4 PAIR OF STRAIGHT LINES

Synopsis

4.1 Slope of Line:

i. If line has inclination θ (angle made by line with positive direction of X-axis) then slope of the line is, $m = \tan\theta$.

ii. If line passes through points (x_1, y_1) and (x_2, y_2) the slope of line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

iii. If line has equation $ax + by + c = 0$, then slope of the line is

$$m = \frac{-a}{b}$$

iv. If intercepts of line on coordinate axes are given, then

$$m = -\left(\frac{y-\text{intercept}}{x-\text{intercept}}\right)$$

v. If lines are parallel then their slopes are equal $(m_1 = m_2)$

vi. If lines are perpendicular then product of their slopes is –1. $(m_1m_2 = -1)$

4.2 Equations of Line:

i. Equation of X - axis: $y = 0$

ii. Equation of Y - axis: $x = 0$

iii. Equation of line parallel to X - axis: $y = b$ (b is constant)

iv. Equation of line parallel to Y - axis: $x = a$ (a is constant)

v. Slope point form: If line has slope m and passes through point (x_1, y_1), then equation of line is $y - y_1 = m(x - x_1)$

vi. Two point form: If line passes through points (x_1, y_1) and (x_2, y_2), then equation of line is $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

vii. Slope intercept form: If line has slope m and makes y intercept of c), then equation of line is $y = mx + c$

viii. Double intercept form: If line has intercepts a and b on coordinate axes, then equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

ix. Normal form: If distance of point from origin is p and inclination of perpendicular is then equation of line is $x\cos\alpha + y\sin\alpha = p$.

x. Parametric form: If (x_1, y_1) is one point on line and (x, y) be another point on line which is at a distance of r units from (x_1, y_1) and making an angle θ with positive direction of x–axis then equation of line is $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r,$ is inclination of the line.

i.e. Coordinates of any point on line at a distance of r units from the point (x_1, y_1) is $(x_1 + r\cos\theta, y_1 + r\sin\theta)$.

xi. Equation of line passing through origin is $y = mx$.

xii. General equation of line: $ax + by + c = 0$.

4.3 Angle between the lines:

i. If m_1 and m_2 be the slopes of the lines and be the angle between the lines, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

ii. The perpendicular distance between the point (x_1, y_1) and line $ax + by + c = 0$ is,

$$P = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

iii. The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by,

$$P = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$$

4.4 Position of point with respect to line:

Let (x_1, y_1) be given point and $ax + by + c = 0$ is a given line.

i. If $ax_1 + by_1 + c$ has same sign as of c then point lie on origin side of the given line.

ii. If $ax_1 + by_1 + c$ has opposite sign as of c then point lie on non-origin side of the given line.

iii. If $ax_1 + by_1 + c = 0$ then point lie on the given line.

4.5 Angle bisector of two lines:

Equations of angle bisectors between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

i. Angle bisectors containing origin:

Write the equations of the lines

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

4.6 Concurrent Lines:

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

4.7 Family of Lines:

Let $u \equiv a_1x + b_1y + c_1 = 0$ and $v \equiv a_2x + b_2y + c_2 = 0$ be two lines, then the equation of any line passing through point of intersection of $u = 0$ and $v = 0$ is given by

$u + kv = 0$, k = constant.

4.8 Pair of Lines:

A) Pair of Lines passing through origin:

i. Equation: A second degree homogeneous equation $ax^2 + 2hxy + by^2 = 0$ always represents pair of lines passing through origin.

ii. Nature of lines:

Conditions	Nature
$h^2 - ab = 0$	Lines are real and coincident
$h^2 - ab > 0$	Lines are real and distinct
$h^2 - ab < 0$	Lines are imaginary

iii. If m_1 and m_2 are slopes of lines, then

$$m_1 + m_2 = \frac{-2h}{b}$$

$$m_1 . m_2 = \frac{a}{b}$$

iv. If θ is the angle between the lines then it is given by

$$\tan = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

v. If lines are parallel then $h^2 - ab = 0$ and if lines are perpendicular then $a + b = 0$.

vi. Angle bisectors of pair of lines is given by

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

B) Pair of Lines not passing through origin:

i. Equation: A second degree non homogeneous equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ always represents pair of lines not passing through origin.

ii. Nature of lines:

Conditions	Nature
$h^2 - ab = 0$	Lines are real and coincident
$h^2 - ab > 0$	Lines are real and distinct
$h^2 - ab < 0$	Lines are imaginary

iii. If m_1 and m_2 are slopes of lines, then

$$m_1 + m_2 = \frac{-2h}{b}$$

$$m_1 \cdot m_2 = \frac{a}{b}$$

iv. If θ is the angle between the lines then it is given by

$$\tan = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$$

v. If lines are parallel then $h^2 - ab = 0$ and if lines are perpendicular then $a + b = 0$.

vi. Condition for non homogeneous equation of second degree representing pair of lines is,

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= 0$$

vii. Point of intersection of lines represented by equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

is given by $\left(\frac{hf - bg}{ab - h^2}, \frac{hg - af}{ab - h^2} \right)$

viii. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents pair of parallel lines then distance between the lines is given by,

$$2\sqrt{\frac{g^2 - ac}{a(a+b)}} \quad \text{or} \quad 2\sqrt{\frac{f^2 - bc}{b(a+b)}}$$

CLASS WORK

Multiple Choice Questions

4.1 Formation of joint equation of two lines and separation of equation from given equation :-

(1) The equation of the lines represented by ab $(x^2 - y^2) + (a^2 - b^2) xy = 0$ are

(a) $ax + by = 0$ and $bx + ay = 0$

(b) $ax + by = 0$ and $bx - ay = 0$

(c) $ax - by = 0$ and $bx + ay = 0$

(d) $ax - by = 0$ and $bx - ay = 0$

(2) The separate equation of the lines represented by the equations $x^2 + 2xy \tan \alpha - y^2 = 0$ are

(a) $(1 \pm \cos\alpha) x - y \sin\alpha = 0$

(b) $(1 \pm \cos\alpha) x + y \sin\alpha = 0$

(c) $(\tan \alpha \pm \sec\alpha) x - y = 0$

(d) $(\tan \alpha \pm \cos\alpha) x + y = 0$

(3) The joint equation of the lines trisecting the angle between the axes OX and OY is

(a) $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$

(b) $\sqrt{3}x^2 + 4xy + \sqrt{3}y^2 = 0$

(c) $3x^2 - 4xy + 3y^2 = 0$

(d) $3x^2 + 4xy + 3y^2 = 0$

(4) The joint equation of pair of lines through point (1, 2) and perpendicular to the lines given by $2x^2 - 5xy + 3y^2 = 0$ is

(a) $3x^2 + 5xy + 2y^2 - 16x + 13y - 21 = 0$

(b) $3x^2 + 5xy + 2y^2 + 16x - 13y - 21 = 0$

(c) $3x^2 + 5xy + 2y^2 - 16x - 13y - 21 = 0$

(d) $3x^2 + 5xy + 2y^2 - 16x - 13y + 21 = 0$

(5) The combined equation of lines passing through the point (–1, 2) one is parallel to $x + 3y - 1 = 0$ and other is perpendicular to $2x - 3y - 1 = 0$ is

(a) $3x^2 + 11xy + 6y^2 + 16x + 13y + 5 = 0$

(b) $3x^2 + 11xy + 6y^2 + 13x + 16y + 5 = 0$

(c) $3x^2 + 11xy + 6y^2 - 16x - 13y - 5 = 0$

(d) $3x^2 + 11xy + 6y^2 - 13x - 16y - 5 = 0$

(6) The joint equation of pair of lines passing through the origin and perpendicular to the lines represented by $5x^2 - 8xy + 3y^2 = 0$ is

(a) $3x^2 + 8xy + 3y^2 = 0$ (b) $3x^2 - 8xy + 3y^2 = 0$

(c) $3x^2 + 4xy + 3y^2 = 0$ (d) $3x^2 - 4xy + 3y^2 = 0$

(7) The joint equation of pair of lines through the origin and making an angle of 30^0 with the line $3x + 2y - 11 = 0$ is

(a) $23x^2 + 48xy + 3y^2 = 0$

(b) $3x^2 + 48xy + 23y^2 = 0$

(c) $23x^2 + 24xy + 3y^2 = 0$

(d) $3x^2 + 24xy + 23y^2 = 0$

(8) The joint equation of pair of lines through the origin and making an equilateral triangle with the line $x = 3$, is

(a) $x^2 + 3y^2 = 0$ (b) $x^2 - 3y^2 = 0$

(c) $3x^2 + y^2 = 0$ (d) $3x^2 - y^2 = 0$

(9) Equation $(x + y - 1)^2 - 4x^2 = 0$ jointly represents two lines, drawn from the point

(a) (1, 0) (b) (0, 1) (c) (0, 0) (d) (1, 1)

(10) Separate equations of lines jointly given by the equation $hxy + gx + \frac{fh}{g}y + f = 0$ we

(a) $x = \frac{-f}{g}, y = \frac{-g}{h}$ (b) $x = \frac{f}{g}, y = \frac{-g}{h}$

(c) $x = \frac{-f}{h}, y = \frac{-g}{h}$ (d) $fg = ch$

(11) Joint equation of two lines, through (x_1, y_1), perpendicular to two lines $ax^2 + 2hxy + by^2 = 0$, is

(a) $b(x - x_1)^2 + 2h(x - x_1)(y - y_1) + a(y - y_1)^2 = 0$

(b) $b(x - x_1)^2 - 2h(x - x_1)(y - y_1) + a(y - y_1)^2 = 0$

(c) $a(x - x_1)^2 - 2h(x - x_1)(y - y_1) + b(y - y_1)^2 = 0$

(d) $a(y_1 - y_2) + 2h(x - x_1)(y - y_1) + b(x - x_1) = 0$

(12) Let PQR be a right angled isosceles triangle, right angled at P(2, 1). If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is

(a) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$

(b) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

(c) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$

(d) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

(13) The point of intersection of the lines represented by equation

$2(x + 2)^2 + 3(x + 2)(y - 2) - 2(y - 2)^2 = 0$ is

(a) (2, 2) (b) (–2, –2)

(c) (–2, 2) (d) (2, –2)

(14) Joint equation of X-axis and the line through the origin having slope 1, is :

(a) $xy + y^2 = 0$ (b) $xy - x^2 = 0$

(c) $xy - y^2 = 0$ (d) $xy + x^2 = 0$

4.2 Sum of slopes, product of slopes, Auxiliary equation Angle bet lines, 11^l (coincident) and $\perp$ lines :-

(15) If the equation $(k + 1)x^2 - 6xy + (k - 7)y^2 = 0$ represents a pair of coincident lines, then $k =$

(a) 8, – 2 (b) – 8, 2 (c) – 8, –2 (d) 8, 2

(16) If the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is k times the slope of other, then

(a) $kh^2 = 4ab(1 + k)^2$ (b) $kh^2 = 2ab(1 + k)^2$

(c) $4kh^2 = ab(1 + k)^2$ (d) $2kh^2 = ab(1 + k)^2$

(17) If $\frac{x^2}{a} + \frac{2xy}{h} + \frac{y^2}{b} = 0$ represents a pair of straight lines and slope of one of the line is twice the other, then $ab : h^2 =$

(a) 1 : 2 (b) 2 : 1 (c) 8 : 9 (d) 9 : 8

(18) If the slopes of the lines represented by $3x^2 + kxy - y^2 = 0$ differ by 4, then $k =$

(a) 2 (b) – 2 (c) ± 2 (d) 4

(19) If the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is square of the slope of the other line, then

(a) $a^2b + ab^2 + 8h^3 + 6abh = 0$

(b) $a^2b + ab^2 - 8h^3 + 6abh = 0$

(c) $a^2b + ab^2 - 8h^3 - 6abh = 0$

(d) $a^2b + ab^2 + 8h^3 - 6abh = 0$

(20) If the lines represented by $ax^2 + 2hxy + by^2 = 0$ make angles of equal measures with the co-ordinate axes, then

(a) $ab = \pm 1$ (b) $a = b$

(c) $a = -b$ (d) $a = \pm b$

(21) Difference of the slopes of the lines given by $x^2(\sec^2\theta - \sin^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

(22) If the line $3x - 2y = 0$ coincides with one of the lines given by $ax^2 + 2hxy + by^2 = 0$, then

(a) $4a + 12h + 9b = 0$ (b) $4a + 12h - 9b = 0$

(c) $4a - 12h + 9b = 0$ (d) $4a - 12h - 9b = 0$

(23) If one of the lines given by $ax^2 + 2hxy + by^2 = 0$ bisects the angle between the co-ordinate axes, then

(a) $(a + b)^2 = 2h$ (b) $(a - b)^2 = 2h$

(c) $(a + b)^2 = 4h^2$ (d) $(a - b)^2 = 4h^2$

(24) If the pair of lines given by $ax^2 + 2hxy + y^2 = 0$ and $x^2 + 3xy + y^2 = 0$ have exactly one line common, then

(a) $a^2 + a + 1 = 0$ (b) $a^2 - a - 1 = 0$

(c) $a^2 + a - 1 = 0$ (d) $a^2 - a + 1 = 0$

(25) If the angle between the lines given by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between lines given by $2x^2 - 5xy + 3y^2 = 0$, then

(a) $100(h^2 - ab) = (a + b)^2$

(b) $100(h^2 - ab) = (a - b)^2$

(c) $25(h^2 - ab) = (a + b)^2$

(d) $25(h^2 - ab) = (a - b)^2$

(26) The equation $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0, \lambda \in R$, represents a pair of straight lines. If θ is the angle between these lines, then $\operatorname{cosec}^2 \theta =$

(a) 10 (b) 9 (c) 1 (d) 5

(27) If the lines given by $ax^2 + 2hxy + by^2 = 0$ form an equilateral triangle with the line $lx + my = 1$, then $(3a + b)(a + 3b) =$

(a) $-2h$ (b) $2h$ (c) $-4h^2$ (d) $4h^2$

(28) Lines represented by $px^2 - qy^2 = 0$ are real and distinct, if

(a) p and q have same sign

(b) p and q have opposite sign

(c) p or q is zero

(d) none of these

(29) Lines represented by $px^2 - qy^2 = 0$ are imaginary, if

(a) p and q have same sign

(b) p and q have opposite sign

(c) p or q is zero

(d) none of these

(30) If the lines represented by $6x^2 + 41xy - 7y^2 = 0$ makes angle α and β with X-sxis, then $\tan \alpha \times \tan \beta =$

(a) $\frac{-6}{7}$ (b) $\frac{6}{7}$ (c) $\frac{-7}{6}$ (d) $\frac{7}{6}$

(31) If acute angle between lines $3x^2 - 4xy + by^2 = 0$ is $\cot^{-1} 2$, then $b =$

(a) 1, – 55 (b) –1, 55 (c) 15, –5 (d) 1, –54

(32) If angle between lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{\pi}{4}$, then $2h =$

(a) $\sqrt{a^2 + b^2 + 3ab}$ (b) $\sqrt{a^2 + b^2 - 3ab}$

(c) $\sqrt{(a+b)^2 + 4ab}$ (d) $\sqrt{(a+b)^2 + ab}$

(33) If the two lines $(3x - y)^2 = k(x^2 + y^2)$ are mutually perpendicular, then : $k =$

(a) 5 (b) 6 (c) –5 (d) – 6

(34) If the two lines $kx^2 + 5xy + 9y^2 = 0$

(a) 5 (b) –5 (c) ± 9 (d) ± 3

(35) Measure of angle between the two lines $3xy - 4y = 0$ is

(a) 30^0 (b) 60^0 (c) 60^0 (d) 120^0

(36) If the lines represented by the equation $6x^2 + 41xy - 7y^2 = 0$ make angles α and β with

(a) $-\frac{6}{7}$ (b) $\frac{6}{7}$ (c) $\frac{7}{6}$ (d) $-\frac{7}{6}$

(37) If acute angle between lines $ax^2 + 2hxy + by^2 = 0$ is congruent to that lines $2x^2 - 5xy + 3y^2 = 0$, and $k(h^2 - ab) = (a + b)^2$, then $k =$

(a) $-(10)^2$ (b) $(-10)^2$ (c) –10 (d) 10

(38) If $3x^2 + 18xy + by^2 = 0$ represents a pair of lines, making an angle π with each other, then $b =$

(a) 3 (b) 9 (c) 27 (d) 81

(39) If slope of one of the lines $ax^2 + 2hxy + by^2 = 0$ is 5 times of the other, then $5h^2$

(a) ab (b) 2ab (c) 7ab (d) 9ab

(40) Joint equation of two lines, through (2, –3), perpendicular to two lines $3x^2 + xy - 2y^2 = 0$

(a) $2x^2 + xy - 3y^2 - 5x - 20y - 25 = 0$

(b) $-2x^2 - xy + 3y^2 - 5x - 20y - 25 = 0$

(c) $3x^2 + xy - 2y^2 - 5x - 20y - 25 = 0$

(d) $3x^2 - xy + 2y^2 + 5x - 20y - 25 = 0$

(41) If the slope of one of the lines $3x^2 + 4xy + \lambda y^2 = 0$ is three times the slope of the other line, then the value of λ is :

(a) 2 (b) 1 (c) – 1 (d) – 2

(42) If the angle between the lines $ax^2 + xy + by^2 = 0$ is 45^0, then

(a) $a = 2, b = 3$ (b) $a = 2, b = -6$

(c) $a = 4, b = 5$ (d) $a = 0, b = 5$

4.3 General 2nd degree equation of pair of lines :-

(43) If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of parallel lines, then

(a) $\frac{a}{h} = \frac{b}{h} = \frac{f}{g}$ (b) $\frac{a}{b} = \frac{b}{h} = \frac{g}{f}$

(c) $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$ (d) $\frac{h}{a} = \frac{b}{h} = \frac{f}{g}$

(44) If the equatio $\lambda x^2 - 5xy + 6y2 + x - 3y = 0$ represents a pair of straight lines, then their point of intersection is

(a) $(-3, -1)$ (b) $(-3, 1)$

(c) $(3, -1)$ (d) $(3, 1)$

(45) If the lines $x^2 - y^2 - 2x + 2y = 0$ and $x + 2y + k = 0$ are concurrent, then $k =$

(a) -1 (b) -3 (c) 1 (d) 3

(46) If the equation $2x^2 + 8xy + py^2 + qx + 2y - 15 = 0$ represented a pair of parallel lines, then

(a) $p = -8, q = -1$ (b) $p = -8, q = 1$

(c) $p = 8, q = -1$ (d) $p = 8, q = 1$

(47) If the quation $hxy + gx + fy + c = 0$, where $h \neq 0$, represents a pair of lines, then

(a) $2\,fgh = c^2$ (b) $2\,fg = ch$

(c) $fgh = c^2$ (d) $fg = ch$

(48) Two lines jointly given by the equation $xy - 2x + y - 2 = 0$ are

(a) $\parallel$ to coordinate axes separately, and $\perp$ to each other

(b) $\perp$ to coordinate axes separately, and $\perp$ to each other

(c) $\parallel$ as well as $\perp$ to coordinate axes

(d) $\parallel$ and $\perp$ to coordinate axes, and $\perp$ to each other

(49) The equation of the perpendiculars drawn from the origin to the lines represented by the equation

$2x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ is

(a) $6x^2 + 5xy + y^2 = 0$ (b) $6y^2 + 5xy + x^2 = 0$

(c) $6x^2 - 5xy + y^2 = 0$ (d) $6x^2 - xy - y^2 = 0$

(50) The equation $(x + y^2) - (x^2 - y^2) = 0$ represents

(a) a circle.

(b) two lines.

(c) two parallel lines.

(d) two mutually perpendicular lines.

(51) If $ax^2 + 2xy - 3y^2 + 4x + c = 0$ represents a pair of perpendicular lines, then

(a) $a = 3, c = 2$ (b) $a = 3, c = \frac{6}{5}$

(c) $a = 3, c = \frac{4}{5}$ (d) $a = 3, c = 6$

(52) The point of intersection of the lines represented by equation

$2(x + 2)^2 + 3(x + 2)(y - 2) - 2(y - 2)^2 = 0$

(a) $(2, 2)$ (b) $(-2,-2)$ (c) $(-2, 2)$ (d) $(2, -2)$

(53) If the angle between the pair of straight lines represented by the equation

$x^2 - 3xy + \lambda y^2 + 3x + 5y + 2 = 0$ is $\tan^{-1} 3$, where λ is a non negative real number, then $\lambda =$

(a) 2 (b) 0 (c) 3 (d) 1

(54) If the pair of lines

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on the Y-axis, then

(a) $2fgh = bg^2 + ch^2$ (b) $bg^2 \neq ch^2$

(c) $abc = 2fgh$ (d) None of these

(55) The point of intersection of the lines represented by the equation

$2x^2 + 3y^2 + 7xy + 8x + 14y + 8 = 0$ is

(a) $(0, 2)$ (b) $(1, 2)$ (c) $(-2, 0)$ (d) $(-2, 1)$

(56) The equation $4x^2 + 12xy + 9y^2 + 2gx + 2fy + c = 0$ will represent two real parallel straight lines, if

(a) $g = 4, f = 9, c = 0$

(b) $g = 2, f = 9, c = 1$

(c) $g = 2, f = 3$, c is any number

(d) $g = 4, f = 9, c > 1$

(57) The equation $ax^2 + by^2 + cx + cy = 0$, $c \neq 0$ represents a pair of lines, if :

(a) $a + b = 0$ (b) $a + c = 0$

(c) $b + c = 0$ (d) $a + b + c = 0$

(58) If l, m, n are in A.P., then the line $lx + my + n = 0$ will always pass through the point

(a) $(-1, 2)$ (b) $(1, -2)$ (c) $(2, 4)$ (d) $(3, 2)$

4.4 Mise.

(59) The triangle formed by the lines $x^2 - 3y^2 = 0$ and $x = 4$ is

(a) an equilateral (b) an isosceles

(c) a right angled (d) none of these

(60) The triangle formed by the lines $10x^2 + 21xy - 10y^2 = 0$ and $7x + 3y = 4$ is

(a) an equilateral

(b) an isosceles with base angle 30^0

(c) a right angled with one angle 30^0

(d) a right angled isosceles

(61) The area of the triangle formed by the lines $x^2 + 3xy - y^2 = 0$ and $x + y = 1$

(a) $\frac{\sqrt{13}}{2}$ sq. units (b) $\frac{\sqrt{13}}{4}$ sq. units

(c) $\frac{\sqrt{13}}{6}$ sq. units (d) $\frac{\sqrt{13}}{5}$ sq. units

(62) The length of each perpendicular side of an isosceles right angled triangle formed by $4x^2 + 6xy - 4y^2 = 0$ and $x - 3y + 7 = 0$ is

(a) $\frac{7}{\sqrt{5}}$ (b) $\frac{4}{\sqrt{5}}$ (c) $\frac{3}{\sqrt{5}}$ (d) $\frac{2}{\sqrt{5}}$

(63) Let ΔOAB is formed by teh lines $x^2 - 4xy + y^2 = 0$ and the line AB given by $2x + 3y - 1 = 0$. Then the equation of the median of the triangle drawn from O is

(a) $x + y = 0$ (b) $x - y = 0$

(c) $7x + y = 0$ (d) $7x - 8y = 0$

(64) Circumcentre of the triangle formed by the lines $xy = 0$ and $x + y = 1$ is

(a) $(0, 0)$ (b) $(1, 0)$ (c) $(0, 1)$ (d) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(65) A diagonal of the rectangle formed by the lines given by $x^2 - 7x + 6 = 0$ and $y^2 - 14y + 40 = 0$ is

(a) $6x - 5y - 14 = 0$ (b) $6x - 5y + 14 = 0$

(c) $5x - 6y = 0$ (d) $5x + 6y = 0$

(66) The joint equation of bisector of angles between the lines given by $5x^2 + 6xy - y^2 = 0$ is

(a) $x^2 + 2xy + y^2 = 0$ (b) $x^2 - 2xy + y^2 = 0$

(c) $x^2 + 2xy - y^2 = 0$ (d) $x^2 - 2xy - y^2 = 0$

(67) If the pair of lines $x^2 - 2nxy - y^2 = 0$ and $x^2 - 2mxy - y^2 = 0$ are such that one of them represents the disectors of the angles between the other, then

(a) $mn = -1$ (b) $mn = 1$

(c) $\frac{1}{m} + \frac{1}{n} = 0$ (d) $\frac{1}{m} - \frac{1}{n} = 0$

(68) The product of lengths of the perpendiculars from point (4, 1) one the lines given by $3x^2 - 4xy - y^2 = 0$ is

(a) $\frac{31\sqrt{2}}{4}$ (b) $\frac{31\sqrt{2}}{8}$ (c) $\frac{31}{4}$ (d) $\frac{31}{8}$

(69) If distance of a point (x_1, y_1) from each of two lins L_1 and L_2, through the origin, is δ, then joint equation of L_1 and L_2 is

(a) $(x_1y - xy_1)^2 = \delta^2(x^2 + y^2)$

(b) $(x_1y + xy_1)^2 = \delta^2(x^2 + y^2)$

(c) $(x_1x - yy_1)^2 = \delta^2(x^2 + y^2)$

(d) $(xx_1 + yy_1)^2 = \delta$

(70) If distance of a given point (a, b) from each of two lines through the origin, is d, then

(a) $(ax - by)^2 = d^2(x^2 + y^2)$

(b) $(ay - bx)^2 = d^2(x^2 + y^2)$

(c) $(ax + by)^2 = d^2(x^2 + y^2)$

(d) $(ax + by)^2 = d^2(x^2 + y^2)$

(71) Diagonals of a square are along the pair of lines $2x^2 - 3xy - 2y^2 = 0$. If (2, 1) is a vertex of the square, then another vertex adjacent to this can be

(a) (1, 4) (b) (1, – 2)

(c) (2, – 1) (d) (1, 2)

(72) Area of the triangle formed by the pair of lines $x^2 - 4y^2 = 0$ and the line $x = a$ is

(a) $2a^2$ (b) $\frac{a^2}{2}$ (c) $\frac{\sqrt{3}}{2}a^2$ (d) $\frac{2}{\sqrt{3}}a^2$

(73) One bisector of the angle between the lines $a(x - 1)^2 + 2h(x - 1)y + by^2 = 0$ is $2x + y - 2 = 0$. The the other bisector is

(a) $x - 2h + 1 = 0$ (b) $2x + y - 1 = 0$

(c) $x + 2h - 1 = 0$ (d) $x - 2y - 1 = 0$

HOME WORK

Multiple Choice Questions

4.1 Formation of joint equation of two lines and separation of equations from given equation :-

(1) The separate equations of the lines represented by the equations $5x^2 - 9y^2 = 0$ are

(a) $5x - 3y = 0$ and $5x + 3y = 0$

(b) $5x - 3y = 0$ and $5x + 9y = 0$

(c) $\sqrt{5}\,x - 3y =$ and $\sqrt{5}\,x + 3y = 0$

(d) $\sqrt{5}\,x - 9y =$ and $\sqrt{5}\,x + 9y = 0$

(2) The separate equations of the lines represented by the equations $3y^2 + 7xy = 0$ are

(a) $y = 0$ and $3x - 7y = 0$

(b) $y = 0$ and $3x + 7y = 0$

(c) $y = 0$ and $7x - 3y = 0$

(d) $y = 0$ and $7x + 3y = 0$

(3) The separate equations of the lines represented by the equations $3x^2 - 2\sqrt{3}\,xy - 3y^2 = 0$ are

(a) $x - \sqrt{3}\,y = 0$ and $\sqrt{3}\,x + y = 0$

(b) $x + \sqrt{3}\,y = 0$ and $\sqrt{3}\,x - y = 0$

(c) $x - 3y$ and $3x + y = 0$

(d) $x + 3y = 0$ and $3x + y = 0$

(4) The separate equations of the lines represented by the equations $x^2 + 2(\operatorname{cosec}\alpha)\,xy + y^2 = 0$ are

(a) $(1 \pm \sin\alpha)\,x + 2y\cos\alpha = 0$

(b) $(1 \pm \cos\alpha)\,x + 2y\sin\alpha = 0$

(c) $(1 \pm \sin\alpha)\,x + y\cos\alpha = 0$

(d) $(1 \pm \cos\alpha)\,x + y\sin\alpha = 0$

(5) The separate equations of the lines represented by the equations $(x - 2)^2 - 3(x - 2)(y + 1) + 2(y + 1)^2 = 0$ are

(a) $x - 2y - 4 = 0$ and $x - y - 3 = 0$

(b) $x - 2y + 4 = 0$ and $x - y + 3 = 0$

(c) $x + 2y - 4 = 0$ and $x + y - 3 = 0$

(d) $x + 2y + 4 = 0$ and $x + y + 3 = 0$

(6) The joint equation of pair of lines passing through the origin and perpendicular to the lines represented by $2x^2 - 3xy - 9y^2 = 0$ is

(a) $9x^2 - 3xy - 2y^2 = 0$ (b) $9x^2 - 3xy + 2y^2 = 0$

(c) $9x^2 + 3xy - 2y^2 = 0$ (d) $9x^2 + 3xy + 2y^2 = 0$

(7) The joint equation of pair of lines passing through the origin and perpendicular to the lines represented by $x^2 + 4xy - 5y^2 = 0$ is

(a) $5x^2 + 4xy + y^2 = 0$ (b) $5x^2 + 4xy - y^2 = 0$

(c) $5x^2 - 4xy + y^2 = 0$ (d) $5x^2 - 4xy - y^2 = 0$

(8) The combined equation of pair of lines through point $(2, -1)$ and parallel to the lines given by $3x^2 - 4xy + 2y^2 = 0$ is

(a) $3x^2 - 4xy + 2y^2 + 16x + 12y - 22 = 0$

(b) $3x^2 - 4xy + 2y^2 - 16x + 12y + 22 = 0$

(c) $3x^2 - 4xy + 2y^2 + 16x - 12y + 22 = 0$

(d) $3x^2 - 4xy + 2y^2 + 16x + 12y + 22 = 0$

(9) The joint equation of the lines $x - y = 0$ and $5x - 3y = 0$ is

(a) $15x^2 - 19xy + 6y^2 - 25x - 15y = 0$

(b) $15x^2 - 19xy + 6y^2 + 25x - 15y = 0$

(c) $15x^2 - xy + 6y^2 + 25x - 15y = 0$

(d) $15x^2 + xy + 6y^2 + 25x - 15y = 0$

(10) Fine the combine equation of the lines passing through the origin and having inclinations $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ is

(a) $x^2 + 3y^2 = 0$ (b) $x^2 + y^2 = 0$

(c) $3x^2 - y^2 = 0$ (d) $x^2 - 3y^2 = 0$

(11) Fine the combine equation of the lines passing through the origin and each of which makes an angle of 60^0 with the Y-axis.

(a) $x^2 - 3y^2 = 0$ (b) $x^2 - \sqrt{3}\,y^2 = 0$

(c) $3x^2 - y^2 = 0$ (d) $\sqrt{3}\,x^2 - y^2 = 0$

(12) The combined equation of lines passing through the point (2, 3) and perpendicular to the lines $3x + 2y - 1 = 0$ and $x - 3y + 2 = 0$ is

(a) $6x^2 - 7xy + 3y^2 - 3x - 32y - 45 = 0$

(b) $6x^2 - 7xy - 3y^2 - 3x + 32y - 45 = 0$

(c) $6x^2 - 7xy - 3y^2 + 3x + 32y - 45 = 0$

(d) $6x^2 - 7xy + 3y^2 - 3x + 32y - 45 = 0$

(13) The joint equation of pair of lines passing through the origin and forming an equilateral triangle with the line $3x + 4y = 5$ is

(a) $39x^2 + 48xy + 11y^2 = 0$

(b) $39x^2 - 48xy + 11y^2 = 0$

(c) $39x^2 + 96xy + 11y^2 = 0$

(d) $39x^2 - 96xy + 11y^2 = 0$

(14) Lines represented by $px^2 - qy^2 = 0$ are real and coincident, if

(a) p and q have same sign

(b) p and q have opposite sign

(c) p or q is zero

(d) None of these

(15) Joint equation of two lines both parallel to Y-axis, and each at a distance of 3 units from it, is

(a) $x^2 - 9 = 0$ (b) $y^2 - 9 = 0$

(c) $x^2 - y^2 = 0$ (d) $y^2 + 9 = 0$

(16) Joint equation of two lines, through the origin, having slopes $\sqrt{3}$ and $\frac{-1}{\sqrt{3}}$

(a) $\sqrt{3}\ (x^2 - y^2) + 2xy = 0$

(b) $\sqrt{3}\ (x^2 + y^2) - 2xy = 0$

(c) $\sqrt{3}\ (x^2 - y^2) - 2xy = 0$

(d) $\sqrt{3}\ (x^2 + y^2) + 2xy = 0$

(17) Joint equation of lines, trisecting angles in second and fourth quadrants, is

(a) $\sqrt{3}\ (x^2 + y^2) - 4xy = 0$

(b) $\sqrt{3}\ (x^2 - y^2) - 4xy = 0$

(c) $\sqrt{3}\ (x^2 + y^2) + 4xy = 0$

(d) $4\ (x^2 + y^2) + \sqrt{3}\,xy = 0$

(18) Joint equation of two lines throught the origin, each making angle of 45^0 with line $3x + y = 0$, is

(a) $2x^2 - 3xy - 2y^2 = 0$ (b) $2x^2 + 3xy + 4y^2 = 0$

(c) $2x^2 + 3xy - 2y^2 = 0$ (d) $2x^2 + 2xy - 3y^2 = 0$

(19) Joint equation of lines passing through the origin, and parallel to the lines $y = m_1x + c_1$ and $y = m_1x + c_2$, is

(a) $m_1m_2\ x^2 - (m_1 + m_2)\ xy + y^2 = 0$

(b) $m_1m_2\ x^2 + (m_1 + m_2)\ xy + y^2 = 0$

(c) $m_1m_2\ y^2 - (m_1 + m_2)\ xy + x^2 = 0$

(d) $m_1m_2\ y^2 + (m_1 + m_2)\ xy + x^2 = 0$

(20) Combined equation of the two lines, passing through the origin, forming an equilateral triangle with the line $x + y + \sqrt{3} = 0$ is

(a) $x^2 + 4xy - y^2 = 0$ (b) $x^2 - 4xy + y^2 = 0$

(c) $x^2 - 4xy + 2y^2 = 0$ (d) $x^2 + 4xy + 2y^2 = 0$

(21) The combined equation of lines, through the origin forming an equilateral triangle with the line $x + y + \sqrt{3} = 0$ is

(a) $x^2 + 4xy - y^2 = 0$ (b) $x^2 - 4xy + y^2 = 0$

(c) $x^2 - 4xy + 2y^2 = 0$ (d) $x^2 + 4xy + 2y^2 = 0$

(22) The pair of straight lines passes through the point (1, 2) and perpendicular to the pair of straight lines $3x^2 - 8xy + 5y^2 = 0$, is

(a) $(5x + 3y + 11)\ (x + y + 3) = 0$

(b) $(5x + 3y - 11)\ (x + y - 3) = 0$

(c) $(3x + 5y - 11)\ (x + y + 3) = 0$

(d) $(3x - 5y + 11)\ (x + y - 3) = 0$

(23) The combined equation of the lines which pass through the origin and each of which makes an angle of 30^0 with the line $2x - y = 0$ is

(a) $11x^2 + 16xy - y^2 = 0$

(b) $11x^2 + 16xy + y^2 = 0$

(c) $11x^2 - 16xy + y^2 = 0$

(d) $11x^2 - 16xy - y^2 = 0$

(24) The combined equation of lines through the origin forming an equilateral triangle with the line $x - y = 4$ is :

(a) $x^2 + 4xy + y^2 = 0$ (b) $x^2 - 4xy + y^2 = 0$

(c) $x^2 + 4xy - y^2 = 0$ (d) $x^2 - 4xy - y^2 = 0$

(25) Combined equation of pair of lines through the point (2, 0) and perpendicular to the pair of lines $2x^2 - xy - y^2 = 0$ is :

(a) $x^2 - xy - 2y^2 - 4x + 2y + 4 = 0$

(b) $x^2 - xy + 2y^2 + 4x - 2y + 4 = 0$

(c) $x^2 - xy + 2y^2 - 4x + 2y + 4 = 0$

(d) $x^2 - xy - 2y^2 - 4x + 2y - 4 = 0$

(26) Joint equation of two lines through the origin having slopes 1 and 5, is :

(a) $5x^2 - 6xy + y^2 = 0$ (b) $5x^2 + 6xy + y^2 = 0$

(c) $5x^2 - 6xy - y^2 = 0$ (d) $5x^2 + 6xy - y^2 = 0$

(27) Combined equation of pair of lines through (–3, 4) and parallel to co-ordinate axes, is :

(a) $xy - 4x + 3y - 12 = 0$

(b) $xy + 4x - 12 = 0$

(c) $xy - 4x - 3y - 12 = 0$

(d) $xy + 3y - 12 = 0$

(28) The combined equation of lines, through the origin and forming an equilateral triangle with the line $2x + 3y = 5$ is :

(a) $7x^2 + 48xy + 3y^2 = 0$

(b) $23x^2 - 48xy - 3y^2 = 0$

(c) $23x^2 - 48xy + 3y^2 = 0$

(d) $7x^2 - 48xy + 3y^2 = 0$

(29) The equation of pair of lines through the origin and inclined at an angle of 30^0 to the line $x + 2y - 1 = 0$ is :

(a) $x^2 - 16xy - 11y^2 = 0$

(b) $x^2 + 16xy - 11y^2 = 0$

(c) $x^2 - 16xy + 11y^2 = 0$

(d) $x^2 + 16xy + 11y^2 = 0$

4.2 Sum of slopes, product of slopes, Auxiliary eqn. Angle bet^n. Lines, ll^l (coincident) and ⊥ lines :-

(30) The product of length of perpendiculars from $P(x_1, y_1)$ to the lines represented by $ax^2 + 2hxy + by^2 = 0$ is

(a) $\left|\dfrac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 - 4h^2}}\right|$

(b) $\left|\dfrac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}\right|$

(c) $\left|\dfrac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a+b)^2 - 4h^2}}\right|$

(d) $\left|\dfrac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a+b)^2 + 4h^2}}\right|$

(31) If m_1 and m_2 are the slopes of the lines represented by $ax^2 + 2hxy + by^2 = 0$, then $m_1 + m_2 =$

(a) $\dfrac{2h}{a}$ (b) $\dfrac{-2h}{a}$ (c) $\dfrac{2h}{b}$ (d) $\dfrac{-2h}{b}$

(32) If the equation $k(x^2 + y^2) = (3x - y)^2$ represents a pair of perpendicular lines, then $k =$

(a) – 5 (b) 5 (c) – 9 (d) – 1

(33) If the lines represented by $(x^2 + y^2) \sin^2 \alpha = (x \cos \alpha - y \sin \alpha)^2$ are perpendicular to each other, then $\alpha =$

(a) 0 (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{4}$

(34) If the lines given by $x^2 - 4xy + y^2 = 0$ make angles α and β with X-axis then $\tan^2 \alpha + \tan^2 \beta =$

(a) 2 (b) – 2 (c) 14 (d) – 14

(35) If the sum of the slopes given by $ax^2 + 8xy + 5y^2 = 0$ is twice their product, then $a =$

(a) – 8 (b) 2 (c) 4 (d) – 4

(36) If the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is 3 times the slope of other, then $3h^2 =$

(a) $16ab$ (b) $4ab$ (c) $2ab$ (d) ab

(37) If the slope of one of the lines given by $kx^2 + 4xy - y^2 = 0$ exceeds the slope of the other by 8, then $k =$

(a) 4 (b) 16 (c) 48 (d) 12

(38) If m_1 and m_2 are the slopes of the lines represented by $(\tan^2 \theta + \cos^2 \theta)\, x^2 - 2xy \tan \theta + (\sin^2 \theta)\, y^2 = 0$, then $|m_1 - m_2| =$

(a) -2 (b) 2 (c) -4 (d) 4

(39) If the line $4x + 5y = 0$ coincides with one of the lines given by $ax^2 + 2hxy + by^2 = 0$, then

(a) $25a + 40h + 16b = 0$

(b) $25a + 40h - 16b = 0$

(c) $25a - 40h + 16b = 0$

(d) $25a - 40h - 16b = 0$

(40) If the line $4x - 5y = 0$ coincides with one of the lines given by $ax^2 + 2hxy + by^2 = 0$, then

(a) $25a - 40h - 16b = 0$

(b) $25a + 40h + 16b = 0$

(c) $25a - 40h + 16b = 0$

(d) $25a + 40h - 16b = 0$

(41) If one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is perpendicular to $px + qy = 0$, then

(a) $ap^2 - 2hpq + bq^2 = 0$

(b) $ap^2 + 2hpq + bq^2 = 0$

(c) $aq^2 - 2hpq + bp^2 = 0$

(d) $ap^2 + 2hpq + bq^2 = 0$

(42) The acute angle θ between the lines represented by $(a^2 - 3b^2)\, x^2 + 8abxy + (b^2 - 3a^2)\, y^2 = 0$ is

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

(43) The acute angle between the lines given by $3y^2 = x\,(7y - 2x)$ is

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

(44) The acute angle θ between the lines represented by $3x^2 + 2xy - y^2 = 0$ is

(a) $\tan^{-1}\left(\frac{1}{2}\right)$ (b) $\tan^{-1}\left(\frac{3}{2}\right)$

(c) $\tan^{-1}\left(\frac{2}{3}\right)$ (d) $\tan^{-1} 2$

(45) If the pair of lines $3x^2 - 5xy + ky^2 = 0$ and $6x^2 - xy - 5y^2 = 0$ have one line in common, then $k =$

(a) $2, \frac{25}{4}$ (b) $-2, \frac{25}{4}$

(c) $2, \frac{-25}{4}$ (d) $-2, \frac{-25}{4}$

(46) The acute angle between the lines represented by $(x - 3)^2 + (x - 3)(y - 4) - 2\,(y - 4)^2 = 0$ is

(a) $\tan^{-1} 2\sqrt{2}$ (b) $\tan^{-1} 2\sqrt{3}$

(c) $\tan^{1} 3$ (d) $\tan^{1} (-3)$

(47) If the angle 2θ is acute, then the acute angle between the pair of lines given by $x^2 (\cos \theta - \sin \theta) + 2xy \cos \theta + y^2 (\cos \theta + \sin \theta)$ is

(a) θ (b) 2θ (c) $\frac{\theta}{2}$ (d) $\frac{\theta}{3}$

(48) If the acute angle between the lines $ax^2 + 2hxy + bh^2 = 0$ is congruent to the acute angle between the lines $3x^2 - 7xy + 4y^2 = 0$, then

(a) $4\,(h^2 - ab) = (a + b)^2$

(b) $7\,(h^2 - ab) = (a + b)^2$

(c) $49\,(h^2 - ab) = (a + b)^2$

(d) $196\,(h^2 - ab) = (a + b)^2$

(49) The angle between the lines given by $ay^2 + (-1 - \lambda^2)\, xy - ax^2$ is same as the angle between the lines

(a) $xy = 0$

(b) $5x^2 + 2xy - 3y^2 = 0$

(c) $5x^2 + 16xy + 5y^2 = 0$

(d) $x^2 - 2xy - 3y^2 = 0$

(50) If $x^2 + 2hxy + y^2 = 0$ represents the equations of straight lines through origin which make an angle α with line $x + y = 0$, then $h =$

(a) $\sin 2\alpha$ (b) $\cos 2\alpha$

(c) $\text{cosec } 2\alpha$ (d) $\sec 2\alpha$

(51) Lines represented by $px^2 - qy^2 = 0$ are real and coincident, if

(a) p and q have same sign

(b) p and q have opposite sign

(c) p or q is zero

(d) none of these

(52) If the lines represented by $ax^2 + 4xy + 4y^2 = 0$ are real and distinct, then

(a) $a = 0$ (b) $a = 1$ (c) $a < 1$ (d) $a > 1$

(53) If the equatio $a^2x^2 + bcy^2 = a(b + c)xy$ represents a pair of coincident lines, then

(a) $b = a$ (b) $b = c$ (c) $c = a$ (d) $a + b = 0$

(54) The joint equation of pair of lines through the origin and making an equilateral triangle with the line $y = 3$, is

(a) $x^2 + 3y^2 = 0$ (b) $x^2 - 3y^2 = 0$

(c) $3x^2 + y^2 = 0$ (d) $3x^2 - y^2 = 0$

(55) If line $lx + my + n = 0$ is perpendicular to one of the lines $ax^2 + 2hxy + by^2 = 0$, then

(a) $am^2 + 2lhm + bl^2 = 0$

(b) $al^2 + 2lhm + bm^2 = 0$

(c) $bm^2 - 2lhm + al^2 = 0$

(d) $la^2 + 2hm + nb^2 = 0$

(56) Measure of angle between the lines $xy - 5x + 4y - 20 = 0$, is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(57) Measure of angle between lines $(3 + 2\sqrt{3})\, x^2 - 2xy - y^2 = 0$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(58) If acute angle between lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{\pi}{3}$, then $4h^2 =$

(a) $a^2 + 4ab + b^2$ (b) $a^2 + 6ab + b^2$

(c) $(a + 2b)(a + 3b)$ (d) $(a - 2b)(2a + b)$

(59) If acute angle between lines $ax^2 + 2hxy + by^2 = 0$ is congruent to that between lines $3x^2 - 7xy + 4y^2 = 0$, and $(a + b)^2 + k(h^2 - ab) = 0$, then $= k$

(a) $-(14)^2$ (b) $(-14)^2$

(c) -14 (d) 14

(60) If q is the angle between the lines $x^2 - 3xy + 2y^2 + \lambda x - 5y + 2 = 0$, the $\csc^2\theta =$

(a) 3 (b) 9 (c) 10 (d) 100

(61) If the lines $2x^2 - 3xy + y^2 = 0$ make angles α and β with X-axis, then : $\cot^2\alpha + \cot^2\beta =$

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{7}{4}$ (d) $\frac{5}{4}$

(62) Joint equation of lines passing through the origin, and parallel to the lines $y = m_1x + c_1$ and $y = m_1x + c_2$, is

(a) $m_1m_2\, x^2 - (m_1 + m_2)\, xy + y^2 = 0$

(b) $m_1m_2\, x^2 + (m_1 + m_2)\, xy + y^2 = 0$

(c) $m_1m_2\, y^2 - (m_1 + m_2)\, xy + x^2 = 0$

(d) $m_1m_2\, y^2 + (m_1 + m_2)\, xy + x^2 = 0$

(63) If the angle between the two lines $ax^2 + xy + by^2 = 0$ is 45^0, then

(a) $a = 2, b = 3$ (b) $a = 1, b = -6$

(c) $a = 4, b = 5$ (d) $a = 3, b = 2$

(64) If the angle between the lines $3x^2 - 8xy + 4y^2 = 0$ is $\tan^{-1} k$, then : $k =$

(a) $\frac{7}{4}$ (b) $\frac{7}{3}$ (c) $\frac{4}{7}$ (d) $\frac{1}{7}$

(65) If the slope of one of the lines represented by $ax^2 + (3a + 1)\, xy + 3y^2 = 0$ be reciprocal of the slope of the other, then the slope of the lines are

(a) $\frac{3}{2}, \frac{2}{3}$ (b) $\frac{1}{2}, \frac{2}{1}$ (c) $\frac{1}{3}, 3$ (d) $\frac{-1}{3}, -3$

(66) Which of the following pair of lines is perpendicular ?

(a) $2x^2 = y(x + 2y)$ (b) $(x + y)^2 = x(y + 3x)$

(c) $2y(x + y) = xy$ (d) $y = \pm 2x$

(67) If one line of the pair of lines $ax^2 + 2hxy + by^2 = 0$ bisects the angle between co-ordinate axes in positive quadrant, then

(a) $a + b = 2|h|$ (b) $a + b = -2h$

(c) $a - b = 2|h|$ (d) $(a - b)^2 = 4h^2$

(68) If m_1, m_2 are slopes of lines represented by $2x^2 - 5xy + 3y^2 = 0$ then equation of lines passing through origin with slopes $\frac{1}{m_1}, \frac{1}{m_2}$ will be

(a) $3x^2 - 5xy + 2y^2 = 0$ (b) $3x^2 + 5xy + 2y^2 = 0$

(c) $2x^2 + 5xy - 3y^2 = 0$ (d) $2x^2 - 5xy - 3y^2 = 0$

(69) If the slope of one of the lines given by $a^2x^2 + 2hxy + b^2y^2 = 0$ be three times of the other then 'h' is equal to

(a) $2\sqrt{3}$ ab (b) $-2\sqrt{3}$ ab

(c) $\frac{2}{\sqrt{3}}\sqrt{ab}$ (d) $-\frac{2}{\sqrt{3}}$ ab

(70) The angle between the pair of straight lines $3x^2 + 10xy + 8y^2 = 0$ is $\tan^{-1}$ (p), where p =

(a) $\frac{-5}{11}$ (b) $\frac{-3}{11}$ (c) $\frac{2}{11}$ (d) $\frac{8}{11}$

(71) The angle between the lines represented by the equation $(x^2 + y^2)\sin\theta + 2xy = 0$ is

(a) θ (b) $\frac{\theta}{2}$ (c) $\frac{\pi}{2} - \theta$ (d) $\frac{\pi}{2} - \frac{\theta}{2}$

(72) If lines $a^2x^2 + bcy^2 = a(b + c)xy$ are mutually perpendicular, then

(a) $c^2 + ab = 0$ (b) $b^2 + ca = 0$

(c) $a^2 + bc = 0$ (d) $a^2 + b^2 + c^2 = 0$

(73) $6x^2 + hxy + 12y^2 = 0$ represents pair of parallel straight lines, if h is

(a) $\pm 6\sqrt{2}$ (b) $\pm\sqrt{2}$

(c) $\pm 12\sqrt{2}$ (d) $\pm\sqrt{6}$

(74) The equation $(x^2 + y^2)(h^2 + k^2 - a^2) = (hx + ky)^2$ represents a pair of perpendicular lines if

(a) $h^2 + k^2 = 2a^2$ (b) $(h + k)(h - k) = 2a^2$

(c) $h^2 + k^2 = a^2$ (d) $h^2 + k^2 = 0$

(75) If one of the line is given by $kx^2 - 5xy - 6y^2 = 0$ is $4x + 3y = 0$, then value of k is

(a) 1 (b) 3 (c) 4 (d) 2

(76) If the lines represented by the equation $ax^2 - bxy - y^2 = 0$ make angles α and β with the X-axis, then the $(\alpha + \beta) =$

(a) $\frac{b}{1+a}$ (b) $\frac{-b}{1+a}$ (c) $\frac{a}{1+b}$ (d) $\frac{b}{1-a}$

(77) If the lines represented by the equation $2x^2 - 3xy + y^2$ make angles α and β with the X-axis, then $\cot^2\alpha + \cot^2\beta =$

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{7}{4}$ (d) $\frac{5}{4}$

(78) If two lines $ax^2 + 2hxy + b^2 = 0$ make equal and opposite angles with X-axis, then

(a) h = 0 and ab > 0 (b) $h \neq 0$ and ab < 0

(c) $h \neq 0$ and ab > 0 (d) h = 0 and ab < 0

(79) If the angle between the lines represented by the equation $y^2 + kxy - x^2 \tan 2A = 0$ be 2A, then k =

(a) 0 (b) 1 (c) 2 (d) tan A

(80) If the equation $4x^2 + hxy + y^2 = 0$ represents coincident lines, then the value of 'h' is

(a) ± 2 (b) ± 4 (c) ± 1 (d) 0

(81) The pair of lines represented by $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ are perpendicular to each other for

(a) two values of a (b) $\forall$ a

(c) one values of a (d) no value of a

(82) If one of the lines given by $kx^2 - 5xy - 3y^2 = 0$ is perpendicular to the line $x - 2y + 3 = 0$, then the value of k is

(a) 2 (b) $-\frac{11}{2}$ (c) $-\frac{2}{3}$ (d) 3

(83) If the ratio of gradients of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is 1 : 3, then the value of the ratio h^2 : ab is

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 1

(84) If $\frac{x^2}{a} + \frac{y^2}{b} + \frac{2xy}{h} = 0$ represent pair of straight lines and slope of one line is twice the other. Then ab : h² is

(a) 9 : 8 (b) 8 : 9 (c) 1 : 2 (d) 2 : 1

(85) Acute angle between the lines represented by $(x^2 + y^2)\sqrt{3} = 4xy$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) None of these

(86) The angle between the lines represented by the equation $4x^2 - 24xy + 11y^2 = 0$ is

(a) $\tan^{-1}\frac{3}{4}, \tan^{-1}\left(-\frac{3}{4}\right)$

(b) $\tan^{-1}\frac{1}{3}, \tan^{-1}\left(-\frac{1}{3}\right)$

(c) $\tan^{-1}\frac{4}{3}, \tan^{-1}\left(-\frac{4}{3}\right)$

(d) $\tan^{-1}\frac{1}{2}, \tan^{-1}\left(-\frac{1}{2}\right)$

(87) The gradient of one of the lines $x^2 + hxy + 2y^2 = 0$ is twice that of the other, then h =

(a) ± 3 (b) $\pm\frac{3}{2}$ (c) ± 2 (d) ±1

(88) The sum of slopes of the lines given by $3x^2 + 5xy - 2y^2 = 0$ is

(a) $\frac{5}{2}$ (b) $-\frac{3}{2}$ (c) $-\frac{5}{2}$ (d) $\frac{1}{2}$

(89) The angle between the lines $x^2 + 2xy \sec\theta + y^2 = 0$ is

(a) 2θ (b) 3θ (c) θ (d) $\frac{\theta}{2}$

(90) The equation $x^2 + ky^2 + 4xy = 0$ represents two coincident lines, if k =

(a) 0 (b) 1 (c) 4 (d) 16

(91) The nature of straight lines represented by the equation $4x^2 + 12xy + 9y^2 = 0$ is

(a) Real and coincident

(b) Real and different

(c) Imaginary and different

(d) None of the above

(92) If the lines represented by $(k^2 + 2)x^2 + 3xy - 6y^2 = 0$ are perpendicular to each other, then the value of k is :

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

(93) If one line of the equation $3x^2 + kxy - y^2 = 0$ bisects the angle between the coordinate axes, then k =

(a) ± 1 (b) ± 3 (c) – 2 (d) $\frac{1}{2}$

(94) If the slopes of the lines represented by $6x^2 - \lambda xy + y^2 = 0$ are in the ratio 2 : 3, then λ =

(a) ± 2 (b) ± 3 (c) ± 5 (d) ± 6

(95) If the angles made by the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ with X-axis are α and β, then $\tan(\alpha + \beta)$ is :

(a) $\frac{h}{a+b}$ (b) $\frac{2h}{a-b}$ (c) $\frac{2h}{a+b}$ (d) $\frac{h}{a-b}$

(96) Out of the two lines $5x^2 + 3xy - y^2 = 0$, the equation of one line is $kx + 3y = 0$, then $k^2 + 9k$ =

(a) 40 (b) 45 (c) – 45 (d) – 40

(97) If one of the lines given by $x^2 + 2kxy + 4y^2 = 0$ is perpendicular to $x - 3y = 0$, then $k =$

(a) $\frac{37}{6}$ (b) $\frac{-37}{6}$ (c) $\frac{41}{6}$ (d) $-\frac{41}{6}$

(98) If one of the lines $kx^2 + xy - y^2 = 0$ bisects the angle between the coordinate axis, then $k =$

(a) 0 or 2 (b) 1 or 2

(c) –1 or 2 (d) 2 or 3

(99) If the slopes of the lines $kx^2 - 4xy + y^2 = 0$ differ by 2, then $k =$

(a) 1 (b) 2 (c) 3 (d) 4

(100) The angle between the lines $xy = 0$ is

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

(101) If $y = mx$ be one of the two lines given by the equation $2x_2 + 4xy + y^2 = 0$, then

(a) $m^2 + 2m + 4 = 0$ (b) $m^2 + 4m + 2 = 0$

(c) $2m^2 + 4m + 1 = 0$ (d) $2m^2 + 4m = 0$

(102) The slopes of the lines $x^2 - 3xy + 2y^2 = 0$ differ by:

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 3

(103) The angle between the lines

$(x^2 + y^2)\sqrt{3} = 6xy$ is

(a) $\tan^{-1}\sqrt{3}$ (b) $\tan^{-1}\sqrt{2}$

(c) $\tan^{-1}\frac{1}{2}$ (d) $\tan^{-1}\frac{1}{3}$

(104) The angle between the lines $3xy - 5x = 0$ is of measure :

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

(105) If the pair of straight lines given by $Ax^2 + 2Hxy + By^2 = 0$, (H2 > AB) forms an equilateral triangle with line $ax + by + c = 0$, then (A + 3B) (3A + B) is :

(a) H^2 (b) $-H^2$ (c) $2H^2$ (d) $4H^2$

4.3 General 2nd degree equation of pair of lines :-

(106) In general second degree equation $f(x, y) = 0$, if $D = 0$ and $h^2 - ab = 0$, then the equation represents

(a) two straight lines

(b) two parallel lines

(c) two perpendicular lines

(d) none of these

(107) The point of intersection of the lines represented by $9x^2 - 6xy + y^2 + 18x - 6y + 3 = 0$ is

(a) (0, 0) (b) (1, 2)

(c) (2, 1) (d) none of these

(108) The point of intersection of the lines represented by $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ is

(a) $\left(\frac{14}{25}, \frac{11}{25}\right)$ (b) $\left(\frac{11}{25}, \frac{14}{25}\right)$

(c) $\left(\frac{14}{5}, \frac{11}{5}\right)$ (d) $\left(\frac{11}{5}, \frac{14}{5}\right)$

(109) The distance between the parallel lines given by

$x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$ is

(a) 2 (b) $2\sqrt{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{2\sqrt{3}}$

(110) If the equation $x^2 + 3xy + 2y^2 + x - y + k = 0$ represented a pair of line, then $k =$

(a) – 12 (b) – 6 (c) 6 (d) 12

(111) If the equation $2x^2 + 4xy - py^2 + 4x + qy + 1 = 0$ represented a pair of perpendicular line, then

(a) $p = 2, q = 0, 4$ (b) $p = 2, q = 0, -4$

(c) $p = 2, q = 0, 8$ (d) $p = 2, q = 0, -8$

(112) If one of the lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ passes through the origin, then

(a) $a = -b$ (b) $c = 0$

(c) $a = b$ (d) $h = 0$

(113) If the equation $k^2x^2 + 10xy + 3y^2 - 15x - 2ly + 18 = 0$ represents a pair of mutually perpendicular lines, then

(a) $k = 5$ (b) $k = \pm\sqrt{2}$

(c) $k = 3$ (d) k is not real

(114) If the equation $ax^2 + hxy + by^2 + 4gx + 6fy + 4c = 0$ represents of pair of lines, then

(a) $4abc + 4fgh = 4.5af^2 + 4bg^2 + h^2$

(b) $4abc + 6fgh = 9af^2 + 4bg^2 + ch^2$

(c) $4abc + 2fgh = 9af^2 + 2bg^2 + h^2$

(d) $4abc + 12fgh = 9af^2 + 4bg^2 + 2h^2$

(115) If the angle between the two lines $ax^2 + xy + by^2 = 0$ is 45^0, then

(a) $f^2 + g^2 = ac$ (b) $f^2 + g^2 + ac$

(c) $g^2 = f^2 + ac$ (d) $c^2 = a^2 + fg$

(116) Angle between straight lines given by $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) parallel lines

(117) If equation $x^2 + y^2 + 2gx + 2fy + 1 = 0$ represents a pair of lines, then

(a) $g^2 + f^2 = \frac{1}{2}$ (b) $f^2 - g^2 = 1$

(c) $f^2 + g^2 = 1$ (d) $g^2 - f^2 = 0$

(118) If the equation $ax^2 + by^2 + cx + cy = 0$ represents a pair of straight lines, then

(a) $a(b + c) = 0$ (b) $b(c + a) = 0$

(c) $c(a + b) = 0$ (d) $a + b + c = 0$

(119) If the angle between the pair of straight lines represented by the equation $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$ is $\tan^{-1}\left(\frac{1}{3}\right)$, where '$\lambda$' is a non-negative real number. Then λ is

(a) 2 (b) 0 (c) 3 (d) 1

(120) The equation

$(x - 5)^2 + (x - 5)(y - 6) - 2(y - 6)^2 = 0$ represents

(a) a circle

(b) Two straight lines passing through origin

(c) Two straight lines passing through the point (5, 6)

(d) An ellipse

(121) Distance between the pair of lines represented by the equation

$x^2 - 6xy + 9y^2 + 3x - 9y - 4 = 0$ is

(a) $\frac{15}{\sqrt{10}}$ (b) $\frac{1}{2}$ (c) $\sqrt{\frac{5}{2}}$ (d) $\frac{1}{\sqrt{10}}$

(122) The equation $xy + a^2 = a(x + y)$ represents

(a) a parabola

(b) a pair of straight lines

(c) an ellipse

(d) two parallel straight lines

(123) Two lines are given by $(x - 2y)^2 + k(x - 2y) = 0$. The value of k so that the distance between them is 3, is

(a) $\frac{1}{\sqrt{5}}$ (b) $\pm\frac{2}{\sqrt{5}}$

(c) $\pm 3\sqrt{5}$ (d) None of these

(124) The slopes of the lines given by the equation $2x^2 + xy - y^2 + x + 4y - 3 = 0$, are

(a) -1 and -2 (b) 1 and 2

(c) -1 and 2 (d) -1 and 3

(125) If the lines $x^2 - y^2 - 2x + 2y = 0$ and $x + 2y + k = 0$ are concurrent, then the value of k is :

(a) -3 (b) 3 (c) 2 (d) -2

(126) The distane between the parallel lines $4x^2 - 12xy + 9y^2 + 16x - 24y + 7 = 0$:

(a) $\frac{5}{\sqrt{13}}$ (b) $\frac{2}{\sqrt{13}}$ (c) $\frac{7}{\sqrt{13}}$ (d) $\frac{6}{\sqrt{13}}$

(127) The angle between the lines $3xy - 5x = 0$ is of measure :

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

(128) If the pair of lines $x^2 + axy + y^2 = 0$ and $x^2 - xy - y^2 = 0$ have one line common, then

(a) $a^2 = 2$ (b) $a^2 = 3$

(c) $a^2 = 4$ (d) $a^2 = 5$

4.4 Mise.

(129) The perimeter of an equilateral triangle formed by the lines $x^2 - 4xy + y^2 = 0$ and $x + y = \sqrt{6}$ is

(a) $3\sqrt{3}$ sq. units (b) 3 sq. units

(c) 2 sq. units (d) 6 sq. units

(130) Let ΔOAB is formed by the lines $x^2 - 4xy + y^2 = 0$ and the line AB given by $x + y - 2 = 0$. Then the equation of the median of the triangle drawn from O is

(a) $x + y = 0$ (b) $x - y = 0$

(c) $7x + 8y = 0$ (d) $7x - 8y = 0$

(131) The joint equation of a pair of lines which bisects angle between the lines given by $x^2 + 3xy + 2y^2 = 0$, is

(a) $3x^2 + 2xy + 3y^2 = 0$

(b) $3x^2 - 2xy - 3y^2 = 0$

(c) $3x^2 + 2xy - 3y^2 = 0$

(d) $3x^2 - 2xy + 3y^2 = 0$

(132) The product of lengths of the perpendiculars from point (2, 3) on the lines given by $2x^2 + 6xy - y^2 = 0$ is

(a) $\frac{7}{9\sqrt{3}}$ (b) $\frac{7}{3\sqrt{5}}$ (c) $\frac{7\sqrt{5}}{9}$ (d) $\frac{7\sqrt{5}}{3}$

(133) The distance between the point of intersection of the two lines $2009x^2 - 2010xy + 2011y^2 = 0$ and the point (1, 1)

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $2 + \sqrt{3}$

(134) If $y = mx$ is one of the bisectors of an angle between the lines $ax^2 - 2hxy + by^2 = 0$, then

(a) $h(1 + m^2) + m(a - b) = 0$

(b) $h(1 - m^2) + m(a + b) = 0$

(c) $h(1 - m^2) + m(a - b) = 0$

(d) $h(1 + m^2) + m(a + b) = 0$

(135) If the area of the Δ formed by the pair of lines $8x^2 - 6xy + y^2 = 0$ and the line $2x + 3y = a$ is 7 sq. units, then : $a =$

(a) 14 (b) $14\sqrt{2}$

(c) 28 (d) none of these

(136) The equation of the disector of the angle between the lines $3x - 4y + 7 = 0$ and $12x - 5y - 8 = 0$

(a) $99x - 77y + 51 = 0$, $21x + 27y - 131 = 0$

(b) $99x - 77y + 51 = 0$, $21x + 27y + 131 = 0$

(c) $99x - 77y + 131 = 0$, $21x + 27y - 51 = 0$

(d) $99x - 77y + 131 = 0$, $21x + 27y + 51 = 0$

(137) The equation of the bisectors of the angle between lines represented by equation

$4x^2 - 16xy - 7x^2 = 0$ is

(a) $8x^2 + 11xy - 8y^2 = 0$

(b) $8x^2 - 11xy - 8y^2 = 0$

(c) $16x^2 + 11xy - 16y^2 = 0$

(d) $16x^2 + 11xy + 16y^2 = 0$

(138) The diagonals of a parallelogram PQRS are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then PQRS must be a

(a) rectangle.

(b) square.

(c) cyclic quadrilateral.

(d) rhombus.

(139) The area of the triangle formed by the lines $4x^2 - 9xy - 9y^2 = 0$ and $x = 2$ is

(a) 2 (b) 3 (c) $\frac{10}{3}$ (d) $\frac{20}{3}$

(140) If one of the lines given by the equation $2x^2 + axy + 3y^2 = 0$ coincide with one of those given by $2x2 + bxy - 3y^2 = 0$ and the other lines represented by them be perpendicular, then

(a) a = – 5, b = 1 (b) a = 5, b = – 1

(c) a = 5, b = 1 (d) none of these

(141) The area of triangle formed by the lines $11x^2 - 16xy - y^2 = 0$ and $x + 2y = \sqrt{10}$ is :

(a) $\frac{1}{\sqrt{3}}$ Sq. units (b) $\frac{2}{\sqrt{3}}$ Sq. units

(c) $\sqrt{3}$ Sq. units (d) $2\sqrt{3}$ Sq. units

(142) The joint equation of the angle bisectors of the angles between the lines

$4x^2 - 16xy + 7y^2 = 0$ is :

(a) $8x^2 - 3xy + 8y^2 = 0$ (b) $8x^2 + 3xy + 8y^2 = 0$

(c) $8x^2 + 3xy - 8y^2 = 0$ (d) $8x^2 - 3xy - 8y^2 = 0$

(143) The orthocentre of the triangle formed by the lines $5x^2 + 4xy - 5y^2 = 0$ and $x + 2y = 5$ is

(a) (1, 5) (b) (– 5, 4)

(c) (– 4, – 5) (d) (0, 0)

(144) If the lines $2x^2 + 2kxy - 3y^2 = 0$ and $3x + 2y = 5$ form an isosceles triangle, then k =

(a) 4 (b) 5 (c) 6 (d) 7

(145) The area of triangle formed by the lines

$x^2 - 4xy - y^2 = 0$ and $x - 2y - 5 = 0$ is :

(a) $\sqrt{5}$ Sq. Units (b) $5\sqrt{5}$ Sq. Units

(c) $3\sqrt{5}$ Sq. Units (d) $\sqrt{\frac{5}{2}}$ Sq. Units

(146) The product of the perpendicular distances of the point (1, – 5) from the lines $5x^2 - 3xy + y^2 = 0$ is

(a) 8 (b) 7 (c) 9 (d) 10

(147) The length of each perpendicular side of an isosceles right angled triangle formed by $15x^2 - 16xy - 15y^2 = 0$ and $4x - y + 12 = 0$

(a) $\frac{12\sqrt{2}}{\sqrt{17}}$ (b) $\frac{9\sqrt{2}}{\sqrt{17}}$ (c) $\frac{8\sqrt{2}}{\sqrt{17}}$ (d)

(148) If the lines given by $6x^2 + hxy + ky^2 = 0$ and $5x + y = 3$ form an isosceles right angled triangle right angled at the origin, then the values of h and k respectively, are :

(a) 5, – 6 (b) – 5, 6 (c) 10, 3 (d) 6, – 5

(149) One of the lines represented by the equation $y^2 + 3xy = 0$ is

(a) x - axis (b) y = axis

(c) Parallel to x - axis (d) Parallel to Y - axis

CLASS WORK - ANSWER KEY

1 **c**	2 **c**	3 **a**	4 **d**	5 **c**	6 **a**	7 **a**	8 **b**	9 **b**	10 **a**
11 **b**	12 **b**	13 **c**	14 **c**	15 **a**	16 **c**	17 **d**	18 **c**	19 **d**	20 **d**
21 **b**	22 **a**	23 **c**	24 **c**	25 **a**	26 **a**	27 **d**	28 **a**	29 **b**	30 **a**
31 **a**	32 **c**	33 **a**	34 **c**	35 **c**	36 **a**	37 **b**	38 **c**	39 **d**	40 **a**
41 **b**	42 **b**	43 **c**	44 **a**	45 **b**	46 **d**	47 **d**	48 **d**	49 **a**	50 **d**
51 **b**	52 **c**	53 **b**	54 **a**	55 **c**	56 **c**	57 **a**	58 **b**	59 **a**	60 **d**
61 **a**	62 **a**	63 **d**	64 **d**	65 **b**	66 **d**	67 **a**	68 **b**	69 **a**	70 **b**
71 **b**	72 **b**	73 **d**							

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HOME WORK - ANSWER KEY

1 **c**	2 **d**	3 **a**	4 **d**	5 **a**	6 **a**	7 **b**	8 **b**	9 **b**	10 **c**
11 **a**	12 **c**	13 **d**	14 **c**	15 **a**	16 **a**	17 **c**	18 **c**	19 **a**	20 **b**
21 **b**	22 **b**	23 **d**	24 **a**	25 **a**	26 **a**	27 **a**	28 **c**	29 **a**	30 **b**
31 **d**	32 **b**	33 **d**	34 **c**	35 **d**	36 **b**	37 **d**	38 **b**	39 **c**	40 **b**
41 **b**	42 **c**	43 **b**	44 **d**	45 **c**	46 **c**	47 **a**	48 **d**	49 **a**	50 **d**
51 **c**	52 **c**	53 **b**	54 **d**	55 **b**	56 **d**	57 **c**	58 **b**	59 **a**	60 **c**
61 **d**	62 **a**	63 **b**	64 **c**	65 **d**	66 **a**	67 **b**	68 **a**	69 **d**	70 **c**
71 **c**	72 **c**	73 **c**	74 **a**	75 **c**	76 **b**	77 **d**	78 **d**	79 **a**	80 **b**
81 **a**	82 **a**	83 **c**	84 **a**	85 **a**	86 **c**	87 **a**	88 **a**	89 **c**	90 **c**
91 **a**	92 **b**	93 **c**	94 **c**	95 **b**	96 **b**	97 **a**	98 **a**	99 **c**	100 **d**
101 **b**	102 **c**	103 **b**	104 **d**	105 **d**	106 **b**	107 **d**	108 **d**	109 **a**	110 **b**
111 **c**	112 **b**	113 **d**	114 **b**	115 **a**	116 **a**	117 **c**	118 **c**	119 **a**	120 **c**
121 **c**	122 **b**	123 **c**	124 **c**	125 **a**	126 **d**	127 **d**	128 **d**	129 **d**	130 **b**
131 **c**	132 **d**	133 **c**	134 **c**	135 **c**	136 **a**	137 **a**	138 **d**	139 **c**	140 **c**
141 **b**	142 **d**	143 **d**	144 **c**	145 **b**	146 **c**	147 **a**	148 **a**	149 **a**	

✪✪✪✪✪

CLASS WORK

Hints and Solution

(1) (c) $ax - by = 0$ and $bx + ay = 0$

$ab(x^2 - y^2) + (a^2 - b^2)xy = 0$

$\Rightarrow \quad abx^2 + a^2xy - aby^2 - b^2xy = 0$

$\Rightarrow \quad ax(bx + ay) - by(ay + bx) = 0$

$\Rightarrow \quad (bx + ay)(ax - by) = 0$

$\Rightarrow \quad bx + ay = 0$ and $ax - by = 0$

(2) (c) $(\tan\alpha \pm \sec\alpha)x - y = 0$

$x^2 + 2xy\tan\alpha - y^2 = 0$

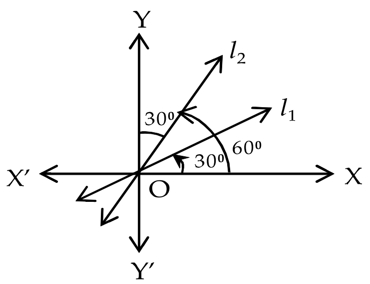
$$y = \left(\frac{-\tan\alpha \pm \sqrt{\tan^2\alpha + 1}}{-1}\right)x$$

$$\Rightarrow \quad y = \left(-\tan\alpha \pm \sqrt{\sec^2\alpha}\right)x$$

$\Rightarrow \quad y = (\tan\alpha \pm \sec\alpha)x$

$\Rightarrow \quad (\tan\alpha \pm \sec\alpha)x - y = 0$

(3) (a) $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$



Let l_1 and l_2 the lines trisecting angle between OX and OY

From figure,

Slope of line $l_1 = m_1 = \tan 30^0 = \dfrac{1}{\sqrt{3}}$

Slope of line $l_2 = m_2 = \tan 60^0 = \sqrt{3}$

Their equations are

$y = m_1x = \dfrac{x}{\sqrt{3}}$ and $y = m_2x \ \sqrt{3}x$

Joint equation is

$$\left(\sqrt{3}y - x\right)\left(y - \sqrt{3}x\right) = 0$$

$\Rightarrow \quad \sqrt{3}y^2 - 3xy - xy + \sqrt{3}x^2$

$\Rightarrow \quad \sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$

(4) (d) $3x^2 + 5xy + 2y^2 - 16x - 13y + 21 = 0$

Required equation is

$3(x - 1)^2 + 5(x - 1)(y - 2) + 2(y - 2)^2 = 0$

$\Rightarrow \quad 3x^2 - 6x + 3 + 5xy - 10x - 5y + 10 + 2y^2 - 8y + 8 = 0$

$\Rightarrow \quad 3x^2 + 5xy + 2y^2 - 16x - 13y + 21 = 0$

(5) (c) $3x^2 + 11xy + 6y^2 - 16x - 13y - 5 = 0$

Equation of line parallel to the line

$x + 3y - 1 = 0$ is

$x + 3y + c_1 = 0$ ...(ii)

Equation of lines perpendicular to line

$2x - 3y - 1 = 0$ is

$3x + 2y + c_2 = 0$ ...(ii)

Lines (i) and (ii) passess through point $(-1, 2)$, then $c_1 = -5$ and $c_2 = -1$

Lines are $x + 3y - 5 = 0$ and $3x + 2y - 1 = 0$

Joint equation is

$(x + 3y - 5)(3x + 2y - 1) = 0$

$\Rightarrow \quad 3x^2 + 11xy + 6y^2 - 16x - 13y - 5 = 0$

(6) (a) $3x^2 + 8xy + 3y^2 = 0$

Here $a = 5$, $2h = -8$, $b = 3$

Required joint equation is

$bx^2 - 2hxy + ay^2 = 0$

$\Rightarrow \quad 3x^2 + 8xy + 3y^2 = 0$

(7) (a) $23x^2 + 48xy + 3y^2 = 0$

Let $y = mx$ be one of the line passing through the origin and making an angle of 30^0 with line $3x + 2y - 11 = 0$ whose slope is $\dfrac{-3}{2}$, then

$$\tan 30^0 = \left|\frac{m-\left(\frac{-3}{2}\right)}{1+m\left(\frac{-3}{2}\right)}\right| \Rightarrow \frac{1}{\sqrt{3}} = \left|\frac{2m+3}{2-3m}\right|$$

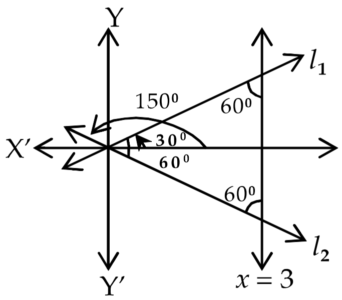
$\Rightarrow$ $(2 - 3m)^2 = 3(2m + 3)^2$

$\Rightarrow$ $4 - 12m + 9m^2 = 12m^2 + 36m + 27$

$\Rightarrow$ $3\left(\frac{y}{x}\right)^2 + 48\left(\frac{y}{x}\right) + 23 = 0$

$\Rightarrow$ $3y^2 + 48xy + 23x^2 = 0$

(8) (b) $x^2 - 3y^2 = 0$



l_1 and l_2 be the line through origin making an equilateral triangle with line x = 3

From figure

Slope of line $l_1 = m_1 = \tan 30^0 = \frac{1}{\sqrt{3}}$

Slope of line $l_2 = m_2 = \tan 150^0 = \frac{-1}{\sqrt{3}}$

Equation of line l_2 and l_2 are

$y = m_1x = \frac{x}{\sqrt{3}}$ and $y = m_2x = \frac{-x}{\sqrt{3}}$

Joint equation is

$\left(x-\sqrt{3}y\right)\left(x+\sqrt{3}y\right) = 0$

$\Rightarrow$ $x^2 - 3y^2 = 0$

(9) (b) (0, 1)

$L_1 \cup L_2 : (x + y - 1)^2 - 4x^2 = 0$

$\therefore$ $[(x + y - 1) + (2x)]\,[(x + y - 1) - 2x] = 0$

$\therefore$ $(3x + y - 1)(-x + y - 1) = 0$

$\therefore$ $L_1 : y = 1 - 3x,\ L_2 : y = x + 1$

$\therefore$ equating y's : $1 - 3x = x + 1$

$\therefore$ $0 = 4x$

$\therefore$ $x = 0$

$\therefore$ $y = x + 1 = 0 + 1 = 1$

$\therefore$ $L_1 \cap L_2 = (0, 1)$

(10) (a) $x = \frac{-f}{g}, y = \frac{-g}{h}$

$L_1 \cup L_2 : hxy + gx + \frac{f\,h}{g}y + f = 0$

$\therefore$ $(gx + f) + \left(hxy + \frac{fh}{g}y\right) = 0$

$\therefore$ $g\,(gx + f) + hy\,(gx + f) = 0$

$\therefore$ $(gx + f)(g + hy) = 0$

$\therefore$ $L_1 : gx + f = 0,\ L_2 : g + hy = 0$

$\therefore$ $L_1 : x = -\frac{f}{g},\ L_2 : y = -\frac{g}{h}$

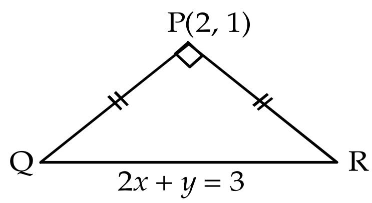
(11) (b) $b\,(x - x_1)^2 - 2h\,(x - x_1)(y - y_1) + a\,(y - y_1)^2 = 0$

(12) (b) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

Slope of QR = – 2.

Suppose slope of PQ = m_1

$\therefore$ $\tan 45^0 = \left|\frac{m_1+2}{1+m_1(-2)}\right|$



$\therefore$ $1 = \left|\frac{m_1+2}{1-2m_1}\right| \Rightarrow m_1 = -\frac{1}{3}$

$\therefore$ Equation of PQ passing through point P (2, 1) and having slope m_1 is

$y - 1 = -\frac{1}{3}(x - 2) \Rightarrow 3\,(y - 1) + (x - 2) = 0$ ...(i)

Let slope of PR = $m_2 = 3$ ....[$\because$ PQ $\perp$ PR]

$\therefore$ equation of PR is

$y - 1 = 3(x - 2)$

$\therefore$ $(y - 1) - 3(x - 2) = 0$ ...(ii)

Joint equation of (i) and (ii) is

$[3(y - 1) + (x - 2)]\ [(y - 1) - 3(x - 2)] = 0$

$\Rightarrow$ $3(y - 1)^2 - 8(y - 1)(x - 2) - 3(x - 2)^2 = 0$

$\Rightarrow$ $3(x - 2)^2 + 8(x - 2)(y - 1) - 3(y - 1)^2 = 0$

$\Rightarrow$ $3(x^2 - 4x + 4) + 8(xy - x - 2y + 2) - 3(y^2 - 2y + 1) = 0$

$\Rightarrow$ $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

(13) (c) (–2, 2)

Only (–2, 2) is satisfying the given equation.

(14) (c) $xy - y^2 = 0$

Equation of X-axis is $y = 0$ and equation of line through the origin and having slope 1 is $y = x$.

$\therefore$ Joint equation is $y\,(x - y) = 0$

(15) (a) 8, – 2

$(k + 1)\,x^2 - 6xy + (k - 7)\,y^2 = 0$

$\Rightarrow$ $a = k + 1,\ h = -3,\ b = k - 7$

For coincident lines $h^2 = ab$

$\Rightarrow$ $9 = (k + 1)\,(k - 7) \Rightarrow k^2 - 6k - 16 = 0$

$\Rightarrow$ $(k - 8)\,(k + 2) = 0 \Rightarrow k = 8, -2$

(16) (c) $4kh^2 = ab\,(1 + k)^2$

$ax^2 + 2hxy + by^2 = 0$

$\Rightarrow$ $m_1 + m_2 = \dfrac{-2h}{b}$ and $m_1m_2 = \dfrac{a}{b}$

Here $m_1 = km_2$, then

$(1 + k)m_2 = \dfrac{-2h}{b} \Rightarrow m_2 = \dfrac{-2h}{b(1+k)}$ and

$km_2^2 = \dfrac{a}{b} \Rightarrow k\left(\dfrac{-2h}{b(1+k)}\right)^2 = \dfrac{a}{b}$

$\Rightarrow$ $4kh^2 = ab\,(1 + k)^2$

(17) (d) 9 : 8

$\dfrac{x^2}{a} + \dfrac{2xy}{h} + \dfrac{y^2}{b} = 0$

$\Rightarrow$ $m_1 + m_2 = \dfrac{\frac{-2}{h}}{\frac{1}{b}} = \dfrac{-2b}{h}$ and

$m_1m_2 = \dfrac{\frac{1}{a}}{\frac{1}{b}} = \dfrac{b}{a}$

Here $m_1 = 2m_2$, then

$3m_2 = \dfrac{-2b}{h} \Rightarrow m_2 = \dfrac{-2b}{3h}$ and

$2m_2^2 = \dfrac{b}{a} \Rightarrow 2\left(\dfrac{-2b}{3h}\right)^2 = \dfrac{b}{a}$

$\Rightarrow \dfrac{8b^2}{9h^2} = \dfrac{b}{a} \Rightarrow \dfrac{8}{9} = \dfrac{h^2}{ab}$

$\Rightarrow \dfrac{ab}{h^2} = \dfrac{9}{8}$

(18) (c) ± 2

$3x^2 + kxy - y^2 = 0$

$\Rightarrow$ $m_1 + m_2 = \dfrac{-2h}{b} = k$ and $m_1m_2 = \dfrac{a}{b} = -3$

Here $|m_1 - m_2| = 4$

$\Rightarrow$ $(m_1 - m_2)^2 = 16$

$\Rightarrow$ $(m_1 + m_2)^2 - 4\,m_1m_2 = 16$

$\Rightarrow$ $k^2 - 4\,(-3) = 16 \Rightarrow k^2 + 12 = 16$

$\Rightarrow$ $k^2 = 4 \Rightarrow k = \pm 2$

(19) (d) $a^2b + ab^2 + 8h^3 - 6abh = 0$

$ax^2 + 2hxy + by^2 = 0$

$\Rightarrow$ $m_1 + m_2 = \dfrac{-2h}{b}$ and $m_1m_2 = \dfrac{a}{b}$

Here, $m_1 = m_2^2$, then $m_2^2 + m_2 = \dfrac{-2h}{b}$

$\Rightarrow$ $m_2\,(m_2 + 1) = \dfrac{-2h}{b}$ ...(i)

$$m_2^3 = \frac{a}{b} \qquad ...(ii)$$

From (i), we get

$$m_2^3 \ (m_2 + 1)^3 = \frac{-8h^3}{b^3}$$

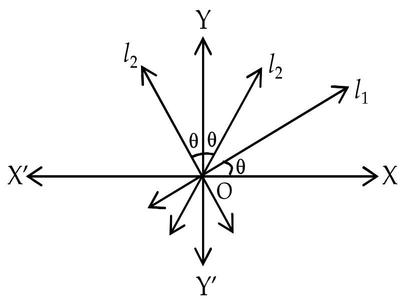
$$\Rightarrow \quad m_2^3 \ (m_2^3 + 3m_2 (m_2 + 1) + 1)^3 = \frac{-8h^3}{b^3}$$

From (i) and (ii), we get

$$\frac{a}{b}\left(\frac{a}{b} + 3\left(\frac{-2h}{b}\right) + 1\right) = \frac{-8h^3}{b^3}$$

$$\Rightarrow \quad a^2b - 6hab + ab^2 = -8h^3$$

(20) (d) $a = \pm b$



$ax^2 + 2hxy + by^2 = 0$

$$\Rightarrow \quad m_1 + m_2 = \frac{-2h}{b} \text{ and } m_1m_2 = \frac{a}{b}$$

Let θ be the angle made by line l_1 through origin with X-axis

Then the inclination of line l_2 is

$$\frac{\pi}{2} - \theta \text{ or } \frac{\pi}{2} + \theta$$

Slope of line $l_1 = m_1 = \tan\theta$

Slope of line $l_2 = m_2 = \tan\left(\frac{\pi}{2} \mp \theta\right) = \pm \cot\theta$

Now $m_1m_2 = (\tan\theta)(\pm\cot\theta)$

$$\Rightarrow \quad \frac{a}{b} = \pm 1 \qquad \Rightarrow \quad a = \pm b$$

(21) (b) 2

$$x^2 (\sec^2\theta - \sin^2\theta) - 2xy \tan\theta + y^2 \sin^2\theta = 0$$

$$m_1 + m_2 = \frac{2\tan\theta}{\sin^2\theta} \text{ and } m_1m_2 = \frac{\sec^2\theta - \sin^2\theta}{\sin^2\theta}$$

Now $(m_1 - m_2)^2 = (m_1 + m_2)^2 - m_1m_2$

$$\Rightarrow \quad (m_1 - m_2)^2 = \frac{4\tan^2\theta}{\sin^4\theta} - 4\left(\frac{\sec^2\theta - \sin^2\theta}{\sin^2\theta}\right)$$

$$\Rightarrow \quad (m_1 - m_2)^2 = \frac{4}{\sin^2\theta}\left(\frac{\tan^2\theta}{\sin^2\theta} - \sec^2\theta + \sin^2\theta\right)$$

$$\Rightarrow \quad (m_1 - m_2)^2 = \frac{4}{\sin^2\theta}$$

$$(\sec^2\theta - \sec^2\theta + \sin^2\theta) = 4$$

$$\Rightarrow \quad |m_1 - m_2| = 2$$

(22) (a) $4a + 12h + 9b = 0$

Auxiliary equation of $ax^2 + 2hxy$ _ $by^2 = 0$ is $bm^2 + 2hm + a = 0$

Slope of lines $3x - 2y = 0$ is $\frac{3}{2}$, then $m = \frac{3}{2}$

$$\text{Now } \frac{9b}{4} + 2h\left(\frac{3}{2}\right) + a = 0$$

$$\Rightarrow \quad 9b + 12h + 4a = 0$$

(23) (c) $(a + b)^2 = 4h^2$

Auxiliary equation of $ax^2 + 2hxy + by^2 = 0$ is $bm^2 + 2hm + a = 0$

One of the line given by $ax^2 + 2hxy + by^2 = 0$ bisects the angle between the co-ordinate axes, then $m = \pm 1$

How $b(\pm 1)^2 + 2h(\pm 1) + a = 0$

$$b \pm 2h + a = 0 \qquad \Rightarrow \quad a + b = \pm 2h$$

$$\Rightarrow \quad (a + b)^2 = 4h^2$$

(24) (c) $a^2 + a - 1 = 0$

Let $y = mx$ be the common line of $ax^2 + 2hxy + y^2 = 0$ and $x^2 + 3xy + y^2 = 0$, then

$m^2 + 2m + a = 0$ ...(i)

$m^2 + 3m + 1 = 0$ ...(ii)

Solving (i) and (ii), we get

$m = a - 1$

From (i), we get

$(a - 1)^2 + 2(a - 1) + a = 0$

$\Rightarrow a^2 - 2a + 1 + 2a - 2 + a = 0$

$\Rightarrow a^2 + a - 1 = 0$

(25) (a) $100(h^2 - ab) = (a + b)^2$

Let θ_1 be the angle between the lines given by $ax^2 + 2hxy + by^2 = 0$ and θ_2 be the angle between the lines given by $2x^2 - 5xy + 3y^2 = 0$.

Here $\theta_1 = \theta_2$, then

$\tan\theta_1 = \tan\theta_2$

$$\Rightarrow \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right| = \left|\frac{2\sqrt{\frac{25}{4} - (2)(3)}}{2 + 3}\right|$$

$$\Rightarrow \frac{4(h^2 - ab)}{(a + b)^2} = \frac{4(25 - 24)}{4(25}$$

$$\Rightarrow \frac{h^2 - ab}{(a + b)^2} = \frac{1}{100}$$

$\Rightarrow 100(h^2 - ab) = (a + b)^2$

(26) (a) 10

Here $a = 1,\ h = \frac{-3}{2},\ b = \lambda,\ g = \frac{3}{2},\ f = \frac{-5}{2},\ c = 0$

$$\Delta = 0 \Rightarrow \begin{vmatrix} 1 & -3/2 & 3/2 \\ -3/2 & \lambda & -5/2 \\ 3/2 & -5/2 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 2\lambda - \frac{25}{4} + \frac{3}{2}\left(-3 + \frac{15}{4}\right) + \frac{3}{2}\left(\frac{15}{4} - \frac{3\lambda}{2}\right) = 0$$

$$\Rightarrow 2\lambda - \frac{25}{4} + \frac{9}{8} + \frac{45}{8} - \frac{9\lambda}{4} = 0$$

$$\Rightarrow \frac{-\lambda}{4} = \frac{-1}{2} \quad \Rightarrow \lambda = 2$$

$$\text{Now } \tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{2\sqrt{\frac{9}{4} - 2}}{1 + 2}\right| = \left|\frac{\sqrt{9 - 8}}{3}\right| = \frac{1}{3}$$

$\Rightarrow \cot\theta = 3$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + 9 = 10$

(27) (d) $4h^2$

Let $y = mx$ one of the line given by $ax^2 + 2hxy + by^2 = 0$ form an equilateral triangle with the line $lx + my = 1$

$$\text{Then } \tan 60^0 = \left|\frac{4\sqrt{h^2 - ab}}{a + b}\right|$$

$$\Rightarrow \sqrt{3} = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right| \Rightarrow 3 = \left|\frac{4(h^2 - ab)}{(a + b)^2}\right|$$

$\Rightarrow 3a^2 + 6ab + 3b^2 = 4h^2 - 4ab$

$\Rightarrow 3a^2 + 10ab + 3b^2 = 4h^2$

$\Rightarrow (3a + b)(a + 3b) = 4h^2$

(28) (a) p and q have same sign

(29) (b) p and q have opposite sign

Here $a = p,\ h = 0,\ b = -q$

$\Delta = h^2 - ab = 0 - p(-q) = pq$

$\Delta > 0$, if p, q are of same sign

$\Delta = 0$, if $p = 0$ or $q = 0$

$\Delta < 0$, if p, q are of opposite sign.

(30) (a) $\frac{-6}{7}$

Let m_1 and m_2 be the slopes of the lines represented by $6x^2 + 41xy - 7y^2 = 0$ then $m_1 = \tan\alpha$ and $m_2 = \tan\beta$

Now $\tan\alpha \tan\beta = m_1 m_2 = \frac{a}{b} = \frac{-6}{7}$

(31) (a) 1, – 55

If $\theta = \cot^{-1}2$, then $\cot\theta = 2$, i.e., $\tan\theta = 1/2$.

$\therefore \left|\frac{2\sqrt{4-3b}}{3+b}\right| = \frac{1}{2} \quad \therefore \quad 16(4-3b) = (3+b)^2$

$\therefore \ b^2 + 54b - 55 = 0 \quad \therefore \quad (b+55)(b-1) = 0$

$\therefore \ b = -55, 1.$

(32) (c) $\sqrt{(a+b)^2 + 4ab}$

(33) (a) 5

(34) (c) ± 9

(35) (c) 60^0

Coefficients of x^2 + foefficients of $y^2 = 0$

(36) (a) $-\frac{6}{7}$

$\tan\alpha \tan\beta = m_1 m_2 = \frac{a}{b} = - -\frac{6}{7}$

(37) (b) $(-10)^2$

$\tan\theta_1 = \tan\theta_2$

$$\Rightarrow \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{\frac{25}{6}-6}}{5}\right| = \left|\frac{1}{5}\right|$$

Squaring both sides, we get

$4 \times 25(h^2 - ab) = (a+b)^2$

i.e. $100(h^2 - ab) = (a+b)^2$

Comparing with given condition,

$k(h^2 - ab) = (a+b)^2$, we get $k = 100$

(38) (c) 27

$L_1 \parallel L_2$

$a = 3, h = 9, b = b$

$$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{81-3b}}{3+b}\right|$$

Since $\tan\pi = 0$, we have

$81 - 3b = 0 \quad \Rightarrow \quad 81 = 2b \quad \Rightarrow \quad b = 27$

Alternate method :

If $L_1 \parallel L_2$

$\Rightarrow \quad h^2 = ab$

$\therefore \ 81 = 3b \qquad \therefore \ b = 27$

(39) (d) 9ab

$4\lambda h^2 = ab(1+\lambda)^2$

Put $\lambda = 5$,

$4(5)h^2 = 36ab \qquad \therefore \ 5h^2 = 9ab$

(40) (a) $2x^2 + xy - 3y^2 - 5x - 20y - 25 = 0$

Perpendicular to $3x^2 + xy - 2y^2 = 0$ and through origin is $-2x^2 - xy + 3y^2 = 0$.

If line passes through (2, –3), then

$-2(x-2)^2 - (x-2)(y+3) + 3(y+3)^2 = 0$

$\therefore \ 2(x-2)^2 + (x-2)(y+3) - 3(y+3)^2 = 0$

$\therefore \ 2x^2 + xy - 3y^2 - 5x - 20y - 25 = 0$

(41) (b) 1

$3x^2 + 4xy + \lambda y^2 = 0$

If the slope of one line is p times the slope of other line, then : $4ph^2 = ab(1+p)^2$

Here $a = 3, b = l, h = 2, p = 3$

Substitule these values and solve.

(42) (b) $a = 2, b = -6$

$ax^2 + xy + by^2 = 0$

$$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$$

$$= \left|\frac{2\sqrt{\frac{1}{4}-ab}}{a+b}\right| = \left|\frac{\sqrt{1-4ab}}{a+b}\right|$$

If $\theta = 450$, then $\left|\dfrac{\sqrt{1-4ab}}{a+b}\right| = 1$

From the given option, If $a = 1$, $b = -6$

$$\left|\frac{\sqrt{1+24}}{1-6}\right| = \left|\frac{5}{-5}\right| = 1$$

$\therefore \quad a = 1, b = -6.$

(43) (c) $\dfrac{a}{h} = \dfrac{h}{b} = \dfrac{g}{f}$

(44) (a) $(-3, -1)$

Here $a = \lambda$, $h = \dfrac{-5}{2}$, $b = 6$, $g = \dfrac{1}{2}$,

$f = \dfrac{-3}{2}$, $c = 0$

$$\text{Now } \Delta = 0 \Rightarrow \begin{vmatrix} \lambda & -5/2 & 1/2 \\ -5/2 & 6 & -3/2 \\ 1/2 & -3/2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \lambda\left(0-\frac{9}{4}\right) + \frac{5}{2}\left(0+\frac{3}{4}\right) + \frac{1}{2}\left(\frac{15}{4}-3\right) = 0$$

$$\Rightarrow \frac{-9\lambda}{4} + \frac{15}{8} + \frac{3}{8} = 0$$

$$\Rightarrow \frac{-9\lambda}{4} + \frac{-18}{8} \qquad \Rightarrow \lambda = 1$$

Point of intersection is

$$\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$$

$$\equiv \left(\frac{\left(\frac{-5}{2}\right)\left(\frac{-3}{2}\right)-(6)\left(\frac{1}{2}\right)}{(1)(6)-\frac{25}{4}}, \frac{\left(\frac{1}{2}\right)\left(\frac{-5}{2}\right)-(1)\left(\frac{-3}{2}\right)}{(1)(6)-\frac{25}{4}}\right)$$

$$\equiv \left(\frac{\frac{15}{4}-3}{6-\frac{25}{4}}, \frac{\frac{-5}{4}+\frac{3}{2}}{6-\frac{25}{4}}\right) \equiv \left(\frac{3}{-1}, \frac{1}{-1}\right) \equiv (-3, -1)$$

(45) (b) -3

$x^2 - y^2 - 2x + 2y = 0 \qquad \ldots$(i)

$a = 1, h = 0, b = -1, g = -1, f = 1, c = 0$

Point of intersection of lines given by (i) is

$$\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$$

$$= \left(\frac{0-(-1)(-1)}{(1)(-1)-0}, \frac{0-(1)(1)}{(1)(-1)-0}\right) \equiv (1, 1)$$

Line $x + 2y + k = 0$ and the lines given by (i) are concurrent, then $1 + 2(1) + k = 0$

$\Rightarrow \quad k = -3$

(46) (d) $p = 8, q = 1$

Here $a = 2$, $h = 4$, $b = p$, $g = \dfrac{q}{2}$, $f = 1$, $c = -15$

Line are parallel, then

$h^2 - ab = 0 \quad \Rightarrow \quad 16 - 2p = 0$

$\Rightarrow \quad -2p = -16 \quad \Rightarrow \quad p = 8$

$$\text{Now } \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2 & 4 & q/2 \\ 4 & 8 & 1 \\ q/2 & 1 & -15 \end{vmatrix} = 0$$

$$\Rightarrow 2(-120-1) - 4\left(-60-\frac{q}{2}\right) + \frac{q}{2}(4-4q) = 0$$

$\Rightarrow \quad -242 + 240 + 2q + 2q - 2q^2 = 0$

$\Rightarrow \quad -2q^2 + 4q - 2 = 0 \quad \Rightarrow \quad q^2 - 2q + 1 = 0$

$\Rightarrow \quad (q-1)^2 = 0 \qquad \Rightarrow \quad q = 1$

(47) (d) $fg = ch$

Compare the given eq. with

$Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + C = 0$,

and the use: $ABC + 2FGH = AF^2$
$= BG^2 + CH^2$

(48) (d) $\|$ and $\perp$ to coordinate axes, and $\perp$ to each other

$xy - 2x + y - 2 = 0 \quad \therefore \; x(y-2) + (y-2) = 0$

$\therefore \quad (x+1)(y-2) = 0 \quad \therefore \; x + 1 = 0, y - 2 = 0$

$\therefore \quad L_1 : x = -1$ .... $\|$ Y-axis, $\perp$ X-axis

$L_2 : y = 2$ .... $\|$ X-ais, $\perp$ Y-axis

Also, $L_1 \perp L_2$.

(49) (a) $6x^2 + 5xy + y^2 = 0$

Equation of perpendicular drawn from origin on $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $bx^2 - 2hxy + ay^2 = 0$

Thus, required equation is

$12x^2 + 10xy + 2y^2 = 0$

i.e., $6x^2 + 5xy + y^2 = 0$

(50) (d) two mutually perpendicular lines.

$x^2 + 2xy + y^2 - x^2 - y^2 = 0$

$2xy = 0 \quad \Rightarrow \quad x = 0$ and $y = 0$

Hence, these are two mutually perpendicular lines (both axes).

(51) (b) $a = 3,\ c = \dfrac{6}{5}$

Since the given lines are perpendicular,

$\therefore \quad a + b = 0$

Here $a = a,\ b = -3$

$\therefore \quad a + b = 0 \quad \Rightarrow \quad a - 3, = 0 \quad \Rightarrow \quad a = 3$

Also we have $h = 1,\ g = 2,\ f = 0,\ c = c$

The condition is

$3(-3)\,c + 3(2^2) - c(1^2) = 0$

$\therefore \quad -10c + 12 = 0 \quad \Rightarrow \quad c = \dfrac{6}{5}$

(52) (c) $(-2, 2)$

Only $(-2, 2)$ is satisfying the given equation.

(53) (b) 0

Here, $a = 1,\ h = -\dfrac{3}{2},\ b = \lambda$

$$\therefore \quad \tan\theta = \left|\frac{2\sqrt{\left(-\frac{3}{2}\right)^2 - (1)\lambda}}{1+\lambda}\right|$$

$\therefore \quad$ But $\theta = \tan^{-1} 3$, i.e., $\tan\theta = 3$

$$\therefore \quad 3 = \left|\frac{2\sqrt{\frac{9-4\lambda}{4}}}{1+\lambda}\right| = \left|\frac{\sqrt{9-4\lambda}}{1+\lambda}\right|$$

$$\therefore \quad \frac{9-4\lambda}{(1+\lambda)^2} = 2 \qquad \Rightarrow \quad 9 - 4\lambda = 9\,(1+\lambda)^2$$

$\Rightarrow \quad 9 - 4\lambda = 9 + 18\lambda + 9\lambda^2$

$\therefore \quad 9\lambda^2 + 22\lambda = 0 \qquad \therefore \quad \lambda\,(9\lambda + 22) = 0$

$\therefore \quad \lambda = 0$ or $\lambda = -\dfrac{22}{9}$

But λ is non-negative

$\therefore \quad \lambda = 0$

(54) (a) $2fgh = bg^2 + ch^2$

for the pair of straight line

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

Put, $a = 0$, we get $2fgh - bg^2 - ch^2 = 0$

$\therefore \quad 2fgh = bg^2 + ch^2$

(55) (c) $(-2, 0)$

$a = 2,\ b = 3,\ h = \dfrac{7}{2},\ g = 4,\ f = 7$

Points of intersection are given by

$$\left(\frac{hf - bg}{ab - h^2}, \frac{hg - af}{ab - h^2}\right) \equiv (-2, 0)$$

(56) (c) $g = 2,\ f = 3$, c is any number

(57) (a) $a + b = 0$

Here $A = a,\ B = b,\ C = 0,\ F = \dfrac{c}{2},\ G = \dfrac{c}{2}$, $H = 0$.

Using $ABC + 2\,FGH - AF^2 - BG^2 - CH^2 = 0$

$(a)\,(b)\,(0) + 2\left(\dfrac{c}{2}\right)\left(\dfrac{c}{2}\right)(0)$

$= (a)\,\dfrac{c^2}{4} - b\,\dfrac{c^2}{4} = 0 \quad \therefore \quad -\dfrac{ac^2}{4} - \dfrac{bc^2}{4} = 0$

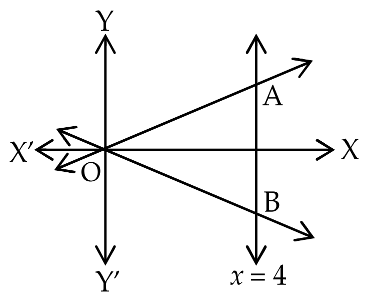
$\therefore \quad a + b = 0,$ (since $c \neq 0$, given)

(58) (b) (1, –2)

l, m, n are in A.P., then $2m = l + n$

For point (1, – 2), $lx + my + n = 0$ becomes $l - 2m + n = 0 \Rightarrow 2m = l + n$

(59) (a) an equilateral



Given lines are $x^2 - 3y^2 = 0$ ...(i)

$x = 4$ ...(ii)

From (i) and (ii), we get

$16 - 3y^2 = 0 \Rightarrow y^2 = \frac{16}{3} \Rightarrow y = \pm\frac{4}{\sqrt{3}}$

$\Rightarrow A \equiv \left(4, \frac{4}{\sqrt{3}}\right), B \equiv \left(4, \frac{-4}{\sqrt{3}}\right)$

$OA = OB = \sqrt{16 + \frac{16}{3}} = \frac{8}{\sqrt{3}}$

$AB = \left|\frac{4}{\sqrt{3}} + \frac{4}{\sqrt{3}}\right| = \frac{8}{\sqrt{3}}$

ΔOAB is an equilateral

(60) (d) a right angled isosceles

$10x^2 + 21xy - 10y^2 - 0$

$\Rightarrow 10x + 25xy - 4xy - 10y^2 = 0$

$\Rightarrow (5x - 2y)(2x + 5y) = 0$

$\Rightarrow 5x - 2y = 0$ and $2x + 5y = 0$

These lines are perpendicular to each other. Triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is an isosceles if $\frac{l^2 - m^2}{lm} = \frac{a-b}{h}$.

Here $a = 10, h = \frac{21}{2}, b = -10, l = 7, m = 3$

Now $\frac{l^2 - m^2}{lm} = \frac{49 - 9}{(7)(3)} = \frac{40}{21}$ and

$\frac{a-b}{h} = \frac{10-(-10)}{\frac{21}{2}} = \frac{40}{21}$

Triangle is a right angled isosceles.

(61) (a) $\frac{\sqrt{13}}{2}$ sq. units

Here $a = 1, h = \frac{3}{2}, b = -1, l = 1, m = 1, n = -1$

$$\text{Area} = \left|\frac{n^2\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2}\right|$$

$$= \left|\frac{(-1)^2\sqrt{\left(\frac{3}{2}\right)^2 - (1)(-1)}}{(1)(1)^2 - 2\left(\frac{3}{2}\right)(1)(1) + (1)(1)^2}\right|$$

$$= \left|\frac{(1)\sqrt{\frac{9}{4} + 1}}{1 - 3 + 1}\right| = \left|\frac{\sqrt{13}}{-2}\right| = \frac{\sqrt{13}}{2} \text{ sq. units}$$

(62) (a) $\frac{7}{\sqrt{5}}$

Area of triangle formed by the line $4x^2 + 6xy - 4y^2 = 0$ and $x - 3y + 7 = 0$ and $x - 3y + 7 = 0$ is

$$\text{Area} = \left|\frac{(7)^2\sqrt{(3)^2 - (4)(-4)}}{(4)(-3)^2 - 2(3)(1)(-3) + (-4)(1)^2}\right|$$

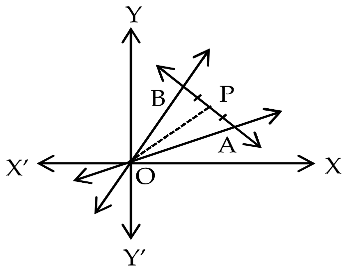
$$\text{Area} = \left|\frac{49\sqrt{9+16}}{36 + 18 - 4}\right| = \left|\frac{49(5)}{50}\right| = \frac{49}{10}$$

Triangle is an isosceles right angled, then

$\text{Area} = \frac{1}{2}(\text{side})^2 \Rightarrow \frac{49}{10} = \frac{1}{2}(\text{side})^2$

$\Rightarrow \quad (\text{side})^2 = \dfrac{49}{5} \quad \Rightarrow \quad \text{side} = \dfrac{7}{\sqrt{5}}$

(63) (d) $7x - 8y = 0$



Joint equation of lines OA and OB is

$x^2 - 4xy + y^2 = 0$ ...(i)

Equation of line AB is

$y = \dfrac{1-2x}{3}$

Let A $\equiv (x_1, y_1)$ and B $\equiv (x_2, y_2)$

From (i) and (ii), we get

$x^2 - 4x\left(\dfrac{1-2x}{3}\right) + \left(\dfrac{1-2x}{3}\right)^2 = 0$

$\Rightarrow \quad 9x^2 - 12x + 24x^2 + 1 - 4x + 4x^2 = 0$

$\Rightarrow \quad 37x^2 - 16x + 1 = 0$

This is quadratic equation in x having roots say x_1 and x_2

Then $x_1 + x_2 = \dfrac{16}{37}$

Let P be mid-points AB, then

$P \equiv \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right) \equiv \left(\dfrac{8}{37}, \dfrac{y_1 + y_2}{2}\right)$

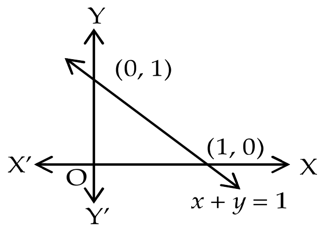
From (ii), we get

$y = \dfrac{1}{3}\left(1 - \dfrac{16}{37}\right) = \dfrac{7}{37} \quad \Rightarrow P \equiv \left(\dfrac{8}{37}, \dfrac{7}{37}\right)$

Equation of OP is

$y = \dfrac{7}{8}x \quad \Rightarrow \quad 7x - 8y = 0$

(64) (d) $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$



Triangle formed by the lines $xy = 0$ and $x + y = 1$ is a right angled.

Circumcentre of a right angled triangle is the mid-point of its hypoteneous which is $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$.

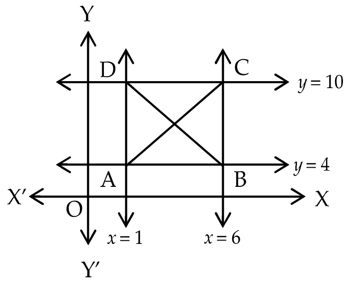
(65) (b) $6x - 5y + 14 = 0$

$x^2 - 7x + 6 = 0\ (x - 1) \quad \Rightarrow \quad (x - 1)(x - 6) = 0$

$\Rightarrow \quad x = 1$ and $x = 6$

$y^2 - 14y + 40 = 0 \quad \Rightarrow \quad (y - 4)(y - 10) = 0$

$\Rightarrow \quad y = 4$ and $y = 10$



From figure,

A $\equiv$ (1, 4), B $\equiv$ (6, 4), C $\equiv$ (6, 10), D $\equiv$ (1, 10)

Equation of diagonal AC is

$\dfrac{y-4}{10-4} = \dfrac{x-1}{6-1} \quad \Rightarrow \quad \dfrac{y-4}{6} = \dfrac{x-1}{5}$

$\Rightarrow \quad 5y - 20 = 6x - 6 \quad \Rightarrow \quad 6x - y + 14 = 0$

(66) (d) $x^2 - 2xy - y^2 = 0$

Here $a = 5$, $h = 3$, $b = -1$

Joint equation of angle bisector of lines $ax^2 + 2hxy + by^2$ is

$$\frac{x^2-y^2}{a-b} = \frac{xy}{h} \Rightarrow \frac{x^2-y^2}{5+1} = \frac{xy}{3}$$

$\Rightarrow \quad 3x^2 - 6xy - 3y^2 = 0$

$\Rightarrow \quad x^2 - 2xy - y^2 = 0$

(67) (a) $mn = -1$

Equation of bisectors of angle between lines $x^2 - 2nxy - y^2 = 0$ is

$$\frac{x^2-y^2}{1+1} = \frac{xy}{-n}$$

$\Rightarrow \quad x^2 + \frac{2}{n}xy - y^2 = 0$ ...(i)

Comparing (i) with $x^2 - 2mxy - y^2 = 0$,

we get

$$\frac{\frac{2}{n}}{-2m} = 1 \quad \Rightarrow \quad \frac{-1}{mm} = 1 \quad \Rightarrow \quad mm = -1$$

(68) (b) $\frac{31\sqrt{2}}{8}$

Here $a = 3$, $h = -2$, $b = -1$, $P(x_1, y_1) \equiv P(4, 1)$

$$\text{Product} = \frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}$$

$$= \frac{(3)(4)^2 + 2(-2)(4)(1) + (-1)(1)^2}{\sqrt{(3+1)^2 + 4(-2)^2}}$$

$$= \frac{48-16-1}{\sqrt{16+16}} = \frac{31}{\sqrt{32}}$$

$$= \frac{31}{4\sqrt{2}} = \frac{31\sqrt{2}}{8}$$

(69) (a) $(x_1y - xy_1)^2 = \delta^2(x^2 + y^2)$

Let L_1 be $y = m_1x$, i.e., $m_1x - y = 0$

$\because$ distance of (x_1, y_1) form L_1 is δ.

$$\therefore \quad \left|\frac{m_1x_1 - y_1}{\sqrt{m_1^2 + (-1)^2}}\right| = \delta$$

$$\therefore \quad |m_1x_1 - y_1| = \delta\sqrt{m_1^2 + 1}$$

But $y = m_1x \quad \Rightarrow \quad m_1 = y/x$

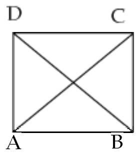
$$\therefore \quad \left|\frac{y}{x}x_1 - y_1\right| = \delta \cdot \sqrt{\frac{y^2}{x^2} + 1}$$

$$\therefore \quad |x_1y - xy_1| = \delta\sqrt{x^2 + y^2}$$

$\therefore \quad (x_1y - xy_1)^2 = \delta^2(x^2 + y^2)$

(70) (b) $(ay - bx)^2 = d^2(x^2 + y^2)$

(71) (b) $(1, -2)$



AC BD : $2x^2 - 3xy - 2y^2 = 0$

$\therefore \quad 2x^2 - 4xy + xy - 2y^2 = 0$

$\therefore \quad 2x(x - 2y) + y(x - 2y) = 0$

$\therefore \quad (x - 2y)(2x \;\; y) = 0$

$\therefore$ AC : $x - 2y = 0$, BD : $2x + y = 0$

(2, 1) satisfies equation of AC

$\therefore$ let : A $\equiv$ (2, 1)

$\therefore$ vertex adjacent to A is B or D

option (b) $(1, -2)$ satisfies the equation of BD : $2x + y = 0$

$\therefore$ correct option is $(1, -2)$

(72) (b) $\frac{a^2}{2}$

Given three lines are $x = a$, $x = 2y$ and $x = -2y$

Solving them in pairs, the three vertices of the triangle are A (0, 0), B $(a, -a/2)$ and C $(a, a/2)$.

$\therefore$ area of ΔABC

$$= \begin{vmatrix} 0 & 0 & 1 \\ a & -a/2 & 1 \\ a & a/2 & 1 \end{vmatrix} = \ldots = \frac{a^2}{2}$$

(73) (d) $x - 2y - 1 = 0$

Let : $X = x - 1$, $Y = y$

Then the equation of the pair of lines is

$aX^2 + 2yXY + bY^2 = 0$,

and one bisector is $2x + y - 2 = 0$,

i.e., $2(x - 1) + y = 0$, i.e., $2X + Y = 0$

Other bisector is perpendicular to this so that its equation is $X - 2Y = 0$,

i.e., $(x - 1) - 2y = 0$,

i.e., $x - 2y - 1 = 0$

HOME WORK

Hints and Solution

(1) (c) $\sqrt{5}\,x - 3y =$ and $\sqrt{5}\,x + 3y = 0$

$5x^2 - 9y^2 = 0 \quad \Rightarrow \left(\sqrt{5}x - 3y\right)\left(\sqrt{5}x + 3y\right) = 0$

$\Rightarrow \sqrt{5}\,x - 3y =$ and $\sqrt{5}\,x + 3y = 0$

(2) (d) $y = 0$ and $7x + 3y = 0$

$3y^2 + 7xy = 0 \Rightarrow \; y(3y + 7x) = 0$

$\Rightarrow y = 0$ and $7x + 3y = 0$

(3) (a) $x - \sqrt{3}\,y = 0$ and $\sqrt{3}\,x + y = 0$

$3x^2 - 2\sqrt{3}\,xy - 3y^2 = 0$

$\Rightarrow \sqrt{3}\,x^2 - 2xy - \sqrt{3}\,y^2 = 0$

$\Rightarrow \sqrt{3}\,x^2 - 3xy + xy - \sqrt{3}\,y^2 = 0$

$\Rightarrow \sqrt{3}\,x\left(x - \sqrt{3}y\right) + y\left(x + \sqrt{3}y\right) = 0$

$\Rightarrow \left(x - \sqrt{3}y\right)\left(\sqrt{3}x + y\right) = 0$

$\Rightarrow x - \sqrt{3}\,y = 0$ and $\sqrt{3}\,x + y = 0$

(4) (d) $(1 \pm \cos\alpha)\,x + y\sin\alpha = 0$

$$y = \left(\frac{-\operatorname{cosec}\alpha \pm \sqrt{\operatorname{cosec}^2\alpha - 1}}{1}\right)x$$

$\Rightarrow y = \left(-\operatorname{cosec}\alpha \pm \sqrt{\cot^2\alpha}\right)x$

$\Rightarrow y = (\operatorname{cosec}\alpha \pm \cot\alpha)\,x = 0$

$\Rightarrow \left(\frac{1}{\sin\alpha} \pm \frac{\cos\alpha}{\sin\alpha}\right)x + y = 0$

$\Rightarrow (1 \pm \cos\alpha)\,x + y\sin\alpha = 0$

(5) (a) $x - 2y - 4 = 0$ and $x - y - 3 = 0$

Put $X = x - 2$ and $Y = y + 1$, then

$X^2 - 3XY + 2Y^2 = 0$

$\Rightarrow X^2 - 2XY - XY + 2Y^2 = 0$

$\Rightarrow X(X - 2Y) - Y(X - 2Y) = 0$

$\Rightarrow (X - 2Y)(X - Y) = 0$

$\Rightarrow X - 2Y = 0$ and $X - Y = 0$

$\Rightarrow x - 2 - 2(y + 1) = 0$ and $x - 2 - 2(y + 1) = 0$

$\Rightarrow x - 2y - 4 = 0$ and $x - y - 3 = 0$

(6) (a) $9x^2 - 3xy - 2y^2 = 0$

Here $a = 2$, $2h = -3$, $b = -9$

Required joint equation is

$bx^2 - 2hxy + ay^2 = 0$

$\Rightarrow -9x^2 + 3xy + 2y^2 = 0$

$\Rightarrow 9x^2 - 3xy - 2y^2 = 0$

(7) (b) $5x^2 + 4xy - y^2 = 0$

Here $a = 1$, $2h = 4$, $b = -5$

Required joint equation is

$bx^2 - 2hxy + ay^2 = 0$

$\Rightarrow -5x^2 - 4xy + y^2 = 0$

$\Rightarrow 5x^2 + 4xy - y^2 = 0$

(8) (b) $3x^2 - 4xy + 2y^2 - 16x + 12y + 22 = 0$

Required equation is

$3(x - 2)^2 - 4(x - 2)(y + 1) + 2(y + 1)^2 = 0$

$\Rightarrow 3x^2 - 12x + 12 - 4xy - 4x + 8y + 8 + 2y^2 + 4y + 2 = 0$

$\Rightarrow 3x^2 - 4xy + 2y^2 - 16x + 12y + 22 = 0$

(9) (b) $15x^2 - 19xy + 6y^2 + 25x - 15y = 0$

Joint equation is

$(3x - 2y + 5)(5x - 3y) = 0$

$\Rightarrow 15x^2 - 9xy - 10xy + 6y^2 + 25x - 15y = 0$

$\Rightarrow 15x^2 - 19xy + 6xy^2 + 25x - 15y = 0$

(10) (c) $3x^2 - y^2 = 0$

Slope of lines are

$m_1 = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ and $m_2 = \tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$

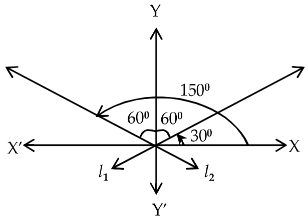
Equation of the lines through origin are

$y = m_1x = \sqrt{3}\,x$ and $y = m_2x = -\sqrt{3}\,x$

Joint equation is

$\left(\sqrt{3}x - y\right)\left(\sqrt{3}x + y\right) = 0 \quad \Rightarrow \quad 3x^2 - y^2 = 0$

(11) (a) $x^2 - 3y^2 = 0$



From figure, slope l_1 is m_1 tan $30^0 = \frac{1}{\sqrt{3}}$

slope of l_1 is m_2 tan $150^0 = \frac{-1}{\sqrt{3}}$

Equation of lines are

$y = m_1x = \frac{x}{\sqrt{3}}$ and $y = m_2x = \frac{-x}{\sqrt{3}}$

Joint equation is

$\left(x - \sqrt{3}y\right)\left(x + \sqrt{3}y\right) = 0 \quad \Rightarrow \quad x^2 - 3y^2 = 0$

(12) (c) $6x^2 - 7xy - 3y^2 + 3x + 32y - 45 = 0$

Equation on of lines perpendicular to the lines $3x + 2y - 1 = 0$ and $x - 3y + 2 = 0$ are

$2x - 3y + c_1 = 0$ ...(i)

$3x + y + c_2 = 0$ ...(ii)

Lines (i) and (ii) passes through point (2, 3), then $c_1 = 5$ and $c_2 = -9$

Lines are $2x - 3y + 5 = 0$ and $3x + y - 9 = 0$

Joint equation is

$(2x - 3y + 5)(3x + y - 9) = 0$

$\Rightarrow \quad 6x^2 - 7xy - 3y^2 + 3x + 32y - 45 = 0$

(13) (d) $39x^2 - 96xy + 11y^2 = 0$

Slope of line $3x + 4y = 5$ is $-\frac{3}{4}$

Let $y = mx$ be one of line through origin and forming an equilateral triangle with line $3x + 4y = 3$, then

$\tan 60^0 = \left|\frac{m + \frac{3}{4}}{1 - \frac{3m}{4}}\right| \quad \Rightarrow \sqrt{3} = \left|\frac{4m+3}{4-3m}\right|$

$\Rightarrow \quad 3(4 - 3m)^2 = (4m + 3)^2$

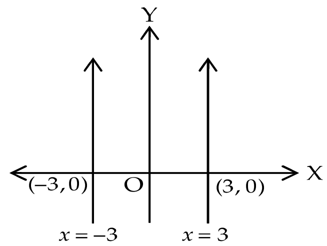
$\Rightarrow \quad 48 - 72m + 27m^2 = 16m^2 + 24m + 9$

$\Rightarrow \quad 11\left(\frac{y}{x}\right)^2 - 96\left(\frac{y}{x}\right) + 36 = 0$

$\Rightarrow \quad 11y^2 - 96xy + 39x^2 = 0$

(14) (c) p or q is zero

(15) (a) $x^2 - 9 = 0$



$u \equiv x - 3 = 0,\ v \equiv x + 3 = 0$

$\therefore \quad uv = 0 \quad \Rightarrow \quad x^2 - 9 = 0$

(16) (a) $\sqrt{3}\ (x^2 - y^2) + 2xy = 0$

$y = \sqrt{3}\,x,\ y = (-1/\sqrt{3})\,x$

$\therefore \quad u \equiv \sqrt{3}\,x - y = 0,\ v \equiv x + \sqrt{3}\,y = 0$

$\therefore \quad uv = 0 \quad \Rightarrow (\sqrt{3}\,x - y)(x + \sqrt{3}\,y) = 0$

$\therefore \quad \sqrt{3}\ (x^2 - y^2) + 2xy = 0$

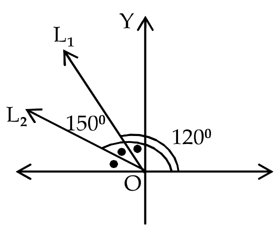
(17) (c) $\sqrt{3}\ (x^2 + y^2) + 4xy = 0$

$m_1 = \tan 120^0 = \tan(90^0 + 30^0)$

$= -\cot 30^0 = -\sqrt{3}$

$m_2 = \tan 150^0 = \tan(90^0 + 60^0)$

$= -\cot 60^0 = -1/\sqrt{3}$



$\therefore \quad L_1 : y = -\sqrt{3}\,x,\ L_2 : y = (-1/\sqrt{3})\,x$

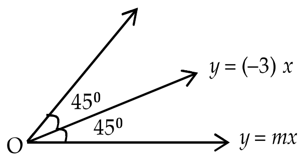
$\therefore\quad u \equiv \sqrt{3}\,x + y = 0,\ v \equiv x + \sqrt{3}\,y = 0$

$\therefore\quad L_1 \cup L_2 : uv = 0$

$\therefore\quad (\sqrt{3}\,x + y)(x + \sqrt{3}\,y) = 0$

$\therefore\quad \sqrt{3}\,(x^2 + y^2) + 4xy = 0$

(18) (c) $2x^2 + 3xy - 2y^2 = 0$



$$\tan 45^0 = \left|\frac{m-(-3)}{1+m(-3)}\right| = 1$$

$$\therefore\quad \left|\frac{\frac{y}{x}+3}{1-\frac{3y}{x}}\right| = 1 \qquad \therefore\quad |3x+y| = |x-3y|$$

$\therefore\quad 9x^2 + 6xy + y^2 = x^2 - 6xy + 9y^2$

$\therefore\quad 8x^2 + 12xy + 8y^2 = 0$

$\therefore\quad 2x^2 + 3xy - 2y^2 = 0$

(19) (a) $m_1m_2\,x^2 - (m_1 + m_2)\,xy + y^2 = 0$

Required two lines are $y = m_1x$ and $y = m_2x$,

i.e., $m_1x - y = 0$ and $m_2x - y = 0$

Their joint equation is :

$(m_1x - y)(m_2x - y) = 0$

$\therefore$ i.e., $m_1m_2\,x^2 - (m_1 + m_2)\,xy + y^2 = 0$

(20) (b) $x^2 - 4xy + y^2 = 0$

(21) (b) $x^2 - 4xy + y^2 = 0$

Let $y = mx$ be the equation of line.

Slope of the given line $y = -x - \sqrt{3}$ is -1

Since, the pair of straight lines and the given line form an equilateral traingle, angle between them is 60^0.

$$\therefore\quad \tan\frac{\pi}{3} = \left|\frac{m+1}{1-m}\right| \quad\Rightarrow\quad \sqrt{3} = \left|\frac{m+1}{1-m}\right|$$

$\therefore\quad 3(1 - m)^2 = (1 + m)^2$

$\therefore\quad 3(1 + m^2 - 2m) = (1 + m^2 + 2m)$

$\therefore\quad 3 + 3m^2 - 6m = 1 + m^2 + 2m$

$\therefore\quad m^2 - 4m + 1 = 0 \qquad$ ...(i)

Now, $y = mx \quad\Rightarrow\quad m = \dfrac{y}{x}$

Substituting the value of m in (i), we get

$$\left(\frac{y}{x}\right)^2 - 4\left(\frac{y}{x}\right) + 1 = 0 \quad\Rightarrow\quad x^2 - 4xy + y^2 = 0$$

(22) (b) $(5x + 3y - 11)(x + y - 3) = 0$

The equation of lines represented by the equatiion $3x^2 - 8xy + 5y^2 = 0$ are $3x - 5y = 0$ and $x - y = 0$.

$\therefore$ equation of lines passing through (1, 2) and perpendicular to given lines are

$x + y - 3 = 0$ and $5x + 3y - 11 = 0$

(23) (d) $11x^2 - 16xy - y^2 = 0$

Given line $2x - y = 0 \quad\Rightarrow\quad$ Slope = 2

Let the slope of required line be m

$$\therefore\quad \tan 30^0 = \left|\frac{m-2}{1+2m}\right| \qquad \therefore\quad \frac{1}{\sqrt{3}} = \left|\frac{m-2}{1+2m}\right|$$

$\therefore\quad m^2 + 16m - 11 = 0$

Line passes through origin

$\therefore\quad y = mx \quad\Rightarrow\quad m = \dfrac{y}{x}$

$$\therefore\quad \left(\frac{y}{x}\right)^2 + 16\left(\frac{y}{x}\right) - 11 = 0$$

$\therefore\quad 11x^2 - 16xy - y^2 = 0$

(24) (a) $x^2 + 4xy + y^2 = 0$

Let $y = mx$ be the line forming an equilateral triangle with the line $x - y = 4$. Now using

$$\text{Tan}\,\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$$

where $\theta = 60^0$, $m_1 = m$, $m_2 = 1$

$$\therefore\quad \tan 60 = \left|\frac{m-1}{1+m}\right|$$

$\therefore\quad 3\,(1 + m)^2 = (m - 1)^2$

$$\therefore \quad 3 + 6m + 3m^2 = m^2 - 2m + 1$$

$$\therefore \quad 2m^2 + 8m + 2 = 0$$

$$\therefore \quad m^2 + 8m + 1 = 0$$

But $y = mx$ i.e. $m = \dfrac{y}{x}$

$$\therefore \quad \frac{y^2}{x^2} + 4\frac{y}{x} + 1 = 0$$

$$\therefore \quad x^2 + 4xy + y^2 = 0$$

(25) (a) $x^2 - xy - 2y^2 - 4x + 2y + 4 = 0$

(26) (a) $5x^2 - 6xy + y^2 = 0$

Equations of line through the origin having slopes 1 and 5 are : $y = x$ and $y = 5x$.

$\therefore$ Combined equation is $(x - y)(5x - y) = 0$, etc.

(27) (a) $xy - 4x + 3y - 12 = 0$

Eqn. of lines through (–3, 4) and parallel to x-axis and y-axis respectively are $y = 4$ and $x = -3$

$\therefore$ combined eq : $(x + 3)(y - 4) = 0$, etc.

(28) (c) $23x^2 - 48xy + 3y^2 = 0$

(29) (a) $x^2 - 16xy - 11y^2 = 0$

Let the line $y = mx$ inclined at an angle of 300 to the line $x + 2y - 1 = 0$.

$$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\therefore \quad \tan 30 = \left|\frac{m + \frac{1}{2}}{1 - \frac{m}{2}}\right|$$

$$\therefore \quad \frac{1}{\sqrt{3}} = \left|\frac{2m+1}{2-m}\right|$$

$$\therefore \quad 3(4m^2 + 4m + 1) = 4 - 4m + m^2$$

$$\therefore \quad 11m^2 + 16m - 1 = 0$$

$$\therefore \quad 11\frac{y^2}{x^2} + 16\frac{y}{x} - 1 = 0$$

$$\therefore \quad x^2 - 16xy - 11y^2 = 0$$

(30) (b) $\left|\dfrac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}\right|$

(31) (d) $\dfrac{-2h}{b}$

(32) (b) 5

$k(x^2 + y^2) = (3x - y)^2$

$\Rightarrow kx^2 + ky^2 - 9x^2 - 6xy + y^2 = 0$

$\Rightarrow (k - 9)x^2 + 6xy + (k - 1)y^2 = 0$

$\Rightarrow a = k - 9,\ h = 3,\ b = k - 1$

For perpendicular lines $a + b = 0$

$\Rightarrow k - 9 + k - 1 = 0 \quad \Rightarrow 2k = 10$

$\Rightarrow k = 5$

(33) (d) $\dfrac{\pi}{4}$

$(x^2 + y^2)\sin^2\alpha = (x\cos\alpha - y\sin\alpha)^2$

$\Rightarrow x^2\sin^2\alpha + y^2\sin^2\alpha = x^2\cos^2\alpha + y^2\sin^2\alpha - 2xy\sin\alpha\cos\alpha$

$(\cos^2\alpha - \sin^2\alpha)x^2 - xy\sin 2\alpha = 0$

Lines are perpendicular, then

$\cos^2\alpha - \sin^2\alpha = 0 \quad \Rightarrow \cos^2\alpha = \sin^2\alpha$

$\Rightarrow \alpha = \dfrac{\pi}{4}$

(34) (c) 14

Let m_1 and m_2 be the slopes of the lines given by $x^2 - 4xy + y^2 = 0$, then $m_1 = \tan\alpha$ and $m_2 = \tan\beta$

Here $m_1 + m_2 = 4$ and $m_1m_2 = 1$

Now $\tan^2\alpha + \tan^2\beta = m_1^2 + m_2^2$

$= (m_1 + m_2)^2 - 2m_1m_2 = 16 - 2 = 14$

(35) (d) -4

$ax^2 + 8xy + 5y^2 = 0$

$\Rightarrow \; m_1 + m_2 = \dfrac{-2h}{b} = \dfrac{-8}{5}$ and

$m_1 m_2 = \dfrac{a}{b} = \dfrac{a}{5}$

Here $m_1 + m_2 = 2m_1 m_2$

$\Rightarrow \; \dfrac{-8}{5} = 2\left(\dfrac{a}{5}\right) \; \Rightarrow \; a = -4$

(36) (b) $4ab$

$ax^2 + 2hxy + by^2 = 0$

$\Rightarrow \; m_1 + m_2 = \dfrac{-2h}{b}$ and $m_1 m_2 = \dfrac{a}{b}$

Here $m_1 = 3m_2$, then

$4m_2 = \dfrac{-2h}{b} \; \Rightarrow \; m_2 = \dfrac{-h}{2b}$ and

$3m_2^2 = \dfrac{a}{b} \; \Rightarrow \; 3\left(\dfrac{-h}{2b}\right)^2 = \dfrac{a}{b}$

$\Rightarrow \; 3h^2 = 4ab$

(37) (d) 12

$kx^2 + 4xy - y^2 = 0$

$\Rightarrow \; m_1 + m_2 = \dfrac{-2h}{b} = 4$ and $m_1 m_2 = \dfrac{a}{b} = -k$

Here $m_1 = m_2 + 8 \; \Rightarrow \; m_1 - m_2 = 8$

$\Rightarrow \; (m_1 - m_2)^2 = 64$

$\Rightarrow \; (m_1 + m_2)^2 - 4\, m_1 m_2 = 64$

$\Rightarrow \; 16 - 4\,(-k) = 64 \; \Rightarrow \; 4k = 48k = 12$

(38) (b) 2

$(\tan^2\theta + \cos^2\theta)\, x^2 - 2xy \tan\theta + (\sin^2\theta)\, y^2 = 0$

$m_1 + m_2 = \dfrac{-2h}{b} = \dfrac{2\tan\theta}{\sin^2\theta}$ and

$m_1 m_2 = \dfrac{a}{b} = \dfrac{\tan^2\theta + \cos^2\theta}{\sin^2\theta}$

Now $(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1 + m_2$

$\Rightarrow \; (m_1 - m_2)^2 = \dfrac{4\tan^2\theta}{\sin^2\theta} - 4$

$\left(\dfrac{\tan^2\theta + \cos^2\theta}{\sin^2\theta} - \cos^2\theta\right)$

$\Rightarrow \; (m_1 - m_2)^2 = \dfrac{4}{\sin^2\theta}$

$\left(\dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}\cos^2\theta\right)$

$\Rightarrow \; (m_1 - m_2)^2 = \dfrac{4}{\sin^2\theta}\left(\dfrac{1-\sin^2\theta}{\cos^2\theta} - \cos^2\theta\right)$

$\Rightarrow \; (m_1 - m_2)^2 = \dfrac{4}{\sin^2\theta}\left(\dfrac{\cos^2\theta}{\cos^2\theta} - \cos^2\theta\right)$

$\Rightarrow \; (m_1 - m_2)^2 = \dfrac{4}{\sin^2\theta}\,(1 - \cos^2\theta) = 4$

$\Rightarrow \; |m_1 - m_2| = 2$

(39) (c) $25a - 40h + 16b = 0$

Auxiliary equatio of $ax^2 + 2hxy + by^2 = 0$ is $am^2 + 2hm + a = 0$

Slope of lines $4x + 5y = 0$ is $\dfrac{-4}{5}$,

then $m = \dfrac{-4}{5}$

Now $\dfrac{16b}{25} + 2h\left(\dfrac{-4}{5}\right) + a = 0$

$\Rightarrow \; 16b - 40h + 25a = 0$

(40) (b) $25a + 40h + 16b = 0$

Auxiliary equatio of $ax^2 + 2hxy + by^2 = 0$ is $am^2 + 2hm + b = 0$

Slope of lines $4x - 5y = 0$ is $\dfrac{4}{5}$, then $m = \dfrac{4}{5}$

Now $\dfrac{16b}{25} + 2h\left(\dfrac{-4}{5}\right) + a = 0$

$\Rightarrow \; 16b + 40h + 25a = 0$

(41) (b) $ap^2 + 2hpq + bq^2 = 0$

Auxiliary equatio of $ax^2 + 2hxy + by^2 = 0$ is $am^2 + 2hm + a = 0$

Slope of lines $px + qy = 0$ is $\frac{-p}{q}$, then $m = \frac{q}{p}$

Now $\frac{bq^2}{p^2} + 2h\left(\frac{p}{q}\right) + a = 0$

$\Rightarrow\ bq^2 + 2hpq + ap^2 = 0$

(42) (c) 60^0

$(a^2 - 3b^2)\,x^2 + 8abxy + (b^2 - 3a^2)\,y^2 = 0$

$A = a^2$, $H = 4ab$, $B = b^2 - 3a^2$

$$\text{Now } \tan\theta = \left|\frac{2\sqrt{H^2 - AB}}{A + B}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{2\sqrt{16a^2b^2 - (a^2 - 3b^2)(b^2 - 3a^2)}}{a^2 - 3b^2 + b^2 - 2a^2}\right|$$

$$= \left|\frac{2\sqrt{16a^2b^2 - a^2b^2 + 3a^4 + 3b^4 - 9a^2b^2}}{-2a^2 - 2b^2}\right|$$

$$= \left|\frac{\sqrt{3a^4 + 6a^2b^2 + 3b^4}}{-(a^2 + b^2)}\right| = \left|\frac{-\sqrt{3}(a^2 + b^2)}{a^2 + b^2}\right|$$

$$= \sqrt{3} \Rightarrow \theta = \tan^{-1}\sqrt{3} = 60^0$$

(43) (b) 45^0

$3y^2 = x(7y - 2x) \Rightarrow 2x^2 - 7xy + 3y^2 = 0$

$\Rightarrow\ a = 2,\ h = \frac{-7}{2},\ b = 3$

$$\text{Now } \tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{2\sqrt{\frac{49}{4} - 6}}{2 + 3}\right| = \left|\frac{\sqrt{49 - 24}}{5}\right|$$

$$= \left|\frac{\sqrt{25}}{5}\right| = \left|\frac{5}{5}\right| = 1 \Rightarrow \theta = 45^0$$

(44) (d) $\tan^{-1} 2$

$3x^2 + 2xy - y^2 = 0 \Rightarrow a = 3,\ h = 1,\ b = -1$

$$\text{Now } \tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{2\sqrt{1 - (3)(-1)}}{3 - 1}\right| = \left|\frac{2\sqrt{1 + 3}}{2}\right|$$

$$= \left|\sqrt{4}\right| = 2 \Rightarrow \theta = \tan^{-1} 2$$

(45) (c) $2, \frac{-25}{4}$

Let $y - mx$ be the common line of $3x^2 - 5xy + ky^2 = 0$ and $6x^2 - xy - 5y^2 = 0$, then

$km^2 - 5m + 3 = 0$ ...(i)

$5m2 + m - 6 = 0 \Rightarrow (m - 1)(5m + 6) = 0$

$\Rightarrow\ m - 1,\ m = \frac{-6}{5}$

If $m = 1$, then from (i), we get

$k - 5 + 3 = 0 \Rightarrow k = 2$

If $m = \frac{-6}{5}$, then from (i), we get

$\frac{36k}{25} + 6 + 3 = 0 \Rightarrow \frac{36k}{25} = -9$

$\Rightarrow\ k = \frac{-25}{4}$

(46) (c) $\tan^1 3$

Here $a = 1,\ h = \frac{1}{2},\ b = -2$

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right| = \left|\frac{2\sqrt{\frac{1}{4} - (1)(-2)}}{1 - 2}\right|$$

$$= \left|\frac{\sqrt{1+8}}{-1}\right| = \left|-\sqrt{9}\right| = \left|-3\right| = 3$$

$\Rightarrow \theta = \tan^{-1} 3$

(47) (a) θ

Here $a = \cos\theta - \sin\theta$, $h = \cos\theta$, $b = \cos\theta + \sin\theta$

$$\tan\phi = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

$$= \left|\frac{2\sqrt{\cos^2\theta - (\cos\theta - \sin\theta(\cos\theta + \sin\theta)}}{\cos\theta - \sin\theta + \cos\theta + \sin\theta}\right|$$

$$= \left|\frac{2\sqrt{\cos^2\theta - \cos^2\theta + \sin^2\theta}}{2\cos\theta}\right| = \left|\frac{\sin\theta}{\cos\theta}\right|$$

$= \tan\theta \quad \Rightarrow \quad \phi = \theta$

(48) (d) $196\,(h^2 - ab) = (a + b)^2$

Let θ_1 be the acute angle between lines given by $ax^2 + 2hxy + bh^2 = 0$ and θ_2 be the acute angle between lines given by $3x^2 - 7xy + 4y^2 = 0$ Here $\theta_1 = \theta_2$, then $\tan\theta_1 = \tan\theta_2 \Rightarrow$

$$\left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{2\sqrt{\frac{49}{4} - 12}}{3+4}\right|$$

$$\Rightarrow \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{\sqrt{49-48}}{7}\right|$$

$$\Rightarrow \frac{4(h^2 - ab)}{(a+b)^2} = \frac{1}{49}$$

$\Rightarrow 196\,(h^2 - ab) = (a + b)^2$

(49) (a) $xy = 0$

For the equation $ay^2 + (-1 - \lambda^2)\,xy - ax^2 = 0$, $A + B = -a + a = 0$, then the lines are perpendicular to each other.

$xy = 0$ represents the joint equation of co-ordinate axes.

(50) (d) $\sec 2\alpha$

Slope of line $x + y = 0$ is -1

Let m_1 and m_2 be the slopes of lines given by $x^2 + 2hxy + y^2 = 0$

α is the angle between the lines of $x^2 + 2hxy + y^2 = 0$ and $x + y = 0$, then

$$\tan\alpha = \left|\frac{m_1 + 1}{1 - m_1}\right| = \left|\frac{-1 - m_2}{1 - m_2}\right|$$

$$\Rightarrow m_1 = \frac{\tan\alpha - 1}{\tan\alpha + 1} \text{ and } m_2 = \frac{\tan\alpha + 1}{\tan\alpha - 1}$$

$$\text{Now } m_1 + m_2 = \frac{(\tan\alpha - 1)^2 + (\tan\alpha + 1)^2}{\tan^2\alpha - 1}$$

$$\Rightarrow -2h = \frac{-2\sec^2\alpha\cos^2\alpha}{\cos^2\alpha - \sin^2\alpha}$$

$$\Rightarrow h = \frac{1}{\cos^2\alpha} = \sec 2\alpha$$

(51) (c) p or q is zero

(52) (c) $a < 1$

For real and distinct lines $\Delta > 0$

$\Rightarrow h^2 - ab > 0 \Rightarrow 4 - a\,(4) > 0$

$\Rightarrow 1 - a > 0 \quad \Rightarrow a < 1$

(53) (b) $b = c$

$a^2x^2 - (ab + ac)\,xy + bcy^2 = 0$

$$\Rightarrow A = a^2,\ H = \frac{-a(b+c)}{2},\ B = bc$$

For coincident lines $H^2 = AB$

$$\Rightarrow \frac{a^2(b+c)^2}{4} = a^2bc$$

$\Rightarrow b^2 + 2bc + c^2 = 4bc$

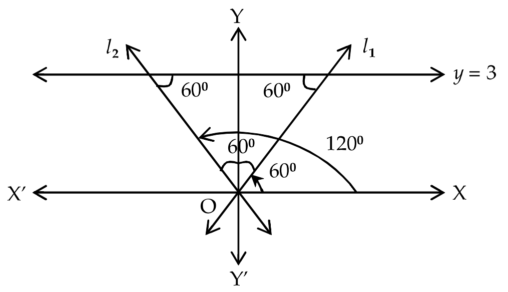
$\Rightarrow b^2 - 2bc + c^2 = 0$

$\Rightarrow (b - c)^2 = 0$

$\Rightarrow b - c = 0$

$\Rightarrow b = c$

(54) (d) $3x^2 - y^2 = 0$



l_1 and l_2 be the line through origin making an equilateral triangle with line $y = 3$

From figure,

Slope of line $l_1 = m_1 = \tan 60^0 = \sqrt{3}$

Slope of line $l_2 = m_2 = \tan 120^0 = -\sqrt{3}$

Equation of lines l_2 and m_2 are

$y = m_1 x = \sqrt{3}\,x$ and $y = m_2 x = -\sqrt{3}\,x$

Joint equation is

$\left(\sqrt{3}x - y\right)\left(\sqrt{3}x + y\right) = 0 \quad \Rightarrow \quad 3x^2 - y^2 = 0$

(55) (b) $al^2 + 2lhm + bm^2 = 0$

Let $L_1 \cup L_2 : ax^2 + 2hxy + by^2 = 0$ ...(i)

and $L_1 : mx - ly = 0$, i.e.,

$y = (mx/l)$, etc.

(56) (d) $\dfrac{\pi}{2}$

$a + b = 0 + 0 = 0 \quad \therefore \quad L_1 \perp L_2$

(57) (c) $\dfrac{\pi}{4}$

$h^2 - ab = 1 - (3 + 2\sqrt{3})(-1)$

$= 1 + 3 + 2\sqrt{3}$

$= 1^2 + \sqrt{3}^2 + 2 \cdot 1 \cdot \sqrt{3} = (1 + \sqrt{3})^2$

(58) (b) $a^2 + 6ab + b^2$

$\because \quad \tan\dfrac{\pi}{4} = 1$

$\therefore \quad \left|\dfrac{2\sqrt{h^2 - ab}}{a + b}\right| = 1$

$\therefore \quad 4(h^2 - ab) = (a + b)^2$

$\therefore \quad 4h^2 - 4ab = a^2 + 2ab + b^2$

$\therefore \quad 4h^2 = a^2 + 6ab + b^2$

(59) (a) $-(14)^2$

(60) (c) 10

$\tan\theta = \left|\dfrac{2\sqrt{(9/4) - (2)}}{3}\right| = \dfrac{1}{3}$

$\therefore \quad \cot\theta = 3$

(61) (d) $\dfrac{5}{4}$

$\because \quad 2x^2 - 3xy + y^2 = 0$

$\therefore \quad (x - y)(2x - y) = 0$

$\therefore \quad y = (1)\,x \qquad \therefore\; m_1 = \tan\alpha = 1$

$y = (2)\,x \qquad \therefore\; m_2 = \tan\beta = 2$

$\therefore \quad \cot^2\alpha + \cot^2\beta = \dfrac{1}{1^2} + \dfrac{1}{2^2} = \dfrac{5}{4}$

(62) (a) $m_1 m_2 x^2 - (m_1 + m_2)xy + y^2 = 0$

Required two lines are $y = m_1 x$ and $y = m_2 x$,

i.e., $m_1 x - y = 0$ and $m_2 x - y = 0$

Their joint equation is :

$(m_1 x - y)(m_2 x - y) = 0$,

i.e., $m_1 m_2 x^2 - (m_1 + m_2)xy + y^2 = 0$

(63) (b) $a = 1, b = -6$

(64) (c) $\dfrac{4}{7}$

$\therefore \quad \frac{h^2}{4} = 6(12) \Rightarrow h^2 = (24)(12)$

$\Rightarrow \quad h^2 = \pm 12\sqrt{2}$

(74) (a) $h^2 + k^2 = 2a^2$

$(x^2 + y^2)(h^2 + k^2 - a^2) = (hx + ky)^2$

$\therefore \quad x^2(h^2 + k^2 - a^2) + y^2(h^2 + k^2 - a^2)$

$= h^2x^2 + k^2y^2 + 2hkxy$

$\therefore \quad x^2(k^2 - a^2) + y^2(h^2 - a^2) - 2hkxy = 0$

Here $A = k^2 - a^2$, $B = h^2 - a^2$, $2H = -2hk$

Since the given pair of lines are perpendicular

$\therefore \quad A + B = 0$

$\therefore \quad k^2 - a^2 + h^2 - a^2 = 0 \Rightarrow \quad h^2 + k^2 = 2a^2$

(75) (c) 4

Slope of the line $4x + 3y = 0$ is $-\frac{4}{3}$

and $-6x^2 - 5m + k = 0$

$\therefore \quad -6\left(-\frac{4}{3}\right)^2 - 5\left(-\frac{4}{3}\right) + k = 0$

$\therefore \quad k - \frac{32}{3} + \frac{20}{3} = 0$

$\therefore \quad k = \frac{12}{3} \quad \Rightarrow \quad k = 4$

(76) (b) $\frac{-b}{1+a}$

Equation is $ax^2 - bxy - y^2 = 0$ and $m_1 = \tan\alpha$, $m_2 = \tan\beta$

$m_1 + m_2 = \frac{b}{-1} = \tan\alpha + \tan\beta$

and $m_1m_2 = \frac{a}{-1} = \tan\alpha\tan\beta$

Now $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{-b}{1-(-a)} = \frac{-b}{1+a}$

(77) (d) $\frac{5}{4}$

$m_1 = \tan\alpha$ and $m_2 = \tan\beta$

$\therefore \quad \cot\alpha = \frac{1}{m_1}$ and $\cot\beta = \frac{1}{m_2}$

$m_1 + m_2 = \frac{-2h}{b} = 3; \; m_1m_2 = \frac{a}{b} = 2$

$\therefore \quad \cot^2\alpha + \cot^2\beta = \frac{1}{m_1^2} + \frac{1}{m_2^2} = \frac{m_1^2 + m_2^2}{(m_1m_2)^2}$

$= \frac{(m_1 + m_2)^2 - 2m_1m_2}{(m_1m_2)^2}$

$= \frac{(3)^2 - 2(2)}{(2)^2} = \frac{5}{4}$

(78) (d) $h = 0$ and $ab < 0$

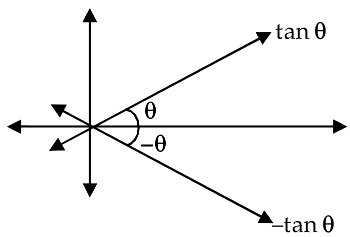
If one makes angle θ with X-axis, then other should also make angle θ with X-axis

$\therefore \quad m_1 = \tan\theta$ and $m_2 = -\tan\theta$

$\therefore \quad m_1 + m_2 = 0$

$\therefore \quad \frac{-2h}{b} = 0 \quad \Rightarrow \quad h = 0$

$\therefore \quad h^2 - ab > 0 \quad \Rightarrow \quad h^2 > ab \quad \Rightarrow \quad ab < 0$



(79) (a) 0

Here, $\tan 2A = \frac{2\sqrt{\frac{k^2}{4} + \tan^2 A}}{1 - \tan^2 A}$

$\Rightarrow \quad \frac{2\tan A}{1 - \tan^2 A} = \frac{2\sqrt{\frac{k^2}{4} + \tan^2 A}}{1 - \tan^2 A}$

(65) (d) $\frac{-1}{3}, -3$

Let m_1 and m_2 be the slopes of the lines $ax^2 + (3a + 1)\,xy + 3y^2 = 0$

Given that $m_1 = \frac{1}{m_2} \Rightarrow m_1m_2 = 1$

Also, $m_1m_2 = \frac{a}{b} = \frac{a}{3} \Rightarrow \frac{a}{3} = 1 \Rightarrow a = 3$

$m_1 + m_2 = -\frac{2h}{b} = -\left(\frac{3a+1}{2}\right) = \frac{-10}{3}$

$m_1 + \frac{1}{m_1} = \frac{-10}{3} \Rightarrow 3m_1^2 + 10m_1 + 3 = 0$

$\therefore\ m_1 = \frac{-1}{3}$ or -3.

(66) (a) $2x^2 = y(x + 2y)$

$2x^2 = xy + 2y^2 \Rightarrow a = 2, b = -2$

$\therefore\ a + b = 0$

Hence, the lines are perpendicular.

(67) (b) $a + b = -2h$

Let the equation of one of the line be $y = x$

$\therefore\ m_1 = 1$

Let m_2 be the slope of the other line.

$\therefore\ m_1 + m_2 = \frac{-2h}{b}$; $m_1m_2 = \frac{a}{b}$; Since $m_1 = 1$,

We have

$m_2 = \frac{a}{b}$; Also, $m_1 + m_2 = \frac{-2h}{b}$

$\Rightarrow 1 + \frac{a}{b} = \frac{-2h}{b} \qquad \therefore\ a + b = -2h$

(68) (a) $3x^2 - 5xy + 2y^2 = 0$

Interchange the coefficients of x^2 and y^2,

$3x^2 - 5xy + 2y^2 = 0$

(69) (d) $-\frac{2}{\sqrt{3}}\,ab$

Let m_1 and m_2 be the slopes of the given lines Given that $m_1 = 3m_2$

$\therefore\ m_1 + m_2 = 4m_2 = \frac{-2h}{b^2} \Rightarrow m_2 = \frac{-h}{2b^2}$

Also, $m_1m_2 = \frac{a^2}{b^2} = 3m_2^2 \Rightarrow m_2^2 = \frac{a^2}{3b^2}$

$m_2^2 = \frac{a^2}{4b^2} = \frac{a^2}{3b^2} \Rightarrow h^2 = \frac{4}{3}a^2b^2$

$\therefore\ h = -\frac{2}{\sqrt{3}}\,ab$

(70) (c) $\frac{2}{11}$

$$p = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{25-24}}{11}\right| = \left|\frac{2}{11}\right| = \frac{2}{11}$$

(71) (c) $\frac{\pi}{2} - \theta$

$$\theta = \tan^{-1}\left(\frac{2\sqrt{1-\sin^2\theta}}{2\sin\theta}\right) = \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow \tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\theta\right)\right\} = \frac{\pi}{2} - \theta$$

(72) (c) $a^2 + bc = 0$

Since, lines are $\perp^r$ we have

$\therefore$ coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow a^2 + bc = 0$

(73) (c) $\pm 12\sqrt{2}$

Comparing $6x^2 + hxy + 12y^2 = 0$

with $Ax^2 + 2Hxy + By^2 = 0$

we get $A = 6$, $B = 12$ and $H = \frac{h}{2}$

Since lines are parallel,

$\Rightarrow \quad \dfrac{k^2}{4} + \tan^2 A = \tan^2 A \quad \Rightarrow \quad k = 0$

(80) (b) ± 4

Coincident lines $h^2 = ab$

$\therefore \quad \dfrac{h^2}{4} = 4(1)$

$\therefore \quad h = \pm 4$

(81) (a) two values of a

Lines are perpendicular,

if coefficient of x^2 + coefficient of $y^2 = 0$

i.e., $3a + (a^2 - 2) = 0$

$\Rightarrow \quad a^2 + 3a - 2 = 0$

Since, the equation is a quadratic equation in 'a' and B2 – 4 aC > 0,

The roots of 'a' are real and distinct.

$\therefore$ Lines are perpendicular to each other for two values of 'a'.

(82) (a) 2

Given equation of line is

$kx^2 - 5xy - 3y^2 = 0$

$\Rightarrow \quad k - 5\dfrac{y}{x} - 3\left(\dfrac{y}{x}\right)^2 = 0$

$\Rightarrow \quad k - 5m - 3m^2 = 0 \qquad \ldots(i)$

Now, slope of line $x - 2y + 3 = 0$ is $\dfrac{1}{2}$.

$\therefore$ Slope of the line perpendicular to $x - 2y + 3 = 0$ is -2.

Put m = – 2 in equation (i), we get

$k - 5(-2) - 3(-2)^2 = 0$

$\Rightarrow \quad k = -10 + 12$

$\Rightarrow \quad k = 2$

(83) (c) $\dfrac{4}{3}$

Gradients $\dfrac{m_1}{m_2} = 1 : 3$

$m_1 = m, m_2 = 3m$

$m_1 + m_2 = -\dfrac{2h}{b} \qquad \ldots(i)$

and $m_1 \cdot m_2 = \dfrac{a}{b} \qquad \ldots(ii)$

From equation (i), $m + 3m = -\dfrac{2h}{b}$

$\Rightarrow \quad m = \dfrac{-h}{2b}$

From equation (ii), $m.3m = \dfrac{a}{b}$

$\Rightarrow \quad 3.\dfrac{h^2}{4b^2} = \dfrac{a}{b} \quad \Rightarrow \quad \dfrac{h^2}{ab} = \dfrac{4}{3}$

(84) (a) 9 : 8

Let m_1, m_2 be the slopes

$\therefore \quad m_1 + m_2 = -\dfrac{2b}{h}$ and $m_1 m_2 = \dfrac{b}{a}$

Given, $m_2 = 2m_1$

$\therefore \quad 3m_1 = -\dfrac{2b}{h}$ and $2\, m_1^2 = \dfrac{b}{a}$

$\therefore \quad \dfrac{9m_1^2}{2m_1^2} = \dfrac{4b^2}{h^2} \times \dfrac{a}{b} \quad \Rightarrow \quad ab : h^2 = 9 : 8$

(85) (a) $\dfrac{\pi}{6}$

$\sqrt{3}\; x^2 - 4xy + \sqrt{3}\; y^2 = 0$

$\tan \theta = \pm \dfrac{2\sqrt{4-3}}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = 30^0$

Hence, acute angle is 30^0 or $\dfrac{\pi}{6}$

(86) (c) $\tan^{-1}\frac{4}{3}$, $\tan^{-1}\left(-\frac{4}{3}\right)$

$$\tan\theta = \pm 2\ \frac{\sqrt{h^2-ab}}{a+b} = \pm 2\ \frac{\sqrt{144-44}}{15} = \pm\ \frac{4}{3}$$

$$\Rightarrow\ \theta = \tan^{-1}\left(-\frac{4}{3}\right)$$

(87) (a) $\pm$ 3

If the slope of one of the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ be λ times that of the other, then $41h^2 = ab\ (1 + \lambda)^2$.

$$\therefore\quad 4 \times 2 \times \left(\frac{h}{2}\right)^2 = 1 \times 2(1+2)^2$$

$$\Rightarrow\ h^2 = 9$$

$$\Rightarrow\ h = \pm 3$$

(88) (a) $\frac{5}{2}$

Here, a = 3, h = $\frac{5}{2}$, b = – 2

$$\text{Now, } m_1 + m_2 = \frac{-2h}{b} = \frac{-2(5/2)}{-2} = \frac{5}{2}$$

(89) (c) θ

Here, a = 1, h = sec θ, b = 1

Let ϕ be the angle between the lines,

$$\therefore\quad \tan\phi = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$$

$$= \left|\frac{2\sqrt{\sec^2\theta-1}}{2}\right|$$

$$\Rightarrow\ \tan\phi = \tan\theta$$

$$\Rightarrow\ \phi = \theta$$

(90) (c) 4

$$h^2 = ab \Rightarrow \quad \frac{k^2}{4} = 4(1)$$

$\therefore$ k = $\pm$ 4

(91) (a) Real and coincident

$4x^2 + 12xy + 9y^2 = 0$

$h^2 - ab = 36 - 36 = 0$

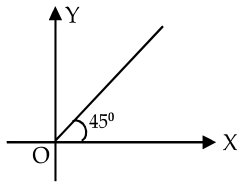
Hence, lines are real and coincident.

(92) (b) $\pm$ 2

For $\perp$ lines $a + b = 0$ $\quad\therefore\quad k^2 + 2 - 6 = 0$, etc.

(93) (c) – 2

$3x^2 + kxy - y^2 = 0$. One line is bisecting the angle between the coordinate axes, its slope if tan 45 = 1 $\quad\therefore$ Its equation is $y = x$ i.e. $x - y = 0$.



Hence $x - y = 0$ coincides with one of the lines $3x^2 + kxy - y^2 = 0$. now proceed as in, If the line $lx + my = 0$ coincides with one of the lines $ax^2 + 2hxy + by^2 = 0$,

then $am^2 - 2hlm + bl^2 = 0$

Here $l = 2$, $m = -3$

$\therefore$ (1) $(-3)^2 - 2(-2)(2)(-3) + k(2)^2 = 0$ etc.

(94) (c) $\pm$ 5

$6x^2 - \lambda xy + y^2 = 0$

$$\frac{m_1}{m_2} = \frac{2}{3}$$

i.e. $m_1 = \frac{2}{3} m_2$

i.e. slope of one line is $\frac{2}{3}$ times of the slope of other line.

If the slope of one of the lines $ax^2 + 2hxy + by^2 = 0$ is p times the slope of other line, then :

$$4ph^2 = ab(1 + p)^2$$

here $a = 5$, $h = \frac{-3}{2}$, $b = k$, $p = 5$

substitute the values and solve.

(95) (b) $\frac{2h}{a-b}$

Since α and β are the angles made by the lines with X-axis, slope of the lines are $m_1 = \tan\alpha$, $m_2 = \tan\beta$

Now $m_1 + m_2 = \frac{-2h}{b}$, $m_1m_2 = \frac{a}{b}$

$\therefore$ $\tan\alpha + \tan\beta = \frac{-2h}{b}$,

$\tan\alpha \,.\, \tan\beta = \frac{a}{b}$,

$\therefore$ $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$ etc.

(96) (b) 45

$5x^2 + 3xy - y^2 = 0$, $kx + 3y = 0$

$a = 5$, $h = \frac{3}{2}$, $b = -1$, $l = k$, $m = 3$

Using short cut :

$am^2 - 2hlm + bl^2 = 0$

$\therefore$ $(5)(9) - 2\left(\frac{3}{2}\right)(k)(3) + (-1)k^2 = 0$

$\therefore$ $45 - 9k - k^2 = 0$

$\therefore$ $45 = k^2 + 9k$

(97) (a) $\frac{37}{6}$

Since one line is $\perp$ to $x - 3y = 0$, its eq^n. is $3x + y = 0$

Hence $3x + y = 0$ coincides with one of the lines $x^2 + 2kxy + 4y^2 = 0$.

(98) (a) 0 or 2

$kx^2 + xy - y^2 = 0$

If one line disects the angle between the coordinate axes, its slope is tan 450, i.e. 1.

$\therefore$ Its eqn is $y = x$.

Hence $x - y = 0$ coincides with one of the lines $kx^2 + xy - y^2 = 0$.

Given : $m_1 + m_2 = 4m_1m_2$

$\therefore$ $\frac{2c}{-7} = 4\left(-\frac{1}{7}\right)$; $\therefore$ $2c = 4$, etc.

(99) (c) 3

$kx^2 - 4xy + y^2 = 0$

If the slopes of the lines differ by p, then :

$4(h^2 - ab) = p^2b^2$.

Here $a = k$, $h = -2$, $b = 1$, $p = 2$, etc.

(100)(d) 90^0

$xy = 0$

$x = 0$ and $y = 0$ are the eqs. of y-axis and x-axis respectively. The angle between them is 90^0.

(101)(b) $m^2 + 4m + 2 = 0$

$y = mx$ is one of the two lines given by the eqn. $2x^2 + 4xy + y^2 = 0$

$\therefore$ Using short cut : $aM^2 - 2hlM + bl^2 = 0$

Where $a = 2$, $b = 1$, $h = 2$, $l = m$, $M = -1$.

$\therefore$ $2(-1)^2 - 2(2)(m)(-1) + (1)m^2 = 0$

$\therefore$ $2 + 4m + m^2 = 0$

(102)(c) $\frac{1}{2}$

$x^2 - 3xy + 2y^2 = 0$

We have : $m_1 + m_2 = \frac{-2h}{b} = \frac{3}{2}$

and $m_1m_2 = \frac{a}{b} = \frac{1}{2}$

Now $(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4\,m_1m_2$. etc.

(103) (b) $\tan^{-1}\sqrt{2}$

As above.

(104) (d) 90^0

$a = 0, b = 0, \Rightarrow a + b = 0$

$\therefore$ The lines are $\perp$ to each other, etc.

(105)(d) $4H^2$

$$Ax^2 + 2Hxy + By^2 = 0 \qquad ...(i)$$

$$ax + by + c = 0$$

Since the triangle formed by these lines is an equilateral triangle.

$\therefore$ The angle between the pair of lines (1) is 60^0.

$$\therefore \quad \tan 60 = \left|\frac{2\sqrt{H^2 - AB}}{A+B}\right|$$

$$\sqrt{3} = \left|\frac{2\sqrt{H^2 - AB}}{A+B}\right|$$

Squaring both sides

$$3(A+B)^2 = 4(H^2 - AB)$$

$$3A^2 + 6AB + 3B^2 = 4H^2 - 4AB$$

$$\therefore \quad 3A^2 + 10AB + 3B^2 = 4H^2$$

$$\therefore \quad (A + 3B)(3A + B) = 4H^2$$

(106) (b) two parallel lines

(107)(d) none of these

Here $a = 9, h = -3, b = 1, g = 9, f = -3, c = 3$

As $h^2 - ab = 0$, then lines are parallel

(108)(d) $\left(\frac{11}{5}, \frac{14}{5}\right)$

Here $a = 2$, $h = \frac{-1}{2}$, $b = -3$, $g = -3$, $f = \frac{19}{2}$, $c = -20$

Point of intersection is

$$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right) \equiv \left(\frac{\frac{-19}{4} - 9}{-6 - \frac{1}{4}}, \frac{\frac{3}{2} - 19}{-6 - \frac{1}{4}}\right)$$

$$\equiv \left(\frac{-19 - 36}{-24 - 1}, \frac{6 - 79}{-24 - 1}\right) \equiv \left(\frac{-55}{-25}, \frac{-70}{-25}\right)$$

$$\equiv \left(\frac{11}{5}, \frac{14}{5}\right)$$

(109) (a) 2

Here $a = 1$, $h = \sqrt{2}$, $b = 2$, $g = 2$, $f = 2\sqrt{2}$, $c = 1$

$$\text{Distance} = 2\sqrt{\frac{g^2 - ac}{a(a+b)}} = 2\,\frac{4-1}{1+2}$$

$$= 2\sqrt{\frac{3}{3}} = 2$$

(110)(b) -6

Here $a = 1$, $h = \frac{3}{2}$, $b = 2$, $g = \frac{1}{2}$, $f = \frac{-1}{2}$, $c = k$

$$\text{Now} \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 3/2 & 1/2 \\ 3/2 & 2 & -1/2 \\ 1/2 & -1/2 & k \end{vmatrix} = 0$$

$$\Rightarrow 1\left(2k - \frac{1}{4}\right) - \frac{3}{2}\left(\frac{3k}{2} + \frac{1}{4}\right) + \frac{1}{2}\left(\frac{-3}{4} - 1\right) = 0$$

$$\Rightarrow 2k - \frac{1}{4} - \frac{9k}{4} - \frac{3}{8} - \frac{7}{8} = 0$$

$$\Rightarrow \frac{-k}{4} = \frac{12}{8} \qquad \Rightarrow k = -6$$

(111) (c) $p = 2, q = 0, 8$

Here $a = 2, h = 2, b = -p, g = 2, f = \frac{q}{2}, c = 1$

Lines are perpendicular, then

$a + b = 0 \quad \Rightarrow \quad 2 - p = 0 \quad \Rightarrow \quad p = 2$

Now $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2 & 2 & 2 \\ 2 & -2 & q/2 \\ 2 & q/2 & 1 \end{vmatrix} = 0$

$\Rightarrow \; 2\left(-2-\frac{q^2}{4}\right) - 2(2-q) + 2(q+4) = 0$

$\Rightarrow \; -4 - \frac{q^2}{2} - 4 + 2q + 2q + 8 = 0$

$\Rightarrow \; \frac{-q^2}{2} + 4q = 0$

$\Rightarrow \; q\,(q-8) = 0$

$\Rightarrow \; q = 0\,,\,8$

(112)(b) $c = 0$

If a curve passes through the origin, then its cartesian equation does not contain any constant term.

$\therefore \quad c = 0$

(113)(d) k is not real

If $L_1 \perp L_2$, then : $a + b = 0$

$\therefore \quad k^2 + 3 = 0$

$\therefore \quad k^2 = -3 < 0$

$\therefore \quad k$ is not real

(114)(b) $4abc + 6fgh = 9af^2 + 4bg^2 + ch^2$

$A = a, B = b, C = 4c,$

$H = h/2, G = 2g, F = 3f$

$\because \quad ABC + 2FGH = AF^2 + BG^2 + CH^2$

$\therefore \quad ab\,(4c) = 2\,(3f)\,(2g)\,(h/2)$

$\quad = a\,(3f)^2 + b\,(2g)^2 + 4c\,(h/2)^2$

$\therefore \quad 4abc + 6fgh = 9af^2 + 4bg^2 + ch^2$

(115)(a) $f^2 + g^2 = ac$

(116)(a) $\frac{\pi}{2}$

$a = -3, b = 3$

Since, $a + b = -3 + 3 = 0$, given lines are $\perp^r$ to each other.

(117) (c) $f^2 + g^2 = 1$

Condition for pair of line,

$\begin{vmatrix} 1 & 0 & g \\ 0 & 1 & f \\ g & f & 1 \end{vmatrix} = 0 \quad .... \begin{bmatrix} \because a = 1, b = 1, c = 1 \\ h = 0, g = g, f = f \end{bmatrix}$

$\therefore \quad 1 - f^2 + g(-g) = 0 \quad \Rightarrow \quad f^2 + g^2 = 1$

(118) (c) $c(a + b) = 0$

$ab(0) + 2\left(\frac{c}{2}\right)\left(\frac{c}{2}\right)(0) - a\left(\frac{c}{2}\right)^2 - b\left(\frac{c}{2}\right)^2$

$- 0(0)^2 = 0$

$\therefore \quad ac^2 + bc^2 = 0 \quad \Rightarrow c^2\,(a + b) = 0$

$\Rightarrow \quad c(a + b) = 0$

(119)(a) 2

Comparing given equation with standard form,

$a = 1,\; h = -\frac{3}{2},\; b = \lambda,$

$\theta = \tan^{-1}\left(\frac{1}{3}\right) \quad \Rightarrow \quad \tan\theta = \frac{1}{3}$

Since, $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$

$\therefore \quad \frac{1}{3} = \frac{2\sqrt{\left(\frac{-3}{2}\right)^2 - \lambda}}{\lambda + 1}$

$\therefore \quad (\lambda + 1)^2 = 9(9 - 4\lambda)$

$\therefore \quad \lambda^2 + 38\lambda - 80 = 0$

$\Rightarrow \quad (\lambda + 40)\,(\lambda - 2) = 0 \quad \Rightarrow \quad l = 2$

(120) (c) Two straight lines passing through the point (5, 6)

$(x-5)^2 + (x-5)(y-6) - 2(y-6)^2 = 0$

$x^2 + 25 - 10x + xy + 30 - 6x - 5y - 2y^2 - 72 + 24y = 0$

$\Rightarrow \; x^2 + xy - 2y^2 - 16x + 19x - 17 = 0$

Obviously, it is not a circle as $a \neq b$ and xy is present. On checking for pair of straight lines, we get that the equation represents a pair of straight lines.

Also for $x = 5$, $y = 6$, equation vanishes.

Therefore, it passes through (5, 6).

(121)(c) $\sqrt{\dfrac{5}{2}}$

$x^2 - 6xy + 9y^2 + 3x - 9y - 4 = 0$

$\therefore \; (x-3y)^2 + 3(x-3y) - 4 = 0$

$$\therefore \; x - 3y = \frac{-3 \pm \sqrt{9+16}}{2} \quad \left(\because x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{-3 \pm 5}{2} = -4 \text{ or } 1$$

$\therefore$ Lines $x - 3y + 4 = 0$ and $x - 3y - 1 = 0$ are parallel lines

$$\therefore \text{ required distance } = \left|\frac{4-(-1)}{\sqrt{1+9}}\right| = \frac{5}{\sqrt{10}} = \sqrt{\frac{25}{10}} = \sqrt{\frac{5}{2}}$$

(122)(b) a pair of straight lines

We have, $xy + a^2 = ax + ay$

$\therefore \; ax + ay - xy - a^2 = 0$

Comparing it with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$$a = 0,\ h = -\frac{1}{2},\ b = 0,\ g = \frac{a}{2},\ f = \frac{a}{2},\ c = -a^2$$

$\therefore \; abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\therefore$ the given equation represents a pair of straight lines.

(123)(c) $\pm 3\sqrt{5}$

(124)(c) -1 and 2

$2x^2 + xy - y^2 + x + 4y - 3 = 0 \qquad \ldots(i)$

Since the lines given by the homogeneous eq^n.

$2x^2 + xy - y^2 = 0$ are parallel to the lines given by eq^n (1).

Hence slopes of the lines given by eq^n (1) are the same as the slopes of the lines $2x^2 + xy - y^2 = 0$.

Factorize and find the slopes.

(125)(a) -3

$x^2 - y^2 - 2x + 2y = 0$

$\therefore \; (x-y)(x+y) - 2(x-y) = 0$

$(x-y)(x+y-2) = 0$

(126)(d) $\dfrac{6}{\sqrt{13}}$

(127)(d) 90^0

$a = 0, b = 0, \Rightarrow a + b = 0$

$\therefore$ The lines are $\perp$ to each other. etc.

(128)(d) $a^2 = 5$

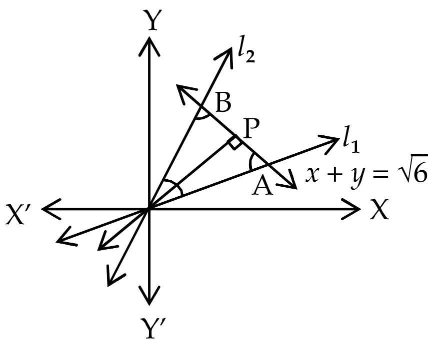
(129)(d) 6 sq. units

The joint equation of pair of lines passing through origin and forming an equilateral triangle with the line $ax + by + c = 0$ is $(ax+by)^2 - 3(bx-ay)^2 = 0$

Here $a = 1$, $b = 1$

Required joint equation is

$x^2 - 4xy + y^2 = 0$



From figure,

$$OP = \left|\frac{0+0-\sqrt{6}}{\sqrt{1+1}}\right| = \left|\frac{-\sqrt{6}}{\sqrt{2}}\right| = \left|-\sqrt{3}\right| = \sqrt{3}$$

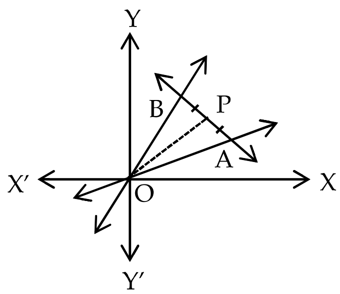
In ΔOAP

$$\sin 60^0 = \frac{OP}{OA} \quad \Rightarrow \quad OA = \sqrt{3}\cdot\frac{2}{\sqrt{3}} = 2$$

Area of $\Delta OAB = \frac{\sqrt{3}}{4}\ (2)^2 = \sqrt{3}$ sq. units

Perimeter of $\Delta OAB = 3\ (OA) = 6$ units

(130) (b) $x - y = 0$



Joint equation of lines OA and OB is

$x^2 - 4xy + y^2 = 0$ ...(i)

Equation of line AB is

$y = 2 - x$ ...(ii)

Let A $\equiv (x_1, y_1)$ and B $\equiv (x_2, y_2)$

From (i) and (ii), we get

$x^2 - 4x\ (2 - x) + (2 - x)^2 = 0$

$\Rightarrow$ $x^2 - 8x + 4x^2 + 4 - 4x + x^2 = 0$

$\Rightarrow$ $6x^2 - 12x + 4 = 0$ $\Rightarrow$ $3x^2 - 6x + 2 = 0$

This is quadratic equation is x having roots say x_1 and x_2

Then $x_1 + x_2 = 2$

Let P be mid-points AB, then

$$P \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2{}^2}{2}\right) \equiv \left(1, \frac{y_1+y_2}{2}\right)$$

From (ii), we get

$y = 2 - 1 = 1$ $\Rightarrow P \equiv (1, 1)$

Equation of OP is

$$y = \left(\frac{1-0}{1-0}\right)x \quad \Rightarrow y = x$$

(131)(c) $3x^2 + 2xy - 3y^2 = 0$

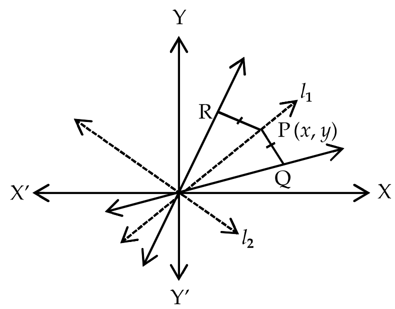
$x^2 + 3xy + 2y^2 = 0$

$\Rightarrow$ $x^2 + 2xy + xy^2 + 2y^2 = 0$

$\Rightarrow$ $(x + 2y)\ (x + y) = 0$

$x + 2y = 0$ and $x + y = 0$

Let P(x, y) be point on one of the line l_1 which bisects the angle between the lines $x + 2y = 0$ and $x + y = 0$



From figure,

$$PQ = PR \quad \Rightarrow \quad \left|\frac{x+2y}{\sqrt{1+1}}\right| = \left|\frac{x+y}{\sqrt{1+1}}\right|$$

$$\Rightarrow \quad \frac{(x+2y)^2}{5} = \frac{(x+y)^2}{2}$$

$\Rightarrow$ $2\ (x^2 + 4xy + 4y^2) = 5\ (x^2 + 2xy + y_2)$

$\Rightarrow$ $3x^2 + 2xy - 3y^2 = 0$

(132) (d) $\frac{7\sqrt{5}}{3}$

Here $a = 2$, $h = 3$, $b = -1$, $P(x_1, y_1) \equiv P(2, 3)$

$$\text{Product} = \frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}$$

$$= \frac{(2)(2)^2 + 2(3)(2)(3) + (-1)(3)^2}{\sqrt{(2+1)^2 + 4(3)^2}}$$

$$= \frac{8+36-9}{\sqrt{9+36}} = \frac{35}{\sqrt{45}}$$

$$= \frac{35}{3\sqrt{5}} = \frac{7\sqrt{5}}{3}$$

(133) (c) $\sqrt{2}$

$$\sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}$$

(134) (c) $h(1 - m^2) + m(a - b) = 0$

$\because$ one bisector is $mx - y = 0$

$\therefore$ other bisector which is perpendicular to it is

$x + my = 0$

$\therefore$ their joint equation is : $(mx - y)(x + my) = 0$

$\therefore$ $mx^2 + (m^2 - 1)xy - my^2 = 0$ ...(i)

But joint equation of bisectors of $ax^2 - 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{-h}$$

i.e., $hx^2 + (a - b)xy - hy^2 = 0$ ...(ii)

$\because$ (i) and (ii) are same equations

$\therefore$ $\frac{m}{h} = \frac{m^2 - 1}{a - b}$

$\therefore$ $m(a - b) = (m^2 - 1)h$

$\therefore$ $h(1 - m^2) + m(a - b) = 0$

(135) (c) 28

$\because$ $8x^2 - 6xy + y^2 \equiv (2x - y)(4x - y)$

$\therefore$ two sides of Δ are $y = 2x$ and $y = 4x$

Solving each with the third side $2x + 3y = a$, vertices of Δ are $(a/8, a/4)$, $(a/14, 2a/7)$ and $(0, 0)$

$\because$ its area is 7

$$\therefore \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a/8 & a/4 & 1 \\ a/14 & 2a/7 & 1 \end{vmatrix} = 7$$

$\therefore$ simplifying : $\frac{a^2}{117} = 7$

$\therefore$ $a^2 = 7(112) = 7(7 \times 16) = 7^2 . 4^2 = (28)^2$

$\therefore$ $a = 28$

(136) (a) $99x - 77y + 51 = 0$, $21x + 27y - 131 = 0$

The equation of the bisector of the angle between the given lines is,

$$\frac{3x - 47 + 7}{\sqrt{3^2 + 4^2}} = \pm\frac{12x - 5y - 8}{\sqrt{144 + 25}}$$

$$\Rightarrow \frac{3x - 4y + 7}{5} = \pm\frac{12x - 5y - 8}{13}$$

Hence, the equations are

$39x - 52y + 91 = 60x - 25y - 40$

and $39x - 52y + 91 = -60x + 25y + 40$

i.e., $21x + 27y - 131 = 0$ and

$99x - 77y + 51 = 0$

(137) (a) $8x^2 + 11xy - 8y^2 = 0$

Equation of bisector of two lines is given by

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

$$\Rightarrow \frac{x^2 - y^2}{11} = \frac{xy}{-8}$$

$$\Rightarrow 8x^2 + 11xy - 8y^2 = 0$$

(138) (d) rhombus.

$$m_1 = \frac{-1}{3} \text{ and } m_2 = 3 \Rightarrow m_1 m_2 = -1$$

Hence, lines $x + 3y = 4$ and $6x - 2y = 7$ are perpendicular to each other.

$\therefore$ The parallelogram is a rhombus.

(139)(c) $\frac{10}{3}$

(140)(c) a = 5, b = 1

(141)(b) $\frac{2}{\sqrt{3}}$ Sq. units

$11x^2 - 16xy - y^2 = 0$

$x + 2y = \sqrt{10}$

Use short cut.

Area of triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is given by

$$\text{Area} = \left| \frac{n^2\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2} \right|$$

Here : $a = 11$, $h = -8$, $b = -1$, $l = 1$.

$m = 2$, $n = -\sqrt{10}$

Substitute and simplify.

(142)(d) $8x^2 - 3xy - 8y^2 = 0$

$4x^2 - 16xy + 7y^2 = 0$

Here $a = 4$, $h = -8$, $b = 7$

Equation of angle bisectors is :

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}, \text{ etc.}$$

(143)(d) (0, 0)

$5x^2 + 4xy - 5y^2 = 0$

observe that the two lines of the above eqn. are perpendicular to each other. The triangle formed by the given lines is a right angled triangle at origin.

Hence the origin (0, 0) is the orthocenter.

(144)(c) 6

$2x^2 + 2kxy - 3y^2 = 0$, $3x + 2y = 5$

Short cut : If the lines

$ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$

form an isosceles traiangle, then

$$\frac{l^2 - m^2}{lm} = \frac{a - b}{h}$$

$$\therefore \frac{(3)^2 - (2)^2}{(3)\,(2)} = \frac{2 + 3}{k}, \text{ etc.}$$

(145)(b) $5\sqrt{5}$ Sq. Units

(146)(c) 9

As above.

(147)(a) $\frac{12\sqrt{2}}{\sqrt{17}}$

As above.

(148)(a) 5, – 6

$6x^2 + hxy + ky^2 = 0$

For $\perp$ lines, $a + b = 0$, i.e. $6 + k = 0$

$\therefore$ $k = -6$

Now use the condition for isosceles triangle.

(149)(a) x - axis

$y^2 + 3xy = 0$

$\therefore$ $y(y + 3x) = 0$

$\therefore$ $y = 0$ is the eqn. of x-axis, etc.

Points to remember

- The combined equation of coordinate axes is $xy = 0$.
- The combined equation of angle bisectors of coordinate axes is $x^2 - y^2 = 0$.
- The combined equation of angle trisectors of coordinate axes in

  a) 1st and 3rd quadrant is $\sqrt{3}\,x^2 - 4xy + y^2 = 0$ and

  b) 2nd and 4th quadrant is $x^2 + 4xy + y^2 = 0$.
- The combined equation of lines perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$.
- If lines $ax^2 + 2hxy + by^2 = 0$ and line $lx + my + n = 0$ forms isosceles triangle, then $= \dfrac{l^2 - m^2}{lm} = \dfrac{a-b}{h}$
- If slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is k times the other, then $4kh^2 = ab(1 + k)^2$.
- The equation $|x| + |y| = a$ represents a square of area $2a^2$ sq. units.
- The following combination of pair of lines forms equilateral triangle with line mention against them.

  a) $x^2 + 4xy + y^2 = 0$ and $x - y = k$

  b) $x^2 - 4xy + y^2 = 0$ and $x + y = k$

  c) $x^2 - 3y^2 = 0$ and $x = k$

  d) $3x^2 - y^2 = 0$ and $y = k$

EVALUATION PAPER - PAIR OF STRAIGHT LINES

Time : 30 Min. **Marks : 25**

(1) The separate equations of the lines represented by the equations $2x^2 + 2xy - y^2 = 0$ are

(a) $\left(1+\sqrt{3}\right)x + y = 0$ and $\left(1-\sqrt{3}\right) x + y = 0$

(b) $\left(1+\sqrt{3}\right)x - y = 0$ and $\left(1-\sqrt{3}\right) x - y = 0$

(c) $x - \left(1+\sqrt{3}\right)y = 0$ and $x - \left(1-\sqrt{3}\right) y = 0$

(d) $x - \left(\sqrt{3}+1\right)y = 0$ and $x + \left(\sqrt{3}-1\right) y = 0$

(2) Find the combine equation of the lines passing through the origin and which are at a distance of 9 units from the Y-axis

(a) $y^2 - 81 = 0$ (b) $y^2 + 81 = 0$ (c) $x^2 - 81 = 0$ (d) $x^2 + 81 = 0$

(3) The combined equation of lines passing through the point (1, 2) and perpendicular to the lines $3x + 2y - 5 = 0$ and $2x - 5y + 1$ is

(a) $10x^2 + 11xy - 6y^2 + 2x - 35y - 36 = 0$ (b) $10x^2 + 11xy - 6y^2 - 2x + 3y - 36 = 0$

(c) $10x^2 - 11xy - 6y^2 + 2x - 35y - 36 = 0$ (d) $10x^2 - 11xy - 6y^2 + 2x + 35y - 36 = 0$

(4) The joint equation of pair of lines through (2, – 3) and parallel to $x^2 + xy - y^2 = 0$ is

(a) $x^2 + xy - y^2 - x - 8y - 11 = 0$ (b) $x^2 + xy - y^2 - 8x - y - 11 = 0$

(c) $x^2 + xy - y^2 - x - 8y + 11 = 0$ (d) $x^2 + xy - y^2 - 8x - y + 11 = 0$

(5) The joint equation of pair of lines passing through the origin and making an angle of 60^0 with the line $x - y = 0$ is

(a) $x^2 - 4xy + y^2 = 0$ (b) $x^2 + 4xy + y^2 = 0$

(c) $x^2 - 3xy + y^2 = 0$ (d) $x^2 + 3xy + y^2 = 0$

(6) If pairs of opposite sides of a quadrilateral are $x^2 - 7x + 6 = 0$ and $y^2 - 14y + 40 = 0$, then equations of its diagonals are

(a) $6x + 5y = 56$, $5x + 6y = 14$ (b) $6x + 5y = 56$, $5x - 6y = 14$

(c) $6x - 5y = 56$, $6x + 5y = 14$ (d) $6x - 5y = 56$, $6x - 5y = 14$

(7) Joint equation of two lines through the origin and parallel to the pair of lines $2x^2 - xy - y^2 + 5x + y + 2 = 0$

(a) $2x^2 - xy + y^2 = 0$ (b) $5x^2 + xy + 2y^2 = 0$ (c) $2x^2 - xy - y^2 = 0$ (d) $2x^2 + xy - y^2 = 0$

(8) The separate equations of the lines whose combined equation is $x^2 + 4xy + y^2 = 0$, are :

(a) $y = (-2 \pm \sqrt{3})\, x$ (b) $y = (2 \pm \sqrt{3})\, x$ (c) $y = (\sqrt{3} \pm 2)\, x$ (d) $y = (\sqrt{3} \pm 2)$

(9) Equation of pair of lines through (1, 1) nd perpendicular to the pair of lines $3x^2 - 7xy - 2y^2 = 0$, is:

(a) $2x^2 + 7xy + 3y^2 = 0$

(b) $2(x + 1)^2 + 7(x - 1)(y - 1) + 3(y - 1)^2 = 0$

(c) $2(x - 1)^2 - 7(x - 1)(y - 1) - 3(y - 1)^2 = 0$

(d) $-2x^2 + 7xy + 3y^2 = 0$

(10) Combined equation of two lines passing through (0, 2) and each making an angle of 45^0 with X-axis, is

(a) $x^2 - (y - 2)^2 = 0$ (b) $x^2 + (y - 2)^2 = 0$ (c) $y^2 - (x - 2)^2 = 0$ (d) $y^2 + (x - 2)^2 = 0$

(11) If the lines represented by $ax^2 - bxy - y^2 = 0$ make angle α and β with X-axis, then $\tan(a + b) =$

(a) $\frac{-a}{1+b}$ (b) $\frac{a}{1+b}$ (c) $\frac{-b}{1+a}$ (d) $\frac{b}{1+a}$

(12) If the slopes of the lines given by $6x^2 + 2hxy + y^2 = 0$ are in the ratio 1 : 2, then $h =$

(a) $\frac{3\sqrt{3}}{2}$ (b) $\frac{3}{2\sqrt{3}}$ (c) $\frac{3}{2}$ (d) $\frac{27}{4}$

(13) If the angle between the lines given by $x^2 \tan^2 A - kxy - y^2 = 0$ is 2A, then $k =$

(a) 0 (b) 1 (c) 2 (d) tan A

(14) The joint equation of pair of lines passing through the origin and making an angle of 60^0 with the line $x - y = 0$ is

(a) $x^2 - 4xy + y^2 = 0$ (b) $x^2 + 4xy + y^2 = 0$

(c) $x^2 - 3xy + y^2 = 0$ (d) $x^2 + 3xy + y^2 = 0$

(15) If slope of one of the lines $ax^2 + 2hxy + by^2 = 0$ is square of the slope of the other, then the value of $\frac{a+b}{h} + \frac{8h^2}{ab} =$

(a) 4 (b) -6 (c) 6 (d) -4

(16) If one of hte lines of the pair $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then : $m =$

(a) $\frac{1}{2}$ (b) -2 (c) ± 1 (d) 2

(17) The angle between the lines represented by the equation $ax^2 + xy + by^2 = 0$ will be 45^0, if

(a) a = 1, b = 6 (b) a = 1, b = − 6 (c) a = 6, b = 1 (d) a = 1, b = 1

(18) If the sum of the slopes of the lines represented by the equation $x^2 - 2xy \tan A - y^2 = 0$ be 4, the $\angle A =$

(a) 0^0 (b) 45^0 (c) 60^0 (d) $\tan^{-1}(-2)$

(19) Two lines represented by equation $x^2 + xy + y^2 = 0$ are
(a) Coincident (b) Parallel
(c) Mutually perpendicular (d) Imaginary

(20) The distance between the parallel lines given by $x^2 + 2xy + y^2 - 8ax - 8ay - 9y^2 = 0$ is
(a) $5a$ (b) 10a (c) $2\sqrt{5}\,a$ (d) $5\sqrt{2}\,a$

(21) If the equation $px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$ represents a pair of perpendicular lines, then
(a) $p = -3,\ q = -8$ (b) $p = 3,\ q = -8$ (c) $p = -3,\ q = 8$ (d) $p = 3,\ q = 8$

(22) Parts of lines given by $3x^2 - 10xy - 3y^2 + 15x + 21y - 18 = 0$ can be taken as
(a) two sides of an equilateral triangle (b) diagonals of a rhombus
(c) opposite sides of a parallelogram (d) opposite sides of a trapezium

(23) The equation $(x + y)^2 - (x^2 + y^2) = 0$ represents
(a) a circle. (b) two lines.
(c) two parallel lines. (d) two mutually perpendicular lines.

EVALUATION PAPER - PAIR OF STRAIGHT LINES ANSWER KEY

1 **b**	2 **c**	3 **d**	4 **a**	5 **b**	6 **b**	7 **c**	8 **a**	9 **c**	10 **a**
11 **c**	12 **a**	13 **a**	14 **b**	15 **c**	16 **c**	17 **b**	18 **d**	19 **d**	20 **d**
21 **a**	22 **b**	23 **d**							

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