5 VECTORS

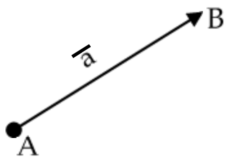
Synopsis

5.1. Vector :

A physical quantity having both magnitude and direction is called a vector.

Representation of vectors

Geometrically a vector is represented by a line segment. For eg. $\bar{a} = \overline{AB}$. Here A is called initial point & B is called termial point or tip.



Magnitude or modulus of $\bar{a}$ is expressed as

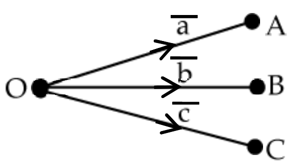
$a = |\overline{AB}|$

5.2. Types of vector :

i. Co – Intitial Vectors :

The Vectors having the same initial point are called co–intitial vectors.

ii. Position Vector :



It is a fixed point and A,B,C are any three points in space, then the vectores $\overline{OA}$, $\overline{OB}$, $\overline{OC}$ are called the position vectors of the points A, B, C respectively with refernce to point to point .

5.3. Addition of Vectors :

If $\bar{a}$ and $\bar{b}$ are any two vectors, then their addition is denoted by $\bar{a} + \bar{b}$.

i. Triangle Law :

If two co–intial vectors are represented in magnitude and direction by the two sides of a triangle, then their sum is represented in magnitude and direction by the third side of the triangle.

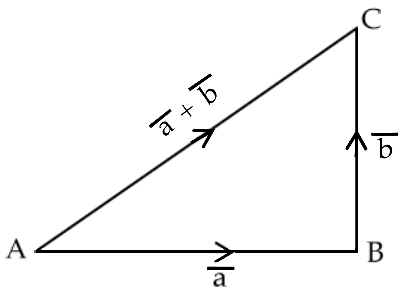


Fig. 6

In figure 6, let $\overline{AB}$ = = $\bar{b}$, then $\overline{AC}$

$= \overline{AB} + \overline{BC} = \bar{a} + \bar{b}$.

$\bar{a} + \bar{b}$ is a vector whose initial point is the initial point of vecotr $\bar{a}$ and the terminal point is the terminal point of $\bar{b}$.

$|\bar{a} + \bar{b}| \ddagger |\bar{a}| + |\bar{b}|$

ii. Parallelogram, law :

If two co–intitial vectors are represented in magnitude and direction by the adjacent sides of a parallelogram, then their sum is represented in magnitude and direction by the diagonal passing through their common point.

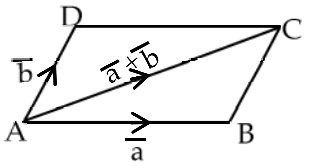


Fig : 7

In Figure, 7, let $\overline{AB} = \bar{a}$, $\overline{AD} = \bar{b}$, then

$$\overline{AC} = \overline{AB} + \overline{AD} = \bar{a} + \bar{b}.$$

5.4. Scalar multiple :

If m is a scalar (a real number) and $\bar{a}$ is a vector, then m$\bar{a}$ is a vector called the Scalar multiple of $\bar{a}$.

If m = o, then m$\bar{a}$ is a zero vector.

If m > O, them m$\bar{a}$ is in the same direction as that of $\bar{a}$ & has magnitude ma.

If m < O, then m$\bar{a}$ is in the direction opposite to that of $\bar{a}$.

5.5. Liner combination of vectors :

Let $\bar{a}$ & $\bar{b}$ any two vector and x,y be any two scalar then the vector

$x\bar{a} + y\bar{b}$ is called a linear combintion of vector $\bar{a}$ & $\bar{b}$.

A linear combination of vectors is itself a vector.

5.6. Condition for collinear vectors :

If two vectors $\bar{a}$ & $\bar{b}$. are collinear, then each of them is a scalar multiple of the other and vice versa.

5.7. Condition for coplanar vectors :

If three vectors $\bar{a}$, $\bar{b}$, $\bar{c}$ are coplanar, then each of them can be uniquely expressed as linear combination of the other two.

$\bar{a}$, $\bar{b}$, $\bar{c}$ are coplanar if and only if,

$$x\bar{a} + y\bar{b} + z\bar{c} = O$$

where, x,y,z are scalar not all zero.

5.8. Linear dependence & independence of vectors :

i. Linearly dependent vectors : A set of vectors $\bar{a}_1, \bar{a}_2, \ldots \bar{a}_n$ is said to be linearly dependent if there exist scalars x_1, $x_2, \ldots x_n$ NOT ALL ZERO

Such that $x_1\bar{a} + x_2\bar{a}_2 + \ldots\ldots + x_n\bar{a}_n = O$

ii. Linearly independent vectors : A set of vectors $\bar{a}_1, \bar{a}_2, \ldots \bar{a}_n$ is said to be linearly independent if

$x_1\bar{a} + x_2\bar{a}_2 + \ldots\ldots + x_n\bar{a}_n = O$

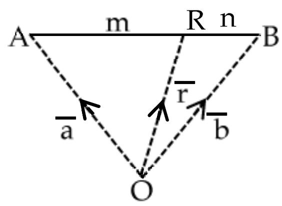
Wher $x, x_2, \ldots x_n$ are scalars

Such that $x_1 = x_2 = \ldots\ldots = x_n =$ O ALL ZERO

5.9. Section formula for internal division :

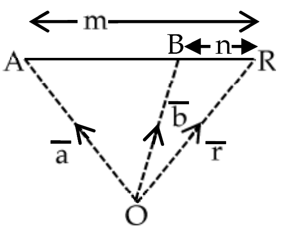
If R $(\bar{r})$ divides the line segment joining the points A $(\bar{a})$ and B$(\bar{b})$ internally in the ratio m : n

then $\bar{r} = \dfrac{m\bar{b} + n\bar{a}}{m+n}$



5.10. Section formula for external division :

Let A $(\bar{a})$ and B$(\bar{b})$ be any two points and R $(\bar{r})$ be a point on the line AB dividing the segment AB externally in the ratio m : n then $\bar{r} = \dfrac{m\bar{b} - n\bar{a}}{m - n}$



5.11. Mid point formula :

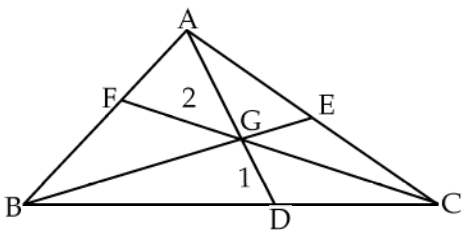
If R$(\bar{r})$ is the mid–point of the line segment joining the points A $(\bar{a})$ and B$(\bar{b})$ then

$$\bar{r} = \frac{\bar{a} + \bar{b}}{2}.$$

5.12. Centroid of a triangle formula :

The point of intersection of three medians of a triangle is called centroid. Centroid divides the medians internally in the ratio 2 : 1, the large part is towards the vertex of the triangle. If G is the centroid, then its

position vector $\vec{g}$ is given by : $\vec{g} = \dfrac{\vec{a} + \vec{b} + \vec{c}}{3}$



5.13. Centroid of a triangle formula :

If A (x_1, y_1, z_1); B (x_2, y_2, z_2); C(x_3, y_3, z_3) are the co–ordinate of vertices of a triangle ABC, then the co–ordinates of centroid of Δ ABC is

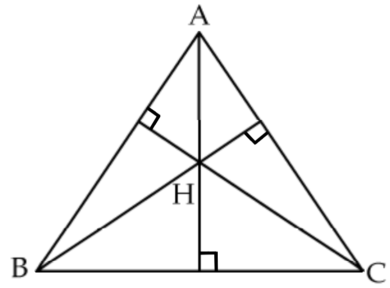
$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

5.14. Orthocentre of a triangle formula :

Orthocentre of the triangle is the point of intersection of three altitudes.

If H is the orthocentre, then the position vector of H is given by ,

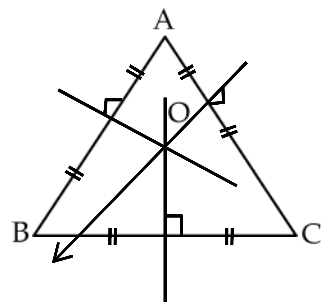
$$\vec{h} = \frac{(\tan A)\vec{a} + (\tan B)\vec{b} + (\tan C)\vec{c}}{\tan A + \tan B + \tan C}$$



5.15. Circumcentre of a triangle formula :

The point of intersection of perpendicular bisectors of the sides of a triangle is called circum – centre of triangle ABC. If O is the circum–centre if ΔABC, then its position vector is given by :

$$\vec{o} = \frac{\vec{a}\sin 2A + \vec{b}\sin 2B + \vec{c}\sin 2C}{\sin 2A + \sin 2B + \sin 2C}$$



5.16. Scalar product (Dot product) :

Def .: If $\bar{a}$ and $\bar{b}$ are two vectors and $m < (\bar{a}, \bar{b}) = \theta$, then :

$\bar{a} \cdot \bar{b} = ab\cos\theta$

Notes :

i. The Scalar product is commutative,

i.e. $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$

ii. For any two vectors $\bar{a}$ and $\bar{b}$, we have

$-ab \le \bar{a} \cdot \bar{b} \le ab.$

If $\bar{a} \uparrow\uparrow \bar{b}$, then $\bar{a} \cdot \bar{b} = ab.$

If $\bar{a} \uparrow\downarrow \bar{b}$, then $\bar{a} \cdot \bar{b} = -ab.$

iii. If $\bar{a}$ and $\bar{b}$ are non–zero vectors, then

$\bar{a} \perp \bar{b}$ iff $\bar{a} \cdot \bar{b} = 0$

iv. $(\bar{a})^2 = \bar{a} \cdot \bar{a} = |\bar{a}|^2$

Remember : $(\bar{a}+\bar{b})^2 = (\bar{a})^2 + (\bar{b})^2 + 2\bar{a} \cdot \bar{b}$

$$(\bar{a}-\bar{b})^2 = (\bar{a})^2 + (\bar{b})^2 - 2\bar{a} \cdot \bar{b}$$

$$(\bar{a}+\bar{b}) \cdot (\bar{a}-\bar{b}) = (\bar{a})^2 - (\bar{b})^2$$

v. $i.\,i. = j = k\,.\,k = 1$

vi. $i.\,j = j.\,k = k.\,i = 0$

vii. Component form of scalar product

$\bar{a} = a_1 i + a_2 j + a_3 k,\ \bar{b} = b_1 i + b_2 j + b_3 k$

then : $\bar{a} \cdot \bar{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

viii. If $m < (\bar{a}, \bar{b}) = \theta$, then $\cos\theta = \dfrac{\bar{a} \cdot \bar{b}}{ab}$

ix. The projection of $\bar{a}$ on the line of $\bar{b}$ is

$$\frac{\bar{a} \cdot \bar{b}}{b} = \bar{a} \cdot \hat{b}$$

Similarly, the projection of $\bar{b}$ on the line of $\bar{a}$ is $\dfrac{\bar{a} \cdot \bar{b}}{b} = \hat{a} \cdot \bar{b}$

5.17. Vector product (Cross product) :

Def.: Let $\bar{a} = \overline{OA}$ and $\bar{b}$ $\overline{OB}$ and be two vectors and θ be the angle through which $\bar{a}$ must be rotated about O so as to lie along $\bar{b}$. If $\hat{n}$ is the unit vector perpendicular ot the plane of $\bar{a}$ and $\bar{b}$ so that $\bar{a}, \bar{b}, \hat{n}$ form a right–handed set then :

$\bar{a} \times \bar{b} = (ab\sin\theta)\,\hat{n}$

also, $|\bar{a} \times \bar{b}| = ab\sin\theta$

Notes :

i. $\bar{b} \times \bar{a} = -\bar{a} \times \bar{b}$

ii. $|\bar{a} \times \bar{b}| = |\bar{b} \times \bar{a}| = ab\sin\theta$

iii. If $m < (\bar{a}, \bar{b}) = \theta$, then $\sin\theta = \dfrac{|\bar{a} \times \bar{b}|}{ab}$

iv. Test of Collinearity :

If $\bar{a}$ and $\bar{b}$ are non–zero vectors, then

$\bar{a} \,||\, \bar{b}$, iff $\bar{a} \times \bar{b} = \bar{0}$.

In particular, $\bar{a} \times \bar{a} = \bar{0}$

v. $i \times i = j \times j = k \times k = \bar{0}$

vi. The unit vectors, i , j , k form a right handed triad, i.e.

$i \times j = k,\ j \times k = i,\ k \times i = j.$

vii. $j \times i = -k,\ k \times j = -i,\ i \times k = -j.$

viii. "Determinant form" of vector product

If $\bar{a} = a_1 i + a_2 j + a_3 k$

and $\bar{b} = b_1 i + b_2 j + b_3 k$, then:

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

viii. A vector which is perpendicular to both $\bar{a}$ and $\bar{b}$ is along $\bar{a} \times \bar{b}$

5.18. Scalar triple product (Box product) :

If $\bar{a}, \bar{b}, \bar{c}$ are three vectors, then the scalar product of $\bar{a}$ and $\bar{b} \times \bar{c}$ is called the scalar triple product of vectors in $\bar{a}, \bar{b}$ and $\bar{c}$ in that order and is denoted by $\bar{a}.(\bar{b} \times \bar{c})$ or $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

$\therefore\ \bar{a}.(\bar{b} \times \bar{c}) = \left[\bar{a}\ \bar{b}\ \bar{c}\right]$

Scalar triple product is also know as box product.

If $\bar{a} = a_1 \tilde{i} + a_2 \tilde{j} + a_3 \tilde{k}$,

$\bar{b} = b_1 \tilde{i} + b_2 \tilde{j} + b_3 \tilde{k}$ and

$\bar{c} = c_1 \tilde{i} + c_2 \tilde{j} + c_3 \tilde{k}$, then $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

5.19. Propertise of scalar triple product :

i. $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left[\vec{b}\ \vec{c}\ \vec{a}\right] = \left[\vec{c}\ \vec{a}\ \vec{b}\right]$ i.e. vector can be changed in cyclic order.

Note that $\left[\vec{b}\ \vec{c}\ \vec{a}\right] \neq \left[\vec{a}\ \vec{b}\ \vec{c}\right]$.

But $\left[\vec{b}\ \vec{c}\ \vec{a}\right] = \vec{b} \cdot \left(\vec{a}\ \vec{c}\right) = \vec{b} \cdot -\left(\vec{c}\ \vec{a}\right)$

$= -\left[\vec{b}\ \vec{c}\ \vec{a}\right] = -\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

i.e. $\left[\vec{b}\ \vec{c}\ \vec{a}\right] = -\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

i.e. in a scalar triple product, any two vectors are interchanged, then the value of box product changes in sign only.

ii. $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \times \vec{b}\right) \cdot \vec{c}$ i.e. dot and cross can be interchanged.

iii. If one of the vectors $\vec{a}, \vec{b}, \vec{c}$ is a zero vector, then $\left(\vec{a}, \vec{b}, \vec{c}\right) = 0$

iv. If $\vec{a}, \vec{b}, \vec{c}$ are co–planar, then $\left(\vec{a}, \vec{b}, \vec{c}\right) = 0$

v. if any two vectors are identical then the value of box product is zero,

i.e. $\left[\vec{a}, \vec{a}, \vec{b}\right] = 0$

vi. $\left[\vec{a}, \vec{b}\left(\vec{c} + \vec{d}\right)\right] = \left[\vec{a}, \vec{b}, \vec{c}\right] + \left[\vec{a}, \vec{b}, \vec{d}\right]$

{ Distributive property}

vii. If x is a scalar , then $x\left[\bar{a}\ \bar{b}\ \bar{c}\right] = x\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

5.20. Lagrange's Identity :

$$\left(\bar{a}.\bar{b}\right) + \left(\bar{a}\times\bar{b}\right)^2 = a^2 b^2$$

5.21. Volume of parallelopiped :

Volume of parallelopied with co – terminus edges $\bar{a}, \bar{b}, \bar{c}$ is $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$.

5.22. Volume of tetrahedron :

Volume of a tetrahedron with co–terminus edges $\bar{a}, \bar{b}, \bar{c}$ is $\frac{1}{6}\left[\bar{a}\ \bar{b}\ \bar{c}\right]$.

5.23. Application of vectors to geometry :

i. The medians of a triangle are concurrent.

ii. The angle bisecotors of a triangle are concurrent.

iii. The altitudes of triangle are concurrent.

iv. The perpendicular bisecotrs of sides of a triangle are concurrent.

v. The segment joining the mid – points of two sides of triangle is parallel to the third side and its length is half of third side.

vi. If diagonals of a quadrilateral bisect each other, then it is parallelogram.

vii. The diagonals of a parallelogram bisect each other.

viii. The median of a trapezium is parallel to parallel sides of the trapezium and its length is half the sum of parallel sides.

ix. The segment joining the mid–points of diagonals of trapezium is parallel to parallel sides and its length is the difference of parallel sides.

x. The angle substended on a semicircle is a right angle.

5.23. Vector triple product :

Let $\bar{a}, \bar{b}, \bar{c}$ any three vectors, then the vectors $\bar{a}\times(\bar{b}\times\bar{c})$ and $(\bar{a}\times\bar{b})\times\bar{c}$ are called vector triple product of $\bar{a}, \bar{b}, \bar{c}$ thus,

$$\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c}$$

$$(\bar{a}\times\bar{b})\times\bar{c} = (\bar{a}.\bar{c})\bar{b} - (\bar{b}.\bar{c})\bar{a}$$

CLASSWORK

Multiple Choice Questions

Collinearity & Coplanarity :

(1) If the points A (4, 5, 2) B (3, 2, p) and C (5, 8, 0,) are collinear, then p =

(a) -4 (b) 4 (c) 0 (d) 2

(2) If the position vectors of the points A,B,C are $\bar{a}$, $\bar{b}$, $3\bar{a} - 2\bar{b}$ respectively, then the points A,B,C are

(a) collinear

(b) non–collinear

(c) form a right angled triangle

(d) none of these

(3) If $\bar{a} = \hat{i} + 3\hat{j}$, $\bar{b} = 2\hat{i} + 5\hat{j}$, $\bar{c} = 4\hat{i} + 2\hat{j}$ and $\bar{c} = t_1 \bar{a} + t_2 \bar{b}$

(a) $t_1 = -16$, $t_2 = -10$ (b) $t_1 = -16$, $t_2 = 10$

(c) $t_1 = 16$, $t_2 = -10$ (d) $t_1 = 16$, $t_2 = -10$

(4) If $\bar{a}, \bar{b}, \bar{c}$ are non – coplanar and the vectors $\bar{p} = 3\bar{a} + \bar{b} + \bar{c}$, $\bar{q} = 2\bar{a} + 2\bar{b} + 3\bar{c}$, $\bar{r} = \bar{a} + 3\bar{b} + m\bar{c}$ are collinear, then m =

(a) 3 (b) -3 (c) 5 (d) -5

(5) If $\bar{a}$ and $\bar{b}$ are non–collinear vectors, then

(a) only $\bar{a} = \bar{0}$

(b) only $\bar{b} = \bar{0}$

(c) both $\bar{a} \neq \bar{0}$ and $\bar{b} \neq \bar{0}$

(d) both $\bar{a} = \bar{b} = \bar{0}$

(6) Given vectors $\bar{a}, \bar{b}$ are non–collinear.

If $\bar{c} = (x-2)\bar{a} + \bar{b}$ and $\bar{d} = (2x+1)\bar{a} - \bar{b}$ are collinear, then x =

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $-\frac{4}{3}$

(7) Let $\bar{a} = i - k$, $\bar{b} = xi + j(1-x)k$, and $\bar{c} = yi + xj + (1+x-y)k$.then $\bar{a}, \bar{b}$ and $\bar{c}$ are non–coplanar for

(a) some values of x

(b) some values of y

(c) no values of x and y

(d) for all values of x and y

(8) If A $(-\hat{i} + 3\hat{j} + 2\hat{k})$, B $(-4\hat{i} + 2\hat{j} - 2\hat{k})$ and C $(5\hat{i} + \lambda\hat{j} + \mu\hat{k})$ are collinear then

(a) $\lambda = 5, \mu = 10$ (b) $\lambda = 10, \mu = 5$

(c) $\lambda = -5, \mu = 10$ (d) $\lambda = 5, \mu = -10$

(9) If the vectors $3\hat{i} + 2\hat{j} - \hat{k}$ and $6\hat{i} - 4x\hat{j} + y\hat{k}$ are parallel, then the value of x and y will be

(a) $-1, -2$ (b) $1, -2$

(c) $-1, 2$ (d) $1, 2$

(10) The value of λ, for which the four points $\hat{i} - \hat{j} + \hat{k}$, $2\hat{i} + 2\hat{j} - 3\hat{k}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$, $\hat{i} + \lambda\hat{j} + 2\hat{k}$ are coplanar, is :

(a) $\frac{7}{4}$ (b) $\frac{-7}{4}$ (c) $\frac{4}{7}$ (d) $\frac{-4}{7}$

Section formula & other formula :

(11) If $\bar{a} = 2\hat{i} - \hat{j} + 5\hat{k}$, $\bar{b} = 3\hat{i} + 2\hat{j}$ are the position vectors of points A,B respectively and point C ($\bar{c}$) divides the line segement AB internally in the ratio 1 : 4, then $\bar{c}$ =

(a) $\hat{i} - \frac{2}{5}\hat{j} - 4\hat{k}$ (b) $\frac{11}{5}\hat{i} - \frac{2}{5}\hat{j} + 4\hat{k}$

(c) $\hat{i} + \frac{2}{5}\hat{j} - 4\hat{k}$ (d) $\hat{i} + \frac{2}{5}\hat{j} + 4\hat{k}$

(12) If the point C divides the line segement joining the points A (4 –2 , 5) and B (–2 , 3 , 7) externally in the ratio 8 : 5, then point C is

(a) $\left(-12, \frac{34}{3}, \frac{31}{3}\right)$ (b) $\left(-12, \frac{31}{3}, \frac{34}{3}\right)$

(c) $\left(-12, \frac{14}{3}, \frac{31}{3}\right)$ (d) $\left(-12, \frac{31}{3}, \frac{14}{3}\right)$

(13) In Δ ABC,D divides BC in the ratio $l : m$, G divides AD in the ratio $(l + m) : n$, then the position vector of G is

(a) $\dfrac{m\bar{c}+n\bar{b}+l\bar{a}}{l+m+n}$ (b) $\dfrac{l\bar{a}+m\bar{b}+n\bar{a}}{l+m+n}$

(c) $\dfrac{n\bar{c}+l\bar{b}+m\bar{a}}{l+m+n}$ (d) $\dfrac{l\bar{c}+m\bar{b}+n\bar{a}}{l+m+n}$

(14) If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of points A,B,C respectively such that $5\bar{a} - 3\bar{b} - 2\bar{c} = 0$, then

(a) C divides BA internally in ratio 5 : 3

(b) C divides BA externally in ratio 5 : 3

(c) C divides AB internally in ratio 5 : 3

(d) C divides AB externally in ratio 5 : 3

(15) The incentre of triangle whose vertices are A (0 , 3 , 0) , B (0 , 0 , 4) , C (0 , 3 , 4) is

(a) (0 , 24 , 36) (b) (0 , 36 , 24)

(c) (0 , 3 , 2) (d) (0 , 2 , 3)

(16) If A (a , 2 , 2) , B (a , b , 1) and (1 , 2 , –2) are the vertices of triangle ABC and G (2 , 1 , c) is its centroid, then values of 'a' , 'b' and 'c' are

(a) $a = \dfrac{1}{2}, b = 1, c = 1$

(b) $a = \dfrac{5}{2}, b = -1, c = \dfrac{1}{3}$

(c) $a = -1, b = 1, c = \dfrac{3}{2}$

(d) $a = \dfrac{1}{2}, b = \dfrac{1}{2}, c = -1$

(17) D , E , F are the mid–points of sides BC, CA, AB respectively of ΔABC. which of the following is true?

(a) $\overline{AB} = 2\overline{ED}$ (b) $\overline{AB} = 2\overline{OC}$

(c) $\overline{AB} = \overline{ED}$ (d) $\overline{AB} = 2\overline{DF}$

(18) If O is origin and C is the mid point of A (2 , –1) and B (– 4 , 3). Then value of $\overline{OC}$ is

(a) $\hat{i} + \hat{j}$ (b) $\hat{i} - \hat{j}$ (c) $-\hat{i} + \hat{j}$ (d) $-\hat{i} - \hat{j}$

(19) If the position vector of a point A is $\bar{a} + 2\bar{b}$ and $\bar{a}$ divides AB in the ratio 2:3, then the position vector of B is

(a) $\bar{a} + \bar{b}$ (b) $\bar{a}$

(c) $\bar{a} - 3\bar{b}$ (d) $\bar{b}$

(20) The position vector of the point P is $6\bar{b} - 2\bar{a}$ and R is $\bar{a} - \bar{b}$. If R divides segment PQ internally in the ratio 1 : 2, then position vector of Q is given by :

(a) $7\bar{a} - 15\bar{b}$ (b) $7\bar{a} + 15\bar{b}$

(c) $15\bar{a} - 7\bar{b}$ (d) $15\bar{a} + 7\bar{b}$

Position vectors & addition of vectors :

(21) If A , B ,C , D , E are five coplanar points, then $\overline{DA} + \overline{DB} + \overline{DC} + \overline{AE} + \overline{BE} + \overline{CE} =$

(a) $\overline{DE}$ (b) $3\overline{DE}$ (c) $2\overline{DE}$ (d) $4\overline{DE}$

(22) $\overline{PQ} - \overline{TQ} + \overline{PS} + \overline{ST} =$

(a) $2\overline{PT}$ (b) $6\overline{OC}$ (c) $\overline{CB}$ (d) $\overline{CD}$

(23) If P is orthocentre, Q is circumcentre and G is centriod of triangle ABC, then

(a) $\overline{QP} = \overline{GQ}$ (b) $\overline{QP} = \overline{QG}$

(c) $\overline{QP} = 3\overline{GQ}$ (d) $\overline{QP} = 3\overline{QG}$

(24) ABCDE is a pentagon. The resultant of the vectors $\overline{AB}, \overline{AE}, \overline{BC}, \overline{DC}, \overline{ED}$ and $\overline{AC}$ in terms of $\overline{AC}$ is

(a) $4\overline{AC}$ (b) $2\overline{AC}$ (c) $3\overline{AC}$ (d) $5\overline{AC}$

(25) If G and G′ are centroids of Δ ABC and Δ A′B′C′ respectively, then $\overline{AA'} + \overline{BB'} + \overline{CC'}$

(a) $2\overline{GG'}$ (b) $3\overline{GG'}$ (c) $4\overline{GG'}$ (d) $\dfrac{1}{3}\overline{GG'}$

Scalar & vector product :

(26) If $\bar{c} = 5\bar{a} - 4\bar{b}$ and $\bar{a}$ is perpendicular to $\bar{b}$, then $c^2 =$

(a) $25a^2 + 16b$ (b) $5a^2 + 16b^2$

(c) $25a^2 + 16b^2$ (d) 0

(27) If $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_1 + \hat{e}_2$ are unit vectors, then angle between $\hat{e}_1$ and $\hat{e}_2$ is

(a) 90^o (b) 120^o (c) 450^o (d) 135^o

(28) If $\bar{a} = \bar{b} + 4\bar{c}$ and angle between $\bar{a}$ and $\bar{c}$ is $\frac{\pi}{6}$ and $a = 2$, $c = 1$ then $b^2 =$

(a) $20 - \sqrt{3}$ (b) $20 - 8\sqrt{3}$

(c) $16 - 4\sqrt{3}$ (d) $15 - \sqrt{3}$

(29) $\begin{vmatrix} \bar{a}.\bar{a} & \bar{a}.\bar{b} \\ \bar{a}.\bar{b} & \bar{b}.\bar{b} \end{vmatrix} =$

(a) 0 (b) $a^2 - b^2$

(c) $\left(\bar{a} \times \bar{b}\right)^2$ (d) $\left(\bar{a}.\bar{b}\right)^2$

(30) If $\bar{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\bar{b} = -\hat{i} + 2\hat{j} - 4\hat{k}$ and $\bar{c} = \hat{i} + \hat{j} + \hat{k}$, then $(\bar{a} \times \bar{b}).(\bar{a} \times \bar{c}) =$

(a) 60 (b) 68 (c) -60 (d) -74

Scalar triple product :

(31) If $\bar{a} = 3\hat{i} - \hat{j} + 4\hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\bar{c} = 5\hat{i} + 2\hat{j} + 3\hat{k}$, then $\bar{a}.(\bar{b} \times \bar{c}) =$

(a) 109 (b) 110 (c) 42 (d) 44

(32) If the vectors $\bar{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\bar{b} = 2\hat{i} + 5\hat{j} - p\hat{k}$, $\bar{c} = 5\hat{i} + 9\hat{j} + 4\hat{k}$, are coplanar, then $p =$

(a) 3 (b) -3 (c) 11 (d) -11

(33) If $\bar{a}$, $\bar{b}$, are non–coplanar vectors and λ a real number, then $\left[\lambda\left(\bar{a}+\bar{b}\right) \ \lambda^2\bar{b} \ \lambda\bar{c}\right] = \left[\bar{a} \ \ \bar{b}+\bar{c} \ \ \bar{b}\right]$ for

(a) exactly three values of λ

(b) exactly two values of λ

(c) exactly one value of λ

(d) no real value of λ

(34) The volume of the parallelopiped with coterminous edges given by the vector

$3\hat{i} + 5$, $4\hat{i} + 2\hat{j} - 3\hat{k}$, $3\hat{i} + \hat{j} + 4\hat{k}$, is

(a) 23 cu. units (b) 33 cu. units

(c) 10 cu. units (d) 43 cu. units

(35) The volume of parallelopiped with vector $\bar{a} + 2\bar{b} - \bar{c}$, $\bar{a} - \bar{b}$ and $\bar{a} - \bar{b} - \bar{c}$ is equal to K $[\bar{a} \ \ \bar{b} \ \ \bar{c}]$.

Then K =

(a) -3 (b) 3 (c) 2 (d) -2

(36) The volume of the tetrahedron whose vertices are A $(-1, 2, 3)$, B $(3, -2, 1)$, C $(2, 1, 3)$ and D $(-1, -2, 4)$

(a) $\frac{2}{3}$ cu. units (b) $\frac{32}{3}$ cu. units

(c) $\frac{8}{3}$ cu. units (d) $\frac{16}{3}$ cu. units

(37) If $\bar{u} = -\hat{i} - 2\hat{j} + \hat{k}$, $\bar{v} = 3\hat{i} + \hat{k}$, $\bar{w} = 4\hat{i} + 5\hat{k}$, then $\left(\bar{u} + \bar{w}\right).\left(\left(\bar{u} \times \bar{r}\right) \times \left(\bar{r} \times \bar{w}\right)\right) =$

(a) 66 (b) 330 (c) 198 (d) 132

(38) If $\bar{a}$, $\bar{b}$, $\bar{c}$ are three non–coplanar vectors, then $\left(\bar{a} - 2\bar{b} + \bar{c}\right) \cdot \left(\left(\bar{a} + \bar{b} - \bar{c}\right) \times \left(\bar{a} - \bar{b} + \bar{c}\right)\right) =$

(a) $\left[\bar{a} \ \bar{b} \ \bar{c}\right]$ (b) $3\left[\bar{a} \ \bar{b} \ \bar{c}\right]$

(c) $2\left[\bar{a} \ \bar{b} \ \bar{c}\right]$ (d) $-6\left[\bar{a} \ \bar{b} \ \bar{c}\right]$

(39) If $\bar{a}, \bar{b}, \bar{c}$ are three non–coplanar vectors and $\bar{p}, \bar{q}, \bar{r}$ are vectors defined by the relations

$$\bar{p} = \frac{\bar{b}\times\bar{c}}{[\bar{a}\ \bar{b}\ \bar{c}]},\ \bar{q} = \frac{\bar{c}\times\bar{a}}{[\bar{a}\ \bar{b}\ \bar{c}]},\ \bar{r} = \frac{\bar{a}\times\bar{b}}{[\bar{a}\ \bar{b}\ \bar{c}]},$$

then $(\bar{a}+\bar{b}).\bar{p} + (\bar{b}+\bar{c}).\bar{p} + (\bar{c}+\bar{a}).\bar{r} =$

(a) 0 (b) 1 (c) 2 (d) 3

(40) Volume of parallelopiped determined by vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$ is 5.

Then the volume of the parallelopiped determined by the vectors $3(\bar{a}+\bar{b})$, $(\bar{b}+\bar{c})$ and $(\bar{c}+\bar{a})$ is

(a) 100 (b) 30 (c) 24 (d) 60

(41) If $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors and mutually perpendicular, then $[\hat{i}\ \hat{k}\ \hat{j}]$ is equal to

(a) 0 (b) –1 (c) 1 (d) 2

(42) $\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{i}\times\hat{k}) + \hat{k}.(\hat{i}\times\hat{j})$ is equal to

(a) 3 (b) 0 (c) – 3 (d) 1

(43) If $\bar{a}, \bar{b}, \bar{c}$ are three coplanar vectors, then $[\bar{a}+\bar{b}\ \ \bar{b}+\bar{c}\ \ \bar{c}+\bar{a}] =$

(a) $(\bar{a}\ \bar{b}\ \bar{c})$ (b) $2(\bar{a}\ \bar{b}\ \bar{c})$

(c) $3(\bar{a}\ \bar{b}\ \bar{c})$ (d) 0

(44) $\bar{a}\ (\bar{b}\times\bar{a}) =$

(a) $\bar{b}\ \bar{b}$ (b) $\bar{a}^2\ \bar{b}$

(c) 0 (d) $\bar{a}^2 + \bar{a}\ \bar{b}$

(45) The value of λ, for which the four points $2\hat{i}+3\hat{j}-\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$, $3\hat{i}+4\hat{j}-2\hat{k}$, $\hat{i}-\lambda\hat{j}+6\hat{k}$, are coplanar, is :

(a) – 2 (b) 8 (c) 6 (d) 0

(46) Let $\bar{a} = \hat{i}-\hat{k}$, $\bar{b} = x\hat{i}+\hat{j}+(1-x)\hat{k}$, and $= \bar{c}\ y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$ and $(\bar{a}\ \bar{b}\ \bar{c})$ depends on

(a) only X (b) Only y

(c) neither x and y (d) both x and y

(47) If $\bar{a}, \bar{b}$ and $\bar{c}$ are unit coplanar vectors, then $[2\bar{a}-\bar{b}\ \ 2\bar{b}-\bar{c}\ \ 2\bar{c}-\bar{a}] =$

(a) 0 (b) 1 (c) $-\sqrt{3}$ (d) $\sqrt{3}$

Applications of vector to geometry :

(48) If ABCD is a parallelogram, then $\overline{AC} - \overline{BD}$

(a) $3\overline{AB}$ (b) $4\overline{AB}$

(c) $2\overline{AB}$ (d) $\overline{AB}$

(49) If the vectors $\overline{AB} = 3\hat{i}+4\hat{k}$ and $\overline{AC} = 5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is

(a) $\sqrt{18}$ (b) $\sqrt{72}$ (c) $\sqrt{33}$ (d) $\sqrt{288}$

(50) If $7\hat{i}+10\hat{k}$, $\hat{i}+6\hat{j}+6\hat{k}$ and $4\hat{i}+9\hat{j}+6\hat{k}$ are vertices of a triangle, then it is

(a) only isosceles

(b) only right angled

(c) equilateral

(d) isosceles right angled

HOME WORK

Multiple Choice Questions

Collinearity & Coplanarity :

(1) The system of vectors $\hat{i}, \hat{j}, \hat{k}$ is

(a) Orthogonal (b) coplanar

(c) collinear (d) parallel

(2) If a and b are two non–collinear vectors, then $x\,a + y\,b$, where x and y are scalars represents a vector which is

(a) parallel to b

(b) parallel to a

(c) coplanar with $\bar{a}$ and $\bar{b}$

(d) none of these

(3) If the points A (1, –2, 3), B (3, 1, 1) and C(–1, p, 3) are collinear, then p =

(a) 5 (b) – 5 (c) 1 (d) – 1

(4) If the vectors $2\hat{i} - p\hat{j} + 3\hat{k}$ and $4\hat{i} - 5\hat{j} + 6\hat{k}$ are collinear, then p =

(a) $\frac{5}{4}$ (b) $\frac{-5}{4}$ (c) $\frac{5}{2}$ (d) $\frac{-5}{2}$

(5) If $\bar{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, $\bar{c} = 4\hat{i} + 3\hat{j} - 18\hat{k}$ and $\bar{c} = x\bar{a} + y\bar{b}$, then

(a) $x = 2, y = 3$ (b) $x = -2, y = -3$

(c) $x = 2, y = -3$ (d) $x = -2, y = 3$

(6) If $\bar{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\bar{b} = \hat{i} - 2\hat{j} + 4\hat{k}$, $\bar{c} = -\hat{i} + 3\hat{j} + 5\hat{k}$, $\bar{d} = -\hat{i} + 4\hat{j} - 4\hat{k}$ and $\bar{d} = x\bar{a} + y\bar{b} + z\bar{c}$, then

(a) $x = 3, y = 2, z = 1$ (b) $x = 2, y = 3, z = 1$

(c) $x = 1, y = 2, z = 3$ (d) $x = 1, y = 3, z = 2$

(7) If the vectors $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = \hat{i} - \hat{j} + \hat{k}$, $\bar{c} = 2\hat{i} + 3\hat{j} + \lambda\hat{k}$ are coplanar, then λ =

(a) 2 (b) – 2 (c) 4 (d) – 4

(8) If $\bar{a}$, $\bar{b}$ are non–collinear vectors and x, y are scalars such that $x\bar{a} + y\bar{b} = \bar{0}$, then

(a) $x = 1, y = -1$

(b) $x = -2, y = 2$

(c) $x = 0, y = 0$

(d) x, y are any real number such that $x + y$

(9) If points $P(\bar{p})$, $Q(\bar{q})$, $R(\bar{r})$ and $S(\bar{s})$ are such $\bar{p} - \bar{q} = 2(\bar{s} - \bar{r})$, then segments

(a) PQ and RS bisect each other

(b) PQ and PR bisect each other

(c) PQ and RS trisect each other

(d) QS and PR trisect each other

(10) Points $A(4, 5, 1)$, $B(0, -1, -1)$, $C(3, 9, 4)$ and $D(-4, 4, 4)$ are

(a) collinear

(b) coplanar

(c) non–coplanar

(d) non–collinear and non–coplanar

(11) Points with position vectors $60\hat{i} + 3\hat{j}$, $40\hat{i} - 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ are collinear if

(a) $a = -40$ (b) $a = 40$

(c) $a = 20$ (d) none of these

(12) If $\bar{a} = 2\bar{p} + 3\bar{q} - \bar{r}$, $\bar{b} = \bar{p} - 2\bar{q} + 2\bar{r}$ and $\bar{c} = -2\bar{p} + \bar{q} + \bar{p} - 2\bar{r}$ and $\bar{R} = 3\bar{p} - \bar{q} + 2\bar{r}$, where $\bar{p}$, $\bar{q}$, $\bar{r}$ are non–coplanar vectors, then R in terms of $\bar{a}$, $\bar{b}$, $\bar{c}$ is

(a) $5\bar{a} + 2\bar{b} + 3\bar{c}$ (b) $3\bar{a} + 5\bar{b} + 2\bar{c}$

(c) $2\bar{a} + 5\bar{b} + 3\bar{c}$ (d) $5\bar{a} + 3\bar{b} + 2\bar{c}$

(13) Three points whose position vectors $\bar{a} + \bar{b}$, $\bar{a} - \bar{b}$ and $\bar{a} + k\bar{b}$ are said to be collinear, then the value of k is

(a) zero

(b) Only negative real number

(c) Only positive real number

(d) Every real number

(14) If three points A,B and C have position vectors $(1, x, 3)$, $(3, 4, 7)$ and $(y, -2, -5)$ respectively and if they are collinear, then $(x,y) =$

(a) $(2, -3)$ (b) $(-2, 3)$

(c) $(2, 3)$ (d) $(-2, -3)$

(15) If $\bar{a}$, $\bar{b}$, and $\bar{c}$ be three non–zero vectors, no two of which are collinear. If the vector $\bar{a} + 2\bar{b}$ is collinear with $\bar{c}$ and $\bar{b} + 3\bar{c}$ is collinear with $\bar{a}$, then (λ being some non–zero scalar) $\bar{a} + 2\bar{b} + 6\bar{c}$ is equal to

(a) $\lambda\bar{a}$ (b) $\lambda\bar{b}$ (c) $\lambda\bar{c}$ (d) $\bar{0}$

(16) A vector perpendicular to $2\hat{i} + \hat{j} + \hat{k}$ and coplanar with $\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + 2\hat{k}$ is

(a) $5(\hat{j} - \hat{k})$ (b) $\hat{i} + 7\hat{j} - \hat{k}$

(c) $5(\hat{j} + \hat{k})$ (d) $2\hat{i} - 7\hat{j} - \hat{k}$

(17) If a, b, c are non–collinear vectors such that for some scalars x, y, z, $xa + yb$, $zc = 0$, then

(a) $x = 0, y = 0, z = 0$

(b) $x \neq 0, y \neq 0, z = 0$

(c) $x = 0, y \neq 0, z \neq 0$

(d) $x \neq 0, y \neq 0, z \neq 0$

(18) If points $A(\bar{a})$, $B(\bar{b})$ and $C(\bar{c})$ are such that $2\bar{a} + 3\bar{b} - 5\bar{c} = \bar{0}$, then their relative positions are

(a) $A - B - C$ (b) $B - A - C$

(c) $A - C - B$ (d) $C - A - B$

(19) If the points A (1, 2, 3), B (–1, 1, 2), C (2, 3, 4) and D (–1, x, 0) are coplanar, then the value of x :

(a) 1 (b) 2 (c) -1 (d) -2

(20) If position vectors of four points A,B,C,D are $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 3\hat{j}$, $3\hat{i} + 5\hat{j} - 2\hat{k}$, $-\hat{j} + \hat{k}$ respectively, then $\overline{AB}$ and $\overline{CD}$ are related as

(a) perpendicular (b) parallel

(c) independent (d) none of these

Section formula & other formulae :

(21) If $\bar{p} = \hat{i} - 2\hat{j} + \hat{k}$, $\bar{q} = \hat{i} + 4\hat{j} - 2\hat{k}$ are the poition vectors of points P,Q respectively and point R ($\bar{r}$) divides the line segment PQ internally in the ratio 2 : 1 $\bar{r} =$

(a) $\hat{i} + 2\hat{j} - \hat{k}$ (b) $\hat{i} - 2\hat{j} + \hat{k}$

(c) $\hat{i} - 2\hat{j} - \hat{k}$ (d) $\hat{i} + 2\hat{j} + \hat{k}$

(22) If the point C divides the line segment joining the points A (2, –6,8) and B (–1, 3, –4) externally in the ratio 1 : 3, then point C is

(a) $\left(\frac{-7}{2}, \frac{-21}{2}, 14\right)$ (b) $\left(\frac{7}{2}, \frac{-21}{2}, -14\right)$

(c) $\left(\frac{-7}{2}, \frac{21}{2}, -14\right)$ (d) $\left(\frac{7}{2}, \frac{-21}{2}, 14\right)$

(23) Of D ABC,D divides BC in the ratio $l : m$, G divides AD in the $(l+m)$: n, then the position vector of D is

(a) $\dfrac{m\bar{c}+l\bar{b}}{m+l}$ (b) $\dfrac{m\bar{c}-l\bar{b}}{m-l}$

(c) $\dfrac{l\bar{c}+m\bar{b}}{l+m}$ (d) $\dfrac{l\bar{c}-m\bar{b}}{l-m}$

(24) If the position vectors of points A ,B, C are $\bar{a}, \bar{b}, \bar{c} = 5\bar{a} - 4\bar{b}$, then

(a) C divides BA internally in the ratio 5 : 4

(b) C divides BA externally in the ratio 5 : 4

(c) C divides AB internally in the ratio 5 : 4

(d) C divides AB internally in the ratio 5 : 4

(25) If origin is the centroid of triangle whose vertices are A $(2,p,-3)$ B $(q,-2,5)$ and C $(-5,1,r)$ then

(a) $p=1, q=-3, r=-2$

(b) $p=1, q=3, r=-2$

(c) $p=1, q=3, r=2$

(d) $p=-1, q=-3, r=-2$

(26) If G $\left(r\dfrac{-4}{3},\dfrac{1}{3}\right)$ is centroid of the triangle having vertices A $(5, 1, p)$ B $(1, q, p)$ C $(1,-2, 3)$, then

(a) $p=-1, q=-3, r=\dfrac{7}{3}$

(b) $p=1, q=-3, r=\dfrac{7}{3}$

(c) $p=-1, q=3, r=\dfrac{7}{3}$

(d) $p=1, q=3, r=\dfrac{7}{3}$

(27) If A $(1, 4, 2)$ B $(-2, 3, -5)$ are two vertices A and B and G $\left(\dfrac{4}{3},0,\dfrac{-2}{3}\right)$ is the centroid of the ΔABC, then the midpoint of side BC is

(a) $\left(\dfrac{3}{2},-2,-2\right)$ (b) $\left(2,1,\dfrac{3}{2}\right)$

(c) $(-3, 1, -1)$ (d) $(3, 1, 1)$

(28) If C is mid point of AB and P is a point outside AB, then

(a) $\overline{PA}+\overline{PB}=\overline{PC}$ (b) $\overline{PA}+\overline{PB}+\overline{PC}=0$

(c) $\overline{PA}+\overline{PB}=2\overline{PC}$ (d) $\overline{PA}+\overline{PB}+2\overline{PC}=0$

(29) In parallelogram ABCD, if P , Q are the mid points of BC and CD respectively, then $\overline{AP}+\overline{AQ}=$

(a) $\dfrac{3}{2}\overline{AC}$ (b) $\dfrac{5}{4}\overline{AC}$

(c) $\overline{AC}$ (d) $2\overline{AC}$

(30) If G is the centroid of ΔABC, then $\overline{CA}+\overline{CB}=$

(a) $3\overline{GC}$ (b) $3\overline{CG}$ (c) $3\overline{AB}$ (d) $3\overline{GA}$

(31) If D,E,F are mid–points of sides BC, CA and AB (of ΔABC) respectively, then $\overline{AD}+\dfrac{2}{3}\overline{BE}+\dfrac{1}{3}\overline{CF}=$

(a) $\dfrac{1}{2}\overline{AC}$ (b) $2\overline{AC}$ (c) $4\overline{AC}$ (d) $6\overline{AC}$

(32) Curve $x=4$ divides the join of A $(3,-2,5)$ and B $(7,3,-2)$ in the ratio

(a) $-1:3$ (b) $1:3$

(c) $-3:1$ (d) $3:1$

(33) If the four points $A(\bar{a})$, $B(\bar{b})$, $C(\bar{c})$ and $D(\bar{d})$ are such that $(\bar{a}-\bar{d})\cdot(\bar{b}-\bar{c})=0=(\bar{b}-\bar{d})\cdot(\bar{c}-\bar{a})$, then the point D is

(a) centroid of ΔABC

(b) orthocentre of ΔABC

(c) circumcentre of ΔABC

(d) incentre of ΔABC

(34) $\bar{a}$, $\bar{b}$ are position vectors of points A , B. If P divides AB in the ratio 3 : 1 and Q is the mid point of AP, then position vector of Q will be

(a) $\frac{1}{2}(\bar{a}-\bar{b})$ (b) $\frac{1}{2}(\bar{a}+\bar{b})$

(c) $\frac{1}{8}(5\bar{a}+3\bar{b})$ (d) $\frac{1}{8}(5\bar{a}-3\bar{b})$

(35) The position vector of four points A , B , C , D are $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ respectively. If $\bar{a}-\bar{b}=2(\bar{d}-\bar{c})$ then

(a) AB and CD bisect

(b) BD and AC bisect

(c) AB and CD trisect

(d) BD and AC trisect

(36) The position vector of the point which divides internally in the ratio 2 : 3, the join of the points $2\bar{a}-3\bar{b}$ and $2\bar{a}-2\bar{b}$, is

(a) $\frac{12}{5}\bar{a}+\frac{13}{5}\bar{b}$ (b) $\frac{12}{5}\bar{a}-\frac{13}{5}\bar{b}$

(c) $\frac{3}{5}\bar{a}-\frac{2}{5}\bar{b}$ (d) $\frac{2}{5}\bar{a}-\frac{3}{5}\bar{b}$

(37) The position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}-\hat{k}$ respectively, in the ratio 2 : 1 externally is

(a) $-3\hat{i}-\hat{k}$ (b) $3\hat{i}+\hat{k}$

(c) $2\hat{i}+\hat{j}$ (d) None of these

(38) If A = (2, 3, – 4) B = (a, 1, – 1), C = (3, 2 , 2) and G (3, 2 , c) is the centroid of ΔABC, then the value of a and c respectively, are :

(a) – 4 , 1 (b) 3 , 4

(c) 4 , 3 (d) 4 , – 1

(39) The position vector of the point which divides the join of the points having position vectors $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}+2\hat{k}$ internally in the ratio 3 : 4, is :

(a) $\frac{1}{7}(12\hat{i}+5\hat{j}-10\hat{k})$

(b) $\frac{1}{7}(3\hat{i}+5\hat{j}+10\hat{k})$

(c) $\frac{1}{7}(9\hat{i}-5\hat{j}+5\hat{k})$

(d) $\frac{1}{7}(11\hat{i}+5\hat{j}+10\hat{k})$

(40) A point P is in the plane of the ΔABC such that $\overline{PA}+\overline{PB}+\overline{PC}=\overline{O}$, then of the ΔABC, P is a ;

(a) orthocenter (b) Incentre

(c) circumcentre (d) centroid

Position vectors & addition of vectors :

(41) If P is orthocentre, Q is circumcentre and G is the centroid of atriangle ABC, then $\overline{QP}$ =

(a) $3\overline{QG}$ (b) $\overline{QG}$ (c) $\overline{GQ}$ (d) $3\overline{GQ}$

(42) In parallelogram ABCD, if $\overline{AB}=\bar{a}$ and $\overline{AD}=\bar{b}$, then the diagonals in terms of $\bar{a}$ and $\bar{b}$

(a) $\bar{a}+\bar{b}$, $\bar{a}-\bar{b}$ (b) $\bar{a}$, $\bar{b}$

(c) $\bar{a}+\bar{b}$, $\bar{b}-\bar{a}$ (d) $2\bar{a}$, $2\bar{b}$

(43) If ABCD is a quadrilateral, then $2\overline{AB}+3\overline{BC}+2\overline{CD}+\overline{DA}+\overline{CA}+\overline{DB}$ =

(a) $\overline{AC}$ (b) $2\overline{AC}$ (c) $\overline{AD}$ (d) $\overline{O}$

(44) Let *PQRS* be a parallelogram whose diagonals *PR* and *QS* intersect at *O'* IF *O* is the origin then : $\overline{OP}+\overline{OQ}+\overline{OR}+\overline{OS}$ =

(a) $4\overline{OO'}$ (b) $3\overline{OO'}$

(c) $2\overline{OO'}$ (d) $\overline{OO'}$

(45) If ABCDE is a regular pentagon, then: $\overline{AB} + \overline{BC} + \overline{AD} + \overline{ED} + \overline{AE}$

(a) $\overline{AC} - 2\overline{DC}$ (b) $3\overline{AC} - 2\overline{DC}$

(c) $\overline{AC} - \overline{DC}$ (d) $3\overline{AC} - \overline{DC}$

(46) ABCD is a parallelogram and P is point of intersection of its diagonals. If O is any point, then $\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} =$

(a) $\overline{OP}$ (b) $4\,\overline{OP}$ (c) $\overline{O}$ (d) $2\overline{OP}$

(47) ABCD is a square,

Then $2\overline{AB} + 4\overline{BC} + 5\overline{CD} + 7\overline{DA} =$

(a) $\overline{CA}$ (b) $3\overline{CA}$ (c) $2\overline{CA}$ (d) $\overline{O}$

(48) ABCD is a regular hexagon and O is its centre, then $\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} + \overline{OE} + \overline{OF}$ is :

(a) $2\overline{AD}$ (b) $3\overline{AC}$ (c) $\overline{O}$ (d) $4\overline{AD}$

(49) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$, then a vector of magnitude 5 and perpendicular to bothe the vectors $\vec{a}$ and $\vec{b}$, is :

(a) $\frac{5}{\sqrt{3}}(\hat{j} - \hat{k})$ (b) $\frac{10}{\sqrt{2}}(\hat{j} - \hat{k})$

(c) $\frac{5}{\sqrt{2}}(\hat{j} - \hat{k})$ (d) $\frac{5}{\sqrt{2}}(\hat{j} + \hat{k})$

(50) If ABCD is a parallelogram, then $\overline{AC} - \overline{BD}$ is equal to :

(a) $\overline{AB}$ (b) $2\overline{AB}$ (c) $3\overline{AB}$ (d) $4\overline{AB}$

Scalar & vector product :

(51) Given vectors $\bar{a} = 3\hat{i} - 6\hat{j} - \hat{k}$, $\bar{b} = \hat{i} + 3\hat{j} + 4\hat{k}$, $\bar{c} = 3\hat{i} - 4\hat{j} - 12\hat{k}$, then the projection of $\bar{a} \times \bar{b}$ on vector $\bar{c}$ is

(a) 14 (b) – 14 (c) 12 (d) 15

(52) If $\bar{c} = 2\bar{a} + 5\bar{b}$, $|\bar{a}| = a$, $|\bar{a}| = b$ and the angle between $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{3}$, then $c^2 =$

(a) $4a^2 + 10ab + 25ab^2$ (b) $a^2 + 10ab + 5b^2$

(c) $4a + 10ab + 25ab^2$ (d) $4a + 10ab + b^2$

(53) $(\bar{a}.\hat{i})\,\hat{i} + (\bar{a}.\hat{j})\,\hat{j} + (\bar{a}.\hat{k})\,\hat{k} =$

(a) $\bar{a}$ (b) $2\bar{a}$ (c) $3\bar{a}$ (d) 0

(54) If $\bar{a}$ is collinear with $\bar{b} = 3\hat{i} + 6\hat{j} + 6\hat{k}$ and $\bar{a}.\bar{b} = 27$, then $\bar{a} =$

(a) $\hat{i} + 2\hat{j} - 2\hat{k}$ (b) $\hat{i} + 2\hat{j} + 2\hat{k}$

(c) $\hat{i} - 2\hat{j} + 6\hat{k}$ (d) $3\hat{i} - 2\hat{j} - 2\hat{k}$

(55) Given that $\bar{a}$ and $\bar{b}$ are not mutually perpendicular. If $\bar{c}$ and $\bar{d}$ are two vectors such that $\bar{b} \times \bar{c} = \bar{b} \times \bar{d}$ and $\bar{a}.\bar{d} = 0$, then : $\bar{d} =$

(a) $\bar{c} + \left(\frac{\bar{a}.\bar{c}}{\bar{a}.\bar{b}}\right)\bar{b}$ (b) $\bar{b} + \left(\frac{\bar{b}.\bar{c}}{\bar{a}.\bar{b}}\right)\bar{c} +$

(c) $\bar{c} - \left(\frac{\bar{a}.\bar{c}}{\bar{a}.\bar{b}}\right)\bar{b}$ (d) $\bar{b} - \left(\frac{\bar{b}.\bar{c}}{\bar{a}.\bar{b}}\right)\bar{c}$

(56) For any vector $\bar{a}$, $(\bar{a} \times i)^2 + (\bar{a} \times j)^2 + (\bar{a} \times k)^2 =$

(a) $4\bar{a}^2$ (b) $2\bar{a}^2$ (c) $\bar{a}^2$ (d) $3\bar{a}^2$

(57) If $\bar{a}, \bar{b}, \bar{c}$ are three vectors such that $\bar{a} + \bar{b} + \bar{c} = 0$, where $|\bar{a}| = 1$, $|\bar{b}| = 2$, $|\bar{c}| = 3$,

then : $\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a} =$

(a) 0 (b) – 7 (c) 7 (d) 1

(58) A tetrahedron has vertices at 0, (0, 0, 0), A (1, 2, 1), B (2, 1, 3) and C (–1, 1, 2). Then the angle between the faces OAB and ABC will be

(a) $\cos^{-1}\left(\frac{1}{35}\right)$ (b) $\cos^{-1}\left(\frac{17}{31}\right)$

(c) 30º (d) 90º

(59) Let $\bar{a}, \bar{b}, \bar{c}$ be unit vectors such that $\bar{a}.\bar{b} = 0 = \bar{a}.\bar{c}$.

If $m < (\bar{b}, \bar{c}) = \frac{\pi}{6}$, then : $\bar{a}$

(a) $\pm 2(\bar{b}, \bar{c})$ (b) $2(\bar{b}, \bar{c})$

(c) $\pm\frac{1}{2}(\bar{b}, \bar{c})$ (d) $-\frac{1}{2}(\bar{b}, \bar{c})$

(60) Angle between vectors $\bar{a}$ and $\bar{b}$, where $\bar{a}, \bar{b}, \bar{c}$ are unit vectors satisfying $\bar{a} + \bar{b} + \sqrt{3}.\bar{c} = \bar{0}$, is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Scalar triple product :

(61) If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$, and $\hat{i} + \hat{j} + c\hat{k}$, where $a, b, c \neq 1$, are coplanar,

then : $\frac{1}{1-a} + \frac{1}{1+b} + \frac{1}{1-c} = ...$

(a) – 1 (b) 0 (c) 1 (d) 3

(62) If $\bar{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$, $\bar{b} = 6\hat{i} + 4\hat{j} - 2\hat{k}$ and $\bar{c} = 3\hat{i} - 2\hat{j} - 4\hat{k}$, then $\bar{a}.(\bar{b} \times \bar{c})$ is :

(a) 122 (b) – 144 (c) 120 (d) – 120

(63) $\left[\bar{a}\ \bar{b}\ \bar{a}\times\bar{b}\right]$ is equal to

(a) $|\bar{a} \times \bar{b}|$ (b) $|\bar{a} \times \bar{b}|^2$

(c) 0 (d) None of these

(64) If $\bar{a}, \bar{b}, \bar{c}$ are three vectors, then $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$ is not equal to

(a) $\left[\bar{b}\ \bar{c}\ \bar{a}\right]$ (b) $\left[\bar{c}\ \bar{a}\ \bar{b}\right]$

(c) $-\left[\bar{b}\ \bar{c}\ \bar{a}\right]$ (d) none of these

(65) The volume of the tetrahedron whose vertices are A (1 , – 1 ,10) B (–1, –3,) C (5, –1, 1) and D (7, – 4, 7) is

(a) 26 (b) 29

(c) 32 (d) none of these

(66) $\bar{p}\ \bar{q}\ \bar{r}$, are three vectors. Then the scalar triple product $\left(\bar{p}-\bar{q}\right).\left[\left(\bar{p}-\bar{r}\right)\times\left(\bar{r}-\bar{q}\right)\right] =$

(a) 0 (b) $2\bar{p}.\left(\bar{p}-\bar{r}\right)$

(c) $\bar{p}.\left(\bar{p}-\bar{r}\right)$ (d) $3\bar{p}.\left(\bar{p}-\bar{r}\right)$

(67) The volume of parallelopiped with vector $\bar{a} + 2\bar{b} - \bar{c}$, $\bar{a} - \bar{b}$ and $\bar{a} - \bar{b} - \bar{c}$ is equal to $k\left[\bar{a}\ \bar{b}\ \bar{c}\right]$, then k = k

(a) – 3 (b) 3 (c) 2 (d) – 2

(68) $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{i} \times \hat{k}) + \hat{k}.(\hat{i} \times \hat{j})$ is equal to

(a) 0 (b) – 3 (c) – 1 (d) 3

(69) If $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - b\hat{j} + \hat{k}$, $\hat{i} + \hat{j} + c\hat{k}$, are coplanar, then abc + 2 is equal to

(a) $a + b - c$ (b) $a - b - c$

(c) $a + b + c$ (d) $a - b + c$

(70) If $\bar{a}, \bar{b}, \bar{c}$ are three non–coplanar vector, then

$$\frac{\bar{a}.\bar{b}\times\bar{c}}{\bar{c}\times\bar{a}.\bar{b}} + \frac{\bar{b}.\bar{a}\times\bar{c}}{\bar{c}.\bar{a}\times\bar{b}} =$$

(a) 0 (b) 2

(c) – 2 (d) None of these

(71) If a vector $\bar{\alpha}$ lie in the plane $\bar{\beta}$ and $\bar{\gamma}$ then which is correct

(a) $[\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}] = 0$ (b) $[\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}] = 1$

(c) $[\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}] = 3$ (d) $[\bar{\beta}\ \bar{\gamma}\ \bar{\alpha}] = 1$

(72) If $\bar{u}$, $\bar{v}$ and $\bar{w}$ are three non–coplanar vectors, then $(\bar{u}+\bar{v}-\bar{w}).[(\bar{u}-\bar{v})\times(\bar{v}-\bar{w})]$ equals

(a) 0 (b) $\bar{u}.(\bar{v}\times\bar{w})$

(c) $\bar{u}.(\bar{w}\times\bar{v})$ (d) $3\bar{u}.(\bar{v}\times\bar{w})$

(73) If $\bar{p} = \dfrac{\bar{b}\times\bar{c}}{[\bar{a}\,\bar{b}\,\bar{c}]}$, $\bar{q} = \dfrac{\bar{c}\times\bar{a}}{[\bar{a}\,\bar{b}\,\bar{c}]}$, $\bar{r} = \dfrac{\bar{c}\times\bar{b}}{[\bar{a}\,\bar{b}\,\bar{c}]}$, where $\bar{a}$ $\bar{b}$, $\bar{c}$ are three non–coplanar vectors, then the value of $(\bar{a}+\bar{b}+\bar{c}).(\bar{p}+\bar{q}+\bar{r})$ is given by

(a) 3 (b) 2 (c) 1 (d) 0

(74) If $\bar{a},\bar{b},\bar{c}$ are non–coplanar vectors and λ is a real number then $\left[\lambda\left(\bar{a}+\bar{b}\right)\lambda^2\bar{b}\ \lambda\bar{c}\right] = \left[\bar{a}\ \bar{b}+\bar{c}\ \bar{b}\right]$ for

(a) exactly three values of λ

(b) exactly two values of λ

(c) exactly one values of λ

(d) no values of λ

(75) For three vectors $\bar{u},\bar{v},\bar{w}$ which of the following expressions is not equal to any of the remaining three

(a) $\bar{u}.(\bar{v}\times\bar{w})$ (b) $(\bar{v}\times\bar{w}).\bar{u}$

(c) $\bar{v}.(\bar{u}\times\bar{w})$ (d) $(\bar{u}\times\bar{v}).\bar{w}$

(76) The volume of the parallelopiped whose edges are represented by $-12\hat{i}+\alpha\hat{k}, 3\hat{j}-\hat{k}$ and $2\hat{i}+\hat{j}-15\hat{k}$ is 546. Then α =

(a) 3 (b) 2 (c) -3 (d) -2

(77) If $\bar{a}$ is perpendicular to $\bar{b}$ and $\bar{c}$, $|\bar{a}| = 2$ $|\bar{b}| = 3$, $|\bar{c}| = 4$ and the angle between $\bar{b}$ and c is $\dfrac{2\pi}{3}$, then $[\bar{a}\ \bar{b}\ \bar{c}]$ is equal to

(a) $4\sqrt{3}$ (b) $6\sqrt{3}$ (c) $12\sqrt{3}$ (d) $18\sqrt{3}$

(78) The number of distinct real values of λ for which the vectors $\bar{a} = \lambda^3\hat{i}+\hat{k}$, $\bar{b} = \hat{i}.-\lambda^3\hat{j}$ and $\bar{c} = \hat{i}+(2\lambda-\sin\lambda)\hat{j}-\lambda\hat{k}$ are coplanar is

(a) 0 (b) 1 (c) 2 (d) 3

(79) If $\bar{a} = \dfrac{1}{\sqrt{10}}(3\hat{i}+\hat{k})$, $\bar{b} = \dfrac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})$, then the value of $(2\bar{a}-\bar{b}).\{(\bar{a}\times\bar{b})\times(\bar{a}+2\bar{b})\}$ is

(a) -5 (b) -3 (c) 5 (d) 3

(80) If $\bar{a} = 2\hat{i}+\hat{j}-\hat{k}$, $\bar{b} = \hat{i}+2\hat{j}+3\hat{k}$, $\bar{c} = -\hat{i}+4\hat{j}+2\hat{k}$ represent the co – terminus edges of a parallelopiped, then its volume is :

(a) 126 cub. units (b) 25 cub. Units

(c) 27 cub. Units (d) 28 Cub. Units

(81) If $\bar{a},\bar{b},\bar{c}$ are non co planar and $\bar{p} = \dfrac{\bar{a}\times\bar{b}}{[\bar{a}\,\bar{b}\,\bar{c}]}$, $\bar{q} = \dfrac{\bar{c}\times\bar{a}}{[\bar{a}\,\bar{b}\,\bar{c}]}$, $\bar{r} = \dfrac{\bar{a}\times\bar{b}}{[\bar{a}\,\bar{b}\,\bar{c}]}$, then $\bar{a}.\bar{p}+\bar{b}.\bar{q}+\bar{c}.\bar{r}$ is equal to :

(a) 1 (b) 2 (c) 3 (d) 4

(82) The value of $\hat{i}\times(\hat{j}\times\hat{k}) =$

(a) $\hat{i}$ (b) $\hat{k}$ (c) $\hat{j}$ (d) $\vec{0}$

(83) If $\bar{p}, \bar{q}, \bar{r}$ are any three vectors which of the following statements is not true ?

(a) $(\bar{q} \times \bar{r}).\ \bar{p} = \bar{p}.(\bar{q} \times \bar{r})$

(b) $(\bar{p} \times \bar{q}).\ \bar{r} = \bar{r}.(\bar{p} \times \bar{q})$.

(c) $(\bar{p} \times \bar{q}).\ \bar{r} = \bar{q} \times \bar{p}$

(d) $(\bar{p} \times \bar{q}).\ \bar{r}$ represents the volume of the parallelopiped with coterminous edges $\bar{p}, \bar{q}, \bar{r}$

(84) For non–zero vector $\bar{a}, \bar{b}, \bar{c}$, $(\bar{a} \times \bar{b}).\bar{c} = |\bar{a}|\ |\bar{b}|\ |\bar{c}|$ holds, iff

(a) $\bar{a}.\bar{b} = 0$, $\bar{b}.\bar{c} = 0$, $\bar{c}.\bar{a} \neq 0$

(b) $\bar{b}.\bar{c} = 0$, $\bar{c}.\bar{a} = 0$, $\bar{a}.\bar{b} \neq 0$

(c) $\bar{c}.\bar{c} = 0$, $\bar{a}.\bar{b} = 0$, $\bar{b}.\bar{c} \neq 0$

(d) $\bar{a}.\bar{b} = \bar{b}.\bar{c} = \bar{c}.\bar{a} \neq 0$

(85) If $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = 2\hat{i} + q\hat{j} + \hat{k}$, $\bar{c} = \hat{i} - \hat{j} + 4\hat{k}$ and $\bar{a}.(\bar{b} \times \bar{c}) = = 1$, then $q =$

(a) 3 (b) – 3 (c) 9 (d) – 9

(86) If the vectors $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = \hat{i} + \hat{j} + \hat{k}$, $\bar{c} = 2\hat{i} + 3\hat{j} + m\hat{k}$ are coplanar, then $m =$

(a) – 2 (b) – 4 (c) 2 (d) 4

(87) If $\bar{a}, \bar{b}, \bar{c}$ are non–coplanar vectors and the vectors $\bar{p} = 2\bar{a} - 5\bar{b} + 2\bar{c}$, $\bar{q} = \bar{a} + 5\bar{b} - \bar{c}$, $\bar{r} = 3\bar{a} - \bar{c}$ are coplanar such that $\bar{p} = m\,\bar{q} - n\,\bar{r}$, then

(a) $m = 1, n = 1$ (b) $m = 1, n = -1$

(c) $m = -1, n = 1$ (d) $m = -1, n = -1$

(88) If A (4, 2, 1), B (2, 1, 0), C (3, 1, –1) , D (1, –1, 2), then the volume of the parallelopiped with segments AB, AC and AD as concurrent edges is

(a) 7 cu. units (b) 6 cu. units

(c) 3 cu. units (d) 5 cu. units

(89) If $[\hat{i} + 4\hat{j} + 6\hat{k} \quad 2\hat{i} + a\hat{j} + 3\hat{k} \quad \hat{i} + 2\hat{j} - 3\hat{k}] = 18$, then a =

(a) 4 (b) 3 (c) 6 (d) 2

(90) If four points A $(\bar{a})$, B $(\bar{b})$, C $(\bar{c})$ and D $(\bar{d})$ are coplanar, then $[\bar{a}\ \bar{b}\ \bar{d}] + [\bar{b}\ \bar{c}\ \bar{d}] + [\bar{c}\ \bar{a}\ \bar{d}] =$

(a) $[\bar{a}\ \bar{c}\ \bar{d}]$ (b) $[\bar{a}\ \bar{b}\ \bar{c}]$

(c) $[\bar{c}\ \bar{b}\ \bar{d}]$ (d) $[\bar{d}\ \bar{b}\ \bar{c}]$

(91) If $\bar{a}, \bar{b}, \bar{c}$ are coplanar unit vectors, then $[2\bar{a} - \bar{b} \quad 2\bar{b} - \bar{c} \quad 2\bar{c} - \bar{a}] =$

(a) 0 (b) 1 (c) $-\sqrt{3}$ (d) $\sqrt{3}$

(92) Let : $\bar{v} = 2\hat{i} + \hat{j} - \hat{k}$ and $\bar{w} = \hat{i} + 3\hat{k}$.

If $\bar{u}$ is a unit vector, then maximum value of $(\bar{u}\ \bar{v}\ \bar{w})$ is

(a) – 1 (b) $\sqrt{10} + \sqrt{6}$

(c) $\sqrt{59}$ (d) $\sqrt{60}$

Applications of vector to ceometry :

(93) If $\overline{AB} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\overline{BC} = -\hat{j} - 2\hat{k}$ are adjacent sides of a parallelogram , then angle between its diagonals can be

(a) $\cos^{-1}\left(\frac{2}{\sqrt{13}}\right)$ (b) $\frac{\pi}{3}$

(c) $\frac{4\pi}{3}$ (d) $\frac{2\pi}{3}$

(94) If ABCDEF is regular hexagon, then $\overline{AD} + \overline{EB} + \overline{FC} =$

(a) 0 (b) $2\overline{AB}$ (c) $3\overline{AB}$ (d) $4\overline{AB}$

(95) The area of triangle whose vertices are A (1, –1, 2), B(2 ,1 ,–1) and C (3, –1, 2) is

(a) 13 (b) $\sqrt{13}$ (c) 6 (d) $\sqrt{6}$

(96) If $\overline{e_1}$ and $\overline{e_2}$ are unit vectors and θ is the angle between them, then $\left|\overline{e_1} - \overline{e_2}\right|$ is equal to :

(a) $2\cos\left(\dfrac{\theta}{2}\right)$ (b) $2\tan\left(\dfrac{\theta}{2}\right)$

(c) $2\sin\left(\dfrac{\theta}{2}\right)$ (d) $\tan\left(\dfrac{\theta}{2}\right)$

(97) In ΔABC, AD is the median, then $\overline{AB} + \overline{AC}$ is

(a) $4\overline{AD}$ (b) $2\overline{AD}$ (c) $3\overline{AD}$ (d) $-2\overline{AD}$

(98) If the area of the parallelogram with $\overline{a}$ and $\overline{b}$ as two adjacent sides is 15 sq units, then the area of the parallelogram having, $3\overline{a} + 2\overline{b}$ and $\overline{a} + 3\overline{b}$ as two adjacent sides in sq. unit is :

(a) 120 (b) 105 (c) 75 (d) 45

(99) If the position vectors of the vertices of a triangle be $2\hat{i} + 4\hat{j} - \hat{k}$, $4\hat{i} + 5\hat{j} + \hat{k}$ and $3\hat{i} + 6\hat{j} - 3\hat{k}$, then the triangle is

(a) right angled

(b) isosceles

(c) equilateral

(d) right angled isosceles

(100) The perimeter of a triangle with sides $3\hat{i} + 4\hat{j} + 5\hat{k}$, $4\hat{i} - 3\hat{j} - 5\hat{k}$ and $7\hat{i} + \hat{j}$ is

(a) $\sqrt{450}$ (b) $\sqrt{150}$ (c) $\sqrt{50}$ (d) $\sqrt{200}$

CLASS WORK - ANSWER KEY

1	**b**	2	**a**	3	**b**	4	**c**	5	**c**	6	**c**	7	**d**	8	**a**	9	**a**	10	**b**
11	**b**	12	**a**	13	**d**	14	**b**	15	**d**	16	**b**	17	**a**	18	**c**	19	**c**	20	**c**
21	**b**	22	**a**	23	**d**	24	**c**	25	**b**	26	**c**	27	**b**	28	**b**	29	**c**	30	**d**
31	**b**	32	**a**	33	**d**	34	**a**	35	**b**	36	**d**	37	**b**	38	**c**	39	**d**	40	**d**
41	**b**	42	**d**	43	**d**	44	**c**	45	**a**	46	**c**	47	**a**	48	**c**	49	**c**	50	**d**

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HOME WORK - ANSWER KEY

1	**a**	2	**c**	3	**b**	4	**c**	5	**d**	6	**c**	7	**a**	8	**c**	9	**d**	10	**b**
11	**a**	12	**c**	13	**d**	14	**a**	15	**d**	16	**a**	17	**a**	18	**c**	19	**c**	20	**b**
21	**a**	22	**d**	23	**c**	24	**b**	25	**b**	26	**a**	27	**a**	28	**c**	29	**a**	30	**b**
31	**a**	32	**b**	33	**b**	34	**c**	35	**d**	36	**b**	37	**a**	38	**d**	39	**d**	40	**d**
41	**a**	42	**c**	43	**d**	44	**a**	45	**b**	46	**b**	47	**b**	48	**c**	49	**c**	50	**b**
51	**b**	52	**a**	53	**a**	54	**b**	55	**c**	56	**b**	57	**b**	58	**a**	59	**a**	60	**c**
61	**c**	62	**b**	63	**b**	64	**d**	65	**b**	66	**a**	67	**b**	68	**d**	69	**b**	70	**a**
71	**a**	72	**b**	73	**a**	74	**d**	75	**c**	76	**c**	77	**c**	78	**b**	79	**a**	80	**c**
81	**c**	82	**d**	83	**c**	84	**d**	85	**a**	86	**c**	87	**c**	88	**a**	89	**c**	90	**b**
91	**a**	92	**c**	93	**a**	94	**d**	95	**b**	96	**c**	97	**b**	98	**b**	99	**d**	100	**a**

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CLASSWORK

Hints & Solutions

(1) **(b)** 4

Hera $\bar{a} = 4\hat{i} + 5\hat{j} + 2\hat{k}$, $\bar{b} = 3\hat{i} + 2\hat{j} + p\hat{k}$,

$\bar{c} = 5\hat{i} + 8\hat{j}$

$\overline{AB} = -\left(\overline{AB}\right)$

$\overline{AB} = \bar{b} - \bar{a} = \hat{i} - 3\hat{j} + (p-2)\hat{k}$

$\overline{AC} = \bar{c} - \bar{a} = \hat{i} + 3\hat{j} - 2\hat{k}$

$-2 = -(p-2)$

$p = 4$

(2) **(a)** collinear

$\overline{AB} = \bar{b} - \bar{a}$

$\overline{AC} = 3\bar{a} - 2\bar{b} - \bar{a} = 2\bar{a} - 2\bar{b} - 2(\bar{b} - \bar{a}) =$

$\Rightarrow \overline{AC} = -2\,\overline{AB}$

(3) **(b)** $t_1 = -16$, $t_2 = 10$

$\bar{c} = t_1\bar{a} + t_2\bar{b}$

$\Rightarrow 4\hat{i} + 2\hat{j} = t_1(\hat{i} + 3\hat{j}) + t_2(2\hat{i} + 5\hat{j})$

$\Rightarrow t_1 + 2t_2 = 4$

$3t_1 + 5t_2 = 2$

Solving (i) and (ii) , we get

$t_1 = -16$, $t_2 = 10$

(4) **(c)** 5

$\bar{p}, \bar{q}, \bar{r}$ are collinear, then $\overline{PQ} = k\overline{PR}$

$\Rightarrow (\bar{q} - \bar{p}) = k(\bar{r} - \bar{p})$

$\Rightarrow -\bar{a} + \bar{b} + 2\bar{c} = k(-2\bar{a} + 2\bar{b}(m-1)\bar{c})$

$\Rightarrow -2k = -1$ and $k(m-1) = 2$

$\Rightarrow k = \frac{1}{2} \Rightarrow \frac{1}{2}(m-1) = 2$

$\Rightarrow m - 1 = 4 \Rightarrow m = 5$

(5) **(c)** both $\bar{a} \neq \bar{0}$ and $\bar{b} \neq \bar{0}$

(6) **(c)** $\frac{1}{3}$

$\frac{x-2}{2x+2} = \frac{1}{-1}$

$\therefore -x + 2 = 2x + 1$

$\therefore 3x = 1$

$\therefore x = \frac{1}{3}$

(7) **(d)** for all values of x and y

$$\begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$= (1 + x - y) - x(1 - x) - (x^2 - y)$

$= 1 \neq 0, \forall\ x, y$

(8) (a) $\lambda = 5$, $\mu = 10$

$\overline{AB} = m.\overline{BC}$

(9) **(a)** $-1, -2$

$\frac{3}{6} = \frac{2}{-4x} = -\frac{1}{y}$

$\Rightarrow x = -1$ and $y = -2$

(10) (b) $\frac{-7}{4}$

$\overline{AB} = 2\bar{i} + 2\bar{j} - 4\bar{k}$

$\overline{AC} = 2\bar{i} + 3\bar{j} - 4\bar{k}$

$\overline{AD} = (\lambda + 1)\hat{j} + \hat{k}$

$\therefore \overline{AB}\ \overline{AC}\ \overline{AD}$ are coplanar

$\therefore \left[\overline{AB}\ \overline{AC}\ \overline{AD}\right] = 0$

$$\therefore \begin{vmatrix} 1 & 3 & -4 \\ 2 & 3 & -4 \\ 0 & (\lambda+1) & 1 \end{vmatrix} = 0$$

$\therefore \lambda = -\frac{7}{4}$

(11) **(b)** $\frac{11}{5}\hat{i} - \frac{2}{5}\hat{j} + 4\hat{k}$

Here $\bar{c} = \frac{\bar{b}+4\bar{a}}{1+4} = \frac{3\hat{i}+2\hat{j}+8\hat{i}-4\hat{j}+20\hat{k}}{5}$

$= \frac{11}{5}\hat{i} - \frac{2}{5}\hat{j} + 4\hat{k}$

(12) **(a)** $\left(-12, \frac{34}{3}, \frac{31}{3}\right)$

Here $\bar{a} = 4\hat{i} - 2\hat{j} + 5\hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j} + 7\hat{k}$

$\bar{c} = \frac{8\bar{b}-5\bar{a}}{8-5}$

$= \frac{-16\hat{i}+24\hat{j}+56\hat{k}-20\hat{i}+10\hat{j}-25\hat{k}}{3}$

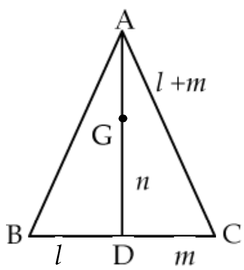
$= \frac{-36\hat{i}+34\hat{j}+31\hat{k}}{3} = -12\hat{i} + \frac{34}{3}\hat{j} + \frac{31}{3}\hat{k} \Rightarrow$

$C = \left[-12, \frac{34}{3}, \frac{31}{3}\right]$

(13) **(d)** $\frac{l\bar{c}+m\bar{b}+n\bar{a}}{l+m+n}$

Let $\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{g}$ be the position vectors of points A, B, C, D, G. Form figure, we get

$\bar{d} = \frac{l\bar{c}+m\bar{b}}{l+m}$



$\bar{g} = \frac{(l+m)\bar{d}+n\bar{a}}{l+m+n} = \frac{l\bar{c}+m\bar{b}+n\bar{a}}{l+m+n}$

(14) **(b)** C divides BA externally in ratio 5 : 3

$5\bar{a} - 3\bar{b} - 2\bar{c} = 0 \Rightarrow 2\bar{c} = 5\bar{a} - 3\bar{b}$

$\Rightarrow \bar{c} = \frac{5\bar{a}-3\bar{b}}{2} = \frac{5\bar{a}-3\bar{b}}{5-3}$

(15) **(d)** (0 , 2 , 3)

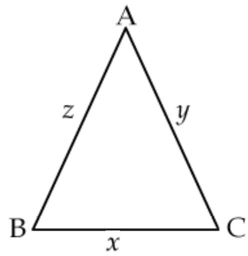
Here $\bar{a} = 3\hat{j}$, $\bar{b} = 4\hat{k}$, $c = 3\hat{j} + 4\hat{k}$

$x = \sqrt{3^2}$, $y = \sqrt{4^2} = 4$

$z = \sqrt{3^2+(-4)^2} = \sqrt{9+16}$

In centre H $\left(\bar{h}\right)$ is

$\bar{h} = \frac{x\bar{a}+y\bar{b}+z\bar{c}}{x+y+z} = \frac{9\hat{j}+16\hat{j}+15\hat{j}+20\hat{k}}{3+4+5}$



$\frac{24\hat{j}+36\hat{k}}{12} = 2\hat{j} + 3\hat{k}$

$\Rightarrow H \equiv (0, 2, 3)$

(16) **(b)** $a = \frac{5}{2}$, $b = -1$, $c = \frac{1}{3}$

$$G = \left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right]$$

$$\Rightarrow (2, 1, c) \equiv \left[\frac{2a+1}{3}, \frac{4+b}{3}, \frac{1}{3}\right]$$

$$\Rightarrow 6 = 2a + 1 , 3 = 4 + b , c = \frac{1}{3}$$

$$\Rightarrow a = \frac{5}{2} , b = -1 , c = \frac{1}{3}$$

(17) **(a)** $\overline{AB} = 2\overline{ED}$

Consider $= 2\overline{ED} \quad = 2\left(\bar{d} - \bar{e}\right)$

$$= 2\left[\frac{\bar{b}+\bar{c}}{2} - \frac{\bar{c}+\bar{a}}{2}\right]$$

$$= 2\left[\frac{\bar{b}-\bar{a}}{2}\right] = \overline{AB}$$

(18) (c) $-\hat{i} + \hat{j}$

Co – ordinate of C is $\left[\frac{2-4}{2}, \frac{-1+3}{2}\right] \equiv$

$(-1, 1)$

$\therefore \quad \overline{OC} = -\hat{i} + \hat{j}$

(19) **(c)** $\bar{a} - 3\bar{b}$

A is $\bar{a} + 2\bar{b}$, let $\bar{B}$ be $(\bar{r})$

Since , a divides AB in the ratio 2 : 3

$$\therefore \quad \frac{2\bar{r} + 3(\bar{a} + 2\bar{b})}{2+3} = \bar{a}$$

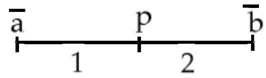
$$\therefore \quad 2\bar{r} = 5\bar{a} - 3\bar{a} - 6\bar{b} = 2\bar{a} - 6\bar{b}$$

$$\therefore \quad \bar{r} = \bar{a} - 3\bar{b}$$

(20) **(c)** $15\vec{a} - 7\vec{b}$

Let $\bar{a} = 2\bar{i} - \bar{j} + \bar{k}$, $\bar{b} = \bar{i} + 3\bar{j} + 2\bar{k}$

$$\bar{p} = \frac{3\bar{b} + 4\bar{a}}{7}, \text{ etc}$$



(21) **(b)** $3\overline{DE}$

$$\overline{DA} + \overline{DB} + \overline{DC} + \overline{AE} + \overline{BE} + \overline{CE}$$

$$= \overline{DA} + \overline{AE} + \overline{DB} + \overline{BE} + \overline{DC} + \overline{CE}$$

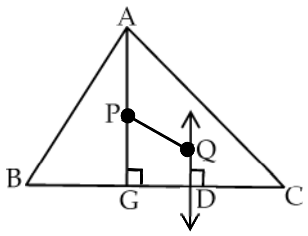
$$= \overline{DE} + \overline{DE} \ \overline{DE} = 3\overline{DE}$$

(22) **(a)** $2\overline{PT}$

$$\overline{PQ} - \overline{TQ} + \overline{PS} + \overline{ST} = \overline{PQ} - \overline{QT} + \overline{PT} =$$

$$= \overline{PT} + \overline{PT} = 2\overline{PT}$$

(23) **(d)** $\overline{QP} = 3\overline{QG}$



Let $\bar{a}$, $\bar{b}$, $\bar{c}$ $\bar{d}$, $\bar{p}$, $\bar{g}$ be position vectors of points A , B, C, D, P, G respectively w.r.t.point Q. D is mid–point of BC, then $2\bar{d} = \bar{b} + \bar{c}$

P is orthocentre and Q is circumcentre of Δ ABC, then

$\therefore$ Centrold of a triangle divides the join of circumcentre & othocentre internally in the ration 1:2

$$\therefore \quad \bar{g} = \frac{\bar{p} + 2\bar{q}}{1+2}$$

(29) **(c)** $\left(\bar{a}\times\bar{b}\right)^2$

$$\begin{vmatrix} \bar{a}.\bar{a} & \bar{a}.\bar{b} \\ \bar{a}.\bar{b} & \bar{b}.\bar{b} \end{vmatrix} = \begin{vmatrix} a^2 & \bar{a}.\bar{b} \\ \bar{a}.\bar{b} & b^2 \end{vmatrix} = a^2b^2 - \left(\bar{a}.\bar{b}\right)^2$$

$= a^2b^2 - (ab\cos\theta)^2 = a^2b^2(1-\cos^2\theta)$

$= a^2b^2\sin^2\theta$

$= \left(\left|\bar{a}\right|\left|\bar{b}\right|\sin\theta\right)^2 = \left(\bar{a}\times\bar{b}\right)^2$

(30) **(d)** -74

$\bar{a}\times\bar{b} = ...$

$\bar{a}\times\bar{c} = ...$

$\therefore \left(\bar{a}\times\bar{b}\right).\left(\bar{a}\times\bar{c}\right) = ...$

(31) **(b)** 110

$$\bar{a}\,(\bar{b}\times\bar{c}) = \begin{vmatrix} 3 & -1 & 4 \\ 2 & 3 & -1 \\ -5 & 2 & 3 \end{vmatrix}$$

$= 3(9+2)+(6-5)+4(4+15)$

$= 33+1+76 = 110$

(32) **(a)** 3

Here $\left[\bar{a}\,\bar{b}\,\bar{c}\right] = 0 \Rightarrow \begin{vmatrix} 1 & -2 & 1 \\ 2 & 5 & -p \\ 5 & 9 & 4 \end{vmatrix} = 0$

(33) **(d)** no real value of λ

$$\left[\lambda\left(\bar{a}+\bar{b}\right)\lambda^2\bar{b}\,\lambda\bar{c}\right] = \left[\bar{a}\;\bar{b}+\bar{c}\;\bar{b}\right]$$

$$\Rightarrow \lambda^4\left[\bar{a}+\bar{b}\;\;\bar{b}\;\bar{c}\right] = \left[\bar{a}\;\bar{b}+\bar{c}\;\bar{b}\right]$$

$$\Rightarrow \lambda^4\left(\left[\bar{a}\;\bar{b}\bar{c}\right]+\left[\bar{b}\;\bar{b}\bar{c}\right]\right) = \left[\bar{a}\;\bar{b}\;\bar{b}\right] + \left[\bar{a}\;\bar{c}\bar{b}\right]$$

$$\Rightarrow \lambda^4\left[\bar{a}\;\bar{b}\;\bar{c}\right] = -\left[\bar{a}\;\bar{b}\;\bar{c}\right]$$

$$\Rightarrow (\lambda^4+1)\left[\bar{a}\;\bar{b}\;\bar{c}\right] = 0$$

But $\left[\bar{a}\;\bar{b}\;\bar{c}\right] \neq 0$ $\lambda^4+1=0$ $\lambda^4=-1$ which does not give real value of λ

(34) **(a)** 23 cu. units

$\bar{a} = 3\hat{i}+5\hat{k}$, $\bar{b} = 4\hat{i}+2\hat{j}-3\hat{k}$, $\bar{c} = 3\hat{i}+\hat{j}+4\hat{k}$, Volume $= \left[\bar{a}\;\bar{b}\;\bar{c}\right] = \begin{vmatrix} 3 & 0 & 5 \\ 4 & 2 & -3 \\ 3 & 1 & 4 \end{vmatrix}$

$= 3(8+3)+5(4-6) = 33-10$

$= 23$ cu. units

(35) **(b)** 3

Volume $= \left[\bar{a}+2\bar{b}-\bar{c}\;\;\;\bar{a}-\bar{b}\;\;\;\bar{a}-\bar{b}-\bar{c}\right]$

$$= \begin{vmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{vmatrix}\left[\bar{a}\;\bar{b}\;\bar{c}\right]$$

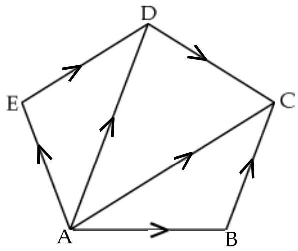
$= ((1-0)-2(-1-0)+(-1+1))\left[\bar{a}\;\bar{b}\;\bar{c}\right]$

$= (1+2+0)\left[\bar{a}\;\bar{b}\;\bar{c}\right]$

$= 3\left[\bar{a}\;\bar{b}\;\bar{c}\right]$

$$\bar{g} = \frac{\bar{p} + 2\bar{q}}{3}$$

(24) **(c)** $3\overline{AC}$



$$\overline{AB} + \overline{AE} + \overline{BC} + \overline{DC} + \overline{ED} + \overline{AC}$$

$$= \overline{AB} + \overline{BC} + \overline{AE} + \overline{ED} + \overline{DC} + \overline{AC}$$

$$= \overline{AC} + \overline{AD} + \overline{DC} + \overline{AC}$$

$$= 2\,\overline{AC} + \overline{AC} = 3\,\overline{AC}$$

(25) **(b)** $3\overrightarrow{GG'}$

$$\overline{AA'} + \overline{BB'} + \overline{CC'}$$

$$= \bar{a}' - \bar{a} + \bar{b}' - \bar{b} + \bar{c}' - \bar{c}$$

$$= (\bar{a}' + \bar{b}' + \bar{c}') - (\bar{a} + \bar{b} + \bar{c})$$

$$= 3\left[\frac{(\bar{a}' + \bar{b}' + \bar{c}')}{3} - \frac{(\bar{a} + \bar{b} + \bar{c})}{3}\right]$$

$$= 3\,[\,\bar{g}' - \bar{g}\,] = 3\,\overline{GG'}$$

(26) **(c)** $25a^2 + 16b^2$

$$\bar{c} = 5\bar{a} - 4\,\bar{b}$$

$$\Rightarrow \bar{c}\cdot\bar{c} = (5\bar{a} - 4\,\bar{b}).(5\bar{a} - 4\,\bar{b})$$

$$\Rightarrow c^2 = 25\,a^2 - 20\,\bar{a}.\bar{b} - 20\,\bar{b}.\bar{a} + 16\,b^2$$

$$\Rightarrow c^2 = 25\,a^2 - 40\,\bar{a}.\bar{b} + 16\,b^2$$

$$\Rightarrow c^2 = 25\,a^2 + 16\,b^2$$

(27) (b) 120°

Here $\left|\hat{e}_1\right| = \left|\hat{e}_2\right| = \left|\hat{e}_1 + \hat{e}_2\right| = 1$

$$\Rightarrow \left|\hat{e}_1 + \hat{e}_2\right| = 1 \quad \left(\hat{e}_1 + \hat{e}_2\right).\left(\hat{e}_1 + \hat{e}_2\right) = 1$$

$$\Rightarrow \hat{e}_1.\hat{e}_1 + \hat{e}_1.\hat{e}_2 + \hat{e}_2.\hat{e}_1 + \hat{e}_2.\hat{e}_2 = 1$$

$$\Rightarrow \left|\hat{e}_1\right|^2 + \hat{e}_1.\hat{e}_2 + \hat{e}_1.\hat{e}_2 + \left|\hat{e}_2\right|^2 = 1$$

$$\Rightarrow 1 + 2\,\hat{e}_1.\hat{e}_2 + 1 = 1 \Rightarrow 2\hat{e}_1.\hat{e}_2 = -1$$

$$\Rightarrow \hat{e}_1.\hat{e}_2 = \frac{-1}{2} \Rightarrow \left|\hat{e}_1\right|\left|\hat{e}_2\right|\cos\theta = \frac{-1}{2}$$

$$\Rightarrow (1)\,(1)\cos\theta = \frac{-1}{2} \Rightarrow \cos\theta = \frac{-1}{2}$$

$$\Rightarrow \theta = 120^\circ$$

(28) **(b)** $20 - 8\sqrt{3}$

$$\bar{b} = \bar{a} - 4\bar{c}$$

$$\Rightarrow \bar{b}\cdot\bar{b} = (\bar{a} - 4\bar{c}).(\bar{a} - 4\bar{c})$$

$$\Rightarrow b^2 = a^2 - 4\bar{a}.\bar{c} - 4\bar{c}.\bar{a} + 16\,c^2$$

$$\Rightarrow b^2 = a^2 - 8\bar{a}.\bar{c} + 16\,c^2$$

$$\Rightarrow b^2 = (2)^2 - 8(2)\,(1)\cos 30^\circ + 16\,(1)^2$$

$$b^2 = 4 - 8\sqrt{3} + 16$$

$$= 20 - 8\sqrt{3}$$

(36) **(d)** $\frac{16}{3}$ cu. units

Here $\bar{a} = -\hat{i} - 2\hat{j} + 3\hat{k}$, $\bar{b} = 3\hat{i} - 2\hat{j} + \hat{k}$,

$\bar{c} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\bar{d} = -\hat{i} - 2\hat{j} + 4\hat{k}$

Now $\overline{AB} = \bar{b} - \bar{a} = 4\hat{i} - 4\hat{j} - 2\hat{k}$

$\overline{AC} = \bar{c} - \bar{a} = 3\hat{i} - \hat{j}$

$\overline{AD} = \bar{d} - \bar{a} = 4\hat{j} + \hat{k}$

$$\text{Volume} = \frac{1}{6}\left[\overline{AB}\ \overline{AC}\ \overline{AD}\right] = \frac{1}{6}\begin{vmatrix} 4 & -4 & -2 \\ 3 & -1 & 0 \\ 0 & -4 & -1 \end{vmatrix}$$

$$= \frac{1}{6}\left(4(-1-0) + 4(3-0) - 2(-12-0)\right)$$

$$= \frac{1}{6}(-4 + 12 + 24) = \frac{32}{6}$$

$$= \frac{16}{3} \text{ cu. units}$$

(37) **(b)** 330

$\bar{u} + \bar{w} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

$$\bar{u} \times \bar{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & 1 \\ 3 & 0 & 1 \end{vmatrix}$$

$= \hat{i}\,(-2-0) - \hat{j}\,(-1-3) + \hat{k}\,(0+6)$

$= 2\hat{i} + 4\hat{j} + 6\hat{k}$

$$= \bar{u} \times \bar{w}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ 4 & 0 & 5 \end{vmatrix} = -\hat{j}\ (15-4) = -11\hat{j}$$

$$\left(\bar{u} + \bar{w}\right).\left(\left(\bar{u} \times \bar{r}\right) \times \left(\bar{r} \times \bar{w}\right)\right) = \begin{vmatrix} 3 & -2 & 6 \\ -2 & 4 & 6 \\ 0 & -11 & 0 \end{vmatrix}$$

$= 3\,(0+66) + 6(22-0) = 198 + 132 = 330$

(38) **(c)** $2\left(\bar{a}\ \bar{b}\ \bar{c}\right)$

$$\left[\bar{a} - 2\bar{b} + \bar{c}\right] . \left(\left(\bar{a} + \bar{b} - \bar{c}\right) \times \left(\bar{a} - \bar{b} + \bar{c}\right)\right)$$

$$\begin{vmatrix} 1 & -2 & 2 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix}\left(\bar{a}\ \bar{b}\ \bar{c}\right)$$

$$= \left((1-1) + 2\,(1+1) + (-1-1)\right)\left(\bar{a}\ \bar{b}\ \bar{c}\right)$$

$$= (0 + 4 - 2)\left(\bar{a}\ \bar{b}\ \bar{c}\right)$$

$$= 2\left(\bar{a}\ \bar{b}\ \bar{c}\right)$$

(39) **(d)** 3

$$\left(\bar{a} + \bar{b}\right) \cdot \bar{p} = \bar{a} \cdot \bar{p} + \bar{b} \cdot \bar{p}$$

$$= \frac{\bar{a}\left(\bar{b} \times \bar{c}\right)}{\left[\bar{a}\ \bar{b}\ \bar{c}\right]} + \frac{\bar{b}\left(\bar{b} \times \bar{c}\right)}{\left[\bar{a}\ \bar{b}\ \bar{c}\right]} = 1 + 0 = 1$$

Similarly $\left(\bar{b} + \bar{c}\right) \cdot \bar{q} = \left(\bar{c} + \bar{a}\right) \cdot \bar{r} = 1$

Now $\left(\bar{a} + \bar{b}\right) \cdot \bar{p} = \left(\bar{b} + \bar{c}\right) \cdot \bar{q} = \left(\bar{c} + \bar{a}\right) \cdot \bar{r}$

$= 1 + 1 + 1 = 3$

(40) **(d)** 60

$\because \quad \left[\bar{a}\,\bar{b}\,\bar{c}\right] = 5$

$\therefore \quad \left[\left(\bar{a}+\bar{b}\right)\left(\bar{b}+\bar{c}\right)2\left(\bar{c}+\bar{a}\right)\right]$

$= 6\left[\left(\bar{a}+\bar{b}\right)\left(\bar{b}+\bar{c}\right)2\left(\bar{c}+\bar{a}\right)\right]$

$= 6\left\{2\left[\bar{a}\ \bar{b}\ \bar{c}\right]\right\}$

$= 12\,(5)$

$= 60$

(41) **(b)** -1

$\left(\hat{i}\ \hat{k}\ \hat{j}\right) = \hat{i}\cdot\left(\hat{k}\times\hat{j}\right)$

$= \hat{i}\cdot\left(-\hat{i}\right)$

$= -1$

(42) **(d)** 1

$\hat{i}\cdot\left(\hat{k}\times\hat{j}\right) + \hat{j}\left(\hat{i}\times\hat{k}\right) + \hat{k}\left(\hat{i}\times\hat{j}\right)$

$= 1 - 1 + 1$

$= 1$

(43) **(d)** 0

$\left[\bar{a}+\bar{b}\ \ \bar{b}+\bar{c}\ \ \bar{c}+\bar{a}\right]$

$= \left[\bar{a}\ \bar{b}\ \bar{c}\right] + \left[\bar{a}\ \bar{b}\ \bar{a}\right] + \left[\bar{a}\ \bar{c}\ \bar{c}\right] + \left[\bar{a}\ \bar{c}\ \bar{a}\right]$

$+ \left[\bar{b}\ \bar{b}\ \bar{c}\right] + \left[\bar{b}\ \bar{b}\ \bar{a}\right] + \left[\bar{b}\ \bar{b}\ \bar{c}\right]$

$+ \left[\bar{b}\ \bar{c}\ \bar{a}\right]\left[\bar{b}\ \bar{c}\ \bar{a}\right]$

$= \left[\bar{a}\ \bar{b}\ \bar{c}\right] + \left[\bar{b}\ \bar{c}\ \bar{a}\right] = 2\left[\bar{b}\ \bar{c}\ \bar{a}\right]$

$= 0$ .... [$\because$ a , b, c are coplanar]

(44) **(c)** 0

Since $\bar{a}\times\bar{b}$ is perpendicular to $\bar{a}$ and $\bar{b}$ both, therfore $\bar{a}\cdot(\bar{a}\times\bar{b}) = 0$

($\therefore$ Scalar product of two perpendicular vector is zero)

(45) **(a)** -2

$\overline{AB} = -\hat{i} - \hat{j} + 4\hat{k}$

$\overline{AC} = \bar{i} + \bar{j} - \bar{k}$

$\overline{AD} = \hat{i} + \left(-\lambda - 3\right)\hat{j} + 7\hat{k}$

Since $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar

$[\overline{AB}\ \ \overline{AC}\ \ \overline{AD}] = 0$

$\therefore \quad \begin{vmatrix} -1 & -1 & 4 \\ 1 & 1 & -1 \\ -1 & -\lambda-3 & 7 \end{vmatrix} = 0$ etc.

(46) **(c)** neither x and y

$\left(\bar{a}\ \bar{b}\ \bar{c}\right) = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1 \\ y & x & 1+x \end{vmatrix}$ ..... [$C_3 \to C_3 + C_1$]

$= 1 + x - x = 1$

(47) **(a)** 0

$\bar{a}, \bar{b}, \bar{c}$ are unit coplanar vectors then $2\bar{a} - \bar{b}$, $2\bar{b} - \bar{c}$, $2\bar{c} - \bar{a}$ are also coplanar vectors,

Hence $\left[2\bar{a} - \bar{b}\ \ 2\bar{b} - \bar{c}\ \ 2\bar{c} - \bar{a}\right] = 0$

(48) **(c)** $2\overline{AB}$

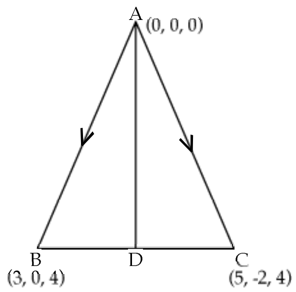
For, Parallelogram ABCD,

$\overline{AC} = \overline{AB} + \overline{BC}$

$\overline{BD} = \overline{BC} - \overline{AB}$

$\overline{AC} - \overline{BD} = 2\overline{AB}$

(49) **(c)** $\sqrt{33}$



Here $\overline{AB} = 3\hat{i} + 4\hat{k}$,

$\overline{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

Let A (0,0,0), then

B (3,0,4) and C (5,–2,4)

Let D is the mid–point of BC, then

AD is the median through A.

by mid–point formula

D $(4\hat{i}, -1, 4)$ $\Rightarrow$

$\overline{AD} = 4\hat{i} - \hat{j} + 4\hat{k} \Rightarrow \left|\overline{AD}\right| = \sqrt{16+1+16}$

$= \sqrt{33}$

(50) **(d)** isosceles right angled

Let $\bar{a} = 7\hat{i} + 10\hat{k}$, $\bar{b} = \hat{i} + 6\hat{j} + 6\hat{k}$,

$\bar{c} = 4\hat{i} + 9\hat{j} + 6\hat{k}$

Now $\overline{AB} = \bar{b} - \bar{a} = -\hat{i} - \hat{j} - 4\hat{k} = \left|\overline{AB}\right| = \sqrt{18}$

$\overline{BC} = \bar{c} - \bar{b} = -3\hat{i} - 3\hat{j} \Rightarrow = \left|\overline{BC}\right| = \sqrt{18}$

$\overline{CA} = \bar{a} - \bar{c} = -4\hat{i} - 2\hat{j} + 4\hat{k} \Rightarrow = \left|\overline{CA}\right| = \sqrt{36}$

Here $\left|\overline{AB}\right| = \left|\overline{BC}\right|$ and $\left|\overline{CA}\right|^2 = \left|\overline{AB}\right|^2 + \left|\overline{BC}\right|^2$

Δ ABC is right angled isosceles

HOME WORK

Hints & Solutions

(1) **(a)** Orthogonal

Informative

(2) **(c)** coplanar with $\bar{a}$ and $\bar{b}$

$x\bar{a} + y\bar{b}$ represents a vector coplanar with $\bar{a}$ and $\bar{b}$.

(3) **(b)** -5

Here $\bar{a} = \hat{i} - 2\hat{j} + 2\hat{k}$, $\bar{b} = 3\hat{i} + \hat{j} + \hat{k}$,

$\bar{c} = -\hat{i} - p\hat{j} + 3\hat{k} \Rightarrow$

$\overline{AB} = \bar{b} - \bar{a} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\overline{AC} = \bar{c} - \bar{a} = -2\hat{i} + (p+2)\hat{j} + \hat{k}$

Points A,B,C are collinear, then $\overline{AB} = m\,\overline{AC} \Rightarrow$

$2\hat{i} + 3\hat{j} - \hat{k} = m(-2\hat{i} + (p+2)\hat{j} + \hat{k}) \Rightarrow$

$-2m = 2$ and $m(p+2) = 3 \Rightarrow$

$m = -1 \Rightarrow -p-2 = 3 \Rightarrow p = -5$

(4) **(c)** $\frac{5}{2}$

Let $\bar{a} = 2\hat{i} - p\hat{j} + 3\hat{k}$, $\bar{b} = 4\hat{i} - 5\hat{j} + 6\hat{k}$,

$\bar{a}$ and $\bar{b}$ are collinear, then $\bar{a} = m\bar{b} \Rightarrow$

$2\hat{i} - p\hat{j} + 3\hat{k} = m(4\hat{i} - 5\hat{j} + 6\hat{k}) \Rightarrow$

$4m = 2$ *and* $-p = -5m \Rightarrow$

$m = \frac{1}{2} \Rightarrow p = \frac{5}{2}$

(5) **(d)** $x = -2, y = 3$

$\bar{c} = x\bar{a} + y\bar{b} \Rightarrow$

$4\hat{i} + 13\hat{j} - 8\hat{k}$

$= x(\hat{i} - 2\hat{j} + 3\hat{k}) + y(2\hat{i} + 3\hat{j} - 4\hat{k}) \Rightarrow$

$4\hat{i} + 13\hat{j} - 8\hat{k}$

$= (x+2y)\hat{i} + (-2x+3y)\hat{j} + (3x-4y)\hat{k} \Rightarrow$

$x + 2y = 4$ ... (i)

$-2x + 3y = 13$ ... (ii)

$3x - 4y = -18$ ... (iii)

Solving (i) and (ii), we get

$x = -2, y = 3$

These value satisfies (iii)

(6) **(c)** $x = 1, y = 2, z = 3$

$\bar{d} = x\bar{a} + y\bar{b} + z\bar{c} \Rightarrow$

$\hat{i} + 4\hat{j} - \hat{k} = (2x + y - z)\hat{i} + (-x - 2y + 3z)\hat{j}$

$+ (3x + 4y - 5z)\hat{k} \Rightarrow$

$2x + y - z = 1$

$-x - 2y + 3z = 4$

$3x + 4y - 5z = -4$

By cramer's we, get

$$D = \begin{vmatrix} 2 & 1 & -1 \\ -1 & 2 & 3 \\ 3 & 4 & -5 \end{vmatrix} = -2, \quad D_x = \begin{vmatrix} 1 & 1 & -1 \\ 4 & -2 & 3 \\ -4 & 4 & -5 \end{vmatrix} = -2$$

$$D_y = D_y = \begin{vmatrix} 2 & 1 & 1 \\ -1 & 2 & 3 \\ 3 & -4 & -5 \end{vmatrix} = -4 \Rightarrow$$

$$D_z = \begin{vmatrix} 2 & 1 & 1 \\ -1 & -2 & 4 \\ 3 & 4 & -4 \end{vmatrix} = -6$$

$x = 1, y = 2, z = 3$

(7) **(a)** 2

Here $\left[\bar{a}\ \bar{b}\ \bar{c}\right] = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow$

$(-\lambda-3) - (\lambda-2) + (3+2) = 0 \Rightarrow$

$-\lambda - 3 - \lambda + 2 + 5 = 0 \Rightarrow -2\lambda = -2\lambda = -4 \Rightarrow$

$\lambda = 2$

(8) **(c)** $x = 0, y = 0$

Standard Results

(9) **(d)** QS and PR trisect each other

$\bar{p} - \bar{q} = 2(\bar{s} - \bar{r}) \therefore \bar{p} - \bar{q} = 2\bar{s} - 2\bar{r}$

$\therefore 2\bar{r} + \bar{p} = 2\bar{s} + \bar{q}$

$\therefore \dfrac{2\bar{r} + \bar{p}}{2+1} = \dfrac{2\bar{s} + 1\bar{q}}{2+1} = T(\bar{t})$, say.

$\therefore$ PR and QS intersect each other in the point T which divides each of them internally in the ratio 2 : 1 .

$\therefore$ point T trisects each of PR and QS.

$\therefore$ QS and PR trisect each other.

(10) **(b)** coplanar

$\left[\overline{AB}\ \overline{AC}\ \overline{AD}\right] = 0$

(11) **(a)** $a = -40$

If $A(\bar{a})$, $B(\bar{b})$ and $C(\bar{c})$ are the given collinear points,

then : $\overline{AB} \parallel \overline{AC}$

$\because \overline{AB} = 20i - 11j$

$\overline{AC} = (a-60)i - 55j$

$\therefore \dfrac{a-60}{-20} = \dfrac{-55}{-11} \qquad \therefore a - 60 = -100$

$\therefore a = -40$

(12) **(c)** $2\bar{a} + 5\bar{b} + 3\bar{c}$

Let us suppose that

$\bar{R} = x\bar{a} + y\bar{b} + z\bar{c}$

$\Rightarrow \bar{R} = x(2\bar{p} + 3\bar{q} - \bar{r}) + y(\bar{p} - 2\bar{q} + 2\bar{r})$

$+ z(-2\bar{p} + \bar{q} - 2\bar{r})$

$\Rightarrow 3\bar{p} - \bar{q} + 2\bar{r} = (2x + y - 2z)\bar{p} + (3x - 2y - z)$

$\bar{p} + (-x + 2y - 2z)\bar{r}$

Comparing we get

$2x + y - 2z = 3,\ 3x - 2y + z = -1,$

$-x + 2y - 2z = 2$

Solving above equations, we get

$x = 2,\ y = 5,\ z = 3$

$\therefore \bar{R} = 2\bar{a} + 5\bar{b} + 3\bar{c}$

(13) **(d)** Every real number

$\overrightarrow{AB} = \lambda\,\overrightarrow{BC}$

Here, $\overrightarrow{AB} = -2b$, $\overrightarrow{BC} = (k+1)b$

Hence $\forall\, k \in R \neq \overrightarrow{AB} = \lambda\,\overrightarrow{BC}$

(14) **(a)** $(2, -3)$

If A , B , C are collinear. Then $\overrightarrow{AB} = \lambda\,\overrightarrow{BC}$

$\Rightarrow 2\hat{i} + (4-x)\hat{j} + 4\hat{k}$

$= \lambda\,[(y-3)\hat{i} - 6\hat{j} - 12\hat{k}]$

$\Rightarrow 2 = (y-3)\lambda \qquad \text{... (i)}$

and $4 - x = -6\lambda \qquad \text{... (ii)}$

Also, $4 = -12\lambda \quad \Rightarrow \lambda = \dfrac{-1}{3}$

Form (i), $y = -3$ and from (ii), $x = 2$

$\therefore (x,y) = (2, -3)$

(15) (d) $\bar{0}$

Since, $\bar{a} + 2\bar{b}$ is collinear with $\bar{c}$ and $\bar{b} + 3\bar{c}$ is collinear with $\bar{a}$.

$\therefore\ \bar{a} + 2\bar{b} = x\bar{c}$ and $\bar{b} + 3\bar{c} = y\,\bar{a}$ for some $x, y \in R$

$\therefore\ \bar{a} + 2\bar{b} + 6\bar{c} = (x+6)\bar{c}$

Also, $\bar{a} + 2\bar{b} + 6\bar{c} = (1+2y)\,\bar{a}$

$\therefore\ (x+6)\bar{c} = (1+2y)\,\bar{a}$

Since, $\bar{a}$ and $\bar{c}$ are non–collinear.

$\therefore\ x + 6 = 0$ and $1 + 2y = 0$

$\Rightarrow x = -6$ and $y = \dfrac{-1}{2}$

$\therefore\ \bar{a} + 2\bar{b} = x\bar{c} \Rightarrow \bar{a} + 2\bar{b} + 6\bar{c} = \bar{0}$

(16) (a) $5(\hat{j} - \hat{k})$

Let the vector be $a\hat{i} + b\hat{j} + c\hat{k}$.

It is perpendicular to $2\hat{i} + \hat{j} + \hat{k}$.

$\therefore\ 2a + b + c = 0$ ... (i)

It is coplanar with $\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + 2\hat{k}$.

$$\therefore \begin{vmatrix} a & b & c \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

$\therefore\ 3a - b - c = 0$ ... (ii)

On solving (i) and (ii), $a = 0$, $b = 5$, $c = -5$

$\therefore$ Required vector is $5(\hat{j} - \hat{k})$

(17) (a) $x = 0,\ y = 0,\ z = 0$

(18) (c) $A - C - B$

$$\bar{c} = \frac{3\bar{b} + 2\bar{a}}{3+2}$$

$\therefore$ C divides AB internally

$\therefore A - C - B$.

(19) (c) -1

$\overline{AB} = -2\bar{i} - \bar{j} - \bar{k}$

$\overline{AC} = \bar{i} + \bar{j} + \bar{k}$

$\overline{AD} = -2\bar{i} + (x-2)\bar{j} - 3\bar{k}$

Since $\overline{AB}$, $\overline{AC}$, $\overline{AD}$ are coplanar

$\left[\overline{AB}\ \overline{AC}\ \overline{AD}\right] = 0$

$$\therefore \begin{vmatrix} -2 & -1 & -1 \\ 1 & 1 & 1 \\ -2 & x-2 & x-3 \end{vmatrix} = 0 \text{ etc.}$$

(20) (b) parallel

Here $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j}$, $\bar{c} = 3\hat{i} + 5\hat{j} - 2\hat{k}$,

$\bar{d} = -\hat{j} + \hat{k}$

Now $\overline{AB} = \bar{b} - \bar{a} = \hat{i} + 2\hat{j} - \hat{k}$

$\overline{CD} = \bar{d} - \bar{c} = -3\hat{i} - 6\hat{j} + 3\hat{k} = -3(\hat{i} + 2\hat{j} - \hat{k}) \Rightarrow$

$\overline{CD} = -3\,\overline{AB} \Rightarrow$

$\overline{AB}$ and $\overline{CD}$ are parallel

(21) **(a)** $\hat{i} + 2\hat{j} - \hat{k}$

(22) **(d)** $\left(\frac{7}{2}, \frac{-21}{2}, 14\right)$

Here $\bar{a} = 2\hat{i} - 6\hat{j} + 8\hat{k}$, $\bar{b} = \hat{i} + 3\hat{j} - 4\hat{k}$

$$\bar{c} = \frac{\bar{b} - 3\bar{a}}{1 - 3} = \frac{-\hat{i} + 3\hat{j} - 4\hat{k} - 6\hat{i} + 18\hat{j} - 24\hat{k}}{-2}$$

$$= \frac{-7\hat{i} + 21\hat{j} - 28\hat{k}}{-2} = \frac{7}{2}\hat{i} - \frac{21}{2}\hat{j} + 14\hat{k} \Rightarrow$$

$$C \equiv \left(\frac{7}{2}, \frac{-21}{2}, 14\right)$$

(23) **(c)** $\frac{l\bar{c} + m\bar{b}}{l+m}$

(24) **(b)** C divides BA externally in the ratio 5 : 4

$$\bar{c} = 5\bar{a} - 4\bar{b} = \frac{5\bar{a} - 4\bar{b}}{1} = \frac{5\bar{a} - 4\bar{b}}{5-4}$$

(25) **(b)** $p = 1, q = 3, \mathrm{r} = -2$

$$(0, 0, 0) \equiv \left(\frac{q-3-5}{3}, \frac{p-2-1}{3}, \frac{-3+5+r}{3}\right) \Rightarrow$$

$$(0, 0, 0) \equiv \left(\frac{q-3}{3}, \frac{p-1}{3}, \frac{r+2}{3}\right)$$

$$\frac{p-1}{3} = 0, \quad \frac{q-3}{3} = 0, \quad \frac{r+2}{3} = 0 \Rightarrow$$

$p = 1, q = 3, \mathrm{r} = -2$

(26) **(a)** $p = -1, q = -3, r = \frac{7}{3}$

$$\left(r\frac{-4}{3}, \frac{1}{3}\right) \equiv \left(\frac{5+1+1}{3}, \frac{1+q-1}{3}, \frac{p+p+3}{3}\right) \Rightarrow$$

$$\left(r\frac{-4}{3}, \frac{1}{3}\right) \equiv \left(\frac{7}{3}, \frac{q-1}{3}, \frac{2p+3}{3}\right) \Rightarrow$$

$$r = \frac{7}{3}, \frac{q-1}{3} = \frac{-4}{3}, \frac{2p+3}{3} = \frac{1}{3} \Rightarrow$$

$$p = -1, q = -3, r = \frac{7}{3}$$

(27) **(a)** $\left(\frac{3}{2}, -2, -2\right)$

Let $C \equiv (x, y, z)$, then

$$\left(\frac{4}{3}, 0, \frac{-2}{3}\right) \equiv \left(\frac{1-2+x}{3}, \frac{4+3+y}{3}, \frac{2-5+z}{3}\right) \Rightarrow$$

$$\left(\frac{4}{3}, 0, \frac{-2}{3}\right) \equiv \left(\frac{x-1}{3}, \frac{y+7}{3}, \frac{z-3}{3}\right) \Rightarrow$$

$$\frac{x-1}{3} = \frac{4}{3}, \frac{y+7}{3} = 0, \frac{z-3}{3} = \frac{-2}{3} \Rightarrow$$

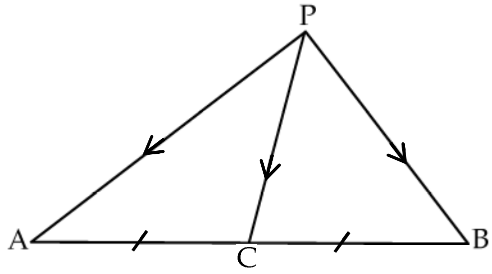
$x = 5, y = -7, z = 1 \Rightarrow C \equiv (5, -7, 1)$

Let D is mid–point of BC, then

$$D \equiv \left(\frac{-2+51}{2}, \frac{3-7}{2}, \frac{-5+1}{2}\right)$$

$$\equiv \left(\frac{3}{2}, -2, -2\right)$$

(28) **(c)** $\overline{PA} + \overline{PB} = 2\overline{PC}$



C is mid–point of AB, then

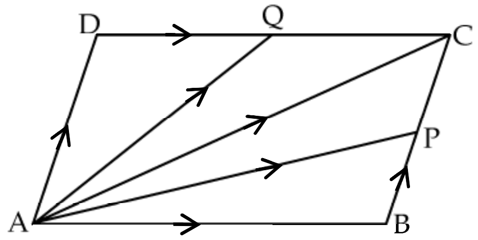
$\overline{AC} = \overline{CB}$

Now $\overline{PA} + \overline{PB} = \overline{PA} + \overline{AC} + \overline{PB} + \overline{BC} - (\overline{AC} + \overline{BC})$

$= \overline{PC} + \overline{PC} - (\overline{AC} + \overline{CB}) = 2\overline{PC} - (\overline{CB} - \overline{CB})$

$= 2\overline{PC}$

(29) **(a)** $\frac{3}{2}\overline{AC}$



$$\overline{AP} = \overline{AB} + \overline{BP} = \overline{AB} + \frac{1}{2}\overline{BC} = \overline{AB} + \frac{1}{2}\overline{AD}$$

$$\overline{AQ} = \overline{AD} + \overline{DQ} = \overline{AD} + \frac{1}{2}\overline{DC} = \overline{AD} + \frac{1}{2}\overline{AB}$$

Adding these, we get

$$\overline{AP} + \overline{AQ} = \frac{3}{2}\overline{AB} + \overline{AD} = \frac{3}{2}(\overline{AB} + \overline{BC})$$

$$= \frac{3}{2}\overline{AC}$$

(30) **(b)** $3\overline{CG}$

$$\overline{CA} + \overline{CB} = (\bar{a} - \bar{c}) + (\bar{b} - \bar{c})$$
$$= (\bar{a} + \bar{b}) - 2\bar{c}$$
$$= (\bar{a} + \bar{b} + \bar{c}) - 3\bar{c}$$
$$= 3\bar{g} - 3\bar{c}$$
$$= 3(\bar{g} - \bar{c}) = 3\overline{CG}$$

(31) **(a)** $\frac{1}{2}\overline{AC}$

Write $\overline{AD} = \bar{d} - \bar{a}$, $\overline{BE} = \bar{e} - \bar{b}$, $\overline{CF} = \bar{f} - \bar{c}$

and, then, simplify.

(32) **(b)** 1 : 3

Required ratio must give the x–coordinate of point of division as 4.

In (b), ... $\frac{1(7)+3(3)}{1+3} = 4.$

$\therefore$ option (b) is correct.

(33) **(b)** orthocentre of ΔABC

$\overline{AD} \perp \overline{BC}$ and $\overline{BD} \perp \overline{AC}$

$\therefore$ D is orthocentre.

(34) **(c)** $\frac{1}{8}(5\bar{a} + 3\bar{b})$

$$\bar{p} = \frac{3\bar{b} + \bar{a}}{4}$$

$$\bar{q} = \frac{\bar{a} + \bar{p}}{2} = \frac{\bar{a} + \frac{3\bar{b} + \bar{a}}{4}}{2} = \frac{5\bar{a} + 3\bar{b}}{8}$$

(35) **(d)** BD and AC trisect

$\bar{a} - \bar{b} = 2(\bar{d} - \bar{c})$

$\therefore \quad 2\bar{c} + \bar{a} = 2\bar{d} + \bar{b}$

$$\therefore \frac{2\bar{c}+\bar{a}}{2+1} = \frac{2\bar{d}+\bar{b}}{2+1}$$

$= \bar{p}$ (Let)

Point 'P' trisects AC & BD both

(36) **(b)** $\frac{12}{5}\bar{a} - \frac{13}{5}\bar{b}$

Position vector of the line joining the given points which divides internally in the ratio 2 : 3 is

$$\frac{3(2\bar{a}-3\bar{b})+2(3\bar{a}-2\bar{b})}{5} = \frac{12\bar{a}-13\bar{b}}{5}$$

(37) **(a)** $-3\hat{i} - \hat{k}$

$\vec{P} = \hat{i} + 2\hat{j} - \hat{k}, \ \vec{Q} = -\hat{i} + \hat{j} - \hat{k}$

Point R divides PQ externally in the ratio 2 : 1.

$$\therefore \vec{R} = \frac{2(-\hat{i}+\hat{j}-\hat{k})-1(\hat{i}+2\hat{j}-\hat{k})}{2-1}$$

$$\Rightarrow \vec{R} = -2\hat{i} + 2\hat{j} - 2\hat{k} - \hat{i} - 2\hat{j} + \hat{k} \quad = -3\hat{i} - \hat{k}$$

(38) **(d)** 4 , – 1

$$\text{Centroid} = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$$\therefore (3,2,c) = \left(\frac{2+a+3}{3}, \frac{3+1+2}{3}, \frac{-4-1+2}{3}\right)$$

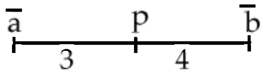
$\therefore \frac{a+5}{3} = 3 \qquad \& \quad c = \frac{-3}{3}$

$\therefore a + 5 = 9 \qquad \& \quad c = -1$

$a = 4$

(39) **(d)** $\frac{1}{7}(11\hat{i} + 5\hat{j} + 10\hat{k})$

Let $\bar{a} = 2\hat{i} - \hat{j} + \hat{k}, \ \bar{b} = \bar{i} + 3\bar{j} - 2\bar{k}$



$$\bar{p} = \frac{(3+8)\hat{i} + (9-4)\hat{j} + (6+4)\hat{k}}{3+4}$$

(40) **(d)** centroid

$\overline{PA} + \overline{PB} + \overline{PC} = \bar{0}$, given

$\therefore \bar{a} - \bar{p} + \bar{b} - \bar{p} + \bar{c} - \bar{p} = \bar{0}$

$\therefore \bar{a} + \bar{b} + \bar{c} = 3\bar{p}$

$$\therefore \bar{p} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}, \text{(centroid)}$$

(41) **(a)** $3\overline{QG}$

Let $\bar{p}$ and $\bar{g}$ be the position vectors of orthocentre P and centriod G. w.r.t circumcentre Q, then

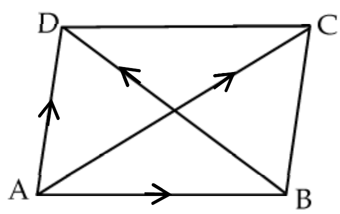
$\overline{QP} = \bar{p}, \ \overline{QG} = \bar{g}$

Q ,G , P are collinear and G divides QP internally in ratio 1 : 2, then

$$\bar{g} = \frac{\bar{p}+2\bar{q}}{1+2} = \frac{\bar{p}+0}{3} = \frac{\bar{p}}{3} \Rightarrow 3\bar{g} = \bar{p} \Rightarrow$$

$3\overline{QG} = \overline{QP}$

(42) **(c)** $\bar{a} + \bar{b}, \ \bar{b} - \bar{a}$



$\overline{AB} = \bar{a}, \ \overline{AD} = \bar{b}$

Diagonals are

$\overline{AC} = \bar{a} + \bar{b}$ and $\overline{BD} = \bar{b} - \bar{a}$

(43) **(d)** $\bar{O}$

Let $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ be the position vectors of vertices of $\square$ABCD

$2\,\overline{AB} + 3\,\overline{BC} + 2\overline{CD} + \overline{DA} + \overline{CA} + \overline{DB}$

$= 2\bar{b} - 2\bar{a} + 3\bar{c} - 3\bar{b} + 2\bar{d} - 2\bar{c} + \bar{a} - \bar{d} + \bar{a} - \bar{c} + \bar{b} - \bar{a}$

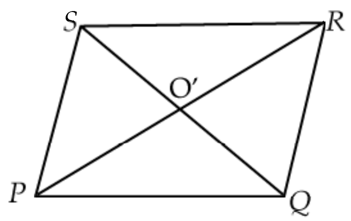
$= \bar{O}$

(44) **(a)** $4\overline{OO'}$

$\overline{OP} = \bar{p}, \overline{OQ} = \bar{q}, \overline{OR} = \bar{r}, \overline{OS} = \bar{s}, \overline{OO'} = \bar{o}'$

$\because$ diagonals PR and QS bisect each other ar o′

$\therefore\ \bar{o}' = \dfrac{\bar{p} + \bar{r}}{2} = \dfrac{\bar{q} + \bar{s}}{2}$ .... (i)



$\therefore\ \overline{OP} + \overline{OQ} + \overline{OR} + \overline{OS} = \bar{p} + \bar{q} + \bar{r} + \bar{s}$

$= (\bar{p} + \bar{r}) + (\bar{q} + \bar{s})$

$= 2\bar{o}' + 2\bar{o}'$

$= 4\bar{o}' = 4\overline{OO'}$

(45) **(b)** $3\overline{AC} - 2\overline{DC}$

$\because$ *ABCDE* is a regular pentagon

$\therefore\ \overline{AB} + \overline{BC} + \overline{AD} + \overline{ED} + \overline{AE}$

$= (\overline{AB} + \overline{BC}) + \overline{AD} + (\overline{AE} + \overline{ED})$

$= \overline{AC} + \overline{AD} + \overline{AD}$

$= \overline{AC} + 2\overline{AD}$

$= \overline{AC} + 2(\overline{CD} - \overline{CA})$ ... Note This Step

$= \overline{AC} + 2\overline{CD} - 2\overline{CA}$

$= \overline{AC} + 2(-\overline{DC}) - 2(-\overline{AC})$

$= \overline{AC} - 2\overline{DC} + 2\overline{AC}$

$= 3\overline{AC} - 2\overline{DC}$

(46) **(b)** $4\,\overline{OP}$

Let $\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{p}$ be the position vectors of points A, B, C, D, P respectively w.r.t.O.

P is the point of intersection of diagonals of parallelogram ABCD, then

$2\bar{p} = \bar{a} + \bar{c}$ and $2\bar{p} = \bar{b} + \bar{d} \Rightarrow$

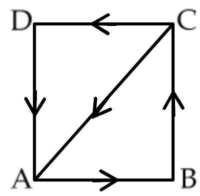
$2\overline{OP} = \overline{OA} + \overline{OC}$ and $\overline{OP} = \overline{OB} + \overline{OD}$

Adding these, we get

$4\overline{OP} = \overline{OA} + \overline{OB} + \overline{OC} + \overline{OD}$

(47) **(b)** $3\overline{CA}$

The option are given in terms of $\overline{CA}$



Now $2\overline{AB} + 4\overline{BC} + 5\overline{CD} + 7\overline{DA}$

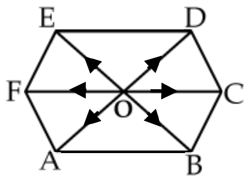
$\therefore\ 2\overline{AB} + 4\overline{BC} + 5\overline{CD} + 7\overline{DA}$

$= 2\overline{CD} - 4\overline{DA} + \overline{CD} + 7\overline{DA}$

$= 3\overline{CD} + 3\overline{DA} = 3(\overline{CD} + \overline{DA})$

$= 3\,\overline{CA}$

(48) **(c)** $\overline{O}$



$\overline{OA} + \overline{OD} = \overline{O}$

$\overline{OB} + \overline{OE} = \overline{O}$ etc.

(49) **(c)** $\frac{5}{\sqrt{2}}(\hat{j} - \hat{k})$

Find unit vector along $\overline{a} \times \overline{b}$

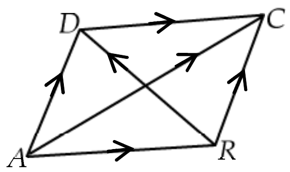
i.e. $\frac{\overline{a} \times \overline{b}}{|\overline{a} \times \overline{b}|}$

The required vector $= 5 \frac{\overline{a} \times \overline{b}}{|\overline{a} \times \overline{b}|}$.

(50) **(b)** $2\overrightarrow{AB}$

$\overline{AC} = \overline{AB} - \overline{BD}$

$\overline{BD} = \overline{BA} + \overline{AD}$



$\therefore \overline{AC} - \overline{BD} = (\overline{AB} - \overline{BC}) - (\overline{BA} + \overline{AD})$

$= (\overline{AB} + \overline{BC}) - (\overline{AB} + \overline{BC})$

$= \overline{AB} + \overline{BC} + \overline{AB} - \overline{BC}$

$= 2\overline{AB}$

(51) **(b)** -14

Projection of $(\overline{a} \times \overline{b})$ on $\overline{c}$ $= \frac{(\overline{a} \times \overline{b}).\overline{c}}{|\overline{c}|}$

$= \frac{1}{\sqrt{9+16+144}} \begin{vmatrix} 3 & -6 & -1 \\ 1 & 4 & -3 \\ 3 & -4 & -12 \end{vmatrix}$

$= \frac{1}{\sqrt{169}}$ $(3(-48-12) + 6(-12+9) - (-4-12))$

$= \frac{1}{\sqrt{13}}$ $(-180 - 18 + 16) = \frac{-182}{13} = -14$

(52) **(a)** $4a^2 + 10ab + 25ab^2$

$\overline{c} = 2\overline{a} + 5\overline{b} \Rightarrow \overline{c}.\overline{c} = (2\overline{a} + 5\overline{b}).(2\overline{a} + 5\overline{b})$

$c^2 = 4a^2 + 10\overline{a}.\overline{b} + 10\,\overline{b}\,\overline{a} + 25b^2 \Rightarrow$

$c^2 = 4a^2 + 20\overline{a}.\overline{b} + 25b^2 \Rightarrow$

$c^2 = 4a^2 + 20\,|\overline{a}|\,|\overline{b}|\cos 60^\circ + 25b^2 \Rightarrow$

$c^2 = 4a^2 + 10\,ab + 25b^2 \Rightarrow$

(53) **(a)** $\overline{a}$

Let $\overline{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, then

$\overline{a}.\hat{i} = a_1,\ \overline{a}.\hat{j} = a_2,\ \overline{a}.\hat{k} = a_3$

$(\overline{a}.\hat{i})\,\hat{i} + (\overline{a}.\hat{j})\,\hat{j} + (\overline{a}.\hat{k})\,\hat{k} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$= \overline{a}$

(54) **(b)** $\hat{i} + 2\hat{j} + 2\hat{k}$

a vertor parallel to $\overline{b}$

(55) **(c)** $\bar{c} - \left(\dfrac{\bar{a}.\bar{c}}{\bar{a}.\bar{b}}\right)\bar{b}$

$\bar{a}.\bar{b} \neq 0,\ \bar{a}.\ \bar{d} = 0$ .... (i)

also : $\bar{b} \times \bar{c} = \bar{b} \times \bar{d}$ $\quad \therefore\ \bar{b} \times (\bar{c} - \bar{d}) = \bar{0}$

$\therefore\ \bar{b} \,||\, (\bar{c} - \bar{d})$ $\quad \therefore\ \bar{c} - \bar{d} = \lambda \bar{b}$

$\therefore\ \bar{d} = \bar{c} - \lambda \bar{b}$ .... (ii)

$\therefore\ \bar{a}.\bar{d} = \bar{a}.(\bar{c} - \lambda \bar{b})$ $\quad \therefore\ 0 = \bar{a}.\bar{c} - \lambda \bar{a}.\bar{b}$

$$\therefore \lambda = \frac{\bar{a}.\bar{c}}{\bar{a}.\bar{b}}$$

$$\therefore \Rightarrow \bar{d} = \bar{c} - \left(\frac{\bar{a}.\bar{c}}{\bar{a}.\bar{b}}\right)\bar{b}$$

(56) **(b)** $2\bar{a}^2$

Taking $\bar{a} = i$,

$(i \times i)^2 + (i + j)^2 + (i \times k)^2$

$= \bar{a}^2 + k^2 + j^2$

$= 0 + 1 + 1$

$= 2 = 2\,(1) = 2i^2 = 2\bar{a}^2$

(57) **(b)** -7

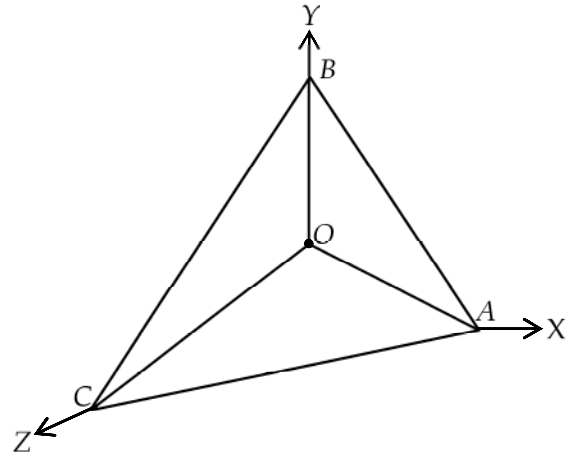
$\because\ \left(\bar{a} + \bar{b} + \bar{c}\right)^2 \bar{0}^2$

$\therefore\ \left(\bar{a}\right)^2 + \left(\bar{b}\right)^2 + \left(\bar{c}\right)^2 + 2\bar{a}.\bar{b} + 2\bar{b}.\ \bar{c} + 2\bar{c}.\bar{a} = 0$

$\therefore\ 1 + 4 + 9 + 2\,(\bar{a}.\bar{b} + ...) = 0$

$\therefore\ \bar{a}.\bar{b} + ... = -7$

(58) **(a)** $\cos^{-1}\left(\dfrac{19}{35}\right)$



$\overline{n_1}$ = prependicular to face OAB

$= \overline{OA} \times \overline{OB}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

$= 5\hat{i} - \hat{j} - 3\hat{k}$

$\overline{n_2}$ = perpendicular to face ABC

$= \overline{AB} \times \overline{AC}$

$= (\hat{i} - \hat{j} - 2\hat{k}) \times (-2\hat{i} - \hat{j} - \hat{k})$

$= \hat{i} - 5\hat{j} - 3\hat{k}$

$\because$ angle between faces = angle between normals

$$\therefore \cos\theta = \frac{\overline{n_1}.\overline{n_2}}{\overline{n_1}\,\overline{n_2}}$$

$$= \frac{(5)(1) + (-1)(-5) + (-3)(-3)}{\sqrt{5^2 + (-1)^2 + (-3)^2}\ \sqrt{1^2 + (-5)^2 + (-3)^2}}$$

$$= \frac{5+5+9}{\sqrt{35}\sqrt{35}} = \frac{19}{35}$$

(59) **(a)** $\pm 2(\bar{b}, \bar{c})$

$\because \bar{a}.\bar{b} = 0$ and $\bar{a}.\bar{c} = 0$

$\therefore \bar{a} \perp \bar{b}$ and $\bar{a} \perp \bar{c}$

$\therefore \bar{a} \parallel (\bar{b} \times \bar{c})$

$$\therefore \bar{a} = \pm \frac{\bar{b} \times \bar{c}}{|\bar{b} \times \bar{c}|} = \pm \frac{\bar{b} \times \bar{c}}{\sin(\pi/6)}$$

$= \pm 2(\bar{b}, \bar{c})$

(60) **(c)** $\frac{\pi}{3}$

$\because \bar{a} + \bar{b} + \sqrt{3}.\bar{c} = \bar{0}$

$\therefore \bar{a} + \bar{b} = -\sqrt{3}.\bar{c}$

$\therefore \left(\bar{a} + \bar{b}\right)^2 = \left(\bar{c}\right)^2$

$\therefore \left(\bar{a}\right)^2 + \left(\bar{b}\right)^2 + 2(\bar{a}.\bar{b}) = 3\,(1)$

$\therefore 1 + 1 + 2 + (1)(1) \cos\theta = 3$

$\therefore \cos\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}$

(61) **(c)** 1

$$\because \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$\therefore$ by $C_1 - C_2$ and $C_2 - C_3$

$$\therefore \begin{vmatrix} a-1 & 0 & 1 \\ 1-b & b-1 & 1 \\ 0 & 1-c & c \end{vmatrix} = 0$$

$\therefore (a-1)\,[c\,(b-1) - (1-c)] + [(1-b)\,(1-c)] = 0$

$\therefore c\,(a-1)\,(b-1) + (1-b)\,(1-c) - (a-1)\,(1-c) = 0$

$\therefore$ dividing throughout by $(1-a)\,(1-c)$,

$$\frac{c}{1-c} + \frac{1}{1-a} + \frac{1}{1-b} = 0$$

$$\therefore \left(\frac{c}{1-c} + 1\right) + \frac{1}{1-a} + \frac{1}{1-b} = 1$$

$$\therefore \frac{1}{1-a} + \frac{1}{1-b} + \frac{c}{1-c} = 1$$

(62) **(b)** -144

$$= \bar{a}.(\bar{b} \times \bar{c}) = \begin{vmatrix} 3 & -2 & 2 \\ 6 & 4 & -2 \\ 3 & -2 & -4 \end{vmatrix}$$

$= -144$

(63) **(b)** $\left|\bar{a} \times \bar{b}\right|^2$

We have $[\bar{a}\ \bar{b}\ \bar{a} \times \bar{b}\ ] = (\bar{a} \times \bar{b}).(\bar{a} \times \bar{b})$

$$= \left|\bar{a} \times \bar{b}\right|^2$$

(64) **(d)** none of these

Since $\left[\bar{a}\ \bar{b}\ \bar{c}\right] = \left[\bar{b}\ \bar{c}\ \bar{a}\right]$

$$= \left[\bar{c}\ \bar{a}\ \bar{b}\right]$$

$$= -\left[\bar{b}\ \bar{a}\ \bar{c}\right]$$

($\because$ The vectors are changed in cyclic order then value of box product remains same)

($\because$ If any two vectors of a box product are inter changed then its value changes in sing.)

(65) **(b)** 29

$\overline{AB}$, $\overline{AC}$ and $\overline{AD}$ are edges of tetrahedron.

$\therefore$ Volume of tetrahedron = $\frac{1}{6}\left[\overline{AB}\ \overline{AC}\,\overline{AD}\right]$

Now, $\overline{AB} = -2\hat{i} - 2\hat{j} - 3\hat{k}$

$\overline{AC} = 4\hat{i} - 9\hat{k}$

$\overline{AD} = 6\hat{i} - 3\hat{j} - 3\hat{k}$

$$\therefore \left[\overline{AB}\ \overline{AC}\,\overline{AD}\right] = \begin{vmatrix} -2 & -2 & -3 \\ 4 & 0 & -9 \\ 6 & -3 & -3 \end{vmatrix}$$

$= -2(0-27) + 2(-12+54) - 3(-12-0)$

$= -2(-27) + 2(42) + 36$

$= 54 + 84 + 36 = 174$

$\therefore$ Volume of tetrahedron = $\frac{1}{6}\left[\overline{AB}\ \overline{AC}\,\overline{AD}\right]$

$= \frac{1}{6}(174)$

$= 29$

(66) **(a)** 0

$$\left(\bar{p}-\bar{q}\right).\left[\left(\bar{p}-\bar{r}\right)\times\left(\bar{r}-\bar{q}\right)\right]$$

$$= \left[\bar{p}-\bar{q}\ \ \bar{q}-\bar{r}\ \ \bar{r}-\bar{p}\right] = 0$$

(67) **(b)** 3

$$\text{Volume} = \begin{vmatrix} 1 & 2 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{vmatrix} \left[\vec{a}\ \vec{b}\ \vec{c}\right] = k\left[\vec{a}\ \vec{b}\ \vec{c}\right]$$

$\Rightarrow 1(1-0) - 2(-1-0) - 1(-1+1) = k$
$\Rightarrow 1 + 2 - 0 = k$
$\Rightarrow k = 3$

(68) **(d)** 3

$\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{i}\times\hat{k}) + \hat{k}.(\hat{i}\times\hat{j})$

$= \hat{i}.\hat{i} + \hat{j}.\hat{j} + \hat{k}.\hat{k}$

$= 3$

(69) **(b)** a – b – c

$$\begin{vmatrix} a & 2 & 1 \\ 1 & -b & 1 \\ 1 & 1 & -c \end{vmatrix} = 0$$

$\therefore a(bc-1) - 1(-c-1) + 1(1+b) = 0$

$abc - a + c + 1 + 1 + b = 0$

$abc + 2$

$= a - b - c$

(70) **(a)** 0

$$\frac{\bar{a}.\bar{b}\times\bar{c}}{\bar{c}\times\bar{a}.\bar{b}} + \frac{\bar{b}.\bar{a}\times\bar{c}}{\bar{c}.\bar{a}\times\bar{b}} = \frac{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}{\left[\bar{c}\ \bar{a}\ \bar{b}\right]} + \frac{\left[\bar{b}\ \bar{a}\ \bar{c}\right]}{\left[\bar{c}\ \bar{a}\ \bar{b}\right]}$$

$$= \frac{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}{\left[\bar{c}\ \bar{a}\ \bar{b}\right]} - \frac{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}{\left[\bar{c}\ \bar{a}\ \bar{b}\right]} = 0$$

(71) **(a)** $[\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}] = 0$

As vector $\bar{\alpha}$ lies in the plane $\bar{\beta}$ and $\bar{\gamma}$

$\Rightarrow \bar{\alpha}\ \bar{\beta}\ \bar{\gamma}$ are coplanar $\Rightarrow [\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}] = 0$

(72) **(b)** $\bar{u}.(\bar{v}\times\bar{w})$

$(\bar{u}+\bar{v}-\bar{w}).[(\bar{u}-\bar{v})\times(\bar{v}-\bar{w})]$

$= \bar{u}\ (\bar{u}-\bar{v}) - \bar{u}.\ (\bar{v}-\bar{w}) + \bar{u}.(\bar{v}\times\bar{w})$

$+\bar{v}.(\bar{u}\times\bar{v})-(\bar{u}-\bar{v})+\bar{v}.(\bar{v}\times\bar{w})-\bar{w}.(\bar{u}-\bar{v})$

$+\bar{w}.(\bar{u}\times\bar{w})-\bar{w}.(\bar{u}\times\bar{w})$

$= \bar{u}.(\bar{v}\times\bar{w})-\bar{v}.(\bar{u}\times\bar{w})-\bar{w}(\bar{u}\times\bar{v})$

$= [\bar{u}\ \bar{v}\ \bar{w}]+[\bar{v}\ \bar{u}\ \bar{w}]-[\bar{w}\ \bar{u}\ \bar{v}]$

$= \bar{u}.(\bar{v}\times\bar{w})$

(73) **(a)** 3

$$\bar{p}+\bar{q}+\bar{r} = \frac{\bar{b}\times\bar{c}+\bar{c}\times\bar{a}+\bar{a}\times\bar{b}}{[\bar{a}\ \bar{b}\ \bar{c}]}$$

$$(\bar{a}+\bar{b}+\bar{c}).(\bar{p}+\bar{q}+\bar{r})$$

$$= \frac{[\bar{a}\ \bar{b}\ \bar{c}]+[\bar{b}\ \bar{c}\ \bar{a}]+[\bar{c}\ \bar{a}\ \bar{b}]}{[\bar{a}\ \bar{b}\ \bar{c}]}$$

$= 3$

(74) **(d)** no values of λ

$$\left[\lambda\left(\bar{a}+\bar{b}\right)\lambda^2\ \bar{b}\ \lambda\bar{c}\right] = \left[\bar{a}\ \bar{b}+\bar{c}\ \bar{b}\right]$$

$$\Rightarrow \lambda^4\left(\bar{a}+\bar{b}\ \bar{b}\ \bar{b}\right) = \left(\bar{a}\ \bar{b}+\bar{c}\ \bar{b}\right)$$

$$\Rightarrow \lambda^2\left\{\left(\bar{a}\bar{b}\bar{c}\right)+\left[\bar{b}\ \bar{b}\bar{c}\right]\right\} = \left\{\left[\bar{a}\ \bar{b}\bar{b}\right]+\left[\bar{a}\bar{c}\ \bar{b}\right]\right\}$$

$$\Rightarrow \lambda^2\left(\bar{a}\bar{b}\bar{c}\right) = -\left(\bar{a}\bar{b}\bar{c}\right)$$

$$\Rightarrow (\lambda^2+1)\left(\bar{a}\bar{b}\bar{c}\right) = 0$$

This is not true for any real value of l as

$$\left(\bar{a}\bar{b}\bar{c}\right) \neq 0$$

(75) **(c)** $\bar{v}.(\bar{u}\times\bar{w})$

(76) **(c)** -3

$$\text{Since, } \begin{vmatrix} -12 & 0 & \alpha \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix} = 546$$

$\therefore\ \alpha = -3$

(77) **(c)** $12\sqrt{3}$

$$\left(\bar{a}\bar{b}\bar{c}\right) = a.\ (b\times c) = a\ (|b|\ |c| \sin\theta\ \hat{n})$$

$$= a(\ 3\times 4\sin\frac{2\pi}{3}.\hat{n}) = a.\left(12\times\frac{2\pi}{3}\hat{n}\right)$$

$$= 6\sqrt{3}\ \ |a|\ |n| = 6\sqrt{3}\times 2\times 1$$

$$\Rightarrow 12\sqrt{3}$$

(78) **(b)** 1

For $\bar{a}\ \bar{b}\ \bar{c}$ to be coplanar,

$$\left(\bar{a}\bar{b}\bar{c}\right) = 0$$

$$\Rightarrow \begin{vmatrix} \lambda^3 & 0 & 1 \\ 1 & -\lambda^3 & 0 \\ 1 & 2\lambda-\sin\lambda & -\lambda \end{vmatrix} = 0$$

Þ $\lambda^7+\lambda^3+2\lambda =$ son λ

This is true of $\lambda = 0$.

For non–zero values of λ it gives

$$\lambda^6+\lambda^2+2 = \frac{\sin\lambda}{\lambda} \qquad \dots \text{(i)}$$

We konw that $\dfrac{\sin x}{x} < 1$ for all $x \neq 0$.

Therefore, L.H.S. of (i) is greater than 2 and R.H.S. is less than 1. So, (i) is not true for any non–zero λ.

Hence, there is only one value of λ.

(79) **(a)** – 5

Here, $\bar{a}$ and $\bar{b}$ are perpendicular unit vectors.

Now,

$$(2\bar{a}-\bar{b}).\{(\bar{a}\times\bar{b})\times(\bar{a}+2\bar{b})\}$$
$$=[\,2\bar{a}-\bar{b}\ \ \bar{a}\times\bar{b}\ \ \bar{a}+2\bar{b}\,]$$
$$=[\,\bar{a}\times\bar{b}\ \ 2\bar{a}-\bar{b}\ \ \bar{a}+2\bar{b}\,]$$
$$=(\bar{a}\times\bar{b}).\{(2\bar{a}-\bar{b})\times(\bar{a}+2\bar{b})\}$$
$$=-(\bar{a}\times\bar{b}).5\,(\bar{a}\times\bar{b})$$
$$=-5\,|\bar{a}\times\bar{b}|=-5\,|\bar{a}|^2\,|\bar{b}|^2 \qquad \ldots\left[\because \bar{a}\perp\bar{b}\right]$$
$$=-5 \qquad \ldots\left[\because |\bar{a}|=|\bar{b}|=1\right]$$

(80) **(c)** 27 cub. Units

(81) **(c)** 3

$$\bar{a}\,.\,\bar{p}+\bar{b}\,.\,\bar{q}+\bar{c}\,.\,\bar{r}$$
$$=\bar{a}\,.\frac{\bar{b}\times\bar{c}}{[\bar{a}\ \bar{b}\ \bar{c}]}+\bar{b}\,.\frac{\bar{c}\times\bar{a}}{[\bar{a}\ \bar{b}\ \bar{c}]}+\bar{c}\,.\frac{\bar{a}\times\bar{b}}{[\bar{a}\ \bar{b}\ \bar{c}]}$$
$$=\frac{[\bar{a}\ \bar{b}\ \bar{c}]}{[\bar{a}\ \bar{b}\ \bar{c}]}+\frac{[\bar{b}\ \bar{c}\ \bar{a}]}{[\bar{a}\ \bar{b}\ \bar{c}]}+\frac{[\bar{c}\ \bar{a}\ \bar{b}]}{[\bar{a}\ \bar{b}\ \bar{c}]}=3$$

(82) **(d)** $\bar{0}$

$$\hat{i}\times(\hat{j}\times\hat{k})$$

We have $\hat{j}\times\hat{k}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}=\hat{i}$

$$\therefore\ \hat{i}\times\hat{i}=\vec{0}$$

(83) **(c)** $(\bar{p}\times\bar{q}).\,\bar{r}=\bar{q}\times\bar{p}$

Informative

(84) **(d)** $\bar{a}\,.\,\bar{b}=\bar{b}\,.\,\bar{c}=\bar{c}\,.\,\bar{a}=0$

$$(\bar{a}\times\bar{b})\,.\,\bar{c}=|\bar{a}|\,|\bar{b}|\,|\bar{c}| \Rightarrow$$

$\bar{a},\bar{b},\bar{c}$ are mutaually perpendicular vectors

$\bar{a}\,.\,\bar{b}=\bar{b}\,.\,\bar{c}=\bar{c}\,.\,\bar{a}=0$

(85) **(a)** 3

$$\bar{a}\,.\,(\bar{b}\times\bar{c})=\begin{vmatrix}1 & 0 & 2\\ 2 & 1 & 0\\ 0 & 1 & 2\end{vmatrix} \Rightarrow$$

$1=(4q+1)-(8-1)+(-2-q) \Rightarrow$

$1=4q+1-7-2-q \Rightarrow$

$3q=9 \Rightarrow q=3$

(86) **(c)** 2

$$\text{Here } [\bar{a}\ \bar{b}\ \bar{c}]=0 \Rightarrow \begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 2 & 3 & m\end{vmatrix}=0 \Rightarrow$$

$(-m-3)-(m-2)+(3+2)=0 \Rightarrow$

$-m-3-m+2+5=0 \Rightarrow -2m=-4 \Rightarrow$

$m=2$

(87) **(c)** $m=-1,\ n=1$

$\bar{p}=m\,\bar{q}+n\,\bar{r} \Rightarrow$

$2\bar{a}-5\bar{b}+2\bar{c}=(m+3n)\,\bar{a}+(5m)\,\bar{b}+(-6m-4n)\bar{c} \Rightarrow$

$m+3n=2$ ... (i)

$5m=-5$ ...(ii)

$-6m-4n=2$ ...(iii)

Form (i) and (ii), we get

$m=-1,\ n=1$

These values satisfies equation (iii)

(88) **(a)** 7 cu. units

Here $\bar{a} = 4\hat{i} + 2\hat{j} + \hat{k}$, $\bar{b} = 2\hat{i} + \hat{j}$, $\bar{c} = 3\hat{i} + \hat{j} + \hat{k}$,

$\bar{d} = \hat{i} - \hat{j} + \hat{k}$,

Now $\overline{AB} = \bar{b} - \bar{a} = -2\hat{i} - \hat{j} - \hat{k}$

$\overline{AC} = \bar{c} - \bar{a} = -\hat{i} - \hat{j} - 2\hat{k}$

$\overline{AD} = \bar{d} - \bar{a} = -3\hat{i} - 3\hat{j} + \hat{k}$

(89) **(c)** 6

Here $\begin{vmatrix} 1 & 4 & 6 \\ 2 & a & 3 \\ 2 & 2 & -3 \end{vmatrix} = 0$

$-3a - 6 - 4(-6 - 3) + 6(4 - a) = 0 \Rightarrow$

$-3a - 6 + 36 + 24 - 6a = 0 \Rightarrow$

$-9a = -54 \Rightarrow a = 6$

(90) **(b)** $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

Points A,B,C,D are coplanar, then $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, are coplanar and

$\overline{AB} \cdot (\overline{AC} \times \overline{AD}) = 0 \Rightarrow$

$$\left(\bar{b} - \bar{a}\right) \cdot \left(\left(\bar{c} - \bar{a}\right) \times \left(\bar{d} - \bar{a}\right)\right) = 0 \Rightarrow$$

$$\left(\bar{b} - \bar{a}\right) \cdot \left(\bar{c} \times \bar{d} - \bar{c} \times \bar{a} - \bar{a} \times \bar{d} + \bar{a} \times \bar{a}\right) = 0 \Rightarrow$$

$$\left(\bar{b} - \bar{a}\right) \cdot \left(\bar{c} \times \bar{d} - \bar{c} \times \bar{a} - \bar{a} \times \bar{d}\right) = 0 \Rightarrow$$

$$\bar{b} \cdot \left(\bar{c} \times \bar{d}\right) - \bar{b}\left(\bar{c} \times \bar{a}\right) - \bar{b}\left(\bar{a} \times \bar{d}\right)$$

$$-\bar{a}\left(\bar{c} \times \bar{d}\right) - \bar{a}\left(\bar{c} \times \bar{a}\right) - \bar{a}\left(\bar{a} \times \bar{d}\right) = 0 \Rightarrow$$

$$\left[\bar{b}\ \bar{c}\ \bar{d}\right] - \left[\bar{b}\ \bar{c}\ \bar{d}\right] - \left[\bar{b}\ \bar{a}\ \bar{d}\right] - \left[\bar{a}\ \bar{c}\ \bar{d}\right] = 0 \Rightarrow$$

$$-\left[\bar{b}\ \bar{a}\ \bar{d}\right] + \left[\bar{b}\ \bar{c}\ \bar{d}\right] - \left[\bar{a}\ \bar{c}\ \bar{d}\right] = \left[\bar{b}\ \bar{c}\ \bar{a}\right] \Rightarrow$$

$$\left[\bar{a}\ \bar{b}\ \bar{d}\right] + \left[\bar{b}\ \bar{c}\ \bar{d}\right] + \left[\bar{c}\ \bar{a}\ \bar{d}\right] = \left[\bar{a}\ \bar{b}\ \bar{c}\right]$$

(91) **(a)** 0

If $\bar{A} = x_1\bar{a} + y_1\bar{b} + z_1\bar{c}$,

$\bar{B} = x_2\bar{a} + y_2\bar{b} + z_2\bar{c}$

$\bar{C} = x_3\bar{a} + y_3\bar{b} + z_3\bar{c}$

Then : $\left[\bar{A}\ \bar{B}\ \bar{C}\right] = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \left[\bar{a}\ \bar{b}\ \bar{c}\right]$

$\because$ $\bar{a}, \bar{b}, \bar{c}$ are coplanar vectors.

$\therefore \left[\bar{a}\ \bar{b}\ \bar{c}\right] = 0$

(92) **(c)** $\sqrt{59}$

$\bar{v} \times \bar{w} = \ldots = 3i - 7j - k$

$\therefore \left|\bar{v} \times \bar{w}\right| = \sqrt{9 + 49 + 1} = \sqrt{59}$

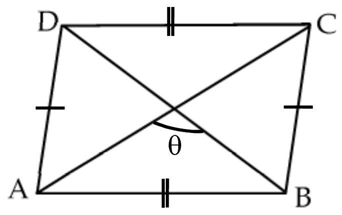
$\therefore \left[\bar{u}\ \bar{v}\ \bar{w}\right] = \bar{u} \cdot \left(\bar{v} \times \bar{w}\right)$

$= \left|\bar{u}\right| \left(\sqrt{59}\right) \cos\theta$

$= (1)\left(\sqrt{59}\right) \cos\theta$

$\leq \sqrt{59}$ .... $\because \cos\theta \leq 1$

(93) **(a)** $\cos^{-1}\left(\frac{2}{\sqrt{13}}\right)$



Form given $\overline{AC} = \overline{AB} + \overline{BC} = 3\hat{i} - 3\hat{j}$

Also $\overline{BC} = \overline{AD}$

and $\overline{AD} + \overline{DB} = \overline{AB}$

$\Rightarrow \overline{DB} = \overline{AB} - \overline{AD}$

$= \overline{AB} - \overline{BC}$

$= 3\hat{i} - \hat{j} + 4\hat{k}$

$\therefore$ Angle between $\overline{AC}$ and $\overline{BD}$

$$= \cos^{-1}\left(\frac{\overline{AC}.(\overline{BD})}{|\overline{AC}|.|\overline{BD}|}\right)$$

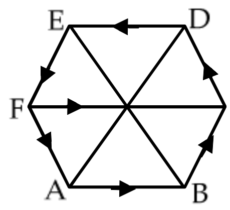
$$= \cos^{-1}\left(\frac{12}{\sqrt{9+9}\sqrt{9+1+16}}\right)$$

$$= \cos^{-1}\left(\frac{12}{3\sqrt{2}\sqrt{2}\sqrt{13}}\right)$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{13}}\right)$$

(94) **(d)** $4\overline{AB}$

A regular hexagon ABCDEF.



We know from the hexagon that $\overrightarrow{AD}$ is parallel to $\overrightarrow{BC}$ or $\overrightarrow{AD} = 2\overrightarrow{BC}$; $\overrightarrow{EB}$ is parallel to $\overrightarrow{FA}$ or $\overrightarrow{EB} = 2\overrightarrow{FA}$, and $\overrightarrow{FC}$ is parallel to $\overrightarrow{AB}$ or $\overrightarrow{FC} = 2\overrightarrow{AB}$.

Thus $\overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC} = 2\overrightarrow{BC} + 2\overrightarrow{FA} + 2\overrightarrow{AB}$

$= 2(\overrightarrow{FA} + \overrightarrow{AB} + \overrightarrow{BC})$

$= 2(\overrightarrow{FC}) = 2(2\overrightarrow{AB})$

$= 4\overrightarrow{AB}$.

(95) **(b)** $\sqrt{13}$

Here $\overrightarrow{OA} = \hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{OB} = 2\hat{i} + \hat{j} - \hat{k}$

and $\overrightarrow{OC} = 3\hat{i} - \hat{j} + 2\hat{k}$

These implies $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{i} + 2\hat{j} - \hat{k}$

and $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 2\hat{i}$

Hence required area is given by $= \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 0 & 0 \end{vmatrix} = -2(3\hat{j} + 2\hat{k})$$

$\Rightarrow$ Area of parallelogram

$$= \frac{1}{2} ´ 3 | 3\hat{j} + 2\hat{k} | = \sqrt{13}$$

(96) **(c)** $2\sin\left(\frac{\theta}{2}\right)$

$$\left|\bar{e}_1 - \bar{e}_2\right|^2 \left|\bar{a} \times \bar{b}\right| = \left|\bar{e}_1\right|^2 - 2\bar{e}_1 \cdot \bar{e}_2 + \left|\bar{e}_2\right|^2$$

$$= 1 - 2(1)(1)\cos\theta + 1$$

$$= 2 - 2\cos\theta$$

$$= 2(2\sin^2(\theta/2))$$

$$= 4\sin^2(\theta/2)$$

$$\therefore \left|\bar{e}_1 - \bar{e}_2\right|^2 = 2\sin^2(\theta/2)$$

(97) **(b)** $2\overline{AD}$

Basic formula

(98) **(b)** 105

Given : $\left|\bar{a} \times \bar{b}\right| = 15$

$$\therefore \left|3\bar{a} + 2\bar{b}\right| \times \bar{a} + 3\bar{b}$$

$$= \left|3\bar{a} \times \bar{a} + 9\bar{a} \times \bar{b} + 2\bar{b} \times \bar{a} + 6\bar{b} \times \bar{b}\right|$$

$$= \left|0 + 9\bar{a} \times \bar{b} - 2\bar{a} \times \bar{b} + 0\right|$$

$$= \left|7\bar{a} \times \bar{b}\right| = 7 \times 15 = 105.$$

(99) **(d)** right angled isosceles

Let $\bar{a} = 2\hat{i} + 4\hat{j} - \hat{k}$, $\bar{b} = 4\hat{i} + 5\hat{j} + \hat{k}$,

$\bar{c} = 3\hat{i} + 6\hat{j} - 3\hat{k}$

Now $\overline{AB} = \bar{b} - \bar{a} = 2\hat{i} + \hat{j} + 2\hat{k} \Rightarrow \left|\overline{AB}\right| = 3$

$\overline{BC} = \bar{c} - \bar{a} = \hat{i} + \hat{j} - 4\hat{k} \Rightarrow \left|\overline{BC}\right| = 3\sqrt{2}$

(100) **(a)** $\sqrt{450}$

Let $\bar{a} = 3\hat{i} + 4\hat{j} + 5\hat{k}$, $\bar{b} = 4\hat{i} - 3\hat{j} - 5\hat{k}$,

$\bar{c} = 7\hat{i} + \hat{j}$

$$\left|\bar{a}\right| = \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$$

$$\left|\bar{b}\right| = \sqrt{16+9+25} = \sqrt{50} = 5\sqrt{2}$$

$$\left|\bar{c}\right| = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

Perimeter $= 5\sqrt{2} + 5\sqrt{2} + 5\sqrt{2}$

$= 15\sqrt{2} = \sqrt{450}$

Points to remember

- Internal division formula $\Rightarrow \bar{p} = \dfrac{m\bar{b}+n\bar{a}}{m+n}$
- External division formula $\Rightarrow \bar{p} = \dfrac{m\bar{b}-n\bar{a}}{m-n}$
- Midpoint formula $\Rightarrow \bar{p} = \dfrac{\bar{a}+\bar{b}}{2}$
- Centroid of a triangle formula $\Rightarrow \bar{g} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$
- Centroid of a tetrahedron formula $\Rightarrow \bar{g} = \dfrac{\bar{a}+\bar{b}+\bar{c}+\bar{d}}{4}$
- Incentre formula $\Rightarrow \bar{i} = \dfrac{a\bar{a}+b\bar{b}+c\bar{c}}{a+b+c}$
- Two unit vectors may not be equal unless they have the same direction.
- Internal bisector of angle between $\bar{a}$ & $\bar{b}$ is $\lambda\left(\hat{a}+\hat{b}\right)$
- External bisector of angle between $\bar{a}$ & $\bar{b}$ is $\lambda\left(\hat{a}-\hat{b}\right)$
- If $\bar{a}\cdot\bar{b} > 0$, then angle between $\bar{a}$ & $\bar{b}$ is acute.
- If $\bar{a}\cdot\bar{b} < 0$, then angle between $\bar{a}$ & $\bar{b}$ is obtuse.
- Collinear vectors need not be along the same line.
- Area of a triangle = $\dfrac{1}{2}\left|\bar{a}\times\bar{b}\right|$ where $\bar{a}$ & $\bar{b}$ are adjacent sides.
- Area of a parallelogram = $\left|\bar{a}\times\bar{b}\right|$ where $\bar{a}$ & $\bar{b}$ are adjacent sides

- Area of a parallelogram = $\frac{1}{2}\left|\bar{d}_1 \times \bar{d}_2\right|$ where $\bar{d}_1$ & $\bar{d}_2$ are diagonals.
- Vector triple product.

$$\bar{a} \times \left(\bar{b} \times \bar{c}\right) = \left(\bar{a} . \bar{c}\right)\bar{b} - \left(\bar{a} . \bar{b}\right)\bar{c}$$

$$\left(\bar{a} \times \bar{b}\right) \times \bar{c} = \left(\bar{a} . \bar{c}\right)\bar{b} - \left(\bar{b} . \bar{c}\right)\bar{a}$$

- Volume of a parallelopiped = $\left(\bar{a}\,\bar{b}\,\bar{c}\right)$
- Volume of tetrahedron = $\frac{1}{6}\left(\bar{a}\,\bar{b}\,\bar{c}\right)$
- If $\bar{a}, \bar{b}, \bar{c}$ are three vectors, then

$$\left(\bar{a}-\bar{b}\right) . \left[\left(\bar{b}-\bar{c}\right) \times \left(\bar{c}-\bar{a}\right)\right] = 0$$

- If $\bar{a}, \bar{b}, \bar{c}$ are non–coplanar vectors, then

$$\begin{vmatrix} \bar{a}.\bar{a} & \bar{a}.\bar{b} & \bar{a}.\bar{c} \\ \bar{b}.\bar{a} & \bar{b}.\bar{b} & \bar{b}.\bar{c} \\ \bar{c}.\bar{a} & \bar{c}.\bar{b} & \bar{c}.\bar{c} \end{vmatrix} = \left[\bar{a}\,\bar{b}\,\bar{c}\right]^2$$

- If O is the circum centre of ΔABC,

then $\overline{OA} + \overline{OB} + \overline{OC} = 3\overline{OG} = \overline{OH}$,

where G is the centroid &

H is the orthocentre of triangle.

- If $\bar{a}, \bar{b}, \bar{c}$ are coplanar, then

$\bar{a}+\bar{b},\ \bar{b}+\bar{c},\ \bar{c}+\bar{a}$; $\bar{a}-\bar{b},\ \bar{b}-\bar{c},\ \bar{c}-\bar{a}$

and $\bar{a} \times \bar{b},\ \bar{b} \times \bar{c},\ \bar{c} \times \bar{a}$ are also coplanar.

- Projection of $\bar{a}$ on $\bar{b} = \frac{\bar{a}.\bar{b}}{b}$
- Projection of $\bar{b}$ on $\bar{a} = \frac{\bar{a}.\bar{b}}{a}$
- A vector perpendicular to the plane of $\bar{a}$ & $\bar{b}$ $\left(\bar{a} \times \bar{b}\right)$

EVALUATION PAPER - VECTORS

Time : 30 Min. **Marks : 25**

(1) If I is the incentre of ΔABC then its position vector $\bar{i}$ is

(a) $\dfrac{a\bar{a}+b\bar{b}+c\bar{c}}{a+b+c}$ (b) $\dfrac{1}{3}\left(\bar{a}+\bar{b}+\bar{c}\right)$ (c) $\dfrac{a\bar{a}+b\bar{b}+c\bar{c}}{\sqrt{a^2+b^2+c^2}}$ (d) $\dfrac{\bar{a}+\bar{b}+\bar{c}}{a+b+c}$

(2) $\bar{a} = \hat{i}+\hat{j}+\hat{k}$, $\bar{b}=\hat{i}-\hat{j}+\hat{k}$, $\bar{c}= 2\hat{i}+\hat{j}-\lambda\hat{k}$ Coplanar, then λ =

(a) – 2 (b) 2 (c) 3 (d) – 3

(3) If $\bar{a}, \bar{b}, \bar{c}$ are unit vector such that $\bar{a}+\bar{b}+\bar{c}= 0$ then $\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a}$ =

(a) $\dfrac{1}{2}$ (b) $-\dfrac{1}{2}$ (c) $\dfrac{3}{2}$ (d) $-\dfrac{3}{2}$

(4) Points (2 , – 7, β) , (5, 6, – 1), (– 1, – 20, 7) are collinear then β =

(a) 3 (b) – 3 (c) 1 (d) 2

(5) $\left[\bar{a}-\bar{b}\ \ \bar{b}-\bar{c}\ \ \bar{c}-\bar{a}\right]$ is equal to

(a) 1 (b) 2 (c) 0 (d) – 1

(6) If $2\bar{a}+3\bar{b}-5\bar{c} = 0$, then the ratio in which C divide AB is

(a) 1 : 2 externally (b) 2 : 4 internally (c) 3 : 2 internally (d) 2 : 5 internally

(7) The volume of parallelopiped, if A (0, 0, 0) , B (2, – 2, 3), C (10, 0, 3) , D (1, – 1, 2) form its co–terminus edges is

(a) 9 (b) 11 (c) 10 (d) 12

(8) If (X, 2, 3) B (1, Y, 3) C (4, 1, 2) are vertices of ΔABC with centroid G (2, 1, 1) then x, y, z are equal to

(a) x = 1 , y = 2 , z = – 3 (b) x = 1 , y = 0 , z = – 3

(c) x = 2 , y = 0 , z = – 3 (d) None of these

(9) If O (0, 0, 0) A (1, 2, 3) ,B (1, 2, 3) , and P (1, 2, 3) , are coplanar points, then which of the following is true.

(a) x – 2y + z = 0 (b) x – 2y – z = 0

(c) 2x – y + z = 0 (d) None of these

(10) If $\bar{p} = \dfrac{\bar{a}\times\bar{b}}{\left(\bar{a}+\bar{b}+\bar{c}\right)}$, $\bar{q} = \dfrac{\bar{c}\times\bar{a}}{\left(\bar{a}+\bar{b}+\bar{c}\right)}$, $\bar{r} = \dfrac{\bar{b}\times\bar{a}}{\left(\bar{a}+\bar{b}+\bar{c}\right)}$ then $\bar{a}\cdot\bar{p}+\bar{b}\cdot\bar{q}+\bar{c}\cdot\bar{r}$

(a) 3 (b) – 1 (c) 1 (d) 0

(11) If C is mid point AB and P is a point outside AB, then

(a) $\overline{PA}+\overline{PB}=\overline{PC}$ (b) $\overline{PA}+\overline{PB}+\overline{PC}=0$

(c) $\overline{PA}+\overline{PB}=2\,\overline{PC}$ (d) $\overline{PA}+\overline{PB}+2\,\overline{PC}=0$

(12) $\bar{a}\cdot\left[\left(\bar{b}+\bar{c}\right)\times\left(\bar{a}+\bar{b}+\bar{c}\right)\right]$ equals

(a) 0 (b) $\left(\bar{a}\ \bar{b}\ \bar{c}\right)\left(\bar{b}\ \bar{c}\ \bar{a}\right)$ (c) $\left(\bar{a}\ \bar{b}\ \bar{c}\right)$ (d) 1

(13) If $\bar{a}\cdot\bar{b}\cdot\bar{c}$ are non–coplanar vectors and λ is a real number, then the vectors $\bar{a}+2\bar{b}+3\bar{c}$, $\lambda\bar{b}+2\bar{c}$ and $(2\lambda-1)\,\bar{c}$ are non–coplanar for

(a) No value of λ (b) All except one value of λ

(c) All except two value of λ (d) All value of λ

(14) The incentre of triangle whose vertices are P (0, 4, 0), Q (– 2, 0, 0), R (– 2, 0, 2)

(a) $\left(\dfrac{-3}{2},\dfrac{1}{2},1\right)$ (b) $\left(\dfrac{3}{2},\dfrac{-1}{2},1\right)$ (c) $\left(\dfrac{-3}{2},\dfrac{-1}{2},1\right)$ (d) $\left(\dfrac{3}{2},\dfrac{1}{2},1\right)$

(15) If D, E, F are the mid–points of sides BC, CA, AB respectively of triangle ABC, then $\overline{AD}+\overline{BE}+\overline{CF}=$

(a) $\overline{O}$ (b) $\bar{a}+\bar{b}+\bar{c}$ (c) $2\left(\bar{a}+\bar{b}+\bar{c}\right)$ (d) $-\left(\bar{a}+\bar{b}+\bar{c}\right)$

(16) If $\bar{a}=3\hat{i}-2\hat{j}+7\hat{k}$, $\bar{b}=5\hat{i}+\hat{j}-2\hat{k}$, $\bar{c}=\hat{i}+\hat{j}-\hat{k}$, then $\bar{a}\cdot\left(\bar{b}\times\bar{c}\right)=$

(a) – 25 (b) 37 (c) 25 (d) 31

(17) If $\bar{a}, \bar{b}, \bar{c}$ be distinct non–negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$, $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is

(a) the arithmetic mean of a and b (b) the geometric mean of a and b

(c) the harmonic mean of a and b (d) equal to zero.

(18) If the points A (2, – 1, 0), B (– 3, λ, 4), C = (– 1,– 1, 4), D (0, – 5, 2) are coplanar, then λ =

(a) 16 (b) – 16 (c) 17 (d) – 17

(19) If $\bar{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\bar{b} = x\hat{i} - 5\hat{j} + 3\hat{k}$, $\bar{c} = 5\hat{i} - 9\hat{j} + 4\hat{k}$ and $\begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix} = 0$, then x =

(a) 3 (b) 4 (c) 2 (d) – 1

(20) If $\bar{u}$, $\bar{u}$ and $\bar{w}$ are three non–coplanar vectors, then $\left(\bar{u} + \bar{v} - \bar{w}\right) \cdot \left(\left(\bar{u} - \bar{v}\right) \times \left(\bar{v} - \bar{w}\right)\right) =$

(a) 0 (b) $\bar{u} \cdot \left(\bar{v} \times \bar{w}\right)$ (c) $\bar{u} \cdot \left(\bar{v} \times \bar{w}\right)$ (d) $3\bar{u} \cdot \left(\bar{v} \times \bar{w}\right)$

(21) If $\bar{a} = \hat{i} - 5\hat{j}$ and $\bar{b} = 6\hat{i} + 3\hat{j}$ are two vectors, and $\bar{c}$ is a vector such that $\bar{c} = \bar{a} \times \bar{b}$, then :

$\left|\bar{a}\right| : \left|\bar{b}\right| : \left|\bar{c}\right| =$

(a) $\sqrt{34} : \sqrt{45} : \sqrt{39}$ (b) $\sqrt{34} : \sqrt{45} : 39$ (c) 34 : 39 : 45 (d) 39 : 35 : 34

(22) $\bar{a}$ and $\bar{b}$ are two non–collinear vectors, then $x\bar{a} + y\bar{b}$ (where x and y are scalars) represents a vector which is

(a) Parallel to $\bar{b}$ (b) parallel to $\bar{a}$

(c) Coplanar with $\bar{a}$ and $\bar{b}$ (d) None of these

(23) If three points A, B, C are collinear, whose position vectors are $\hat{i} - 2\hat{j} - 8\hat{k}$ and $11\hat{i} + 3\hat{j} + 7\hat{k}$ respectively, then the ratio in which B divides AC is

(a) 1 : 2 (b) 2 : 3 (c) 2 : 1 (d) 1 : 1

(24) $\bar{a} \cdot \left[\left(\bar{b}+\bar{c}\right) \times \left(\bar{a}+\bar{b}+\bar{c}\right)\right]$ is equal to

(a) $\left(\bar{a}\,\bar{b}\,\bar{c}\right)$ (b) $2\left(\bar{a}\,\bar{b}\,\bar{c}\right)$ (c) $3\left(\bar{a}\,\bar{b}\,\bar{c}\right)$ (d) 0

(25) The volume of tetrahedron whose coterminus edges are $2\hat{i}+\hat{j}-\hat{k}$, $\hat{i}-2\hat{k}$ and $\hat{i}+3\hat{j}-+\hat{k}$ is :
(a) 3 Cu. units (b) 2 Cu. units (c) 4 Cu. units (d) 5 Cu. units

EVALUATION PAPER - VECTORS

1 **a**	2 **a**	3 **d**	4 **a**	5 **c**	6 **c**	7 **c**	8 **b**	9 **a**	10 **a**
11 **c**	12 **b**	13 **c**	14 **b**	15 **a**	16 **c**	17 **b**	18 **d**	19 **c**	20 **b**
21 **b**	22 **c**	23 **b**	24 **d**	25 **b**					

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