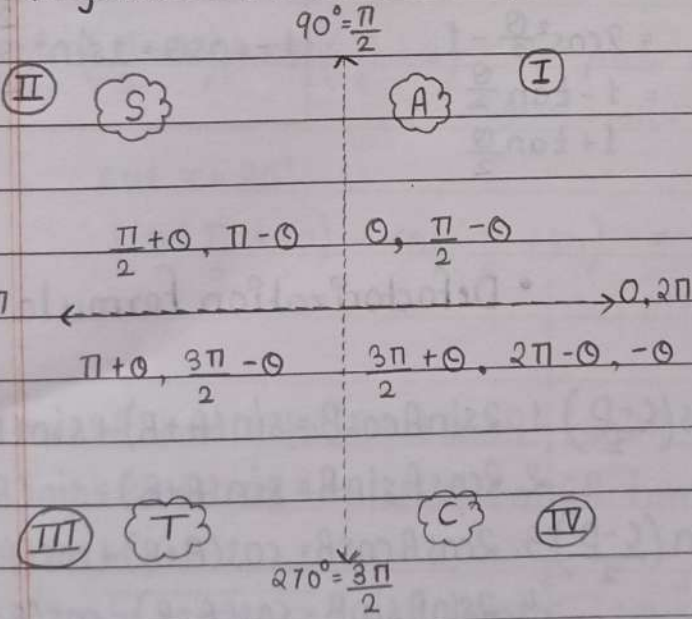
# Formula Sheet

## Trigonometry

- Sum and difference formula:

1. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
2. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
3. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
4. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
5. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$
   $= \cos^2 B - \sin^2 B$
6. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
   $= \cos^2 A - \cos^2 B$
7. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$
8. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$
9. $\cot(A-B) = \dfrac{1 + \cot A\cot B}{\cot A - \cot B}$
10. $\cot(A+B) = \dfrac{1 - \cot A\cot B}{\cot A + \cot B}$.

- Trigonometric functions for allied angles:




- ASTC
- $\sin(-\theta) = -\sin\theta$
  $\cos(-\theta) = \cos\theta$
- funtⁿ change (निचे 2 होतो)
  sin ↔ cos
  tan ↔ cot
  cosec ↔ sec
- Sign change quandrant wise.

- Short trick: If $A+B = 90°$ then $\tan A \times \tan B = 1$
  $\tan A + \tan B = 90 \quad \therefore \tan B = 90 - \tan A$
  $\tan A \times \tan B = \tan A \times \tan(90-A) = \tan A \times \cot A = 1$.
  Ex. $\tan 1° \times \tan 2° \times \ldots\ldots \times \tan 89° = 1$

- Trignometric ratio of multiple angle:

$\sin 2\theta = 2\sin\theta\cos\theta$

$= \frac{2\tan\theta}{1+\tan^2\theta}$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

$1+\cos 2\theta = 2\cos^2\theta$

$1-\cos 2\theta = 2\sin^2\theta$

- Trigonometric function of half angle:

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$= \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

$= 1 - \sin^2\frac{\theta}{2}$

$= 2\cos^2\frac{\theta}{2} - 1$

$= \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$

$\tan\theta = \frac{2\tan\theta/2}{1-\tan^2\theta/2}$

$1+\cos\theta = 2\cos^2\frac{\theta}{2}$

$1-\cos\theta = 2\sin^2\frac{\theta}{2}$

- Factorization formula:

1. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
2. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
3. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
4. $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

- Defactorization formula:

1. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
2. $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
3. $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
4. $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

- Trigonometric function of angle of triangle:

| | | |
|---|---|---|
| 1. $\sin(B+C) = \sin A$ | 1. $\cos(B+C) = -\cos A$ | 1. $\tan(B+C) = -\tan A$ |
| 2. $\sin(C+A) = \sin B$ | 2. $\cos(C+A) = -\cos B$ | 2. $\tan(C+A) = -\tan B$ |
| 3. $\sin(A+B) = \sin C$ | 3. $\cos(A+B) = -\cos C$ | 3. $\tan(A+B) = -\tan C$ |
| 1. $\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$ | 1. $\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$ | |
| 2. $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$ | 2. $\cos\left(\frac{B+C}{2}\right) = \sin\frac{A}{2}$ | |
| 3. $\sin\left(\frac{C+A}{2}\right) = \cos\frac{B}{2}$ | 3. $\cos\left(\frac{C+A}{2}\right) = \sin\frac{B}{2}$ | |

- Short tricks with examples:

1. $\sin\left(\frac{\pi}{3}+x\right) - \cos\left(\frac{\pi}{6}+x\right)$ ... put $x$ = something value

अशी value put करायची ज्याने आहिती असलेल्या function value मिळतील

$\therefore$ put $x = 30°$

$\therefore \sin\left(\frac{\pi}{3}+30\right) - \cos\left(\frac{\pi}{6}+30\right) = \sin(60+30) - \cos(30+30)$

$\therefore \sin 90° - \cos 60° = 1 - 1/2 = \underline{\underline{1/2}}$

2. $\tan3A \cdot \tan2A \cdot \tan A = \tan3A - \tan2A - \tan A$

$\tan2A + \tan2B + \tan2C = \tan2A \cdot \tan2B \cdot \tan2C$

3. $\sqrt{2+\sqrt{2+2\cos4\theta}} = 2\cos\theta$

$4\theta \xrightarrow{\sqrt{}} 2\theta \xrightarrow{\sqrt{}} \theta \longrightarrow 2\cos\theta$

$8\theta \xrightarrow{\sqrt{}} 4\theta \xrightarrow{\sqrt{}} 2\theta \longrightarrow 2\cos2\theta$

4. $\sin\theta \sin(60-\theta)\sin(60+\theta) = \frac{\sin3\theta}{4}$

5. $\cos\theta \cos(60-\theta) \cos(60+\theta) = \frac{\cos 3\theta}{4}$

6. $\tan\theta \tan(60-\theta) \tan(60+\theta) = \tan 3\theta$.

Qu- $\sin\underset{\theta}{20^\circ} \sin\underset{60-\theta}{40^\circ} \sin 60^\circ \sin\underset{60+\theta}{80^\circ} \rightarrow \sin 60 \cdot \frac{\sin 3\theta}{4} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \times \frac{1}{4} = \frac{3}{10}$

Que- $\cos\underset{\theta}{20^\circ} \cos\underset{60-\theta}{40^\circ} \cos 60^\circ \cos\underset{60+\theta}{80^\circ} \rightarrow \cos 60^\circ \frac{\cos 3\theta}{4} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{16}$

Que- $\tan\underset{\theta}{20^\circ} \tan\underset{60-\theta}{40^\circ} \tan 45^\circ \tan\underset{60+\theta}{80^\circ} \rightarrow \tan 45^\circ \times \tan 3\theta = 1 \times \sqrt{3} = \sqrt{3}$

7. $\Delta ABC$, $\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C + \cot A$

$A+B+C = 180^\circ$ ∴ put value of ABC so that sum is $180^\circ$

$\therefore A = B = C = 60$ $\therefore \cot 60 \cdot \cot 60 + \cot 60 \cdot \cot 60 + \cot 60 \cdot \cot 60$

$1/\sqrt{3} \cdot 1/\sqrt{3} + 1/\sqrt{3} \cdot 1/\sqrt{3} + 1/\sqrt{3} \cdot 1/\sqrt{3}$

$\therefore \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$

8. $\sin\theta + \cos\theta = x$ then $\sin\theta - \cos\theta = \frac{1}{x}$.