Chapter

2 Mechanical Properties of Fluids

Quick Review

Fluids

Fluids are a state of matter distinguished by their ability to flow and take the shape of the container they are placed in. Their flow behaviour is described by the concept of viscosity, which is a measure of a fluid's resistance to flow. Understanding dynamics and behaviour of flow is essential in diverse fields like engineering, meteorology and biology among others.

Liquids

They have a definite volume and take the shape of their container they are filled in. They are generally considered considered to be incompressible, i.e., their volume does not change significantly under pressure.

Gases

Gases do not have a definite shape or definite volume. They occupy the entire space of the container they are in and are highly compressible.

➤ **Liquid Pressure:**

Pressure due to liquid column

- The pressure exerted by liquid column depends on height and density of liquid column.
- It is independent of the shape of the containing vessel or total mass of the liquid.
- **Hydrostatic Paradox:** The liquid pressure at a point is independent of quantity of liquid and depends upon the depth of point below the liquid surface. This is known as Hydrostatic paradox.

Atmosphere Pressure

- The gaseous envelope surrounding the earth is called the earth's atmosphere and the pressure exerted by the atmosphere is called atmospheric pressure.
- 1 atm = 1.01×10^5 Pa = 1.01 bar = 760 torr
- The atmospheric pressure is maximum at the surface of earth and goes on decreasing as we move up into the earth's atmosphere.

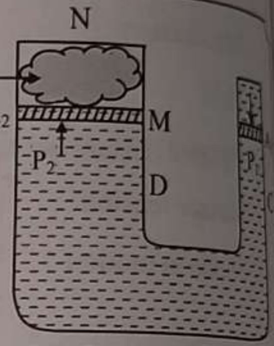
Absolute Pressure and Gauge Pressure

- Absolute pressure refers to the total pressure exerted be a fluid and includes both the atmospheric as well as any other additional pressure due to the fluid itself.
- Gauge pressure at a point in a liquid is the difference between the absolute pressure and the atmospheric pressure.

➢ **Pascal's Law:**

- If gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same.

Applications of Pascal's Law



i. **Hydraulic press:**
It consists of two cylinders C and D filled with water. A large platform M is fitted to the piston P_2 and strong metal frame (N) fixed at its top. Load is kept between M and N. A_2 is greater than A_1.
In hydraulic press, a smaller force is applied on a column of liquid and it is converted to the very large force in upward direction on the load.

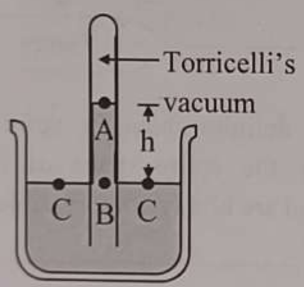
ii. **Hydraulic lift:**
Hydraulic lift is used to lift or support heavy objects such as cars, trucks etc.

iii. **Hydraulic brakes:**
Small force applied to brake pedal is immediately transmitted equally by the brake; fluid in the cylinders produces a large thrust on the wheels and vehicle stops.

➢ **Pressure Measurement:**

The instruments used to measure pressure are called pressure meters or pressure gauges or vacuum gauges.

Mercurry Barometer

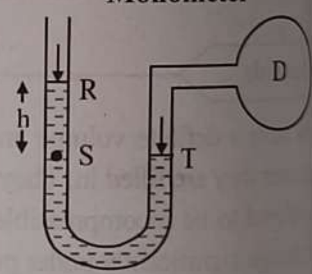


- Invented by Torricelli.
- Level of Mercury in a glass tube gives the pressure

Pressure Gauges and Meters

- The pressure to be measured is done indirectly (i.e., other physical parameters like bending of a material or change in capacitance is used)
- More accurate

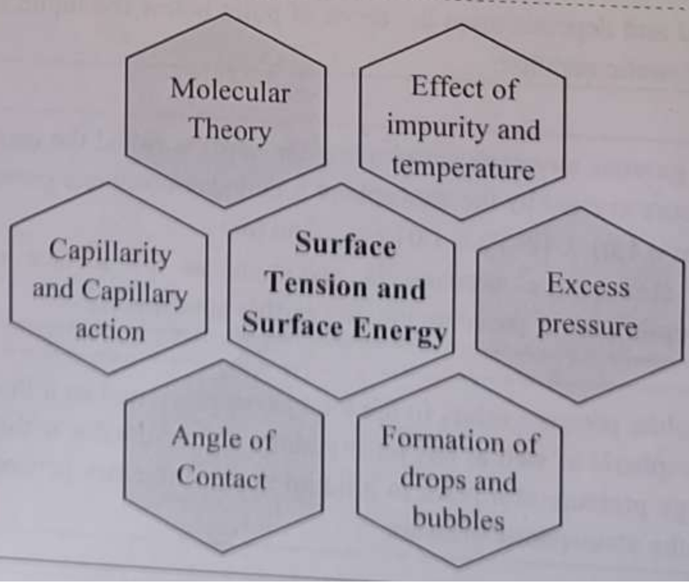
Monometer



Open tube manometer

- Utilises Pascal's law and height of Mercury column to measure pressure difference
- Gives gauge pressure

➢ **Surface Tension:**



➢ Molecular Theory of Surface Tension:

Surface Tension

- **Sphere of Influence** → An imaginary sphere drawn round a molecule (taken as centre) with a radius equal to the range of molecular attraction is called the sphere of molecular attraction. It is also called the sphere of influence of that molecule.
- **Surface Film** → A thin film of liquid near its surface having thickness equal to molecular range of attraction is called as surface film.
- **Surface Energy** → Surface energy is the work per unit area done by the force that creates the new surface.

Angle of Contact ⇒ *The angle between the surface of the solid and the tangent drawn to the surface of the liquid at the point of contact on the side of liquid is called the angle of contact of that liquid with that solid.*

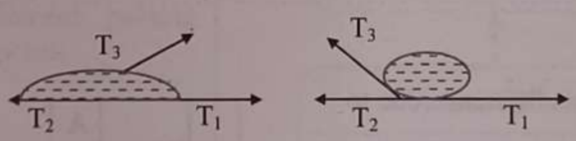
Characteristics of angle of contact

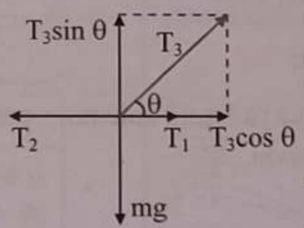
	θ	Shape of meniscus	Relative force strength	Level in Capillary tube	Wetness	Example
i.	$< 90°$	Concave	Adhesive > Cohesive	Rises	Liquid wets glass surface	Water in glass capillary
ii.	$90°$	Horizontal	Adhesive = Cohesive	Remains unchanged	Liquid wets the glass surface	Water in silver capillary
iii.	$> 90°$	Convex	Adhesive < Cohesive	Falls	Liquid does not wet glass surface	Mercury in glass capillary

➢ Formation of bubbles and drops and Excess Pressure:

Formation of bubbles and drops

When a small quantity of liquid is poured on a solid, it may take the shape of drop or spread on the surface of solid.



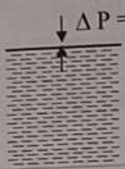
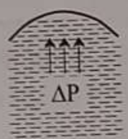
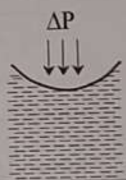


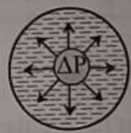
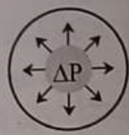
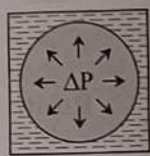
For drop in equilibrium, $\cos\theta = \dfrac{T_2 - T_1}{T_3}$

	Cases	cos θ	θ	Wetness	Equilibrium	Example
i.	$T_2 > T_1$	+ve	Acute	Liquid partially wets solid	Possible	Kerosene in contact with clean glass
ii.	$T_2 < T_1$	–ve	Obtuse	Liquid does not wet solid	Possible	Mercury in contact with clean glass
iii.	$T_1 = T_2$	0	90°	Liquid wets solid	Possible	Pure water in contact with silver
iv.	$T_2 - T_1 = T_3$	1	0	Liquids wets solid completely	Possible	Pure water contact with clean glass
v.	$T_2 - T_1 > T_3$	>1	Not Possible	–	Not possible (No drop formation possible and liquid spreads over a solid surface)	

Excess Pressure

Due to the property of surface tension, a drop or bubble tends to contract and thus compresses the matter enclosed. This in turn increases the internal pressure, which prevents further contraction and equilibrium is achieved. At equilibrium, the pressure inside a bubble or drop is greater than outside pressure. This difference of pressure between two sides of the liquid surface is called excess pressure.

Surface	Shapes	Excess Pressure
Plane		$\Delta P = 0$
convex surface		$\Delta P = \frac{2T}{R}$
Concave Surface		$\Delta P = \frac{2T}{R}$

Surface	Shapes	Excess Pressure
Drop		$\Delta P = \frac{2T}{R}$
Bubble in air		$\Delta P = \frac{4T}{R}$
Bubble in liquid (soap bubble)		$\Delta P = \frac{2T}{R}$

➤ **Capillarity (Capillary Action):**

- The rise or fall of level of liquid in a capillary tube is called as capillary action or capillarity.
- Capillarity does not violate law of conservation of energy.

Capillarity

A tube with a hole of very small diameter is called as capillary tube or capillary.

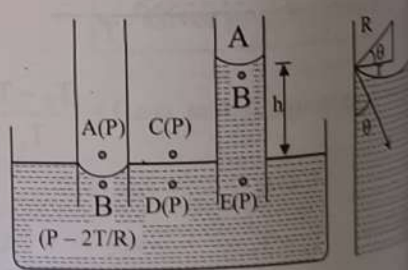
Capillary Rise | Capillary Fall

➤ **Ascent formula:**

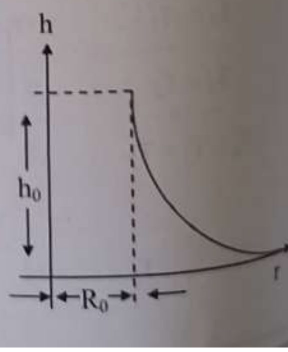
- Pressure due to liquid column = pressure-difference due to surface tension

$\therefore \quad hdg = \frac{2T}{R}$

$\therefore \quad h = \frac{2T}{Rdg} = \frac{2T\cos\theta}{rdg} \qquad ....(\because R = \frac{r}{\cos\theta})$



- The capillary rise depends on the nature of liquid and solid both i.e., on T, d, θ and R.
- The liquid rises in a capillary tube when the angle of contact is acute and it falls when the latter is obtuse.
- It is important to note that in equilibrium, the height h is independent of the shape of capillary if the radius of meniscus remains the same. That is why the vertical height h of a liquid column in capillaries of different shapes and sizes will be same if the radius of meniscus remains the same.



Factors affecting Surface Tension:

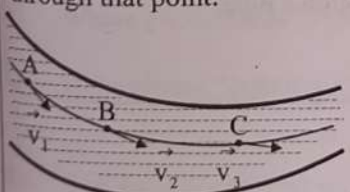
i.	**Nature of liquid**	The value of surface tension at interface depends on the nature of liquid under consideration but independent of area of surface or length of line considered.
ii.	**Impurities**	• If impurity is highly soluble, the surface tension of liquid increases. • If impurity is partially soluble, the surface tension of liquid decreases. • Surface tension increases by adding a waterproofing agent. • Addition of detergent decreases surface tension.
iii.	**Temperature**	• Variation of surface tension with temperature is given by, $T = T_0 (1 - \alpha\, \theta)$ where T and T_0 is the surface tension at θ °C and 0 °C respectively and α is the temperature – coefficient of surface tension. • Surface tension decreases with increase in temperature.
iv.	**Electrification**	When a soap bubble is charged either positively or negatively, its radius always increases as a force starts acting outwards normally to the surface of the liquid. Thus, surface tension decreases.

Fluid Dynamics:

Fluid Dynamics

Streamline Flow
($0 < R_n < 1000$)

Streamline flow of liquid is the flow in which each element of the liquid passing through a point travels along the same path and with the same velocity as the preceding element passing through that point.



Streamline flow

Path ABC in the figure is streamline flow and v_1, v_2, v_3 are velocities of particles at A, B, C respectively.

Reynold's Number

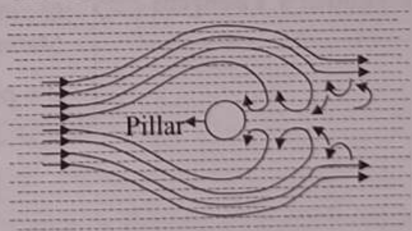
Reynold's number (R_n) is a pure number which determines the type of flow or nature of flow of liquid through a pipe. $R_n = \frac{v_c \rho D}{\eta}$

Critical Velocity

Critical velocity of a liquid is that velocity upto which its flow is streamlined and above which its flow becomes turbulent.

Turbulent Flow
($R_n > 2000$)

When a liquid moves with a velocity greater than its critical velocity, then the motion of the particles of liquid becomes disordered or irregular. Such a flow is called as a turbulent flow.



Turbulent flow at pillar

The path and the velocity of the particles of the liquid changes continuously and haphazardly with time from point to point.

Viscosity

1.
- The coefficient of viscosity is defined as the viscous force acting per unit area between two layers moving with unit velocity gradient.
- Viscosity of liquid is much greater (about 100 times more) than that of gases i.e., $\eta_L > \eta_G$
  e.g. selection of suitable lubricant in machine.

2.
Stokes Law:
The viscous force acting on a small sphere falling through a medium is directly proportional to the radius (r) of the sphere, its velocity (v) through fluid and coefficient of viscosity (η) of the fluid.
$F = 6\pi\eta rv$

3.
- The constant maximum velocity acquired by a body while falling through viscous fluid is called terminal velocity.
- Terminal velocity (v) is given by, $v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

  e.g. Rain drops attain terminal velocity when they fall through air.

Equation of continuity

For a non-viscous liquid in streamline flow passing through a tube of varying cross-section,

$av = \text{constant}$ or $a \propto \frac{1}{v}$

where, a = area of cross-section of tube, v = velocity of flow

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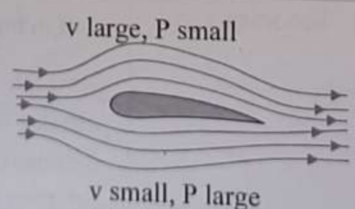
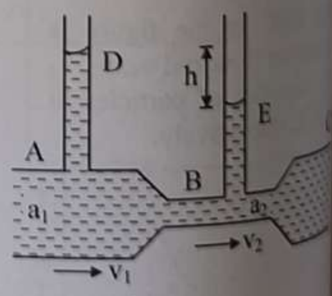
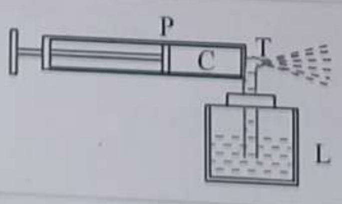
➢ Bernoulli's Theorem:

Bernoulli's Theorem

The total energy (pressure energy, potential energy and kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow, provided there is no source or sink of the fluid along the length of the pipe.
Mathematically, for unit volume of liquid flowing through a pipe,

$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant.}$

Applications of Bernoulli's Theorem

i. Lift on air craft wing:		iii. Venturimeter:	
ii. Atomiser:			

Formulae

1. Pressure: $P = \frac{F}{A}$
2. Absolute pressure: $P = P_0 + h\rho_g$
3. Pressure difference in liquid column: $P_2 - P_1 = h\rho g$
4. Surface tension: $T = \frac{F}{l}$
5. Force due to surface tension: $F = T \times l$
6. Surface energy: $E = dW = T(dA)$
7. Excess pressure inside a drop or air bubble: $P = \frac{2T}{r}$
8. Excess pressure inside a soap bubble: $P = \frac{4T}{r}$
9. Rise or fall of liquid in capillary tube: $h = \frac{2T\cos\theta}{r\rho g}$
10. Velocity gradient: $v_g = \frac{dv}{dx}$
11. Viscous force: $F = \eta A \frac{dv}{dx}$
12. Coefficient of viscosity: $\eta = \frac{Fdx}{Adv}$
13. Stokes' formula: $F = 6\pi\eta rv$
14. Terminal velocity: $v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g$
15. Reynold's number: $R_n = \frac{\rho v_c D}{\eta}$
16. Critical velocity: $v_c = \frac{N \times \eta}{\rho D}$
17. Equation of continuity for an incompressible fluid: $A_1v_1 = A_2v_2$
18. Bernoulli's equation: $P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 =$ constant
19. Speed of efflux: $v = \sqrt{2gh}$ ....[For a tank open to atmosphere]

Shortcuts

1. For a wire or ring lying on the surface of the liquid, always replace l by $2l$ for the wire and replace $2\pi R$ by $4\pi R$ for the ring.
2. If the phrase "isothermal change" appears in the question, then apply the formula, $r^2 = r_1^2 + r_2^2$. Where r = radius of bigger bubble and r_1 and r_2 = radii of smaller bubbles
3. If the phrase "common interface" appears in the question, then apply the formula, $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$.
4. The force F required to separate two glass plates in which a liquid film of area A, thickness t is enclosed is $F = \frac{2AT}{t}$.
5. In case of liquids which do not wet the walls of the containing vessel, the force of adhesion is less than $1/\sqrt{2}$ times the force of cohesion.
6. If the drop of liquid of density ρ floats half immersed in a liquid of density d and surface tension T, then radius of the drop is $R = \sqrt{\frac{3T}{(2\rho - d)g}}$
7. Excess pressure (P), radius (r) and volume V of a liquid drop are related by the relation, $\frac{PV}{r^2} =$ constant.

   For two different drops, $\frac{P_1V_1}{r_1^2} = \frac{P_2V_2}{r_2^2}$

8. If a capillary tube dipped vertically in a liquid kept in a vessel is placed in a lift at rest, then when the lift is moving up with an acceleration a, the height h′ through which the liquid will rise in the capillary tube

$h' = \frac{hg}{g+a}$;where h is capillary rise when the lift is at rest.

9. The height of the liquid column in a capillary tube on the surface of moon is six times than that on the earth.

10. If h′ is the height through which a liquid rises in a capillary tube at depth d below the surface of the earth then

$\frac{h}{h'} = \left(1 - \frac{d}{R}\right)$; where, R is radius of earth

11. The work done in breaking a big drop of liquid of radius R into small n droplets of equal radius r is

$W = 4\pi R^2 T (n^{1/3} - 1)$

12. An air bubble has surface area $8\pi R^2$ outside water and $4\pi R^2$ inside water.

13. For a drop outside liquid, surface area should be written as $4\pi R^2$ and the formula for work done should be $dW = T\ dA$ where dA is change in surface area.

14. If total pressure is P′ atmosphere then pressure due to liquid column is (P′ − 1) atmosphere.

15. When two capillary tubes are connected parallel under the same pressure head, the rates of flow are added.

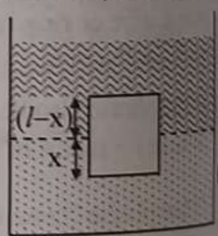
16. The vertical velocity of liquid at the orifice is zero.

17. The horizontal range of liquid coming out of orifice is maximum if the depth of orifice (h) from the free surface of liquid in vessel is half the total height (H) of the liquid in vessel (i.e., if h = H/2, then horizontal range is maximum = H)

18. If two holes at heights h_1 and h_2 are such that the liquid from these holes covers the same horizontal distance, then the heights of the liquid column is such that $H = h_1 + h_2$.

19. If a cube of side l remains suspended at the interface of two liquids in a vessel of densities ρ_1 and ρ_2 where $\rho_2 > \rho_1$ then, the true weight of the cube is,

$W = l^2 [x\rho_2 + (l - x)\rho_1]g$



Mindbenders

1. Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

2. Rain drop falling under gravity do not acquire high velocity. But in accordance with Stokes' law, they attain terminal velocity.

3. In most liquids, surface tension (surface energy per unit area) decreases with increase of temperature. However, for molten copper or molten cadmium surface tension increases with temperature.