Chapter

4 Laws of Motion

Quick Review

Motion

Kinematics
- Describes various motions without their causes.
- Parameters: Distance, displacement, speed, velocity and acceleration

Dynamics
- Describes the motion along with its cause,
- Parameters: Force, momentum, energy, power etc. along with that in kinematics.

Inertial frame of reference
- Inertial frame of reference is a co-ordinate system in which Newton's laws of motion hold good.

Non-inertial frame of reference
- A frame of reference in which Newton's laws of motion do not hold good is called non-inertial frame of reference.

Types of Motion

Linear motion
- Initial velocity $\vec{u} = 0 \Rightarrow$ Acceleration $\vec{a}$ in any direction results into linear motion
- $\vec{u} \neq 0 \Rightarrow$ for motion to be linear, $\vec{a}$ must be collinear with $\vec{u}$

Circular motion
- $\vec{u} \neq 0$ and $\vec{a} \perp \vec{u}$ throughout the motion

Parabolic motion
- $\vec{a}$ = constant and $\vec{u}$ is not in line with $\vec{a}$ then the motion is parabolic e.g. projectile motion

➢ Newton's laws of Motion:

	Newton's first law	Newton's second law	Newton's third law
Statement	Every inanimate object continues to be in a state of rest or of uniform unaccelerated motion along a straight line, unless it is acted upon by an external, unbalanced force.	The rate of change of linear momentum of a rigid body is directly proportional to the applied (external unbalanced) force and takes place in the direction of force.	To every action (force) there is always an equal and opposite reaction (force).
Importance	i. Shows equivalence between state of rest uniform and state of motion. The distance lies in the choice of frame of reference.	i. Gives mathematical formulation for quantitative measure of force.	i. Defines action and reaction forces.
	ii. Defines force as an entity that brings change in state of motion.	ii. Defines momentum	ii. Action and reaction forces always act on different objects.
	iii. Defines inertia as a fundamental property of every physical object.	iii. Aristotle's fallacy is overcome	

➢ Forces and their types:

Fundamental Forces

1. **Gravitational Force**
   Force of attraction between two (point) masses separated by a distance

2. **Electromagnetic Force**
   The attractive and repulsive force between electrically charged particles

3. **Strong Nuclear force**
   The strong force which binds protons and neutrons (nucleons) together in the nucleus of an atom

4. **Weak Nuclear force**
   The force of interaction between subatomic particles which results in the radioactive decay of atoms

Types of Forces

1. **Contact**
   The forces experienced by a body due to physical contact are called contact forces.

2. **Non-Contact**
   The forces experienced by a body without any physical contact are called non-contact

3. **Real**
   A force which is produced due to interaction between the objects is called real force.

4. **Pseudo**
   A pseudo force is one which arises due to the acceleration of the observer's frame of reference.

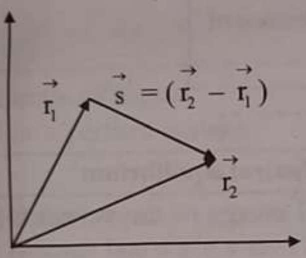
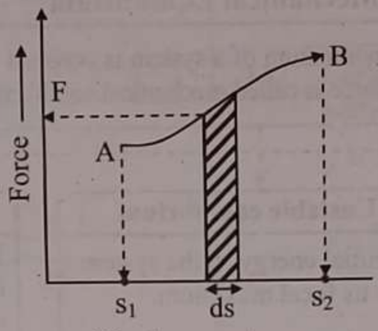
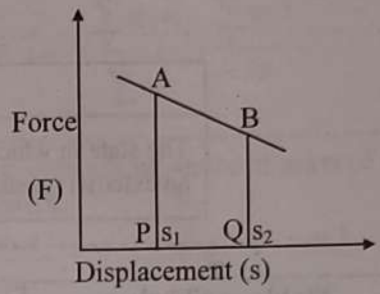
5. **Conservative**
   If work done by or against a force is independent of the actual path, the force is said to be a conservative force.

6. **Non-conservative**
   If work done by or against a force is dependent of the actual path, the force is said to be a non- conservative force.

Caution

If in a numerical, gravitational units of force are given, convert them into absolute units of force
1 kg wt or 1 kg f = 9.8 N and 1 g wt or 1 g f = 980 dyne

Work done: The product of applied force and the displacement produced in the direction of the force is called as work done.

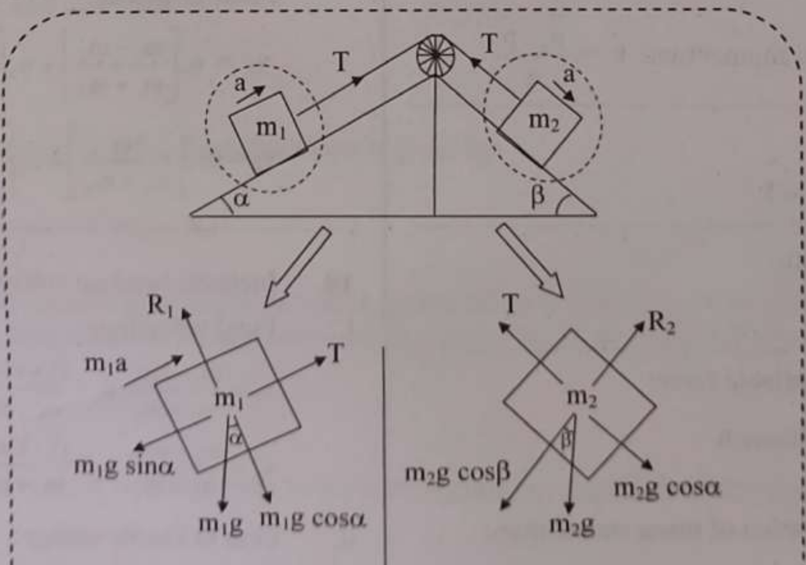
Work done (Using vectors)	Work done due to variable force (Method of integration)	Work done due to variable force (Graphical Method)
• When a constant force $\vec{F}$ changes the position of an object from $\vec{r}_1$ to $\vec{r}_2$, the work done by the force is given by the dot product between, $\vec{F}$ and $\vec{s}$.  Here, $\vec{s} = (\vec{r}_2 - \vec{r}_1)$ $\therefore\ W = \vec{F}\cdot\vec{s} = Fs\cos\theta$.	• The formula for work done is applicable only if both force $\vec{F}$ and displacement $\vec{s}$ are constant and finite i.e., it cannot be applied when the force is variable.  • The work done is calculated using method of integration in case of variable force. $W = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s}$	• In case of linearly variable force also the area under the curve gives the value of work done.  • The work done in this case is, W = Area APQB

Work-energy Theorem: Decrease in the potential energy due to work done by a conservative force is entirely converted into kinetic energy. Vice versa, for an object moving against a conservative force, its kinetic energy decreases by an amount equal to the work done against the force.

Conservation of linear momentum	If no external force acts on a system (isolated system) of constant mass, the total momentum of the system remains constant with time.

➢ **Free Body Diagram (FBD):**

A free body diagram refers to forces acting on only one body at a time, and its acceleration.



Free body diagram of mass m_1

Free body diagram of mass m_2

Collisions

Elastic collisions

- Linear momentum is conserved
- K.E. is conserved.
- Coefficient of restitution e = 1

Inelastic collision

- Linear momentum is conserved
- K.E. is not conserved
- For perfectly inelastic collision, e = 0

Mechanical Equilibrium

The state in which the momentum of a system is constant in the absence of an external unbalanced force is called mechanical equilibrium.

Stable equilibrium

Potential energy of the system is at its local minimum.

Unstable equilibrium

potential energy of the system is at its local maximum.

Neutral equilibrium

Potential energy of the system is constant over a plane and remains same at any position.

Centre of mass	A point about which the summation of moments of masses in the system is zero
Centre of Gravity	The point around which the resultant torque due to force of gravity on the body is zero.

Formulae

1. Force:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(m\vec{v}\right) = m\vec{a}$$

2. Gravitational force between two bodies:

$$F = \frac{Gm_1m_2}{r^2}$$

3. Force in terms of momentum: $\vec{F} = \frac{\vec{p_2} - \vec{p_1}}{t}$

4. Impulse:

$$\vec{J} = \vec{F}t = m(\vec{v} - \vec{u})$$

5. Linear momentum:

$$\vec{p} = m\vec{v}$$

6. Work done by variable force:

$$W = \int_b^a \vec{F}.\overline{ds} = \int_b^a F ds\cos\theta$$

7. Laws of conservation of linear momentum:

i. $\sum_{i=1}^{n} mv = \text{constant}$

ii. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

8. Coefficient of restitution:

$$e = -\frac{v_s}{u_a} = -\frac{v_2 - v_1}{u_2 - u_1} = \frac{v_1 - v_2}{u_2 - u_1}$$

9. Elastic head-on collision:

Final velocities:

$$v_1 = u_1\left[\frac{m_1 - m_2}{m_1 + m_2}\right] + u_2\left[\frac{2m_2}{m_1 + m_2}\right]$$

$$v_2 = u_1\left[\frac{2m_1}{m_1 + m_2}\right] + u_2\left[\frac{m_2 - m_1}{m_1 + m_2}\right]$$

10. Inelastic head-on collision:

i. Final velocities:

$$v_1 = \frac{m_1 - em_2}{m_1 + m_2}u_1 + \frac{(1+e)m_2}{m_1 + m_2}u_2$$

$$v_2 = \frac{m_1 - em_2}{m_1 + m_2}u_1 + \frac{(1+e)m_2}{m_1 + m_2}u_2$$

ii. Loss in kinetic energy:

$$\Delta K = \frac{(1 - e^2)m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$$

11. Perfectly inelastic head-on collision:

i. Final velocity: $v = \frac{m_1v_1 + m_2v_2}{(m_1 + m_2)}$

ii. Loss in kinetic energy:

$$\Delta K.E. = \frac{m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$$

12. Magnitude of moment of force (Torque):

$$\tau = \vec{r} \times \vec{F} = r\,F \sin\theta$$

13. Moment of couple:

$$\vec{\tau} = \vec{r}_{12} \times \vec{F}_1 = \vec{r}_{21} \times \vec{F}_2$$

14. Centre of mass:

i. For n particle system,

Position vector $\vec{r}_{C.M.} = \frac{\sum_1^n m_i \vec{r}_i}{\sum_1^n m_i} = \frac{\sum_1^n m_i \vec{r}_i}{M}$

ii. For continuous distribution,

Position vector $\vec{r}_{C.M.} = \frac{\int \vec{r}\, dm}{M}$

iii. velocity $\vec{v}_{cm} = \frac{\sum_1^n m_i \vec{v}_i}{\sum_1^n m_i} = \frac{\sum_1^n m_i \vec{v}_i}{M}$

iv. acceleration $\vec{a}_{cm} = \frac{\sum_1^n m_i \vec{a}_i}{\sum_1^n m_i} = \frac{\sum_1^n m_i \vec{a}_i}{M}$

15. Cartesian co-ordinate of centre of mass of two particles system:

$$x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \quad y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2},$$

$$z = \frac{m_1z_1 + m_2z_2}{m_1 + m_2}$$

Shortcuts

1. If a person climbs down along the rope with acceleration a, then tension in the rope will be $m(g - a)$.
2. If a person climbs up along the rope with acceleration a, then tension in the rope will be $m(g + a)$.
3. If $\vec{a}$ is the acceleration of the centre of mass and $\vec{a}_1$, $\vec{a}_2$ are the accelerations of masses m_1 and m_2 then

   $\vec{a} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$.

4. In elastic collisions, if Ki and Kf are the initial and final kinetic energies of mass m1, the fractional decrease in its kinetic energy is given by

   $$\frac{K_i - K_f}{K_i} = 1 - \frac{v_1^2}{u_1^2}$$

   Further, if $m_2 = nm_1$ and $u_2 = 0$, then

   $$\frac{K_i - K_f}{K_i} = \frac{4n}{(1+n)^2}$$

5. The loss of kinetic energy in an inelastic collision is given by,

   $$K_i - K_f = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(1 - e^2)(u_1 - u_2)^2$$

6. Suppose a body is dropped from a height h0 and it strikes the ground with a speed v0. Let, after the inelastic collision, the speed with which it rebounds be v_1 and h_1 be the height to which it rises, then

   $$e = \frac{v_1}{v_0} = \frac{\sqrt{2gh_1}}{\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}}$$

7. If a small part of mass m_2 is removed from larger part of mass m_1, then CM of remaining part is

   $$X_{cm} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

8. Centre of mass of two point system divides the line joining them in inverse ratio of their masses.