CHAPTER 02

Straight Line

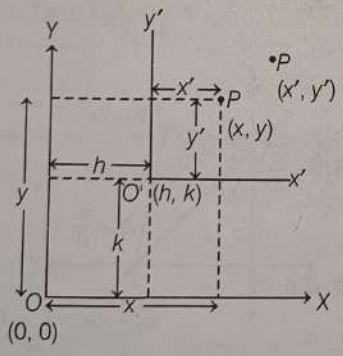
Locus

It is the path or curve traced by a moving point satisfying the given condition.

Equation of Locus The equation of locus of a point is the algebraic relation which is satisfied by the coordinates of every point on the locus of the point.

Shifting of Origin

Let $P(x, y)$ is the point with respect to the origin O. Now, the origin $O(0, 0)$ is shifted to a new point say $O'(h, k)$, then the coordinates of same point P with respect to the new origin O' be $P'(x', y')$ in such a way that



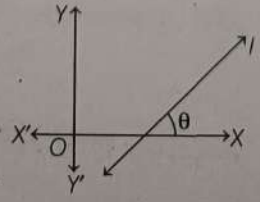
Hence, if the origin is shifted to point (h, k) without rotation of axes, then the new equation of the curve can be obtained by putting $(x + h)$ and $(y + k)$ in place of x and y, respectively.

Straight Line

A straight line defined as the curve which is such that the segment joining any two points on it lies wholly on it.

Inclination of a Line

An angle θ made by the line with positive X-axis in anti-clockwise direction is called inclination of a line.

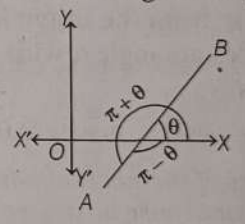


When $\theta = 0°$, then line is parallel to X-axis (horizontal line).

When $\theta = 90°$, then line is perpendicular to X-axis, i.e. parallel to Y-axis (vertical line).

Slope of a Line

The trigonometrical tangent of the angle that a line makes with the positive direction of X-axis in anti-clockwise sense, is called the slope or gradient of the line. The slope of a line is generally denoted by m.



$m = \tan\theta$

- The slope of X-axis is $m = \tan 0° = 0$
- The slope of Y-axis is $m = \tan 90°$, which is not defined.

Angle between Intersecting Lines

The angle θ between the lines having slopes m_1 and m_2 is given by $\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|$.

Perpendicular Lines

If two lines of slopes m_1 and m_2 are perpendicular, then the angle θ between them is $90°$.

$$\therefore \quad \cot\theta = 0 \Rightarrow \frac{1 + m_1 m_2}{m_1 - m_2} = 0 \Rightarrow m_1 \cdot m_2 = -1$$

Thus, when two lines are perpendicular, the product of their slopes is -1.

If m is the slope of a line, then the slope of a line perpendicular to it is $\left(-\dfrac{1}{m}\right)$.

Equation of Line in Standard Forms

There are various forms of equation of line as follows

Slope Intercept Form

- The equation of a line having slope m and making an intercept c on Y-axis is $y = mx + c$.
- The equation of line having slope m and making an intercept d on X-axis is $y = m(x - d)$.

Point Slope Form

The equation of a line which passes through the point (x_1, y_1) and has the slope m, is

$$y - y_1 = m(x - x_1)$$

Two Points Form

The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is $(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$.

Double Intercept Form of a Line

The equation of a line which cuts-off intercepts a and b respectively from X and Y-axes, is $\frac{x}{a} + \frac{y}{b} = 1$. $(a, b \neq 0)$.

Normal Form

The equation of the straight line upon which the length of the perpendicular from the origin is p and this perpendicular makes an angle α with the positive direction of X-axis is

$$x\cos\alpha + y\sin\alpha = p, \text{ where } 0 < \alpha < 2\pi.$$

Note *The equations of straight lines which pass through a point (x_1, y_1) and make an angle α with the straight line*

$y = mx + c$ *are* $y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$

General form of Equation of a Line

An equation of the form $Ax + By + C = 0$, where A, B, C are equal constants and atleast one of A and B is non-zero, is called **general linear equation** or **general equation** of a line.

If $B \neq 0$, then $Ax + By + C = 0$ can be written as

$$y = \frac{-A}{B}x - \frac{C}{B} \quad \text{or} \quad y = mx + c \qquad \ldots(i)$$

where, slope $(m) = -\frac{A}{B}$ and y-intercept $(c) = \frac{-C}{B}$

If $C \neq 0$, then $Ax + By + C = 0$ can be written as

$$\frac{x}{\frac{-C}{A}} + \frac{y}{\frac{-C}{B}} = 1 \text{ or } \frac{x}{a} + \frac{y}{b} = 1$$

where, X-intercept is $-\frac{C}{A}$ if $A \neq 0$

and Y-intercept is $-\frac{C}{B}$ if $B \neq 0$

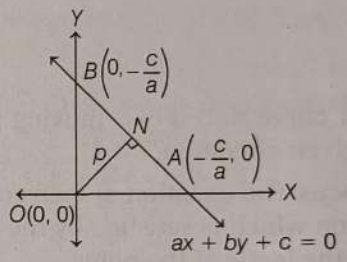
- If $A = 0$, then the line is parallel to the X-axis.
- If $B = 0$, then the line is parallel to the Y-axis.

Point of Intersection of Lines

The coordinates of the point of intersection of two intersecting lines can be obtained by solving their equations simultaneously.

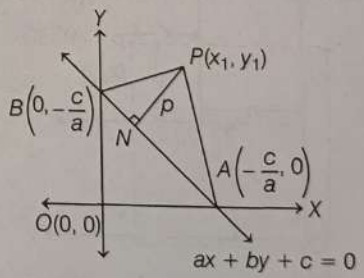
Distance of the Origin from a Line

The perpendicular distance of the origin from the line $ax + by + c = 0$ is given by $p = \left|\frac{c}{\sqrt{a^2 + b^2}}\right|$



The Distance of the Point (x_1, y_1) from a Line

The distance of the point $P(x_1, y_1)$ from line $ax + by + c = 0$ is given by $p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$



The Distance between Two Parallel Line

The distance between the parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $p = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$