CONCEPT MAP

PERMUTATIONS AND COMBINATIONS

Class XI

PERMUTATIONS AND COMBINATIONS

Permutations

Arranging r objects out of n different things

- When repetition is not allowed $= {}^nP_r = \dfrac{n!}{(n-r)!}$, where $0 \le r \le n$
- When repetition is allowed $= n^r$

Properties

- ${}^nP_n = n! = n(n-1)\cdot(n-2) \ldots 3\cdot2\cdot1 = {}^nP_{n-1}$
- ${}^nP_1 = n$
- ${}^nP_0 = \dfrac{n!}{(n-0)!} = 1$
- ${}^nP_r = n\cdot{}^{n-1}P_{r-1} = n(n-1)\,{}^{n-2}P_{r-2}$
  $= n(n-1)(n-2)\,{}^{n-3}P_{r-3}$ and so on
- ${}^{n-1}P_r + r\cdot{}^{n-1}P_{r-1} = {}^nP_r$
- $\dfrac{{}^nP_r}{{}^nP_{r-1}} = n - r + 1$

Restricted Permutations

The number of ways in which r objects can be arranged from n dissimilar objects if k particular objects are

- Always included (or never excluded) $= {}^{n-k}C_{r-k}\, r! = {}^rP_k\, {}^{n-k}P_{r-k}$.
- Always excluded (never included) $= {}^{n-k}C_r\, r! = {}^{n-k}P_r$

Circular Permutations

(i) Arrangement of n different things taken all at a time in form of circle is

- $(n-1)!$, if sense matter.
- $1/2(n-1)!$, if sense doesn't matter

(ii) Number of circular permutations of n dissimilar things taken r at a time

$= \dfrac{{}^nP_r}{r}$ if clockwise and anticlockwise orders are considered as different.

$= \dfrac{{}^nP_r}{2r}$ if clockwise and anticlockwise order is considered as same.

Distributions into Groups

Distribution of n distinct things into r groups $G_1, G_2, \ldots, G_r$ containing $P_1, P_2, \ldots, P_r$ elements respectively.

- Groups are distinct : $\dfrac{n!}{P_1!\,P_2!\ldots P_r!}\,r!$
- Groups are identical : $\dfrac{n!}{P_1!\,P_2!\ldots P_r!}$

De-arrangements

Any change in the existing order of things is called De-arrangement. If m things are arranged in a row, the number of ways in which they can be dearranged so that none of them occupies its original place (no one of them occupies the place assigned to it)

$$= m!\left[1 - \frac{1}{1!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^m \frac{1}{m!}\right]$$

$$= m! \sum_{r=0}^{m} (-1)^r \frac{1}{r!}, \text{ we denote it by } D(m)$$

$$= {}^mP_m - {}^mP_{m-1} + {}^mP_{m-2} - \ldots + (-1)^m\, {}^mP_0$$

Factorial Notation

Product of first n natural numbers is denoted by $n!$

i.e., $n! = n(n-1)(n-2) \ldots 3\cdot2\cdot1$

Fundamental Principle of Counting

In an operation A can be performed in m different ways and another operation B can be performed in n different ways, then

- Both the operations can be performed in $m \times n$ ways.
- Either of the two operations can be performed in $(m+n)$ ways.

Combinations

- Selecting r objects out of n different things given by ${}^nC_r = \dfrac{n!}{r!\,(n-r)!}, 0 \le r \le n$

Properties

- ${}^nP_r = {}^nC_r\, r!,\ 0 \le r \le n$
- For $0 \le r \le n$, ${}^nC_r = {}^nC_{n-r}$
- For $1 \le r \le n$, ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_a = {}^nC_b \Rightarrow a = b$ or $n = a + b$
- ${}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 2^n$

Restricted Combinations

The number of ways in which r objects can be selected from n dissimilar objects if k particular objects are

- Always included $= {}^{n-k}C_{r-k} = {}^{n-k}C_{n-r}$
- never included (Always excluded) $= {}^{n-k}C_r$