# Continuity

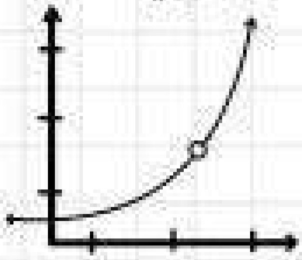
**A function f is continuous at c if**

$f(x)$ is defined at $x = c$ AND $\lim_{x \to c} f(x)$ exists AND $\lim_{x \to c} f(x) = f(c)$

## types of discontinuities

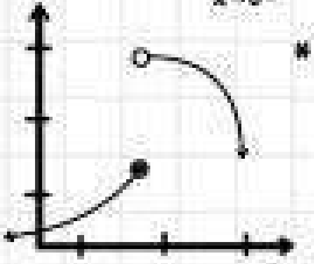
① **Hole** $\lim_{x \to c} f(x)$ exists, but $f(c)$ is undefined

*removable



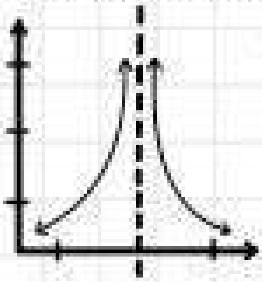
② **Jump** $\lim_{x \to c} f(x)$ does not exist because $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$

*non-removable

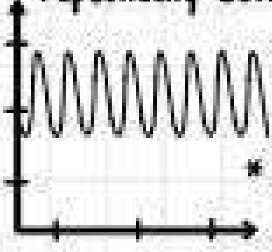


③ **Asymptote** $\lim_{x \to c} f(x) = \pm\infty$

*non-removable



④ **Oscillating** function moves repeatedly between two or more values



*non-removable

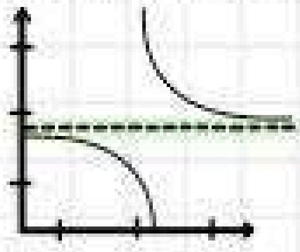
## Intermediate Value Theorem

Given a continuous function $f(x)$ on $[a,b]$ and a number $k$ between $f(a)$ and $f(b)$:

There exists 1 or more numbers $c$ on $[a,b]$ such that $f(c) = k$

## vertical asymptotes

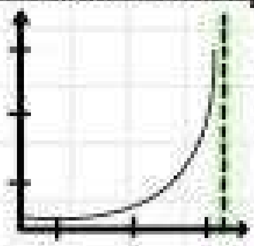
f increases/decreases without bound as $x \to c$



graph approaches positive or negative infinity

occurs when denominator equals 0.

## horizontal asymptotes



As $x \to \pm\infty$, f approaches but never touches a horizontal line

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If $\lim_{x \to c} f(x)$ exists, but the $\lim_{x \to c} f(x) \neq f(c)$, $f(x)$ has a removable discontinuity at $x = c$

We can remove the discontinuity by redefining $f(c)$

Cancel common factors in the numerator and denominator.

## L'Hôpital's Rule

If $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$$