Formula sheet

Trigonometric Funtion.

• Principal solⁿ :- $0 \le \theta \le 2\pi$.

• General solⁿ :-

1) $\sin\theta = \sin\alpha = n\pi + (-1)^n\alpha$, $n \in Z$.
2) $\cos\theta = \cos\alpha = 2n\pi \pm \alpha$, $n \in Z$.
3) $\tan\theta = \tan\alpha = n\pi + \alpha$, $n \in Z$.

• Remark's :-

1) $\sin\theta = 0 \longrightarrow \theta = n\pi$, $n \in Z$.
2) $\cos\theta = 0 \longrightarrow \theta = (2n+1)\frac{\pi}{2}$, $n \in Z$.
3) $\tan\theta = 0 \longrightarrow \theta = n\pi$, $n \in Z$.

$$\left.\begin{array}{l} 4)\ \sin^2\theta = \sin^2\alpha \\ 5)\ \cos^2\theta = \cos^2\alpha \\ 6)\ \tan^2\theta = \tan^2\alpha \end{array}\right\} \theta = n\pi \pm \alpha$$

• Relation between polar and cartesian co-ordinates:-

$x = r\cos\theta$ and $y = r\sin\theta$.

$r = \sqrt{x^2+y^2}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r} = \frac{y}{x}$

$$\theta = \tan^{-1}\frac{y}{x}.$$

• Sine Rule:-

In ΔABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a : b : c = \sin A : \sin B : \sin C$.

- Cosine Rule:-

In ΔABC, 1) $\cos A = \frac{b^2+c^2-a^2}{2bc}$ | $a^2 = b^2+c^2-2bc\cos A$.

2) $\cos B = \frac{c^2+a^2-b^2}{2ac}$ | $b^2 = a^2+c^2-2ac\cos B$

3) $\cos C = \frac{a^2+b^2-c^2}{2ab}$ | $c^2 = a^2+b^2-2ab\cos C$.

- Projection Rule:-

In ΔABC, 1) $a = b\cos C + c\cos B$

2) $b = c\cos A + a\cos C$.

3) $c = a\cos B + b\cos A$.

- Application:- 1] Half angle formula:-

i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$, $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

ii) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$, $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

iii) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, $\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$, $\tan\frac{C}{D} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

2] Heron's formula:- $A(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

3] Napier's Analogy:- In ΔABC,

1) $\tan\left(\frac{B-C}{2}\right) = \frac{(b-c)}{(b+c)} \cot\frac{A}{2}$

2) $\tan\left(\frac{C-A}{2}\right) = \frac{(c-a)}{(c+a)} \cot\frac{B}{2}$

3) $\tan\left(\frac{A-B}{2}\right) = \frac{(a-b)}{(a+b)} \cot\frac{C}{2}$

- Area of $\triangle ABC$:- $\frac{1}{2} ab \sin C$, $\frac{1}{2} bc \sin A$, $\frac{1}{2} ac \sin B$.

- Inverse of funtion :-

Function	Domain (x)	Range (y)
$y = \sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \tan^{-1}x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y = \cot^{-1}x$	R	$(0, \pi)$
$y = \sec^{-1}x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \operatorname{cosec}^{-1}x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

- Properties :-

a) 1) $\sin^{-1}(-x) = -\sin^{-1}x$
2) $\cos^{-1}(-x) = \pi - \cos^{-1}x$
3) $\tan^{-1}(-x) = -\tan^{-1}x$
4) $\cot^{-1}(-x) = \pi - \cot^{-1}x$
5) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$
6) $\sec^{-1}(-x) = \pi - \sec^{-1}x$

b) 1) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$
2) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$
3) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$

c) 1) $\sin^{-1}(\sin x) = x$ - in Range
2) $\cos^{-1}(\cos x) = x$ - in Range
3) $\tan^{-1}(\tan x) = x$ - in Range
4) $\cot^{-1}(\cot x) = x$ - in Range
5) $\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$ - in Range
6) $\sec^{-1}(\sec x) = x$ - in Range

d) 1) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $x, y > 0$ and $xy < 1$

2) $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $x, y > 0$ and $xy > 1$.

3) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $x \geq 0$ and $y > 0$.

e) 1> $2\tan^{-1}x = \sin^{-1}\left(\frac{2x^2}{1+x^2}\right)$, For $|x| \le 1$.

2> $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, For $x \geq 0$

3> $2\tan^{-1}x = \tan^{-1}\left(\frac{2x^2}{1-x^2}\right)$, For, $-1 < x < 1$.

f) 1> $\sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}x$, For $x \in [-\infty, -1] \cup [1, \infty]$

2> $\cos^{-1}\frac{1}{x} = \sec^{-1}x$, For $x \in [-\infty, -1] \cup [1, \infty)$

3> $\tan^{-1}\frac{1}{x} = \cot^{-1}x$, For all $x > 0$

4> $\tan^{-1}\frac{1}{x} = -\pi + \cot^{-1}x$, For all $x < 0$

- Always :-

$A+B+C = 180$ and $A+C = 2B$ ··· in A.P.

$\therefore A+C+B = 180$ $\therefore 2B+B = 180$

i.e. $3B = 180$ $\therefore B = \frac{180}{3}$ $\therefore \underline{\underline{B = 60^\circ}}$