Chapter

12 Electromagnetic Induction

Quick Review

Electromagnetic Induction

The phenomenon of producing an induced e.m.f. in a conductor or conducting coil due to changing magnetic flux is called electromagnetic induction.

Faraday's laws

First Law	Whenever there is a change of magnetic flux through a coil an e.m.f. is induced in it
Second Law	The magnitude of induced e.m.f. produced in the circuit is directly proportional to the rate of change of magnetic flux through it.

Lenz's laws

The direction of induced current in a circuit is such that the magnetic field produced by the induced current opposes the change in the magnetic flux that induces the current. The direction of induced e.m.f. is same as that of induced current.

Motional e.m.f. When a conductor of length 'l' moves across or rotates in a magnetic field, an e.m.f. is induced.

Induction and Energy Transfer Whenever a coil is pulled through a magnetic field with a speed 'v', the work done in the process appears as heat energy.

Eddy current

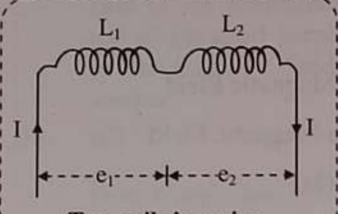
- Eddy current are circulating currents induced in conductive materials when exposed to changing magnetic fields
- Eddy current can lead to energy loss in the form of heat and it can be reduced by making a discontinuity in the structure of conducting plate

Self Inductance

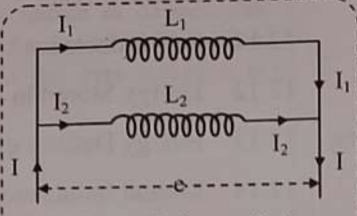
- The self-inductance of a circuit is the ratio of magnetic flux (produced due to current in the circuit) linked with the circuit to the current flowing in it.
- Self-inductance of a circuit is the ratio of induced e.m.f. (caused by changing current in the circuit) produced around the circuit to the rate of change of

Mutual Inductance

- The mutual inductance of two circuits is equal to the magnetic flux linked with one circuit per unit current in the other circuit.
- Mutual inductance is defined as the value of induced e.m.f. produced in the secondary circuit per unit rate of change in current in the primary circuit.



Two coils in series



Two coils in parallel

Coefficient of coupling

- It is a measure of the portion of flux that reaches secondary coil which is in vicinity of primary coil.
- There are three conditions possible:

i. If $K > 0.5$, the two coils are tightly coupled.

ii. If $K < 0.5$, the coils are loosely coupled.

iii. If $K = 1$ then coil is coupled to itself.

Transformer

Transformer is an electrical device which converts low alternating voltage at high current to high alternating voltage at low current or vice-versa.

Principle:

It is based on the principle of mutual induction i.e., whenever the magnetic flux linked with a coil changes, an e.m.f. is induced in the neighbouring coil.

Formulae

1. Magnetic flux: $\phi = BA\cos\theta$

2. Magnitude of induced e.m.f.:

i. $|e| = \frac{d\phi}{dt}$ (for one turn)

ii. $|e| = n\frac{d\phi}{dt}$ (for n turns)

3. E.M.F. induced in a straight conductor in translational motion perpendicular to magnetic field: $e = Blv$

where, v = velocity of the conductor

4. E.M.F. induced in a straight conductor in rotational motion about its end in a plane perpendicular to the magnetic field:

i. $e = Bf\pi l^2$ ii. $e = \frac{1}{2}Bl^2\omega$

iii. $e = Bl\,v\sin\theta$

where, l = length of the conductor

5. Peak e.m.f. induced in a magnet-coil system:

$$|e_0| = \left(\frac{d\phi}{dt}\right)_{max} \approx \left(\frac{d\phi}{d\theta}\right)_{max} \cdot \left(\frac{2\pi\theta_0}{T}\right)$$

6. Rate of mechanical work done in pulling a loop through the magnetic field:
$$P = \frac{B^2L^2v}{R}$$

7. Induced e.m.f. in a rotating coil:
$e = nAB\omega \sin \omega t = 2\pi fBNA \sin \omega t$

8. Peak value or maximum value of induced e.m.f.:
$e_0 = nAB\omega = 2\pi fBnA$

9. Induced current:

i. $I = \frac{e}{R} = \frac{e_0}{R} \sin \omega t = I_0 \sin \omega t$

ii. $I_0 = \frac{e_0}{R} = \frac{2\pi fBnA}{R}$

where,
e_0 = maximum or peak value of e.m.f.
I_0 = maximum or peak value of current

10. Instantaneous e.m.f. in a rotating coil
$e = e_0 \sin \omega t$

11. Magnetic flux due to self inductance: $\phi = LI$

12. Induced e.m.f. in a coil (Self induction):
$$|e| = L\frac{dI}{dt}$$
where, L = coefficient of self induction or self inductance
I = current through the coil

13. Inductance per unit length near the middle of a long solenoid: $\frac{L}{l} = \mu_0 n^2 A = \mu_0 n^2\left(\frac{\pi d^2}{4}\right)$

14. Self inductance for 'n' coils in series:
$L_{eq} = L_1 + L_2 \ldots + L_n$

15. Self inductance for 'n' coils in parallel:
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \ldots + \frac{1}{L_n}$$

16. Energy density (u_B) stored in magnetic field:
$$u_B = \frac{B^2}{2\mu_0}$$

17. Magnetic flux due to mutual inductance: $\phi_S = MI_P$

18. Induced e.m.f. in the secondary coil (Mutual induction):
$$|e_s| = M\frac{dI_P}{dt}$$
where, M = coefficient of mutual induction or mutual inductance
I_P = current through the primary coil

19. Coefficient of coupling: $K = \frac{M}{\sqrt{L_1 L_2}}$

20. Turns per unit length:
$n = \frac{N}{l}$ where, N = total turns

21. Transformer:

i. Induced e.m.f. in secondary coil, $e_S = -N_S\frac{d\phi}{dt}$

ii. Induced e.m.f. in primary coil, $e_P = -N_P\frac{d\phi}{dt}$

iii. Turns ratio $= \frac{N_S}{N_P}$

iv. Efficiency, $\eta = \frac{\text{output power}}{\text{input power}} = \frac{e_S I_S}{e_P I_P}$
where $\eta < 1$

22. Relation between voltage and current in transformer:

i. For an ideal transformer: $\frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$

ii. For step-up transformer: $\frac{e_S}{e_P} = \frac{N_S}{N_P} > 1$

iii. For a step-down transformer: $\frac{e_S}{e_P} = \frac{N_S}{N_P} < 1$

where, N_p = number of turns of primary coil
N_s = number of turns of secondary coil
I_p = current through primary coil
I_s = current through secondary coil

Shortcuts

1. Mutual inductance of coaxial solenoids: The mutual inductance (M) between them can be calculated using the formula $M = \frac{\mu_0 N_1 N_2 A}{L}$

where μ_0 is permeability of free space, N_1 and N_2 are the number of turns in the two solenoids, A is the area of cross-section and L is the length of the solenoids.

2. The inductance of parallel wires each of length 'l' in the same circuit can be calculated using the formula $L = \frac{\mu_0 l}{\pi} \ln\left(\frac{d}{a}\right)$, where 'a' is the radius of wire 'd' is the separation between axes of wires.

3. The mutual potential energy of two circuits is,

$W = MI_1I_2$

$\therefore \quad M = \frac{W}{I_1I_2}$

$\therefore \quad M = W \quad$ ....[when, $I_1 = I_2$]

Therefore, the mutual inductance (M) may also be defined as the mutual potential energy (W) of two circuits corresponding to unit current in each circuit.

4. $W = \frac{1}{2}LI^2$

If $I = 1$ A

$\therefore \quad L = 2W$ [Numerically]