Chapter

7 Wave Optics

Quick Review

Light exhibits dual nature

Using wave nature → **Huygens' Wave Theory**, **Maxwell's Electromagnetic Theory**, **Photon Model**

Using particle nature → **Newton's Corpuscular Theory**

Huygens' Wave Theory

- Light energy from a source is propagated in the form of waves.
- The theory could explain laws of reflection, refraction, interference, diffraction and polarization of light and formation of images by mirrors and lenses assuming wave nature of the light.
- It proved that velocity of light in rarer medium is greater than that in a denser medium.
- The theory assumed existence of ether medium exists everywhere in the universe.
- Huygens' theory is an empirical theory. It assumes that the backward travelling waves will not be effective which is incorrect.

Newton's Corpuscular Theory

i. Light is made up of particles, i.e., corpuscles which travel along straight lines in the absence of any external force.
ii. The theory could explain rectilinear propagation and reflection of light.
iii. The theory predicted that speed of light in a denser medium is more than the speed of light in a rarer medium which was experimentally proved wrong. Hence Newton's corpuscular theory was rejected.
iv. It could not explain phenomena such as interference, diffraction or polarization of light.

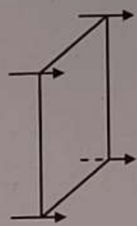
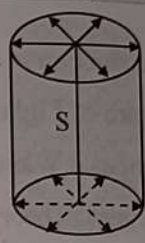
Huygens' principle:

Each point on a wavefront acts as a secondary source of light emitting secondary light waves called wavelets in all directions which travel with the speed of light in the medium. The new wavefront can be obtained by taking the envelope of these secondary wavelets travelling in the forward direction and is thus, the envelope of the secondary wavelets in forward direction. The wavelets travelling in the backward direction are ineffective.

➤ **Wavefront:**

i. A wavefront is a surface of constant phase.

ii. There are three types of wavefronts:

Parameters	Spherical	Plane	Cylindrical
Shape			
Obtained by	Keeping a point source at finite distance, which is radius of the wavefront.	Keeping a point source of light at infinity or at a sufficiently large distance.	Using an extended source of light like a slit or a linear source.
Example	Tip of candle flame	Sun light	Tube light
Wave normal	Along the radius of the wavefront. They are diverging from the source.	Parallel to each other	Along the radius
Variation of amplitude (A) with distance (r)	$A \propto \frac{1}{r}$	A = constant	$A \propto \frac{1}{\sqrt{r}}$
Variation of intensity (I) with distance (r)	$I \propto \frac{1}{r^2}$	I = constant	$I \propto \frac{1}{r}$

Polarization

Definition:
The phenomenon of restriction of the vibration of light waves in a particular plane perpendicular to direction of wave motion is called polarization of light.

Produced in:
Transverse waves only

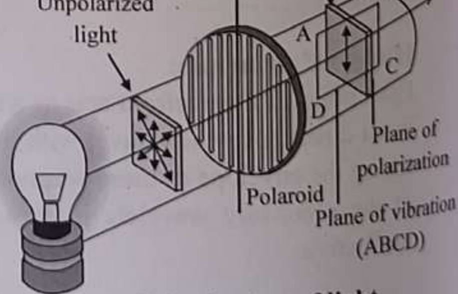
Brewster's law:
based upon polarization by reflection.
Statement: The tangent of the polarizing angle is equal to the refractive index of the refracting medium at which partial reflection takes place.

Produced using:
polarizers (materials which allow only those light waves which have their electric field along a particular direction to pass through and block all other waves)

Produced by:
i. reflection ii. scattering
iii. refraction iv. polaroid

Malus' law:
Gives the intensity of a linearly polarized wave after it passes through a polarizer.

Diagrammatic representation:



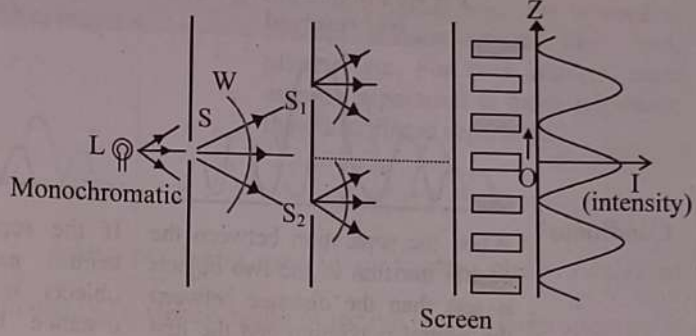
Polarization of light

- **Interference of light:**
  It is the phenomenon of redistribution of energy on account of superposition of light waves from two coherent sources.
  Thomas Young, first demonstrated the phenomenon of interference of light with the help of a slit.

Young's double slit experiment (YDSE):

Bright fringe
Constructive interference

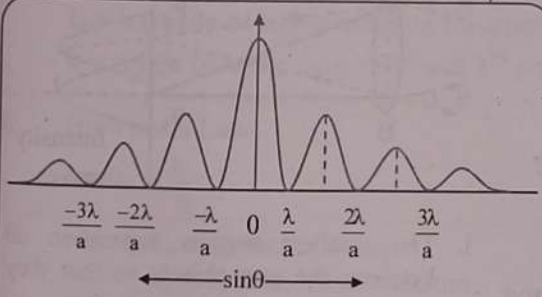
Dark fringe
Destructive interference



- **Comparison between constructive and destructive interference:**

No.	Constructive interference	Destructive interference
i.	The resultant amplitude of two waves is equal to the sum of amplitudes of individual waves i.e., $a = a_1 + a_2$	The resultant amplitude of two waves is equal to the difference of amplitudes of individual waves i.e., $a = a_1 - a_2$
ii.	The amplitude of resultant wave is maximum.	The amplitude of resultant wave is minimum.
iii.	The path difference between two waves is an even multiple of $\frac{\lambda}{2}$. i.e., $\Delta l = 2n\frac{\lambda}{2}$ where, n = 0, 1, 2, 3 ….	The path difference between two waves is an odd multiple of $\frac{\lambda}{2}$ i.e., $\Delta l = (2n-1)\frac{\lambda}{2}$ where, n = 1, 2, 3 ……
iv.	The crests or troughs of two waves coincide.	The crest of one wave coincides with the trough of another and vice versa.

Using single slit



Graph of intensity against sin θ

Diffraction

The bending of light near the edge of an obstacle or slit and spreading into the region of geometrical shadow is called diffraction of light.

Using double slit

In double slit diffraction the pattern will be decided by the diffraction pattern of the individual slits, as well as by the interference between them.

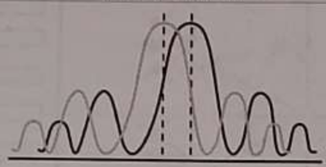
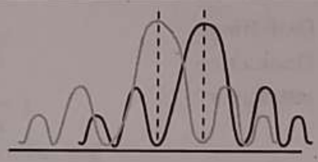
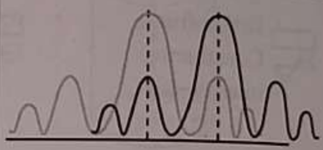
Fresnel diffraction

- The source is at a finite distance.
- No optical is required.
- Fringes are not sharp and well-defined.

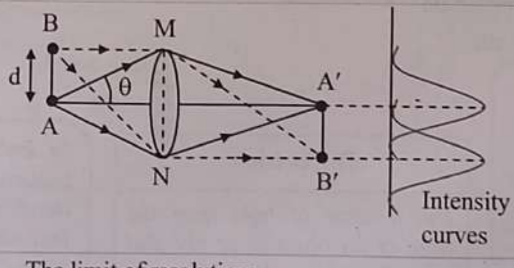
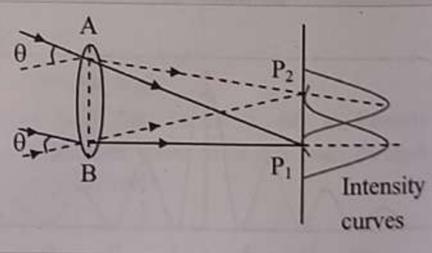
Fraunhofer diffraction

- The source is effectively at an infinite distance.
- No optical is required.
- Fringes are not sharp and well-defined.

➢ Rayleigh's criterion for resolution:

Definition	i. According to Lord Rayleigh, the images of two point objects close to each other are regarded as resolved (separated), if the central maximum of one falls on the first minimum of the other. ii. The reasoning of Rayleigh's criterion is given by considering intensity distribution in the diffraction pattern produced by two objects.		
Conditions	**Not resolved**	**Just resolved**	**Well resolved**
			
	When the separation between the central maxima of the two objects is less than the distance between the central maximum and the first minimum of any of the two objects, they are said to be 'not resolved' or unresolved.	If the separation between the central maxima of the two objects is just equal to the distance between the central maximum and first minimum of any of the two objects, they are said to be just resolved.	If the separation between the central maxima of two objects is greater than the distance between the central maximum and first minimum of any of the two objects, they are said to well resolved.
Application	This criterion of resolution is equally applicable for resolution of spectral lines of equal intensity.		

➢ Resolving power of an optical instrument:

Definition	i. *The ability of an optical instrument to produce distinctly separate images of two objects very close to each other is called the resolving power of the instrument.* ii. The minimum distance of separation between two objects when they can be observed as separate by an optical instrument is called the limit of resolution of that instrument. iii. The reciprocal of the limit of resolution is called its resolving power. Mathematically, R.P. = $\frac{1}{\text{Limit of resolution}}$	
Types of optical instrument	**Microscope**	**Telescope**
Diagram		
Limit of resolution/angular separation	i. The limit of resolution: a. for self-luminous objects $d = \frac{1.22\lambda}{2\sin\theta} = \frac{0.61\lambda}{\sin\theta}$ and b. for objects illuminated by light of wavelength λ, $d = \frac{\lambda}{2\sin\theta}$ ii. If there is a liquid of refractive index μ between the object and objective of the microscope, then $d = \frac{\lambda}{2\mu\sin\theta}$, where θ is the angle subtended by an object at the objective.	i. The smallest angular separation dθ between the two objects so that they appear just separated is given by: a. Circular aperture: $d\theta = \frac{1.22\lambda}{D}$, where D is the aperture (diameter) of objective of the telescope. b. Rectangular aperture: $d\theta = \frac{d}{D}$, where, d is slit separation, D distance
Resolving power	R.P. $= \frac{1}{d} = \frac{2\mu\sin\theta}{\lambda}$, where, μ sin θ is called the numerical aperture (N.A.) of the objective of the microscope.	R.P. $= \frac{1}{d\theta} = \frac{D}{1.22\lambda}$

Deciding factors	R.P. of a microscope is increased i. with increase in the value of the refractive index of the medium between the object and the objective. ii. by using lower wavelength of light to illuminate the object. iii. by using ultraviolet light and quartz lenses.	i. R.P. of a telescope is directly proportional to the aperture (diameter) of the objective of telescope and inversely proportional to the wavelength of light. ii. An astronomical telescope is used to observe distant objects like stars, planets, etc. For such objects, their angular separation is more important than their linear separation.

Caution

Though according to mathematical equation R.P. $\propto \frac{1}{\lambda}$, telescopes being used in sunlight to observe celestial objects, practically there is no control on wavelength incident (λ). Hence, using objective lens of large aperture is effective way to increase R.P. of telescope.

Formulae

1. Refractive Index:

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{v_1}{v_2}$$

2. Relation between refractive index (n) and wavelength:

$$n = \frac{\lambda_0}{\lambda}$$

where, λ_0 is wavelength is vacuum
λ is wavelength in medium of refractive index n

3. Malus' Law:
Intensity of transmitted component from analyser,

$$I = I_0 \cos^2 \theta$$

I_0 = Intensity of light emerging from polariser
θ = Angle between axes of 1st and 2nd polarizer

4. Brewster's Law:

$$\tan \theta_B = \frac{n_2}{n_1}$$

where, θ_B is Brewster's angle,
n_2 is R.I. of denser medium,

5. For Interference:

i. Path difference of n^{th} bright fringe,

$$\Delta l \text{ or } \Delta x = n\lambda = 2n\frac{\lambda}{2}$$

where, n = 0, 1, 2, 3,…

ii. Path difference of n^{th} dark fringe,

$$\Delta l \text{ or } \Delta x = (2n-1)\frac{\lambda}{2}$$

where, n = 1, 2, 3,….

iii. Phase difference of n^{th} bright fringe,
$\Delta\phi = 2n\pi$
where, n = 0, 1, 2, 3,…

iv. Phase difference of n^{th} dark fringe
$\Delta\phi' = (2n-1)\pi$
where, n = 1, 2,3,….

v. Relation between phase difference and path difference: $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta l$

vi. Fringe (band) width: $W = \frac{\lambda D}{d}$

vii. Distance of n^{th} bright fringe from centre of screen: $y_n = \frac{n\lambda D}{d} = nW$
where n = 0, 1, 2, 3, ….

viii. Distance of n^{th} dark fringe from centre of screen:

$$y'_n = (2n-1)\frac{D\lambda}{2d} = (2n-1)\frac{W}{2}$$

where n = 1, 2, 3,….

ix. Fringe shift on covering one of the slits with a thin transparent plate of thickness 't' and refractive index n, $y_0 = \frac{D}{d}(n-1)t$

x. Angular width of fringe: $\theta = \frac{W}{D} = \frac{W}{f} = \frac{\lambda}{d}$
where, f = focal length of the lens held very close to the slit

xi. Wavelength: $\lambda = \frac{Wd}{D}$

xii. Angular position of n^{th} bright fringe:

$$\theta_n = \frac{n\lambda}{d}$$

xiii. Angular position of n^{th} dark fringe:

$$\theta'_n = (2n-1)\frac{\lambda}{2d}$$

6. For diffraction at a single slit:

i. Condition for n^{th} secondary minimum:
Path difference = $a \sin\theta = n\lambda$
where n = 1, 2, 3, ..

ii. Condition of n^{th} secondary maximum:
Path difference = $a \sin\theta = (2n+1)(\lambda/2)$
where n = 1, 2, 3, …

iii. Angular position of n^{th} minimum:
$$\theta_n = \frac{n\lambda}{a}$$

iv. Distance of n^{th} minimum from the centre of the screen, $y_{n_d} = \frac{nD\lambda}{a} = \frac{nf\lambda}{a}$

v. Angular position of n^{th} secondary maximum,
$$\theta'_n = (2n+1)\frac{\lambda}{2a}$$
where $n = \pm 1, \pm 2, \pm 3, ...$

vi. Distance of n^{th} secondary maximum from the centre of the screen,
$$y_{n_b} = (2n+1)\frac{\lambda D}{2a}$$

vii. Linear width of central maximum,
$$W_c = 2W = \frac{2\lambda D}{a} = \frac{2f\lambda}{a}$$

viii. Angular width of central maximum (2θ):
$$2\theta = \frac{2\lambda}{d}$$

7. Limit of resolution:
$$d\theta = \frac{\lambda}{a}$$
where, a is separation distance between two objects.

8. Minimum separation between the two linear objects that are just resolved:
$$y = D(d\theta) = \frac{D\lambda}{a}$$
where, D is distance between objects and instrument.

9. For microscope:

i. Path difference = $2a \sin\alpha$
where, $\alpha = \frac{1}{2}$(angular separation between objects at the objective)

ii. For nonluminous objects:
a. $a = \frac{\lambda}{2n\sin\alpha} = \frac{\lambda}{2(\text{N.A.})}$
where $n \sin\alpha$ is numerical aperture (N.A.)
b. Resolving power $R = \frac{1}{a} = \frac{2(\text{N.A.})}{\lambda}$

iii. For self-luminous objects:
a. $a = \frac{1.22\lambda}{2n\sin\alpha} = \frac{0.61\lambda}{(\text{N.A.})}$
b. $R = \frac{1}{a} = \frac{(\text{N.A.})}{0.61\lambda}$

10. For telescope:

i. Path difference = 1.22λ

ii. Limit of resolution $\theta = \frac{1.22\lambda}{D}$
where, D is aperture of telescope

iii. Resolving power $R = \frac{1}{\theta} = \frac{D}{1.22\lambda}$

Shortcuts

1. If it is given in the question that reflected and refracted rays are perpendicular to each other, it means $i + r = 90°$.

2. An unpolarised beam of light is incident on a group of 'n' polarising sheets which are arranged in such a way that the characteristic direction of each polarising sheet makes an angle of θ with that of the preceding sheet then, fraction of incident unpolarised light emerging out of last sheet has intensity $\frac{I_0}{2}(\cos^2\theta)^{n-1}$

3. If n^{th} bright band of λ_1 coincides with m^{th} bright band of λ_2 then apply $n\lambda_1 = m\lambda_2$. For dark bands use $(2n+1)\lambda_1 = (2m+1)\lambda_2$ where n_1 and n_2 are measured from initial value of 0.

4. On immersing apparatus in liquid with refractive index n, the fringe width decreases to W′, where,
$$W' = \frac{W}{n}$$

5. If in the question it is given that a transparent sheet of refractive index n is inserted and central fringe shifts to a place of N^{th} bright fringe, then apply the formula, $(n-1) \times t = N\lambda$. If in the question central fringe shifts to N^{th} dark fringe is given, just replace N in formula above by $(N - \frac{1}{2})$.