CHAPTER 06

Optics

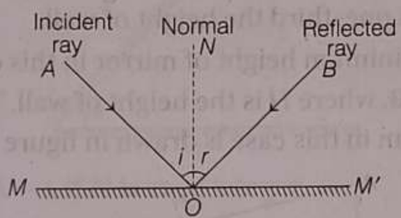
Light is a form of energy which produces sensation of vision to our eyes and makes objects visible. The branch of physics which deals with the study of nature of light, its properties, effects and propagation is known as optics.

Here, we will study two important phenomena of optics (light) namely reflection and refraction.

Reflection of Light

When light is incident on a polished smooth surface, then most of the incident light returns to the same medium. This phenomenon of returning of light after striking a smooth polished surface is called reflection of light.

After reflection, velocity, wavelength and frequency of light remain same but intensity decreases.



Reflection of light from a plane surface

Some Important Terms Related with Reflection of Light

(i) **Reflecting surface** The surface, after striking which, light reflects is called reflecting surface (MM').

(ii) **Incident ray** The incident ray (AO) on the reflecting surface is called incident ray.

(iii) **Reflected ray** The reflected ray (OB) after striking the reflecting surface is called reflected ray.

(iv) **Point of incidence** It is the point (O) on the reflecting surface at which incident light strikes.

(v) **Normal** It is the normal (ON) to the surface drawn from point of incidence (O) on the reflecting surface.

(vi) **Angle of incidence** It is the angle (i) between incident ray (AO) and the normal (ON). In the given figure, $\angle AON = i$.

(vii) **Angle of reflection** It is the angle (r) between reflected ray (OB) and the normal (ON). In the given figure, $\angle NOB = r$.

Laws of Reflection

There are two laws for reflection of light from a smooth reflecting surface

(i) **First law** The angle of incidence is equal to the angle of reflection, *i.e.* $\angle i = \angle r$.

(ii) **Second law** The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.

Image

Light rays emerging from an object (or a point) after reflection or refraction meets or appears to meet at a point is called image of object (or first point).

Images are of two types

(i) **Real image** When light rays emerging from a point object really meets at a point, the later point is called real image of the first point. These types of images can be obtained on a screen.

(ii) **Virtual image** When light rays emerging from a point object appears to meet at a point, the later point is called virtual image of the first point. These types of images cannot be obtained on a screen.

Note

- *If the incident rays diverge from a point object, the object is real object.*
- *If rays are converging towards a mirror, the point where these rays would meet, if there was no mirror is virtual object.*

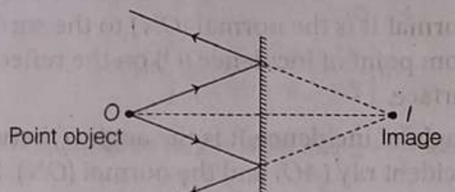
Reflection by a Plane Mirror

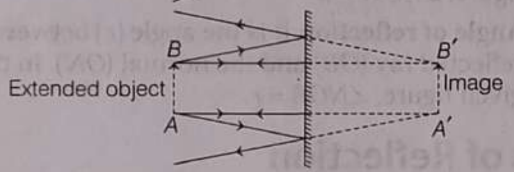
The mirror whose reflecting surface is plane, called plane mirror. Its one side is silvered and reflection take place from other side.

Characteristics of a plane mirror are given below

1. Properties of Image formed by Plane Mirror

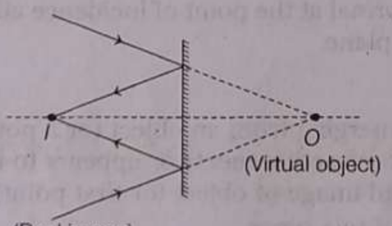
The image formed by a plane mirror is virtual, erect, of same size and at the same distance in opposite side from the mirror as the distance of object from mirror. The ray diagram of the image of a point object and of an extended object is as shown below





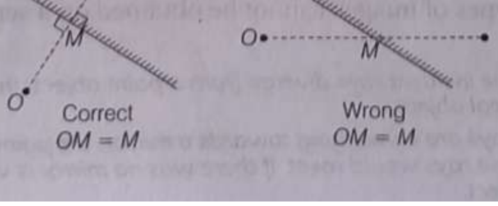
Formation of images by a plane mirror

When a convergent beam is incident on the plane mirror, the object (O) is virtual as shown in figure, in this case image (I) formed by the plane mirror is real.



Formation of image by plane mirror of a virtual object

Note *To find the location of image of an object from an inclined plane mirror, you have to see the perpendicular distance of the object from the mirror.*

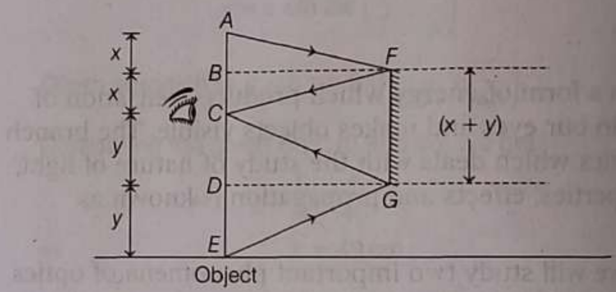


2. Field of View of an Object for a Plane Mirror

The field of view is the region between the extreme reflected rays and depends on the location of the object in front of the mirror.

Two important results based on the field of view are given below

(i) The minimum height of a plane mirror to see object's full height in it is $H/2$, where H is the height of object. But the mirror should be placed in a fixed position as shown in figure.



Minimum height of plane mirror to see full height

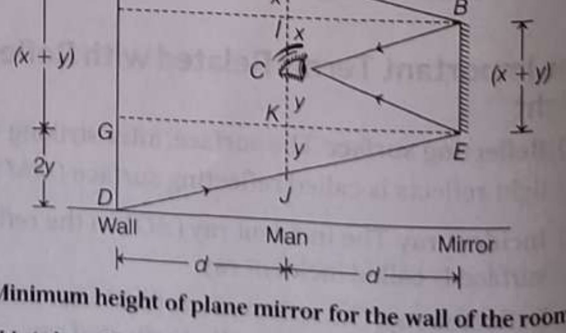
Height of the object $AE = 2(x + y) = H$

Height of the mirror $= (x + y) = \dfrac{H}{2}$

Note *The mirror can be placed anywhere between the centre line BF (of AC) and DG (of CE).*

(ii) Minimum height of the plane mirror fixed on the wall of a room in which an observer at the centre of the room can see the full image of the wall behind him, is one-third the height of wall.

The minimum height of mirror in this case should be $H/3$, where H is the height of wall. The ray diagram in this case is drawn in figure



Minimum height of plane mirror for the wall of the room

Height of the wall is $3(x + y)$ while that of the mirror is $(x + y)$.

3. Special Cases of Image Formation through a Plane Mirror

(i) If the object is displaced by a distance a towards or away from the plane mirror, image will also be displaced by a distance a towards or away from the mirror.

(ii) If an object approaches or recedes from the plane mirror with velocity v, then the image also moves in the same manner from the mirror with velocity v.

But the velocity of the image with respect to object will be $2v$.

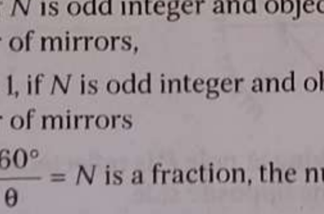
(iii) If plane mirror moves a distance x towards or away from the object, the image will move a distance $2x$ towards or away from the mirror/object.

(iv) If both plane mirror and object are moved by a distance x, each in opposite directions, the image will be displaced by a distance $3x$ in the direction of the displacement of the mirror.

(v) If a luminous object is placed in front of a thick glass mirror, multiple images are formed due to multiple reflections. The second image formed by the first reflection by the polished surface is much brighter than the others. The intensity of the other images rapidly fade away.

(vi) If an object is placed between two mirrors facing each other at an angle θ, the number of images is given by

$\frac{360^\circ}{\theta} = N$ and actual number of images are n, where



(a) $n = N - 1$, if N is even integer,

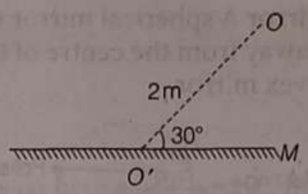
(b) $n = N$, if N is odd integer and object is not on the bisector of mirrors,

(c) $n = N - 1$, if N is odd integer and object is on the bisector of mirrors

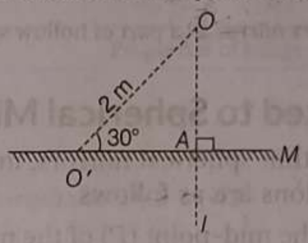
(d) and if $\frac{360^\circ}{\theta} = N$ is a fraction, the number of images will be equal to its lower boundary of integral part.

Example 1. A point object O is at an angle of 30° from the plane mirror M, as shown in figure. If $OO' = 2$ m, then the location of image will be

a. 4 m b. 6 m
c. 1 m d. 2 m



Sol. (c) To find the location of image, we have to draw a perpendicular on the mirror M from point O as shown in figure



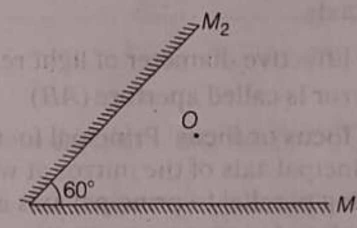
Now from the triangle $O'AO$, we have

$\sin 30^\circ = \frac{OA}{2} \Rightarrow OA = 2 \times \sin 30^\circ = 1$ m

$\therefore$ Distance of the image from the mirror is $AI = OA = 1$ m.

Example 2. Consider two plane mirrors inclined at an angle 60° as shown in figure. The number of images of object O formed will be

a. 5 b. 2 c. 8 d. 10



Sol. (a) Two mirrors M_1 and M_2 are inclined at angle θ.

Let, $N = \frac{360^\circ}{\theta}$

For $\theta = 60^\circ$, $N = \frac{360^\circ}{60^\circ} = 6$ = even integer

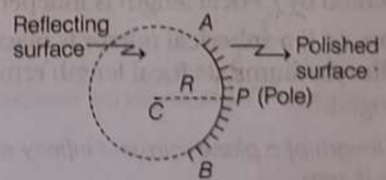
$\therefore$ Number of images formed $n = N - 1 = 6 - 1 = 5$

Reflection from Curved Mirror (Spherical Mirror)

Spherical mirror is a mirror whose reflecting surface is a part of a hollow sphere.

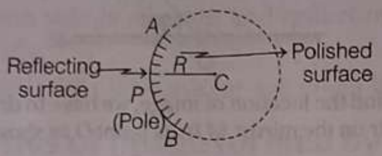
Spherical mirrors are of two types

(i) **Concave Mirror** A spherical mirror whose reflecting surface is towards the centre of the sphere is called concave mirror.



Concave mirror as a part of hollow sphere

(ii) **Convex Mirror** A spherical mirror whose reflecting surface is away from the centre of the sphere is called convex mirror.



Convex mirror as a part of hollow sphere

Terms Related to Spherical Mirror

In case of these thin spherical mirrors, important terms and their definitions are as follows

(i) **Pole** It is the mid-point (P) of the mirror.

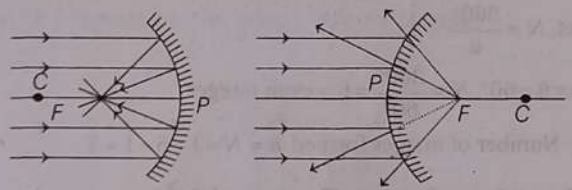
(ii) **Centre of curvature** It is the centre C of the sphere of which the mirror is a part.

(iii) **Radius of curvature** Distance between pole and centre of curvature is called radius of curvature. In given diagram, the distance CP is radius of curvature.

(iv) **Principal axis** The line joining the pole P and centre of curvature C when produced is called principal axis.

(v) **Aperture** Effective diameter of light reflecting area of the mirror is called aperture (AB).

(vi) **Principal focus or focus** Principal focus is a point on the principal axis of the mirror at which the light rays coming parallel to principal axis actually meet after reflection (or appears to meet).



(a) Concave mirror (a converging mirror) (b) Convex mirror (a diverging mirror)

Principal focus of (a) concave mirror (b) convex mirror

Focus of concave mirror is real, while focus of convex mirror is virtual.

(vii) **Focal length** The distance between pole and focus of a spherical mirror is called its focal length. It is represented by f. Focal length is independent of medium. *i.e.* If a spherical mirror is placed in water or any other mediums, its focal length remains unchanged.

Note *Focal length of a plane mirror is infinity and hence its power is zero.*

(viii) Focal length of a mirror is equal to half the radius of curvature of the mirror.

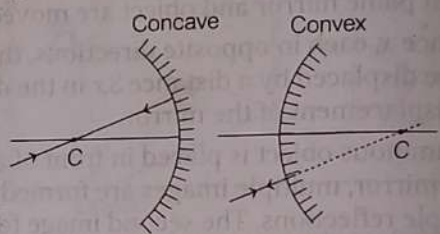
i.e. $$f = \frac{R}{2}$$

Rules for Image Formation in Spherical Mirrors

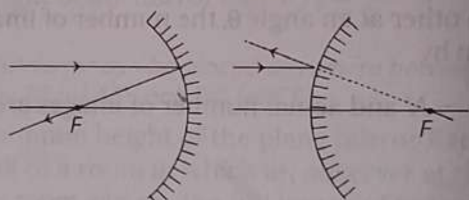
In ray optics, to locate the image of an object, tracing of a ray as it reflects is very important.

Following four types of rays are used for image formation

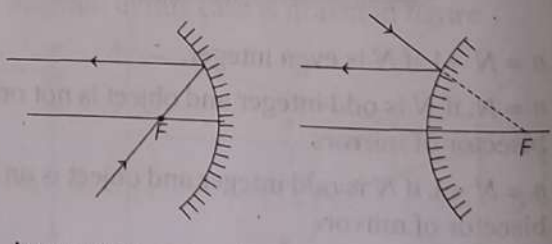
Ray 1. A ray through the centre of curvature which strikes the mirror normally and is reflected back along the same path.



Ray 2. A ray parallel to principal axis after reflection either actually passes through the principal focus F or appears to diverge from it.



Ray 3. A ray passing through the principal focus F or a ray which appears to converge at F is reflected parallel to the principal axis.



Ray 4. A ray striking at pole P is reflected symmetrically back in the opposite side.

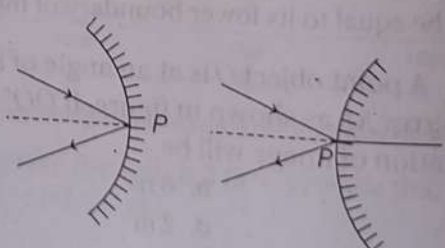


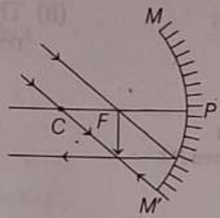
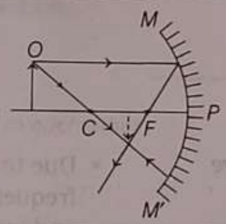
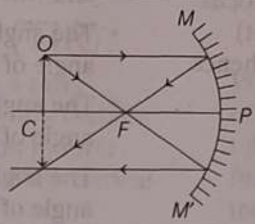
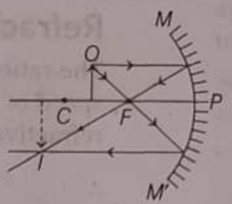
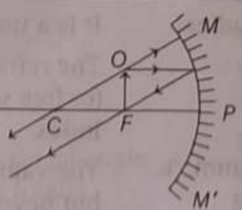
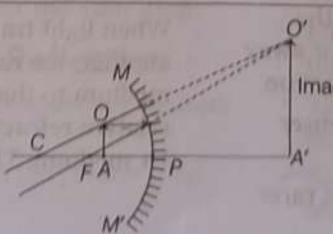
Image Formation by Spherical Mirrors

Ray diagrams of image formation by spherical mirrors are discussed below

1. Image Formation by Concave Mirror

In case of a concave mirror, the image is erect and virtual when the object is placed between F and P. In all other positions of object, the image is real and inverted as shown in the table given below

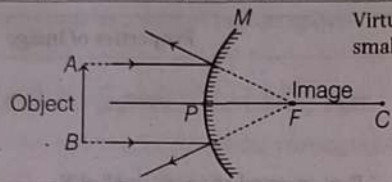
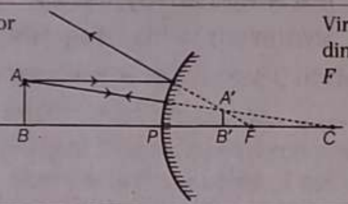
Image Formation by Concave Mirror

S.No.	Position of Object	Ray Diagram	Properties of Image
(i)	At infinity		Real, inverted and very small at F
(ii)	Between infinity and C		Real, inverted and diminished between F & C
(iii)	At C		Real, inverted and equal in size at C
(iv)	Between F and C		Real, inverted and very large between 2 F & infinity
(v)	At F		Real, inverted and very large at infinity
(vi)	Between F and P		Virtual, erect and large in size behind the mirror

2. Image Formation by Convex Mirror

Image formed by convex mirror is always virtual, erect and diminished no matter where the object is. All the images formed by this mirror will be between pole and focus as shown in the table given below

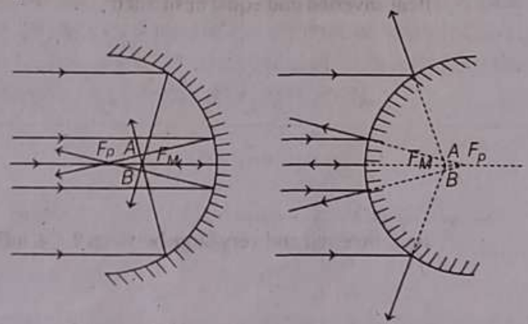
Image Formation by Convex Mirror

S.No.	Position of Object	Ray Diagram	Properties of Image
(i)	At infinity		Virtual, erect and very small in size at F
(ii)	In front of mirror		Virtual, erect and diminished between P & F

Spherical Aberration of Mirror

Spherical aberration is applicable only for small aperture spherical mirror and for paraxial rays.

When the rays are farther from the principal axis, the focus gradually shifts towords pole. This phenomenon (defect) arises due to spherical shape of the reflecting surface, hence called as spherical aberration.



Spherical aberration for curved mirrors

Here, the distance between F_M and F_p gives the longitudinal spherical aberration.

Refraction of Light

When a light passes from one medium to another medium, a part is reflected back into the first medium and the rest passes into the second medium. When it passes into the second medium, it either bends towards the normal or away from the normal. This phenomenon is known as refraction.

- When ray of light goes from a rarer medium to a denser medium, it bends towards the normal.
- When a ray of light goes from a denser medium to a rarer medium, it bends away from the normal.

Laws of Refraction

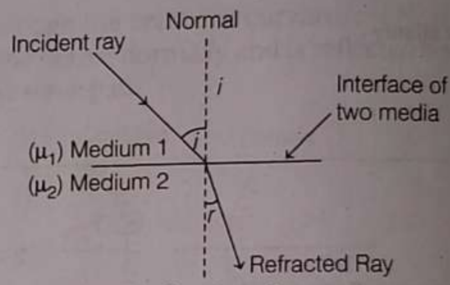
There are two laws of refraction as follows

(i) For two particular media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant.

i.e. $$\frac{\sin i}{\sin r} = \text{constant} = \frac{\mu_2}{\mu_1}$$

This is known as **Snell's law.**

(ii) The incident ray, the reflected ray and the refracted ray all lie in the same plane.



Refraction of ray

- Due to refraction of light, their is no change in frequency of light but there is a change in velocity and wavelength.
- The angle made by incident ray with the normal is angle of incidence.
- The angle made by refracted ray with the normal is angle of refraction.
- The angle made by incident ray with the surface is angle of glancing.

Refractive Index

The ratio of the speed of light in vacuum (c) to the speed of light in a given medium (v) is known as the refractive index of that medium.

$$\therefore \qquad \mu = \frac{c}{v}$$

It is a unitless, dimensionless and a scalar quantity.

The refractive index of a medium w.r.t. vacuum (or free space) is known as its **absolute refractive index**.

The value refractive index can be 1 or more than 1 but never less than 1.

When light travels from one material medium to another, the ratio of the speed of light in the 1st medium to that in the 2nd medium is known as the **relative refractive index of 2nd medium, w.r.t. the 1st medium.** Thus,

$$\mu_{21} = \frac{v_1}{v_2} = \frac{c/v_2}{c/v_1} = \frac{\mu_2}{\mu_1}$$

Note

- *Refractive index (μ) of a material changes with respect to wavelength of light as* $\mu = A + \frac{B}{\lambda^2}$, *where A and B are constants.* ***(Cauchy's formula)***
- *When a light ray, after suffering any number of reflections and refractions, has its final path reversed, it travels back along its entire initial path. This is called* ***principle of reversibility of light.***

Lateral Shift

When refraction of light takes place through a rectangular glass slab, then lateral shift,

$$L = \frac{t \sin(i_1 - r_1)}{\cos r_1}$$

Here, i_1 = angle of incidence during Ist refraction,

r_1 = angle of reflection during Ist refraction

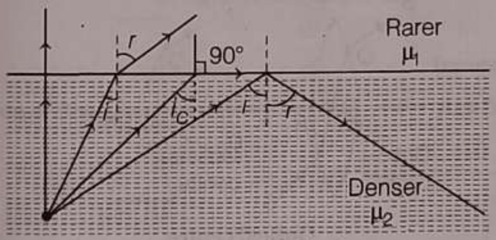
and t = thickness of the glass slab.

Note

- *Apparent depth and real depth are related to each other as* $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$
where, μ = *refractive index of water.*

Total Internal Reflection (TIR)

When a ray of light goes from a denser to a rarer medium, it bends away from the normal. For a certain angle of incidence i_C, the angle of refraction in rarer medium becomes 90°. The angle i_C is called the **critical angle.**



Total internal reflection

$$\sin i_C = \frac{\mu_1}{\mu_2} = \frac{1}{\mu_{21}}$$

For the angle of incidence greater than the critical angle ($i > i_C$) in the denser medium, the light ray is totally internally reflected back into the denser medium itself.

Conditions for Total Internal Reflection

(i) The light ray should travel from the denser medium to the rarer medium.

(ii) The angle of incidence should be the greater than the critical angle.

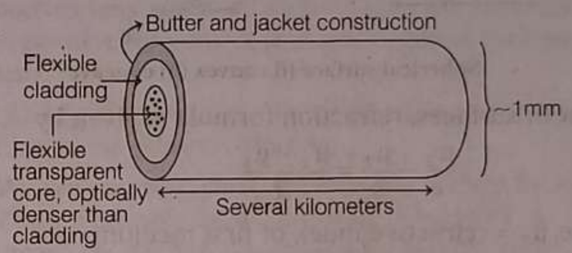
Applications of Total Internal Reflection

1. Diamond

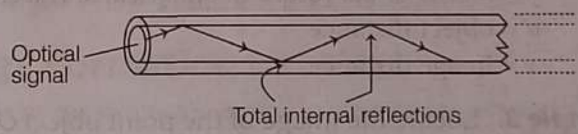
Spectacular brilliance of diamond is due to its total internal reflection.

2. Optical Fibre

They are fabricated with high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of cladding.



Optical fibre construction



Optical fibre working

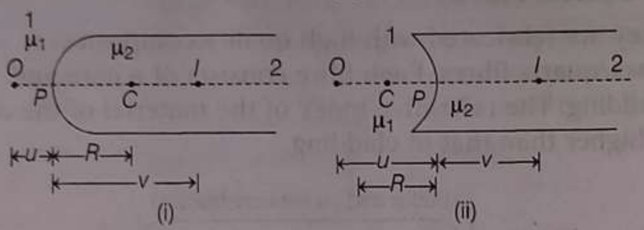
When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflection along the length of the fibre and finally comes out at the other end. Since, light undergoes total internal reflection at each stage there is no appreciable loss in intensity of light. Hence, these are used for transmission of optical signals.

Some of the advantages of optical fibre communication are listed below

- **Broad bandwidth (frequency range)** For TV signals, a single optical fibre can carry over 90000 channels (independent signals).
- **Immune to EM interference** Being electrically non-conductive, it is not able to pick up nearby EM signals.
- **Low attenuation loss** The loss is lower than 0.2 dB/km, so that a single long cable can be used for several kilometers.
- **Electrical insulator** No issue with ground loops of metal wires or lightning.
- **Theft prevention** It is does not use copper or other expensive material.
- **Security of information** Internal damage is most unlikely.

Refraction at Single Curved Surface

When two transparent media are separated by a spherical surface, light incident on the surface from one side get refracted into the medium on the other side. Spherical surfaces are of two types as shown in figure



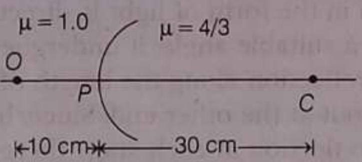
Spherical surface (i) convex (ii) concave

For both surfaces, refraction formula is given by

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

where, μ_1 = refractive index of first medium,
μ_2 = refractive index of second medium,
R = radius of curvature of spherical surfaces,
u = object distance
and v = image distance.

Example 3. Locate the image of the point object O. The point C is centre of curvature of the spherical surface, is



a. –15 cm b. 15 cm
c. 10 cm d. -10 cm

Sol (a) Here, $\mu_1 = 1, \mu_2 = 4/3, u = -10$ cm and $R = 30$ cm

Using the relation, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Substituting the values,

$$\Rightarrow \frac{4/3}{v} - \frac{1}{-10} = \frac{(4/3) - 1}{30}$$

$$\Rightarrow \frac{4}{3v} + \frac{1}{10} = \frac{1}{90}$$

$$\Rightarrow \frac{4}{3v} = \frac{1}{90} - \frac{1}{10} = \frac{-8}{90}$$

$$\Rightarrow v = -15 \text{ cm}$$

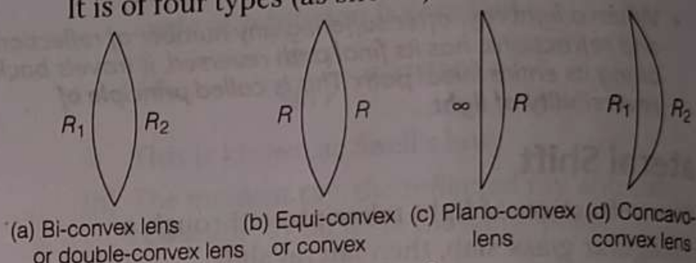
The image is formed 15 cm left of spherical surface and is virtual.

Refraction at a Spherical Surface (Lenses)

Lens is a transparent medium (material) bounded by two surfaces atleast one of which should be spherical.

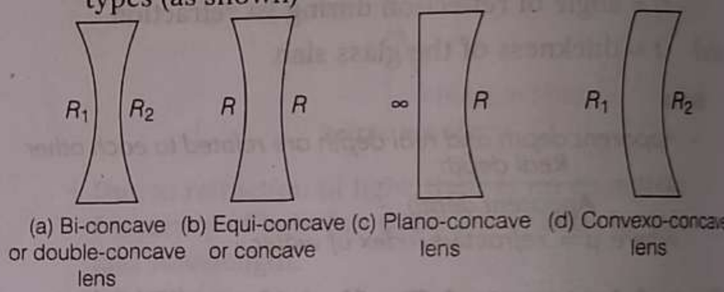
Lenses are of two types

(i) **Convex or converging lens** In this lens, a transparent medium is bounded by two surfaces such that it is thicker in the middle.
It is of four types (as shown)



Types of convex lens

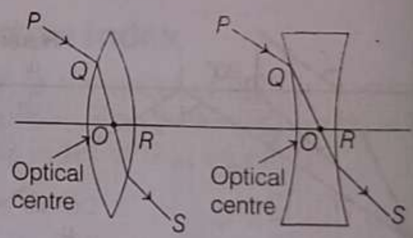
(ii) **Concave or diverging lens** In this lens, a transparent medium is bounded by two surfaces such that it is thinner in the middle. It is of four types (as shown)



Types of concave lens

Some Definitions Related to Lenses

(i) **Optical centre** The optical centre is a point within or outside the lens, at which incident rays refract without deviation in its path.



Optical centre

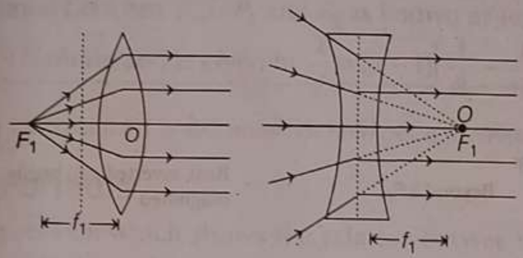
(ii) **Centre of curvature** The centres of the two imaginary spheres of which the lens is a part are called centres of curvature of the lens. A lens has two centres of curvature with respect to its two curved surfaces.

(iii) **Radii of curvature** The radii of the two imaginary spheres of which the lens is a part are called radii of curvature of the lens. A lens has two radii of curvature. These may or may not be equal.

(iv) **Principal axis** The straight line passing through the optical centre of lens is called principal axis of lens.

(v) **Principal focus** Lens has two principal focus

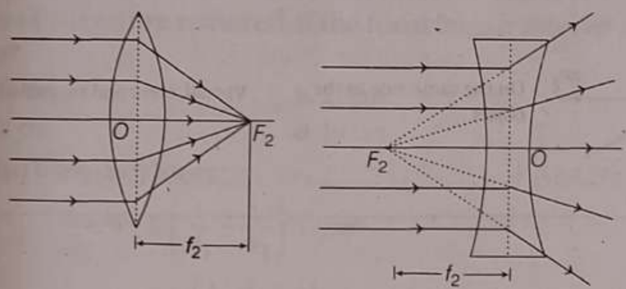
(a) **First principal focus** It is a point on the principal axis of lens, the rays starting from which (convex lens) or appear to converge at which (concave lens) become parallel to principal axis after refraction.



First principal focus

(b) **Second principal focus** It is the point on the principal axis at which the rays coming parallel to the principal axis converge (convex lens) or appear to diverge (concave lens) after refraction from the lens.

Both the focus of convex lens are real, while that of concave lens are virtual.



Second principal focus

(vi) **Focal length** The distance between focus and optical centre of lens is called focal length of lens.

(vii) **Aperture** The effective diameter of circular boundary of the lens is called aperture.

Intensity $\propto$ (Aperture)^2

Sign Convention for Lenses

(i) Sign convention for lenses are same as for spherical surfaces.

(ii) From the figures, we can see that f_1 is negative for a convex lens and positive for a concave lens. But f_2 is positive for convex lens and negative for concave lens.

(iii) $|f_1| = |f_2|$, if the media on the two sides of a thin lens have same refractive index.

(iv) We are mainly concerned with the second focus f_2. Thus, wherever we write the focal length f, it means the second principal focal length. Thus, $f = f_2$ and hence f is positive for a convex lens and negative for a concave lens.

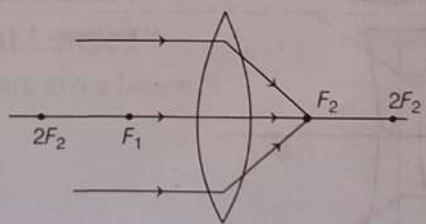
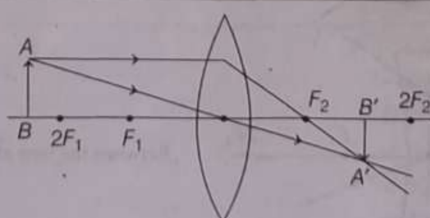
Image Formation by Lens

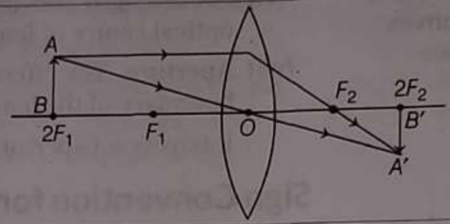
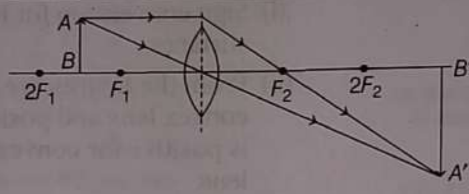
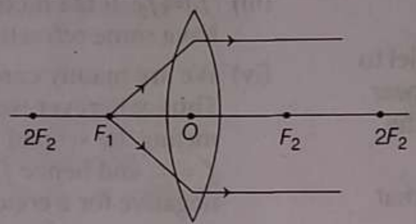
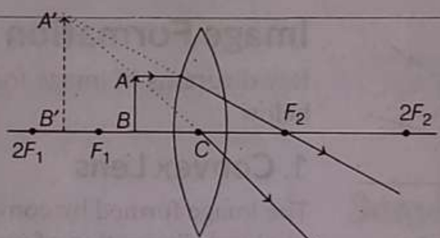
Ray diagrams of image formation by lenses are discussed below

1. Convex Lens

The image formed by convex lens depends on the position of object. Formation of image by convex lens for different positions of object is shown in the table given below

Image Formation by Convex Lens

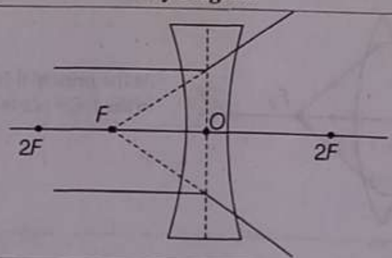
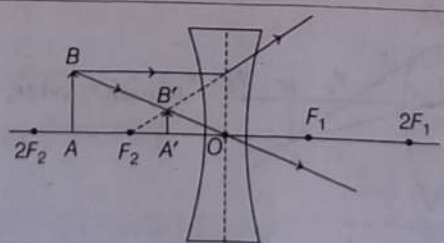
S.No.	Position of Object	Ray Diagram	Position of Image	Nature and Size of Image
(i)	At infinity		At the principal focus (F_2) or in the focal plane	Real, inverted and extremely diminished
(ii)	Beyond $2F_1$		Between F_2 and $2F_2$	Real, inverted and diminished

S.No.	Position of Object	Ray Diagram	Position of Image	Nature and Size of Image
(iii)	At $2F_1$		At $2F_2$	Real, inverted and of same size as the object
(iv)	Between F_1 and $2F_1$		Beyond $2F_2$	Real, inverted and highly magnified
(v)	At F_1		At infinity	Real, inverted and highly magnified
(vi)	Between F_1 and optical centre		On the same side as the object	Virtual, erect and magnified

2. Concave Lens

The image formed by a concave lens is always virtual, erect and diminished (like a convex mirror). The image formation by concave lens for different positions of object is shown in the table given below

Image Formation by Concave Lens

S.No.	Position of Object	Ray Diagram	Position of Image	Nature and Size of Image
(i)	At infinity		At the focus	Virtual, erect and point size
(ii)	Anywhere on the principal axis		Between the lens and F_2	Virtual, erect and diminished

Lens Maker's Formula

If R_1 and R_2 are the radii of curvature of first and second refracting surfaces of a thin lens of focal length f and refractive index μ (with respect to surrounding medium), the relation between f, μ, R_1 and R_2 is known as lens maker's formula and is given by $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Note *The relation $f = R/2$ does not hold good for lenses.*

Lens Formula

The expression which shows the relation between u, v and f is called lens formula and given by

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

where, u is distance of object from the lens, v is distance of image from the lens and f is focal length of the lens.

Example 4. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required, if the focal length is to be 20 cm?

a. 20 cm b. 21 cm
c. 22 cm d. 19 cm

Sol (c) Using the relation,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \text{ we get}$$

$$\frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} - \left(\frac{1}{-R}\right)\right]$$

$$= 0.55\left[\frac{1}{R} + \frac{1}{R}\right] = 0.55 \times \frac{2}{R}$$

or $\quad R = 1.10 \times 20 = 22$ cm

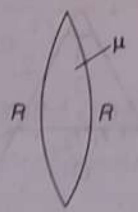
Focal Lengths of Different Lenses

Focal lengths of different lenses are given below

(i) **For equi-biconvex lens,**

$R_1 = +R$ and $R_2 = -R$

$$\therefore \quad f = \frac{R}{2(\mu - 1)}$$

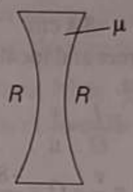


Equi-biconvex lens

(ii) **For equi-biconcave lens,**

$R_1 = -R$ and $R_2 = +R$

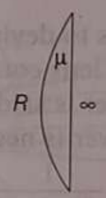
$$\therefore \quad f = -\frac{R}{2(\mu - 1)}$$



Equi-biconcave lens

(iii) **For plano-convex lens** $R_1 = -R$ and $R_2 = \infty$

$$\therefore \quad f = \frac{R}{\mu - 1}$$

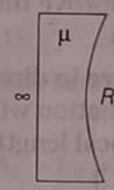


Plano-convex lens

(iv) **For plano-concave lens,** $R_1 = \infty$, $R_2 = R$

$$\therefore \quad f = \frac{-R}{(\mu - 1)}$$

For a lens



Plano-concave lens

Magnification Produced by a Lens

The ratio of the size of the image to the size of object is called magnification.

Magnification m produced by a lens,

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{I}{O} = \frac{v}{u}$$

$$m = \frac{f}{f + u} = \frac{f - u}{f}$$

Example 5. An object of size 3.0 cm is placed at 14 cm from concave lens of focal length 21 cm. Describe the image produced by the lens. What happens, if the object is moved further away from the lens?

a. Virtual, image moves towards the focus
b. Erect, image moves towards the centre
c. Both (a) and (b)
d. None of the above

Sol (a) Using the relation,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}, \text{ we get}$$

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{-21}+\frac{1}{(-14)}$$

$\therefore$ $v=-8.4$ cm

So, the image is virtual, erect and located at 8.4 cm from the lens on the same side as the object.

Also, we know that, $m=\frac{I}{O}=\frac{v}{u}$

$$\therefore \quad I=\frac{v}{u}\times O=\frac{-8.4}{-14}\times 3=1.8 \text{ cm}$$

i.e. The image is of diminished size.

If the object is moved away from the lens, the virtual image moves towards the focus of the lens (but never beyond focus).

Power of a Lens

It is the ability of the lens to deviate the path of rays passing through it. If the lens converges the rays parallel to principal axis, its power is said positive and if it diverges the rays, its power is negative.

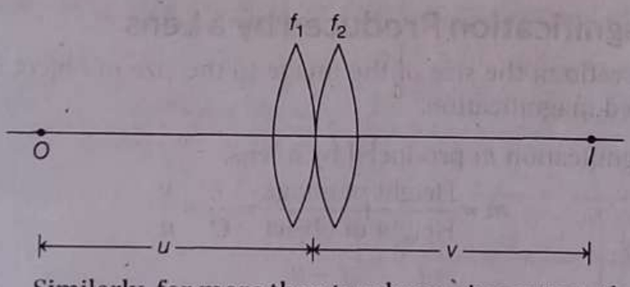
$$P(\text{in dioptre})=\frac{1}{f(\text{metre})}$$

Combination of Two or More Thin Lenses

Combinations of lenses in contact are used in many optical instruments to improve their performance. It is studied in two conditions

(i) **When two lenses are in direct contact** (zero separation) Combination will behave as a lens, which has lesser focal length.

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$



Similarly, for more than two lenses in contact, the equivalent focal length is given by the formula,

$$\frac{1}{F}=\sum_{i=1}^{n}\frac{1}{f_i}=\frac{1}{f_1}+\frac{1}{f_2}+\ldots$$

Note *Here f_1, f_2, etc., are to be substituted with sign.*

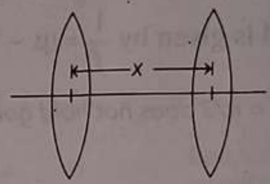
(a) Power of combination, $P=P_1+P_1+\ldots=\sum_{i=1}^{n}P_i$

(b) Magnification of combination,

$$M=m_1\times m_2\times\ldots=\prod_{i=1}^{n}m$$

(ii) **When lenses are separated by a distance** If two lenses of focal lengths f_1 and f_2 are separated by a distance x, then its equivalent focal length,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2}$$



Combination of two convex lenses with x separation

(a) Power of combination,

$$P=P_1+P_2-xP_1P_2$$

(b) Total magnification remains unchanged, *i.e.*

$$m=m_1\times m_2$$

Note ***Conjugate focii*** Two points so related to a lens or concave mirror that an image at one point is focused at the other and *vice-versa*, this phenomena is called the conjugate focii.

Example 6. A converging lens of focal length 5.0 cm is placed in contact with a diverging lens of focal length 10.0 cm. Find the combined focal length of the system.

a. +10.0 cm **b.** –10.0 cm

c. 5.0 cm **d.** –5 cm

Sol (*a*) Here, $f_1=+5.0$ cm and $f_2=-10.0$ cm

Therefore, the combined focal length F is given by

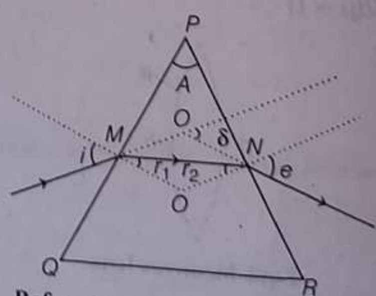
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{5.0}-\frac{1}{10.0}=+\frac{1}{10.0}$$

$\therefore$ $F=+10.0$ cm

i.e. The combination behaves as a converging lens of focal length 10.0 cm.

Deviation by Prism

A prism is a homogeneous and transparent medium bounded by two plane surfaces inclined at an angle A with each other. These surfaces are called as **refracting surfaces** and the angle between them is called **angle of prism** A.



Refraction of light through prism

The above figure shows the refraction of monochromatic light through a prism. Here, i & e represent the angle of incidence & angle of emergence respectively and r_1 & r_2 are two angles of refraction. If μ is the refractive index of the material of the prism, then

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin e}{\sin r_2}$$

The angle between the incident ray and the emergent ray is known as the **angle of deviation** (δ). For refraction through a prism, it is found that

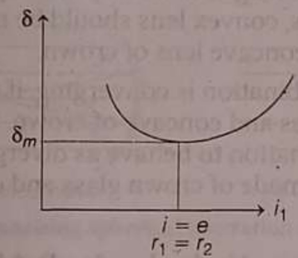
$$i + e = A + \delta$$

where, $r_1 + r_2 = A$.

Minimum Deviation

It is found that the angle of deviation δ varies with the angle of incidence i of the ray incident on the first refracting face of the prism. The variation is shown in given figure and for one angle of incidence, it has a minimum value (δ_{min}).

At this value,



$$i = e$$

It therefore, follows that,

$$r_1 = r_2 = r$$

$$r = \frac{A}{2}$$

Further at $\delta = \delta_m = (i + i) - A$

$$i = \frac{A + \delta_m}{2}$$

$$\mu = \frac{\sin i}{\sin r}$$

or $$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\left(\sin\frac{A}{2}\right)}$$

The above expression is also called **prism formula.**

Note *For very small angle of prism A (thin prism), deviation is given by*

$$\delta \approx (\mu - 1)A$$

*where, μ is **refractive index of material of prism.***

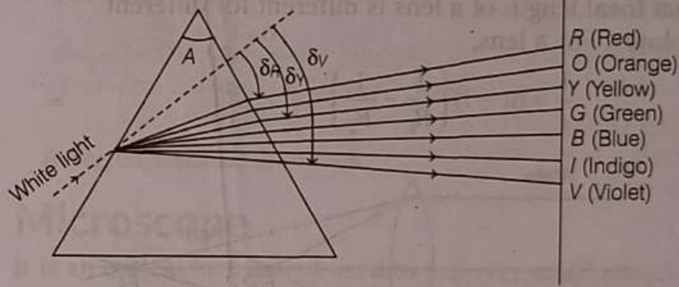
Dispersion of Light by a Prism

When a white light enters into a prism, it splits into its seven components in the decreasing sequence of deviation, *i.e.* violet, indigo, blue, green, yellow, orange and red. This phenomenon is called dispersion of light by a prism.

It is caused due to variation of refractive index of prism with wavelength.

As, $\lambda_V > \lambda_R$, we get

$$\delta_V > \delta_R$$



Formation of VIBGYOR by prism

Mean deviation, $\delta_{VR} = \frac{\delta_V + \delta_R}{2} = \delta_Y = (\mu_Y - 1)A$

Angular Dispersion

It is the angular separation between the two extreme rays.

Angular dispersion, $\theta = \delta_V - \delta_R = (\mu_V - \mu_R)A$

Dispersive Power

The dispersive power of a prism material is measured by the ratio of angular dispersion to the mean deviation (δ_y) suffered by light beam.

$\therefore$ Dispersive power, $\omega = \frac{\delta_V - \delta_R}{\delta_Y} = \frac{\mu_V - \mu_R}{\mu - 1}$

where, $\mu = \mu_Y$ = mean refractive index.

It is unitless and dimensionless quantity.

Example 7. White light is passed through a prism of angle 4°. If the refractive indices for red and blue colours are 1.641 are 1.659 respectively, then the angle of deviation and the dispersive power are

a. 2.6°, 0.028 **b.** 3.6°, 0.038 **c.** 4.2°, 0.048 **d.** 3.6°, 0.048

Sol. (*a*) Angle of deviation,

$$\delta = (\mu_Y - 1)A = \left(\frac{\mu_R + \mu_B}{2} - 1\right)4°$$

$$= \left(\frac{1.641 + 1.659}{2} - 1\right) \times 4° = (1.650 - 1) \times 4° = 2.6°$$

Dispersive power,

$$\omega = \frac{\mu_B - \mu_R}{\mu_Y - 1} = \frac{1.659 - 1.641}{1.650 - 1} = 0.028$$

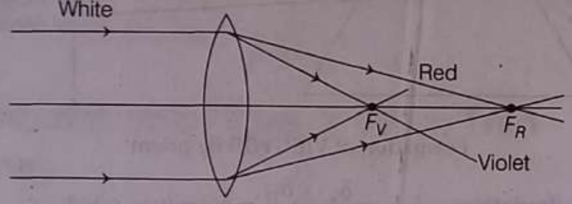
Defects of Lenses

Actual image formed by an optical system is usually imperfect. The defects of images are called **aberrations.** The defect may be due to light or optical system. If the defect is due to light, it is called **chromatic aberration,** and if due to optical system, **monochromatic aberration.**

1. Chromatic Aberration

The image of an object formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration. This defect arises due to the fact that focal length of a lens is different for different colours. For a lens,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$



Chromatic aberration

As, μ is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it.

The difference between f_R and f_V is a measure of longitudinal chromatic aberration. Thus,

$$\text{LCA}=f_R-f_V=\omega f_Y$$

where, $$\omega=\frac{\mu_V-\mu_R}{\mu_Y-1}$$

Here, $$\mu_Y=\frac{\mu_V+\mu_R}{2}$$

Condition of Achromatism

To get achromatism, we use a pair of two lenses in contact. For two thin lenses in contact, we have

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\therefore \quad \frac{-dF}{F^2}=-\frac{df_1}{f_1^2}-\frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration, if

$$dF=0$$

$$\therefore \quad \frac{df_1}{f_1^2}+\frac{df_2}{f_2^2}=0$$

$$\therefore \quad \frac{\omega_1 f_1}{f_1^2}+\frac{\omega_2 f_2}{f_2^2}=0$$

$$\therefore \quad \frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$$

This is the **condition of achromatism.** From the condition of achromatism, following conclusions can be drawn

(i) As ω_1 and ω_2 are positive quantities, f_1 and f_2 should have opposite signs, *i.e.* if one lens is convex, the other must be concave.

(ii) If $\omega_1=\omega_2$, means both the lenses are of same material, then

$$\frac{1}{f_1}+\frac{1}{f_2}=0 \text{ or } \frac{1}{F}=0$$

or $$F=\infty$$

Thus, the combination behaves as a plane glass plate. So, we can conclude that both the lenses should be of different materials or $\omega_1 \neq \omega_2$.

(iii) Dispersive power of crown glass (ω_C) is less than that of flint glass (ω_F).

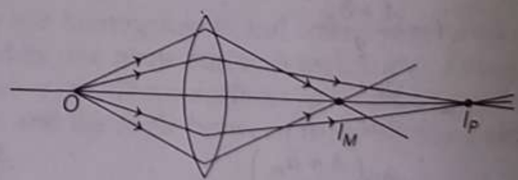
(iv) If we want the combination to behave as a convergent lens, then convex lens should have lesser focal length or its dispersive power should be more. Thus, convex lens should be made of flint glass and concave lens of crown.

Thus, combination is converging, if convex is made of flint glass and concave of crown. Similarly, for the combination to behave as diverging lens, convex is made of crown glass and concave of flint glass.

2. Monochromatic (spherical) Aberration

This is the defect in image due to optical system. Monochromatic aberration is of many types such as, spherical, distortion, curvature and antigmatism. Here, we shall limit ourselves to spherical aberration only.

Spherical aberration arises due to spherical nature of lens (or mirror).

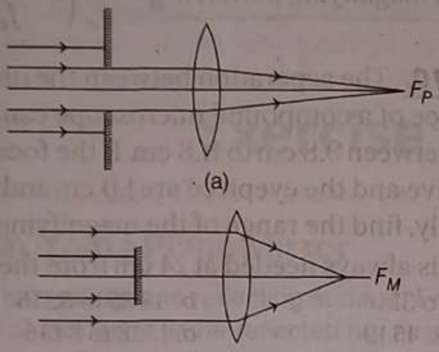


Spherical aberration

The paraxial rays (close to optic axis) get focused at I_P and marginal rays (away from the optic axis) are focused at I_M. Thus, image of a point object O is not a point.

The inability of the lens to form a point image of an axial point object is called **spherical aberration.** Spherical aberration can never be eliminated but can be minimised by the following methods

(i) **By using stops** By using stops, either paraxial or marginal rays are cut-off.

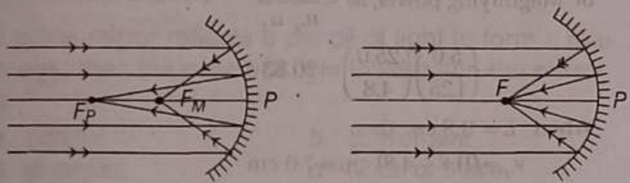


(a)
(b)

Minimising spherical aberration using stops

(ii) **Using two thin lenses separated by a distance** Two thin lenses separated by a distance $d = f_2 - f_1$, has the minimum spherical aberration.

(iii) **Using parabolic mirrors** If spherical mirror is replaced by parabolic mirror, spherical aberration is minimised.

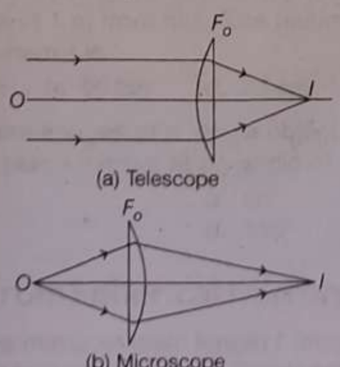


(a) Spherical mirror (b) Parabolic mirror

Minimising spherical aberration using mirrors

(iv) **Using lens of large focal length** It has been found that spherical aberration varies inversely as the cube of the focal length. So, if f is large, spherical aberration will be reduced.

(v) **Using plano-convex lens** In case of plano-convex lens spherical aberration is minimised, if its curved surface faces, the incident or emergent ray whichever is more parallel.



(a) Telescope

(b) Microscope

Minimising spherical aberration using plano-convex lens

Example 8. A combination is made of two lenses of focal lengths are f_1 & f_2 and dispersive powers are ω_1 & ω_2, respectively. The combination will be achromatic, if

a. $\omega_1 = 2\omega_2$ and $f_1 = 2f_2$ b. $2\omega_1 = \omega_2$ and $f_1 = 2f_2$

c. $\omega_1 = 2\omega_2$ and $f_1 = -2f_2$ d. $2\omega_1 = \omega_2$ and $2f_1 = f_2$

Sol (c) Coordinate of achromatic combination is satisfied, if

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \quad \text{...(i)}$$

$\therefore$ We check the options, in option (c), we get

$$\omega_1 = 2\omega_2 \text{ and } f_1 = -2f_2$$

So, from Eq. (i), we get

$$\frac{2\omega_2}{-2f_2} + \frac{\omega_2}{f_2} = 0$$

$$\Rightarrow \quad -\frac{\omega_2}{f_2} = -\frac{\omega_2}{f_2}$$

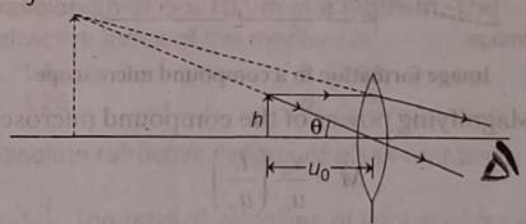
Hence, the condition is satisfied.

Microscope

It is an optical instrument used to see very small object.

Simple Microscope

A simple microscope consists of a single convex lens (converging lens) of small focal length. The image formed by this microscope is erect, virtual enlarged and on same side of object.



Object in front of a simple microscope

(i) When final image is formed at infinity.

Magnifying power, $M_\infty = \dfrac{D}{f}$

This is also called **magnifying power for normal adjustment.**

(ii) When final image is formed at D.

The magnifying power forms at the near point.

$$M_D = 1 + \frac{D}{f}$$

Example 9. A 20 D lens is used as a magnifier. Where should the object be placed to obtain maximum angular magnification? (Given, $D = 25$ cm)

a. 4.17 cm b. 4.7 cm

c. 4.07 cm d. None of these

Sol (a) Focal length of the lens,

$$f = \frac{1}{20}\text{ m} = \frac{100}{20}\text{ cm} = 5\text{ cm}$$

Maximum angular magnification is obtained when final image is formed at D, hence

$$\frac{1}{-25} - \frac{1}{-u_o} = \frac{1}{5}$$

$$\Rightarrow \quad u_o = 4.17\text{ cm}$$

Compound Microscope

Simple microscope has a limited maximum magnification. So, for larger magnification, one can use compound microscope.

It consists of two converging lenses arranged coaxially. The one facing the object is called **objective** and the one close to eye is called **eyepiece**. The objective has a smaller aperture and smaller focal length than those of the eyepiece.

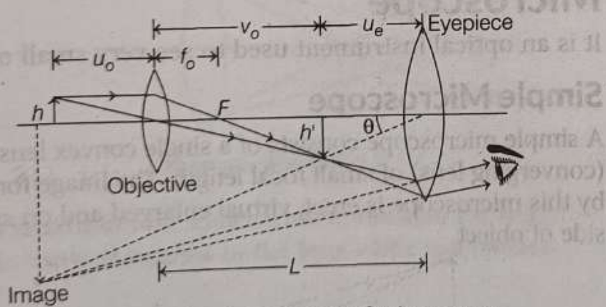


Image formation in a compound microscope

(i) Magnifying power of the compound microscope,

$$M = \frac{v_o}{u_o}\left(\frac{D}{u_e}\right)$$

(ii) When the final image (by eyepiece) is formed at D, then magnifying power, $M_D = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$

Example 10. The separation between the objective and the eyepiece of a compound microscope can be adjusted between 9.8 cm to 11.8 cm. If the focal length of the objective and the eyepiece are 1.0 cm and 6 cm respectively, find the range of the magnifying power, if the image is always needed at 24 cm from the eye.

a. 20.83 to 31.16
b. 18.35 to 32.16
c. 33.16 to 45.19
d. 15.18 to 34.16

Sol (a) For eyepiece, it is given that, $f_e = 6$ cm, $v_e = -24$ cm

$$\therefore \quad \frac{1}{-24} - \frac{1}{-u_e} = \frac{1}{6}$$

$$\therefore \quad u_e = 4.8\text{ cm}$$

When $L = 9.8$ cm, then $v_o = (9.8 - 4.8)$ cm $= 5.0$ cm

$$\therefore \quad \frac{1}{5.0} - \frac{1}{-u_o} = \frac{1}{1.0}$$

$$\Rightarrow \quad u_o = 1.25\text{ cm}$$

or Magnifying power, $M = \frac{v_o}{u_o}\frac{D}{u_e}$

$$= \left(\frac{5.0}{1.25}\right)\left(\frac{25.0}{4.8}\right) = 20.83$$

When $L = 11.8$ cm, then

$$v_o = (11.8 - 4.8)\text{ cm} = 7.0\text{ cm}$$

$$\therefore \quad \frac{1}{7.0} - \frac{1}{-u_o} = \frac{1}{1.0}$$

or $\quad u_o = 1.17$ cm

$$\therefore \quad M = \frac{v_o}{u_o}\cdot\frac{D}{f_e} = \left(\frac{7.0}{1.17}\right)\left(\frac{25.0}{4.8}\right) = 31.16$$

Therefore, range of magnifying power is from 20.83 to 31.16.