Relations and Functions

Relation : If A and B are two non-empty sets, then any subset R of $A \times B$ is called Relation from set A to B. i.e. $R : A \to B \Leftrightarrow R \subseteq A \times B$

If $(x, y) \in R$ then we write $x R y$ (read as x is R related to y) and
If $(x, y) \notin R$ then we write $x \not R y$ (read as x is not R related to y)

Domain and Range of a Relation : If R is any relation from set A to set B then,

- **Domain** of R is the set of all first coordinates of elements of R and is denoted by Dom(R).
- **Range** of R is the set of all second coordinates of R and it is denoted by Range (R)

A relation R on set A means, the relation from A to A i.e., $R \subseteq A \times A$

Empty Relation : A Relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e. $R = \phi \subset A \times A$

Universal Relation : A Relation R in a set A is called universal relation each of A is related to every element of A, i.e. $R = A \times A$

Identity Relation : $R = \{(x, y) : x \in A,\ y \in A,\ x = y\}$ OR $R = \{(x, x) ;\ x \in A\}$

A Relation R in a set A is called –

Reflexive Relation : If $(a, a) \in A$, for every $a \in A$

Symmetric Relation : If $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$ for all $a_1, a_2 \in A$

Transitive Relation : If $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies $(a_1, a_3) \in R$ for all $a_1, a_2, a_3 \in A$

Equivalence Relation : If R is reflexive, symmetric and transitive

Antisymmetric Relation : A relation R in a set A is antisymmetric.
if $(a, b) \in R, (b, a) \in R \Rightarrow a = b \ \forall \ a, b \in R$ or aRb and $bRa \Rightarrow a = b, \forall \ a, b \in R$.

Inverse Relation : If A and B are two non-empty sets and R be a relation from A to B, such that $R = \{(a, b) : a \in A, b \in B\}$, then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$

Equivalence class : Let R be an equivalence relation on a non-empty set A. For all $a \in A$, the equivalence class of 'a' is defined as the set of all such elements of A which are related to 'a' under R. It is denoted by [a].
i.e. $[a]$ = equivalence class of 'a' = $\{x \in A : (x, a) \in R\}$

Function : Let X and Y be two non-empty sets. Then a rule f which associates to each element $x \in X$, a unique element, denoted by $f(x)$ of Y, is called a function from X to Y and written as $f : X \to Y$ where, $f(x)$ is called image of x and x is called the pre-image of $f(x)$ and set Y is called the co-domain of f and $f(X) = \{f(x) : x \in X\}$ is called the range of f.

One-One OR Injective Function : A function $f : X \to Y$ is defined to be one-one if the images of distinct element of X under f are distinct ;
i.e $x_1, x_2 \in X : f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ Otherwise f is called many-one.