 rad/s. The velocity of the body at 20 mm displacement, when the amplitude of motion is 60 mm, is

a) 40 mm/s b) 60 mm/s

c) 113 mm/s d) 120 mm/s

114. A particle executes S.H,M. with a period of 6 sand amplitude of 3 cm. Its maximum speed in cm/s, is

a) $\pi/2$ b) π

c) 2π d) 3π

115. The maximum speed of a particle executing S.H.M. is 1 m/s and its maximum acceleration is 1.57 m/s^2. Then the time period of the particle will be

a) 1/1.57 s b) 1.57 s

c) 2 s d) 4 s

116. A particle executs S.H.M. of period 1.2 s and amplitude 8 cm. Find the time it takes to travel 5 cm from the position of extremity of its oscillation

a) 1.7 s b) 0.27 s

c) 0.17 s d) 2.7 s

117. A particle is executing S.H.M. The graph of acceleration as a function of angular displacement is

a) a straight line b) a circle

c) an ellipse d) sinusoidal

118. The motion of a particle varies with time according to the relation,

$x = A(\sin \omega t + \cos \omega t)$, then

a) the motion is oscillatory but not S.H.M.

b) the motion is S.M.H. with amplitude A

c) the motion is S.M.H. with amplitude $A\sqrt{2}$

d) the motion is S.M.H. with amplitude 2 A

119. A body is executing S.H.M. when its displacement from the mean position is 4 cm and 5 cm, the corresponding velocity of the body is 10 cm/s and 8 cm/s. Then the time period of oscillation of the body is

a) 2π s b) $\pi/2$s

c) π s d) $3\pi/2$ s

120. The height of a swing changes during its motion from 0.1 m to 2.5 m. The minimum velocity of a boy who swings in this swing is

a) 5.4 m/s b) 4.95 m/s

c) 3.14 m/s d) zero

121. The displacement of a particle executing SHM is given by $x = 0.30 \sin(220t + 0.64)$ metre. Then the frequency and maximum velocity of the particle is

a) 35 Hz, 66 m/s b) 45 Hz, 66 m/s

c) 58 Hz, 113 m/s d) 35 Hz, 132 m/s

122. A particle moves according to equation $x = A\cos \pi t$. The distance covered by it in 3.5 s is

a) 3 A b) 7 A

c) A d) zero

123. The equation of S.H.M. of a particle is $a + 4\pi^2 x = 0$, where a is instantaneous linear acceleration at displacement x. Then the frequency of motion is

a) 1 Hz b) 4π Hz

c) 1/4 Hz d) 4 Hz

124. The velocity of a particle performing S.H.M. are 0.13 m/s and 0.12 m/s. When it is at 0.12 mV and 0.13 m from the mean position respectively. Then the amplitude is

a) 0.117 m b) 0.177 m

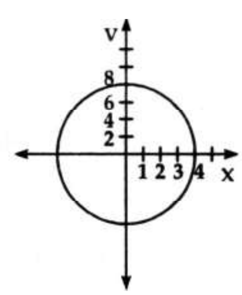
c) 11.7 cm d) 10 m

125. The ratio of instantaneous speed at centre to average speed of a particle performing S.H.M. is

a) 2π b) $2/\pi$

c) $\frac{\pi}{2}$ d) $\frac{\pi}{2}\sqrt{1-\frac{x^2}{A^2}}$

126. A graph is plotted between the instantaneous velocity of a particle performing S.H.M. and displacement. Then the v period of S.H.M. is



a) 2 s   b) π s

c) $\frac{1}{\pi}$s s   d) 0.5 s

127. The ratio of instantaneous velocity and the average speed of the particle performing S.H.M. is

a) 2π b) $\frac{2}{\pi}$

c) $\frac{\pi}{2}$ d) $\frac{\pi}{2}\sqrt{1-\frac{x^2}{A^2}}$

128. The velocities of a body executing S.H.M. are 3 cm/s and 4 cm/s when the displacements from the mean position are 4 cm and 3 cm, then the period of oscillation is

a) $\frac{1}{2\pi}$s   b) $\frac{x}{2}$s

c) $\frac{2}{x}$s   d) 2πs

129. The acceleration of a particle executing SHM at a distance of 3 cm from equilibrium position is 5 cm/s^2. Its acceleration at a distance of 2 cm from equilibrium position is

a) 10/3 cm/s^2 b) 10 cm/s^2

b) 7.5 cm/s^2 d) 4.5 cm/s^2

130. A particle of mass 0.25 kg vibrates with a period of 2 s. If its greatest displacement is 0.4 m, its maximum velocity in m/s will be .

a) $\pi/5$ b) $\pi/10$

c) $2\pi/5$ d) $\pi/2$

131. A particle executing SHM has a frequency of 10 Hz when it crosses its equilibrium position with a velocity of 2π m/s. Then the amplitude of vibration is

a) 0.1 m   b) 0.2 m

c) 0.4 m   d) 1 m

132. A particle describes SHM along a straight line with a period of 2 s and amplitude 10 cm. Its velocity when it is at a distance of 6 cm from mean position is

a) 2π cm/s   b) 4π cm/s

c) 6π cm/s   d) 8π cm/s

133. The period of a simple harmonic oscillator is 2 s. If it crosses mean position at an instant. The time after which its displacement from the mean position will be half of the amplitude is.

a) (1/8) s   b) (1/6) s

c) (1/4) s   d) (1/2) s

134. A simple harmonic oscillator has amplitude 2 A and maximum veiocity 2 V. Then its displacement at which its velocity is V and the velocity at displacement A are

a) A,V   b) $\frac{A}{2},\frac{v}{2}$

c) $\frac{A}{\sqrt{2}},\frac{v}{\sqrt{2}}$ d) $\sqrt{3}A,\sqrt{3}V$

135. Velocity of a simple harmonic oscillator in the mean position is v_0. If its amplitude is doubled, without changing its period of oscillation, its velocity in the mean position will be

a) $v_0/2$ b) $2v_0$

c) v_0 d) $v_0/4$

136. A body executing S.H.M has a period of 3 sunder one force and 4 s under another force. Then time period under the action of both the forces together in the same direction is

a) 1.2 s   b) 2.4 s

c) 3.6 s   d) 4.8 s

137. A simple harmonic oscillator has its displacement varying as $x = 15 \sin(2\pi t + \pi/4)$ metre. Then its initial displacement is

a) 15 m   b) $15/\sqrt{2}$ m

c) $15\sqrt{2}$ m   d) 7.5 m

138. A particle executes S.H.M., according to the displacement equation $x = 6 \sin(3\pi t + \pi/6)$ m. Then the magnitude of its acceleration at t = 2 s is

a) $3\pi^2$ m/s^2 b) $9\pi^2$ m/s^2

c) $18\pi^2$ m/s^2 d) $27\pi^2$ m/s^2

139. The displacement equation of a particle performing S.H.M. is $x = 10 \sin(2\pi t + \frac{\pi}{6})$ m. Then the initial displacement of a particle is

a) 5 m   b) 2.5 m

c) 0.5 m   d) 0.25 m

140. If the displacement of a particle executing S.H.M. is given by $x = 0.24 \sin(400 t + 0.5)$ m, then the

maximum velocity of the particle is

a) 24 m/s b) 48 m/s

c) 96 m/s d) 72 m/s

141. Two simple harmonic motions are given by $x_1 = A \sin [(\pi/2)t + \phi]$ and $x_2 = B \sin [(2\pi/3)t + \phi]$ Then the phase difference between these two after 1 s is

a) π b) $\pi/2$

c) $\pi/4$ d) $\pi/6$

142. The displacement of a simple harmonic oscillator is, $x = 5 \sin(\pi t/3)$ m. Then its velocity at t = 1 s, is

a) $\frac{\pi}{6}$ m/s b) $\frac{5\pi}{6}$ m/s

c) $\frac{6\pi}{6}$ m/s d) $\frac{\pi}{2}$ m/s

143. A particle oscillates, according to the equation $x = 5 \cos(\pi t/2)$ m. Then the particle moves from equilibrium position to the position of maximum displacement in time of

a) 1 s b) 2 s

c) 1/2 s d) 4 s

144. A particle executing S.H.M. is given by $x = 10 \sin(8t + \pi/3)$ m.

Its velocity when it is at a distance 6 m from the mean position is,

a) 36 m/s b) 10 m/s

c) 80 m/s d) 64 m/s

145. The displacement equation of an oscillator is given by $x = 5 \sin(2\pi t + 0.5\pi)$ m. Then its time period and initial displacement are

a) 0.5 s, 5 m b) 1 s, 2.5 m

c) 0.5 s, 2.5 m d) 1 s, 5 m

146. The acceleration of a simple harmonic oscillator is 1 m/s^2 when its displacement from mean position is 0.5 m. Then its frequency of oscillation is

a) $\sqrt{2}\pi$ Hz b) $\pi/\sqrt{2}$ Hz

c) $\frac{1}{\sqrt{2}\pi}$ Hz d) $\frac{\sqrt{2}}{\pi}$ Hz

147. The displacement of a simple harmonic oscillator is given by $x = 4 \cos(2\pi t + \pi/4)$ m. Then velocity of the oscillator at t = 2 s is

a) $4\pi\sqrt{2}$ m/s b) $\frac{4\pi}{\sqrt{2}}$ m/s

c) $\frac{\sqrt{2}\pi}{4}$ m/s d) $\frac{4\sqrt{2}}{\pi}$ m/s

148. A particle performing S.H.M. along a straight line with a time period of 12 s. Then the time taken for a displacement equal to the half of its amplitude from its mean position is

a) 0.1 s b) 1 s

c) 0.01 s d) 1.1 s

149. Two bodies of equal mass are hung from two light vertical springs. The springs are elongated by 1 cm and 4 cm. If they are made to oscillate, the ratio of time periods is

a) 1 : 1 b) 1 : 2

c) 2 : 1 d) 1 : 4

150. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy?

a) $\frac{1}{3}$ s b) $\frac{1}{4}$ s

c) $\frac{1}{6}$ s d) $\frac{1}{12}$ s

151. A particle of mass 0.3 kg is subjected to a force F = –kx with K = 15 N/m. What will be its initial acceleration if it is released from a point 20 cm away from the origin?

a) 3 m/s^2 b) 5 m/s^2

c) 10 m/s^2 d) 15 m/s^2

152. The displacement of a SHO is given by $y = 2 \sin(2\pi t + \pi/4)$ m. The ratio of its initial displacement to maximum displacement is

a) $1/\sqrt{2}$ b) $\sqrt{2}/1$

c) 2 d) zero

153. A particle is executing SHM with an amplitude of 2 m. The difference in the magnitudes of its maximum acceleration and maximum velocity is 4. The time period of its oscillation is

a) 2 s b) (7/22) s

c) (22/7) s d) (44/7) s

154. The maximum velocity and maximum acceleration of a particle executing SHM are 20 cm s^{-1} and 100 cm s^{-2}. The displacement of

the particle from the mean position when its speed is 10 cm s^{-1} is

a) 2 cm b) 2.5 cm

c) 2./3 cm d) $2\sqrt{2}$ cm

155. A particle is executing SHM with and frequency of $\frac{1}{8}$ Hz. If it starts from the mean position at time t = 0, the ratio of distances convered by it in 1st and 2nd seconds is

a) 1 b) $1/(\sqrt{2}-1)$

c) $1/(\sqrt{3}-1)$ d) $\sqrt{2}-1$

156. A particle executes SHM with a time period of 8 s. If it starts from the extreme position at time t = 0, the ratio of distances covered by it in 1st and 2nd seconds is

a) 1 b) $1/(\sqrt{2}-1)$

c) $1/(\sqrt{3}-1)$ d) $\sqrt{2}-1$

157. If the maximum speed of a SHO is π ms^{-1}. Its average speed during one oscillation is

a) $\frac{\pi}{2}$ ms^{-1} b) $\frac{\pi}{4}$ ms^{-1}

c) π ms^{-1} d) 2 ms^{-1}

158. A particle executing SHM passes through the mean position with a velocity of 4 ms^{-1}. The velocity of the particle at a point where the displacement is half of the amplitude is

a) 2 ms^{-1} b) $2\sqrt{3}$ ms^{-1}

c) $\sqrt{3}$ ms^{-1} d) 1 ms^{-1}

159. The restoring force acting on a particle executing SHM is 10 N at a displacement of 2 cm from mean position. The restoring force at a displacement of 3 cm is

a) 20 N b) 15 N

c) $\frac{20}{3}$ N d) $\frac{40}{9}$ N

160. The displacement of a SHO is given by, y = 0.4 sin $(10\pi t + \pi/3)$ cos $(10\pi t + \pi/3)$. The ratio of maximum velocity to the maximum acceleration of the particle is

a) 10π s b) $\frac{1}{10\pi}$s

c) 20π s d) $\frac{1}{20\pi}$s

161. A particle executes SHM along x–axis with an amplitude A, time period T with origin as the mean position. At t = 0, if the particle starts in the +ve x–direction from the origin the minimum time in which it will be at x = –A/2 will be

a) $\frac{T}{12}$ b) $\frac{T}{6}$

c) $\frac{T}{4}$ d) $\frac{7T}{12}$

162. If the displacement (y in m) and velocity (v in ms^{-1}) of a particle executing SHM are related by the equation $4v^2 = 16 - y^2$, then the path length of the motion and time period of oscillation respectively are

a) 4 m, 2π s b) 4 m, 4π s

c) 8 m, 2π s d) 8 m, 4π s

163. An object is attached to the bottom of a light vertical spring and set vibrating. The maximum speed of the object is 15 cm/s and the period is 628 ms. The amplitude of the motion in cm is

a) 3.0 b) 2.0

c) 1.5 d) 1.0

164. Two particles A and B execute SHM on a straight line of path length 2 m starting from the two extreme points simultaneously. If their respective time periods are 1 sand 2 s, the minimum time in which they meet is

a) $\frac{1}{4}$ s b) $\frac{1}{3}$ s

c) $\frac{1}{2}$ s d) $\frac{2}{5}$ s

165. A particle executes SHM according to the equation $y = 2 \sin 2\pi t$, where y is displacement in m and t is time in s. The distance covered by the particle in 4 s of its motion is

a) 0 b) 2 m

c) 8 m d) 32 m

166. The ratio of the magnitudes of maximum velocity to the maximum acceleration of a particle making SHM is 1 : 1. The time taken by the particle to move between the extreme points of its path is

a) 1.57 s b) 3.14 s

c) 6.28 s d) 1 s

167. The maximum velocity of a particle executing SHM is v. If its amplitude of oscillation is doubled and its time period is made half of its initial value, the maximum velocity of it will be

a) v b) 2 v

c) 4 v d) 8 v

168. Two simple harmonic oscillators of masses m, 4m have same energy. If their amplitudes are in the ratio 1 : 2, the ratio of their time periods is

a) 1 : 2 b) 1 : 4

c) 1 : 1 d) 2 : 1

169. A particle executing SHM has a velocity of 2 ms^{-1} when its displacement from mean position is 1 cm and a velocity of 1 ms^{-1} when its displacement is 2 cm. Its amplitude of oscillation is

a) 5 cm b) $\sqrt{5}$ cm

c) 3 cm d) $\sqrt{7}$ cm

4.4 Phase, amplitude, period and frequency of S.H.M.

170. The phase of a particle executing simple harmonic motion starts from mean position is (π /2), when it has

a) maximum velocity

b) minimum acceleration

c) maximum kinetic energy

d) maximum displacement

171. The amplitude of particle performing S.H.M. is

a) tensor

b) vector

c) scalar

d) depending upon magnitude

172. The phase quantity depends upon

a) time and displacement.

b) time, direction and displacement

c) displacement and direction

d) displacement

173. The particle performing S.H.M. with epoch a displacement of motion is x = A sin ω t + a.

The term (ω t + α) is known as

a) initial phase angle of motion

b) instantaneous phase of S.H.M.

c) total phase of S.H.M.

d) none of these

174. In S.H.M.,

a) epoch and phase continuously changes with time

b) epoch and phase remains constant at all times

c) epoch remains constant while phase changes continuously with time

d) phase remains constant while epoch change continuously with time

175. The phase of a particle performing S.H.M. when the particle is at a distance of amplitude from mean position is

a) π /2 b) π

c) 3 π /2 d) odd multiple of π /2

176. The period of a body performing S.H.M. of frequency 5 Hz is

a) (2 π /5)s b) (5 π)s

c) (π /5) s d) (0.2) s

177. A particle executing S.H.M. starts from midway between mean and extreme position, the phase is

a) $\frac{\pi}{3}$ rad b) $\frac{\pi}{2}$ rad

c) $\frac{\pi}{6}$ rad d) π rad

178. The displacement of a particle performing S.H.M. is x = A sin (ω t + α). The quantity α is called

a) phase constant b) epoch

c) initial phase d) all of the above

179. The vertical extension in a light spring by a weight of 1 kg suspended from the wire is 9.8 cm. Then the period of oscillation is

a) 20 π s b) 2 π s

c) 2 π /10s d) 200 π s

180. A small spherical steel ball is placed a little away from the centre of a large concave mirror of radius of curvature 2.5 m, if the ball is released, then the period of the motion will be (g = 10 m/s^2)

a) π /4 s b) π /2 s

c) π s d) 2 π s

181. In S.H.M., the phase difference between the displacement and velocity of a particle, at any

instant is

a) π b) 2π

c) $\pi/2$ d) 0

182. Two particles executes S.H.M. of the same amplitude and frequency along the same straight line. They pass one another when going in opposite directions, each time their displacement is root three by two times amplitude. The phase difference between them is

a) $\frac{2\pi}{3}$ rad b) $\frac{\pi}{2}$ rad

c) $\frac{\pi}{6}$ rad d) $\frac{\pi}{3}$ rad

183. Two particles starts vibrating together in S.H.M. starting from their mean position. If their periods are 40 s and 60 s respectively, their phase difference after 20 s from the start will be

a) 60° b) 90°

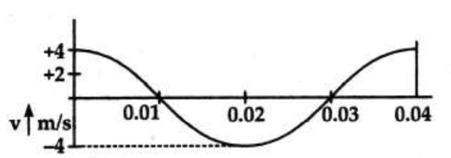
c) 30° d) 120°

184. Two particles P and Q performs S.H.M. of same amplitude 'A' and frequency n along the same straight line the resultant S.H.M. have the amplitude $\sqrt{2}$ times amplitude of individual. The initial phase difference between two particles will be nearly

a) zero b) $\frac{\pi}{6}$ rad

c) $\frac{\pi}{2}$ rad d) $\frac{3\pi}{4}$ rad

185. The velocity time diagram of a harmonic oscillator is shown in the figure, then the frequency of oscillation is



a) 25 Hz b) 12.25 Hz

c) 50 Hz d) 33.3 Hz

186. The epoch of a simple harmonic motion represented by $x = \sqrt{3}\sin\omega t + \cos\omega t$ m is

a) 30° b) 40.3°

c) 60° d) 25°

187. If the mass of an oscillator is numerically equal to its force constant, then the frequency of oscillation will be

a) π b) 2π

c) $1/\pi$ d) $1/2\pi$

188. If a particle performing S.H.M. from the mean position of amplitude 'A' and period 'T', then the lapse of fraction of period in (7/8)th oscillation will be

a) $\frac{T}{12}$ b) $\frac{T}{6}$

c) $\frac{11}{12}T$ d) $\frac{7}{8}T$

189. Two particles are executing S.H.M. according to the equations

$x_1 = 6\sin(10\pi t + \pi/3)$ and $x_2 = 5\cos(8\pi t + \pi/4)$

Then the phase difference between the first and second particle at t = 0.5 s will be

a) $\frac{7\pi}{12}$ b) $\frac{13\pi}{12}$

c) $\frac{25\pi}{12}$ d) $\frac{\pi}{12}$

190. Two particles are executing S.H.M. according to the equations

$x_1 = 6\sin(10\pi t + \pi/3)$ and $x_2 = 6\sin(8\pi t + \pi/4)$

Then the phase difference between the first and second particle at t = 0.5 s will be

a) $\frac{7\pi}{12}$ b) $\frac{13\pi}{12}$

c) $\frac{25\pi}{12}$ d) $\frac{\pi}{12}$

191. A body moves between two points A and B which are at a distance a and b from the point 0 in the same straight line OAB. Then the amplitude of oscillation is

a) $\frac{a+b}{2}$ b) $\sqrt{\frac{a^2+b^2}{2}}$

c) $\frac{b-a}{2}$ d) $b - a$

192. If α and β denote the maximum velocity and maximum acceleration of a simple harmonic oscillator, then its amplitude of vibration is

a) $\alpha^2\beta$ b) $\beta^2\alpha$

c) α^2/β d) β^2/α

193. Two particles execute SHM of the same time period along the same straight lines. They cross each other at the mean position while going in opposite directions. Their phase difference is

a) $\pi/2$ b) π

c) $3\pi/2$ d) 2π

194. The displacement of a particle making S.H.M is given by x = 6 cos $\left(100t+\frac{\pi}{4}\right)$ m then the frequency is

a) 1.592 Hz b) 15.92 Hz

c) 159.2 Hz d) 1592 Hz

195. The equation of S.H.M with amplitude 4m and time period 1/2 s with initial phase $\pi/3$ is x =

a) $4\sin(2\pi t+\pi/3)$ m

b) $4\sin(4\pi t+\pi/3)$ m

c) $4\sin(\pi t+\pi/3)$ m

d) $4\sin(2\pi t+\pi/6)$ m

196. Two S.H.Ms are represented by $x_1 = A\sin(\omega t - kx)$ and $x_2 = B\cos(\omega t - kx)$. Then phase difference between these two is

a) $\pi/2$ b) $\pi/4$

c) $\pi/6$ d) $3\pi/4$

4.5 Kinetic energy, potential energy and total energy of a particle performing S.H.M.

197. The kinetic energy of a particle performing S.H.M. at mean position is

a) minimum b) maximum

c) constant d) $1/2\ m\ \omega^2x^2$

198. The kinetic energy of a particle performing S.H.M. at extreme position is

a) minimum or zero b) maximum

c) constant d) $1/2\ m\ \omega^2A^2$

199. The potential energy of a particle performing S.H.M. at extreme position is

a) minimum b) maximum

c) remains constant d) $1/2\ m\omega^2x^2$

200. The potential energy of a particle performing S.H.M. at mean position is

a) minimum

b) in between minimum and maximum

c) maximum

d) $1/2\ m\ \omega^2x^2$

201. The graph between kinetic energy and displacement of a particle performing S.H.M. is

a) parabola b) straight line

c) ellipse d) circle

202. The graph between potential energy and displacement of a particle performing S.H.M. is

a) parabola b) straight line

c) ellipse d) circle

203. The graph between total energy and displacement of a particle performing S.H.M. is

a) parabola b) straight line

c) ellipse d) circle

204. The total energy of a particle performing S.H.M. is proportional to the

a) square of frequency

b) square of mass

c) square of amplitude

d) both 'a' and 'c'

205. The potential energy of a particle performing S.H.M. at mean position is

a) $\frac{1}{2}\ m\omega^2A^2$ b) 0

c) $\frac{1}{2}\ m\omega^2x^2$ d) $\frac{1}{2}\ m\omega^2(A^2-x^2)$

206. If a particle executes an undamped S.H.M.of period T, then the period with which the potential energy fluctuate is

a) T b) 2T

c) T/2 d) ∞

207. If a particle executes an undamped S.H.M. of period T, then the period with which the kinetic energy fluctuate is

a) T b) 0

c) T/2 d) ∞

208. If a particle executes an undamped S.H.M. of period T, then the period with which the total

energy fluctuate is

a) T b) 2T

c) T/2 d) ∞

209. The particle performing S.H.M. along a straight line about the mean position with an amplitude 'A'. Then the maximum potential energy at a distance A from the extreme position is .

a) A b) 0

c) A/2 d) (3/2) A

210. A particle oscillates simple harmonically, with a frequency, 'n' its kinetic energy varies periodically. Then the frequency of the kinetic energy is

a) 4 π b) n

c) 2 π d) $\pi/2$

211. A particle oscill ates simple harmonically, with a frequency 'n' its kinetic energy varies periodically. The frequency of the potential energy is

a) 4 π b) π

c) 2 π d) $\pi/2$

212. A particle executing simple harmonic motion, the kinetic energy E is, $E = K_0 \cos^2 \omega t$. The maximum value of potential energy is

a) K_0 b) $K_0/2$

c) zero d) not obtained

213. The potential energy of a particle with displacement X is u (x). The motion is simple harmonic, when (k is force constant)

a) $u = \frac{-kx^2}{2}$ b) $u = \frac{1}{2} k x^2$

c) u = k d) u = k x

214. Kinetic energy of the particle performing S.H.M. is

a) harmonic motion and oscillatory

b) periodic motion but not oscillatory

c) oscillatory motion but not periodic

d) periodic and oscillatory motion

215. Potential energy of the particle performing S.H.M. is

a) harmonic motion and oscillatory

b) periodic motion but not oscillatory.

c) oscillatory motion but not periodic

d) periodic and oscillatory motion

216. Kinetic energy of a particle performing S.H.M.

a) leads the potential energy by a phase of π

b) leads the potential energy by a phase of $\pi/2$

c) lags the potential energy by a phase of $\pi/2$

d) lags the potential energy by a phase of π

217. In a period of kinetic energy, the number of times kinetic energy and potential energy are equal in magnitude is

a) single times b) thrice

c) twice d) four times

218. In a period of oscillating particle, the number of times kinetic energy and potential energy are equal in magnitude is

a) single b) thrice

c) twice d) four times

219. The total energy of a particle executing SHM is E. Its kinetic energy is K at the mean position. Then always

a) $E - K = 0$ b) $E - K < 0$

c) $E - K > 0$ d) $E - K \geq 0$

220. A particle of mass m executes SHM with amplitude A and frequency n. The average energy in one time period is

a) $\pi^2 mn^2A^2$ b) $2\pi^2 mn^2A^2$

c) $\pi^2 m^2n^2A^2$ d) $2\pi mn^2A^2$

221. A particle of mass m executes SHM with amplitude A and frequency n. The average kinetic energy during its motion from mean to extreme positions is

a) $\pi^2 mn^2A^2$ b) $2\pi^2 mn^2A^2$

c) $\frac{\pi^2 mn^2 A^2}{2}$ d) zero

222. A particle executing S.H.M. has total energy of 20 J. If the potential energy of the particle at midway between mean and extreme position is 5 J, then the average kinetic energy will be

a) 10 J b) 5 J

c) 15 J d) 7.5 J

223. A particle executing S.H.M. has total energy of 20 J. If the potential energy of the particle midway between mean and extreme position is 5 J, then the average potential energy will be

a) 10 J b) 5 J

c) 15 J d) 7.5 J

224. A particle executing S.H.M. has total energy of 20 J. If the potential energy of the particle midway between mean and extreme position is

5 J, then the average total energy will be

a) 10 J b) 20 J

c) 15 J d) 7.5 J

225. Total energy of a particle executing S.H.M. is proportional to

a) square of amplitude

b) square root of angular velocity

c) amplitude

d) angular velocity

226. In S.H.M. the displacement of a particle is half of the amplitude, the share of the potential energy and kinetic energy are

a) 50% and 50% respectively

b) 59% and 41% respectively

c) 25% and 75% respectively

d) 33% and 66% respectively

227. The ratio of potential energy to kinetic energy of a particle performing S.H.M. at midway between mean and extreme position is

a) 3 : 1 b) 1 : 3

c) 6 : 9 d) 4 : 3

228. A particle of mass 100 g is executing S.H.M. with amplitude of 10 cm. When the particle passes through the mean position at t = 0. Its kinetic energy is 8 mJ. The equation of simple harmonic motion, if initial phase is zero is

a) x = 0.1 sin 4t b) x = 0.1 cos 4t

c) x = 0.1 sin 2t d) x = 0.1 cos 2t

229. A particle is performing simple harmonic motion with an amplitude of 4 cm. At what displacement from the equilibrium position is its energy half potential and half kinetic?

a) 1 cm b) $\sqrt{2}$ cm

c) 3 cm d) $2\sqrt{2}$ cm

230. A particle is' performing linear S.H.M. at a point A, on the path, its potential energy is three times kinetic energy. At another point B on the same path, its kinetic energy is 3 times the potential energy. The ratio of the potential energy at A to its potential energy at B is

a) 9 : 1 b) 1 : 9

c) 1 : 3 d) 3 : 1

231. A particle is performing linear S.H.M. at a point A, on the path, its potential energy is three times kinetic energy. At another point B on the same path, its kinetic energy is 3 times the potential energy. The ratio of the kinetic energy at A to its kinetic energy at B is

a) 9 : 1 b) 1 : 9

c) 1 : 3 d) 3 : 1

232. A body of mass 25 gm performs linear S.H.M. The force constant of the motion is 400 dyne/cm. When the body is at a distance of 10 cm from the equilibrium position has velocity of 40 cm/s, then the total energy of the body will be

a) 40×10^4 J b) 4×10^4 erg

c) 2×10^4 erg d) 2 × 105 J

233. A particle is executing linear simple harmonic motion of amplitude 'A'. The fraction of the total energy is the kinetic when the displacement is half of the amplitude is

a) $\frac{1}{4}$ b) $\frac{1}{2\sqrt{2}}$

c) $\frac{1}{2}$ d) $\frac{3}{4}$

234. The kinetic energy of a body executing S.H.M. is 16 J, when it is at its mean position. If the amplitude of oscillation is 25 cm and the mass of the body is 5.12 kg, then the period of oscillation will be

a) $\pi/5$s b) 2π s

c) 20π s d) 5π s

235. The ratio of kinetic energy and potential energy, of a particle executing S.H.M. when it is at a distance of (1/n)th of its amplitude is

a) n^2 b) $n^2 + 1$

c) $1/n^2$ d) $n^2 - 1$

236. An object of mass 0.2 kg performs simple harmonic oscillation along the x–axis with a frequency of $(25/\pi)$ Hz. If the kinetic energy 0.5 J and potential energy 0.4 J of an object at a distance of 0.04 m from the mean position, then the amplitude of oscillation will be

a) 4 cm b) 8 cm

c) 2 cm d) 6 cm

237. The kinetic energy and potential energy of a particle executing simple harmonic motion will be equal, when displacement of the particle is

a) $\frac{A}{2}$ b) $A\sqrt{2}$

c) $\frac{A}{\sqrt{2}}$ d) $\frac{A\sqrt{2}}{3}$

238. The potential energy of a particle executing S.H.M. is 2.5 J, when its displacement is half of amplitude. Then total energy of the particle will be

a) 7.5 J   b) 10 J

c) 5 J   d) 2.5 J

239. A particle of mass 0.5 kg executes SHM. If its period of oscillation is π seconds and total energy is 0.04 J, then the amplitude of oscillation will be

a) 40 cm   b) 20 cm

c) 15 cm   d) 10 cm

240. The total mechanical energy of a particle performing S.H.M. is 150 J with amplitude 1 m and force constant 200 N/m. Then minimum P.E. is,

a) 50 J   b) 100 J

c) 150 J   d) zero

241. The total energy of simple harmonic oscillator is E and amplitude is 'A'.1f the kinetic energy is 3E/4 without altering the amplitude. The displacement of the oscillator is

a) A/2   b) 3A/4

b) 2 A   d) 4 A

242. The potential energy of a particle performing S.H.M in its rest position is 15 J. If the average kinetic energy is 5 J, then the total energy of the particle peforming S.H.M will be

a) 5 J   b) 10 J

c) 20 J   d) 15 J

243. If a particle of mass 1 kg is moving in a simple harmonic motion with a time period of 1/60 sand an amplitude of 2×10^{-2} m, then the maximum force acting on a particle will be

a) 2.88 π^2N   b) 28.8 π^2N

c) 288 π^2N   d) 2880 π^2N

244. An oscillating mass spring system has a mechanical energy 1 J when it has an amplitude 0.1 m and maximum speed of 1 m/s. Then force constant of the spring is

a) 100 N/m   b) 200 N/m

c) 300 N/m   d) 50 N/m

245. The K.E. of a particle in S.H.M 0.2 s after passing the mean position is half of its total energy. Then its period of oscillation is

a) 0.8 s   b) 1.6 s

c) 0.4 s   d) 1.2 s

246. The PE of a simple harmonic oscillator 0.1 s after crossing the mean position is 1/4 of its total energy. Then the period of its oscillation is

a) 0.2 s   b) 0.3 s

c) 0.9 s   d) 1.2 s

247. If the amplitude of the particle in S.H.M. is doubled. The quantity doubled is

a) time period   b) kinetic energy

c) total energy   d) maximum velocity

248. The potential energy as a function of displacement for an oscillator u = $8x^2$ joule. The magnitude of the force on the oscillator of mass 0.5 kg placed at x = 0.2 m is

a) 1.6 N   b) 3.2 N

c) 0.8 N   d) 4.8 N

249. The displacement of a particle of mass 1 kg in S.H.M is $x = 2\sin(\pi t + \phi)$ m. Then variation of its PE in joule is

a) $U = 4\pi^2 \sin^2(\pi t + \phi)$

b) $U = 2\pi^2 \sin^2(\pi t + \phi)$

c) $U = 2\pi^2 \cos^2(\pi t + \phi)$

d) $U = 4\pi^2 \cos^2(\pi t + \phi)$

250. A particle of mass 1 kg executing S.H.M is given by $y = 2\cos(10t + \pi/3)$ in SI units. Its maximum potential energy is

a) 2 J   b) 20 J

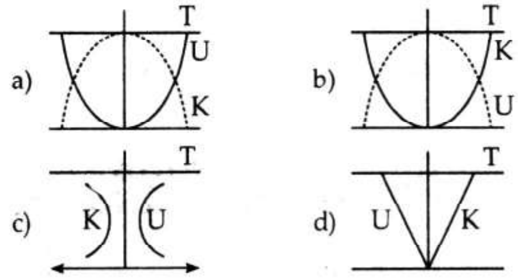
c) 200 J   d) 100 J

251. If A is amplitude of a particle in SHM, its displacement from the mean position when its kinetic energy is thrice that to its potential energy

a) A   b) A/4

c) A/2   d) 3A/4

252. For a simple harmonic oscillator moving along a straight line, the graphs which represent the variation of kinetic energy K, potential energy U and total energy T with displacement are



253. A particle is executing SHM. At a displacement y_1 its potential energy is U_1 and at a displacement y_2 its potential energy is U_2. The potential energy of the particle at displacement $(y_1 + y_2)$ is

a) $U_1 - U_2$ b) $\sqrt{U_1^2 + U_2^2}$

c) $U1 - U_2$ d) $U_1 + U_2 + 2\sqrt{U_1U_2}$

254. A particle executing SHM has a total energy E. Another particle of twice the mass of the first, particle executes SHM with 4 times the frequency arid twice the amplitude of the first, Its total energy is

a) E/2 b) 32 E

c) 64 E d) 128 E

255. The potential energy of a particle executing SHM in its rest position is 15 J. The average kinetic energy of the particle during one oscillation is 5 J. The total energy of the particle is

a) 10 J b) 25 J

c) 15 J d) 5 J

256. The displacement of a particle of mass 100 g executing SHm is given by the equation y = 0.2 $\cos\left(100t + \frac{\pi}{4}\right)$ meter. Its total energy is

a) 1 J b) 10 J

c) 2 J d) 20 J

4.6 Composition of two S.H.M.'s

257. The displacement of two particles executing S.H.M. are represented by

$x_1 = A\sin(\omega t + \phi)$ and $x_2 = A\cos\omega t$.

The phase difference between the velocities of these particle is

a) $(\pi + \phi)$ b) $(\phi - \pi/2)$

c) ϕ^2 d) $(\phi + \pi)^2$

258. Two simple harmonic motions are given by

$y_1 = A_1\sin\omega t$ and $y_2 = A_2\sin(\omega t + \phi)$

are acting on the particles in the same direction. The resultant motion is S.H.M., its amplitude is

a) $\sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

b) $\sqrt{A_1^2 + A_2^2 - 2A_1A_2\cos\phi}$

c) $A_1^2 + A_2^2 - 2A_1A_2\cos\phi$

d) $A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

259. The displacement of a particle performing S.H.M. is, x = 3 sin (314 t) + 4 cos (314 t), where x and tare in CGS units. Then the amplitude of the S.H.M. is,

a) 7 cm b) 3 cm

c) 4 cm d) 5 cm

260. The displacement of a particle performing S.H.M. is x = 3 sin (314 t) + 4 cos (314 t). Then the initial phase angle is,

a) $\tan^{-1}(7)$ b) $\tan^{-1}(4)$

c) $\tan^{-1}(3)$ d) $\tan^{-1}(4/3)$

261. If two particles performing S.H.M. are given by

$X_1 = 10\sin[3\pi t + (\pi/4)]$ and

$x_2 = 5[\sin 3\pi t + \sqrt{3}\cos 3\pi t)$.

Then the ratio of their amplitude is,

a) 2 : 1 b) 1 : 1

c) 1 : 2 d) $1 : \sqrt{2}$

262. The displacement of a harmonic oscillator is given by $x = \alpha\sin\omega t + \beta\cos\omega t$. Then the amplitude of the oscillator is,

a) α b) β

c) $\alpha + \beta$ d) $(\alpha^2 + \beta^2)^{1/2}$

263. If the two particles performing S.H.M. with same amplitude and initial phase angle, then the initial phase angle of resultant motion depends on

a) initial phase angle only

b) initial phase angle and amplitude of individual

c) amplitude of individual only

d) neither amplitude nor initial phase angle

264. If the two particles performing S.H.M. with different initial phase angle and amplitude, then

the initial phase angle of resultant motion depends on

a) initial phase angle only

b) initial phase angle and amplitude of individual

c) amplitude of individual only

d) neither amplitude nor initial phase angle

265. If the two particles performs S.H.M. of same initial phase angle but different amplitudes of individuals, then the resultant motion initial phase angle depends on

a) initial phase angle only

b) initial phase angle and amplitude of individual

c) amplitude of individual only

d) neither amplitude nor initial phase angle

266. The two particles performing S.H.M. have a phase difference of π. If the amplitude of second particle is three times the amplitude of first particle, then the amplitude of resultant motion will be

a) increase by 41.4 %

b) increase by 200 %

c) decrease by 300 %

d) decrease by 441.4 %

267. The displacement of a particle performing S.H.M. is $x = 0.1 \sin(5\pi t) + 0.4 \cos(5\pi t)$ m.

Then the speed of the particle at 0.1 s will be

a) $82\ \pi$ b) $2\ \pi$

c) π d) 0

268. The displacement of a particle performing S.H.M. is $x = 0.1 \sin(5\pi t) + 0.4 \cos(5\pi t)$ m. The acceleration of the particle at 0.1 s is

a) 10π m/s b) $-2\pi^2$ m/s²

c) $-2.5\ \pi^2$m/s² d) $4\pi^2$ m/s²

269. The displacement of a particle executing S.H.M is given by $x = 0.34 \sin(300t + 0.68)$ m. Then its frequency is

a) $\frac{300}{\pi}$ Hz b) $\frac{300}{2\pi}$ Hz

c) $\frac{150}{2\pi}$ Hz d) 300 Hz

270. The minimum phase difference between the two simple harmonic oscillations

$x_1 = (1/2) \sin \omega t + (\sqrt{3}/2) \cos \omega t$ and

$x_2 = (\sqrt{3}/2) \sin \omega t + (1/2) \cos \omega t$ is

a) $\pi/6$ b) $\pi/3$

c) $\pi/4$ d) $\pi/12$

271. If two simple harmonic motions are given as $x_1 = A \sin \omega t$ and $x_2 = (A/2) \sin \omega t + (A/2) \cos \omega t$, then the ratio of their amplitudes is

a) 1 b) 2

c) $1/\sqrt{2}$ d) $\sqrt{2}$

272. The displacement of a particle in S.H.M is given by $x = \sin \omega t + \sqrt{3} \cos \omega$ m. Then at t = 0 its displacement in metres is

a) 0.5 b) $\sqrt{3}$

c) 1 d) $1/\sqrt{2}$

273. The displacement of a particle in SHM is given by $x = \sin \omega t + \cos \omega t$, Then its amplitude and initial displacement are

a) 1, $\sqrt{2}$ units b) $\sqrt{2}$,1 units

c) $\sqrt{2}$, $\sqrt{2}$ units d) 1, 1 units

4.7 Simple pendulum

274. The period of oscillation of a simple pendulum at a given place with acceleration due to gravity 'g' depends on

a) the length of the pendulum '*l*' only

b) both '*l*' and 'm' of the pendulum

c) mass of the bob of the pendulum only

d) '*l*' and 's'

275. Second's pendulum is taken from the surface of the earth to the surface of the moon in order to maintain the period constant

a) the length of the pendulum has to be decreased

b) the length of the pendulum has to be increased

c) the amplitude of the pendulum has to be increased

d) the amplitude of the pendulum has to be decreased

276. The period of a simple pendulum is doubled when

a) its length is doubled

b) the mass of the bob is doubled

c) its length is four times

d) the amplitude of the pendulum is double and the length of the pendulum also double

277. Ideal simple pendulum can not exist practically because

a) heavy point mass and in extensible string is not possible

b) perfectly rigid support is not possible

c) the bob is not symmetric

d) 'a' and 'b'

278. The practical simple pendulum is a

a) heavy metallic sphere suspended from a light weight slightly extensible string

b) suspended from a rigid support

c) 'a' and 'b'

d) light weight in extensible string from rigid support

279. The motion of a simple pendulum when it oscillates with small amplitude is

a) angular S.H.M. only

b) angular and linear S.H.M.

c) linear S.H.M. only

d) linear complex oscillatory motion

280. The period of simple pendulum

a) increases with decrease in length

b) increases with increase in length

c) increases with keeping length constant

d) decrease with decrease in acceleration due to gravity

281. The equation $T = 2\pi\sqrt{l/g}$ is valid only when

a) the length of the pendulum is less than the radius the earth

b) the length of the pendulum is greater than or equal to radius by the earth

c) the length of pendulum is independent of the radius of the earth

d) length of the pendulum is equal to radius and the earth

282. A spring watch would be taken to the moon, it

a) runs faster b) runs slower

c) does not work d) shows no change

283. The graph between the length and square of the period of a simple pendulum is a

a) circle b) parabola

c) straight line d) hyperbola

284. Time period of simple pendulum increase with

a) increase in value of g

b) increase in amplitude

c) decrease in value of g

d) decrease in amplitude

285. If the pendulum is taken inside the mines or on the hills. The time period

a) increases

b) decreases and increases respectively

c) decreases

d) increases and decreases respectively

286. The pendulum made from the wood is replaced by metallic pendulum of same dimensions

a) T increases

b) T remains unchanged

c) T decreases

d) first increases and then decreases

287. Time period of simple pendulum of wire is independent

a) mass of the bob and amplitude of oscillation

b) amplitude of oscillation

c) temperature of the bob

d) acceleration due to gravity

288. Time period of simple pendulum suspended from a metallic wire

a) increases with increase in temperature of the wire

b) increases with decreasing temperature of the wire

c) decreases with decreasing in temperature

d) can not be predicted

289. The time period of a simple pendulum gets increased, if it is made to oscillates in a liquid whose density is

a) less than the density of the material of the bob

b) greater than the density of the material of the bob

c) equal to the density of the material of the bob

d) less than or equal to density of the material at the bob

290. A simple pendulum is suspended in a lift and lift is moving upwards with an acceleration, time period of simple pendulum

a) increases

b) decreases

c) remains constant

d) first increases and then decreases

291. If a simple pendulum is suspended in a lift and lift is moving downwards an acceleration, time period of simple pendulum

a) increases

b) decreases

c) remains constant

d) first increases and then decreases

292. If the rope of the lift breaks, time period of suspended pendulum in a lift is

a) zero

b) abrupt by increases

c) abrupt by decreases

d) infinite

293. If the lift moves up or down with uniform speed, then the time period of pendulum

a) increases

b) there will be no effect

c) decreases

d) can not be predicted

294. When a girl sitting on an oscillating swing, stands up then the period of oscillation will

a) increase b) decrease

c) remains same

d) first increase then decrease

295. The period of oscillation of a simple pendulum of constant length at earths surface is T. Its period inside a mine is

a) greater than T b) less than T

c) equal to T d) cannot be compared

296. If a simple pendulum has charge q and is oscillating in a uniform electric field E in the direction of acceleration due to gravity, then its frequency of oscillations

a) increases

b) decreases

c) first increases and later on it remains constant

d) independent of the effect of electric field

297. The time period of spring pendulum is

A) independent of mass suspended

B) depends upon the mass suspended q independent of acceleration due to gravity

D) depends upon the acceleration due to gravity

The correct statement are

a) A and C b) B and C

c) B and C d) C only

298. Tension in the string is minimum when the pendulum is at

a) mean position

b) extreme position

c) mid way between mean and extreme position

d) any position

299. The period of a simple pendulum is doubled, when

a) its length is doubled

b) the mass of the bob is doubled

c) its length is made four times

d) the mass of the bob and the length of the pendulum are doubled

300. The period of oscillation of a simple pendulum of constant length at earth surface is T. Its period inside a mine is

a) greater than T b) less than T

c) Equal to T d) cannot be compared

301. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will

a) remains unchanged b) increase

c) decrease d) become erratic

302. The work done by a string of simple pendulum during one complete oscillation is equal to

a) total energy of pendulum

b) kinetic energy of pendulum

c) potential energy of pendulum

d) zero

303. A simple pendulum consisting of a ball of mass m tied to a spring of length l is made to swing on a circular arc of angle θ in a vertical plane. At the end of this arc, another ball of mass m is placed at rest. The momentum transferred to this ball at rest by the swinging ball is

a) zero b) $m\theta\sqrt{\frac{g}{l}}$

c) $\frac{m\theta}{l}\sqrt{\frac{l}{g}}$ d) $\frac{m}{l}2\pi\sqrt{\frac{l}{g}}$

304. Identify correct statement among the following

a) the greater the mass of a pendulum bob, the shorter is its frequency of oscillation

b) a simple pendulum with a bob of mass m swings with an angular amplitude of 40° its angular amplitude is 20°, the tension in the string earlier is less than the tension in the string later

c) as the length of a simple pendulum is increased, the maximum veloci ty of its bob during its oscillation will also increases.

d) the fractional change in the time period of a pendulum on changing the temperature is independent of the length of the pendulum

305. A train moving at a uniform speed has a simple pendulum hung from the ceiling of one of the compartments. The train suddenly experiences a uniform acceleration, during this interval, the time period of the pendulum

a) remains same b) decreases
c) increases d) becomes infinite

306. If the length of a simple pendulum is increased, its maximum velocity during oscillation

a) decreases b) increases
c) remains same d) is zero

307. The metal bob of a simple pendulum is positively charged and has certain time period in air. If bob oscillates above a negatively charged metal plate. Its time period now

a) remains same b) increases
c) decreases d) none

308. Clock A is based on oscillations of a spring and clock B is based on pendulum motion. Both clocks run at the same rate on earth. On the surface of moon

a) A will run faster than B
b) B will run faster than A
c) both will run at same rates
d) can not be predicated

309. The steel bob of a simple pendulum is replaced by a wooden bob of same volume. Then its time period

a) increases b) decreases
c) does not change d) is zero

310. A simple pendulum has a bob which is a hallow sphere full of sand and oscillates with certain period. If all that sand is drained out through a hole at its bottom, then its period

a) increases b) decreases
c) remains same d) is zero

311. A simple pendulum is suspended inside a trolly which is sliding down on an inclined plane, which is frictionless. It oscillates, mean position of that pendulum will be

a) exactly vertical
b) exactly horizontal
c) normal to the inclined surface
d) parallel to the inclined surface

312. The phase of simple harmonic oscillator is

a) expressed in time
b) expressed in angle
c) expressed in distance
d) 'a' and 'b'

313. A pendulum clock shows correct time at 0^{o}C. On a summer day

a) it runs slow and gains time
b) it runs fast and loses time
c) it runs slow and loses time
d) it runs fast and gains time

314. The seconds pendulum is taken from earth to moon, to keep the time period constant

a) length of the seconds pendulum should be reduced
b) the length of the seconds pendulum should be raised
c) the amplitude should increase
d) the amplitude should decrease

315. Time period of a pendulum in the vacuum as compared to that in atmosphere,

a) does not oscillate b) decreases
c) increases d) remains same

316. The tension in the string of a simple pendulum is

a) constant
b) maximum in the extreme position
c) zero in the mean position
d) minimum at extreme position

317. A pendulum clock on the earth is taken on the Jupiter, then it will run

a) slow
b) fast
c) at the same rate

d) first slower then faster and remains same rate

318. A girl is swinging a swing in a standing position. If the girl seat and swings, the period will be
a) shorter
b) longer
c) same
d) first shorter and then longer

319. If the earth stops rotating then the time period of a simple pendulum at the equator would be
a) decrease b) increase
c) remains unchanged d) becomes zero

320. For infinitely long length of the pendulum, the time period of the pendulum is
a) 48 min b) 84.6 min
c) 64 min d) 32 min.

321. Two pendulums oscillates with a constant phase difference of 90° and same amplitude. The maximum velocity of one pendulum is v, then the maximum velocity of the other pendulum will be
a) 2 v b) v
c) v $\sqrt{2}$ d) $\sqrt{2v}$

322. Two clocks one working with the principle of oscillating pendulum, the other with that of oscillating spring are taken to the moon, then
a) both show correct time on the moon
b) first one only shows correct time
c) second one only shows correct time
d) both show wrong time

323. The angular displacement of a simple pendulum is increased from 2° to 4°. Its frequency of oscillation
a) remains same b) is doubled
c) is halved d) is quadrupled

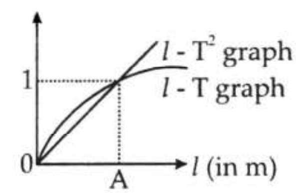
324. A boy is swinging in a cradle in sitting position. If he stands up the time period of the cradle
a) increases b) decreases
c) remains constant d) becomes zero

325. A simple pendulum oscillates with a time period T in a stationary cart on a horizontal road. If the cart moves with acceleration a = g on the road, the new time period of the pendulum will be
a) T b) $\frac{T}{\sqrt{2}}$
c) $\sqrt{2}$ T d) $\frac{T}{2^{1/4}}$

326. The time period of an oscillating spring of mass 630 g and spring constant 100 N/m with a load of 1 kg is
a) 0.2 π s b) 0.21 π s
c) 0.22 π s d) 0.02 π s

327. The length of second's pendulum on the moon is nearly
a) 1/5 m b) 1/6 m
c) 1/3 m d) 1 m

328. $l-T$ and $l-T^2$ graphs of a simple pendulum on earth are as shown in the figure. The x–coordinate (OA) of point of intersection of the graphs is nearly equal to (on the earth g = 9.8 ms^{-2})



a) 20 cm b) 25 cm
c) 50 cm d) 100 cm

329. Time period of simple pendulum of length l at a place, where acceleration due to gravity is g and period T. Then period of simple pendulum of the same length at a place where the acceleration due to gravity 1.02 g will be
a) T b) 1.02 T
c) 0.99 T d) 1.01 T

330. The time period of seconds pendulum is 2 s. The hallow spherical bob of a mass of 50 g. This is now replaced by another solid bob of same radius but of different mass of 100 g. The new period will be
a) 4 s b) 1 s
c) 2 s d) 8 s

331. A body has a time period (4/5) s under the action of one force and (3/5) s under the action of another force. Then time period when both the forces are acting in the same direction simultaneously will be
a) 4/5 s b) 5/4 s
c) 1 s d) 12/25 s

332. A body has a time period (4/5) s under the action

of one force and (3/5) s under the action of another force. Then time period when both the forces are acting in the opposite direction simultaneously will be

a) 4/5 s b) 5/4 s

c) 1 s d) 12/2 $\sqrt{7}$ s

333. The length of simple pendulum is increased by 1%, the time period will

a) increase by 0.5 %

b) increase by 1 %

c) increase by 2 %

d) decrease by 0.5 %

334. If a simple pendulum is taken to a place where g decreases by 2%, then the time period

a) increase by 2 % b) increase by 1 %

c) increase by 4 % d) decrease by 1 %

335. The length of second's pendulum is increased by 21%, then time period will increase by

a) 1.5 % b) 21 %

c) 5.5 % d) 10 %

336. If a pendulum clock keeps correct time at sea level is taken to a place 1 km above sea level the clock will approximately

a) gains 13.5 s/day b) loses 13.5 s/day

c) loses 7 s/day d) gains 7 s/day

337. If a pendulum clock, keeps correct time at sea level is taken to a place 16 km below sea level, the clock will approximately

a) gains 13.5 s/day b) loses 13.5 s/day

c) loses 108 s/day d) gains 7 s/day

338. If the length of a second's pendulum is decreased by 0.1 %, the pendulum gain or lose per day will be

a) gains 43.2 s b) loses 43.2 s

c) loses 7 s d) loses 13.5 s

339. A pendulum clock, keeps correct time at sea level, when taken to a place lkm below sea level, the clock loses or gain per day will be

a) gains 13.5 s b) loses 13.5 s

c) loses 7 s d) gains 7 s

340. Two pendulums have time period T and (5T/4)s. They start S.H.M. at the same time from the mean position, the phase difference between them after the bigger pendulum has completed one oscillation is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$

c) $\frac{3\pi}{3}$ d) $\frac{\pi}{6}$

341. The length of seconds pendulum is increased by 2%. How many seconds it will lose per day?

a) 3927 s b) 392.7 s

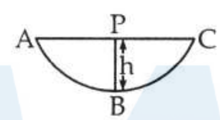
c) 37.38 s d) 864 s

342. If the length of simple pendulum is increased by 44%, then the percentage increase in its time period will be

a) 44 % b) $\sqrt{44}$ %

c) 20 % d) 10 %

343. If a simple pendulum with a bob of mass m oscillates from A to C and back to A such that PB is h. Then the velocity of the bob as it passes from B will be



a) zero b) 2gh

c) mgh d) $\sqrt{2gh}$

344. A simple pendulum suspended from the ceiling of a lift car has a period of oscillation T when the lift car is stationary. Then the period of pendulum when the lift car is ascending vertical with an acceleration a = 3g will be

a) T/2 b) 2T

c) T/3 d) 3T

345. A simple pendulum perform simple harmonic motion about mean position with an amplitude A and time period T. Then the speed of the pendulum at half of the amplitude will be

a) $\pi A\sqrt{3}/T$ b) $\pi A/T$

c) $\pi A\sqrt{3}\ /\ 2T$ d) $3\ \pi^2 A/T$

346. If the length of seconds pendulum is decreased by 1%, the gain or lose time per day by the pendulum will be

a) gain 432 s b) lose 443 s

c) gain 4.4 s d) lose 0.44 s

347. If the length of the simple pendulum is equal to the radius of the earth then period of pendulum is nearly

a) 84 min b) 60 min
c) 75 min d) 48 min

348. A pendulum suspended from the ceiling of the train has a time period of two seconds when the train is at rest, then the time period of the pendulum, if the train accelerates 10 m/s^2 will be ($g = 10\ m/s^2$)

a) 2s b) $2\sqrt{2}$ s
c) $2/\sqrt{2}$ s d) $2^{3/4}$

349. What would be increase in period of seconds pendulum when its length is increased by 21% ?

a) 0.1 s b) 0.2 s
c) 0.4 s d) 0.15 s

350. If the period of a simple pendulum increases by 50% when the length of the pendulum is increased by 0.6 m, then the initial length of the pendulum will be ($g = 9.8\ m/s^2$)

a) 0.48 m b) 0.5 m
c) 0.994 m d) 0.3 m

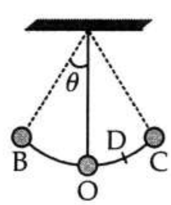
351. The time period of a seconds pendulum on a planet, whose mass is twice that of the earth and radius is equal to the diameter of the earth, will be

a) 2.828 s b) 3.742 s
c) 1.414 s d) 2 s

352. Two simple pendulum of lengths 1 m and 9 m respectively are both given a displacement in the same direction at the same instant. They will again be in phase after the shorter pendulum has completed number of oscillations are

a) 1/4 b) 3/2
c) 4/3 d) 2

353. A simrple pendulum is moving simple harmonically ith a period of 6 s between two extermem position B and C abour a point O. If the distanc ebetween B and C is 10 cm, then the time taken by the pendulum to move from position Cto position D exactly midway between O and C will be



a) 0.5 s b) 1.0 s
c) 1.5 s d) 3 s

354. The time period of a simple pendulum of infinitely long length is (R is radius of the earth)

a) $T = 2\pi\sqrt{\frac{R}{g}}$ b) $T = 2\pi\sqrt{\frac{R}{2g}}$
c) $T = 2\pi\sqrt{\frac{2R}{g}}$ d) T = infinite

355. The length of the pendulum is increased by 90 cm and period is doubled then it's original length will be

a) 30 cm b) 60 cm
c) 90 cm d) 45 cm

356. The frequency of the seconds pendulum is

a) 1 Hz b) 2 Hz
c) 0.5 Hz d) 3 Hz

357. If a simple pendulum of length 1 has maximum angular displacement 8m then the maximum kinetic energy of the bob of mass 'm' is

a) $ml/2g$ b) $mg / 2l$
c) $mgl\ (1 - \cos\theta)$ d) $mgl \sin(\theta/2)$

358. A pendulum clock keeps correct time at 20°C and coefficient of linear expansion of pendulum is $12 \times 10^{-6}/^{\circ}C$. If the room temperature increases to 40°C, how many seconds the clock lose or gain per day?

a) 10.36 s b) 20.6 s
c) 5 s d) 20 minutes

359. A simple pendulum is suspended from the roof of a train. If the train is moving with an acceleration 49 cm/s^2. Then the angle of inclination of the string about the vertical will be

a) 20° b) zero
c) 30° d) 3°

360. Two pendulums start oscillating in the same direction at the same time. Their time periods are 2 s and 1.5 s respectively, then the phase difference between them when the smaller pendulum has completed one vibration, will be

a) $\frac{\pi}{4}$ b) $\frac{2\pi}{3}$

c) $\frac{\pi}{2}$ d) $\frac{3\pi}{2}$

361. If the length of the seconds pendulum is increases by 4%, how many seconds it will lose per day?

a) 3456 s b) 864 s
c) 432 s d) 1728 s

362. A simple pendulum of length 1 m, and energy 0.2 J, oscillates with an amplitude 4 cm. When its length is doubled then the energy of oscillation will be

a) 0.04 J b) 0.1 J
c) 0.02 J d) 0.8 J

363. Two simple pendulums of equal length cross each other at mean position. Then their phase difference is

a) π radian b) $\frac{3\pi}{2}$ radian
c) $\frac{\pi}{2}$ radian d) 0 radian

364. Length of second's pendulum is decreased by 1%, then the gain or loss in time per day will be

a) gain 4.40 s b) gain 432 s
c) loses 4.40 s d) loses 44 s

365. A body of mass 1 kg is suspended from a weightless spring having force constant 600N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity of 3 m/s and gets embedded in it. Then the frequency of oscillation is,

a) $\frac{\pi}{10}$ Hz b) $\frac{10}{\pi}$ Hz
c) $\frac{1}{2\pi}$ Hz d) 3.142 Hz

366. If acceleration due to gravity on the moon is one–sixth that on the earth, then the length and time period of seconds pendulum on the surface of moon are

a) 6 m, 1.5 s b) 1/6 m, 2 s
c) 1/6 m, 0.5 s d) 6 m, 2 s

367. A simple pendulum has time period T. The percentage change in its time period if its amplitude is decreases by 5% is

a) 6 %
b) 3 %
c) 1.5 %
d) it will remain unchanged

368. The pendulum bob has a speed of 3 m/s at its lowest position and the length of pendulum is 0.5 m. The speed of the bob when the length of the pendulum makes an angle of 60° to the vertical will be ($g = 10\ m/s^2$)

a) $\frac{3}{2}$ m/s b) 2 m/s
c) $\frac{1}{2}$ m/s d) 3 m/s

369. A man measures the period of a simple pendulum inside a stationary lift and found to be T s. If the lift accelerates with an acceleration of g/4, then the period of the pendulum will be

a) T b) $\frac{T}{4}$
c) $\frac{2T}{\sqrt{5}}$ d) $2T\sqrt{5}$

370. The periodic time of a simple pendulum of length 1 m and amplitude 2 cm is 5 s. If the amplitude is made 4 cm, its periodic time in seconds will be

a) 2.5 b) 5
c) 10 d) $5\sqrt{2}$

371. If a second's pendulum is placed in a space laboratory orbiting around the earth at a height 3 R, where R is the radius of the earth, then the time period of the pendulum will be

a) zero b) 2/3 s
c) 4 s d) infinite

372. The bob of a simple pendulum of mass in and total energy E will have maximum linear momentum equal to

a) $\frac{\sqrt{2E}}{m}$ b) $\sqrt{2mE}$
c) 2mE d) mE^2

373. The time period of a simple pendulum is two seconds. If its length is increased to 4 times, then the period will become

a) 16 s b) 12 s
c) 8 s d) 4 s

374. Two simple pendulums of length 5 m and 20 m respectively are given small linear displacement in one direction at the same time. They will again be in phase when the pendulum of shorter length has completed the number of oscillations are

a) 5 b) 1
c) 2 d) 3

375. If a simple pendulums oscillate with an amplitude of 50 mm and time period of 2 s, then its maximum velocity is

a) 0.10 m/s b) 0.157 m/s
c) 0.8 m/s d) 0.16 m/s

376. Two simple pendulum oscillates simultaneously from mean positions. The first pendulum made 20 oscillations in certain time in which the second pendulum made 25 oscillations. Then the ratio of the lengths of the pendulum is

a) 5 : 4 b) 4 : 5
c) 25 : 16 d) 16 : 25

377. A simple pendulum with a metallic bob has a time period T. If the bob is immersed in a non viscous liquid and its time period is measured as 4 T, then the ratio of densities of metal bob to that of the liquid will be

a) 15 : 16 b) 16 : 15
c) 1 : 16 d) 16 : 1

378. A pendulum which gives correct time 2 s on the ground. It is moved to an altitude 640 m above the earth. If radius of the earth is 6400 km, how much time will the pendulum lose or gain in one day?

a) loses 8640 s b) gains 4.32 s
c) loses 8.64 s d) gains 8.64 s

379. A simple pendulum oscillates with an amplitude of 2×10^{-2} m. If the force acting on it at extreme position is 4 N. Then the force on particle at a point midway mean and extreme position is

a) 4 N b) 3 N
c) 2 N d) 1 N

380. A simple pendulum of length 121 cm takes 11/3 minutes to execute 100 oscillations. Then the time that another simple pendulum of length 81 cm takes to execute the same number of oscillations is

a) 2 minutes b) 3 minutes
c) 6 minutes d) 9 minutes

381. A simple pendulum of length 11 has frequency 1/4 Hz and another simple pendulum of length 12 has frequency 1/3 Hz. Then time period of pendulum of length $(l_1 - l_2)$ is

a) 5 s b) 1 s
c) $\sqrt{7}$ s d) $\sqrt{12}$ s

382. A simple pendulum of length l_1 has frequency 1/4 Hz and another simple pendulum of length 12 has frequency 1/3 Hz. Then time period of pendulum of length $(l_1 + (l_2)$ is

a) 5 s b) 1 s
c) $\sqrt{7}$ s d) $\sqrt{12}$ s

383. Two identical simple pendulums oscillate with amplitudes 6 cm and 2 cm respectively. Then ratio of their energies of oscillation are

a) 3 : 1 b) 1 : 3
c) 9 : 1 d) 1 : 9

384. The length of the seconds pendulum changes from 1m to 1.21 m. Then the percentage of the change in its period will be

a) 20 % b) 21 %
c) 10 % d) 11 %

385. A second's pendulum has a hollow spherical bob of mass 25×10^{-3} kg. It is replaced by another solid bob of same radius but of mass 50×10^{-3} kg. Then the new time period will be

a) 6s b) 4s
c) 3s d) 2s

386. The length of a seconds pendulum is 100 cm. To have a period half of this value the length is to reduced by

a) 25 cm b) 75 cm
c) 50 cm d) 100 cm

387. The mass and diameter of a planet are half those of the earth. The time period of a simple pendulum on this planet, if it is a seconds pendulum on the earth is

a) 4 s b) $2\sqrt{2}$ s
c) $\sqrt{2}$ s d) $1/\sqrt{2}$ s

388. If length of seconds pendulum is increased by 0.3%, its new time period would be

a) 2.0018 s b) 2.0009 s
c) 2.06 s d) 2.003 s

389. If the length of seconds pendulum on the earth is 1 m. Then the length of second pendulum on the moon will be

a) 1 m b) (1/6) m
c) 6 m d) 36 m

390. If the period of pendulum loses 13.5 slday at 1 km above the surface of earth, then the period of pendulum 1 km below the surface of the earth loses will be

a) 7 s b) 13.5 s
c) 14 s d) 27

391. The ratio of the height of the mountain and the depth of a mine, if a pendulum swings with the same period at the top of the mountain and at the bottom of the mine is

a) 1 : 1 b) 1 : 2
c) 2 : 1 d) 4 : 1

392. A simple pendulum of length 25 cm has a time period of 1 s and another pendulum of length 100cm has a time period of 2 s. The time period of a simple pendulum of length 125 cm will be

a) 3 s b) 2.5 s
c) $\sqrt{5}$ s d) $\sqrt{7}$ s

393. A simple pendulum oscillating in air has a time period of $\sqrt{3}$ s. Now the bob of the pendulum is completely immersed in a non–viscous liquid whose density is equal to $\frac{1}{4}$ th that of the material of the bob. The new time period of simple pendulum will be

a) $\sqrt{2}$ s b) $\sqrt{3}$ s
c) 2 s d) $\sqrt{\frac{7}{2}}$

394. Two simple pendulums are drawn to same side from their mean positions and are released simultaneously. Their time periods are 2 s and 3s. The phase difference between the pendulums when the longer pendulum completes one oscillation is

a) $\frac{\pi}{3}$ rad b) $\frac{\pi}{2}$ rad
c) $\frac{2\pi}{3}$ rad d) π rad

395. Two simple pendulums of lengths 100 cm and 144 cm start oscillating together at time t = 0. The minimum time after which they swing together is (take g = π^2 ms^{-2})

a) 2 s b) 4 s
c) 10 s d) 12 s

396. The pendulum of wall clock is a second's pendulum. The number of oscillations made by it in one day is

a) 3600 b) 21600
c) 43200 d) 86400

397. During summer, the time period of the pendulum of a clock changes to 2.02 s from 2 s. The clock runs

a) slow by 14.4 minutes per day
b) fast by 14.4 minutes per day
c) slow by 28.8 minutes per day
d) fast by 28.8 minutes per day

398. The length of a second's pendulum at the equator is L. The length of second's pendulum at the North pole will be

a) < L b) > L
c) L d) L/2

399. A second's pendulum on the earth is taken to the moon, where acceleration due to gravity is nearly 1/6 th that on the earth. If it is to act as second's pendulum there too on the moon, its length should be

a) decreased by 17% b) increased by 17%
c) decreased by 83% d) increased by 600%

400. The time period of a simple pendulum is T. When the length is increased by 10 cm, its period is Tt. When the length is decreased by 10 cm, its period is T_2. Then, the relation between T, T_1 and T_2 is

a) $\frac{2}{T^2}=\frac{1}{T_1^2}+\frac{1}{T_2^2}$ b) $\frac{2}{T^2}=\frac{1}{T_1^2}-\frac{1}{T_2^2}$
c) $2T^2 = T_1^2 + T_2^2$ d) $2T^2 = T_1^2 - T_2^2$

401. The time period of a simple pendulum at the surface of the earth is T. If it is taken to a height of 640 km, its time period will

a) increase by 20% b) increase by 10%
c) decrease by 10% d) decrease by 5%

4.8 Spring pendulum and Harmonic Oscillator

402. A spring of force constant K is cut into three

equal parts. The force constant of each part is

a) K b) 3 K
c) 2 K d) 4 K

403. A spring has a natural length of 50 cm and a force constant 2×10^3 N/m. A body of mass 10 kg is suspended from it and the spring is stretched.

If the body is pulled down further stretching the string to a length of 58 cm and released, it executes simple harmonic motion. The force on the body when it is at its lowermost position of oscillation will be ($g = 10$ m/s^2)

a) 160 N b) 40 N
c) 60 N d) 80 N

404. The restoring force on the body of the above problem due to the spring when it is at its upper most position of its oscillation is

a) 160 N b) 40 N
c) 60 N d) 80 N

405. A spring when stretched upto 2 cm has potential energy u. If it is stretched upto 10 cm, potential energy would be

a) u/25 b) u/5
c) 5 u d) 25 u

406. For a body of mass m attached to the spring the spring factor is given by

a) m/ω^2 b) $m\,\omega^2$
c) $m^2\,\omega$ d) $m^2\,\omega^2$

407. If a single spring has length 'L' and force constant k. It is cut into two springs of lengths l_1 and l_2 such that $l_1 = nl_2$ (where n is an integer). The force constant of spring of length l_1 is

a) k (n + 1) b) (k/n)(1 + n)
c) k d) k/(n + 1)

408. A simple spring has length l and force constant k. It is cut into two springs of lengths l_1 and l_2 such that $l_1 = nl_2$ where n is an integer.

The force constant of the spring of length l_2 is

a) k (1 + n) b) k/n (1 + n)
c) k d) k/(n + 1)

409. A spring of force constant k is cut into two pieces whose lengths are in the ratio 1 : 2. The force constant of the longer piece?

a) k/2 b) 3k/2
c) 2k d) 3k

410. 1 kg weight is suspended to a weightless spring and it has time period T. If now 4 kg weight is suspended from the same spring the time period will be

a) T b) T/2
c) 2T d) 4T

411. A spring of force constant k is cut into four equal parts. The force constant of each part will be

a) k b) 4 k
c) k/4 d) 16 k

412. When a mass m is attached to a spring. The spring normally extends by 0.2 m. Now the spring is slightly extend and released then its time period will be ($g = 9.8$ m/s^2)

a) $\frac{1}{\pi}$ s b) $\frac{2\pi}{7}$ s
c) 7 s d) $\frac{1}{7}$ s

413. The force constant of an ideal spring is 200 newton per meter. It is loaded with a mass of $200/\pi^2$ kg at the lower end the time period of its vibration is

a) $2\pi^2$s b) 2 s
c) 1 s d) π^2s

414. A loaded spring vibrates with a period T. The spring is divided into four equal parts and the same load is suspended from one end of these parts. The new period is

a) T b) 2 T
c) T/2 d) T/4

415. The springs of spring constant k, 2 k, 4 k, 8 k. ... are connected in series. A mass m kg is attached to the lower end of the last spring and the system is allowed to vibrate. What is the time period of oscillations? (m = 40 gm, k = 2 N/cm)

a) 1 s b) 0.1256 s
c) 0.5 s d) 3.142 s

416. A spring having a spring constant k is loaded with a mass 'm'. The spring is cut into two equal parts and one of these is loaded again with the same mass. The new spring constant is

a) k/2 b) k
c) 2 k d) k^2

417. A spring has a force constant k and a mass m is suspended from it the spring is cut in two equal halves and the same mass is suspended from one

of the halves. If the frequency of oscillation in the first case is n, then the frequency in the second case will be

a) 2n b) $n/\sqrt{2}$

c) n d) $\sqrt{2}$ n

418. A loaded vertical spring executes simple harmonic oscillations with period of 4 s. The difference between kinetic energy and potential energy of this system oscillates with a period of

a) 8 s b) 1 s

c) 2 s d) 4 s

419. When a mass m is suspended from two light springs separately, the periods of vertical oscillations are T_1 and T_2. If the same mass is suspended from the two springs connected in series then the time period of oscillation is

a) T_1 b) T_2

c) $T_1 + T_2$ d) $\sqrt{T_1^2 + T_2^2}$

420. When a mass m is suspended from two light springs separately, the periods of vertical oscillations are T_1 and T_2. If the same mass is suspended from the two springs connected in parallel then the time period of oscillation is

a) T_1 b) T_2

c) $\sqrt{T_1^2 + T_2^2}$ d) $\frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$

421. Two particles P and Q describe S.H.M. of same amplitude 'A' and frequency n along the same straight line. The resultant displacement amplitude of the two S.H.M.s is $(A/\sqrt{2})$. The initial phase difference between the two particles is nearly

a) π b) $\frac{\pi}{2}$

c) $\frac{\pi}{4}$ d) $\frac{3\pi}{4}$

422. The time periods for vertical harmonic oscillations of the three systems of single, two series springs and two parallel springs, each springs have a force constant 'k'. The ratio of time periods of the system is

a) $\frac{1}{\sqrt{2}} : \sqrt{2} : 1$ b) $\sqrt{2} : 1 : \frac{1}{\sqrt{2}}$

c) $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$ d) $1 : \frac{1}{\sqrt{2}} : \sqrt{2}$

423. Two spring of each force constant k is connected in series and parallel combination is provided with mass m. The ratio of time periods of these two combination is

a) 2 : 1 b) $1 : \sqrt{2}$

c) $\sqrt{2}$: 0.5 d) $\sqrt{2}$: 1

424. Two springs of equal lengths and equal cross sectional area are made of materials whose young's moduli are in the ratio of 3 : 2. They are suspended and loaded with the same mass. When stretched and released, they will oscillate with time period in the ratio of

a) $\sqrt{2} : \sqrt{3}$ b) $\sqrt{3} : \sqrt{2}$

c) 3 : 2 d) 9 : 4

425. A mass on the end of a spring undergoes simple harmonic motion with a frequency of 0.5 Hz. If the attached mass is reduced to one quarter of its value, then the new frequency in Hz is

a) 0.25 b) 1.0

c) 2.0 d) 4.5

426. A spring is stretched by 0.20 m, when a mass of 0.50 kg is suspended. When a mass of 0.25 kg is suspended, then its period of oscillation will be ($g = 10$ m/s^2)

a) 0.328 s b) 0.628 s

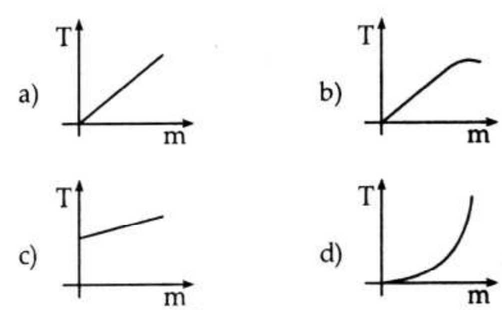
c) 0.137 s d) 1.00 s

427. If a watch with a wound spring watch is taken to the moon, it

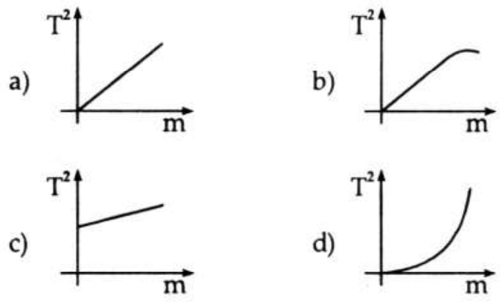
a) runs faster b) runs slower

c) does not work d) shows no change

428. The graph between period of oscillation (T) and mass attached (m) to a spring will be



429. If a graph is plotted for the square of period (T^2) and mass attached to a spring, its shape will be



430. A body of mass 1 kg suspended by a light vertical spring of spring constant 100 N/m executes SHM. Its angular frequency and frequency are

a) $\frac{5}{\pi}$ rad/s, 100 Hz b) 10 rad/s, $\frac{5}{\pi}$ Hz

c) 10 rad/s, $\frac{10}{\pi}$ Hz d) 10 rad/s, $\frac{5}{\pi}$ Hz

431. A body of mass 2 kg hanging from a light vertical spring executes SHM with an amplitude 0.05 m and time period $\frac{5}{\pi}$ s. The maximum force exerted by the spring on the body is ($g = 10\ ms^{-2}$)

a) 10 N b) 20 N

c) 25 N d) 30 N

432. When a block of mass 2 kg is suspended from a light spring, it elongates by 0.05 m. It is pulled further from its equilibrium position by 0.1 m and released. Its time period of oscillation is

a) $\frac{\sqrt{2}\pi}{7}$ s b) $\frac{2\pi}{7}$ s

c) $\frac{\pi}{7}$ s d) $\frac{\pi\sqrt{3}}{7}$ s

433. A block of mass 2 kg executes SHM on a smooth horizontal plane under the action of restoring force of a spring. If the amplitude and time period of oscillation are 0.05 m and $\pi/5$ second, the maximum force exerted by the spring on the block is

a) 5 N b) 10 N

c) 100 N d) 19.6 N

434. A body hanging from a light vertical spring executes SHM with a frequency of 1 Hz. If the body is brought to rest and is removed, the length of the spring decreases by

a) 0.1 m b) 0.15 m

c) 0.2 m d) 0.25 m

435. The time period of vertical oscillations of a light spring with a load of mass M is T. If another load of mass m is attached to the load M, the time period of oscillation is doubled. The value of m is

a) M b) 2 M

c) 3 M d) 4 M

Questions given in MHT-CET

436. The maximum velocity and maximum acceleration of a body moving in a simple harmonic oscillator are 2 m/s and 4 m/s^2. The angular velocity is

a) 1 rad/s b) 2 rad/s

c) 4 rad/s d) 5 rad/s

437. A particle executes simple harmonic motion of amplitude A. At what distance from the mean position is its kinetic energy equal to its potential energy?

a) 0.51 A b) 0.61 A

c) 0.71 A d) 0.81 A

438. Two springs of constants k_1 and k_2 equal maximum velocities, when executing simple harmonic motion. The ratio of their amplitudes (masses are equal) will be

a) $\frac{k_1}{k_2}$ b) $\frac{k_2}{k_1}$

c) $\left(\frac{k_1}{k_2}\right)^{1/2}$ d) $\left(\frac{k_2}{k_1}\right)^{1/2}$

439. The force constant of a wire is K and that of another wire of the same material is 2 K. When both the wires are stretched, then work done is

a) $W_2 = 0.5\ W_1$ b) $W_2 = W_1$

c) $W_2 = 2W_1$ d) $W_2 = 2W_1^2$

440. If a simple pendulum oscillates with an amplitude of 50 mm and time period of 2 s, then its maximum velocity is

a) 0.10 m/s b) 0.16 m/s

c) 0.25 m/s d) 0.5 m/s

441. The time period of a bar magnet in uniform ...

magnetic field $\vec{B}$ is 'T'. It is cut into two halves, by cutting it parallel to its length then the time period of each part in same field is

a) $\sqrt{2}$ T b) T

c) 2 T d) none of these

442. W denotes to the total energy of a particle in linear S.H.M. At a point, equidistant from the mean position and extremity of the path of the particle

a) K. E. of the particle will be w/2 and P. E. will also be w/2

b) K. E. of the particle will be w/4 and P. E. will be 3 w/4

c) K. E. of the particle will be 3w/4 and P. E. will be w/4

d) K. E. of the particle will be w/8 and P. E. will be 7 w/8

443. In S.H.M. path length is 4 cm and maximum acceleration is $2\pi^2$ cm/s^2. Time period of motion is

a) 2 s b) $\sqrt{2}$ s

c) 4 s d) 1/2 s

444. Time period of pendulum is 6.28 sec and amplitude of oscillation is 3 cm. Maximum acceleration of pendulum is

a) 8 cm/s^2 b) 0.3 cm/s^2

c) 3 cm/s^2 d) 58.2 cm/s^2

445. The length of a pendulum is halved. Its energy will

a) decreased to half

b) increased to 2 times

c) decreased to one fourth

d) increased to 4 times

446. Dimensions of force constant are

a) $[L^0 M^1 T^{-2}]$ b) $[L^0 M^{-1} T^{-2}]$

c) $[L^1 M^0 T^{-2}]$ d) $[L M T^{-2}]$

447. If velocity of body is half the maximum velocity. Then what is the distance from the mean position?

a) 2 A b) $\frac{\sqrt{3}}{2} \times A$

c) A d) $\frac{A}{2}$

448. Particle moves from extreme position to mean position, its

a) kinetic energy increases, potential energy decreases

b) kinetic energy decreases, potential increases

c) both remains constant

d) potential energy becomes zero and kinetic energy remains constant

449. Spring is pulled down by 2 cm. What is amplitude of motion?

a) 0 cm b) 6 cm

c) 2 cm d) 4 cm

450. If the length of the simple pendulum is increased by 44%, then what is the change in time period of pendulum?

a) 22 % b) 20 %

c) 33 % d) 44 %

450. A magnet of magnetic moment M oscillates in magnetic field B with time period 2 sec. If now the magnet is cut into two half pieces parallel to the axis, then what is new time period if only one part oscillate in field?

a) 2 s b) $2\sqrt{2}$ s

c) $\frac{1}{\sqrt{2}}$ s d) 2.4 s

452. When a particle performing uniform circular motion of radius 10 cm undergoes the SHM, what will be its amplitude?

a) 10 cm b) 5 cm

c) 2.5 cm d) 20 cm

453. The kinetic energy of a particle executing S.H.M. is 16 J when it is at its mean position. If the mass of the particle is 0.32 kg, then what is the maximum velocity of the particle?

a) 5 m/s b) 15 m/s

c) 10 m/s d) 20 m/s

454. The value of displacement of particle performing SHM, when kinetic energy is (3/4)lh of its total energy is

a) $x = \pm \frac{A}{2}$ b) $x = \pm \frac{A}{4}$

c) $x = \pm \frac{\sqrt{3}A}{2}$ d) $x = \pm \frac{A}{\sqrt{2}}$

455. The frequency of wave is 0.002 Hz. Its time period is

a) 100 s b) 500 s
c) 5000 s d) 50 s

456. The equation of displacement of particle performing SHM is X = 0.25 sin (200 t). The maximum velocity is
a) 100 m/s b) 200 m/s
c) 50 m/s d) 150 m/s

457. The maximum velocity for particle in SHM is 0.16 m/s and maximum acceleration is 0.64 m/s^2. The amplitude is
a) 4×10^{-2} m b) 4×10^{-1} m
c) 4×10 m d) 4×100 m

458. The pendulum is acting as second pendulum on earth. Its time on a planet, whose mass and diameter are twice that of earth, is
a) $\sqrt{2}$ s b) 2 s
c) $2\sqrt{2}$ s d) $1/\sqrt{2}$

459. A simple pendulum of length 1 and mass (bob) m is suspended vertically. The string makes an angle 9 with the vertical. The restoring force acting on the pendulum, is
a) mg tan θ b) mg sin θ
c) – mg sin θ d) – mg cos θ

560. The displacement of a particle performing simple harmonic motion is given by, $x = 8 \sin \omega t + 6 \cos \omega t$, where distance is in cm and time is in second. The amplitude of motion is
a) 10 cm b) 14 cm
c) 2 cm d) 3.5 cm

561. If a bar magnet of magnetic moment M is kept in a uniform magnetic field B, its time period of oscillation is T. The another magnet of same length and breadth is kept in a same magnetic field. If magnetic moment of new magnet is M/4, then its oscillation time period is
a) T b) 2T
c) T/2 d) T/4

562. A particle of mass m is executing SHM about its mean position. The total energy of the particle at given instant is
a) $\frac{\pi^2 mA^2}{T^2}$ b) $\frac{2\pi^2 mA^2}{T^2}$
c) $\frac{4\pi^2 mA^2}{T^2}$ d) $\frac{8\pi^2 mA^2}{T^2}$

563. A load of mass 100 gm increases the length of wire by 10 cm. If the system is kept in oscillation, its time period is (g = 10 m/s^2)
a) 0.314 s b) 3.14 s
c) 0.628 s d) 6.28 s

564. The acceleration due to gravity changes from 9.8 m/s^2 to 9.5 m/s^2. To keep the period of pendulum constant, its length must changes by
a) 3 m b) 0.3 m
c) 0.3 cm d) 3 cm

565. Starting from the extreme position, the time taken by an ideal simple pendulum to travel a distance of half of the amplitude is
a) T/6 b) T/12
c) T/13 d) T/4

566. The period of oscillation of a mass M, hanging from a spring of force constant k is T. When additional mass m is attached to the spring, the period of oscillation becomes 5T/4. m/M =
a) 9 : 16 b) 25 : 16
c) 25 : 9 d) 19 : 9

467. In SHM, graph of which of the following is a straight line?
a) T.E. against displacement
b) P.E. against displacement
c) acceleration against time
d) velocity against displacement

468. A magnet, when suspended in an external magnetic field, has a period of oscillation of 4 s. When it is cut length wise, and suspended in the same magnetic field, the period of vibration will be
a) $2\sqrt{2}$ s b) 2 s
c) $4\sqrt{2}$ s d) 8 s

469. A body performing SHM about its mean position with period 24 s, after 4 s its velocity is π m/s, then its path will be
a) 48 m b) 24 m
c) 52 m d) 12 m

470. In simple harmonic motion, the wrong statement is
a) velocity of the body is maximum at mean position
b) kinetic energy is minimum at extreme position

c) its acceleration is maximum at extreme position and direction away from mean position

d) its acceleration is minimum at mean position

471. Time period of pendulum on earth surface is T}. Its time period at a height equal to radius of earth is T_2, then the ratio of $T_1 : T_2$ is

a) 8 : 10 b) 5 : 10

c) 1 : 1 d) 2 : 10

472. A bar magnet is oscillating in a uniform magnetic field of induction 0.4×10^{-5} T. When the frequency of the oscillating bar magnet is double due to increasing magnetic field, the increase in magnetic induction is

a) 1.2×10^{-4} T b) 1.2×10^{-5} T

c) 1.6×10^{-4} T d) 1.6×10^{-5} T

473. The total work done by a restoring force in simple harmonic motion of amplitude A and angular velocity. ω, in one oscillation is

a) $\frac{1}{2}$ m A^2 ω^2 b) Zero

c) m A^2 ω^2 d) $\frac{1}{2}$ m A ω

474. In simple harmonic motion, acceleration of the particle is zero, when its

a) velocity is zero

b) displacement is zero

c) both velocity and displacement are zero

d) both velocity and displacement are maximum

475. An amplitude of a simple pendulum of a period 'T' and length 'L' is increased by 5%. The new period of that pendulum will be

a) $\frac{T}{8}$ b) $\frac{T}{4}$

c) $\frac{T}{2}$ d) T

476. The maximum velocity of a particle executing S.H.M. is V. If the amplitude is doubled and the time period of oscillation decreases to (1/3) of its original value, then the maximum velocity will be

a) 18 V b) 6 V

c) 12 V d) 3 V

477. For a constant force the work done in stretching if spring constant $k_1 > k_2$ then energy stored in two wire related as

a) $w_1 > w_2$ b) $w_1 = w_2$

c) $w_1 < w_2$ d) $w_2 = 2\,w_1$

478. If A_1 and A_2 are the amplitude of two waves superimpose with other that $A_1 > A_2$ then difference of maximum amplitude and minimum amplitude is

a) $A_1 + A_2$ b) $A_1 - A_2$

c) $2A_1$ d) $2\,A_2$

479. A particle performing S.H.M. about their mean position with the equation of velocity is given by $4v^2 = 25 - x^2$, then the period of motion is

a) $2\,\pi$ b) π

c) $3\,\pi$ d) $4\,\pi$

480. A simple pendulum of length l and mass of a metallic bob is m. If it oscillates with a small amplitude a, then the maximum tension in the string is

a) $T_{max} = mg\left[1+\left(\frac{A}{l}\right)^2\right]$

b) $T_{max} = mg\left[1-\left(\frac{A}{l}\right)^2\right]$

c) $T_{max} = mg\left[1-\left(\frac{l}{A}\right)^2\right]$

d) $T_{max} = mg\left[1+\left(\frac{l}{A}\right)^2\right]$

481. A mass (m) is suspended at the end of a weightless wire of length L, cross–sectional area A and Young's modulus Y. The period of oscillation for the S.H.M. along the vertical direction is,

a) $2\pi\sqrt{\frac{YA}{mL}}$ b) $2\pi\sqrt{\frac{mL}{YA}}$

c) $2\pi\sqrt{\frac{mY}{AL}}$ d) $2\pi\sqrt{\frac{AL}{mY}}$

482. A flat spiral spring of force constant k is loaded with mass M and oscillate about vertical with a time period T. Then the mass suspended to the free end is

a) $\frac{4\pi^2}{kT^2}$ b) $\frac{kT^2}{4\pi^2}$

c) $\frac{kT}{4\pi^2}$ d) $\frac{kT}{4\pi}$

483. A block resting on the horizontal surface executes S.H.M. in horizontal plane with amplitude 'A'. The frequency of oscillation for which the block just starts to slip is (μ = coefficient of friction, g = gravitational acceleration)

a) $\frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$ b) $\frac{1}{4\pi}\sqrt{\frac{\mu g}{A}}$

c) $2\pi\sqrt{\frac{A}{\mu g}}$ d) $4\pi\sqrt{\frac{A}{\mu g}}$

484. A particle performs S.H.M. with amplitude 25 cm and period 3 s. The minimum time required for it to move between two points 12.5 cm on either side of the mean position is

a) 0.6 s b) 0.5 s

c) 0.4 s d) 0.2 s

485. A particle is executing S.H.M. of periodic time 'T'. The time taken by a particle in moving from mean position to half the maximum displacement is (sin 30° = 0.5)

a) $\frac{T}{2}$ b) $\frac{T}{4}$

c) $\frac{T}{8}$ d) $\frac{T}{12}$

486. A simple pendulum is oscillating with amplitude 'A' and angular frequency 'ω'. At displacement 'x' from mean position, the ratio of kinetic energy of potential energy is

a) $\frac{x^2}{A^2 - x^2}$ b) $\frac{x^2 - A^2}{x^2}$

c) $\frac{A^2 - x^2}{x^2}$ d) $\frac{A - x}{x}$

❍❍❍

Answers

1. (a)	2. (b)	3. (c)	4. (d)	5. (a)	6. (a)	7. (d)	8. (a)	9. (d)	10. (b)
11. (c)	12. (d)	13. (a)	14. (a)	15. (c)	16. (b)	17. (c)	18. (a)	19. (b)	20. (d)
21. (a)	22. (d)	23. (a)	24. (b)	25. (c)	26. (d)	27. (c)	28. (c)	29. (d)	30. (c)
31. (b)	32. (b)	33. (c)	34. (d)	35. (d)	36. (a)	37. (d)	38. (b)	39. (b)	40. (c)
41. (d)	42. (a)	43. (c)	44. (a)	45. (b)	46. (c)	47. (a)	48. (a)	49. (c)	50. (a)
51. (b)	52. (c)	53. (d)	54. (c)	55. (d)	56. (c)	57. (c)	58. (c)	59. (d)	60. (d)
61. (c)	62. (a)	63. (b)	64. (a)	65. (b)	66. (a)	67. (c)	68. (a)	69. (b)	70. (d)
71. (b)	72. (c)	73. (d)	74. (d)	75. (c)	76. (b)	77. (b)	78. (a)	79. (d)	80. (c)
81. (b)	82. (b)	83. (b)	84. (a)	85. (b)	86. (d)	87. (d)	88. (b)	89. (c)	90. (b)
91. (a)	92. (b)	93. (d)	94. (b)	95. (b)	96. (b)	97. (b)	98. (a)	99. (b)	100. (c)
101. (b)	102. (b)	103. (a)	104. (c)	105. (b)	106. (d)	107. (c)	108. (c)	109. (c)	110. (a)
111. (b)	112. (a)	113. (c)	114. (b)	115. (d)	116. (c)	117. (d)	118. (c)	119. (c)	120. (d)
121. (a)	122. (b)	123. (a)	124. (b)	125. (c)	126. (b)	127. (d)	128. (d)	129. (a)	130. (c)
131. (a)	132. (d)	133. (b)	134. (d)	135. (b)	136. (b)	137. (b)	138. (d)	139. (a)	140. (c)
141. (d)	142. (b)	143. (a)	144. (d)	145. (d)	146. (c)	147. (a)	148. (b)	149. (b)	150. (c)
151. (c)	152. (a)	153. (c)	154. (c)	155. (b)	156. (b)	157. (d)	158. (b)	159. (b)	160. (d)
161. (d)	162. (d)	163. (c)	164. (b)	165. (d)	166. (b)	167. (c)	168. (b)	169. (b)	170. (d)
171. (b)	172. (b)	173. (c)	174. (c)	175. (d)	176. (d)	177. (c)	178. (d)	179. (c)	180. (c)
181. (c)	182. (d)	183. (a)	184. (c)	185. (a)	186. (a)	187. (d)	188. (c)	189. (a)	190. (b)
191. (c)	192. (c)	193. (b)	194. (b)	195. (b)	196. (a)	197. (b)	198. (a)	199. (b)	200. (a)
201. (a)	202. (a)	203. (b)	204. (d)	205. (b)	206. (c)	207. (c)	208. (d)	209. (b)	210. (c)
211. (c)	212. (a)	213. (b)	214. (b)	215. (b)	216. (b)	217. (c)	218. (d)	219. (d)	220. (b)
221. (a)	222. (a)	223. (a)	224. (b)	225. (a)	226. (c)	227. (b)	228. (a)	229. (d)	230. (d)
231. (c)	232. (b)	233. (d)	234. (a)	235. (d)	236. (d)	237. (c)	238. (b)	239. (b)	240. (a)
241. (a)	242. (b)	243. (c)	244. (b)	245. (b)	246. (d)	247. (d)	248. (b)	249. (b)	250. (c)
251. (c)	252. (a)	253. (d)	254. (d)	255. (c)	256. (d)	257. (b)	258. (a)	259. (d)	260. (d)
261. (b)	262. (d)	263. (a)	264. (b)	265. (a)	266. (c)	267. (b)	268. (c)	269. (b)	270. (a)
271. (d)	272. (b)	273. (b)	274. (a)	275. (a)	276. (c)	277. (d)	278. (c)	279. (c)	280. (b)
281. (a)	282. (d)	283. (c)	284. (c)	285. (a)	286. (b)	287. (a)	288. (a)	289. (a)	290. (b)
291. (a)	292. (d)	293. (b)	294. (b)	295. (a)	296. (a)	297. (b)	298. (b)	299. (c)	300. (a)
301. (b)	302. (d)	303. (a)	304. (b)	305. (b)	306. (a)	307. (c)	308. (a)	309. (c)	310. (c)
311. (c)	312. (d)	313. (c)	314. (a)	315. (d)	316. (d)	317. (b)	318. (b)	319. (a)	320. (b)
321. (b)	322. (c)	323. (a)	324. (b)	325. (d)	326. (c)	327. (b)	328. (b)	329. (c)	330. (c)
331. (d)	332. (d)	333. (a)	334. (b)	335. (d)	336. (b)	337. (c)	338. (a)	339. (c)	340. (b)
341. (d)	342. (c)	343. (d)	344. (a)	345. (a)	346. (a)	347. (b)	348. (d)	349. (b)	350. (a)
351. (a)	352. (b)	353. (b)	354. (a)	355. (a)	356. (c)	357. (c)	358. (a)	359. (d)	360. (c)
361. (d)	362. (b)	363. (a)	364. (b)	365. (b)	366. (b)	367. (d)	368. (b)	369. (c)	370. (b)
371. (d)	372. (b)	373. (d)	374. (c)	375. (b)	376. (c)	377. (b)	378. (c)	379. (c)	380. (b)
381. (c)	382. (a)	383. (c)	384. (c)	385. (d)	386. (b)	387. (c)	388. (d)	389. (b)	390. (a)
391. (b)	392. (c)	393. (c)	394. (d)	395. (d)	396. (c)	397. (a)	398. (b)	399. (c)	400. (c)
401. (b)	402. (b)	403. (c)	404. (b)	405. (d)	406. (b)	407. (b)	408. (a)	409. (b)	410. (c)

Answers

411.	(b)	412.	(b)	413.	(b)	414.	(c)	415.	(b)	416.	(c)	417.	(d)	418.	(c)	419.	(d)	420.	(d)
421.	(d)	422.	(c)	423.	(a)	424.	(a)	425.	(b)	426.	(b)	427.	(d)	428.	(b)	429.	(a)	430.	(d)
431.	(d)	432.	(c)	433.	(b)	434.	(d)	435.	(c)	436.	(b)	437.	(c)	438.	(d)	439.	(c)	440.	(b)
441.	(b)	442.	(c)	443.	(a)	444.	(c)	445.	(b)	446.	(a)	447.	(b)	448.	(a)	449.	(c)	450.	(b)
451.	(a)	452.	(a)	453.	(c)	454.	(a)	455.	(b)	456.	(c)	457.	(a)	458.	(c)	459.	(c)	460.	(a)
461.	(b)	462.	(b)	463.	(c)	464.	(d)	465.	(a)	466.	(a)	467.	(a)	468.	(b)	469.	(b)	470.	(c)
471.	(b)	472.	(b)	473.	(b)	474.	(b)	475.	(d)	476.	(b)	477.	(c)	478.	(d)	479.	(b)	480.	(a)
481.	(b)	482.	(b)	483.	(a)	484.	(b)	485.	(d)	486.	(c)								

39. $F = KA = m\omega^2 A$

40.
$$x = A\sin\omega t$$
$$\frac{A}{2} = A\sin\omega t$$
$$\therefore \quad \omega t = \sin\left(\frac{1}{2}\right)$$
$$\frac{2\pi t}{1} = \frac{\pi}{6} = \frac{1}{12}$$
$$\therefore \quad t = \frac{1}{12}\text{ s}$$

41. Comparing it with the standard equation of S.H.M. $\frac{d^2x}{dt^2} = -\omega^2 x$

We get, $\omega^2 = \alpha$
$$\therefore \quad \omega = \sqrt{\alpha}$$
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$$

43. $T_1 = 2\pi\sqrt{\frac{m_1}{k}}$ and $T_2 = 2\pi\sqrt{\frac{m_2}{k}}$
$$\frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{4}{1}}$$
$$T_2 = 2T_1$$

69.
$$v = \omega\sqrt{A^2 - x^2}$$
$$\therefore \quad \frac{A^2}{4} = A^2 - x^2 \qquad \therefore \quad \frac{\sqrt{3}}{2}A = x$$

70.
$$v = A\omega\cos\omega t$$
$$\therefore \quad \frac{1}{2}A\omega = A\omega\cos\omega t$$
$$\therefore \quad \omega t = \cos^{-1}\left(\frac{1}{2}\right)$$
$$\frac{2\pi}{T}t = \frac{\pi}{3}$$
$$\therefore \quad \frac{t}{T} = \frac{1}{6}$$

71. $T = 1.2$ s. $\quad x_1 = 4\sin\omega t_1$
$$\therefore \quad \omega t_1 = \sin^{-1}\frac{(x_1)}{4} = \sin^{-1}\left(\frac{2}{4}\right) = \frac{\pi}{6}$$
$$x_2 = 4\sin\omega t_2$$
$$\therefore \quad \omega t_2 = \sin\left(\frac{4}{4}\right) = \sin^{-1}(1) = \frac{\pi}{2}$$
$$t_2 - t_1 = \left(\frac{\pi}{2} - \frac{\pi}{6}\right)\frac{1}{\omega} = \frac{\pi}{2\pi}T\left[\frac{1}{2} - \frac{1}{6}\right]$$
$$= \frac{T}{2}\left[\frac{3-1}{6}\right] = \frac{T}{2}\left[\frac{2}{6}\right] = \frac{T}{6} = \frac{1.2}{6}$$
$$= 0.2\text{ s}$$

$\therefore$ Therefore time taken by the particle to move from $x = 2$cm to $x = 4$cm and back again is
$$0.2 + 0.2 = 0.4\text{ s.}$$

72. $T = 4$ s and $A = 1$m.

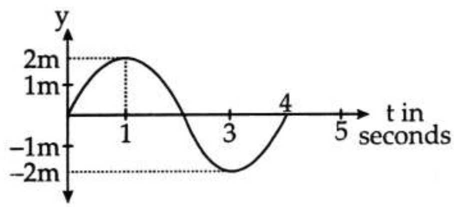
$\therefore$ The maximum acceleration $= \omega^2 A$
$$= \frac{4\pi^2}{T^2}\times A = \frac{4\pi^2}{16}\times 1 = \frac{10}{4} = 2.5$$

73.
$$T = \frac{2\pi}{\omega} \qquad \therefore \omega = \frac{2\pi}{16} = \frac{\pi}{8}$$
$$v = A\omega\cos\omega t$$
$$\therefore \quad \pi = A\frac{\pi}{8}\cos\left(\frac{\pi}{4}\right)$$
$$1 = \frac{A}{8}\times\frac{1}{\sqrt{2}}$$
$$\therefore \quad A = 8\sqrt{2}$$

74.
$$x = A\sin\omega t \qquad \therefore \frac{A}{2} = A\sin\omega t$$
$$\omega t = \sin^{-1}\left(\frac{1}{2}\right) \qquad \therefore \frac{2\pi}{T}t = \frac{\pi}{6} \qquad \therefore t = \frac{T}{12}\text{ s}$$

77. $y = A\sin(\omega t + \phi) = 0.2\sin 90° = 0.2$ m.

78.



$$A = 2\text{m};\ T = 4\text{ s}$$
$$V_{max} = A\omega = A\times\frac{2\pi}{T} = 2\times\frac{2\pi}{4} = \pi\ \text{ms}^{-1}.$$

79. $v_m = A\omega = 0.2 \times \frac{2\pi}{0.01} = 40\pi$.

81. $2A = 2 - 1.5 \quad \therefore A = \frac{0.5}{2} = 0.25$ m

$v = \omega\sqrt{A^2 - x^2} \quad \therefore x = A/2$

$v = \frac{\sqrt{3}}{2} A\omega = \frac{\sqrt{3}}{2} \times \frac{1}{4} \times \frac{2\pi}{\sqrt{3}}$

$= 0.866\, A\omega$

84. $\omega = \pi$

$\therefore 2\pi n = \pi \quad \therefore n = \frac{1}{2} = 0.5$

85. $T = 2\pi\sqrt{\frac{m}{k}} = 2$ s

$T_1 = 2\pi\sqrt{\frac{m+2}{k}} = 3$ s

$\frac{T_1}{T} = \sqrt{\frac{m+2}{m}} = \frac{9}{4} = \frac{m+2}{m}$

$\therefore 9m = 4m + 8 \quad \therefore 5m = 8 \quad \therefore m = 1.6$ kg

86. Acceleration $= -\omega^2 x = \frac{-4\pi^2}{T^2}\frac{A}{2} = \frac{-2\pi^2 A}{T^2}$

87. Acceleration $= \omega^2 x$

$1 = \omega^2 \times 0.01$

$\therefore \omega^2 = 100 \quad \therefore \omega = 10$ but $T = \frac{2\pi}{\omega}$

$\therefore T = \frac{\pi}{5}$ s

88. $v_m = A\omega = 0.05 \times 10 = 0.5$ m/s.

89. Acceleration max $= (A\omega^2)$

$= 2(0.05\pi)^2$

$= 2 \times 25\pi^2 \times 10^{-4}$

$= 50\pi^2 \times 10^{-4}$

$= 0.5\pi^2 \times 10^{-2}$

$= \frac{\pi^2}{200}$

90. $x = A\sin\omega t$

$12.5 = 25\sin\omega t$

$\frac{1}{2} = \sin\omega t \quad \therefore \omega t = \sin^{-1}(0.5)$

$\omega t = \frac{\pi}{6} \quad \therefore \frac{2\pi}{T} t = \frac{\pi}{6}$

$t_1 = \frac{T}{12} = \frac{3}{12} = \frac{1}{4}$ s

One side of 12.5 cm time is 1/4 s

$\therefore$ Other side of 12.5 cm the time is 1/4 s

$\therefore$ The time for either sides of 12.5 cm from mean position is

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2} = 0.5$ s

91. $A = \sqrt{v_1^2 + v_2^2}$

92. $v_1 = \omega\sqrt{A^2 - x_1^2}$

$3 = \omega\sqrt{25 - 16}$

$3 = 3\omega \quad \therefore \omega = 1$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 6.28$ s

95. $2A = 2 - 1.5 \quad \therefore A = \frac{0.5}{2} = 0.25$ m

$v_{max} = A\omega = A\sqrt{\frac{g}{A}}$

$= 0.25\sqrt{\frac{10}{0.25}} = \sqrt{\frac{10 \times 0.25 \times 0.25}{0.25}}$

$= \sqrt{2.5}$ m/s

96. The maximum restoring force $= m\omega^2 A$

$= 4\pi^2 n^2 m A$

$= 4 \times 10 \times 16 \times \frac{1}{2} \times 10 = 3.2 \times 10^3$ N

97. $x = A\cos\omega t \quad \frac{A}{2} = A\cos\omega t$

$\therefore \omega t = \cos^{-1}\left(\frac{1}{2}\right) \quad \therefore \frac{2\pi}{T} t = \frac{\pi}{3}$

$t = \frac{4}{3 \times 2} = \frac{2}{3}$ s

98. $v = \omega\sqrt{A^2 - x^2}$

$\therefore \frac{A\omega}{2} = \omega\sqrt{A^2 - x^2}$

$\frac{A^2}{4} - A^2 = x^2 \quad \therefore \frac{3A^2}{4} = x^2$

$x = \frac{\sqrt{3}}{2} A = 0.866\, A = 86.6\,\%\, A$

99. $v = \omega\sqrt{A^2 - \frac{A^2}{4}} \quad v = A\omega\frac{\sqrt{3}}{2}$

$= A\omega \times 0.866 = 86.6\,\%\, v_m$

100. $x = 4\cos^2\left(\frac{1}{2}t\right)\sin 1000\,t$

$= 2(1+\cos t).\sin 1000\,t$

$\therefore \quad 2\cos^2\theta = 1+\cos 2\theta$

$x = 2\sin 1000\,t + 2\sin 1000\,t \cos t$

$= 2\sin 1000\,t + \sin 100\,t + \sin 999\,t$

$\because \quad 2\sin A.\cos B = \sin(A+B) + \sin(A-B)$

101. $v = \omega\sqrt{A^2 - x^2}$ $\qquad v_m = A\omega$

102. $v_m = A\omega$ $\qquad a = A\omega^2$

$\therefore \quad \omega^2 = a/A$

103. $T = 2\pi\sqrt{\frac{x}{g}}$

107. $A - 2.344 = A\cos\omega t$

$\cos\omega t = \frac{5.656}{8} = 0.707$

$\omega t = \cos^{-1}(0.707)$

$\frac{2\pi}{T}t = \frac{\pi}{4}$

$t = 0.15$ s

108. $T = 2\pi\sqrt{\frac{x}{g}}$

122. $\omega = \pi$

$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

124. Tricks : $A = \sqrt{v_1^2 + v_2^2}$

126. $v_m = 8$ $\qquad A = 4$

$v_m = A\omega$

$\therefore \quad \omega = \frac{v_m}{A} = \frac{8}{4} = 2$

151. $|F| = Kx$

$|ma| = Kx$

$|a| = \frac{Kx}{m} = \frac{15\times 0.2}{0.3}$

$= 10\ m/s^2$.

152. $y_0 = 2\sin\frac{\pi}{4} = \frac{2}{\sqrt{2}} = \sqrt{2}$ m

$y_{max} = 2m; \frac{y_0}{y_{max}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

153. $|A\omega^2| - |A\omega| = 4; 2(\omega^2) - 2\omega - 4 = 0$

$2\omega^2 - 4\omega + 2\omega - 4 = 0$

$2\omega(\omega-2) + 2(\omega-2) = 0$

$\Rightarrow \omega = 2$ rad/s

$\frac{2\pi}{T} = 2; T = \pi = \frac{22}{7}$ s.

154. $V_m = A\omega = 20\ cms^{-1}$

$a_m = A\omega^2 = 100\ cms^{-2}$

$\frac{V_m^2}{a_m} = A = \frac{400}{100} = 4$ cm

$\frac{a_m}{V_m} = \omega = \frac{100}{20} = 5$ rad/s

$V = \omega\sqrt{A^2 - y^2}; 10 = 5\sqrt{4^2 - y^2}$

$4 = 4^2 - y^2; y^2 = 4^2 - 4 = 12; y = 2\sqrt{3}$ cm.

155. $n = \frac{1}{8}\ Hz \Rightarrow T = 8$ s

Mean to extreme t = 2s. The distance covered by it in 1 second is y_1 and in the II second $= A - y_1$.

$$\frac{y_1}{A-y_1} = \frac{A\sin\frac{2\pi}{8}\times 1}{A - A\sin\frac{2\pi}{8}\times 1}$$

$$= \frac{A/\sqrt{2}}{A - A/\sqrt{2}} = \frac{1}{\sqrt{2}-1}$$

156. $\frac{A-y_1}{y_1} = \sqrt{2}-1 \quad \therefore \frac{y_1}{A-y} = \frac{1}{\sqrt{2}-1}$

157. $V_m = A\omega = \pi; \pi = A\omega \Rightarrow A = \frac{\pi}{\omega}$

AV speed $= \frac{4A}{T} = \frac{4}{T}\times\frac{\pi}{\omega} = \frac{4\omega}{2\pi}\times\frac{\pi}{\omega}$

$= 2\ ms^{-1}$.

158. $V = \frac{A\omega\sqrt{3}}{2} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}\ ms^{-1}$.

159. $\frac{F_2}{F_1} = \frac{y_2}{y_1}$

$F_2 = F_1\frac{y_2}{y_1} = \frac{10\times 3}{2} = 15$ N.

160. $y = 0.2\sin 2\left(10\pi t + \frac{\pi}{3}\right)$

$= 0.2\sin\left(20\pi t + \frac{2\pi}{3}\right)$

$\frac{V_{max}}{a_{max}} = \frac{A\omega}{A\omega^2} = \frac{1}{\omega} = \frac{1}{20\pi}$ s.

161. $-\frac{A}{2} = A\sin\omega t;\ \sin\omega t = -\frac{1}{2} \Rightarrow \omega t = \frac{7\pi}{6}$

$$t = \frac{7T}{12}.$$

162.

$$4V^2 = 16 - y^2;\ V^2 = \frac{1}{4}(16 - y^2)$$

$$V = \frac{1}{2}\sqrt{16 - y^2}\ ;\ \omega = \frac{1}{2}\ \text{rad/s}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1} \times 2 = 4\pi\ \text{s}$$

$$A^2 = 16;\ A = \pm 4\ \text{m}$$

$$\text{Path length} = 2A = 8\ \text{m}$$

164.

$$y_1 = y_2$$

$$A\sin\left(\omega_1 t + \frac{\pi}{2}\right) = A\sin\left(\omega_2 t - \frac{\pi}{2}\right)$$

$$\cos\omega_1 t = -\cos\omega_2 t$$

$$\cos\omega_1 t = \cos(\pi - \omega_2 t)$$

$$\omega_1 t = \pi - \omega_2 t$$

$$t = \frac{\pi}{\omega_1 + \omega_2}$$

$$\omega_1 = \frac{2\pi}{T_1} = 2\pi\ \text{s};\ \omega_2 = \frac{2\pi}{2} = \pi\ \text{s}$$

$$t = \frac{\pi}{2\pi + \pi} = \frac{1}{3}\ \text{s}.$$

165.

$$\omega = 2\pi;\ T = \frac{2\pi}{\omega} = 1\ \text{s}$$

distance in a time period

$= 4A = 4 \times 2 = 8\text{m}.$

In 4s the particle makes 4 oscillations.

$\therefore\ d = 8 \times 4$

$= 32\ \text{m}.$

166.

$$\frac{A\omega}{A\omega^2} = \frac{1}{\omega} = 1;\ \frac{T}{2\pi} = 1;\ T = 2\pi\ \text{second}.$$

$$t = \frac{T}{2} = \pi = 3.14\ \text{s}.$$

167.

$$V = A\omega = \frac{2\pi A}{T};\ V' = \frac{(2A)\times 2\pi}{T/2}$$

$$= \frac{4A}{T} \times 2\pi$$

$$= 4\ \text{V}.$$

168.

$$E = \frac{1}{2} m\omega^2 A^2$$

$$E_1 = E_2$$

$$m\left(\frac{2\pi}{T_1}\right)^2 \times (1)^2 = 4\,m \times \left(\frac{2\pi}{T_2}\right)^2 \times (2)^2$$

$$\frac{T_1^2}{T_2^2} = \frac{1}{16} \Rightarrow \frac{T_1}{T_2} = \frac{1}{4}.$$

169.

$$\frac{V_1}{V_2} = \sqrt{\frac{A^2 - y_1^2}{A^2 - y_2^2}}\ ;\ \frac{2}{1} = \sqrt{\frac{A^2 - 1}{A^2 - 2^2}}$$

$$4A^2 - 16 = A^2 - 1$$

$$3A^2 = 15;\ A^2 = 5$$

$$A = \sqrt{\frac{2^2 \times 2^2 - 1^2 \times 1^2}{2^2 - 1}}$$

$$= \sqrt{\frac{15}{3}} = \sqrt{5}.$$

By shortcut method, $A = \sqrt{v_1^2 + v_2^2}$

179. $T = 2\pi\sqrt{\frac{x}{g}} = 2\pi\sqrt{\frac{9.8 \times 10^{-2}}{9.8}} = 0.2\pi\ \text{s}$

180. $T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{2.5}{10}} = \frac{2\pi \times 5}{10} = \pi\ \text{s}$

182. $x_1 = A\sin\omega t$

$$\omega t = \sin^{-1}\left(\frac{x}{A}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\delta_1 = \omega t = \frac{\pi}{3}$$

$$\delta\theta = \pi - 2\delta = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

183. $\delta = \delta_1 - \delta_2 = \omega_1 t - \omega_2 t = \left(\frac{2\pi}{T_1} - \frac{2\pi}{T_2}\right)t$

185. From figure,

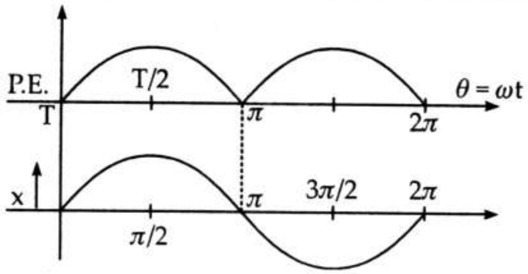
$$T = 0.04 \qquad \therefore\ n = \frac{1}{T} = 25\ \text{Hz}$$

186. $\delta = \tan^{-1}\left[\frac{A_1\sin\alpha_1 + A_2\sin\alpha_2}{A_1\cos\alpha_1 + A_2\cos\alpha_2}\right]$

188. In $(1/8)^{th}$ oscillation the period is T/8

$\therefore$ $(7/8)^{th}$ oscillation the period is 11T/12.

206.



207. Refer above figure.

208. Total energy remains constant.

It should not fluctuate

$\therefore$ It's total change is zero. Hence **T** is ∞

210.

$$K.E. = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$= \frac{1}{2} m\omega^2 (A^2 - A^2 \sin^2 \omega t)$$

$$= \frac{1}{2} m\omega^2 A^2 (1 - \sin^2 \omega t)$$

$$K.E. = \frac{1}{2} m\omega^2 A^2 \frac{(1 + \cos 2\omega t)}{2}$$

$\therefore$ The frequency of its kinetic energy is 2ω

212.

$$K = K_0 \cos^2 \omega t$$

$$K_{max} = K_0$$

$$(P.E.)_{max} = K_{max} = K_0$$

213.

$$P.E. = \frac{1}{2} m\,\omega^2 x^2 \qquad \text{As } \frac{1}{2} m\,\omega^2 = \text{const.}$$

$$\therefore \quad P.E. \propto x^2 = k\,x^2$$

220.

$$E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m(2\pi n)^2 A^2$$

$$= 2\pi^2 m n^2 A^2.$$

221.

$$\frac{KE_{max} + 0}{2} = \frac{1}{2}\left(\frac{1}{2} m\omega^2 A^2\right)$$

$$= \frac{1}{4} m\,(2\pi n)^2 A^2$$

$$= \pi^2 m n^2 A^2.$$

222. $\langle K.E. \rangle = \frac{1}{2} \langle T.E. \rangle$

223. $\langle P.E. \rangle = \frac{1}{2} \langle T.E. \rangle$

224. $\langle T.E. \rangle = T.E.$

227.

$$\frac{P.E.}{K.E.} = \frac{\frac{1}{2} k \frac{A^2}{4}}{\frac{1}{2} k \left(A^2 - \frac{A^2}{4}\right)} = \frac{A^2}{4} \times \frac{4}{3A^2} = \frac{1}{3}$$

228.

$$K.E. = (1/2)\, m\omega^2 A^2$$

$$10^{-3} \times 8 = \frac{1}{2} \times 0.1 \times 0.01\, \omega^2$$

$$\therefore \quad \omega^2 = 16$$

$$\omega = 4$$

$$\therefore \quad x = 0.1 \sin \omega t$$

229.

$$K.E. = P.E.$$

$$\frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2 x^2$$

$$A^2 - x^2 = x^2 \qquad \therefore A^2 = 2x^2$$

$$\therefore \quad x^2 = \frac{A^2}{2} \qquad \therefore x = \frac{a}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2} \text{ cm}$$

230.

$$T.E._A = K.E_A + P.E_A$$

$$= KE_A + 3KE_A$$

$$= 4\,KE_A$$

$$T.E._B = KE_B + KE_B = KE_B + \frac{1}{3} KE_B$$

$$T.E._B = \frac{4KE_B}{3}$$

232.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{25}} = \frac{20}{5} = 4$$

$$v = \omega\sqrt{A^2 - x^2} \qquad \therefore 40 = 4\sqrt{A^2 - 100}$$

$$100 = A^2 - 100 \qquad \therefore A^2 = 200$$

$$T.E. = \frac{1}{2} kA^2 = \frac{1}{2} \times 400 \times 200 = 4 \times 10^4 \text{ erg}$$

233.

$$KE = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$= \frac{1}{2} m\omega^2 \left(A^2 - \frac{A^2}{4}\right) = \frac{3}{4} \times \frac{1}{2} m\omega^2 A^2$$

$$\therefore \quad KE = \frac{3}{4} T.E.$$

234.

$$\frac{1}{2} m\omega^2 A^2 = 16$$

$$\therefore \quad \omega^2 = \frac{16 \times 2}{5.12 \times 25 \times 25 \times 10^{-4}}$$

$$\therefore \quad \omega = \frac{4}{\sqrt{2.56} \times 25 \times 10^{-2}} = \frac{4}{1.6 \times 25 \times 10^{-3}}$$

$$\therefore \quad \omega = \frac{10^3}{10} = 10^2 \qquad \therefore \omega = 10$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = \frac{\pi}{5}$$

253. Potential energy,

$$U = \frac{1}{2} m\omega^2 y^2$$
$$U \propto y^2$$
$$U = K(y_1 + y_2)^2$$
$$= K(Y_1^2 + Y_2^2 + 2y_1 y_2)$$
$$U = U_1 + U_2 + 2\sqrt{U_1 U_2}.$$

254.
$$E = 2\pi^2 mn^2 A^2$$
$$E' = 2\pi^2 (2m)(4n^2)(2A^2)$$
$$\frac{E}{E'} = \frac{1}{128}$$
$$E' = 128\,E.$$

255.
$$PE_{max} = \frac{1}{2} m\omega^2 A^2 + U$$

(U other form of potential energy)

Instantaneous rest position is extreme position where potential energy is maximum which will be the total energy.

256.
$$E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} \times 0.1 \times 100^2 \times (0.2)^2$$
$$= 20\,J.$$

259.
$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\alpha_1 - \alpha_2}$$
$$= \sqrt{9 + 16 + 2\times3\times4\times0} = \sqrt{25} = 5cm$$

260. $\delta = \tan^{-1}\left[\frac{A_1 \sin\alpha_1 + A_2 \sin\alpha_2}{A_1 \cos\alpha_1 + A_2 \cos\alpha_2}\right]$

261.
$$y_2 = 5 \sin 3\pi t + 5\sqrt{3} \cos 3\pi t$$
$$R_1 = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\alpha_1 - \alpha_2}$$
$$= \sqrt{25 + 25\times3 + 25\times\sqrt{3}\times\cos(\pi/2)}$$
$$= \sqrt{25 + 75} = \sqrt{100}$$
$$R_1 = 10\text{ cm } R_2 = 10\text{ cm}$$
$$\frac{R_1}{R_2} = \frac{10}{10} = \frac{1}{1}$$

262.
$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\alpha_1 - \alpha_2}$$
$$= \sqrt{\alpha^2 + \beta^2 + 2\alpha\beta\cos\frac{\pi}{2}} = \sqrt{\alpha^2 + \beta^2}$$

282. The period of spring pendulum is independent of acceleration due to gravity.

283. $T = 2\pi\sqrt{\frac{L}{g}}$ $\quad T^2 = \frac{4\pi^2}{g} L$

i.e. $T^2 \propto L$, $\quad y = m\,x$

285. Either sides of surface of earth g goes on decreasing, hence period increases.

286. T is independent of mass.

320. $T = 2\pi\sqrt{\frac{R}{g}} = 84.6$ min.

321. The phase difference is constant and hence its period must be equal therefore velocities are also equal.

326.
$$T = 2\pi\sqrt{\frac{1 + \frac{0.63}{3}}{100}} = 2\pi\sqrt{\frac{1.21}{100}} = 2\pi \times \frac{1.1}{10}$$
$$= 0.22\pi s.$$

328.

Slope of $l - T^2$ graph $= \frac{T^2}{l}$

But, $\quad g = 4\pi^2 \frac{l}{T^2}$

$$\therefore \quad \frac{T^2}{l} = \frac{4\pi^2}{g}$$

$T = T^2 = 1$ s at the point of intersection

$$\therefore \quad \frac{1}{l} = \frac{4\pi^2}{g};\ l = \frac{g}{4\pi^2} \approx \frac{1}{4}\text{ m} = 25\text{ cm.}$$

329.
$$T_1 = 2\pi\sqrt{\frac{l}{g}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{l}{(1.02)g}}$$
$$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{1}{1.02}}$$
$$T_2 = T\sqrt{\frac{100}{102}} \qquad \therefore\ T_2 = 0.99\,T$$

330. T is independent of mass.

331.
$$F = F_1 + F_2$$
$$T^2 \propto \frac{1}{F} \qquad \therefore\ F \propto \frac{1}{T^2}$$
$$T = \frac{T_1T_2}{\sqrt{T_1^2 + T_2^2}} = \frac{\frac{4}{5}\times\frac{3}{5}}{\sqrt{\frac{16}{25} + \frac{9}{25}}} = \frac{12/25}{1}$$
$$\therefore \quad T = 0.48$$

332.
$$F = F_1 - F_2$$
$$T^2 \propto \frac{1}{F} \qquad \therefore\ F \propto \frac{1}{T^2}$$
$$T = \frac{T_1T_2}{\sqrt{T_1^2 - T_2^2}} = \frac{\frac{4}{5}\times\frac{3}{5}}{\sqrt{\frac{16}{25} - \frac{9}{25}}} = \frac{\frac{12}{25}}{\frac{7}{25}} = \frac{12}{7}$$

333. $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2} \times 1\,\% = 0.5\,\%$

334. $\frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \sqrt{\frac{g_1}{(1-0.02)g_1}}$

By using Binomial theorem

$$\therefore \quad \frac{T_2}{T_1} = [\;1\;]^{1/2} = 1 + \frac{1}{2} \times 0.02$$

$$\frac{T_2}{T_1} - 1 = 0.01$$

$$\frac{T_2 - T_1}{T_1}\,\% = 1\,\%$$

335. $\frac{T_2}{T_1} = \sqrt{\frac{1.21\,l}{l}} = 1.10$

$$\therefore \quad \frac{T_2 - T_1}{T_1} = 0.10 \qquad \therefore \quad \frac{T_2 - T_1}{T}\,\% = 10\,\%$$

336. $g_h = g\left(1 - \frac{2h}{R}\right)$

$$\therefore \quad g_h = g\left(1 - \frac{2\times 10^3}{6.4\times 10^6}\right)$$

$$\frac{T_2}{T_1} = \sqrt{\frac{g}{g_h}} = \sqrt{\frac{g}{g\left(1 - \frac{1}{3.2\times 10^3}\right)}}$$

$$\frac{T_2}{T_1} = \left[1 - \frac{1}{3.2\times 10^3}\right]^{-1/2}$$

$$= \left[1 + \frac{1}{2}\times\frac{1}{3.2\times 10^3}\right]$$

$$\therefore \quad \frac{T_2 - T_1}{T_1} = \frac{10^{-3}}{6.4} = 0.15623$$

This is the time loses per oscillation

The time loses per day will be

$T_2 - T_1 = 0.15625 \times 86400 = 13.5$ s.

Note : Time in one day is 86400 seconds

Alternative method :

$$\frac{\Delta T}{T} = -\frac{1}{2}\frac{\Delta g}{g}$$

338. $l_2 = l_1 - 0.001\,l_1 = 0.999\,l_1$

$$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{0.999} = \sqrt{99.9\times 10^{-2}} = 0.9995$$

$$\therefore \quad T_2 = T_1 \times 0.9995 = 2 \times 0.9995 = 1.999$$

$$T_2 - T_1 = 2.0000 - 1.999 = 10^{-3}$$

In one oscillation the pendulum gain 10^{-3} s

$\therefore$ In one day (oscillations of second pendulum are 43200) it will gains the time is

$10^{-3} \times 43200 = 43.2$ s

339. The period of clock pendulum $T_1 = 1$ s.

$$T_1 = 2\pi\sqrt{\frac{L}{g_1}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{L}{g_2}}$$

Where T_1 period on the earth and g_1 acceleration due to gravity on the earth. g_2 acceleration due to gravity at depth 1km below the earth and T_2 corresponding period.

$$\frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \left(1 - \frac{d}{R}\right)^{1/2}$$

$$\frac{T_2 - T_1}{T_1} = -\frac{d}{2R}$$

$$\therefore \quad \frac{T_1 - T_2}{T_1} = \frac{d}{2R} = \frac{1\times 10^3}{2\times 6.4} \times 10^{-6} = \frac{10^{-3}}{12.8}$$

In one sec. the pen. clock loses time $\frac{10^{-3}}{12.8}$ s

$\therefore$ In one day (86400 osci.) the clock lose time is

$$\frac{86400\times 10^{-3}}{12.8} = 6.750$$

340. A bigger pendulum of time period (5T/4).

A smaller pendulum will complete (5/4) vibration. It means the smaller pendulum will be leading the bigger pendulum by a time of (T/4) s. The phase for (T/4) s is $(\pi/2)$

341. $\frac{T_2}{T_1} = \sqrt{1.02} = (1 + 0.02)^{1/2} = 1 + 0.01$

$$\frac{T_2 - T_1}{T_1} = 0.01 \text{ s}$$

For second's pendulum $T_2 = 2$ s

$\therefore \quad T_2 - T_1 = 0.01 \times 2 = 0.02$ s

In one oscillation the pendulum lose 0.02 s

$\therefore$ 43200 oscillation the pendulum loses is 864 s

342. $\frac{T_2}{T_1} = \sqrt{1.44} = \frac{T_2}{T_1} = 1.20$

$$\frac{T_2 - T_1}{T_1} = 0.20 \qquad \therefore \quad \frac{T_2 - T_1}{T_1}\,\% = 20\,\%$$

343. By the law of conservation of energy

P. B. at A = K. E. at B.

$$mgh_A = \frac{1}{2}\,m\,v^2_B$$

$$V_B = \sqrt{2gh}$$

344. The period of pendulum when lift is stationary

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When the lift is moving upwards, the acceleration of the lift increases.

$$\therefore \quad T_1 = 2\pi\sqrt{\frac{L}{g+a}} = 2\pi\sqrt{\frac{L}{g+3g}} = 2\pi\sqrt{\frac{L}{4g}}$$

$$\frac{T_1}{T} = \sqrt{\frac{g}{4g}} = \frac{1}{2} \qquad \therefore T_1 = \frac{T}{2}$$

345. $$v = \omega\sqrt{A^2 - x^2} = \omega\sqrt{A^2 - \frac{A^2}{4}}$$

$$= \omega\sqrt{\frac{4A^2 - A^2}{2}} = \frac{A\omega}{2}\sqrt{3}$$

$$= \frac{\sqrt{3}}{2} A \frac{2\pi}{T} = \pi A\sqrt{3}/T$$

346. $$l_2 = (l_1 - 0.01)$$

$$T_1 = 2\pi\sqrt{\frac{1}{g}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{0\cdot 99}{g}}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{0\cdot 99}{1}}$$

$$\therefore \quad T_2 = 2\sqrt{99\times 10^{-2}} = 2\times 9.95\times 10^{-1}$$

$$T_2 = 1.990 \text{ s}$$

In 1 osci. gain of time is, 2 – 1.990 = 0.01 s

∴ In one day i.e. 43200 oscillations.

The time gain is 43200 × 0.01 = 432

347. $T = 2\pi\sqrt{\frac{R}{2g}} = \frac{84.6}{1.414} = 59.40$ min.

348. The net acceleration $= \sqrt{(a^2 + g^2)}$

$$= \sqrt{100+100} = \sqrt{200}$$

$$a_R = 1\ \sqrt{2}$$

$$T_1 = 2\pi\sqrt{\frac{l}{g}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{l}{a_R}}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{10}{10\sqrt{2}}} \qquad \therefore T_2 = \frac{2}{\sqrt{\sqrt{2}}}$$

349. $$\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{1.21}$$

$$\therefore \quad \frac{T_2}{T_1} = 1.1$$

$$\frac{T_2 - T_1}{T_1} = 0.1$$

$$\therefore \quad T_2 - T_1 = 0.1\times 2 = 0.2 \text{ s}$$

350. $T_2 = (1.5)T_1$ and $l_2 = l_1 + 0.6$

$$\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{l_1+0.6}{l_1}} \qquad \therefore (1.5)^2 = \frac{l_1+0.6}{l_1}$$

$$\therefore \quad 2.25 = \frac{l_1+0.6}{l_1} \qquad \therefore 1.25\, l_1 = 0.6$$

$$l_1 = \frac{0.6}{1.25} = \frac{0.6\times 4}{5} = \frac{2.4}{5}$$

$$= 0.48 \text{ m}$$

352. $n_1 \propto \frac{1}{\sqrt{l_1}}$ and $n_2 \propto \frac{1}{\sqrt{l_2}}$

$$n_1\sqrt{l_1} = n_2\sqrt{l_2}$$

$$\frac{3}{n_1} = \frac{1}{n_2}$$

$$n_1 = 3n_2 \quad \text{and} \quad n_2 = \frac{n_1}{3}$$

$$n_1 - n_2 = 1$$

$$n_1 - \frac{n_1}{3} = 1$$

$$3n_1 - n_1 = 3$$

$$n_1 = \frac{3}{2}$$

353. T = 6 s A = 5 cm

$$x = A\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore \quad 2.5 = 5\cos\omega t$$

$$\omega t = \cos^{-1}(0.5) = \frac{\pi}{3}$$

$$\therefore \quad \frac{2\pi}{6}t = \frac{\pi}{3} \qquad \therefore t = 1 \text{ s}$$

355. $l_2 = l_1 + 0.9$ and $T_2 = 2T_1$

$$T_1 \propto \sqrt{l_1} \qquad T_2 \propto \sqrt{l_2}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{l_1+0.9}{l_1}}$$

$$2 = \sqrt{\frac{l_1+0.9}{l_1}}$$

$$4\,l_1 = l_1 + 0.9$$

$$3\,l_1 = 0.9$$

$$l_1 = 0.3 = 30 \text{ cm}$$

358. $T = 2\pi\sqrt{\frac{l}{g}}$ $\quad T^2 = \frac{4\pi^2 l}{g}$

$$T^2 = K\,l \quad \text{... (i)}$$

Where $K = \frac{4\pi^2}{g}$

Differentiating equation (i),

$$2TdT = Kdl \quad \text{... (ii)}$$

From equation (i) and (ii),

$$\therefore \quad \frac{dT}{T} = \frac{kdl}{2l} \quad \text{... (iii)}$$

But $\alpha = \frac{dl}{L\,\Delta t}$ $\quad \therefore \alpha\Delta t = \frac{dl}{L}$

Hence equation (iii) becomes,

$$\frac{dT}{T} = \frac{1}{2}\alpha\Delta t$$

$$= \frac{1}{2} \times 12 \times 10^{-6} \times (40 - 20)$$

$$= 12 \times 10^{-5}$$

The period of clock pendulum T = 1 s

$\therefore \quad dT = 12 \times 10^{-5}$ s

Thus the clock gains 12×10^{-5} s every osci.

$\therefore$ In one day (86400 osci.) the clock will gain time of $12 \times 10^{-5} \times 86400 = 10.37$ s

359. $F = mg\sin\theta$

$a = g\sin\theta$

$$\therefore \quad \frac{49}{980} = \sin\theta$$

$$\therefore \quad \theta = 3°$$

360. $\Delta\phi = \omega_2 t - \omega_1 t$

$$= \left(\frac{2\pi}{T_2} - \frac{2\pi}{T_1}\right)t$$

$$= 2\pi t\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$= 2\pi \times 1.5\left(\frac{1}{1.5} - \frac{1}{2}\right) = \frac{\pi}{2}$$

361. $l_2 = 1.02\,l_1$ $\quad \frac{T_2}{T_1} = \sqrt{1.02}$

$$= (1 + 0.02)^{1/2}$$

$$\frac{T_2 - T_1}{T_1} = 0.01$$

$$T_2 - T_1 = 0.02$$

In one oscillation it looses 0.02

$\therefore$ 43200 .... ? $0.04 \times 43200 = 1728$ s

362. $E_1 = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}\frac{mgA^2}{l_1}$

$\therefore \quad E_1 \propto \frac{1}{l_1}$ and $E_2 = \frac{1}{l_2}$

$$\frac{E_2}{E_1} = \frac{l_1}{l_2} = \frac{1}{2}$$

$$\therefore \quad E_1 = \frac{0.2}{2} = 0.1\text{ J}$$

368. By the law of conservation of energy

$$TE_{\text{at }L} = TE_{\text{at }h}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgh$$

$$v^2 = u^2 + 2gh$$

$$u^2 = v^2 - 2gh$$

$$u^2 = 9 - 2 \times 10 \times \frac{1}{2}$$

$$u^2 = 9 - 5 = 4$$

$$u = 2\text{ m/s}$$

392. $T^2 = T_1^2 + T_2^2$

$$T^2 = 1^2 + 2^2$$

$$= 5$$

$$T = \sqrt{5}\text{ s.}$$

393. $T_N = \frac{T}{\sqrt{1 - \frac{d_l}{d_0}}} = \frac{T}{\sqrt{1 - \frac{1}{4}}} = \frac{2T}{\sqrt{3}} = \frac{2}{\sqrt{3}}\sqrt{3}$

$$= 2\text{s.}$$

394. $\Delta\phi = \omega_1 t - \omega_2 t = \left(\frac{2\pi}{2} - \frac{2\pi}{3}\right) \times 3$

$$= 2\pi\left(\frac{1}{6}\right) \times 3 = \pi \text{ radian.}$$

395. 100 cm pendulum is a second's pendulum,

$T = 2$s

$N\sqrt{l} =$ constant

N = Number of oscillations in a given time $N \times \sqrt{100} = (N - 1)\sqrt{144}$

$$10\,N = 12\,N - 12$$

$$2\,N = 12;\; N = 6$$

$$\therefore \quad t = 6 \times 2$$

$$= 12\text{ s.}$$

396. $T = 2$s

$$n = \frac{t}{T} = \frac{86400}{2} = 43200.$$

398. $$l = \frac{g}{\pi^2}$$

$$l \propto g$$

$$g_{pole} > g_{equator}$$

$$g_{pole} > l_{equator}.$$

399. $$\frac{l_2 - l_1}{l_1} \times 100 = \frac{g_2 - g_1}{g_1} \times 100$$

$$= \frac{g/6 - g}{g} \times 100$$

$$= \frac{-5}{6} \times 100 = -83.3.$$

400. $$T \propto \sqrt{l}$$

$$T_1 \propto \sqrt{l+10}\,;\ T_2 \propto \sqrt{l-10}$$

$$\frac{T_1^2}{T^2} = \frac{l+10}{l};\ \frac{T_2^2}{T^2} = \frac{l-10}{l}$$

$$\frac{T_1^2}{T^2} + \frac{T_2^2}{T^2} = \frac{l+10}{l} + \frac{l-10}{l} = 2$$

$$\frac{T_1^2}{T^2} + \frac{T_2^2}{T^2} = 2T^2 \quad \therefore\ T_1{}^2 + T_2{}^2 = 2T^2.$$

401. $$g_h = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{640}{6400+6400}\right)^2$$

$$= g\left(\frac{100}{121}\right)$$

$$T' = \frac{11}{10}\,T$$

$$\frac{T'-T}{T} \times 100 = 10\%.$$

403. $$l = A + x + L$$

$$58 = A + 50 + x$$

$$A + x = 8$$

$$F = Kx \qquad \therefore\ x = \frac{F}{K}$$

$$x = 0.05 \text{ m}$$

$$A = 0.03 \text{ m}$$

404. When the body is at the uppermost position the compression of the spring takes place i.e.

$$0.05 - 0.03 = 0.02$$

$$F = k \times x = 2 \times 10^3 \times 0.02 = 40 \text{ N}$$

405. $$u_1 = \frac{1}{2}\,k x_1^2 \quad \text{and} \quad u_2 = \frac{1}{2}\,k x_2^2$$

$$\frac{u_2}{u_1} = \frac{x_2^2}{x_1^2} \quad \therefore\ u_2 = \frac{u \times 100}{4} = 25u$$

406. $K = m\omega^2$

407. Let k be the force constant of the spring of length l_2. Since $l_1 = n\,l_2$ where n is an integer and

$$l_2 = \frac{l_1}{n} \quad \text{as} \quad l = l_1 + l_2.$$

$$\therefore \quad l = l_1 + \frac{l_1}{n} = l_1\left(1 + \frac{1}{n}\right)$$

but $$F = kx \qquad \therefore\ k = \frac{F}{l}$$

where k is the force constant length l_1.

$$\frac{k}{k_1} = \frac{l_1}{l} = \frac{l_1}{\left(1+\frac{1}{n}\right)} \qquad \therefore\ k_1 = \frac{k}{n}\,(n_1 + 1)$$

408. $$l_1 + l_2 = l$$

$$\therefore \quad l = nl_2 + l_2 = l_2\,(n+1)$$

$$\frac{F}{l} = k \quad \text{and} \quad \frac{F}{l_2} = k_1$$

$$\therefore \quad \frac{k}{k_1} = \frac{l_2}{l_2(n+1)} \qquad \therefore\ k_1 = (n+1)\,K$$

409. If K is the spring constant of the spring then K_1 spring constant of longer parts of the spring is

$$K_1 = \frac{K}{n}(n+1) = \frac{3K}{2}$$

410. $$T_1 = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{k}}$$

$$T_2 = 2\pi\sqrt{\frac{4}{k}} = 2\pi \times 2\sqrt{\frac{1}{k}}$$

$$\frac{T_2}{T_1} = \frac{2}{1} \qquad \therefore\ T_2 = 2T_1 = 2T$$

411. Let k be the force constant for each piece.

$$\frac{1}{k_s} = \frac{1}{k} + \frac{1}{k} + \frac{1}{k} + \frac{1}{k}$$

$$\therefore \quad \frac{1}{k_s} = \frac{4}{k} \qquad \therefore\ k_s = 4\,k$$

412. $$T = 2\pi\sqrt{\frac{x}{g}} = 2\pi\sqrt{\frac{0.2}{9.8}} = \frac{2\pi}{7}\ \text{s}$$

413. $$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\left(\frac{200}{\pi^2}\right)}{200}} = 2\ \text{s}$$

414. $$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{m}{4k}}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{1}{2} \qquad \therefore\ T_2 = \frac{T}{2}$$

415. The effective spring constant k.

By using Geometric progression is given by

$$\frac{1}{k_s} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \ldots..$$

$$= \frac{1}{k}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots.\right] = \frac{1}{k}\left[\frac{1}{1-(1/2)}\right]$$

$$\frac{1}{k_s} = \frac{2}{k} \qquad \therefore\ k_s = \frac{k}{2}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{k/2}} = 2\pi\sqrt{\frac{2m}{k}}$$

$$= 2\pi\sqrt{\frac{2\times0.04}{2\times100}} = 0.1256\ \text{s}$$

418. $E_k - E_P = \frac{1}{2}m\omega^2[a^2 - a^2\sin^2\omega t] + \frac{1}{2}m\omega^2a^2\sin^2\omega t$

$$= (1/2)\,m\omega^2a^2[\cos^2\omega t - \sin^2\omega t]$$

$$E_k - E_p = (1/2)\,m\omega^2a^2\cos 2\omega t$$

$$\omega' = 2\omega \quad \therefore\ T' = \frac{2\pi}{\omega'} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

The loaded spring have a period

$$T = \frac{2\pi}{\omega}$$

$$\frac{T'}{T} = \frac{T}{\omega}\times\frac{\omega}{2\pi} = \frac{1}{2} \quad \therefore\ T' = \frac{T}{2} = \frac{4}{2} = 2\ \text{s}$$

419. $T_1^2 = \frac{4\pi^2m}{k_1}$ and $T_2^2 = \frac{4\pi^2m}{k_2}$

when the springs are connected in series the spring constant are

$$K_s = \frac{k_1k_2}{k_1+k_2} \qquad \text{As} \qquad K_s \propto \frac{1}{T_s^2}$$

$$\therefore\ \frac{1}{T_s^2} = \frac{\frac{1}{T_1^2}\times\frac{1}{T_2^2}}{\frac{1}{T_1^2}+\frac{1}{T_2^2}} = \frac{\frac{1}{T_1^2}\times\frac{1}{T_2^2}}{\frac{T_2^2+T_1^2}{T_1^2\times T_2^2}}$$

$$\frac{1}{T_s^2} = \frac{1}{T_1^2+T_2^2} \qquad \therefore\ T_s = \sqrt{T_1^2+T_2^2}$$

420. $T_1^2 = \frac{4\pi^2m}{k_1}$ and $T_2^2 = \frac{4\pi m}{k_2}$

when springs are connected in parallel effective force constant

$$k_p = k_1 + k_2$$

$$\therefore\ \frac{1}{T_p^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2} = \frac{T_2^2+T_1^2}{T_1^2T_2^2}$$

421. $R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha_1-\alpha_2)}$

$$\left(\frac{A}{\sqrt{2}}\right)^2 = A^2 + A^2 + 2A^2\cos(\alpha_1-\alpha_2)$$

$$\frac{1}{2} = 2 + 2\cos(\alpha_1-\alpha_2)$$

$$\therefore\ 1+\cos(\alpha_1-\alpha_2) = \frac{1}{4} \qquad \therefore\ \cos(\alpha_1-\alpha_2) = \frac{1}{4}-1$$

$$\therefore\ \cos(\alpha_1-\alpha_2) = -\frac{3}{2\times2} \qquad \therefore\ \cos(\alpha_1-\alpha_2) = -\frac{3}{4}$$

$$\therefore\ [\pi - (\alpha_1-\alpha_2)] = \cos^{-1}\left(\frac{3}{4}\right) = 41.40^\circ$$

$$\therefore\ \alpha_1 - \alpha_2 = 180 - 41.40^\circ \approx \frac{3\pi}{4}$$

422. The period of single spring

$$T_1 = 2\pi\sqrt{\frac{m}{k}}$$

Period of two springs in series

$$T_2 = 2\pi\sqrt{\frac{2m}{k}}$$

Period of two springs in parallel

$$T_3 = 2\pi\sqrt{\frac{m}{2k}}$$

$$T_1 : T_2 : T_3 = 1 : \sqrt{2} : \frac{1}{\sqrt{2}}$$

430. $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{100}{1}} = 10\ \text{rad/s}$

$$n = \frac{\omega}{2\pi} = \frac{10}{2\pi} = \frac{5}{\pi}\ \text{Hz.}$$

431. $T = 2\pi\sqrt{\frac{m}{K}}$

$$\frac{\pi}{s} = 2\pi\sqrt{\frac{2}{K}}$$

$$\begin{aligned}
K &= 100\times2 \\
&= 200\ \text{N/m} \\
K &= Mg/x \\
200 &= (2\times10)/x \\
x &= 0.1\ \text{m} \\
F_{max} &= K(x+y) \\
&= 200\,(0.1+0.05) \\
&= 200\,(0.15) \\
&= 30\ \text{N.}
\end{aligned}$$

432. $$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{mx}{mg}} = 2\pi\sqrt{\frac{x}{g}}$$

$$= 2\pi\sqrt{\frac{0.05}{9.8}} = \frac{2\pi}{14} = \frac{\pi}{7}\text{ s.}$$

433. $$\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi/5}$$

$$\omega = 10\text{ rad/s}$$
$$F = m\omega^2 A$$
$$= 2\times 10^2 \times 0.05$$
$$= 10\text{N}$$

434. Extension $x = \dfrac{mg}{K}$

$$T = 2\pi\sqrt{\frac{m}{k}}\ ;\ 1 = 2\pi\sqrt{\frac{m}{k}}$$

$$\pi = \frac{1}{4\pi^2} = \frac{1}{4g}$$

$$\therefore \quad x = \frac{1}{4g}\times g = \frac{1}{4}\text{ m.}$$

435. $$T = 2\pi\sqrt{\frac{M}{K}}$$

$$2T = 2\pi\sqrt{\frac{M+m}{K}}$$

$$\frac{1}{2} = 2\pi\sqrt{\frac{M}{M+m}} \Rightarrow \frac{1}{4} = \frac{M}{M+m}$$

$$m = 3M.$$

436. $$\omega = \frac{a_m}{v_m} = \frac{4}{2} = 2$$

437. $$KE = PE$$

$$\frac{1}{2}mw^2(A^2-x^2) = \frac{1}{2}mw^2x^2$$
$$A^2 - x^2 = x^2$$
$$A = 2x^2$$
$$A = \sqrt{2}\,x$$

$$x = \frac{A}{\sqrt{2}} = 0.707\text{ A.}$$

442. $$KE = \frac{1}{2}m\omega^2(A^2-x^2)\text{ at } x = \frac{A}{2}$$

$$= \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$$

$$= \frac{1}{2}m\omega^2A^2\times\frac{3}{4} = \frac{3}{4}\text{ W}$$

$$PE = \frac{1}{2}mw^2\frac{A^2}{4} = \frac{1}{4}\text{ W.}$$

443. $$A = 2\text{ cm}$$
$$a_m = A\omega^2$$

$$\omega^2 = \sqrt{\frac{a_m}{A}} = \sqrt{\frac{2\pi^2}{2}}$$

$$\therefore \quad \omega = \pi$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{T_1} = 2\text{ s}$$

444. $T = 6.28$ s, $A = 3$ cm, $a_m = ?$

$$a_m = \omega^2 A = \frac{4\pi^2}{T^2}A = \frac{3\times 4\pi^2}{(2\pi)^2}$$

$$a_m = 3\text{ cm/s}^2$$

445. $$\frac{E_2}{E_1} = \frac{l_1}{l_2} = 2$$

$$E_2 = 2E_1$$

446. $$K = \frac{F}{x}$$

$$[L] = \frac{[L^1M^1T^{-2}]}{[L^1]} = [L^0M^1T^{-2}].$$

447. $$V = \frac{V_m}{2}$$

$$\omega\sqrt{A^2-x^2} = \frac{A\omega}{2}$$

$$A^2 - x^2 = \frac{A^2}{4}$$

$$A^2 - \frac{A^2}{4} = x^2$$

$$x = \frac{\sqrt{3}}{2}\text{ A.}$$

449. $A = 2$ cm.

450. $$\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{1.44} = 1.2$$

$$\frac{T_2 - T_1}{T_1} = 1.2 - 1 = 0.2 = 20\%.$$

452. $$A = 10\text{ cm}$$

453. $KE = \frac{1}{2} mv^2$

$\therefore v^2 = \frac{2KE}{m} = \frac{2\times16}{0.32}$

$v^2 = 100$

$v = 10$ m/s

460. $x = 8 \sin \omega t + 6 \cos \omega t$

$R = \sqrt{A_1^2 + A_2^2} = \sqrt{64+36} = \sqrt{100}$

$= 10$ m.

462. $E = \frac{1}{2} m\omega^2A^2 = \frac{m4\pi^2A^2}{T^2} = \frac{4\pi^2mA^2}{T^2}$

463. $T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{0.100}{10}} = 2\pi\sqrt{1\times10^{-2}}$

$= 2\pi \times 10^{-1} = 0.628$ s.

464. $l_1 - l_2 = \frac{g_1 - g_2}{\pi^2} = \frac{9.8-9.5}{10} = \frac{0.3}{10}$

$= 0.03$ m

$= 3$ cm

465. $t = \frac{T}{12}$

466. $T_1 = T, T_2 = \frac{5}{4}T$

$\frac{T_2}{T_1} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{M+m}{M}}$

$\frac{5T}{4T} = \sqrt{\frac{M+m}{M}}$

$\frac{25}{16} = 1 + \frac{m}{M}$

$\frac{25-16}{16} = \frac{m}{M}$

$\frac{9}{16} = \frac{m}{M}$

469. $T = 24$ s

$v = A\omega \cos \omega t$

$\pi = A \times \frac{2\pi}{T} \times \cos\left(\frac{2\pi}{24}\times4\right)$

$1 = A \times \frac{2}{24} \times \cos\frac{\pi}{3}$

$12 = \frac{A}{2}$

$\therefore A = 24$ m

471. $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \sqrt{\left(\frac{R}{R+h}\right)^2}$

$= \left(\frac{R}{R+R}\right) = \frac{1}{2}$

483. Restoring force = force of friction

$KA = \mu mg$

$m\omega^2A = \mu mg$

$\omega^2 = \frac{\mu a}{A}$

$\omega = \sqrt{\frac{\mu g}{A}}$

$2\pi n = \sqrt{\frac{\mu g}{A}}$

$\therefore n = \frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$

484. $t = \frac{T}{12} + \frac{T}{12} = \frac{T}{6}$

$= \frac{3}{6} = \frac{1}{2} = 0.5$ s

485. $t = \frac{T}{12}$.

486. $\frac{KE}{PE} = \frac{\frac{1}{2}m\omega^2[A^2 - x^2]}{\frac{1}{2}m\omega^2x^2} = \frac{A^2 - x^2}{x^2}$