CHAPTER 11

Definite Integration

Fundamental Theorem of Integral Calculus

Let f be a continuous function defined on the closed interval $[a, b]$ and if $\int f(x)dx = g(x)+C$.

Then, $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

In $\int_a^b f(x)dx$, a is called as a **lower limit** and b is called as an **upper limit**.

Properties of Definite Integrals

The properties of definite integral are as follows

(i) $\int_\alpha^\alpha f(x)\,dx = 0$

(ii) $\int_\alpha^\beta f(x)\,dx = \int_\alpha^\beta f(t)\,dt$

(iii) $\int_\alpha^\beta f(x)\,dx = -\int_\beta^\alpha f(x)\,dx$

(iv) (a) $\int_\alpha^\beta f(x)dx = \int_\alpha^\gamma f(x)dx + \int_\gamma^\beta f(x)dx$,

where $\alpha < \gamma < \beta$

(b) $\int_\alpha^\beta f(x)\,dx = \int_\alpha^{C_1} f(x)dx + \int_{C_1}^{C_2} f(x)dx + \dots + \int_{C_n}^\beta f(x)\,dx$, where $\alpha < C_1 < C_2 < \dots < C_n < \beta$

(v) (a) $\int_\alpha^\beta f(x)\,dx = \int_\alpha^\beta f(\alpha + \beta - x)\,dx$

(b) $\int_0^\alpha f(x)\,dx = \int_0^\alpha f(\alpha - x)\,dx$

(c) $\int_0^{b-c} f(x+c)\,dx = \int_c^b f(x)\,dx$

(vi) $\int_0^{2a} f(x)dx = \int_0^a f(x) + \int_0^a f(2a-x)dx$

(vii) $\int_{-\alpha}^\alpha f(x)\,dx$

$$= \begin{cases} 2\int_0^\alpha f(x)\,dx, \text{if } f(-x) = f(x) \\ \qquad \text{i.e. } f(x) \text{ is an even function} \\ 0, \text{ if } f(-x) = -f(x) \\ \qquad \text{i.e. } f(x) \text{ is an odd function} \end{cases}$$

$$= \int_0^\alpha [f(x) + f(-x)]\,dx$$

(viii) $\int_0^{2\alpha} f(x)\,dx = \begin{cases} 2\int_0^\alpha f(x)dx, & \text{if } f(2\alpha - x) = f(x) \\ 0, & \text{if } f(2\alpha - x) = -f(x) \end{cases}$

(ix) If $f(x)$ is a periodic function with period T, then
$\int_\alpha^{\alpha+nT} f(x)\,dx = n\int_0^T f(x)\,dx, n \in I$

(x) If $f(x) = f(a+x)$, then $\int_0^{na} f(x)dx = n\int_0^a f(x)dx$

(xi) $\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|\,dx$

Definite Integration by Substitution

There are several methods for finding the definite integral. One of the important methods for finding the definite integral is the method of substitution. To evaluate $\int_a^b f(x)dx$ by substitution, we use the following steps

***Step* I** Consider, the given integral without limits, i.e. $\int f(x)\,dx$ and substitute some part of integrand as another variable (say t), such that its differentiation exist in the integral, so that the given integral reduces to a known form.

***Step* II** Integrate the new integral with respect to the new variable without mentioning the constant of integration.

***Step* III** Replace the new variable by the original variable in the answer obtained in step II.

***Step* IV** Find the difference of the values, obtained in step III, at the upper and lower limits.