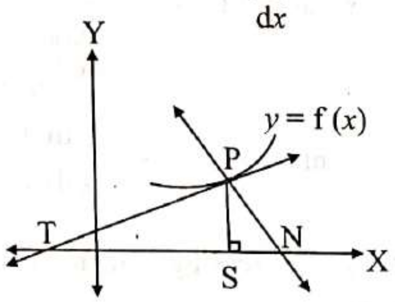
Applications of Derivatives

Formulae

1. **Tangents and Normals:**

i. **Slope of the tangent** to the curve at the point $P(x_1, y_1)$ is $\frac{dy}{dx} = \tan\alpha$,
where α is the angle which the tangent makes with +ve X-axis.

ii. **Slope of normal** $= \frac{-1}{\frac{dy}{dx}}$



iii. **Equation of tangent:**
Equation of tangent at (x_1, y_1) to the curve $y = f(x)$ is

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1)$$

iv. **Equation of Normal:**

$$y - y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}} (x - x_1)$$

a. If tangent is parallel to X-axis, then $\alpha = 0$ or $180°$

$\therefore \tan\alpha = 0$

$\therefore \frac{dy}{dx} = 0$

b. If tangent is parallel to Y-axis, then $\alpha = 90°$

$\therefore \tan\alpha = \infty$

$\therefore \frac{dy}{dx} = \infty$

$\boxed{\therefore \frac{dx}{dy} = 0}$

c. If the tangent line makes equal angles with the axes, then $\alpha = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

$\therefore \tan\alpha = \pm 1$

$\therefore \frac{dy}{dx} = \pm 1$

2. **Rate Measure:**

i. If x and v denotes the displacement and velocity of a particle at any instant, then velocity is $\frac{dx}{dt}$.

ii. Rate of change of velocity w.r.t time is called acceleration.

So $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v.\frac{dv}{dx}$

3. **Approximation:**
If $y = f(x)$ is a differentiable function of x and δx be a change in x.
Then, $f(x + \delta x) \approx f(x) + \delta x f'(x)$ OR
If $y = f(x)$ is a differentiable function and $x = a$ is a point on its domain,
then $f(a + h) \approx f(a) + hf'(a)$, where h is very small.

i. Error in $y = \delta y = \frac{dy}{dx} \cdot \delta x$

ii. Relative error $= \frac{\delta y}{y}$

iii. Percentage error $= \frac{\delta y}{y} \times 100$

4. i. **Rolle's theorem:**
If a function $f(x)$ is

a. Continuous in [a, b]

b. Derivable in (a, b)

c. $f(a) = f(b)$

then $\exists$ at least one point c of x in the interval (a,b) such that $f'(c) = 0$.

ii. **Lagrange's Mean Value theorem:**
If a function $f(x)$ is

a. Continuous in [a,b]

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then $\exists$ at least one point $c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

5. **Increasing and Decreasing function:**

i. **Increasing Function:**
Positive slope of tangent i.e. $f'(x) > 0$

OR

A function $f(x)$ is s.t.b a strictly increasing function on (a, b) if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)\ \forall\ x_1, x_2 \in (a, b)$

ii. **Decreasing function:**

Negative slope of tangent i.e. $f'(x) < 0$

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A function $f(x)$ is s.t.b decreasing on (a,b) if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)\ \forall\ x_1, x_2 \in (a, b)$

iii. A function f is s.t.b **monotonic** in an interval if it is either increasing or decreasing in that interval

i.e.

Monotonic increasing	$\Rightarrow$	$f'(x) \geq 0$
Monotonic decreasing	$\Rightarrow$	$f'(x) \leq 0$
Constant	$\Rightarrow$	$f'(x) = 0$
		$\forall\ x \in (a,b)$
Strictly increasing	$\Rightarrow$	$f'(x) > 0$
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6. **Maxima and Minima:**

i. **Maxima:** A function $f(x)$ is s.t.b maximum at $x = a$ if $f'(a) = 0$ and $f''(a) < 0$.

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iii. The values of x for which $f'(x) = 0$ are called **stationary values or critical values** of x.

iv. **Point of inflexion:** If at point a, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$, then $x = a$, is a point of inflexion (No minima or maxima).

Shortcuts

1. Length of tangent $= \left|\frac{y\sqrt{1+y_1^2}}{y_1}\right|$,

where $y_1 = \frac{dy}{dx}$

2. Length of normal $= \left|y\sqrt{1+y_1^2}\right|$

3. Length of Subtangent $= \left|\frac{y}{\frac{dy}{dx}}\right|$

4. Length of Subnormal $= \left|y \cdot \left(\frac{dy}{dx}\right)\right|$

5. Length of intercepts made on axes by the tangent at (x_1, y_1)

i. x-intercept $= x_1 - \frac{y_1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}$

ii. y-intercept $= y_1 - x_1\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$

6. If $f(x)$ is increasing, then $f^{-1}(x)$ is also increasing.

7. If $f(x)$ is decreasing, then $f^{-1}(x)$ is also decreasing.

8. If $f(x)$ and $g(x)$ are monotonic on $[a, b]$, then $g(f(x))$ is also monotonic of same nature.

9. If $y = f(x)$ is continuous function and its least value is m and greatest value is M, then $m \leq f(x) \leq M$.

10. Of all rectangles of a given perimeter, the square has the largest area.

11. A cone of maximum volume that can be inscribed in a sphere of a given radius r is of height $\frac{4r}{3}$.

12. Of all rectangles of a given area, the square has the least perimeter.

13. The height of cylinder of maximum volume inscribed in a sphere of radius r is $\left(\frac{2r}{\sqrt{3}}\right)$.

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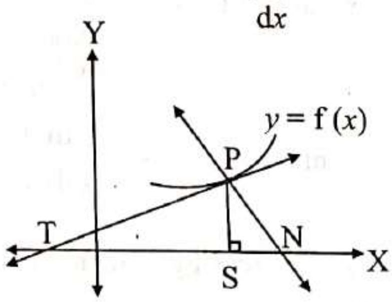

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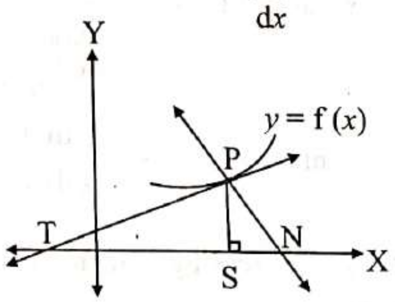

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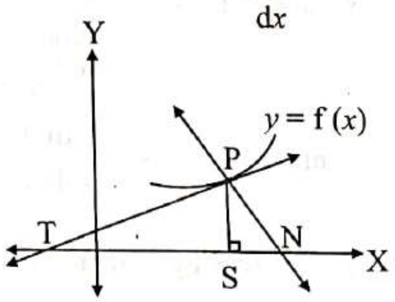

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