2 MATRICES

Synopsis

2.1. MATRICES :

A matrices is an arrangement of $m \times n$ numbers written into m rows and n columns, enclosed in a bracket. A matrices is generally denoted by capital letters A, B, C .... X, Y, Z.

e.g.

$$\begin{array}{c@{}l} A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} & \begin{array}{l} \leftarrow 1^{st}\text{ Row} \\ \leftarrow 2^{nd}\text{ Row} \end{array} \\ \quad\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 1^{st} & 2^{nd} & 3^{rd} \end{array} & \end{array}$$

2.2. TYPES OF MATRICES :

i. Row matrix :

A matrix having only one row is called a row matrix. It is also called a row vector.

Example : A = [2 3 8]

ii. Column matrix :

A matrix having only one column is called a column matrix or a column vector.

Example : $A = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

iii. Zero matrix [Null matrix) :

A matrix in which each element is zero, then their sum can be obtained by adding the corresponding elements

iv. Square matrix :

A matrix is which the number of rows is equal to the number of columns, is called a square matrix. If it has m rows and m columns, it is called a square matrix of order 'm'.

Example : $[5]_{1 \times 1}$ $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$ $\begin{bmatrix} 1 & 2 & 3 \\ 5 & -1 & 2 \\ 3 & 4 & 8 \end{bmatrix}_{3 \times 3}$

v. Diagonal matrix :

A square matrix in which all the element which are not in the leading diagonal, are 'zero', is called a diagonal matrix.

Thus in a diagonal matrix $A = [a_{ij}]$, $a$ij = 0 for all $i \neq j$.

Example : $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

vi. Scalar matrix :

A diagonal matrix in which all the diagonal elements are equal, is called a scalar matrix.

Example : $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

vii. Unit or Identity matrix :

A scalar matrix in which all the diagonal elements are equal to 1, is called a unit matrix or identity matrix, denoted by I.

Example : $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

viii. Upper triangular matrix :

A square matrix $A = [a_{ij}]$ whose element $a_{ij} = 0$ for all $i > j$ is called an upper triangular matrix.

Example : $A = \begin{bmatrix} 5 & 1 & 7 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$

ix. Lower triangular matrix :

A square matrix $A = [a_{ij}]$ whose element $a_{ij} = 0$ for all $i < j$ is called a lower triangular matirx.

Example : $A = \begin{bmatrix} 5 & 0 & 0 \\ 3 & 4 & 0 \\ 5 & 1 & 2 \end{bmatrix}$

x. Singular matrix :

A square matrix A is called a singular matrix if the value of its determinant is zero, i.e. $|A| = 0$

Example :

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 5 & -2 & -3 \\ 2 & 1 & -3 \end{bmatrix} \therefore |A| = \begin{vmatrix} 3 & -1 & -2 \\ 5 & -2 & -3 \\ 2 & 1 & -3 \end{vmatrix}$$

$= 3\,(6 + 3) + 1\,(-15 + 6) - 2\,(5 + 4)$

$= 27 - 9 - 18 = 0$

xi. Non – singular matrix :

A square matrix A is said to be a non – singular if $|A| \neq 0$.

Example : $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $|A| = 4 - 6 = -2 \neq 0$.

xii. Transpose of a matrix :

Let A be any matrix. Then a matrix obtained by interchanging rows and columns of A, is called transpose of A, denoted by A^T or A'.

Example : $A = \begin{bmatrix} 2 & -3 & 5 \\ 1 & 4 & 7 \end{bmatrix}, A' = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 5 & 7 \end{bmatrix}$

xiii. Symmetric matrix :

A square matrix $A = [a_{ij}]$ is said to be a symmertric matrix, if $a_{ij} = a_{ji}$, for all i and j. In symmetric matrix, $A = A^T$

Example : $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & -1 \\ 4 & -1 & 7 \end{bmatrix}$

observe that : $a_{12} = a_{21} = 3$

$a_{13} = a_{31} = 4$

$a_{23} = a_{32} = -1$

xiv. Skew symmetric matrix :

A square matrix $A = [a_{ij}]$ is said to be a skew symmetric matrix, if $a_{ij} = -a_{ji}$ for all i and j. In skew symmetric matrix, $A = -A^T$

Example : $A = \begin{bmatrix} 0 & 3 & -4 \\ -3 & 0 & 5 \\ 4 & -5 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

xv. Orthogonal matrix :

A square matrix 'A' is said to be orghogonal if $A\,A^T = I = A^T\,A$.

Example :

$$\text{Let } A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}, \therefore A' = \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

$$\therefore AA' = \frac{1}{9}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1+4+4 & 2+2-4 & -2+4-2 \\ 2+2-4 & 4+1+4 & -4+2+2 \\ -2+4-2 & -4+2+2 & 4+4+1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\therefore$ A is orthogonal matrix.

xvi. Invertible matrix :

A matrix 'A' is said to be an invertible matrix, if its exists. Hence matrix 'A' will be invertible if $|A| \neq 0$.

5.3. TYPES OF MATRIX :

1. Only a square matrix has a diagonal.
2. If the determinats of two square matrices are equal, then those two matrices are need not be equal.
3. If A is of order $m \times n$, then A′ is of order $m \times n$ matrix.
4. $(A')' = A$
5. $(A \pm B)' = A' \pm B'$ (A and B are of same order)
6. $(kA)' = kA'$ (k is a scalar)
7. Let A is of order $m \times n$ and B is of order, $n \times p$, then $(AB)' = BA'$ (Reversal low for transpose)
8. $(A^{-1})^{-1} = A$
9. $(A')^{-1} = (A^{-1})'$
10. If A and B are two square matrices of same order such that their inverses exist, then $(AB)^{-1} = B^{-1}A^{-1}$ (Reversal law for inverse)
11. If A is a non – singular symmetric matrix, then A^{-1} is also symmetric matrix.
12. If A is a non – singular matrix, then

$$|A^{-1}| = \frac{1}{|A|}$$

13. $(kA)^{-1} = \frac{1}{k} A^{-1}$ $(k \neq o)$
14. If $|A| \neq o$, then $A^{-1} = \frac{1}{|A|}$ (adj A)
15. $AA^{-1} = A^{-1}A = I$
16. The product of two diagonal matrices of the same order is a diagonal matrix.
17. The adjoint of a diagonal matrix is a diagonal matrix.
18. If A is an orthogonal matrix, then A^{-1} is also an orthogonal matrix.
19. $|kA| = k^n |A|$ where n is the order of square matrix A.
20. The diagonal elements of skew symmetric matrix are necessarily zero.
21. A + A′ is always a symmetric matrix.
22. A – A′ is always a symmetric matrix.
23. AA′ is symmetric then A^2 is also symmetric.
24. If A is symmetric then A^2 is also symmetric.
25. If A = B + C, then it is not always true that $|A| = |B| + |C|$
26. If A is symmetric, then B′AB is symmetric.
27. If A is skew symmetric, then B′AB is skew symmetric.
28. If A is skew symmetric matrix, then

    $A = -(A')$
29. Detreminant of a skew symmetric matrix of odd order is zero and of even order is a non – zero perfect square.
30. $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$
31. Inverse of symmetric matrix is symmetric.
32. Inverse of symmetric is symmetric.
33. Every orthogonal matrix has inverse i.e. Every orthogonal matrix is invertible. But every invertible matrix is not necessaryily orthogonal.

34. If A is a square matrix of order n, then A. (adj A) = (adj A) A = | A | I.

35. adj (AB) = (adj B) . (adj A)

36. If A is a square matrix of order n, then adj (adj A) = $|A|^{n-2}$ A. and | adj A | = $|A|^{n-1}$

37. adj (A′) = (adj A′)

38. If A is a square matrix of order n, then adj (KA) = K^{n-1} adj A.

39. If $A=\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ then $A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}$

and $A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$

40. Trace of a matrix :

Let A be a square matrix, then the sum of all diagonal elements of A is called trace of A, denoted by tr (A).

5.4. ALGEBRA OF MATRICES :

1. Addition of two matrices :

Two matrices are said to be conformable for addition if they are of same order.

If A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{m \times n}$ are tow matrices of same order $m \times n$, then their addition A + B is the matrix of the same order $m \times n$ and is given by A + B = $[a_{ij} + b_{ij}]_{m\times n}$ for all i and j.

Order of A, B and A + B are same

For example :

Let $A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & -2 & 4 \\ 7 & -6 & -8 \end{bmatrix}$, then

$$A+B=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}+\begin{bmatrix} 2 & -2 & 4 \\ 7 & -6 & -8 \end{bmatrix}=\begin{bmatrix} 3 & 0 & 7 \\ 11 & -1 & -2 \end{bmatrix}$$

2. Subtraction of matrices :

If A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{m \times n}$ are tow matrices of same order $m \times n$, then A –B is a matrix of the same $m \times n$ whose elements are obtained by subtracting the elements of B from the corresponding elements of A.

∴ A – B = $[a_{ij} + b_{ij}]_{m \times n}$ for all i and j.
A – B = A + (–B)

A + (– A) = 0

For example :

Let $A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & -2 & 4 \\ 7 & -6 & -8 \end{bmatrix}$, then

$$A-B=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}-\begin{bmatrix} 2 & -2 & 4 \\ 7 & -6 & -8 \end{bmatrix}=\begin{bmatrix} -1 & 4 & -1 \\ -3 & 11 & 14 \end{bmatrix}$$

3. Multiplication of a matrices by scalar :

Two matrices A and B are said to be conformable for multiplication AB only if the number of columns in A are equal to the number of rows in B.

If A = $[a_{ij}]_{m \times n}$ and B = $[b_{jk}]_{n \times p}$ are tow matrices, then the product C = AB = $[c_{ik}]_{m \times p}$ where c_{ik} is the sum of the products of the corresponding elements of i^{th} row and A and k^{th} column of B.

∴ $c_{ik} = a_{i1} b_{k1} + a_{i2} b_{2k} + ... + a_{in} b_{nk}$

$$\therefore c_{ik} = \sum_{i=1}^{n} a_{ij}\, b_{jk}$$

In the product AB, A is called the pre–factor of AB and B is called post–factor of AB.

4. Multiplication of matrices :

Let $A_{m \times n}$ and $B_{n \times p}$ be two matrices, then their product (AB) is defined and the order of (AB) will be $m \times p$.

Observe that the number of columns of matrix A is equal to the number of rows of matrix B. This is the required condition for multiplicaiton to exist. Otherwise multiplication does not exist. If the product AB is defined, then A and B are said to be comformable for multiplication.

Example : (i) $A_{3 \times 2} . B_{2 \times 3} = (AB)_{3 \times 3}$
(ii) $A_{3 \times 3} . B_{3 \times 2} = (AB)_{3 \times 2}$
(iii) $A_{2 \times 3} . B_{2 \times 3,}$ product AB does into exist

i. The product of two matrices A and B is obtained as follows :

Let $A=\begin{bmatrix}2 & 3\\ 4 & -1\end{bmatrix}$, $B=\begin{bmatrix}3 & -1\\ 2 & 5\end{bmatrix}$

$$AB=\begin{bmatrix}2 & 3\\ 4 & -1\end{bmatrix}\cdot\begin{bmatrix}3 & -1\\ 2 & 5\end{bmatrix}$$

$$=\begin{bmatrix}(2\times3+3\times2) & (2\times-1+3\times5)\\ (4\times3+-1\times2) & (4\times-1+-1\times5)\end{bmatrix}$$

$$=\begin{bmatrix}12 & 13\\ 10 & -9\end{bmatrix}$$

ii. Product is not commutative. i.e. $AB \neq BA$ in general.

iii. Matrix multiplication is associative, if comformability exists i.e. $(AB)\,C = A\,(BC)$, if order of A is $m \times n$, order of B is $n \times p$, order of C is $p \times q$.

iv. If $AB = 0$, it does not necessarily imply that matrix $A = 0$ or / and matrix $B = 0$.

Example :

Let $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$.

here $A \neq 0$, $B \neq 0$.

But

$$AB=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}=\begin{bmatrix}1-1 & 1-1\\ -1+1 & -1+1\end{bmatrix}$$

$$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=0$$

v. Conjugate of a matrix :

If the elements of a matrix are complex numbers, then replacing the corresponding elements by the conjugate numbers we get the conjugate matrix. If A is the given matrix then its conjugate matrix is denoted by $\bar{A}$.

Example :

Let $A=\begin{bmatrix}3-2i & 4+i & 5i\\ 4+3i & 3+6i & 2i+5\\ 4 & -7 & 2+i\end{bmatrix}$

then $\bar{A}=\begin{bmatrix}3+2i & 4-i & -5i\\ 4-3i & 3-6i & -2i+5\\ 4 & -7 & 2-i\end{bmatrix}$

* $\left(\bar{\bar{A}}\right) = A$

* $\left(\overline{A+B}\right)=\bar{A}+\bar{B}$

* $\left(\overline{AB}\right)=\bar{A}\,\bar{B}$, if comformability of multiplication exists.

5.5. MINOR AND COFACTOR OF ELEMENTS :

Let $D=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$

The position of an element in a given deteerminant is denoted by using two suffix. The first suffix denote the row number and the second suffix denote the coloumn number in which that element occur.

In general an element a_{ij} belongs to i^{th} row and j^{th} column.

MINOR of a_{ij}

The minor of element a_{ij} is defined as the value of the determinant obtained by eleminating the i^{th} row and j^{th} column of D.

$\therefore$ Minor of $a_{11}=M_{11}=\begin{vmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{vmatrix}=a_{22}a_{33}-a_{23}a_{32}$

$\therefore$ Minor of $a_{12}=M_{12}=\begin{vmatrix}a_{21} & a_{23}\\ a_{31} & a_{33}\end{vmatrix}=a_{21}a_{33}-a_{23}a_{31}$

$\therefore$ Minor of $a_{13}=M_{13}=\begin{vmatrix}a_{21} & a_{22}\\ a_{31} & a_{32}\end{vmatrix}=a_{21}a_{32}-a_{22}a_{31}$

$\therefore$ Minor of $a_{21} = M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = a_{12}a_{33} - a_{13}a_{32}$

$\therefore$ Minor of $a_{22} = M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} = a_{11}a_{33} - a_{13}a_{31}$

$\therefore$ Minor of $a_{23} = M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} = a_{11}a_{32} - a_{12}a_{31}$

$\therefore$ Minor of $a_{31} = M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} = a_{12}a_{23} - a_{13}a_{22}$

$\therefore$ Minor of $a_{32} = M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = a_{11}a_{23} - a_{13}a_{21}$

$\therefore$ Minor of $a_{33} = M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

The value of determinant is also given by

$\Delta = a_{11}\, M_{11} - a_{12}\, M_{12} + a_{13}\, M_{13}$ or

$\Delta = -a_{21}\, M_{21} + a_{22}\, M_{22} - a_{23}\, M_{23}$ or

$\Delta = a_{31}\, M_{31} - a_{32}\, M_{32} + a_{33}\, M_{33}$

COFACTOR OF a_{ij}

Cofactor of a_{ij} is defined as $(-1)^{i+j}\, M_{ij}$, where M_{ij} is the minor of element a_{ij}.

The cofactor of element a_{ij} is denoted by A_{ij}

$A_{ij} = (-1)^{i+j}\, M_{ij}$

$\therefore$ $A_{11} = (-1)^{1+1}\, M_{11} = M_{11}$

$\therefore$ $A_{12} = (-1)^{1+2}\, M_{12} = -\, M_{12}$

$\therefore$ $A_{13} = (-1)^{1+3}\, M_{13} = M_{13}$

$\therefore$ $A_{21} = (-1)^{2+1}\, M_{21} = -\, M_{21}$

$\therefore$ $A_{22} = (-1)^{2+2}\, M_{22} = M_{22}$

$\therefore$ $A_{23} = (-1)^{2+3}\, M_{23} = -\, M_{23}$

$\therefore$ $A_{31} = (-1)^{3+1}\, M_{31} = M_{31}$

$\therefore$ $A_{32} = (-1)^{3+2}\, M_{32} = -\, M_{32}$

$\therefore$ $A_{33} = (-1)^{3+3}\, M_{33} = M_{33}$

5.6. COFACTOR MATRIX :

The cofactor matrix of the square matrix A = $[a_{ij}]$ of order n is a matrix of order n, in which each element a_{ij} of the matrix A is replaced by its cofactor A_{ij}.

$\therefore$ Cofactor matrix of A = $[A_{ij}]_{n \times n}$.

5.7. ADJOINT OF A MATRIX :

The transpose of the cofactor matrix of A is called the adjoint of A and is denoted by adj A.

$\therefore$ adj A = $[A_{ij}]^T$

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then

$$\text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T$$

$\therefore$ adj $A = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$, where A_{ij} denotes the cofactor of a_{ij} in matrix A.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is a square matrix of order 2, then co–factor matrix of A is $\begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}$

$\therefore$ adj $A = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$

The adjoint of a square matrix of order 2 can be obtained by interchanging the diagonal elements and changing the signs of non–diagonal elements.

5.8. INVERSE OF MATRIX BY ADJOINT METHOD :

Let A be a non–singular square matrix of order n, If B is a matrix of the same order such that AB = I = BA, where I is the identity matrix of order n, then B is called the inverse of A and is denoted by A^{-1}.

Thus $AA^{-1} = A^{-1}A = I$

The inverse of matrix is defined only for square matrix.

If A is non–singular square matrix, i.e. $|A| \neq 0$, then its inverse exists and it is given by

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

Shortcut for Writing Adjoint of a Matrix :

1. Adjoint of matrix of order 2 × 2.

Interchange the elements of leading diagonal and change the sign of other two elements which are not in leading diagonal.

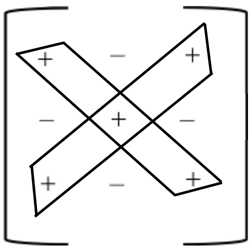
Examples :

(1) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

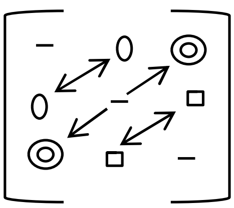
(2) $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$, then adj $A = \begin{bmatrix} 5 & 3 \\ -4 & 2 \end{bmatrix}$

2. Adjoint of matrix of order 3 × 3.

First remember the short cut for the sign of co–factors of each element in the determinant of matrix of order 3 × 3.



Also, the positions of co–factors of each element are shown with identical marking in the adjoining diagram. The co–factors of the diagonal elements remain at the same places.



Now consider the following example :

The adjoint of matrix $\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$ is :

a) $\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 4 & 2 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$

c) $\begin{bmatrix} 3 & -2 & -1 \\ 5 & -6 & 9 \\ 5 & 2 & 8 \end{bmatrix}$ d) $\begin{bmatrix} 3 & 3 & 10 \\ 4 & 5 & 9 \\ 5 & 3 & 8 \end{bmatrix}$

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Cofactor of a_{12} must be at the place of a_{21} and vice versa. Cofactor of a_{13} must be at the place of a_{31} and vice versa. Cofactor of a_{23} must be at the place of a_{32} and vice versa.

Now cofactor of –2 = – [4 + 4] = –8

and cofactor of 2 = – [– 4 + 9] = –5

Hence option at "a" is correct.

5.9. ELEMENTARY TRANSFORMATIONS :

i. Row Transformations :

The following are elementary row transformaitons.

a) Interchanging i^{th} and j^{th} row.

Notation : R_{ij} or $R_i \longleftrightarrow R_j$

b) Multiplying ith row by a non–zero number k.

Notation : $R_i \longrightarrow k.\, R_i$ or simply $k.\, R_i$

c) Adding to the elements of ith row, k times the corresponding elements of ith row.

Notation : $R_i \longrightarrow R_i + k.\, R_j$ or simply

$R_i + k. R_j$

ii. Column Translfrimaton :

a) C_{ij} or $C_i\ C_j$

b) $C_i \longrightarrow k. C_i$ or simply $k. C_i$

c) $C_i \longrightarrow C_i + k. C_j$ or simply $C_i + k. C_j$ are similarly defined.

An Important Result

Suppose AB = P.

If a row transformation on the pre–factor A and the product P gives the matrices A_R and P_R respectively, then $A_R . B = P_R$

Similary, if a column transformation on the post–factor B and product P gives matrices B_C and P_C respectively, then $A. B_C = P_C$

iii. Inverse of a Matrix by Row Transformations

Consider the same matrix A as above.

We have, $A . A^{-1} = I$

$$\therefore \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} . A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, perform a series of row transformations on the pre–factor (A) on the i.h.s. so as to convert it into the identify matrix. Do the same transformations simultaneously on the identity matrix I on the r.h.s.

To convert A into I. apply a suitable row transformation to get :

Step 1 : 1 in the place of a_{11} .... $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Step 2 : 0 in the place of a_{21}, a_{31}

Step 3 : 1 in the place of a_{22} .... $\begin{matrix} 0 \\ 1 \\ 0 \end{matrix}$

Step 4 : 0 in the places of a_{12}, a_{32}

Step 5 : 1 in the place of a_{33} .... $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Step 6 : 0 in the place of a_{13}, a_{23}

The matrix A on the i.h.s. is now converted into I. If, at this stage, the identify matrix I on the r.h.s. is converted into some matrix B, then we get

$I . A^{-1} = B$,

i.e. $A^{-1} = B$.

iv. Inverse of a Matrix by Column Transformations :

Suppose we are given a matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

We have, $A^{-1} . A = I$

$$\therefore A^{-1} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, perform a series of suitable column transformations on the post–factor (A) on the i.h.s. so as to convert it into the identify matrix. Do the same transformations simultaneously on the identity matrix I on the r.h.s.

To convert A into I, apply a suitable column transformation to get :

Step 1 : 1 in the place of a_{11}

Step 2 : 0 in the place of a_{11}, a_{13}

..... 1 0 0

Step 1 : 1 in the place of a_{22}

Step 2 : 0 in the places of a_{22}, a_{23}

..... 0 1 0

Step 1 : 1 in the place of a_{33}

Step 2 : 0 in the place of a_{31}, a_{32}

..... 0 0 1

The matrix A on the i.h.s. is now converted into I. If, at this stage, the identify matrix I on the r.h.s. is converted into some matrix B, then we get

$$A^{-1} . I = B,$$

i.e. $A^{-1} = B$.

Shortcut for writing Inverse of a Matrices :

1. For inverse of order 2×2 matrix :

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $|A|$ ad – bc then

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

i.e. interchange the elements of leading

diagonal and change the signs of elements of other diagonal.

Example :

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} ;\ |A| = 10 + 12 = 22$$

$$\therefore \quad A^{-1} = \frac{1}{22}\begin{bmatrix} 5 & 3 \\ -4 & 2 \end{bmatrix}$$

2. For inverse of order 3×3 matrix :

Consider the following example :

If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ then A^{-1} is :

(a) $\begin{bmatrix} 3 & 2 & 4 \\ 1 & 1 & 2 \\ -2 & -2 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -5 & 6 \\ 2 & 1 & 0 \\ 3 & 4 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \\ -2 & 4 & 5 \end{bmatrix}$

Solution : We know that $AA^{-1} = I$

$\therefore$ Multiplying A by any one of the given options, we must get $\begin{bmatrix} \boxed{1 \quad 0 \quad 0} \\ 0 \quad 1 \quad 0 \\ 0 \quad 0 \quad 1 \end{bmatrix}$

We check for only three elements $\boxed{1 \quad 0 \quad 0}$

We take 1st row of A and 1st column of opetion (a). If we are not getting 1, then obviosuly option (a) is rejected.

1st row of A	1st column of option (a)	1st column of option (b)
1 2 – 2	3	3
	1	2
	– 2	3
	$3 + 2 + 4 \neq 1$ option (a) rejected	$3 + 4 - 6 = 1$

2nd col. of option (b)	1st col. of option (c)	2nd col. of option (c)
– 5	3	2
1	1	1
4	2	2
$-5 + 2 - 8 \neq 0$	$3 + 2 - 4 = 1$	$2 + 2 - 4 = 0$
option (b) rejected		

$\therefore$ Option (c) is correct. (You may check 3rd column) After some practice, you may do all the above product without writing.

5.10. SOLUTION OF SIMULTANEOUS

EQUATIONS :

1. By Method of Inversion :

Consider AX = B

Where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

Pre–multiplying by A^{-1}

$A^{-1}(AX) = A^{-1}B$

$\therefore$ $(A^{-1}A)X = A^{-1}B$

$\therefore$ $IX = A^{-1}B$

$\therefore$ $X = A^{-1}B$

from this, we get the values of x, y, z.

2. By Method of Reduction :

In this method, we need not calculate A^{-1}. We apply elementary row transformations on

AX = B (i.e., on A and B) such that

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

reduces to

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & b_{23} \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b'_1 \\ b'_2 \\ b'_3 \end{bmatrix}$$

Which gives,

$b_{11}x + b_{12}y + b_{13}z = b'_1$ .... (i)

$b_{22}y + b_{23}z = b'_2$ .... (ii)

$b_{33}z = b'_3$ .... (iii)

By solving (i), (ii) and (iii) we will get values of x, y, z.

5.11. CONSISTENCY OF EQUATIONS :

Let AX = B be a system of n linear equation in n variables.

i. If $|A| = 0$ and $(\text{adj } A)B \neq 0$, then the system of equations is inconsistent (on solution).

ii. If $|A| \neq 0$ then the system of equations is consistent and has a unique solution given by $X = A^{-1}B$.

iii. If $|A| = 0$ and $(\text{adj } A)B \neq 0$, then the system of equations is consistent and has infinitely many solutions.

CLASS WORK

Multiple Choice Questions

2.1 Introduction to Matrices

(1) A square matrix $A = [a_{ij}]$ in which $a_{ij} = 0$, for $i \neq j$ and $a_{ij} = k$ (constant), for $i = j$ is called a

(a) unit matrix (b) null matrix

(c) sclar matrix (d) diagonal matrix

(2) Square matrix $[a_{ij}]_{n\times n}$ will be an upper trianglar matrix, if

(a) $a_{ij} = 0$, for $i < j$ **(b)** $a_{ij} = 0$, for $i < j$

(c) $a_{ij} \neq 0$, for $i > j$ (d) none of these

(3) If I is a unit matrix, than $3I$ will be

(a) a unit matrix **(b)** a scalar martix

(c) a triangular matrix (d) none of these

(4) If A is a square matrix, $A + A^T$ is symmetrix matrix, then $A - A^T$

(a) zero matrix

(b) unit matrix

(c) symmetric matrix

(d) skew symmetric matrix

(5) If A is a singular matrix, then A. adj A:

(a) is a scalar matrix

(b) is a zero matrix

(c) is an identity matrix

(d) is an orthogonal matrix

(6) The matrix $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ is :-

(a) symmetric (b) skew - symmetric

(c) orthogonal (d) scalar

(7) If A, B are two $n \times n$ non - singular matrices, then

(a) AB is singular

(b) AB is non - singular

(c) $(AB)^{-1}$ does not exist

(d) $(AB)^{-1} = A^{-1}.B^{-1}$

(8) Choose the correct answer

(a) Every diagonal matrix is an identify matrix

(b) Every identify matrix is a scalar matrix

(c) Every scalar matrix is an identify matrix

(d) A square matirx whose each element is 1 is an identify matrix

2.2 Algebra of Matrices

(9) If A and B are square matrices of same order, then

(a) A-B = B-A (b) A+B = A-B

(c) A+B = B+A (d) $AB = BA$

(10) If A and B are square matrices of order 2, then $(A+B)^2 =$

(a) $A^2 + 2AB + B^2$ (b) $A^2 + 2BA + B^2$

(c) $A^2 + AB + BA + B^2$ (d) none of these

(11) If A^T, B^T are transpose matrices of the square matrices A, B respectively, then $(AB)^T =$

(a) AB^T (b) BA^T

(c) A^TB^T **(d)** B^TA^T

(12) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then A^5

(a) $32A$ **(b)** $16A$

(c) $10A$ (d) $5A$

(13) If A = $\begin{bmatrix} 4 & -1 \\ -1 & k \end{bmatrix}$ such that $A^2 - 6A + 7I = 0$ then $k =$

(a) 1 (b) 3

(c) 2 (d) 4

(14) If A is a square matrix, then which of the following matrices is not symmetic

(a) $A + A'$ **(b)** $A - A'$

(c) $A'\,A$ (d) AA'

(15) If $AX = B$, wher $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 3 \\ 0 & 6 \end{bmatrix}$, then $X =$

(a) $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

(16) If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = I$, then

(a) $1+\alpha^2+\beta\gamma=0$ **(b)** $1-\alpha^2-\beta\gamma=0$

(c) $1-\alpha^2+\beta\gamma=0$ (d) $1+\alpha^2-\beta\gamma=0$

(17) If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, the value of α for which $A^2 = B$ is.

(a) 1 (b) –1

(c) 4 **(d)** no real values

(18) If ω is a complex cube root of unity, and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then : $H^{70} =$

(a) H (b) O

(c) $-H$ (d) H^2

(19) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 5 \end{bmatrix}$, then $R_1 \leftrightarrow R_2$ and $C_1 \to C_1 + 2C_3$ given

(a) $\begin{bmatrix} -13 & 2 & -5 \\ -1 & 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -2 & -1 \\ 13 & -2 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 13 & -2 & 5 \\ -1 & 2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -13 & -5 \\ 2 & -1 & -1 \end{bmatrix}$

(20) $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$, operating $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$ gives

(a) $\begin{bmatrix} 1 & -3 & 2 \\ 0 & 9 & 11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

(b) $\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

(c) $\begin{bmatrix} 1 & 3 & -4 \\ 0 & 9 & 11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

(d) $\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

(21) If I is the identify matrix of order 2 and $X \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} = I$, then $X =$

(a) $\begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{-1}{5} & \frac{2}{5} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{3}{5} & \frac{-1}{5} \end{bmatrix}$

(22) Matrix $A = \begin{bmatrix} 1 & 0 & -k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$ is invertible for :

(a) $k = 1$ (b) $k = -1$

(c) all real k (d) $k = 2$

(23) If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true ?

(a) $A = B$ **(b)** $AB = BA$

(c) Either A or B is a Zero matrix

(d) Either A or B is an identity matrix

(24) If $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$, then for what value of λ is $A^2 = 0$

(a) 2 (b) + 1

(c) –2 **(d)** 1

(25) If A $[a\ b]$, $B = [-b\ -a]$ and $C = \begin{bmatrix} a \\ -a \end{bmatrix}$, then the correct

(a) $A = -B$ (b) $A + B = A - B$

(c) $AC = BC$ (d) $CA = CB$

2.3 Minors, Cofactors & Adjoint of a Martix

(26) If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and A_{ij} denotes the cofactor of element a_{ij}, then

$a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} =$

(a) 0 (b) $|A|$

(c) $-|A|$ (d) $2|A|$

(27) The cofactor of –4 and 9 in $\begin{bmatrix} -1 & -2 & 3 \\ -4 & -5 & -6 \\ -7 & 8 & 9 \end{bmatrix}$ are respectively

(a) –42, 3 **(b)** 42, –3

(c) –42, –3 (d) 42, 3

(28) The cofactor of the elements of the second row of $\begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}$ are

(a) –16, 2, 4 **(b)** –16, 8, 4

(c) 16, –8, 4 (d) –16, 8, –4

(29) Matrix of coofactors of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$ is

(a) $\begin{bmatrix} -3 & 12 & 6 \\ 1 & 3 & -2 \\ -11 & 9 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & -12 & 6 \\ -1 & 3 & -2 \\ -11 & -9 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & -1 & -11 \\ -12 & 3 & -9 \\ 6 & 2 & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} -3 & -12 & 6 \\ -1 & 3 & 2 \\ -11 & -9 & 1 \end{bmatrix}$

(30) If $A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$, then adj A =

(a) $\begin{bmatrix} -5 & -3 \\ 3 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 3 \\ -3 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}$ **(d)** $\begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$

(31) Adjoint of the matrix $N = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ is

(a) $-N$ **(b)** N

(c) $2N$ (d) none of these

(32) If $A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, then A (adj A)

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

(33) If $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$, then $|\text{adj}(\text{adj } A)| =$

(a) 17^2 (b) 17^3

(c) 17^5 **(d)** 17^4

(34) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$, then

(a) $C_{12} + C_{22} + C_{32} = 0$ (b) $C_{13} + C_{23} + C_{33} = 1$

(c) $C_{11} + C_{21} = C_{32}$ (d) $C_{12} + C_{22} = C_{32}$

(35) If $A = \begin{bmatrix} -1/3 & -2/3 & -2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix} = \left[a_{ij}\right]_{3\times3}$, then , of all i and j, the cofactor is such that

(a) $C_{ij} = a_{ji}$ (b) $C_{ij} = -a_{ji}$

(c) $C_{ij} = a_{ij}$ (d) $C_{ij} = (a_{ij})^2$

(36) If $A = \begin{bmatrix} 3 & -6 \\ -4 & 2 \end{bmatrix}$, then minor M_{22} is equal to

(a) 3 (b) –6

(c) –4 (d) 2

(37) If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then A (adj A) =

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

(38) If $A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where A_{11}, A_{12}, A_{13} are cofactor of a_{11}, a_{12}, a_{13} respectively, then the value of $a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13} =$

(a) –1 **(b)** 1

(c) 0 (d) $\frac{1}{2}$

(39) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $|A(\text{adj})A|$ is

(a) 36 **(b)** 216

(c) 6 (d) 0

(40) adj (AB) =

(a) (adj A) (adj B) **(b)** (adj B) (adj A)

(c) adj (BA) (d) A. (adj B)

2.4 Inverse of a Matrix

(41) The matrix having the same matrix as its inverse is

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(42) If for the martix A, $A^3 = I$, then $A^{-1} =$

(a) A **(b)** A^2

(c) A^3 (d) none of these

(43) If A is a square matrix satisfying the equation $A^2 - 4A - 5I = 0$, then $A^{-1} =$

(a) $\dfrac{A+4I}{5}$ **(b)** $\dfrac{A-4I}{5}$

(c) $\dfrac{I-4A}{5}$ (d) $\dfrac{I+4A}{5}$

(44) If $A = \begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}$ and $A^{-1} = \lambda\,(\text{adj } A)$ then λ =

(a) $\dfrac{-1}{6}$ (b) $\dfrac{-1}{3}$

(c) $\dfrac{1}{3}$ (d) $\dfrac{1}{6}$

(45) The element in the second row and third column of the inverse of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is

(a) –2 **(b)** –1

(c) 1 (d) 2

(46) If martix $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ and B is the inverse of matrix A, then α =

(a) –2 (b) –1

(c) 2 **(d)** 5

(47) The inverse of matrix $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ is

(a) $-\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$

(c) $-\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$ **(d)** $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

(48) If $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then

(a) $A^{-1} = B$ (b) B^{-1} exist

(c) A^{-1} does not exist (d) A^{-1} exist

(49) If A = $\begin{bmatrix} 4 & 5 \\ 2 & 5 \end{bmatrix}$, then $|(2A)^{-1}| =$

(a) $\dfrac{1}{30}$ (b) $\dfrac{1}{20}$

(c) $\dfrac{1}{60}$ **(d)** $\dfrac{1}{40}$

(50) The inverse of the matrix $\begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix}$ is

(a) $\begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & -\sec\theta \end{bmatrix}$ (b) $\begin{bmatrix} -\sec\theta & \tan\theta \\ \tan\theta & -\sec\theta \end{bmatrix}$

(c) $\begin{bmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix}$ (d) $\begin{bmatrix} -\sec\theta & -\tan\theta \\ \tan\theta & -\sec\theta \end{bmatrix}$

(51) If $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, then

(a) $6A^{-1} = A - 5I$ (b) $6A^{-1} = A - 5I$

(c) $6A^{-1} = A + 5I$ (d) $6A^{-1} = -A - 5I$

(52) If $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 24 & 7 \\ 21 & 9 \end{bmatrix}$ and $AXB = C$, then $X =$

(a) $-\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$

(c) $-\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

(53) Inverse of the matrix $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & -1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 3 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -3 & 5 \\ 7 & 4 & 6 \\ 4 & 2 & 7 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & -4 \\ 8 & -4 & -5 \\ 3 & 5 & 2 \end{bmatrix}$

(54) Inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ is

(a) $\frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 1 & -1 \\ 4 & -3 & 1 \\ -5 & 3 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 & 1 \\ -4 & 3 & -1 \\ 5 & -3 & 1 \end{bmatrix}$

(55) The inverse of the matrix $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is

(a) $-\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $-\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(56) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -2 \end{bmatrix}$, then

(a) $(AB)^{-1}$ exists

(b) $(AB)^{-1}$ does not exists

(c) $(BA)^{-1}$ exists (d) None of these

(57) If A is a non-singular matrix, then det $(A^{-1}) =$

(a) det (A) (b) $1/\det(A)$

(c) 1 (d) 0

(58) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & 2c \\ 5 & -3 & 1 \end{bmatrix}$, then

(a) $a = 2, c = -1/2$ (b) $a = 1, c = -1$

(c) $a = -1, c = -1$ (d) $a = 1/2, c = 1/2$

(59) Inverse of the matrix $\begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}$ is

(a) $\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$ (b) $\begin{bmatrix} -0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}$

(c) $\begin{bmatrix} -0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -6 \\ 6 & 8 \end{bmatrix}$

(60) If $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$, then : $(A^{-1})^3 =$

(a) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$ (b) $\frac{1}{27}\begin{bmatrix} 1 & 26 \\ 0 & 27 \end{bmatrix}$

(c) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & -27 \end{bmatrix}$ (d) $\frac{1}{27}\begin{bmatrix} -1 & -26 \\ 0 & -27 \end{bmatrix}$

(61) If matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{k}$ adj A, then k is

(a) 7 (b) –7

(c) $\frac{1}{7}$ (d) 11

(62) The inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$ is

(a) $\frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ 14 & -3 & -5 \end{bmatrix}$ (b) $\frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -6 & 19 \\ 14 & -3 & -5 \end{bmatrix}$

(c) $\frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -6 & 19 \\ -3 & 14 & -5 \end{bmatrix}$ (d) $\frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & 19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$

(63) The multiplicative inverse of matrix $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ is

(a) $\begin{bmatrix} 4 & -1 \\ -7 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & -1 \\ 7 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -7 \\ 7 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

(64) $A = \begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}$ and $A^{-1} = \lambda\,(\text{adj}(A))$, then λ =

(a) $\frac{-1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{-1}{3}$ (d) $\frac{1}{6}$

(65) $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then

(a) $A^{-1} = B$ (b) B^{-1} does not exist

(c) A^{-1} does not exist (d) Both (B) and (C)

(66) If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, then $|A^{-1}|$

(a) 1 (b) –1

(c) 0 (d) 2

(67) If $A = \begin{bmatrix} 2 & 2 \\ -3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then $(B^{-1}A^{-1}) =$

(a) $\begin{bmatrix} 2 & -2 \\ 0 & 3 \end{bmatrix}$ (b) $\frac{1}{10}\begin{bmatrix} 2 & 2 \\ -2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -2 \\ 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

(68) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$, then $B^{-1} A^{-1} =$

(a) $\begin{bmatrix} -1 & 2 \\ 5 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 2 \\ -5 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 5 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 2 \\ 5 & -9 \end{bmatrix}$

(69) If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then the values of a and b respectively are :

(a) $\sin 2\theta, \cos 2\theta$ (b) $\cos 2\theta, \sin 2\theta$

(c) $-\cos\theta, \sin 2\theta$ (d) $\cos 2\theta, -\sin 2\theta$

2.5 Applications of Matrices

(70) The values of x, y, z, for the following equations $x + y + z = 6$, $x - y + 2z = 5$, $2x + y - z = 1$ are

(a) $x = 3, y = 2, z = 3$

(b) $x = 2, y = 1, z = 3$

(c) $x = -1, y = 2, z = 3$

(d) $x = 3, y = 2, z = -1$

(71) The values of x, y, z, for the equations $x + y + z = 6$, $3x - y + 3z = 10$, $5x + 5y - 4z = 3$ are

(a) $x = 3, y = 2, z = 5$

(b) $x = 1, y = 4, z = -2$

(c) $x = 1, y = 2, z = 1$

(d) $x = 2, y = -1, z = 5$

(72) The value of a for which the system of equation $ax + y + z = 0$; $x + ay + z = 0$; $x + y + z = 0$ possess a non-null solution is

(a) 1 (b) 2

(c) -1 (d) –2

(73) The solution of the equation $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 14$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ are

(a) $x = 2, y = 5, z = 3$

(b) $x = 2, y = 3, z = 5$

(c) $x = 1, y = 5, z = 1$

(d) $x = 1, y = 3, z = 5$

(74) Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$.

If $AX = B$, then $X =$

(a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$

(75) If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$, then $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is equal

(a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

HOME WORK

Multiple Choice Questions

2.1 INTRODUCTION TO MATRICES

(1) Choose the correct answer

(a) A square matrix whose each element is 1 is an identity matrix

(b) Every identity matrix is a scalar matrix

(c) Every scalar matrix is an identity matrix

(d) Every diagonal matrix is an identity matrix

(2) If A is square matrix for which $a_{ij} = i^2 - j^2$, then matrix A is

(a) skew - symmetric (b) unit

(c) zero (d) symmetric

(3) Whcih of the following is not true

(a) Every skew- symmetric matrix of odd order is non - singular

(b) Adjoint of a diagonal matrix is diagonal

(c) Adjoint of symmetric matrix is symmetric

(d) If denterminant of a square matrix is non-zero, then it is nno - singular

(4) If A, B, C are three square matrices such that AB = AC implies B = C, then the matrix A is always a/an

(a) none-singular matrix

(b) diagonal matrix

(c) orthogonal matrix

(d) singular matrix

(5) If A is square matrix, then $A + A^T$ is a

(a) skew-smmetric matrix

(b) unit matrix

(c) symmetric matrix

(d) non singular matrix

(6) For any square matrix A, AA^T is a

(a) skew- symmetric matrix

(b) unit matrix

(c) diagonal matrix

(d) symmetric matrix

(7) If A = , $\begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}$ then

(a) A^{-1} does not exists (b) A is singular

(c) A^{-1} exists (d) none of these

(8) If A = $\begin{vmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{vmatrix}$, then

(a) A is invertible

(b) A is scalar matrix

(c) A is singular

(d) A does not invertible

(9) If A = $\begin{bmatrix} 3 & 4 & 3 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{bmatrix}$ then,

(a) A is not invertible

(b) A is singular

(c) A is lower triangular matrix

(d) A is invertible

(10) If inverse of matrix A exists , then A:

(a) is a square or a rectangular matrix

(b) may be a square matrix

(c) must be a square matrix

(d) may be a rectangualr matrix

(11) If A is an orthogonal matrix, then

(a) det. A = – 1 (b) det. A = 0

(c) det. A = 1 (d) None of these

(12) If A is symmetric as well skew-symmetric then A is :-

(a) Null (b) Indentity

(c) Diagonal (d) Scalar

(13) If the matrix $\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{vmatrix}$ is singular . then λ =

(a) 2 (b) –3 (c) – 5 (d) 4

(14) The matrix $\begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -5 \\ -1 & 5 & 0 \end{bmatrix}$ is

(a) Symmetric (b) Orthogonal

(c) Null (d) Skew-symmetric

(15) A square matrix can always be expressed as a

(a) Skew- summetric matrix

(b) Sum of symmetric and a skew- symmetric matrix

(c) Skew matrix

(d) Sum of diagonal matrix and a symmetric matrix

2.2 ALGEBRA OF MATRICES

(16) If two matrices A and B are of order $p \times q$ and $r \times s$ respectively, can be subtracted only, if

(a) $p = q$ (b) $p = r, q = s$

(c) $p = q$, $r = s$ (d) none of these

(17) If A and B are two matrices such that A + B and AB are both defined, then

(a) A and B are two matrices not necessarily of same order

(b) A and B are square matrices of same order

(c) number of columns of A = number of rows of B same order

(d) None of these

(18) If A and B ares square matrices of order $n \times n$, then $(A - B)^2 =$

(a) $A^2 - AB - BA + B^2$ (b) $A^2 - B^2$

(c) $A^2 + 2AB + B^2$ (d) $A^2 - 2AB + B^2$

(19) If A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then A^6 =

(a) 32 A (b) 6 A (c) 12 A (d) 16 A

(20) $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \sin\theta & \sin\theta \end{bmatrix} =$

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(21) If $[m \quad n] \begin{bmatrix} m \\ n \end{bmatrix} = [25]$ and $m < n$, then $(m, n) =$

(a) (4,3) (b) (2,3)

(c) (3,4) (d) none of these

(22) If A = $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 5 \\ 2 & -5 & 0 \end{bmatrix}$, then

(a) A′ = 2 A (b) A′ = –A

(c) A′ = A (d) none of these

(23) If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 2 & 3 \end{bmatrix}$, then

(a) A is notn-singular (b) A is not invertible

(c) A is invertible (d) none of these

(24) The matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is invertible, if

(a) $\lambda \neq -15$ (b) $\lambda \neq -18$

(c) $\lambda \neq -17$ (d) $\lambda \neq -16$

(25) Matrix A = $\begin{bmatrix} 1 & 0 & -k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$ is invertible for

(a) $k = 1$ (b) $k = -1$

(c) $k = 0$ (d) all real k

(26) If A = $\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$, B $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ = and AX = B, then X=

(a) $\frac{1}{3}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$ (b) $[5 \ \ 7]$

(c) $\frac{1}{3}$ $[5 \ \ 7]$ (d) $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

(27) The matrix A safisfying A $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 6 & 0 \end{bmatrix}$ is

(a) $\begin{bmatrix} 9 & 4 \\ 36 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -6 \\ 12 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -16 \\ 6 & -30 \end{bmatrix}$ (d) $\begin{bmatrix} 9 & -16 \\ 36 & -45 \end{bmatrix}$

(28) If : $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then : α =

(a) $\pm$ 5 (b) $\pm$ 1 (c) $\pm$ 2 (d) $\pm$ 3

(29) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $a, b \in N$.

Then

(a) there cannot exist any B such that $AB = BA$

(b) there exists more than one but finite number of B's such that $AB = BA$

(c) there exists one B such that $AB = BA$

(d) there exist infinitely many B's such that $AB = BA$

(30) The upper triangualr matrix of the matrix

$\begin{bmatrix} 1 & -2 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & 0 & \frac{-1}{3} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{-1}{3} \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{-1}{3} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{-1}{3} & 0 & 0 \\ 3 & -1 & 0 \\ -1 & 2 & 0 \end{bmatrix}$

(31) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$A, $R_2 \to R_2 - 2R_1$ gives

(a) $\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$ A

(b) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ A

(c) $\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ A

(d) $\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ A

(32) $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$A, then

$R_1 \to R_1 + R_2 - R_3$ gives

(a) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ A

(b) $\begin{bmatrix} 1 & -3 & 4 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ A

(c) $\begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ A

(d) $\begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ A

(33) The matrix $\begin{bmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$ is not invertible , if 'a' has the value

(a) –1 (b) 2 (c) 1 (d) 0

(34) The matrix A satisfying A $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 6 & 0 \end{bmatrix}$ is

(a) $\begin{bmatrix} 3 & -3 \\ 6 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 6 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -16 \\ 6 & -30 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -16 \\ 6 & 30 \end{bmatrix}$

(35) Which of the following matrices is not invertible?

(a) $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 3 \\ 4 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$

(36) If A and B are two matrices such that AB = B and BA = A. then $A^2 + B^2 =$

(a) $2A^2$ (b) 2 Ab (c) AB (d) A + B

(37) If A = $\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$ and AB = I, then B =

(a) $\begin{bmatrix} \tan\theta & 1 \\ 1 & -\tan\theta \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & \cot\theta \\ -\cot\theta & 1 \end{bmatrix}$

(d) $\cos^2\theta \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

(38) If, $\begin{bmatrix} -2 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 14 \end{bmatrix}$ then $\begin{bmatrix} x \\ y \end{bmatrix} =$

(a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(39) If A is a square matrix such that $A^2 = A$, then $|A|$ is :

(a) 0 or 3 (b) 0 or 1 (c) 0 or 2 (d) 1 or 2

(40) If A = $\begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$, then $|A^5| =$

(a) $5|A|$ (b) 1 (c) 5 (d) $6|A|$

(41) If $X = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$ then value of X^n is

(a) $\begin{bmatrix} 3^n & (-3)^n \\ 1^n & (-1)^n \end{bmatrix}$ (b) $\begin{bmatrix} 2n & (-3)n \\ n & -n \end{bmatrix}$

(c) $\begin{bmatrix} 1+n & 4-n \\ n & -n \end{bmatrix}$ (d) None fo these

(42) If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$,$B = \begin{bmatrix} 1 & x \\ 2 & y \end{bmatrix}$ and

$(A + B)^2 = A^2 + B^2$ x and y are

(a) – 1, – 1 (b) 1, 1

(c) 1, – 1 (d) – 1, 1

(43) Find the matrix X, if $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$ and

$C = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ and $3A - 2B + 4X = 5C$.

(a) $\begin{bmatrix} 1/3 & 1/4 \\ 1/4 & 1/3 \end{bmatrix}$ (b) $\begin{bmatrix} 1/3 & -2 \\ -1 & 1/3 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 1/4 \\ 1/4 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1/4 & -2 \\ -1 & 1/4 \end{bmatrix}$

(44) If $A = \begin{bmatrix} 1/3 & 2 \\ 0 & 2x-3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ and $AB = I$,

then x =

(a) 5 (b) – 4 (c) 1 (d) 3

(45) If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $AB = 0$, then B =

(a) $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$

(46) If A = , then A^2 is

(a) 2A (b) a Null matrix

(c) a Unit matrix (d) a symmetric matrix

(47) If $A = [x \ y \ z]$, $B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$, $C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$,

then ABC =

(a) $[ax^2 + by^2 + cz^2]$

(b) $[ax^2 + by^2 + cz^2 + 2hxy + 2gxz + 2fyz]$

(c) $[ax^2 + by^2 + cz^2 + hxz + gxz + fyz]$

(d) None of the above

(48) If $B' = [x \ y \ z]$, $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $A = \begin{bmatrix} 0 & 3 & 3 \\ -3 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$, then

$B'AB$ =

(a) $[3xy - 4yz - 3xz]$ (b) $[0]$

(c) $[3xy - 4yz + 3xz]$ (d) $[3xy + 4yz - 3xz]$

(49) If $A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, then $(A + I)(A - I)$ =

(a) $\begin{bmatrix} 1 & -2 \\ 4 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & -4 \\ 8 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$

(50) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, AB?

(a) A (b) Null (c) Scalar (d) B

2.3 MINORS , COFACTORS & ADJOINT OF A METRIX

(51) If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and A_{ij} denotes the cofactor of element a_{ij} , then $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} =$

(a) $2\,|A|$ (b) 0

(c) $|A|$ (d) $-\,|A|$

(52) If A and B are non - singular square matrices of same order, then adj (AB) =

(a) $(adj\ B^{-1})\,(adj\ A^{-1})$ (b) $(adj\ A)\,(adj\ B)$

(c) $(adj\ B)\,(adj\ A)$ (d) $(adj\ A^{-1})\,(adj\ B^{-1})$

(53) The cofactors of the elemetns of the second row of $\begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{bmatrix}$ are

(a) – 39, 3, 11 (b) 4, 5, 6

(c) – 3, 11, – 39 (d) 11, 3, – 39

(54) The cofactors of the elements of the third row of $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ are

(a) 3, 2, – 6 (b) – 6, 2, 4

(c) 6, – 2, 3 (d) – 6, 1, 3

(55) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$, then

(a) $A_{11} + A_{21} = A_{32}$ (b) $A_{12} + A_{22} + A_{32} = 0$

(c) $A_{13} + A_{23} + A_{33} = 1$ (d) None of these

(56) Matrix of co - factors of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ is

(a) $\begin{bmatrix} -4 & 3 \\ 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$

(57) Matrix of co - factors of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 3 \\ 0 & 3 & -5 \end{bmatrix}$ is

(a) $\begin{bmatrix} -14 & -10 & -6 \\ 6 & -5 & -3 \\ 2 & -7 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -14 & -10 & 6 \\ 6 & -5 & -3 \\ -2 & -7 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -14 & -10 & -6 \\ -6 & 5 & -3 \\ -2 & -7 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -14 & -10 & -6 \\ 6 & -5 & -3 \\ -2 & -7 & 1 \end{bmatrix}$

(58) If $A = \begin{bmatrix} -2 & 6 \\ -5 & 7 \end{bmatrix}$, then $adj\ A =$

(a) $\begin{bmatrix} 2 & -6 \\ 5 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -6 \\ 5 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -6 \\ 5 & -7 \end{bmatrix}$ (d) None of these

(59) If $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$, then $|adj\ A| =$

(a) 16 (b) 6

(c) 10 (d) None of these

(60) If $A = \begin{bmatrix} 3 & 5 & -1 \\ 2 & 0 & 4 \\ 1 & -3 & 0 \end{bmatrix}$, then $\begin{bmatrix} 12 & 4 & -6 \\ 3 & 1 & 14 \\ 1 & -14 & -10 \end{bmatrix}$ is

(a) $-A^{-1}$ (b) $adj\ (A')$

(c) $-(adj\ A)$ (d) $adj\ A$

(61) If $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$, then $adj\ A =$

(a) $\begin{bmatrix} 0 & -1 & -1 \\ -8 & 3 & -7 \\ 4 & -2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -8 & 4 \\ -1 & 3 & -2 \\ 4 & -7 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -1 & 1 \\ 8 & 3 & -7 \\ 4 & -2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 & 1 \\ -8 & 3 & -7 \\ 4 & -2 & 2 \end{bmatrix}$

(62) $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix}$ and $adj\ A = \begin{bmatrix} 5 & x & -2 \\ 1 & 1 & 0 \\ -2 & -2 & y \end{bmatrix}$

then $(x, y) =$

(a) (4, 1) (b) (4, – 1)

(c) (– 4, 1) (d) (– 4, –10)

(63) For any 2×2 matrix A, if $A\ (adj\ A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$,

then $|A| =$

(a) 0 (b) 100 (c) 20 (d) 10

(64) If $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$, then $A\ (adj\ A) =$

(a) I (b) $|A|\,I$

(c) $|A|$ (d) none of these

(65) If $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, then $(adj\ A)\ A =$

(a) $\begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ (d) $\begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{bmatrix}$

(66) If A is square matrix of order n, where $|A| = 5$ and $|A(adj\ A)| = 125$, then $n =$

(a) 4 (b) 3 (c) 2 (d) 1

(67) If A and B are square matrices of the same order, then : $adj\ (AB) =$

(a) $(adj.\ A) - (adj.\ B)$ (b) $(adj.\ A)\ (adj.\ B)$

(c) $(adj.\ B)\ (adj.\ A)$ (d) $(adj.\ A) + (adj.\ B)$

(68) If A is a singular matrix, then : $A\ (adj.\ A) =$

(a) transpose of A (b) identity matrix

(c) null matrix (d) scalar matrix

(69) If a matrix $A = [a_{ij}]_{3 \times 3}$ has first two rows identical, and $a_{31} = 1$, $a_{32} = 2$, $a_{33} = 3$, $A_{31} = 3$, $A_{33} = 1$ then : $A_{32} =$

(a) 6 (b) – 4 (c) 2 (d) – 3

(70) If A is a 3×3 matrix such that $adj.\ A = 0$, then

(a) A is singular

(b) $A = 0$

(c) A is non-singular

(d) all elements of A are equal

(71) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ -1 & 2 & 3 \end{bmatrix}$, then co-factor of 3rd row are

(a) 4, 5, –1 (b) 4, –5, 1

(c) – 4, 5, –1 (d) – 4, 5, 1

(72) If $B = \begin{bmatrix} 2 & 3 \\ -4 & 3 \end{bmatrix}$, then co - factor A_{21} =

(a) – 4 (b) 2

(c) – 3 (d) None of these

(73) If $A = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \\ 2 & -4 & 3 \end{bmatrix}$, then M_{11} =

(a) – 3 (b) 2 (c) – 7 (d) 8

(74) If matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}$ and its inverse is denoted by $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then the value of a_{23} =

(a) $\frac{2}{5}$ (b) $\frac{21}{20}$ (c) $\frac{1}{5}$ (d) $-\frac{2}{5}$

(75) For any 2 × 2 matrix A, if A . (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then $|A|$ =

(a) 100 (b) 0 (c) 10 (d) 20

(76) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and A.adj A = $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k is equal to

(a) $\cos 2\alpha$ (b) 0

(c) 1 (d) $\sin\alpha \cos\alpha$

(77) adj AB – (adj B) (adj A) =

(a) O (b) adj A – adj B

(c) I (d) none of these

(78) If A = adj (adj A) =

(a) A^3 (b) A (c) –A (d) A^2

(79) The adjoint of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ is :

(a) $\begin{bmatrix} 5 & 2 & 7 \\ 1 & 3 & 2 \\ 4 & 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 2 & -8 \\ -2 & -5 & 3 \\ -3 & 3 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 1 & -5 \\ -2 & -5 & 3 \\ -3 & 3 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & 1 & 3 \\ 4 & 2 & 3 \\ 3 & -3 & 1 \end{bmatrix}$

(80) If A is a square matrix of order n and k is a scalar, then adj (kA) is :

(a) k^{n-2} (adj A) (b) k (adj A)

(c) k^{n} (adj A) (d) k^{n-1} (adj A)

2.4 INVERSE OF MATRIX

(81) The inverse of a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

(a) $\frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (b) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(c) $\begin{bmatrix} b & -a \\ d & -c \end{bmatrix}$ (d) $\frac{1}{|A|} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(82) If I_3 is identity matrix of order 3, then I_3^{-1} =

(a) does not exist (b) 0

(c) I_3 (d) $3\,I_3$

(83) If for a matrix A, $A^5 = I$, then A^{-1} =

(a) A^4 (b) A^2 (c) A^3 (d) A

(84) If A and B are square matrices of the same order and AB = 3 I then A^{-1} =

(a) $\frac{1}{3} B^{-1}$ (b) 3 B (c) $3B^{-1}$ (d) $\frac{1}{3}$ B

(85) If $A^2 - A + I = O$, then $A^{-1} =$

(a) $A + I$ (b) A^{-2} (c) $I - A$ (d) $A - I$

(86) If A and B are two square matrices such that $B = -A^{-1}BA$ then $(A + B)^2 =$

(a) $A^2 + 2AB + B^2$ (b) 0

(c) $A + B$ (d) $A^2 + B^2$

(87) If $A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 7/34 & 1/17 \\ -3/34 & 2/17 \end{bmatrix}$ then $x =$

(a) 4 (b) -4 (c) 2 (d) 3

(88) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $|A^{-1}| =$

(a) 4 (b) 1 (c) -1 (d) 2

(89) If $A^{-1} = \frac{-1}{2}\begin{bmatrix} 1 & -4 \\ -1 & 2 \end{bmatrix}$, then A =

(a) $\begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -4 \\ -1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 4 \\ -1 & 1 \end{bmatrix}$

(90) If the multiplicative group of 2×2 matrices of the form $\begin{bmatrix} a & a \\ a & a \end{bmatrix}$, for $a \neq 0$ and $a \in R$, then the inverse of $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1/8 & 1/8 \\ 1/8 & 1/8 \end{bmatrix}$ (b) $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$

(c) $\begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}$ (d) none of these

(91) The inverse of matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is

(a) $\frac{1}{2}\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$ (b) $\frac{-1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(c) $\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ (d) $\frac{-1}{2}\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$

(92) The inverse matrix $\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$ is

(a) $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$ (b) $-\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$ (d) $\frac{-1}{3}\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$

(93) The inverse matrix $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ is

(a) $-\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

(c) $-\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

(94) If $A = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$, then $A^{-1} =$

(a) $\frac{1}{2}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$

(c) $\frac{-1}{2}\begin{bmatrix} 2 & -4 \\ -3 & 5 \end{bmatrix}$ (d) $\frac{1}{2}\begin{bmatrix} -4 & 4 \\ -3 & 5 \end{bmatrix}$

(95) The inverse of $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$ is

(a) $\frac{1}{8}\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$ (b) $\frac{-1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$

(c) $\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (d) $\frac{-1}{8}\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

(96) The multiplicative inverse of matrix $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ is

(a) $\begin{bmatrix} -4 & -1 \\ 7 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -7 \\ 7 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -1 \\ -7 & -2 \end{bmatrix}$

(97) If $A = \begin{bmatrix} i & 0 \\ 0 & i/2 \end{bmatrix}$, then $A^{-1} =$

(a) $\begin{bmatrix} 0 & i \\ 2i & 0 \end{bmatrix}$ (b) $\begin{bmatrix} i & 0 \\ 0 & i/2 \end{bmatrix}$

(c) $\begin{bmatrix} i & 0 \\ 0 & 2i \end{bmatrix}$ (d) $\begin{bmatrix} -i & 0 \\ 0 & -2i \end{bmatrix}$

(98) Let $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = xA + yI$, then

(a) $x = \frac{1}{11},\ y = \frac{2}{11}$ (b) $x = \frac{-1}{11}\ y = \frac{-2}{11}$

(c) $x = \frac{-1}{11},\ y = \frac{2}{11}$ (d) $x = \frac{1}{11},\ y = \frac{-2}{11}$

(99) If $A = \begin{bmatrix} 2 & -3 \\ 5 & -7 \end{bmatrix}$, then $A + A^{-1} =$

(a) $\begin{bmatrix} 4 & 0 \\ 0 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

(100) The inverse of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & -1/2 \end{bmatrix}$ (b) $\begin{bmatrix} -1/2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$

(101) The inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1/3 & 0 \\ 3 & 2/3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1/3 & 0 \\ 3 & 2/3 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1/3 & 0 \\ 3 & 2/3 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1/3 & 0 \\ 3 & 2/3 & -1 \end{bmatrix}$

(102) The inverse of the matrix is $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

(a) $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 1 \\ 2 & 2 & -2 \end{bmatrix}$ (b) $\frac{-1}{6}\begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$

(c) $\frac{1}{6}\begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 1 & -1 \\ 1 & 0 & -1 \\ -2 & -2 & 2 \end{bmatrix}$

(103) The inverse of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ is

(a) $\begin{bmatrix} -3 & 1 & -1 \\ 15 & -6 & 5 \\ -5 & 2 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -1 & 1 \\ 5 & -2 & 2 \\ -15 & 6 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 1 & -1 \\ -5 & 2 & -2 \\ 15 & -6 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

(104) The inverse of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$ is

(a) $\frac{1}{25}\begin{bmatrix} 25 & 10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$

(b) $\frac{1}{5}\begin{bmatrix} 5 & 10 & -15 \\ -2 & 4 & 11 \\ -3 & 1 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & -3 \\ -2 & 4 & 11 \\ -3 & 1 & 9 \end{bmatrix}$

(d) $\frac{-1}{25}\begin{bmatrix} -25 & 10 & -15 \\ 10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$

(105) The inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ is

(a) $\frac{-1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -2 & 1 \\ 2 & 1 & 5 \\ -1 & 0 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -1 & 0 & 1 \end{bmatrix}$ (d) $\frac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$

(106) The inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 1 & \cos\theta & \sin\theta \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{bmatrix}$

(107) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ and A^{-1} exist and not equal to 0, then $(A^2 - 4A)\,A^{-1} =$

(a) $\begin{bmatrix} 5 & 2 & 5 \\ 2 & 5 & 5 \\ 5 & 5 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 2 & 5 \end{bmatrix}$

(108) If $A = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then $(B^{-1}A^{-1})^{-1} =$

(a) $\frac{1}{10}\begin{bmatrix} 2 & 2 \\ -2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$ (d) $\frac{1}{10}\begin{bmatrix} 3 & 2 \\ -2 & 2 \end{bmatrix}$

(109) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ then $(AB)^{-1} =$

(a) $-\begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 3 \\ 7 & 11 \end{bmatrix}$

(c) $-\begin{bmatrix} 2 & 5 \\ 7 & 11 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$

(110) If $A^2 + mA + nI = O$ and $n \neq 0$ $|A| \neq 0$, then $A^{-1} =$

(a) $A + mnI$ (b) $-\frac{1}{m}(A + nI)$

(c) $-\frac{1}{n}(A + mI)$ (d) $-\frac{1}{n}(I + mA)$

(111) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then $19\,A^{-1} =$

(a) $-A$ (b) A (c) $2A$ (d) $\frac{1}{2}A$

(112) Which of the following matrices does not have inverse ?

(a) $\begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

(113) If x is complex cube root of unity and $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & x & x^2 \\ 1 & x^2 & x \end{bmatrix}$, then $A^{-1} =$

(a) $\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & x^2 & x \\ 1 & x & x^2 \end{bmatrix}$ (b) $\frac{1}{4}\begin{bmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{bmatrix}$

(c) $\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & x^2 & x \\ 1 & x & x^2 \end{bmatrix}$ (d) $\frac{1}{6}\begin{bmatrix} 1 & 1 & 1 \\ 1 & x^2 & x \\ 1 & x & x^2 \end{bmatrix}$

(114) If $A = \begin{bmatrix} 1+i & -i \\ i & 1-i \end{bmatrix}$ where $i = \sqrt{-1}$ and

$= A^2 - 2A + I = 0$, then : $A^{-1} =$

(a) $\begin{bmatrix} 1+i & -i \\ i & 1-i \end{bmatrix}$ (b) $\begin{bmatrix} 1-i & i \\ -i & 1+i \end{bmatrix}$

(c) $\begin{bmatrix} 1-i & -i \\ i & 1+i \end{bmatrix}$ (d) $\begin{bmatrix} 1+i & i \\ -i & 1-i \end{bmatrix}$

(115) If A and B are non - singular matrices, then

(a) $(AB)^{-1} = B^{-1}A^{-1}$ (b) $(AB)^{-1} = A^{-1}B^{-1}$

(c) $AB = BA$ (d) $(AB)' = A'B'$

(116) If product of matrix A with $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ is $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, then

A^{-1} is given by

(a) $\begin{bmatrix} 0 & 1 \\ -2 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ 2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -1 \\ -2 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$

(117) If a matrix A is such that

$= 3A^3 + 2A^2 + 5A + I = 0$, then its inverse is

(a) $3A^2 - 2A - 5I$ (b) $-(3A^2 + 2A + 5I)$

(c) $3A^2 + 2A + 5I$ (d) None of these

(118) The inverse of $\begin{bmatrix} 1 & \sin\alpha \\ -\sin\alpha & -1 \end{bmatrix}$ is

(a) $\cos^2\alpha \begin{bmatrix} 1 & \sin\alpha \\ -\sin\alpha & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -\sin\alpha \\ -\sin\alpha & -1 \end{bmatrix}$

(c) $-\sec^2\alpha \begin{bmatrix} 1 & -\sin\alpha \\ \sin\alpha & -1 \end{bmatrix}$

(d) $\sec^2\alpha \begin{bmatrix} 1 & \sin\alpha \\ -\sin\alpha & -1 \end{bmatrix}$

(119) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and

(10) $B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ .If B is the inverse of matrix

A , then α is

(a) -2 (b) 5 (c) -1 (d) 2

(120) $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}^{-1} =$

(a) $\begin{bmatrix} -1 & -3 \\ -3 & -10 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 3 \\ 3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}$

(121) The inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & -a & ac-b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ b & -c & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac & b & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac-b & -c & 1 \end{bmatrix}$

(122) If A $= \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{7}{14} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$, then the value of x is

(a) 4 (b) 2 (c) 3 (d) – 4

(123) If A is an 3×3 non - singular matrix such that $AA' = A'A$ and $B = A^{-1}$, then BB' equlas

(a) I (b) B^{-1} (c) $(B^{-1})'$ (d) I + B

(124) If A $= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ A and $A^{-1}B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ then A^{-1} B is

(a) $\begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ 5 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 1 \\ -5 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -1 \\ 5 & 0 \end{bmatrix}$

(125) If A $= \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, then $(A')^{-1} =$

(a) $\frac{1}{10}\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$ (b) $\frac{1}{10}\begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix}$

(c) $\frac{1}{10}\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ (d) $\frac{1}{10}\begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$

(126) Inverse of the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ is :

(a) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

(b) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

(d) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

(127) If A $= \begin{bmatrix} a & 0 \\ 0 & \frac{1}{b} \end{bmatrix}$, then $A^{-1} =$

(a) $\begin{bmatrix} -a & 0 \\ 0 & b \end{bmatrix}$ (b) $\begin{bmatrix} -a & 0 \\ 0 & -\frac{1}{b} \end{bmatrix}$

(c) $\begin{bmatrix} -\frac{1}{a} & 0 \\ 0 & -\frac{1}{b} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{a} & 0 \\ 0 & b \end{bmatrix}$

(128) If A $= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 5 & 5 \end{bmatrix}$, then A^{-1} is

(a) $\begin{bmatrix} -5 & 20 & 2 \\ 1 & 3 & 0 \\ 3 & 11 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 20 & 2 \\ -1 & 3 & 0 \\ 3 & 11 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 20 & -2 \\ -1 & 3 & 0 \\ 3 & -11 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & 20 & -2 \\ 1 & 3 & 0 \\ 3 & 11 & 1 \end{bmatrix}$

(129) If A is square matrix of order 2×2, then $\left|(3A)^{-1}\right|$ is

(a) $\dfrac{3}{|A|}$ (b) $3\left|A^{-1}\right|$

(c) $9\left|A^{-1}\right|$ (d) $\dfrac{1}{9}\left|A^{-1}\right|$

(130) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -4 \\ 5 & 1 \end{bmatrix}$, then $(A + B)^{-1}$ is

(a) $\dfrac{1}{10}\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$ (b) $\dfrac{1}{10}\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$

(c) $\dfrac{1}{10}\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$ (d) $\dfrac{1}{10}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(131) Let $X \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $D = \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix}$

and $A = \begin{bmatrix} 2 & -1 & -2 \\ 2 & 1 & 1 \\ 4 & -1 & -2 \end{bmatrix}$, if $X = A^{-1}D$, then X is equal to :

(a) $\begin{bmatrix} \frac{8}{3} \\ \frac{1}{3} \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} \frac{8}{3} \\ \frac{-1}{3} \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{-8}{3} \\ 1 \\ 0 \end{bmatrix}$

(e) $\begin{bmatrix} \frac{8}{3} \\ \frac{1}{3} \\ -1 \end{bmatrix}$

(132) If A is an invertible matrix of order 2, then which of the following is not true.

(a) $|kA| = k^2|A|$ (b) $(A^{-1})^{-1} = A$

(c) $|\text{adj } A| = |A|$ (d) $|\text{adj } A| = |A|^2$

(133) If ω is a cube root of unity and

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$, then $A^{-1} =$

(a) $\dfrac{1}{2}\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$ (b) $\dfrac{1}{4}\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ 1 & \omega^2 & 1 \end{bmatrix}$

(c) $\dfrac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ (d) $\dfrac{1}{6}\begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$

(134) 41. Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $\alpha \in R$. then $[F(\alpha)]^{-1}$ is equal to

(a) $F(2\alpha)$ (b) $F(-\alpha)$

(c) $F(\alpha^{-1})$ (d) None of these

(135) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \dfrac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & k \\ 1 & -2 & 3 \end{bmatrix}$

if B is the inverse fo A, then k = ?

(a) $k = 5$ (b) $k = 2$ (c) $k = 3$ (d) $k = 4$

(136) If A = then A^{-1} is

(a) $\frac{1}{12}\begin{bmatrix} 58 & 94 & -12 \\ 0 & 0 & 6 \\ -74 & 54 & -25 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 0 \\ -74 & 54 & -25 \\ 58 & 94 & -12 \end{bmatrix}$

(c) $\frac{1}{4}\begin{bmatrix} 58 & 94 & -12 \\ 0 & 0 & 0 \\ -74 & 54 & -25 \end{bmatrix}$

(d) None of these

(137) Let F (α) = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

G (β) = $\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$, then $(F(\alpha)\, G(\beta))^{-1}$=

(a) $(G(\beta))^{-1}(F(\alpha))^{-1}$ (b) $(F(\alpha))^{-1}\,(G(\beta))^{-1}$

(c) $-F(\alpha) - G(\beta)$ (d) $F(\alpha) - G(\beta)$

(138) If A = $\begin{bmatrix} -2 & 0 & 0 \\ -2 & -1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & -1 \end{bmatrix}$,

then

(a) $(AB)^{-1}$ unit matrix

(b) $(AB)^{-1}$ exists

(c) $(AB)^{-1}$ is null matrix

(d) $(AB)^{-1}$ not exist

(139) The inverse of the matrix $\begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix}$ is

(a) $\begin{bmatrix} 2 & 5 & 2 \\ 3 & 6 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ (b) $-\begin{bmatrix} 3 & 6 & 2 \\ 2 & 5 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 6 & 2 \\ 10 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$

(140) If $\begin{bmatrix} x & y^3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 2 & 0 \end{bmatrix}$, then $\begin{bmatrix} x & y \\ 2 & 0 \end{bmatrix}^{-1} =$

(a) $-\begin{bmatrix} 0 & -2 \\ -2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -8 \\ -2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1/2 \\ 1/2 & -1/4 \end{bmatrix}$

2.5 APPLICATION OF MATRICES

(141) Equations $x + y = 2$, $2x + 2y = 3$ will have

(a) many finite solutions

(b) no solution

(c) only one solution

(d) none of these

(142) If $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 10 & -15 \\ -5 & -2 & -13 \\ 10 & -4 & -6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$, then $x+y+z =$

(a) 1 (b) 3 (c) 0 (d) 2

(143) If $\begin{bmatrix} x-y-z \\ -y-z \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}$, then *the value of* x,y and z are respectively

(a) 11, 8, 3 (b) 0, – 3, 3

(c) 1, – 2, 3 (d) 5, 2, 2

(144) Ler : $A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1-2\sqrt{k} & \\ -2\sqrt{k} & 2\sqrt{k} & -1 \end{bmatrix}$,

$B = \begin{bmatrix} 0 & 2\sqrt{k}-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$

where k is a real number.

If : det (adj$A$) + det (adj$B$) - 10^6,

then : $[k]$ =

(a) 6 (b) 3 (c) 4 (d) 5

(145) If $3x - 4y + 2z = -1$, $2x + 3y + 5z = 7$, $x+z = 2$, then $x =$

(a) – 1 (b) 3 (c) 2 (d) 1

(146) The solution of the equations $5x–7y = 2$, $7x– 5y = 3$ are

(a) $x = 2,\ y = 1$ (b) $x = \frac{11}{24}$, $y = \frac{1}{24}$

(c) $x = \frac{10}{24}, y = \frac{5}{24}$ (d) $x = -6y = -5$

(147) If $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 4 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \\ 13 \end{bmatrix}$, then *t*he value of x,y,z respectively are =

(a) 1, 1, 2 (b) 1, 2, 3

(c) 3, 2, 1 (d) 2, 2, 1

(148) The values of x,y,z in order, in the system of equation $3x + y + 2z = 3$, $2x –3y – z = –3$, $x + 2y + z = 4$, are :

(a) 1, 2, – 1 (b) 2, 1, 5

(c) 1, 1, 1 (d) 1, – 2, – 1

(149) If $2x + 4y + 3z = 9$, $2x + 3y + z = 4$, $4x + 5y + 4z = 15$,then the values of x,y,z respectively, are :

(a) 3, 1, 2 (b) 2, 1, 3

(c) 2, 1, – 3 (d) 2, –1, 3

(150) The values of x,y,z for the equations $2x–y+z = 1$, $x+2y+3z = 8$, $3x + y – 4z = 1$ are

(a) $x = 2, y = -1, z = 5$

(b) $x = 0, y = 5, z = 2$

(c) $x = 3, y = 1, z = -2$

(d) $x = 1, y = 2, z = 1$

CLASS WORK - ANSWER KEY

1 **c**	2 **b**	3 **b**	4 **d**	5 **b**	6 **c**	7 **b**	8 **b**	9 **c**	10 **c**
11 **d**	12 **b**	13 **c**	14 **b**	15 **d**	16 **b**	17 **d**	18 **a**	19 **c**	20 **d**
21 **a**	22 **c**	23 **b**	24 **d**	25 **c**	26 **a**	27 **b**	28 **b**	29 **d**	30 **d**
31 **b**	32 **c**	33 **d**	34 **c**	35 **c**	36 **a**	37 **a**	38 **b**	39 **b**	40 **b**
41 **c**	42 **b**	43 **b**	44 **a**	45 **b**	46 **d**	47 **d**	48 **c**	49 **d**	50 **c**
51 **a**	52 **d**	53 **b**	54 **a**	55 **b**	56 **a**	57 **b**	58 **b**	59 **a**	60 **a**
61 **d**	62 **d**	63 **d**	64 **a**	65 **d**	66 **b**	67 **a**	68 **d**	69 **b**	70 **a**
71 **d**	72 **a**	73 **b**	74 **b**	75 **d**					

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HOME WORK - ANSWER KEY

1 **b**	2 **a**	3 **a**	4 **a**	5 **c**	6 **d**	7 **c**	8 **a**	9 **d**	10 **c**
11 **c**	12 **a**	13 **a**	14 **d**	15 **b**	16 **b**	17 **b**	18 **a**	19 **a**	20 **a**
21 **c**	22 **b**	23 **b**	24 **c**	25 **a**	26 **a**	27 **c**	28 **d**	29 **d**	30 **b**
31 **c**	32 **d**	33 **c**	34 **c**	35 **d**	36 **d**	37 **d**	38 **a**	39 **b**	40 **b**
41 **d**	42 **c**	43 **d**	44 **c**	45 **a**	46 **c**	47 **b**	48 **b**	49 **d**	50 **d**
51 **c**	52 **c**	53 **a**	54 **b**	55 **a**	56 **c**	57 **d**	58 **b**	59 **c**	60 **b**
61 **d**	62 **a**	63 **d**	64 **b**	65 **a**	66 **b**	67 **c**	68 **c**	69 **d**	70 **a**
71 **c**	72 **c**	73 **b**	74 **a**	75 **c**	76 **c**	77 **a**	78 **b**	79 **c**	80 **d**
81 **a**	82 **c**	83 **a**	84 **d**	85 **c**	86 **d**	87 **a**	88 **b**	89 **b**	90 **d**
91 **b**	92 **a**	93 **d**	94 **c**	95 **b**	96 **c**	97 **d**	98 **c**	99 **c**	100 **c**
101 **b**	102 **b**	103 **d**	104 **a**	105 **d**	106 **d**	107 **b**	108 **c**	109 **d**	110 **c**
111 **b**	112 **d**	113 **c**	114 **b**	115 **a**	116 **d**	117 **b**	118 **d**	119 **b**	120 **c**
121 **b**	122 **a**	123 **a**	124 **c**	125 **b**	126 **a**	127 **d**	128 **c**	129 **d**	130 **c**
131 **c**	132 **d**	133 **c**	134 **b**	135 **a**	136 **d**	137 **a**	138 **b**	139 **d**	140 **d**
141 **c**	142 **b**	143 **c**	144 **c**	145 **b**	146 **b**	147 **c**	148 **a**	149 **d**	150 **d**

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CLASSWORK

Hints & Explanation

2.1 Introduction to Matrices

(1) **(c)** sclar matrix

By definition

(2) **(b)** $a_{ij} = 0$, for $i < j$

By defination of an upper triangular matrix

(3) **(b)** a scalar martix

Informative

(4) **(d)** skew symmetric matrix

By property

(5) **(b)** is a zero matrix

W know that $A.\text{adj}A = |A| I_n$

But A is a singular matrix (given)

$\therefore |A| = 0$

$\therefore A.\text{adj}A = 0$

(6) **(c)** orthogonal

Let $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Now $A.A'$=

$$\begin{bmatrix} \cos^2 + \sin^2\alpha & -\sin\alpha\cdot\cos\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cdot\cos\alpha + \cos\alpha\cdot\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ A is Orthogonal.

(7) **(b)** AB is non - singular

$\therefore |A| \neq 0$ *and* $|B| \neq 0$

$\Rightarrow |AB| = |A| \cdot |B| \neq 0$

$\therefore$ AB is non-singular

(8) **(b)** Every identify matrix is a scalar matrix

(9) **(c)** $A+B = B+A$

Commutative law

(10) **(c)** $A^2 + AB + BA + B^2$

$(A + B)^2 = (A + B) = A^2 + AB + BA + B^2$

(11) **(d)** $B^T A^T$

By property

(12) **(b)** $16A$

A is a scalar matrix, then

$$A^5 = \begin{bmatrix} 2^5 & 0 & 0 \\ 0 & 2^5 & 0 \\ 0 & 0 & 2^5 \end{bmatrix} = 2^4 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 16A$$

(13) **(c)** 2

$A^2 - 6A + 7I = 0 \Rightarrow$

$$\begin{bmatrix} 4 & -1 \\ -1 & k \end{bmatrix}\begin{bmatrix} 4 & -1 \\ -1 & k \end{bmatrix} - 6\begin{bmatrix} 4 & -1 \\ -1 & k \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= \ 0 \Rightarrow$

$$\begin{bmatrix} 17 & -4-k \\ -4-k & 1+k^2 \end{bmatrix} + \begin{bmatrix} -17 & 6 \\ 6 & -6k+7 \end{bmatrix} = 0 \Rightarrow$$

$-4 - k + 6 = 0 \Rightarrow k = 2$

(14) **(b)** $A - A'$

$(A - A')' = A' - (A') = A' - A = -(A - A')$

(15) **(d)** $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

Reverse solution.

(16) **(b)** $1 - \alpha^2 - \beta\gamma = 0$

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$\because A^2 = I \quad \therefore A \cdot A = I$

$$\therefore \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = I$$

$$\therefore \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore \alpha^2 + \beta\gamma = 1$

$\therefore 1-\alpha^2-\beta\gamma=0$

(17) **(d)** no real values

$\because A^2=B \quad \therefore AA=B$

$\therefore \begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix}\begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}$

$\therefore \{\alpha^2=1\}$ and $\{\alpha+1=5\}$

$\therefore \{\alpha=\pm1\}$ and $\{\alpha=4\}$

$\because$ the two sets have no common values

$\therefore$ there is no real value

(18) **(a)** H

$H^1=\begin{bmatrix}\omega^1 & 0\\0 & \omega^1\end{bmatrix}$

$H^2=\begin{bmatrix}\omega & 0\\0 & \omega\end{bmatrix}\begin{bmatrix}\omega & 0\\0 & \omega\end{bmatrix}=\begin{bmatrix}\omega^2 & 0\\0 & \omega^2\end{bmatrix},$

...... and so on.

$\therefore H^{70}\begin{bmatrix}\omega^{70} & 0\\0 & \omega^7\end{bmatrix},$

where : $\omega^7=(\omega^3)^2\cdot\omega=(1)(\omega)=\omega,$

$\omega^{70}=(\omega^7)^{10}=\omega^{10}=(\omega^3)^3\cdot\omega=\omega$

$\therefore H^{70}\begin{bmatrix}\omega & 0\\0 & \omega\end{bmatrix}=H$

(19) **(c)** $\begin{bmatrix}13 & -2 & 5\\-1 & 2 & -1\end{bmatrix}$

Applying $R_1\leftrightarrow R_2$

$A=\begin{bmatrix}3 & -2 & 5\\1 & 2 & -1\end{bmatrix}$

Applying $C_1\rightarrow C_1+2C_3$,

$A=\begin{bmatrix}13 & -2 & 5\\-1 & 2 & -1\end{bmatrix}$

(20) **(d)** $\begin{bmatrix}1 & 3 & -2\\0 & 9 & -11\\0 & -1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\-2 & 0 & 1\end{bmatrix}A$

$R_2\rightarrow R_2+3R_1$ and $R_3\rightarrow R_3-2R_1$ gives

$\begin{bmatrix}1 & 3 & -2\\0 & 9 & -11\\0 & -1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\-2 & 0 & 1\end{bmatrix}A$

(21) **(a)** $\begin{bmatrix}\frac{2}{5} & -\frac{1}{5}\\\frac{1}{5} & \frac{2}{5}\end{bmatrix}$

$X\begin{bmatrix}2 & 1\\-1 & 2\end{bmatrix}=I$ (given)

$\Rightarrow X$ is the inverse of $\begin{bmatrix}2 & 1\\-1 & 2\end{bmatrix}$, etc.

(22) **(c)** all real k

Matrix A is invertible, if $|A|\neq 0$

i.e. $\begin{vmatrix}1 & 0 & -k\\2 & 1 & 3\\k & 0 & 1\end{vmatrix}\neq 0$

i.e. $1-k(-k)\neq 0$

i.e. $1+k^2\neq 0$

But for any real K, $1+k^2\neq 0$

(23) **(b)** $AB=BA$

$A^2-B^2=(A-B)(A+B)$ ...(given)

$\therefore A^2-B^2=A^2+AB-BA-B^2$

$\therefore AB-BA=0$

$\therefore AB-BA$

(24) **(d)** 1

$A^2=A\cdot A=\begin{bmatrix}\lambda & 1\\-1 & -\lambda\end{bmatrix}\begin{bmatrix}\lambda & 1\\-1 & -\lambda\end{bmatrix}$

$$\begin{bmatrix} \lambda^2 - 1 & 0 \\ 0 & -1+\lambda^2 \end{bmatrix} = 0 \text{ (As given)}$$

$$\Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$$

(25) **(c)** $AC = BC$

Do yourself

(26) **(a)** 0

Informative

(27) **(b)** 42, –3

$A_{21} = -M_{21} = -(-18 + 21) = 42$

$A_{33} = M_{33} = 5 - 8 = -3$

(28) **(b)** –16, 8, 4

$A_{21} = -M_{21} = -(6 + 10) = -16$

$A_{22} = M_{22} = 2 + 6 = 8$

$A_{23} = -M_{23} = -(5 - 9) = 4$

(29) **(d)** $\begin{bmatrix} -3 & -12 & 6 \\ -1 & 3 & 2 \\ -11 & -9 & 1 \end{bmatrix}$

$A_{11} = M_{11} = -3 - 0 = -3$

$A_{12} = -M_{12} = -(2 + 10) = -12$

$A_{13} = M_{13} = 0 + 6 = 6$

$A_{21} = M_{21} = -(1 - 0) = -1$

$A_{22} = -M_{22} = -1 + 4 = 3$

$A_{23} = M_{23} = -(0 - 2) = 2$

$A_{31} = M_{31} = -5 - 6 = -11$

$A_{32} = -M_{32} = -(5 + 4) = -9$

$A_{33} = M_{33} = (3 - 2) = 1$

(30) **(d)** $\begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$

If $A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$, then adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(31) **(b)** N

Matrix of cofactors of $N = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$$\text{adj } N = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} = N$$

(32) **(c)** $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

$|A| = 1(8 - 6) + 2(0 + 9) + 2(0 - 6)$

$= 2 + 18 - 12 = 8$

$A \text{ (adj } A) = |A| \text{ I} = 8I$

(33) **(d)** 17^4

$|A| = 2(6 - 1) + (3 + 1) + (1 + 2)$

$= 10 + 4 + 3 = 17$

$|\text{adj(adj } A)| = |A|^{(n-1)2} = |A|^{(3-1)2}$

$= |A|^4 = 17^4$

(34) **(c)** $C_{11} + C_{21} = C_{32}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\therefore C_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix} = 2$$

$$\therefore C_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 2 & 3 \\ 2 & 2 \end{vmatrix} = 2$$

$$\therefore C_{32} = (-1)^{3+1} \cdot \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = 4$$

$$\therefore C_{11} + C_{21} = C_{32}$$

(35) **(c)** $C_{ij} = a_{ij}$

With $A = \frac{1}{3}\begin{bmatrix} \ldots\ldots \\ \ldots\ldots \\ \ldots\ldots \end{bmatrix}$, find C_{12}, C_{13} ....and, then,

note that $C_{12} = a_{21}, C_{13} = a_{31}, \ldots\ldots$

(36) **(a)** 3

$M_{22} = 3$ (leaving R_2 and C_2)

(37) **(a)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x$$

Since, $A(\text{adj } A) = |A|.I$

$$\therefore A(\text{adj } A) = 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(38) **(b)** 1

$$a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$$

$$= \cos\theta(\cos\theta - 0) + \sin\theta[-\sin\theta - 0] + 0(0 - 0)$$

$$= \cos^2\theta + \sin^2\theta = 1$$

(39) **(b)** 216

(40) **(b)** (adj B) (adj A)

Basic property. Refer short cut.

(41) **(c)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Inverse of an identiiy matrix of any order is an identity matrix itself

(42) **(b)** A^2

$$A^3 = I \Rightarrow A^{-1} A^3 = A^{-1} I \Rightarrow A^2 = A^{-1}$$

(43) **(b)** $\dfrac{A - 4I}{5}$

$$A^2 - 4A - 5I = 0 \Rightarrow A^{-1}A^2 - 4AA^{-1} - 5I A^{-1} = 0 \Rightarrow$$

$$A - 4I - 5A^{-1} = 0 \Rightarrow A^{-1} = \frac{A - 4I}{5}$$

(44) **(a)** $\dfrac{-1}{6}$

$$A - 1 = \frac{1}{|A|} = (adj\, A)$$

$$|A| = 0 - 6 = -6 \Rightarrow \lambda = \frac{1}{|A|} = \frac{-1}{6}$$

(45) **(b)** -1

$|A| = 1(1 - 0) - 2(2 - 0) + 1(0 + 1)$

$= 1 - 4 + 1 = -2$

Element of second roe and third column in the inverse

$$= \frac{C_{32}}{|A|} = \frac{(-1)^{3+2}(0-2)}{-2} = \frac{(-1)(-2)}{-2} = -1$$

(46) **(d)** 5

$$A^{-1} = B \Rightarrow A^{-1} = \frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$$

α is the cofactor of the element a_{32} in A

$$\alpha = C_{32} = (-1)^{3+2}(-3-2) = (-1)(-5) = 5$$

(47) **(d)** $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \Rightarrow |A| = 8 - 7 = 1$

$$A^{-1} = \frac{1}{|A|} \quad \text{adj } A = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

(48) **(c)** A^{-1} does not exist

$|A| = 4 - 4 = 0$

$|B| = 1 - 1 = 0$

(49) **(d)** $\dfrac{1}{40}$

$|A| = 20 - 10 = 10$

$$(2A)^{-1} = \frac{1}{2}A^{-1}$$

$$|(2A)^{-1}| = \left|\frac{1}{2}A^{-1}\right| = \frac{1}{4}|A^{-1}| = \frac{1}{4}\cdot\frac{1}{|A|} = \frac{1}{40}$$

(50) **(c)** $\begin{bmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix}$

Let $A = \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix}$

$\Rightarrow |A| = \sec^2\theta - \tan^2\theta = 1 \Rightarrow$

$A^{-1} = \begin{bmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix}$

(51) **(a)** $6A^{-1} = A - 5I$

$|A| = 4 - 10 = -6$

$A^{-1} = \frac{1}{|A|}\text{ adj } A = \frac{-1}{6}\begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix} \Rightarrow$

$6A^{-1} = \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$

$A - 5I = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$

(52) (d) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

$|A| = 2 - 1 = 1$

$|B| = 4 - 3 = 1$

$AXB = C \Rightarrow X = A^{-1}CB^{-1} \Rightarrow$

$X = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 24 & 7 \\ 31 & 9 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 17 & 5 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 17-15 & -17+20 \\ 7-6 & -7+8 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

(53) (b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

$|A| = 3(1 - 0) + 2(-4 + 2) - 1(0 - 2)$

$= 3 - 4 + 2 = 1$

Matrix of cofactors of $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$

$\text{adj } A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

(54) (a) $\frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \Rightarrow$

$|A| = 0 - 1(1 - 9) + 2(1 - 6) = 8 - 10 = -2$

$A_{11} = M_{11} = 2 - 3 = -1$

$A_{12} = -M_{12} = -(1 - 9) = 8$

$A_{13} = M_{13} = 1 - 6 = -5$

$A_{21} = -M_{21} = -(1 - 2) = 1$

$A_{22} = M_{22} = 0 - 6 = -6$

$A_{23} = -M_{23} = -(0 - 3) = 3$

$A_{31} = M_{31} = 3 - 4 = -1$

$A_{32} = -\ M_{32} = -(0-2) = 2$

$A_{33} = M_{33} = 0-1 = -1$

$$A^{-1}\ \frac{1}{|A|} = \text{adj } A = \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

(55) (b) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\text{Let } A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

$|A| = \cos^2\theta + \sin^2\theta = 1$

$A_{11} = M_{11} = \cos\theta - 0 = \cos\theta$

$A_{12} = -\ M_{12} = -(\sin\theta - 0) - \sin\theta$

$A_{13} = M_{13} = 0$

$A_{21} = -\ M_{21} = -(-\sin\theta - 0) = \sin\theta$

$A_{22} = M_{22} = \cos\theta - 0 = \cos\theta$

$A_{23} = -\ M_{23} = 0$

$A_{31} = M_{31} = 0$

$A_{32} = -\ M_{32} = 0$

$A_{33} = M_{33} = \cos^2\theta + \sin^2\theta = 1$

$$A^{-1} = \frac{1}{|A|}\ \text{adj } A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(56) (a) $(AB)^{-1}$ exists

$$AB = \begin{bmatrix} 1+2+3 & -1+4-6 \\ 1-2-3 & -1-4+6 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ -4 & 1 \end{bmatrix} \Rightarrow$$

$|AB| = 6 - 12 = -6 \neq 0$

$(AB)^{-1}$ exists

$$BA = \begin{bmatrix} 1-1 & 2+2 & 3+3 \\ 1+2 & 2-4 & 3-6 \\ 1-2 & 2+4 & 3+6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 & 6 \\ 3 & -2 & -3 \\ -1 & 6 & 9 \end{bmatrix} \Rightarrow$$

$|BA| = 0 - 4(27-3) + 6(18-2) = -96 + 96 = 0$

$(BA)^{-1}$ does not exists

(57) (b) $1\ /\ \det(A)$

$\because$ A is non - singular $\quad \therefore |A| \neq 0$

$\because AA^{-1} = I \qquad |AA^{-1}| = |I|$

$\therefore |A|\,|A^{-1}| = 1 \qquad \therefore |A^{-1}| = 1\ /\ |A|$

(58) (b) $a = 1,\ c = -1$

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix} \quad A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & 2c \\ 5 & -3 & 1 \end{bmatrix}$$

$\because\ (R_3 \text{ of } A)(C_1 \text{ of } A^{-1}) = 0$

$\therefore\ (3)\ (1/2) + (a)\ (-8/2) + (1)(5/2) = 0$

$\therefore\ 3 - 8a + 5 = 0 \qquad \therefore\ 8a = 8 \qquad \therefore\ a = 1$

$\because\ (R_1 \text{ of } A)(C_3 \text{ of } A^{-1}) = 0$

$\therefore\ (0)\ (1) + (1)\ (2c) + (2)(1) = 0$

$\therefore\ 2c = -2 \qquad \therefore\ c = -1$

$\therefore\ a = 1,\ c = -1$

(59) (a) $\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$

$$\because\ A = \begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix} \qquad \therefore A = 1$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj. } A) = \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$$

(60) (a) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$

$$\because \quad |A| = \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} \qquad \therefore |A| = 3$$

$$\therefore \quad A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$\therefore \quad (A^{-1})^2 = \frac{1}{9}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix}$$

$$\therefore \quad (A^{-1})^3 = \frac{1}{9}(A^{-1})^3(A^{-1}) = \frac{1}{27}\begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$$

(61) (d) 11

Given $A^{-1} = \frac{1}{k}$ adj A

$$\therefore \quad k = |A|$$

$$\therefore \quad |A| = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 3(2+1) - 2(1-0) + 4(1-0)$$

$$= 9 - 2 + 4 = 11$$

$$\Rightarrow k = 11$$

(62) (d) $\frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad |A| = \begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{vmatrix}$$

$$= 1(1-2) - 3(2-10) + 4(2-5)$$

$$= -1 + 24 - 12$$

$$= 11 \neq 0$$

$$adj\ A = \begin{bmatrix} -1 & 8 & -3 \\ 1 & -19 & 14 \\ 2 & 6 & -5 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

(63) (d) $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

Multipicative inverse of matrix

$$(A^{-1}) = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} = \frac{1}{2 \times 4 - 7 \times 1}\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

(64) (a) $\frac{-1}{6}$

$$|A| = \begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix} = -6 \neq 0$$

$\therefore$ A^{-1} exists

Since $A^{-1} = \frac{1}{|A|}$ adj A

Given , $A^{-1} = \lambda(\text{adj } A)$

$\therefore$ $\lambda = \frac{1}{|A|} = -\frac{1}{6}$

(65) (d) Both (b) and (c)

$$|A| = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$= 4 - 4$

$= 0$

$\therefore$ A^{-1} and B^{-1} does not exist

(66) (b) -1

$|A^{-1}| = \frac{1}{|A|}$ etc .

(67) (a) $\begin{bmatrix} 2 & -2 \\ 0 & 3 \end{bmatrix}$

$(B^{-1} A^{-1})^{-1} = (A^{-1})^{-1} (B^{-1})^{-1}$

(Reverssal law)

$= AB$ etc.

(68) (d) $\begin{bmatrix} -1 & 2 \\ 5 & -9 \end{bmatrix}$

(69) (b) $\cos 2\theta$, $\sin 2\theta$

$$\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix} = \cos^2\theta \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} . \cos^2\theta \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$= \cos^2\theta \begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix}$$

$$= \cos^2\theta \begin{bmatrix} 1-\frac{1-\sin^2\theta}{\cos^2\theta} & \frac{-2\sin\theta}{\cos\theta} \\ 2\frac{\sin\theta}{\cos\theta} & 1-\frac{\sin^2\theta}{\cos^2\theta} \end{bmatrix}$$

$$= \cos^2\theta \begin{bmatrix} \frac{\cos 2\theta}{\cos^2\theta} & \frac{-2\sin\theta}{\cos\theta} \\ \frac{2\sin\theta}{\cos\theta} & \frac{\cos 2\theta}{\cos^2\theta} \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin\theta & \cos 2\theta \end{bmatrix}$$

$\therefore$ $a = \cos 2\theta$, $b = \sin 2\theta$

(70) (a) $x = 3$, $y = 2$, $z = 3$

satisfies the given set of equations

(71) (d) $x = 1, y = 2, z = 3$

satisfies the given set of equations

(72) (a) 1

$$\begin{bmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_3,$

$$\begin{bmatrix} a-1 & 0 & 0 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $(a - 1)\, x + 0 + 0 = 0$

$\therefore$ $a - 1 = 0 \Rightarrow a - 1$

(73) (b) $x = 2, y = 3, z = 5$

The given system of equations can be written in the matrix from as AX = B, where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} 1/x \\ 2/y \\ 3/z \end{bmatrix} \text{and } B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = 1200 \neq 0$$

$\therefore$ the given system of equations has a unique solution whicih is given by $X = A^{-1}\ B$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$X = A^{-1} B$

$$\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$\Rightarrow x = 2, y = 3, z = 5$

(74) (b) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1} B$

$$|A| = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix} = 1(-2)+1(-1)+2(4)=5\neq 0$$

$\therefore$ A^{-1} exist

$$\therefore \quad \text{adj } A = \begin{bmatrix} -2 & 1 & 4 \\ 5 & -5 & -5 \\ -1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{5}\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$$

$$\therefore \quad X = \frac{1}{5}\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -5 \\ 10 \\ 15 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

(75) (d) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

We have , $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$

$x + y + z = 0$

$x - 2y - 2z = 3$

$x + 3y + z = 4$

On solviing $x = 1$, $y = 2$, $z = -3$ i.e. $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

HOME WORK

Hints and Explanation

(1) **(b)** Every identity matrix is a scalar matrix

(2) **(a)** skew - symmetric

$a_{ij} = i^2 - j^2 \Rightarrow a_{ij} = j^2 - i^2 = -(i^2 - j^2) = -a_{ij}$

(3) **(a)** Every skew- symmetric matrix of odd order is non - singular

(4) **(a)** none-singular matrix

If AB = AC implies B=C, then A^{-1} exists Hence A is a non - singular matrix

(5) **(c)** symmetric matrix

$(A\ A^T)^T = A^T + (A^T)^T = A^T + A$

(6) **(d)** symmetric matrix

$(A\ A^T)^T = A^T(A^T)^T = A^T A$

(7) **(c)** A^{-1} exists

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = 3 - 6 = -3 \neq 0$$

(8) **(a)** A is invertible

$\sec^2\theta - \tan^2\theta = 1 \neq 0$

(9) **(d)** A is invertible

$$|A| = \begin{vmatrix} 3 & 4 & 3 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{vmatrix} = 3(5-0) - 4(5-0) + 3(4-1)$$

$= 15 - 20 + 9 = 4 \neq 0$

A^{-1} exists

(10) **(c)** must be a square matrix

(11) **(c)** det. A = 1

Since A is an orthogonal matrix

$\therefore \quad A.A' = I \Rightarrow |AA'| = 1 \Rightarrow |A|.|A'| = 1$

$\Rightarrow |A|.|A| = 1 \left(\because |A'| = |A|\right)$

$\Rightarrow |A|^2 = 1 \Rightarrow |A| = 1$

(12) **(a)** Null

Let $A = [a_{ij}]$

Since A is skew - symmetric

$a_{ij} = 0$ and $a_{ij} = -a_{ij}$ $(\because i \neq j)$

A is symmetric as well, so that $a_{ij} = a_{ij}$ for all i and j

$\Rightarrow a_{ij} = 0$ for all $i \neq j$

Hence $a_{ij} = 0$ for all i and j

Therefore A is a null matrix.

(13) **(a)** 2

The matrix $A = \begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{vmatrix}$ is singular

$\Rightarrow |A| = 0$

$\Rightarrow 0 - 1(-3\lambda) + (-2)(3) = 0$

$\Rightarrow 3\lambda - 6 = 0 \Rightarrow \lambda = 2$

(14) **(d)** Skew-symmetric

(15) **(b)** Sum of symmetric and a skew- symmetric matrix

A square matrix is always expressible as the sum of a symmetric and skew-symmetric matrix

(16) **(b)** $p = r, q = s$

Two matrices can be suntracted only if they are same order

(17) **(b)** A and B are square matrices of same order

A+B exists if A, B are of same order

AB exists if number of columns of A = number of rows of B

Hence A and B are square matrices of same order

(18) **(a)** $A^2 - AB - BA + B^2$

$(A - B)^2 = (A - B)(A - B)\ A^2 - AB - BA + B^2$

(19) **(a)** 32 A

$A = 2I \Rightarrow A^2 = (2I)(2I) = 4I \Rightarrow$

$A^4 = A^2 A^2 = (4I)(4I) = 16I \Rightarrow$

$A^6 = A^2 A^4 = (4I)(16I) = 64I = 32A$

(20) **(a)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \sin\theta & \sin\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(21) **(c)** (3,4)

$$[m \quad n] \begin{bmatrix} m \\ n \end{bmatrix} = [25] \Rightarrow \left[m^2 + n^2\right] = [25]$$

$\Rightarrow m^2 + n^2 = 25 \Rightarrow$

For $m < n$ $(m\ n) = (3,4)$

(22) **(b)** $A' = -A$

$$A' = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -5 \\ -2 & 5 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 5 \\ 2 & -5 & 0 \end{bmatrix} = -A$$

(23) **(b)** A is not invertible

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ 1 & 2 & 8 \end{vmatrix} = 0 \quad \text{..... } [R_1 \text{ identical } R_3]$$

A is not invertible

(24) **(c)** $\lambda \neq -17$

$\lambda(0-1) + 1(-6+1) + 4(-3-0) \neq 0$

$\Rightarrow -\lambda - 5 - 12 \neq 0 \Rightarrow \lambda \neq -17$

(25) **(a)** $k = 1$

$|A| \neq 0 \Rightarrow 1(1-0) - k(0-k) \neq 0$

$\Rightarrow 1 + k^2 \neq 0$

(26) **(a)** $\frac{1}{3}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$AX = B \Rightarrow A^{-1}AX = A^{-1}B \Rightarrow X = A^{-1}B$

$$= 1 - 4 = -3 \Rightarrow A^{-1} = \frac{-1}{3}\begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} \Rightarrow$$

$$X = \frac{-1}{3} = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \frac{-1}{3}\begin{bmatrix} -3 & -2 \\ -6 & -1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

(27) **(c)** $\begin{bmatrix} 3 & -16 \\ 6 & -30 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 6 & 0 \end{bmatrix}$$

$C_2 \to C_2 - 5C_1$

$$A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -16 \\ 6 & -30 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & -16 \\ 6 & -30 \end{bmatrix}$$

(28) **(d)** ± 3

$\because \ A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ $\qquad \therefore \ |A| = \alpha^2 - 4$

$\because \ |A^3| = 125$ $\qquad \therefore \ |A|^3 = 125$

$\therefore \ (\alpha^2 - 4)^3 = 125$ $\qquad \therefore \ \alpha^2 - 4 = 5$

$\therefore \ \alpha^2 = 9$ $\qquad \therefore \ \alpha = \pm 3$

(29) **(d)** there exist infinitely many B's such that $AB = BA$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

$$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$$

$\therefore$ $AB = BA$ if $a = b$, where $a, b \in N$.

$\because$ there are infinitely many natural numbers

$\because$ required values of a, b are infinitely many

$\because$ there are infinitely many matrices B

(30) **(b)** $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{-1}{3} \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$

Applyiing $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 5 & -2 \end{bmatrix}$$

Applyiing $R_3 \to R_3 - \left(\frac{5}{3}\right) R_2$,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{-1}{3} \end{bmatrix}$$

(31) **(c)** $\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

$R_2 \to R_2 - 2R_1$ gives $\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

(32) **(d)** $\begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$R_1 \to R_1 + R_2 - R_3$ gives

$$\begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

(33) **(c)** 1

The matrix is not invertible if $\begin{vmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow 1(2-5) - a(1-10) + 2(1-4) = 0$

$\Rightarrow -3 + 9a - 6 = 0$

$\Rightarrow a = 1$

(34) **(c)** $\begin{bmatrix} 3 & -16 \\ 6 & -30 \end{bmatrix}$

$$A\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 6 & 0 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 3 & -1 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 3 & -1 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 1 & -5 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -16 \\ 6 & -30 \end{bmatrix}$$

(35) **(d)** $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$

Since $\begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} = 4 - 4 = 0$

$\therefore$ $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ does not have inverse, hence not invertible

(36) **(d)** A + B

$AB = B \Rightarrow ABA = BA$

$\Rightarrow AA = A \ (\because BA = A)$

$\Rightarrow A^2 = A$. Similarly $B^2 = B$

$\therefore \quad A^2 + B^2 = A + B$

(37) **(d)** $\cos^2\theta \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

$AB = I$

$\therefore$ $B = A^{-1}$ etc

(38) **(a)** $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Basic concept

(39) **(b)** 0 or 1

$A^2 = A$

$\therefore \quad |A^2| = |A|$

$\therefore \quad |AA| = |A|$

$\therefore \quad |A|.|A| = |A|$

$\therefore \quad |A|^2 - |A| = 0$

$\therefore \quad |A| \left[|A| - 1\right] = 0$

$\therefore \quad |A| = 0$ or $|A| = 1$

(40) **(b)** 1

$A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}, \qquad \therefore \quad |A| = 21 - 20 = 1$

Now $|A^5| = |A|.|A|.|A|.|A|.|A|$

$= |A|^5 = 1$

(41) **(d)** None fo these

We have,

$X^2 = X.X = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}$

for n = 2, matrices in (a), (b) and (c) do not match with $\begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}$

(42) **(c)** 1, – 1

We have , $(A + B)^2 = A^2 + B^2 + AB + BA$

But, $(A + B)^2 = A^2 + B^2 \Rightarrow AB + BA = 0$

So, $AB = -BA$

$BA = \begin{bmatrix} 1 & x \\ 4 & y \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1+2x & -1-x \\ 4+2y & -4-y \end{bmatrix}$

and $AB = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & x \\ 4 & y \end{bmatrix} = \begin{bmatrix} 1-4 & x-y \\ 2-4 & 2x-y \end{bmatrix}$

$\therefore \quad \begin{bmatrix} 1-4 & x-y \\ 2-4 & 2x-y \end{bmatrix} = -\begin{bmatrix} 1+2x & -1-x \\ 4+2y & -4-y \end{bmatrix}$

$\therefore \quad -3 = -1 - 2x \Rightarrow 2x = 2 \Rightarrow x = 1$ and

$2 - 4 = -4 - 2y \Rightarrow 2y = -2 \Rightarrow y = -1$

(43) **(d)** $\begin{bmatrix} 1/4 & -2 \\ -1 & 1/4 \end{bmatrix}$

we have,

$3A - 2B + 4X = 5C \qquad \therefore 4X = 5C + 2B - 3A$

$= 5\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} + 2\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} - 3\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$

$= \begin{bmatrix} 5 & -5 \\ 0 & 10 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 8 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 12 & 21 \end{bmatrix}$

$\therefore \quad X = \begin{bmatrix} 1/4 & -2 \\ -1 & 1/4 \end{bmatrix}$

(44) **(c)** 1

$AB = \begin{bmatrix} 1/3 & 2 \\ 0 & 2x-3 \end{bmatrix}\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 3-2x \end{bmatrix}$

$= I = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (As given)

$\Leftrightarrow 3 - 2x = 1$ or $x = 1$

(45) **(a)** $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

Since $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = AB$

$\Rightarrow B = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

(46) **(c)** a Unit matrix

$A^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

(47) **(b)** $[ax^2 + by^2 + cz^2 + 2hxy + 2gxz + 2fyz]$

(48) **(b)** $[0]$

(49) **(d)** $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$

(50) **(d)** B

(51) **(c)** $|A|$

Informative

(52) **(c)** $(adj\ B)\ (adj\ A)$

By property

(53) **(a)** $-39,\ 3,\ 11$

$A_{21} = -M_{21} = -(18 + 21) = -39$

$A_{22} = M_{22} = 9 - 6 = 3$

$A_{23} = -M_{23} = -(-7 - 4) = 11$

(54) **(b)** $-6,\ 2,\ 4$

$A_{31} = M_{31} = -4 - 2 = -6$

$A_{32} = -M_{32} = -(4 - 6) = 2$

$A_{23} = M_{23} = 1 + 3 = 4$

(55) **(a)** $A_{11} + A_{21} = A_{32}$

$A_{11} = M_{11} = 6 - 4 = 2$

$A_{21} = -M_{21} = -(4 - 6) = 2$

$A_{32} = -M_{32} = -(2 - 6) = 4$

Hence $A_{11} + A_{21} = A_{32}$

(56) **(c)** $\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

$A_{11} = M_{11} = 4$

$A_{12} = -M_{12} = -3$

$A_{21} = -M_{21} = -2$

$A_{22} = M_{22} = 1$

(57) **(d)** $\begin{bmatrix} -14 & -10 & -6 \\ 6 & -5 & -3 \\ -2 & -7 & 1 \end{bmatrix}$

$A_{11} = M_{11} = -5-9 = -14$

$A_{12} = -M_{12} = -(10-0) = -10$

$A_{13} = M_{13} = -6-0 = -6$

$A_{21} = -M_{21} = -(0-6) = 6$

$A_{22} = M_{22} = -5-0 = -5$

$A_{23} = -M_{23} = -(3-0) = -3$

$A_{31} = M_{31} = 0-2 = -2$

$A_{32} = -M_{32} = -(3+4) = -7$

$A_{33} = M_{33} = 1-0 = 1$

(58) **(b)** $\begin{bmatrix} 7 & -6 \\ 5 & -2 \end{bmatrix}$

(59) **(c)** 10

$\text{adj A} = \begin{bmatrix} 4 & -2 \\ -3 & 4 \end{bmatrix} \Rightarrow |adj\ A| = 16-6 = 10$

(60) **(b)** adj (A′)

$A_{11} = M_{11} = 0+12 = 12$

$A_{12} = -M_{12} = -(0-4) = 4$

$A_{13} = M_{13} = -6-0 = -6$

$A_{21} = -M_{21} = -(0-3) = 3$

$A_{22} = M_{22} = 0+1 = 1$

$A_{23} = -M_{23} = -(-9-5) = 14$

$A_{31} = M_{31} = 20-0 = 20$

$A_{32} = -M_{32} = -(12+2) = -14$

$A_{33} = M_{33} = 0-10 = -10$

$\text{adj A} = \begin{bmatrix} 12 & 3 & 20 \\ 4 & 1 & -14 \\ -6 & 14 & -10 \end{bmatrix}$

By property *adj* (A′) = (adj A)′

(61) **(d)** $\begin{bmatrix} 0 & -1 & 1 \\ -8 & 3 & -7 \\ 4 & -2 & 2 \end{bmatrix}$

$A_{11} = M_{11} = 2-2 = 0$

$A_{12} = -M_{12} = -(6+2) = -8$

$A_{13} = M_{13} = 3+1 = 4$

$A_{21} = -M_{21} = -(0+1) = -1$

$A_{22} = M_{22} = 4-1 = 3$

$A_{23} = -M_{23} = -(2-0) = -2$

$A_{31} = M_{31} = 0+1 = 1$

$A_{32} = -M_{32} = (4+3) = -7$

$A_{33} = M_{33} = 2-0 = 2$

adj A = Transpose of Matrix of co - factors

$= \begin{bmatrix} 0 & -1 & 1 \\ -8 & 3 & -7 \\ 4 & -2 & 2 \end{bmatrix}$

(62) **(a)** (4, 1)

x = cofactor of – 1 in A = – (0 – 4) = 4

y = cofactor of 1 in third row and third colum of A = 1 – 0 = 1

(63) **(d)** 10

$A\ (adj\ A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}\ 10\ I$

But $A = (\text{adj A}) = |A|\ I \Rightarrow |A| = 10$

(64) **(b)** $|A|\ I$

By rule

(65) **(a)** $\begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

$$|A| = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = 0 + (9 + 2) + 0 = 11$$

$$(\text{adj } A)\ A = |A|\, I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

(66) **(b)** 3

$A\ (\text{adj } A) = |A|\ I = 5\ I$

$|A(\text{adj } A)| = 125 \Rightarrow |5\ I| = 125 \Rightarrow 5^n$

$= 125 \Rightarrow n = 3$

(67) **(c)** (adj. B) (adj. A)

(68) **(c)** null matrix

(69) **(d)** – 3

$|A| = 0$

$\therefore\ a_{31} A_{31} + a_{32} A_{32} + a_{33} A_{33} = 0$

$\therefore\ (1)\ (3) + (2)\ A_{32} + (3)\ (1) = 0$

$\therefore\ 2A_{32} = -6 \qquad \therefore\ A_{32} = -3$

(70) **(a)** A is singular

$0\ (A) = 3 \times 3$ and adj. $A = 0$

$\therefore\ |\text{adj. } A| = 0 \qquad \therefore\ |A|^{3-1} = 0 \quad \therefore\ |A| = 0$

$\therefore$ matrix A is singular

(71) **(c)** – 4, 5, –1

Co - factor of 1 = $A_{31} = (-1)^{3+1} \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}$

$= -3 - 1 = -4$

Co - factor of 2 = $A_{32} = (-1)^{3+2} \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$

$= -(-3-2)$

$= -(-5) = 5$

Co - factor of 3 = $A_{33} = (-1)^{3+3} \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$= (1 - 2) = -1$

$\therefore$ Co - factor are – 4, 5, –1

(72) **(c)** – 3

Co - factor $A_{21} = (-1)^3 (3) = -(3) = -3$

(73) **(b)** 2

$$M_{11} = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = 6 - 4 = 2$$

(74) **(a)** $\frac{2}{5}$

$|A| = -20$

$$\therefore\ a_{23} = \frac{\text{Co} - \text{factor of } a_{32}}{-20} = \frac{-8}{-20} = \frac{2}{5}$$

(75) **(c)** 10

$A\ (\text{Adj } A) = |A|\ .\ (I_n)$

$$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = |A| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} |A| & 0 \\ 0 & |A| \end{bmatrix}$$

$\Rightarrow |A| = 10$

(76) **(c)** 1

We know that, $A\ (\text{adj } A) = |A|\ .\ I$

$$\Rightarrow \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = (\cos^2 \alpha + \sin^2 \alpha) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow k = 1$

(77) **(a)** O

adj AB – (adj B) (adj A)

= (adj B) (adj A) – (adj B) (adj A)

.... [adj AB = (adj B) (adj A)]

= O

(78) **(b)** A

(79) **(c)** $\begin{bmatrix} 6 & 1 & -5 \\ -2 & -5 & 3 \\ -3 & 3 & -1 \end{bmatrix}$

Use short cut given in theory part.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The cofactor of a_{12} must be in the place of a_{21} and a vice versa.

Now cofactor of a_{12} i.e. cofactor of 2

= – [8 – 6] = – 2

Cofactor of a_{21} = – [8 – 9] = 1

Hence option (c) is correct.

(80) **(d)** k^{n-1} (adj A)

(81) **(a)** $\dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$|A| = ad - bc \Rightarrow A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(82) **(c)** I_3

Inverse of an identity matrix is an identity matrix itself

(83) **(a)** A^4

$A^5 = I \Rightarrow A^5 A^{-1} = I A^{-1} \Rightarrow A^4 = A^{-1}$

(84) **(d)** $\dfrac{1}{3}B$

$AB = 3I \Rightarrow A^{-1} AB = 3A^{-1} I \Rightarrow IB = 3A^{-1} \Rightarrow$

$B = 3A^{-1} \Rightarrow A^{-1} = \dfrac{1}{3}B$

(85) **(c)** I – A

$A^2 - A + I = O \Rightarrow A^{-1}(A^2 - A + I) = O \Rightarrow$

$A^{-1}A^2 - A^{-1}A + A^{-1}I = O \Rightarrow A - I + A^{-1} = O$
$\Rightarrow A^{-1} = I - A$

(86) **(d)** $A^2 + B^2$

$B = -A^{-1}BA \Rightarrow AB = -AA^{-1}BA = -IBA$
$= -BA \Rightarrow (A+B)^2 = A^2 + B^2$

(87) **(a)** 4

$$|A| = 7x + 6 \Rightarrow A^{-1} = \frac{1}{7x+6}\begin{bmatrix} 7 & 2 \\ -3 & x \end{bmatrix}$$

$$\Rightarrow \frac{7}{7x+6} = \frac{7}{34} \Rightarrow 7x + 6 = 34 \Rightarrow 7x = 28$$

$\Rightarrow x = 4$

(88) **(b)** 1

$|A| = \cos^2\theta + \sin^2\theta = 1$

We have $|A^{-1}| = \dfrac{1}{|A|} = 1$

(89) **(b)** $\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

Here adj A = $\begin{bmatrix} 1 & -4 \\ -1 & 2 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

(90) **(d)** none of these

Let A = $\Rightarrow |A| = 4 - 4 = 0$

A^{-1} does not exists

(91) **(b)** $\dfrac{-1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Let A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = |A| \Rightarrow 4 - 6 = -2$

$$A^{-1} = \frac{1}{|A|} = \text{adj } A = \frac{-1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

(92) (a) $\frac{1}{3}\begin{bmatrix}1 & 1\\ 1 & -2\end{bmatrix}$

Let $A = \begin{bmatrix}2 & 1\\ 1 & -1\end{bmatrix} \Rightarrow |A| = -2-1 = -3$

$A^{-1} = \frac{1}{|A|}\text{ adj } A = \frac{-1}{3}\begin{bmatrix}-1 & -1\\ -1 & 2\end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix}1 & 1\\ 1 & -2\end{bmatrix}$

(93) (d) $\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$

Let $A = \begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix} \Rightarrow |A| = 4-3 = 1$

$A^{-1} = \frac{1}{|A|}\text{ adj } A = \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$

(94) (c) $\frac{-1}{2}\begin{bmatrix}2 & -4\\ -3 & 5\end{bmatrix}$

$|A| = 10-12 = -2 \Rightarrow A^{-1} = \frac{-1}{2}\begin{bmatrix}2 & -4\\ -3 & 5\end{bmatrix}$

(95) (b) $\frac{-1}{8}\begin{bmatrix}2 & 3\\ 4 & 2\end{bmatrix}$

$|A| = 4-12 = -8 \Rightarrow A^{-1} = \frac{-1}{8}\begin{bmatrix}2 & 3\\ 4 & 2\end{bmatrix}$

(96) (c) $\begin{bmatrix}4 & -1\\ -7 & 2\end{bmatrix}$

The multiplicative inverse of A is A^{-1}

$|A| = 8-7 = 1 \Rightarrow A^{-1} = \begin{bmatrix}4 & -1\\ -7 & 2\end{bmatrix}$

(97) (d) $\begin{bmatrix}-i & 0\\ 0 & -2i\end{bmatrix}$

$|A| = \frac{i^2}{2} - 0 = \frac{-1}{2} \Rightarrow A^{-1} = \frac{1}{\frac{-1}{2}}\begin{bmatrix}i/2 & 0\\ 0 & i\end{bmatrix}$

$= \begin{bmatrix}-i & 0\\ 0 & -2i\end{bmatrix}$

(98) (c) $x = \frac{-1}{11}, y = \frac{2}{11}$

$|A| = 1+10 = 11 \Rightarrow A^{-1} = \frac{1}{11}\begin{bmatrix}1 & -2\\ 5 & 1\end{bmatrix}$

$A^{-1} = xA + yI \Rightarrow = \frac{1}{11}\begin{bmatrix}1 & -2\\ 5 & 1\end{bmatrix} =$

$\begin{bmatrix}x & 2x\\ -5x & x\end{bmatrix} + \begin{bmatrix}y & 0\\ 0 & y\end{bmatrix} \Rightarrow x + y = \frac{1}{11}$ and

$2x = \frac{-2}{11} \Rightarrow x = \frac{-1}{11},\ y = \frac{2}{11}$

(99) (c) $\begin{bmatrix}-5 & 0\\ 0 & -5\end{bmatrix}$

$|A| = 3-0 = 3 \Rightarrow A^{-1} = \begin{bmatrix}-7 & 3\\ -5 & 2\end{bmatrix} \Rightarrow$

$A + A^{-1} = \begin{bmatrix}2 & -3\\ 5 & -7\end{bmatrix} + \begin{bmatrix}-7 & 3\\ -5 & 2\end{bmatrix}$

$= \begin{bmatrix}-5 & 0\\ 0 & -5\end{bmatrix}$

(100) (c) $\begin{bmatrix} -1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow$

Here A is diagonal matrix , then

$$A^{-1} = \begin{bmatrix} -1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(101) (b) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1/3 & 0 \\ 3 & 2/3 & -1 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix} \Rightarrow$

$|A| = -3$

Now $A A^{-1} = I$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 3 & 2 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 3R_1$ and $R_3 - 5R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$$

$R_2 \rightarrow \frac{R_2}{3}$ and $R_3 \rightarrow -R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1/3 & 0 \\ 5 & 0 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1/3 & 0 \\ 3 & 2/3 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1/3 & 0 \\ 3 & 2/3 & -1 \end{bmatrix}$$

(102) (b) $\frac{-1}{6}\begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} \Rightarrow$

$|A| = (2-6) + (0-2) = -4 - 2 = -6$

$A_{11} = M_{11} = 2 - 6 = -4$

$A_{12} = -M_{12} = -(0-3) = 3$

$A_{13} = M_{13} = 0 - 2 = -2$

$A_{21} = -M_{21} = -(0-2) = 2$

$A_{22} = M_{22} = 1 - 1 = 0$

$A_{23} = -M_{23} = -(2-0) = -2$

$A_{31} = M_{31} = 0 - 2 = -2$

$A_{32} = -M_{32} = -(3-0) = -3$

$A_{33} = M_{33} = 2 - 0 = 2$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{-1}{6}\begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$$

(103) (d) $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \Rightarrow$

$|A| = 2(3-0) - (5-0) = 6 - 5 = 1$

$A_{11} = M_{11} = 3 - 0 = 3$

$A_{12} = -M_{12} = -(15 - 0) = -15$

$A_{13} = M_{13} = 5 - 0 = 5$

$A_{21} = -M_{21} = -(0+1) = -1$

$A_{22} = M_{22} = 6 - 0 = 6$

$A_{23} = -M_{23} = -(2-0) = -2$

$A_{31} = M_{31} = 0 + 1 = 1$

$A_{32} = -M_{32} = -(0+5) = -5$

$A_{33} = M_{33} = 2 - 0 = 2$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

(104) (a) $\frac{1}{25}\begin{bmatrix} 25 & 10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} \Rightarrow$

$|A| = (0+25) - 3(0+10) - 2(-15-0)$

$= 25 - 30 + 30 = 25$

$A_{11} = M_{11} = 0 + 25 = 25$

$A_{12} = -M_{12} = -(0 + 10) = -10$

$A_{13} = M_{13} = -15 - 0 = -15$

$A_{21} = -M_{21} = -(0 - 10) = 10$

$A_{22} = M_{22} = 0 + 4 = 4$

$A_{23} = -M_{23} = -(5-6) = 1$

$A_{31} = M_{31} = -15 - 0 = -15$

$A_{32} = -M_{32} = -(-5-6) = 11$

$A_{33} = M_{33} = 0 + 9 = 9$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{25}\begin{bmatrix} 25 & 10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$$

(105) (d) $\frac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} \Rightarrow$

$|A| = (4-4) - 2(-4-2) + 3(-2-1)$

$= 12 - 9 = 3$

$A_{11} = M_{11} = 4 - 4 = 0$

$A_{12} = -M_{12} = -(-4-2) = 6$

$A_{13} = M_{13} = -2 - 1 = -3$

$A_{21} = -M_{21} = -(8-6) = -2$

$A_{22} = M_{22} = 4 - 3 = 1$

$A_{23} = -M_{23} = -(2-2) = 0$

$A_{31} = M_{31} = 4 - 3 = 1$

$A_{32} = -M_{32} = -(2+3) = -5$

$A_{33} = M_{33} = 1 + 2 = 3$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$$

(106) **(d)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{bmatrix}$

$|A| = -\cos^2\theta - \sin^2\theta = -1 \Rightarrow$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{bmatrix}$$

(107) **(b)** $\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

$(A^2 - 4A)A^{-1} = AAA^{-1} - 4AA^{-1} = AI - 4I$
$= A - 4I$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

(108) **(c)** $\begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$

$(B^{-1}A^{-1})^{-1} = (A^{-1})^{-1}(B^{-1})^{-1} = AB$

$$= \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$$

(109) **(d)** $\begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$

$$AB = \begin{bmatrix} 2+9 & 0+3 \\ 1+6 & 0+2 \end{bmatrix} = \begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix} \Rightarrow$$

$|AB| = 22 - 21 = 2$

$$(AB)^{-1} \frac{1}{|AB|} \text{ adj } (AB) = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$$

(110) **(c)** $-\frac{1}{n}(A + mI)$

$A^2 + mA + nI = O \quad \therefore A^2 + mA = -nI$

$\therefore \quad A(A + mI) = -nI$

$\therefore \quad A[-\frac{1}{n}(A + mI)] = I$

$\therefore \quad A^{-1} = -\frac{1}{n}(A + mI)$

(111) **(b)** A

$\because \quad A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} \qquad \therefore \quad |A| = -19$

$\therefore \quad A^{-1} = \frac{1}{-19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

$\therefore \quad 19A^{-1} = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = A$

(112) **(d)** $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

If $|A| = 0$ then A does not have inverse.

(113) **(c)** $\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & x^2 & x \\ 1 & x & x^2 \end{bmatrix}$

Reverse solution : Check if $A^{-1}A$ is I.

(114) **(b)** $\begin{bmatrix} 1-i & i \\ -i & 1+i \end{bmatrix}$

$\because \quad A^2 - 2A + I = O$

$\therefore \quad 2A - A^2 = I$

$\therefore \quad A(2I - A) = I = AA^{-1}$

$$\therefore \quad A^{-1} = 2I - A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1+i & -i \\ i & 1-i \end{bmatrix}$$

$$= \begin{bmatrix} 1-i & i \\ -i & 1+i \end{bmatrix}$$

(115) (a) $(AB)^{-1} = B^{-1}A^{-1}$

(116) (d) $\begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$

If $AB = C$, then $B^{-1}A^{-1} = C^{-1}$

$\therefore \quad A^{-1} = BC^{-1}$

Here, $A\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$$\therefore \quad A^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$$

(117) (b) $-(3A^2 + 2A + 5I)$

$3A^3 + 2A^2 + 5A + I = 0$

$\Rightarrow I = -3A^3 - 2A^2 - 5A$

$\Rightarrow IA^{-1} = -3A^2 - 2A - 5I$

$\Rightarrow A^{-1} = -(3A^2 + 2A + 5I)$

(118) (d) $\sec^2\alpha \begin{bmatrix} 1 & \sin\alpha \\ -\sin\alpha & -1 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{-1+\sin^2\alpha}\begin{bmatrix} -1 & -\sin\alpha \\ \sin\alpha & 1 \end{bmatrix}$$

$$= \frac{1}{\cos^2\alpha}\begin{bmatrix} 1 & \sin\alpha \\ -\sin\alpha & -1 \end{bmatrix}$$

(119) (b) 5

$$\text{Given, } = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} 10A^{-1}$$

$$\Rightarrow \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$\Rightarrow -5 + \alpha = 0 \Rightarrow \alpha = 5$

(Equating the element of 2nd row and first column.)

(120) (c) $\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix}$

As $\begin{bmatrix} 1 & 3 \\ 3 & 10 \end{bmatrix}\begin{bmatrix} 10 & -3 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(121) (d) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac-b & -c & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$

$\therefore \quad |A| = 1$

$$\text{Adj } A = \begin{bmatrix} 1 & -a & ac-b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac-b & -c & 1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac-b & -c & 1 \end{bmatrix}$$

(122) (a) 4

$$A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 7 & 2 \\ -3 & x \end{bmatrix}$$

$$|A| = 7x + 6$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} \frac{7}{(7x+6)} & \frac{2}{(7x+6)} \\ \frac{-3}{7x+6} & \frac{x}{7x+6} \end{bmatrix} \quad \text{...(i)}$$

$$A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix} \quad \text{........(ii) [Given]}$$

From (i) and (ii), we get

$$\begin{bmatrix} \frac{7}{7x+6} & \frac{2}{7x+6} \\ \frac{-3}{7x+6} & \frac{x}{7x+6} \end{bmatrix} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$$

$$\Rightarrow \frac{7}{7x+6} = \frac{7}{34} \Rightarrow 7x + 6 = 34\ x = 4$$

(123) (a) I

$$\begin{aligned} BB' &= (A^{-1}A')(A^{-1}A')' \\ &= (A^{-1}A')(A(A^{-1}A')') \\ &= A^{-1}AA'(A^{-1})' \\ & \qquad \text{.....}[\because AA' = A'A \text{ (given)}] \\ &= (A^{-1}A)(A'(A^{-1})') \\ &= I\,(A^{-1}A)' = I.I = I^2 = I \end{aligned}$$

(124) (c) $\begin{bmatrix} 3 & 1 \\ -5 & 0 \end{bmatrix}$

(125) (b) $\frac{1}{10}\begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \quad \therefore \quad A' = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$$

use short cut to find $(A')^{-1}$

(126) (a) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

Use short cut

(127) (d) $\begin{bmatrix} \frac{1}{a} & 0 \\ 0 & b \end{bmatrix}$

$$|A| = \frac{a}{b}$$

$$\therefore \quad A^{-1} = \frac{b}{a}\begin{bmatrix} 1/b & 0 \\ 0 & a \end{bmatrix} = \begin{bmatrix} 1/a & 0 \\ 0 & b \end{bmatrix}$$

(128) (c) $\begin{bmatrix} -5 & 20 & -2 \\ -1 & 3 & 0 \\ 3 & -11 & 1 \end{bmatrix}$

Try given option

(129) (d) $\frac{1}{9}|A^{-1}|$

As above

(130) (c) $\frac{1}{10}\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$

$$A + B = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & -4 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$$

Now use short cut to write inverse

(131) (c) $\begin{bmatrix} \frac{8}{3} \\ \frac{-1}{3} \\ 0 \end{bmatrix}$

$X \;\; = A^{-1} D$

$\therefore \; AX = AA^{-1}D$

$\therefore \; AX = D \qquad (\because AA^{-1} = I)$

Now try the given option

$$\begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 1 \\ 4 & -1 & -2 \end{bmatrix} \begin{bmatrix} \frac{8}{3} \\ -\frac{1}{3} \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} = D$$

Hence option (c) is correct

(132) (d) $|\text{adj } A| = |A|^2$

Basic property

(133) (c) $\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$

Since $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \cdot \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega^1 \\ 1 & \omega & \omega^2 \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \; A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega^1 \\ 1 & \omega & \omega^2 \end{bmatrix}$

(134) (b) $F(-\alpha)$

We have $F(\alpha) \; F(-\alpha)$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\therefore \; F(-\alpha) = [F(\alpha)]^{-1}$

(135) (a) $k = 5$

Given $10\,A^{-1} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & k \\ 1 & -2 & 3 \end{bmatrix}$

$\Rightarrow 10A^{-1}.\; A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & k \\ 1 & -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$\Rightarrow -5 + k = 0 \;\Rightarrow\; k = 5$

($\because$ comparing a_{21} element on boths side)

(136) (d) None of these

Given that $A = \begin{bmatrix} 1 & 11 & -3 \\ -9 & 13 & 27 \\ 17 & -7 & -51 \end{bmatrix}$

$= (-663 + 189) - 11\,(459 - 459) - 3\,(63 - 221)$

$= -474 - 0 + 474 = 0$

A is a singular matrix.

Therefore A^{-1} is not defined

(137) (a) $(G(\beta))^{-1}(F(\alpha))^{-1}$

By property

$(F(\alpha)^{-1}\, G(\beta)^{-1} = (G(\beta))^{-1}(F(\alpha))^{-1}$

(138) (b) $(AB)^{-1}$ exists

$$AB = \begin{bmatrix} -2 & 0 & 0 \\ -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -8 \end{bmatrix} \Rightarrow$$

$|A| = 0 + 2 = 2 \neq 0$ $(AB)^{-1}$ exists

(139) (d) $\begin{bmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$

$$\text{Let } A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix} \Rightarrow$$

$|A| = (0 - 3) - 2(0 + 1) - 2(0 - 2)$

$= -3 - 2 + 4 = -1$

$A_{11} = M_{11} = 0 - 3 = -3$

$A_{12} = -M_{12} = -(0 + 1) = -1$

$A_{13} = M_{13} = 0 - 2 = -2$

$A_{21} = -M_{21} = -(0 + 6) = -6$

$A_{22} = M_{22} = 0 - 2 = -2$

$A_{23} = -M_{23} = -(3 + 2) = -5$

$A_{31} = M_{31} = 2 - 4 = -2$

$A_{32} = -M_{32} = -(1 - 0) = -1$

$A_{33} = M_{33} = -2 - 0 = -2$

(140) (d) $\begin{bmatrix} 0 & 1/2 \\ 1/2 & -1/4 \end{bmatrix}$

$$\begin{bmatrix} x & y \\ 2 & 0 \end{bmatrix}^{3} = \begin{bmatrix} 1 & 8 \\ 2 & 0 \end{bmatrix} \Rightarrow x = 1, y = 2 \Rightarrow$$

$$\begin{bmatrix} x & y \\ 2 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}^{-1} = \frac{-1}{4}\begin{bmatrix} 0 & -2 \\ -2 & 1 \end{bmatrix}$$

(141) (c) only one solution

$x + y = 2,\ 2x + 2y = 3 \Rightarrow$

$2(2) = 3 \Rightarrow 4 = 3$ which is not true

(142) (b) 3

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 1 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25 + 0 - 25 \\ -25 + 0 + 65 \\ 50 + 0 + 30 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 0 \\ 40 \\ 80 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \Rightarrow$$

$x + y + z = 0 + 1 + 2 = 3$

(143) (c) 1, – 2, 3

$$\begin{bmatrix} x - y - z \\ -y + z \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix} \Rightarrow z = 3,\ -y + z = 5,$$

$$x - y - z = 0 \Rightarrow$$

$x = 1,\ y = -2,\ z = 3$

(144) (c) 4

$\because$ B is a skew - symmetric matrix of odd order.

$\because$ $|B| = 0$

$\because$ Also : $|A| = ... = (2k + 1)^3$

$\because$ A, B are square matrices of order 3 such that

$|\text{adj } A| + |\text{adj } B| = 10^6$

$\therefore$ $|A|^{3-1} + |B|^{3-1} = 10^6$

$\therefore$ $(2k + 1)^6 + 0^2 = 10^6$

$\therefore$ $2k + 1 = 10$

$\therefore$ $k = \frac{9}{2} = 4.5$

$\therefore$ $[k] = 4$

(145) (b) 3

$$\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$$R_2 - 5R_3 \Rightarrow \begin{bmatrix} 3 & -4 & 2 \\ -3 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$$

$$R_1 - 2R_3 \Rightarrow \begin{bmatrix} 3 & -4 & 0 \\ -3 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ 2 \end{bmatrix}$$

$\Rightarrow x - 4y = -5$ ...... (i)

$-3x + 3y = -3$ ...... (ii)

Solving (i) and (ii) , we get $x = 3$

(146) (b) $x = \dfrac{11}{24}, y = \dfrac{1}{24}$

The given system of equation can be written in the matrix from as AX = B, where

$$A = \begin{bmatrix} 5 & -7 \\ 7 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\text{Now } |A| = \begin{bmatrix} 5 & -7 \\ 7 & -5 \end{bmatrix}$$

$$= 24$$

$$\neq 0$$

$\therefore$ The given system of equations has a unique soltion which is given by $X = A^{-1} B$.

$$\text{adj } A = \begin{bmatrix} 5 & -7 \\ 7 & -5 \end{bmatrix}^T = \begin{bmatrix} -5 & 7 \\ -7 & 5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{24} \begin{bmatrix} -5 & 7 \\ -7 & 5 \end{bmatrix}$$

$$\therefore \quad X = A^{-1}B = \frac{1}{24} \begin{bmatrix} -5 & 7 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 11 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11/24 \\ 1/24 \end{bmatrix}$$

$\Rightarrow x = 11/24 , y = 1/24$

(147) (c) 3 , 2 , 1

Consider option (c)

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 4 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \\ 13 \end{bmatrix}$$

$\therefore$ option (c) is the correct answer.

(148) (a) 1 , 2 , – 1

(149) (d) 2 , –1 , 3

(150) (d) $x = 1, y = 2, z = 1$

$x = 1, y = 2, z = 1$ satisfies the given set of equations

Points to remember

- Minor & Co–factors are defined only for the elements of a square matrix.
- Adjoint of a square matrix of order 2 can be easily obtained by interchanging the diagonal elements and changing the signs of the diagonal elements.
- The inverse of a symmetric matrix is symmetric.
- The inverse of a diagonal matrix is diagonal.
- The inverse of the transpose of a matrix is the transpose of its inverse i.e. $(A^1)^{-1} = (A^{-1})^1$.
- If A is a square matrix, then A^2, AA^1 and A^1A are symmetric matrice.
- If a matrix A is such that $A^2 = A$, then A is called an IDEPOTENT matrix.
- If a matrix is such that $A^2 = I$, then A is called an INVOLUTORY matrix.
- $A^{-1} = \frac{1}{|A|}\ (\text{adj } A)$
- If A is a non–singular matrix, then $|A^{-1}| = \frac{1}{|A|}$
- A square matrix A is called ORTHOGONAL if $AA' = I$
- $(A + B)(A - B) = A^2 - B^2$; if $AB = BA$.
- Determinant of a diagonal matrix = product of its diagonal elements.
- Determinant of a triangular matrix = product of its diagonal elements.
- Determinant of a skew symmetric matrix of odd order = 0.
- Determinant of a skew symmetric matrix of even order = non zero perfect squate.
- $|A| = |A^1|$
- $|AB| = |A| \,.\, |B|$
- If A & B are two square matrices of same order, then $|AB| = |BA|$.
- $(ABC)' = C' B' A'$
- $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$
- $(KA)^{-1} = \frac{1}{K}.A^{-1}$

- If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$; then $A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$

- adj (AB) = (adj B) . (adj A)
- Adj (A^{-1}) = (adj A)$^{-1}$

EVALUATION PAPER - MATHEMATICAL LOGIC

Time : 30 Min. **Marks : 25**

(1) If $A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$, then A^2 is

(a) null matrix (b) unit matrix (c) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(2) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$X, $B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 3 \\ 3 & 0 & 1 \end{bmatrix}$X. $C = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 6 & 5 \end{bmatrix}$X :

(a) A + B = B + A and A + (B + C) = (A + B) + C (b) A + B = B + A and AB = BA
(c) BC = CB (d) AC = CA

(3) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A^{-1} is equals to

(a) $ad - bc$ (b) $\frac{1}{ad-bc}$ (c) $\frac{1}{ad+bc}$ (d) None of these

(4) If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$, then A^{-1} is equals to

(a) $\begin{bmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ (b) $\begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$

(c) $\begin{bmatrix} -\cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$ (d) $\begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(5) If $A = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & -1 \end{bmatrix}$ then AB is

(a) a singular matrix (b) a non singular matris

(c) $|AB| = 4$ (d) None of these

(6) A matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is such that

(a) $A^2 = I$ (b) $A^2 = A$ (c) $A^2 = 0$ (d) $A^3 = 0$

(7) A is square matrix $[a_{ij}]$ such that
$a_{ij} = 0$ for $i \neq j$
$= k$ for $i = j$
where k = constant, then A is called as

(a) Null matrix (b) Unit matrix (c) Diagonal matrix (d) Scalar matrix

(8) If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$, then

(a) $(A + B)^2 = A^2 + AB + BA + B^2$ (b) $(A + B)^2 = A^2 + AB + B^2$

(c) $(A + B)^2 = A^2 + BA + B^2$ (d) $(A + B)^2 = A^2 + B^2$

(9) Matrix A is order $m \times n$; matrix B is order $p \times q$ such that AB exists, then

(a) $m = n$ (b) $p = n$ (c) $m = q$ (d) $p = q$

(10) If $A = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$, then A^{-1} equal to :

(a) $\frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$ (c) $\frac{1}{2}\begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix}$ (d) $-\frac{1}{2}\begin{bmatrix} 5 & -4 \\ -3 & 5 \end{bmatrix}$

(11) If $a = \begin{bmatrix} 4 & -1 \\ -1 & k \end{bmatrix}$ such that $A^2 - 6A + 7I = 0$ than $k =$

(a) 4 (b) 3 (c) 2 (d) 0

(12) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and AB = BA = I then B =

(a) $\begin{bmatrix} \sin\theta & \cos\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ (b) $\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ (c) $\begin{bmatrix} \sin\theta & \cos\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (d) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(13) For the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 4 & 1 \\ 2 & 6 & 3 \end{bmatrix}$, A_{ij} is co–factor of element a_{ij} then $a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13} =$

(a) 8 (b) 7 (c) 6 (d) 9

(14) If A is square matrix of order n then $|KA| =$

(a) $K|A|$ (b) $k^n |A|$ (c) $\dfrac{|A|}{k^n}$ (d) $|A|$

(15) If $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then

(a) $A^{-1} = B$ (b) B^{-1} exist
(c) A^{-1} = does not exist (d) A^{-1} exist

(16) If two matrices A and B are of order $p \times q$ and $r \times s$ respectively, can be subtracted only, if

(a) $p = r, q = s$ (b) $p = q, r = s$ (c) $p = q$ (d) none of these

(17) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$, then

(a) A is non–singular (b) A^{-1} does not exist
(c) A^{-1} exists (d) None of these

(18) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and AX = I, then X =

(a) $\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$ (b) $-\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$ (c) $\dfrac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ (d) $\dfrac{-1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(19) The inverse of the marix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

(a) $\begin{bmatrix} -7 & -3 & 3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 3 & 3 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -7 & 3 & 3 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

(20) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$ then $B^{-1}A^{-1} =$

(a) $\begin{bmatrix} 1 & -2 \\ -5 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 3 \\ 5 & -9 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 2 \\ 5 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 2 \\ 5 & 4 \end{bmatrix}$

(21) If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$, then $\begin{bmatrix} x \\ y \\ z \end{bmatrix} =$

(a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

(22) The values of x, y, z, for the equations $5x - y + 4z = 5$, $2x + 3y + 5x = 2$, $5x - 2y + 6z = -1$ are

(a) $x = 3, y = 3, z = 5$ (b) $x = 1, y = 2, z = 1$

(c) $x = 1, y = 3, z = 5$ (d) $x = 3, y = 1, z = -1$

(23) The element of second row and third column in the inverse of $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is

(a) -2 (b) -1 (c) 1 (d) 2

(24) If $A = \begin{bmatrix} a & c \\ d & b \end{bmatrix}$, then $A^{-1} =$

(a) $\frac{1}{ab-cd}\begin{bmatrix} b & -c \\ -d & a \end{bmatrix}$ (b) $\frac{1}{ad-bc}\begin{bmatrix} b & -c \\ -d & a \end{bmatrix}$ (c) $\frac{1}{ab-cd}\begin{bmatrix} b & d \\ c & a \end{bmatrix}$ (d) None of these

(25) If $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$, then $|\text{adj } A|$ is equal to

(a) 16 (b) 10 (c) 6 (d) None of these

EVALUATION PAPER - MATHEMATICAL LOGIC ANSWER KEY

1 **a**	2 **a**	3 **b**	4 **b**	5 **b**	6 **b**	7 **d**	8 **a**	9 **b**	10 **d**
11 **c**	12 **d**	13 **a**	14 **b**	15 **c**	16 **a**	17 **b**	18 **d**	19 **c**	20 **c**
21 **d**	22 **a**	23 **b**	24 **a**	25 **b**					

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