CHAPTER 04

Thermal Properties of Matter

In this chapter, we will study about thermal expansion, specific heat capacity, calorimetry and heat transfer, one by one as discussed below.

Thermal Expansion

Almost all substances (solid, liquid and gas) expand on heating and contract on cooling. The expansion of a substance on heating is called thermal expansion of substance.

It is of three types

1. Linear Expansion

Thermal expansion along a single dimension of a solid body is defined as the linear expansion.

If a rod is having length l_0 at temperature T, then elongation in length of rod due to rise in temperature by ΔT is

$$\Delta l = l_0 \alpha \Delta T$$

or $$\alpha = \frac{\Delta l}{l_0 \times \Delta T}$$

where, α is the coefficient of linear expansion whose value depends on the nature of the material.

Final length, $l_f = l_0 + l_0 \alpha \Delta T = l_0(1+\alpha \Delta T)$

Example 1. The length of a steel rod is 5 cm longer than that of a brass rod. If this difference in their lengths is to remain the same at all temperatures, then the length of brass rod will be (Take, coefficient of linear expansion for steel and brass are $12\times 10^{-6}/{}^\circ\text{C}$ and $18\times 10^{-6}/{}^\circ\text{C}$, respectively)

a. 10 cm b. 20 cm

c. 5 cm d. 6 cm

Sol. (*a*) Given, $\Delta l_{\text{St}} - \Delta l_{\text{Br}} = \Delta l$

Let $l_{\text{St}} = l \Rightarrow l_{\text{Br}} = (l-5)$ cm

($\because$ steel rod is 5 cm longer than that of a brass rod)

Given, $\alpha_{\text{St}} = 12\times 10^{-6}/{}^\circ\text{C}$ and $\alpha_{\text{Br}} = 18\times 10^{-6}/{}^\circ\text{C}$

We know that, $\Delta l_{\text{St}} = l_{\text{St}}\, \alpha t_1$

$\Rightarrow$ $$\alpha_{\text{St}} = \frac{\Delta l_{\text{St}}}{l_{\text{St}} \times t_1}$$

$\Rightarrow$ $$12\times 10^{-6} = \frac{\Delta l}{l \times t} \quad \text{...(i)}$$

Similarly, $\Delta l_{\text{Br}} = l_{\text{Br}}\, \alpha t_2$

$\Rightarrow$ $$\alpha_{\text{Br}} = \frac{\Delta l_{\text{Br}}}{l_{\text{Br}} \times t_2} \Rightarrow 18\times 10^{-6} = \frac{\Delta l}{(l-5)\times t} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{12\times 10^{-6}}{18\times 10^{-6}} = \frac{\Delta l / l \times t}{\Delta l/(l-5)\times t} \Rightarrow \frac{2}{3} = \frac{l-5}{l}$$

$\Rightarrow$ $2l = 3l - 15 \Rightarrow l = 15$

So, $l_{\text{Br}} = l - 5 = 15 - 5 = 10$ cm

2. Superficial or Areal Expansion

Expansion of solids along two dimensions of solid objects is defined as superficial expansion.

Coefficient of superficial expansion, $\beta = \dfrac{\Delta A}{A_0 \times \Delta T}$

Final area, $A_f = A_0(1+\beta \Delta T)$

where, A_0 is the area of the body at temperature T.

Example 2. A metal ball having a diameter of 0.4 m is heated from 273 to 360 K. If the coefficient of areal expansion of the material of the ball is $0.000034\ \text{K}^{-1}$, then the increase in surface area of the ball will be

a. $2486\times 10^{-3}\ \text{m}^2$ b. $1.486\times 10^{-3}\ \text{m}^2$

c. $2486\times 10^{-4}\ \text{m}^2$ d. $5.486\times 10^{-3}\ \text{m}^2$

Sol. (*b*) Given, diameter = 0.4 m and radius, $r = \frac{0.4}{2} = 0.2$ m

$\therefore$ Area of ball, $A_0 = 4\pi r^2 = 4 \times \pi \times (0.2)^2 = 0.5024\ \text{m}^2$

Temperature, $\Delta T = T_2 - T_1 = 360\ \text{K} - 273\ \text{K} = 87\ \text{K}$

Coefficient of area expansion, $\beta = 0.000034\ \text{K}^{-1}$

Change in area, $\Delta A = \beta A_0 \Delta T$,

$\Rightarrow \quad \Delta A = 0.000034 \times 0.5024 \times 87 = 0.001486$
$= 1.486 \times 10^{-3}\ \text{m}^2$

3. Volume or Cubical Expansion

Expansion of solids along three dimensions of solids objects is defined as cubical expansion.

Coefficient of volume or cubical expansion,

$$\gamma = \frac{\Delta V}{V_0 \times \Delta T}$$

Final volume, $V = V_0(1 + \gamma \Delta T)$.

where, V_0 is the volume of the body at temperature T.

Relation between α, β and γ

The coefficients α, β and γ for a solid are related to each other.

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

Example 3. The volume of mercury in the bulb of a thermometer is 10^{-6}m^3. The area of cross-section of the capillary tube is $2 \times 10^{-7}\ \text{m}^2$. If the temperature is raised by $100°$ C, then the increase in the length of the mercury column will be (Take, $\gamma_{Hg} = 18 \times 10^{-5}/°\text{C}$)

a. 9 cm
b. 10 cm
c. 15 cm
d. 5 cm

Sol. (*a*) By cubical expansion relation, $\Delta V = V \times \gamma \Delta T$ ...(i)

where, $\gamma = 18 \times 10^{-5}/°\text{C}$,

and initial volume, $V = 10^{-6}\text{m}^3$

Temperature, $\Delta T = 100°\text{C}$

Putting these values in Eq. (i), we get

$$\Delta V = 10^{-6} \times 18 \times 10^{-5} \times 10^2$$
$$= 18 \times 10^{-9}\ \text{m}^3$$

Since, $\Delta V = A \times \Delta l$

$\therefore \quad 18 \times 10^{-9} = 2 \times 10^{-7} \times \Delta l$ or $9 \times 10^{-2} = \Delta l$ or $\Delta l = 9$ cm

Note

- *As temperature increases, density decreases according to relation,*

$$\rho = \frac{\rho_0}{1 + \gamma \Delta T}$$

or $\rho = \rho_0(1 - \gamma \Delta T)$ *(valid for small ΔT)*

- *Water at 4°C expands whether it is heated or cooled, i.e. the level of water in a beaker at 4°C increases when the beaker is cooled or heated.*

Heat Capacity (Thermal Capacity)

If an amount of heat ΔQ is needed to change the temperature of a body by ΔT, then heat capacity of the material of the body is given by

Heat capacity, $S = \frac{\Delta Q}{\Delta T}$

Heat capacity is equal to the heat energy required to change the temperature of a body by unity.
Its unit is J/K.

Specific Heat Capacity

The amount of heat needed to raise the temperature of unit mass of a substance by unity is known as the specific heat capacity or specific heat. It is denoted by s and its SI unit is $\text{Jkg}^{-1}\text{K}^{-1}$.

$$s = \frac{S}{m} = \frac{\Delta Q}{m\Delta T}$$

$\Rightarrow$ Specific heat capacity, $s = \frac{1}{m}\frac{\Delta Q}{\Delta T}$

where, m = mass of given substance.

The SI unit of specific heat capacity is $\text{Jkg}^{-1}\text{K}^{-1}$.

Note *Water has the highest specific heat capacity ($4.18 \times 10^3\ \text{J kg}^{-1}\ °\text{C}^{-1}$) compared to other substances. For this reason, water is used as a coolant in automobile radiators as well as a heater in hot water bags.*

Heat Equations

Heat is a form of energy that flow from one body to another because of temperature difference between them.

In general, heat is given by $Q = m \times s \times \Delta T$

This is called heat equation.

The SI unit of heat is joule and CGS unit of heat is calorie (cal).

One calorie is defined as the heat energy required to raise the temperature of one gram of water through 1°C.

$$1\ \text{cal} = 4.18\ \text{J}$$

Example 4. When 400 J of heat are added to a 0.1 kg sample of metal, its temperature increases by 20°C. What is the specific heat of the metal?

a. $200\ \text{J kg}^{-1}{}°\text{C}^{-1}$
b. $300\ \text{J kg}^{-1}{}°\text{C}^{-1}$
c. $400\ \text{J kg}^{-1}{}°\text{C}^{-1}$
d. $800\ \text{J kg}^{-1}{}°\text{C}^{-1}$

Sol. (*a*) Given, heat, $\Delta Q = 400$ J

Mass, $m = 0.1$ kg

Temperature, $\Delta T = 20°\text{C}$

Specific heat, $S = \frac{1}{m} \cdot \frac{\Delta Q}{\Delta T} = \left(\frac{1}{0.1}\right)\left(\frac{400}{20}\right) = 200\ \text{J kg}^{-1}{}°\text{C}^{-1}$

Example 5. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 min? Assuming 50% of power is used up in heating the machine itself or lost to the surroundings.
(Take, specific heat of aluminium $= 0.91\ \text{Jg}^{-1}{}^\circ\text{C}^{-1}$).

a. 203.02°C b. 103.02°C
c. 104.02°C d. 105.02°C

Sol. (b) Given, power, $P = 10\ \text{kW} = 10 \times 10^3\ \text{W}$

Time, $t = 2.5\ \text{min} = 2.5 \times 60\ \text{s}$

As, the power is defined as rate at which energy is consumed.

$$\therefore \quad P = \frac{E}{t}$$

Total energy, $E = Pt = 10 \times 10^3 \times 2.5 \times 60$

$$= 1.5 \times 10^6\ \text{J}$$

Energy absorbed by aluminium block,

$$Q = 1.5 \times 10^6 \times \frac{50}{100}$$

$$= 0.75 \times 10^6\ \text{J}$$

$$\therefore \quad \Delta T = \frac{Q}{ms} = \frac{0.75 \times 10^6}{8.0 \times 10^3 \times 0.91}$$

$$= 103.02^\circ\text{C}$$

Molar Specific Heat Capacity

The amount of heat needed to raise the temperature of one mole of a substance (gas) by unity is known as the **molar heat capacity** of that substance. It is denoted by C, and SI unit is $\text{Jmol}^{-1}\text{K}^{-1}$.

$$C = \frac{S}{\mu} = \frac{\Delta Q}{\mu \Delta T}$$

Molar specific heat capacity,

$$C = \frac{\Delta Q}{\mu\, \Delta T}$$

where, μ = number of moles of substance (gas).

Specific Heat Capacity of Gas

The specific heat of gas is defined in two different situations and accordingly there are two specific heat of gas, one when during heating the volume of the gas is kept constant and the pressure is allowed to increase and the other when pressure of the gas is kept constant and the volume is allowed to increase. These are called respectively the specific heat of constant volume and the specific heat at constant pressure of the gas.

- **Specific heat at constant volume** It is the amount of heat required to raise the temperature of 1 g of the gas through 1°C (or 1 K), when the volume is kept constant. It is denoted by S_V.
- **Specific heat at constant pressure** It is the amount of heat required to raise the temperature of 1 g of the gas through 1°C (or 1 K), when the pressure is kept constant. It is denoted by S_p.

Molar Specific Heat Capacities of Gases

There are two types of molar specific heat capacity

- **Molar specific heat capacity at constant pressure** It is molar heat capacity of a gas at constant pressure, *i.e.* the amount of heat required to raise the temperature of 1 mole of a gas by unity at constant pressure and is denoted by C_p.
- **Molar specific heat capacity at constant volume** It is molar heat capacity of a gas at constant volume, *i.e.* the amount of heat required to raise the temperature of 1 mole of a gas through 1°C at constant volume and is denoted by C_V.
- If C_p and C_V are measured in unit of heat, then $C_p - C_V = R/J$.

Relation between Specific Heat and Molar Specific Heat Capacity

As, number of moles, $\mu = \dfrac{m}{M}$

where, m = mass of the substance
and M = molecular mass.

$$C = \frac{1}{\mu}\left[\frac{\Delta Q}{\Delta T}\right] = M\left[\frac{\Delta Q}{m\Delta T}\right]$$

But, $\dfrac{\Delta Q}{m\Delta T} = s$

$$C = Ms$$

where, s = specific heat capacity,
M = molecular mass of the substance
and C = molar specific heat capacity.

Hence, molar specific heat capacity
= molecular mass × specific heat capacity.

i.e. $C_p = M \times s_p$
and $C_V = M \times s_V$

Calorimetry

The branch of science that deals with the measurement of heat. When a body at higher temperature is brought in contact with another body at lower temperature, then the heat flows from the body kept at higher temperature to the body kept at lower temperature till the both bodies acquire the same temperature.

Principle of Calorimetry

The principle of calorimetry states that total heat given by a hotter body equals to the total heat received by colder body.

i.e. **Heat lost by hotter body = Heat gained by colder body**

If there are two bodies of masses m_1 and m_2 and having values of specific heats s_1 and s_2, respectively. Let these bodies be at temperature T_1 and T_2 respectively, such that $T_1 > T_2$, when they are placed in contact with each other, they obtain common temperature T_1. Then, according to principle of calorimetry,

Heat lost by first body = Heat gained by second body

$m_1 s_1 (T_1 - T) = m_2 s_2 (T - T_2)$

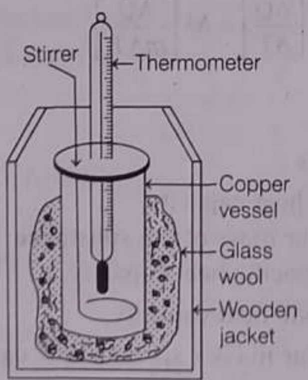
$\therefore$ Common temperature of the mixture at equilibrium,

$$T = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$$

Calorimeter

It is a device used for measuring the quantities of heat. It consists of a cylindrical vessel of copper provided with a stirrer. The vessel is kept inside a wooden jacket. The space between the calorimeter and the jacket is packed with a heat insulating material like glass wool.

Thus, the calorimeter gets thermally isolated from the surroundings. The loss of heat due to radiation is further reduced by polishing the outer surface of the calorimeter and the inner surface of the jacket.



Calorimeter

The lid is provided with holes for inserting a thermometer and a stirrer into the calorimeter.

When bodies at different temperatures are mixed together in the calorimeter, heat is exchanged between the bodies as well as with the calorimeter.

If there is no loss of heat to the surroundings, then according to the principle of calorimetry,

Heat gained by cold bodies = Heat lost by hot bodies

Water Equivalent

It is the quantity of water whose heat capacity is same as the heat capacity of the body. It is denoted by w.

$w = ms$ = Heat capacity of the body.

Example 6. In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm^3 of water at 27°C. The final temperature is 40°C. The specific heat of the metal will be

a. 0.2 cal $g^{-1}°C^{-1}$
b. 0.3 cal $g^{-1}°C^{-1}$
c. 0.1 cal $g^{-1}°C^{-1}$
d. 0.4 cal $g^{-1}°C^{-1}$

Sol. (c) Given, mass of metal block, $m = 0.20 \text{ kg} = 200 \text{ g}$

Fall in temperature of metal block,

$\Delta T = 150° - 40° = 110°C$

Let specific heat of metal block, $c = 5 \text{ cal g}^{-1}°C^{-1}$

$\therefore$ Heat lost by metal block $= mc\,\Delta T = 200 \times 5 \times 110$ cal

Volume of water in calorimeter = 150 cm^3

$\Rightarrow$ Mass of water, $m' = 150$ g

Water equivalent of calorimeter, $w = 0.025 \text{ kg} = 25 \text{ g}$

Specific heat of water, $s' = 1 \text{ cal g}^{-1}°C^{-1}$

$\therefore$ Heat gained by water and calorimeter

$= (m' + w)\, s' \Delta T'$

$= (150 + 25) \times 1 \times (40 - 27)$ cal

$= 175 \times 13$ cal

According to principle of calorimetry, we get

Heat lost = Heat gained

$\therefore \quad 200 \times s \times 110 = 175 \times 13$

or $\quad s = \dfrac{175 \times 13}{200 \times 110} = 0.1 \text{ cal g}^{-1}°C^{-1}$

Heat Transfer

There are three different ways in which heat can be transferred; conduction, convection and radiation.

Conduction

It is a process by which the heat is transferred in solid. In conduction, molecules vibrate about a fixed location and transfer the heat by collision.

When a metallic rod is put in a flame, the other end of rod will soon be so hot that you cannot hold it by your hands. It means heat transfer takes place by conduction from hot end of rod through its different parts to the other end.

Thermal Conductivity

In solids, heat is transferred through conduction. We will study conduction of heat through a solid bar.

Regarding conduction following points are worth noting

- The **amount of heat flowing** in a rod of surface area A in time t is

$$\frac{\Delta Q}{\Delta t} = -KA\frac{\Delta\theta}{\Delta x}$$

Here, K = coefficient of heat conduction

$\frac{\Delta\theta}{\Delta x}$ = temperature gradient between faces of a rod

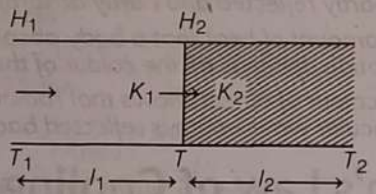
In the above relation, negative sign is used to make $\frac{\Delta Q}{\Delta t}$ a positive quantity, because it is negative.

- The **ratio of thermal and electrical conductivities** is the same for the metals at a particular temperature and is proportional to the absolute temperature of the metal. If T is the absolute temperature, then

$$\frac{K}{\sigma} \propto T$$

or $$\frac{K}{\sigma T} = \text{constant}$$

- Let two rods of thermal conductivities K_1, K_2 lengths l_1, l_2 and cross-sectional area A are connected in series. In steady state, the temperatures of ends of rod are T_1 and T_2 and the **temperature of junction is** T. Then,



$$T = \frac{K_1T_1l_2 + K_2T_2l_1}{K_1l_2 + K_2l_1}$$

- The **thermoelectric conductivity or diffusivity** is defined as the ratio of the coefficients of thermal conductivity to thermal capacity per unit volume. So,

Thermal capacity per unit volume $= \left(\frac{m}{V}\right)s = \rho s$, where ρ is the density of substance.

Diffusivity, $D = \frac{K}{\rho s}$

- The **hindrance offered by a body** to the flow of heat is called its **thermal resistance.**

$$R = \frac{\text{Temperature difference } (\Delta T)}{\text{Heat current } (H)}$$

$$= \frac{\Delta T}{H} = \frac{l}{KA}$$

where, l is length of rod, A the area and ΔT the temperature difference across its ends.

If different rods are connected in series, then heat flowing per second is same.

i.e. $\quad H_1 = H_2 = H_3 = \ldots$

$\therefore \quad R_s = R_1 + R_2 + R_3 + \ldots$

If different rods are connected in parallel, then temperature difference is same, *i.e.*

$$\Delta T_1 = \Delta T_2 = \Delta T_3 = \ldots$$

$$\therefore \quad \frac{1}{R_p} = \frac{1}{R_1} = \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

- **Heat current**

$$H = \frac{dQ}{dt} = \frac{\Delta T}{R} \quad \left(\text{where, } R = \frac{l}{KA}\right)$$

Current flow through a resistance,

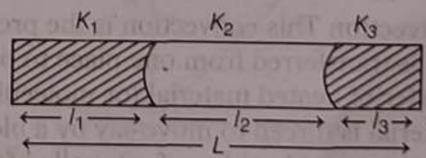
$$i = \frac{dq}{dt} = \frac{\Delta V}{R} \quad \left(\text{where, } R = \frac{l}{\sigma A}\right)$$

We find the following similarities in heat flow through a rod and current flow through a resistance.

Heat flow through a conducting rod	Current flow through a resistance
Heat current, $H = \frac{dQ}{dt}$ = rate of heat flow	Electric current, $i = \frac{dq}{dt}$ = rate of charge flow
$H = \frac{\Delta T}{R}$	$i = \frac{\Delta V}{R}$
$R = \frac{l}{KA}$	$R = \frac{l}{\sigma A}$
K = thermal conductivity	σ = electrical conductivity

From the above table, it is evident that flow of heat through rods in series and parallel is analogous to the flow of current through resistances in series and parallel. This analogy is of great importance in solving complicated problems of heat conduction.

- **In series combination of rods of different materials,** equivalent conductivity



$$\frac{l_1 + l_2 + l_3}{K_s} = \frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3}$$

If lengths of rods are equal, then

$$\frac{1}{K_s} = \frac{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}}{3}$$

- In parallel combination of slabs of different materials, equivalent conductivity, $K_p = \frac{K_1A_1 + K_2A_2 + K_3A_3}{A_1 + A_2 + A_3}$

Example 7. A metal plate 4 mm thick has a temperature difference of 32°C between its faces. It transmits 200 kcal/h through on area of 5 cm^2. Thermal conductivity of the material is

a. 58.33 W/m-°C
b. 33.58 W/m-°C
c. 5×10^{-4} W/m-°C
d. None of these

Sol. (*a*) Here, $\Delta x = 4 \text{ mm} = 4 \times 10^{-3}$ m, $\Delta T = 32°C$

Transmit heat per hours,

$$\frac{\Delta Q}{\Delta t} = 200 \text{ kcal/h} = \frac{200 \times 1000 \times 4.2}{60 \times 60} = 233.33 \text{ J/s}$$

$$A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$$

We know that, $\frac{\Delta Q}{\Delta t} = KA\left(\frac{\Delta T}{\Delta x}\right)$

$\therefore$ Thermal conductivity of material, $K = \frac{\Delta Q / \Delta t}{A(\Delta T / \Delta x)}$

$$\Rightarrow \quad K = \frac{233.33 \times 4 \times 10^{-3}}{5 \times 10^{-4} \times 32} = 58.33 \text{ W/m-°C}$$

Applications of Thermal Conductivity

- The metals are much better conductors than the non-metals. This is because the metals have large number of free electrons which can carry heat from hotter part to colder part.

  The handles of utensils are made of bad conductors of heat, so that they cannot conduct heat from the utensils to our hands.
- To prevent ice from melting it is wrapped in a gunny bag. A gunny bag is a poor conductor of heat.

Convection

It is the process in which heat is transferred from one point to another by the actual motion of matter from a region of high temperature to a region of lower temperature.

There are two types of convections

- **Forced convection** This convection is the process in which heat is transferred from one place to other by actual transfer of heated material (or molecules). If heated material is forced to move say by a blower or a pump, the process of heat transfer is called **forced convection**. The heat transfer in human body is an example of forced convection.
- **Natural or Free convection** In the process of convection, if the heated material moves due to difference in density. This process of heat transfer is **called natural** or **free convection** such as heat transfer in water.

Applications of convections

The mechanism of heating room by a heat convector or heater is entirely based on convection.

The heat of the transformer body loses by convection.

Radiation

It is a mode of heat transfer from one place to another without heating the intervening medium. The heat is transferred by the mean of thermal radiations, radiant energy or simply radiation. Here, the term radiation is used in two meanings. The first is the process by which the energy is emitted by a body, is transmitted in space and falls on another body. The second one is the energy itself is being transmitted in space.

The heat from the sun reaches to the earth by radiation. These are travelling millions of kilometres of empty space (*i.e.* without any material medium).

Thermal Radiation

The electromagnetic radiation emitted by a body by virtue of its temperature like the radiation by a red hot iron or light form filament lamp is called **thermal radiation.**

Note *When thermal radiation falls on some other body it may partly reflected and partly absorbed.*

- *The amount of heat that a body absorbed by radiation depends on the colour of the body.*
- *The colour of object shows that radiation of that particular wavelength is reflected back by the body.*

Newton's Law of Cooling

It states that the rate of cooling of a body (R) is directly proportional to the temperature difference between the body and its surroundings, provided the temperature difference is small,

i.e. Rate of cooling (R) $\propto$ Temperature difference between the body and its surroundings.

$$-\frac{dT}{dt} \propto (T - T_0)$$

Rate of cooling, $(R) = -\frac{dT}{dt} = K(T - T_0)$

where, k is a constant which depends upon surface area and nature of the surface. It can also be written as

$$\left(\frac{T_1 - T_2}{t}\right) = K\left[\left(\frac{T_1 + T_2}{2}\right) - T_0\right]$$

Here, T_1 and T_2 are temperatures of the body in time interval t.

Example 8. A body cools in 10 min from 60°C to 40°C. What will be its temperature after next 10 min? The temperature of the surroundings is 10°C will be

a. 28°C b. 30°C c. 100°C d. 0°C

Sol. (*a*) According to Newton's law of cooling,

$$\left(\frac{T_1 - T_2}{t}\right) = K\left[\left(\frac{T_1 + T_2}{2}\right) - T_0\right]$$

For the given conditions, $\frac{60-40}{10} = k\left[\frac{60+40}{2} - 10\right]$ ...(i)

Let T be the temperature after next 10 min. Then,

$$\frac{40-T}{10} = K\left[\frac{40+T}{2} - 10\right] \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$T = 28°\text{C}$$

Example 9. A body cools down from 52.5°C to 47.5°C in 5 min and to 42.5°C in 7.5 min. The temperature of the surroundings is

a. 135°C b. 35° C c. 90°C d. 40°C

Sol. (*b*) Using Newton's law of cooling,

$$\frac{T_1 - T_2}{t} = K\left[\left(\frac{T_1 + T_2}{2}\right) - T_0\right], \text{ we get}$$

$$\frac{52.5°\text{C} - 47.5°\text{C}}{5\text{ min}} = K\left(\frac{52.5°\text{C} + 47.5°\text{C}}{2} - T_0\right)$$

or $\frac{5°\text{C}}{5\text{min}} = K(50°\text{C} - T_0)$ ...(i)

and $\frac{47.5°\text{C} - 42.5°\text{C}}{7.5\text{min}} = K\left(\frac{47.5°\text{C} + 42.5°\text{C}}{2} - T_0\right)$

or $\frac{5°\text{C}}{7.5\text{min}} = K(45°\text{C} - T_0)$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{50°\text{C} - T_0}{45°\text{C} - T_0} = \frac{7.5}{5} = \frac{3}{2} \text{ or } T_0 = 35°\text{C}$$

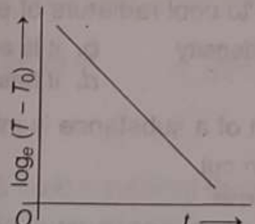
Cooling Curves

- **Curve of $\log(T - T_0)$ *versus* time**

$$\because \quad \frac{dT}{dt} = -K(T - T_0) \Rightarrow \int \frac{dT}{T - T_0} = -\int K dt$$

$$\Rightarrow \log_e(T - T_0) = -Kt + \log_e A$$

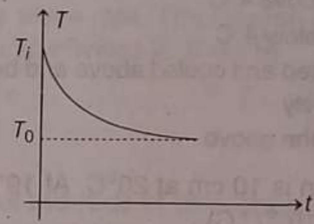
where, $\log_e A$ is a constant



Thus, **graph is a straight line with negative slope.** It means rate of cooling of the body decreases with time.

- **Curve of body temperature *versus* time**

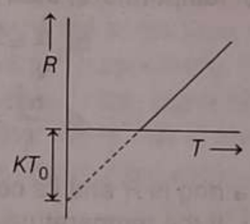
We know that, temperature of the body at any time t is given by $T = T_0 + (T_i - T_0)e^{-\alpha t}$. From this expression, we can say that $T = T_i$ at $t = 0$ and $T = T_0$ at $t = \infty$, *i.e.* **temperature of the body varies exponentially with time from T_i to T_0 ($< T_i$).** The temperature *versus* time graph is as shown in figure.



- **Curve of rate of cooling *versus* body temperature**

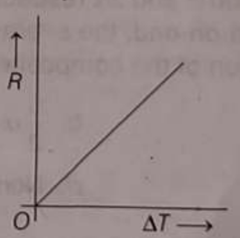
Rate of cooling $(R) = -\frac{dT}{dt} = -K(T - T_0)$

Graph of R *versus* T is a straight line with intercept $-KT_0$ as shown in figure.



- **Curve of rate of cooling *versus* temperature difference between body temperature (T) and surrounding (T_0).**

Rate of cooling $(R) = -\frac{dT}{dt} = K(T - T_0)$



Clearly, $R \propto (T - T_0)$

i.e. Rate of cooling $(R) \propto$ Temperature difference (ΔT)

Therefore, graph between R and ΔT is a straight line passing origin as shown in figure.