Formula Sheet.

Vectors.

- Types of vector:

1. Zero/Null vector ⇒ Initial & Terminal pts co-incide.
   ⇒ $\bar{0}$. Has any direction.

2. Unit vector ⇒ Mag. as unity (1)
   ⇒ $\hat{a} = \dfrac{\bar{a}}{|\bar{a}|}$, $\bar{a} = |\bar{a}|\,\hat{a}.1$

3. Co-initial vector ⇒ Same initial pts.
   Co-terminal vector ⇒ Same terminal pts.

4. Equal vector ⇒ Same mag. and direction.

5. Negative vector ⇒ $\bar{a} = -\bar{a}$ [diff direction]

6. Collinear vector ⇒ $\bar{a} = t\bar{b}$.

7. Free vector ⇒ Cannot change direction.
   Localized vector ⇒ Fixed vector.

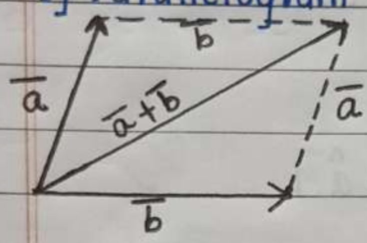
- Scalar multiplication of vectors:
  Magnitude of $k\bar{a}$ = k times $|\bar{a}|$
  1) if $k<0$ then direction opposite
  2) if $k>0$ then direction same.

- Co-planar vector:

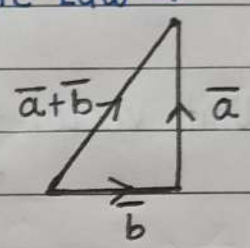
$\Rightarrow$ if they lie in same plane or in parallel plane.

- Addition of two vectors: $\Rightarrow \bar{a}+\bar{b}$

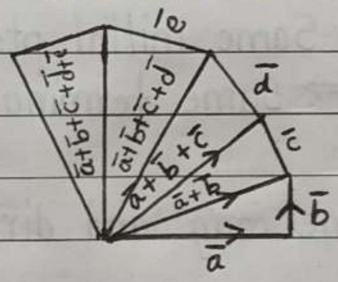
1] Parallelogram Law $\Rightarrow$



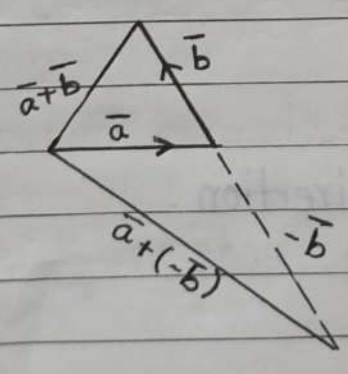
2] Triangle Law $\Rightarrow$



3] Polygon Law $\Rightarrow$ Extension of triangle law.



- Substraction of two vectors: $\Rightarrow \bar{a}-\bar{b}$



•Properties:

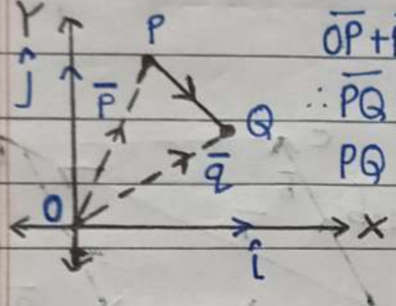
1. $\bar{a}+\bar{b}=\bar{b}+\bar{a}$ [Commutative]
2. $(\bar{a}+\bar{b})+\bar{c} = \bar{a}+(\bar{b}+\bar{c})$ [Associative]
3. $\bar{a}+\bar{0} \neq \bar{a}$
4. $\bar{a}+(-\bar{a})=\bar{0}$

5. $\bar{a}$ & $\bar{b}$ vector, m & n scalers
   i) $(m+n)\bar{a} = m\bar{a}+n\bar{a}$
   ii) $m(\bar{a}+\bar{b}) = m\bar{a}+m\bar{b}$
   iii) $m(n\bar{a}) = (mn)\bar{a} = n(m\bar{a})$

6. $|\bar{a}+\bar{b}| \leq |\bar{a}| + |\bar{b}|$

7. Any 2 vectors $\bar{a}, \bar{b}$ determine plane and vectors $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$ lie in same plane.

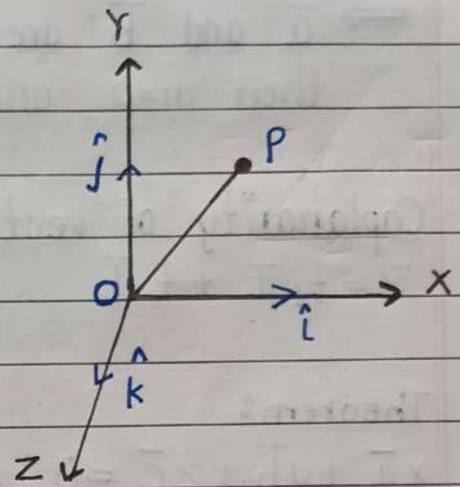
- Vectors in 2D and 3D :-



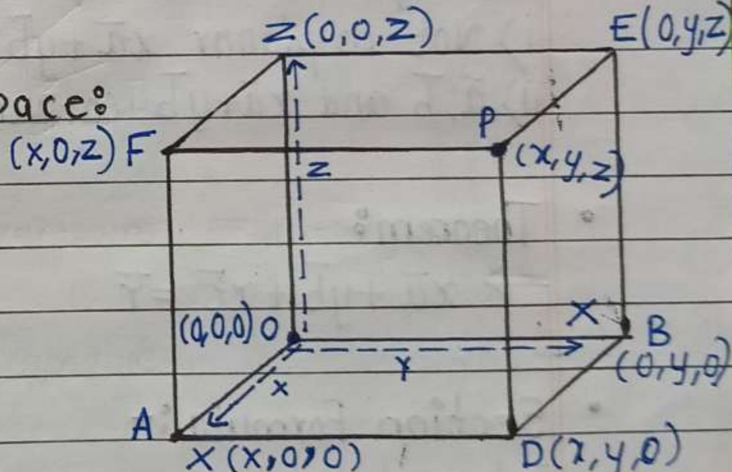
$\overline{OP}+\overline{PQ}=\overline{OQ}$

$\therefore \overline{PQ}=\overline{OQ}-\overline{OP}$

$PQ = \bar{q}-\bar{p}$



- Co-ordinate of points in space:



1] Dist. of any pt. $P(x,y,z)$ from co-ordinate plane. $\Rightarrow$

XY Plane $\Rightarrow |z|$

YZ Plane $\Rightarrow |x|$

XZ Plane $\Rightarrow |y|$

2] Dist of any pt. $P(x,y,z)$ from Origin $(0,0,0)$

$\Rightarrow \sqrt{x^2+y^2+z^2}$

3] Dist. between any two pt. $\Rightarrow \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

4] Dist from X-axis, Y-axis, Z-axis

$\sqrt{y^2+z^2}$ $\sqrt{x^2+z^2}$

- Collinearity of vectors:

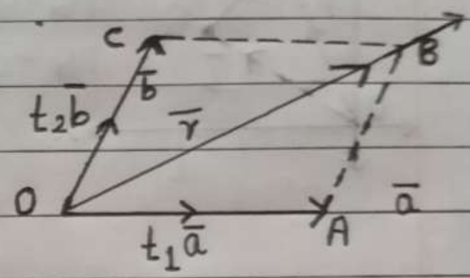
$\Rightarrow m\bar{a} + n\bar{b} = 0$

$\Rightarrow \bar{a}$ and $\bar{b}$ are not collinear and $m\bar{a}+n\bar{b} = \bar{0}$ then $m=0$, $n=0$.

$\Rightarrow \bar{a}$ and $\bar{b}$ are not collinear and $m\bar{a}+n\bar{b} = p\bar{a}+q\bar{b}$ then $m=p$ and $n=q$.

- Coplanarity of vectors:-

$\bar{r} = t_1\bar{a} + t_2\bar{b}$



- Theorem:

$x\bar{a} + y\bar{b} + z\bar{c} = 0$ with $(x,y,z) \neq (0,0,0)$

i) Not co-planar $x\bar{a}+y\bar{b}+z\bar{c} = 0$ then $x=0$ $y=0$ $z=0$

ii) $\bar{a}, \bar{b}$ and $x\bar{a}+y\bar{b}$ co-planar for all value of x and y

- Theorem:

$\Rightarrow x\bar{a} + y\bar{b} + x\bar{c} = \bar{r}$

- Section Formula:-

Internal division:

$\Rightarrow \bar{r} = \dfrac{m\bar{b}+n\bar{a}}{m+n}$

External division:

$\Rightarrow \bar{r} = \dfrac{m\bar{b}-n\bar{a}}{m-n}$

- Mid-point Formula:

$\Rightarrow m:n = 1:1$

$\Rightarrow \bar{r} = \dfrac{1\bar{b}+1\bar{a}}{1+1} = \dfrac{\bar{a}+\bar{b}}{2}$

$\Rightarrow \bar{r} = \dfrac{k\bar{b}+\bar{a}}{k+1}$ [Internal] $\bar{r} = \dfrac{k\bar{b}-\bar{a}}{k-1}$ [External]

- Centroid Formula:

$\Rightarrow$ Ratio 2:1

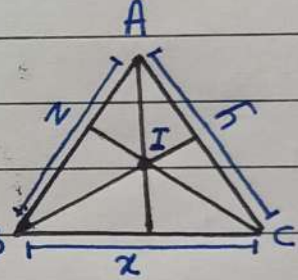
$\Rightarrow \bar{g} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

Tetrahedron: $\Rightarrow$ Ratio 3:1

$\Rightarrow \bar{g} = \dfrac{\bar{a}+\bar{b}+\bar{c}+\bar{d}}{4}$

- Incentre of triangle:

$\bar{h} = \dfrac{x\bar{a}+y\bar{b}+z\bar{c}}{x+y+z}$



- Product of vectors:

1] Dot Product:

i) $\bar{a}.\bar{b} = |\bar{a}||\bar{b}|\cos\theta$

ii) $\bar{a}=0 \,/\, \bar{b}=0$, $\bar{a}.\bar{b}=0$

iii) $\theta = \pi$, $\bar{a}.\bar{b} = |\bar{a}||\bar{b}|$

$\theta = \dfrac{\pi}{2}$, $\bar{a}.\bar{b} = -|\bar{a}||\bar{b}|$

iv) $\bar{a}.(\bar{b}+\bar{c}) = \bar{a}.\bar{b}+\bar{a}.\bar{c}$.

- Finding angle between 2 vector:

i) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

ii) $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

$\bar{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\bar{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$= a_1b_1+a_2b_2+a_3b_3$

$\therefore \cos\theta = \dfrac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|} = \dfrac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$

- Angle between two vectors:

Cond^n $\Rightarrow$ Join tail of two vectors [co-initial] , $0 \le \theta \le \pi$

$\Rightarrow$ Take shortest angle.

Angle between collinear vector $\Rightarrow 0 \Rightarrow$ Same direction.
$\Rightarrow 180^\circ \Rightarrow$ Opposite direction.

- Important Results:

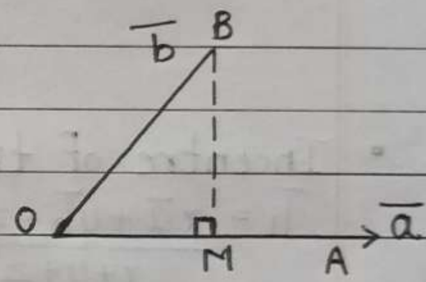
1] $|\bar{a}+\bar{b}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a}\cdot\bar{b}$

2] $|\bar{a}-\bar{b}|^2 = |\bar{a}|^2 + |\bar{b}|^2 - 2\bar{a}\cdot\bar{b}$

3] $|\bar{a}+\bar{b}+\bar{c}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a})$

4] $(\bar{a}+\bar{b})(\bar{a}-\bar{b}) = |\bar{a}|^2 - |\bar{b}|^2$

- Projection of vectors:-



$OM = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|}$

Scalar Projection $\Rightarrow$

1. S.P. of $\bar{a}$ on $\bar{b} \Rightarrow \dfrac{\bar{a}\cdot\bar{b}}{|\bar{b}|}$

2. S.P of $\bar{b}$ on $\bar{a} \Rightarrow \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|}$

Vector Projection $\Rightarrow$

1. V.P of $\bar{a}$ on $\bar{b} \Rightarrow \dfrac{(\bar{a}\cdot\bar{b})\bar{b}}{|b|^2}$

2. V.P. of $\bar{b}$ on $\bar{a} \Rightarrow \dfrac{(\bar{a}\cdot\bar{b})\bar{a}}{|\bar{a}|^2}$

- Direction angle:

$\alpha, \beta, \gamma \Rightarrow$ Direction angle.

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- Direction cosines:-

$l \Rightarrow \cos\alpha$, $m \Rightarrow \cos\beta$, $n \Rightarrow \cos\gamma$

$\therefore l^2+m^2+n^2=1$ i.e. $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$.

Direction cosines $\Rightarrow$ X-axis $\Longrightarrow (1,0,0)$

Y-axis $\Longrightarrow (0,1,0)$

Z-axis $\Longrightarrow (0,0,1)$

- Direction ratios:-

$l=ak$, $m=bk$, $n=ck$, $(ak)^2+(bk)^2+(ck)^2=1$

$k=\dfrac{1}{\sqrt{a^2+b^2+c^2}}$

$l=\dfrac{a}{\sqrt{a^2+b^2+c^2}}$, $m=\dfrac{b}{\sqrt{a^2+b^2+c^2}}$, $n=\dfrac{c}{\sqrt{a^2+b^2+c^2}}$

- 2] Vector Product:

1. $\bar{a}\times\bar{b}=|\bar{a}||\bar{b}|\sin\theta\,\hat{n}$ $\hat{n}\rightarrow$ Unit vector.
2. $|\bar{a}\times\bar{b}|=|\bar{a}||\bar{b}|\sin\theta$ $[\hat{n}=1]$
3. $\bar{a}\times\bar{b}$ $\perp^{ler}$ to plane of $\bar{a}$ & $\bar{b}$.
4. $\hat{n}=\dfrac{\bar{a}\times\bar{b}}{|\bar{a}||\bar{b}|\sin\theta}$
5. $\bar{a}\ ||$ to $\bar{b}$, $\theta=\bar{0}$ $\therefore \bar{a}\times\bar{b}=\bar{0}$
6. $(\bar{a}\times\bar{b})=-(\bar{b}\times\bar{a})$
7. $\bar{a}=k\bar{b}$ then $\bar{a}\times\bar{b}=k\bar{b}\times\bar{b}=k(\bar{0})=\bar{0}$
8. $\bar{a}\times(\bar{b}\ast\bar{c})=\bar{a}\times\bar{b}+\bar{a}\times\bar{c}$ [Left]

   $(\bar{b}\ast\bar{c})\times\bar{a}=\bar{b}\times\bar{a}+\bar{c}\times\bar{a}$
9. $m(\bar{a}\times\bar{b})=(m\bar{a})\times\bar{b}=\bar{a}\times(m\bar{b})$

   $m\bar{a}\times n\bar{b}=mn(\bar{a}\times\bar{b})$
10. $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$

    $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{k}=\hat{i}$, $\hat{k}\times\hat{i}=\hat{j}$

    $\hat{j}\times\hat{i}=-\hat{k}$, $\hat{k}\times\hat{j}=-\hat{i}$, $\hat{i}\times\hat{k}=-\hat{j}$
11. $\bar{a}\times\bar{b}$

    $\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

- Lagrange's identity:

$$(\bar{a}\cdot\bar{b})^2 + |\bar{a}\times\bar{b}|^2 = |a|^2|b|^2$$

- Angle between two vector:-

$\Rightarrow |\bar{a}||\bar{b}|\sin\theta$

$\Rightarrow \cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{ab} \qquad \sin\theta = \dfrac{|\bar{a}\times\bar{b}|}{|\bar{a}||\bar{b}|}$

- Geometrical meaning of vectors Product:

$\Rightarrow A(\text{parallelogram}) = |\bar{a}\times\bar{b}|$

$\Rightarrow d_1$ and d_2 diagonal, $A(\|^{gm}) = \dfrac{1}{2}|\bar{d}_1\times\bar{d}_2|$

$\Rightarrow A(\Delta ABC) = \dfrac{1}{2}|\overline{AB}\times\overline{AC}|$

- Scalar Triple Product: Box Product.

$\Rightarrow$ Order is IMP. $\bar{a},\bar{b},\bar{c}$ is $\bar{a}\cdot(\bar{b}\times\bar{c})$ $[\bar{a}\ \bar{b}\ \bar{c}]$

$\Rightarrow [\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$\Rightarrow [\bar{a}\ \bar{b}\ \bar{c}] = [\bar{c}\ \bar{a}\ \bar{b}] = [\bar{b}\ \bar{c}\ \bar{a}]$ cyclic change.

$\Rightarrow [\bar{a}\ \bar{b}\ \bar{c}] = -[\bar{b}\ \bar{a}\ \bar{c}] = -[\bar{c}\ \bar{b}\ \bar{a}] = -[\bar{a}\ \bar{c}\ \bar{b}]$

$\Rightarrow \bar{a}\cdot(\bar{b}\times\bar{c}) = (\bar{a}\times\bar{b})\cdot\bar{c}$

$\Rightarrow$ Any one is true, Scalar Product is zero.

1) One of vector is zero.
2) Any two collinear.
3) Three coplanar.

- Volume of Parallelepiped:

  Volume $= [\bar{a}\ \bar{b}\ \bar{c}]$

- Volume of tetrahedron:

  Volume $= \frac{1}{6}[\bar{a}\ \bar{b}\ \bar{c}]$

- Vector Triple Product:

1) $\bar{a} \times (\bar{b} \times \bar{c})$

   $\Rightarrow \perp^{ler}$ to $\bar{a}$ and $\bar{b} \times \bar{c}$

   $\Rightarrow$ Lies in plane of $\bar{b}$ & $\bar{c}$

2) $(\bar{a} \times \bar{b}) \times \bar{c}$

   $\Rightarrow \perp^{ler}$ to $\bar{a} \times \bar{b}$ and $\bar{c}$

   $\Rightarrow$ Lies in plane of $\bar{a}$ & $\bar{b}$

- Remark:

1) $\bar{a} \times (\bar{b} \times \bar{c}) \Rightarrow (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$
2) $(\bar{a} \times \bar{b}) \times \bar{c} \Rightarrow (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$

Long Jump.
Short Jaamp

$\Rightarrow$ Dot Product $\Rightarrow$ Scalar
$\Rightarrow$ Cross Product $\Rightarrow$ Vector.
$\Rightarrow$ Perpendicular $\Rightarrow$ Orthogonal.
$\Rightarrow$ Behind $\Rightarrow$ -ve.
$\Rightarrow$ Right $\Rightarrow$ +ve
$\Rightarrow$ above $\Rightarrow$ +ve

$\Rightarrow$ Left $\Rightarrow$ -ve

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- Volume of Parallelepiped:

  Volume $= [\bar{a}\ \bar{b}\ \bar{c}]$

- Volume of tetrahedron:

  Volume $= \frac{1}{6}[\bar{a}\ \bar{b}\ \bar{c}]$

- Vector Triple Product:

1) $\bar{a} \times (\bar{b} \times \bar{c})$

   $\Rightarrow \perp^{ler}$ to $\bar{a}$ and $\bar{b} \times \bar{c}$

   $\Rightarrow$ Lies in plane of $\bar{b}$ & $\bar{c}$

2) $(\bar{a} \times \bar{b}) \times \bar{c}$

   $\Rightarrow \perp^{ler}$ to $\bar{a} \times \bar{b}$ and $\bar{c}$

   $\Rightarrow$ Lies in plane of $\bar{a}$ & $\bar{b}$

- Remark:

1) $\bar{a} \times (\bar{b} \times \bar{c}) \Rightarrow (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$
2) $(\bar{a} \times \bar{b}) \times \bar{c} \Rightarrow (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$

Long Jump.
Short Jump

$\Rightarrow$ Dot Product $\Rightarrow$ Scalar	$\Rightarrow$ Left $\Rightarrow$ -ve
$\Rightarrow$ Cross Product $\Rightarrow$ Vector.	
$\Rightarrow$ Perpendicular $\Rightarrow$ Orthogonal.	
$\Rightarrow$ Behind $\Rightarrow$ -ve.	
$\Rightarrow$ Right $\Rightarrow$ +ve	
$\Rightarrow$ above $\Rightarrow$ +ve	