MOTION IN A PLANE

Projectile motion –

- Initial velocity $u_x = u\cos\theta,\ u_y = u\sin\theta$
- Acceleration $a_x = 0,\ a_y = -g$

Equation of trajectory: It describes the path of particle

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

Displacement of particle:

$$\vec{r} = (u\cos\theta)t\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}$$

Instantaneous velocity:

$$\vec{V} = V_x\hat{i} + V_y\hat{j},\ |\vec{V}| = \sqrt{u^2 + g^2t^2 - 2u\sin\theta\, gt}$$

Time of flight: The time taken to complete the motion

$$T = \frac{2u\sin\theta}{g}$$

Ascent time: (t_a) the time required to reach max height

$$t_a = \frac{T}{2} = (t_d)\ Descent\ time$$

Maximum Height:

$$H_{max} = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$$

Range: The maximum horizontal distance covered

$$R = \frac{u^2\sin 2\theta}{g} = \frac{u\cos\theta.2.u\sin\theta}{g} = \frac{2.U_x.U_y}{g}$$

Maximum Range: For range to be max, $\theta = 45°$

$$R_{max} = \frac{u^2.\sin(2\times 45°)}{g} = \frac{u^2}{g}$$

NOTE: Range will be same for complementary angles, θ and $90 - \theta$

Relation between range and height:

$$\tan\theta = \frac{4H}{R}$$

NOTE: For complementary angles of projection θ and $90° - \theta$
Ratio of maximum height

$$\frac{H_1}{H_2} = \tan^2\theta$$

Change in velocity:

$$\Delta V = V_f - V_i$$
$$\vec{v}_i = \vec{u}_i = u\cos\theta\hat{i} + u\sin\theta\hat{j}$$
$$\Delta\vec{V} = -gt\hat{j}$$

Change in velocity in complete projectile:

$$\Delta\vec{v} = v_f - u_i = -2u\sin\theta\hat{i}$$

Equation of trajectory:

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta} = x\tan\theta\left[1 - \frac{x}{R}\right]$$

HORIZONTAL PROJECTILE MOTION

Horizontal	Vertical
$u_x = u$	$u_y = 0$
$V_x = ?$	$V_y = ?$
$S_x = x$	$S_y = -y$
$a_x = 0$	$a_y = -g$
$t_x = t$	$t_y = t$

Time of flight –

$$T = \sqrt{\frac{2H}{g}}$$

Equation of trajectory:

$$y = \frac{gx^2}{2u^2}$$

Instantaneous velocity:

$$\vec{V} = u\hat{i} - gt\hat{j}$$

Range –

$$R = u.\sqrt{\frac{2H}{g}}$$

Speed –

$$|\vec{V}| = \sqrt{u^2 + (gt)^2}$$

Displacement of particle –

$$|\vec{r}| = \sqrt{(ut)^2 + \left(\frac{1}{2}gt^2\hat{j}\right)^2}$$

$$\alpha = \tan^{-1}\left(\frac{gt}{2u}\right)$$

$$\emptyset = \tan^{-1}\left[\frac{V_y}{V_x}\right]$$

$$\emptyset = \tan^{-1}\left[\frac{gt}{u}\right]$$