Chapter

10 Magnetic Fields due to Electric Current

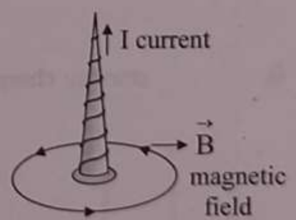
Quick Review

Magnetic effect of electric current	The phenomenon by virtue of which, an electric current in a conductor produces a magnetic field around it is called the magnetic effect of electric current.

Direction of magnetic field:

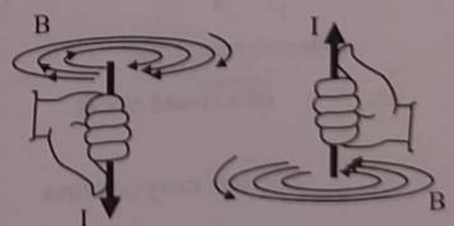
Direction of magnetic field is determined with the help of the following laws:

i. **Maxwell's cork screw rule:** If a right-handed cork screw, placed along the current carrying linear conductor, is rotated such that the screw moves in the direction of current, then the direction of rotation of the screw gives the direction of magnetic field lines.

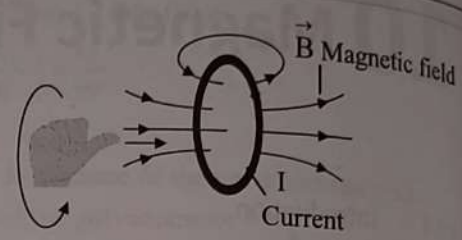
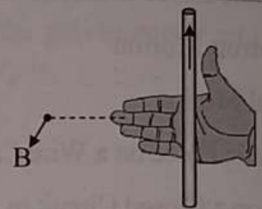
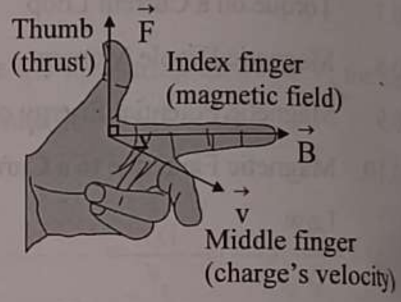


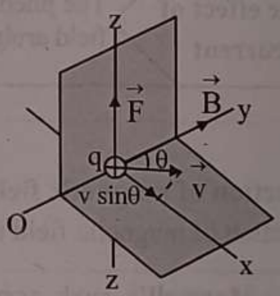
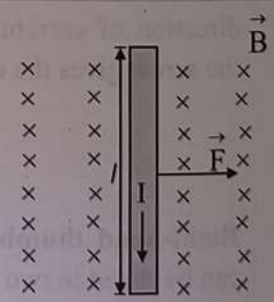
ii. **Right-hand thumb rule:** The right-hand thumb rule can be stated in two ways:

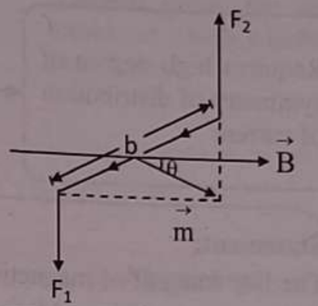
a. If a straight current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current, then the direction of folding fingers will represent the direction of magnetic field lines.



Right hand thumb rule

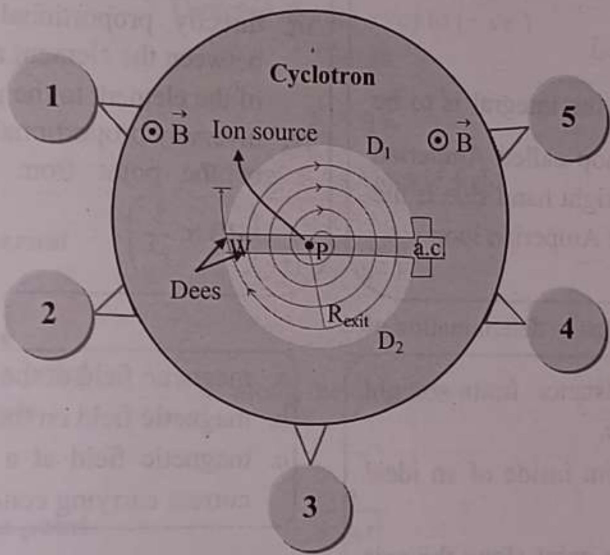
b.	If the direction of current in circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of the stretched thumb.	
iii.	**Right-hand palm rule:** If we stretch our right hand such that fingers point towards the point at which magnetic field is required while thumb is in the direction of current then normal to the palm will show the direction of magnetic field.	
iv.	**Fleming's left hand rule:** If we stretch the index finger, middle finger and thumb of left hand mutually perpendicular to each other such that the index finger points along the direction of magnetic field and the middle finger along the direction of current (moving charge), then the thumb represents the direction of the force $\vec{F}$ experienced by the moving charge.	

		Force	
	experienced by	in uniform	more information
i.	moving charge	magnetic field	
ii.	moving charge	magnetic and electric field	Obeys Lorentz force law
iii.	current carrying conductor	magnetic field	
iv.	on a closed circuit	magnetic field	is always zero
v.	2 current carrying wires	-	Attract each other if carry current in same direction, repel each other if carry current in opposite direction

	Torque	
experienced by	in uniform	more information
i. current loop	magnetic field	a.  b. Forms working principle of moving coil galvanometer (MCG)

Cyclotron
A charged particle accelerator, accelerating charged particles to high energies.

Cyclotron formula
$mv = p = qBR$

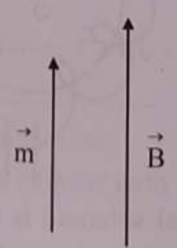


Limitations
It cannot accelerate uncharged particles (neutrons), charged particles with small mass and high velocity (electrons).

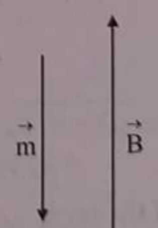
Energy provided
The final energy is proportional to the square of the radius of the outermost circular path (R_{exit})

Resonance condition ($f_a = f_c$)
The frequency of the applied voltage (f_a) between the two Ds is adjusted so that polarity of the two Ds is reversed as the ion arrives at the gap after completing one semi-circle.

Magnetic potential energy of a dipole:
A magnetic dipole freely suspended in a magnetic field possesses magnetic potential energy because of its orientation in the field.



Minimum potential energy



Maximum potential energy

Determination of Magnetic field

using Ampere's law → Requires high degree of symmetry of distribution of current

using Biot-Savart's law → High degree of symmetry of distribution of current not required

Ampere's law

Statement:
The line integral of magnetic field of induction $\vec{B}$ around any closed path in free space is equal to absolute permeability of free space μ_0 times the total current flowing through area bounded by the path.

Mathematically, $\oint \vec{B} \cdot \vec{ds} = \mu_0 I$

The sign $\oint$ indicates that the integral is to be evaluated over a closed loop called Amperian loop. The current I on the right hand side is the net current encircled by the Amperian loop.

helps in determination of

a. magnetic field at a distance from straight current conducting wire,
b. magnetic field at a point inside of an ideal long straight solenoid,
c. magnetic induction at a point along the axis of a toroid.

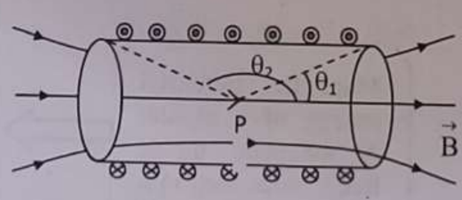
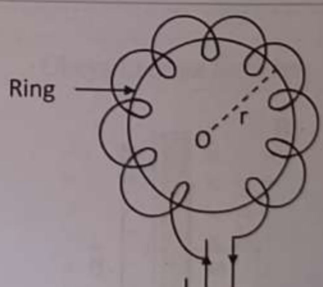
Biot-Savart's law

Statement:
The magnitude of magnetic induction (dB) at a point due to a small element of current carrying conductor is

i. directly proportional to the current ($dB \propto I$).
ii. directly proportional to the length of the element ($dB \propto dl$).
iii. directly proportional to the sine of the angle between the element and the line joining the centre of the element to the point ($dB \propto \sin\theta$) and
iv. inversely proportional to the square of the distance of the point from the centre of the element $\left(dB \propto \frac{1}{r^2}\right)$.

helps in determination of

a. magnetic field at the centre of circular coil,
b. magnetic field on the axis of the coil,
c. magnetic field at a distance from a straight current carrying conductor.

Configuration	Solenoid	Toroid
Diagram		
Meaning of the term	A solenoid consists of an insulated long wire closely wound in the form of a helix. Its length is very large as compared to its diameter.	Toroidal solenoid is an anchor ring (called torus), around which a large number of turns of metallic wire are wound. It is an endless solenoid.
Magnetic field produced	i. Magnetic field inside a long straight solenoid is independent of the length and diameter of the solenoid and is uniform over the cross-section of the solenoid.	i. Magnetic field B inside the toroid is independent of r, provided number of turns per unit length remains same.

	ii. Magnetic field at the ends of a long current-carrying solenoid is half of that at the centre. iii. For a point outside the solenoid, the magnetic field is zero. iv. Magnetic field produced by the solenoid is similar to magnetic field produced by a bar magnet.	ii. For a point outside the toroid, magnetic field is zero because the field is only confined inside the core of toroid, on which winding has been made.

Analogy between Electrostatics and Magnetism:

Quantity		Electrostatics	Magnetism
Basic Physical Quantity		Electrostatic charge (q)	Magnetic pole strength (q_m)
Constant of proportionality		$\frac{1}{4\pi\varepsilon_0}$	$\frac{\mu_0}{4\pi}$
Dipole moment		$p = q\,(2l)$ Directed along (– ve) to (+ ve) charge	$m = q_m\,(2l)$ Directed along S to N pole
Force		$\vec{F} = q\vec{E}$ Where, $\vec{E}$ is Electric field	$\vec{F} = q_m\vec{B}$ Where, $\vec{B}$ is Magnetic field
Energy of a dipole (In external field)		$U = -\vec{p}\cdot\vec{E}$	$U = -\vec{m}\cdot\vec{B}$
For short dipole:	**Field at a point on axis**	$\frac{2\vec{p}}{4\pi\varepsilon_0 r^3}$ along the direction of $\vec{p}$	$\frac{\mu_0\left(2\vec{m}\right)}{4\pi r^3}$ along the direction of $\vec{m}$
	Field at a point on equator	$\frac{\vec{p}}{4\pi\varepsilon_0 r^3}$ opposite to $\vec{p}$	$-\frac{\mu_0\vec{m}}{4\pi r^3}$ opposite to $\vec{m}$

Formulae

1. Force on a moving charge in uniform magnetic field: $F = qvB\sin\theta$

2. Lorentz force: $\vec{F} = q\vec{E} + q\left(\vec{v}\times\vec{B}\right)$

3. Cyclotron motion:
   i. Radius of circular path in a cyclotron:
   $R = \frac{mv}{qB}$
   ii. Time period for charged particle to complete circular path: $T = \frac{2\pi m}{qB}$
   iii. Frequency of charged particle:
   $f = \frac{1}{T} = \frac{qB}{}$
   iv. K.E. of charged particle:
   $\text{K.E.} = \frac{1}{2}mv_{max}^2 = \frac{q^2B^2R_{exit}^2}{2m}$
   where, $v_{max} = \frac{qBR_{exit}}{m}$

4. Force acting on a conductor carrying current:
   i. For straight wire: $F = IlB\sin\theta$
   ii. For arbitrary shaped wire: $\vec{F}_m = I\left[\int d\vec{l}\right]\times\vec{B}$

5. Force on closed circuit in magnetic field:
   $\vec{F}_m = I\left[\oint_c d\vec{l}\right]\times\vec{B}$

6. Torque on a current loop:
   i. $\tau = IAB\sin\theta$
   ii. $\tau = NIAB\sin\theta$ (for N turns)

7. Moving coil galvanometer (M.C.G.):

i. Deflecting torque acting on coil
$\tau_d = NIAB \cos\theta$

ii. Restoring torque: $\tau_r = K\phi$

iii. Deflection: $\phi = \left(\frac{NAB}{K}\right)I$

8. Magnetic moment of current carrying loop:

i. $m = IA$ (For single turn)

ii. $m = nIA$ (For n turns)

9. Torque on a current loop in terms of magnetic dipole moment (m):

i. $\tau = mB \sin\theta$

ii. $\vec{\tau} = \vec{m} \times \vec{B}$ (in vector form)

10. Magnetic potential energy of a dipole:
$U = -\vec{m}\cdot\vec{B} = -mB\cos\theta$

11. Biot-Savart's law:

i. $dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}$

ii. $\overline{dB} = \frac{\mu_0}{4\pi}\left(\frac{I\overline{dl}\times\vec{r}}{r^3}\right)$ (in vector form)

12. Magnetic induction at a point at a perpendicular distance R due to straight conductor carrying current:

i. For infinite wire: $B = \frac{\mu_0 I}{2\pi R}$

ii. For semi-infinite wire:
$B = \frac{\mu_0 I}{4\pi R}$

13. Force on length L of the wire: $F = \frac{\mu_0 I_1 I_2 L}{2\pi d}$

14. Force per unit length between two infinitely long current carrying conductors: $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

15. Magnetic field due to current carrying arc:
$B = \frac{\mu_0 I}{4\pi R}\times\theta$ where, θ is in radian.

16. Magnetic field at the centre of the circular coil:

i. $B = \frac{\mu_0 I}{2r}$ (for single turn)

ii. $B = \frac{\mu_0 nI}{2r}$ (for n turns)

17. Magnetic induction at a point along the axis of a coil carrying current:

i. $B = \frac{\mu_0 IR^2}{2(z^2+R^2)^{3/2}}$ (for one turn)

ii $B = \frac{\mu_0 nIR^2}{2(z^2+R^2)^{3/2}}$ (for n turns)

iii. $B = \frac{\mu_0 nIR^2}{2z^3}$
(at the point far away from centre of coil)

iv. $B = \frac{\mu_0 m}{2\pi z^3}$ (in terms of magnetic moment)

18. Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 I$

19. Magnetic induction at a point along the axis of a long solenoid:

i. Induction at a point inside the solenoid,
$B = \mu_0 ni$
where, $n = \frac{N}{l}$ = turns per unit length,
i = current flowing through wire.

ii. Induction at a point near the end of solenoid,
$B_{end} = \frac{1}{2}\mu_0 ni$

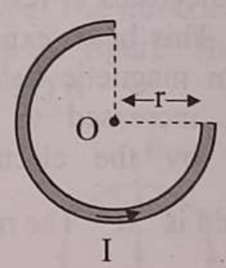
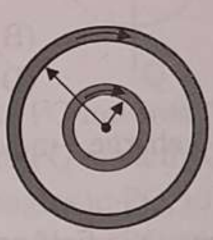
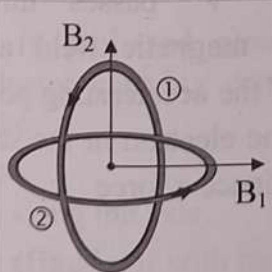
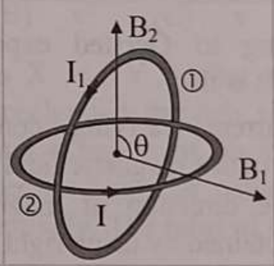
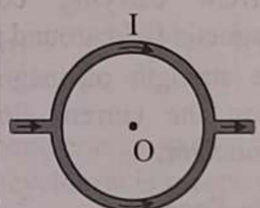
20. Magnetic induction along the axis of a toroid:
$B = \mu_0 ni$
where, $n = \frac{N}{2\pi r}$

Shortcuts

1. If a current carrying circular loop (n = 1) is turned into a coil having n identical turns then magnetic field at the centre of the coil becomes n^2 times the previous field,
i.e., $B_{(n\ turn)} = n^2 B_{(single\ turn)}$

2. The value of magnetic field induction at a point, on the centre of separation of two linear parallel conductors carrying equal currents in the same direction is zero.

3.

Sr. No.	Condition	Figure	Magnetic field
i.	Three quarter semi-circular current carrying arc		$B = \frac{\mu_0}{4\pi} \cdot \frac{\left(2\pi - \frac{\pi}{2}\right) I}{r} = \frac{3\mu_0 I}{8r}$
ii.	Concentric co-planar circular loops carrying current in the same direction		$B = \frac{\mu_0}{2} I \left(\frac{1}{r_1} + \frac{1}{r_2}\right)$
iii.	Concentric co-planar circular loops carrying current in the opposite direction		$B = \frac{\mu_0}{2} I \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$
iv.	Concentric loops but their planes are perpendicular to each other		$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r}\sqrt{I_1^2 + I_2^2}$
v.	Concentric loops but their planes are at an angle θ with each other		$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos\theta}$
vi.	Distribution of current across the diameter		$B = 0$
vii.	Distribution of current between any two points on the circumference		$B = 0$