Formula Sheet
Definite Integration

- Definite integration as limit of sum:

$$\int_a^b f(x)\,dx = \lim_{\substack{n\to\infty \\ h\to 0}} \sum_{r=1}^{n} hf(a+rh)$$

$$nh = b-a \qquad h = \frac{b-a}{n}$$

- Important Result:

1] $\sum_{r=1}^{n} 1 = n$

2] $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

3] $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{2}$

4] $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$

- Fundamental theorem of integral calculus:

$$\int f(x)\,dx = g(x)+c$$

$$\therefore \int_a^b f(x)\,dx = [g(x)+c]_a^b$$

$$= [g(b)+c-g(a)-c]$$

$$\int_a^b f(x)\,dx = g(b)-g(a)$$

- Properties of definite integral:

1] $\int_a^a f(x)\,dx = 0$

2] $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$

3] $\int_a^b F(x)dx = \int_a^b f(t)dt$.

4] $\int_a^b F(x)dx = \int_a^c F(x)dx + \int_c^b F(x)dx$

where, $a<c<b$, $c \in [a,b]$

5] $\int_a^b F(x)dx = \int_a^b F(a+b-x)dx$

6] $\int_0^a F(x)dx = \int_0^a f(a-x)dx$.

- Important Results:-

1. $\int_0^{\pi/2} \log(\tan x)dx = \int_0^{\pi/2} \log(\cot x)dx = 0$

2. $\int_0^{\pi/2} \log(\sin x)dx = \int_0^{\pi/2} \log(\cos x)dx = -\frac{\pi}{2}\log 2$.

3. $\int_0^{\pi/2} \log(\operatorname{cosec} x)dx = \int_0^{\pi/2} \log(\sec x).dx = \frac{\pi}{2}\log 2$.

- Properties:-

1] $\int_{-a}^{a} f(x)dx = 2\int_0^a F(x)$, if $F(x)$ even funt^n.

$= 0$, if $F(x)$ odd funt^n

$F(x)$ even funt^n ; if $f(-x) = f(x)$

and $F(x)$ odd funt^n if $f(-x) = -F(x)$

2] $\int_a^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a F(2a-x)dx$

- Reduction Formula:-

$$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx$$

$$= \frac{(n-1)}{n} \cdot \frac{(n-3)}{(n-2)} \cdot \frac{(n-5)}{(n-4)} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot \text{(odd)}$$

$$= \frac{(n-1)}{n} \cdot \frac{(n-3)}{(n-2)} \cdot \frac{(n-5)}{(n-4)} \cdots \frac{3}{4} \cdot \frac{\pi}{2} \cdot \text{(even)}$$

- Short trick:

$$(x-a)^m \cdot (b-x)^n\,dx = (b-a)^{m+n+1}\ \frac{m!\,n!}{(m+n+1)!}$$