DIFFERENTIAL EQUATIONS

Class XII

CONCEPT MAP

DIFFERENTIAL EQUATIONS

An equation involving dependent variable, independent variable and derivative(s) of dependent variable(s) w.r.t. independent variable(s) is called differential equation.

Order and Degree of Differential Equation

Order : The order of a differential equation is the order of it highest order derivative.

Degree : Degree is the highest power of the highest orde derivative in a polynomial equation of differentials.

Solution of Differential Equation

A function of the form $y = f(x) + c$, which satisfies the given differential equation is the solution of the differential equation.

- **General Solution :** Contains as many arbitrary constants as order of equation.
- **Particular Solution :** Obtained by assigning values to arbitrary constants.

Formation of Differential Equation

- Differentiate the given equation as many times as th number of arbitrary constants and then eliminate th arbitrary constants from them.
- Order of differential equation = Number of arbitrar constants in differential equation.

Orthogonal Trajectory

Any curve which meet each member of a given family of curve at right angle is called an orthogonal trajectory of the family.

Steps to Find the Orthogonal Trajectory :

- Let $g(x, y, c) = 0$ be the equation of the given family of curve
- Differentiate the equation $g(x, y, c) = 0$ w.r.t. x and eliminat the parameter c.
- Put $-\frac{dx}{dy}$ for $\frac{dy}{dx}$ in the differential equation so obtained
- Now, solve this differential equation to get the orthogona trajectories.

Methods of Solving First Order and First Degree Differential Equations

Variable Separable

- If $\frac{dy}{dx} = f(x)g(y)$, then $\int \frac{dy}{g(y)} = \int f(x)dx + c$.
- If $\frac{dy}{dx} = f(ax + by + c)$, then put $ax + by + c = z$ and $b\frac{dy}{dx} = \frac{dz}{dx} - a$, then apply variable separable method.

Homogeneous Differential Equation

f $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$, where $f(x, y)$, $g(x, y)$ are homogeneous unctions of the same degree in x and y, then put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, then apply variable separable method.

Linear Differential Equation

f $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$, where $e^{\int Pdx}$ is the integrating actor (I.F.).

Method of Solving Differential Equations of Second Order

Consider a differential equation of second order.

$$A_0\frac{d^2y}{dx^2} + A_1\frac{dy}{dx} + A_2y = 0 \quad \text{...(}$$

Putting $\frac{d}{dx} = D$ in (i), we get

$$(A_0D^2 + A_1D + A_2)y = 0 \quad \text{...(i}$$

Auxiliary Equation (A.E.) is given by

$$A_0D^2 + A_1D + A_2 = 0 \quad \text{...(ii}$$

which is a quadratic equation in D.

Let roots of (iii) be given by $D = C_1, C_2$

Roots	Solution
$C_1 \neq C_2$	$y = Ae^{C_1x} + Be^{C_2x}$
$C_1 = C_2(C)$	$y = (A + Bx)e^{Cx}$