3. LOGIC

SYNOPSIS

1. **Logic.**

Logic is the study of general patterns of reasoning, without reference to particular meaning or contents.

Acrrording to dictionary, **Logic is a science of reasoning.** Logic is the process with the help of which, we arrive at a conclusion from given statement by using valid laws of logic. Logic plays an important role in any study involving reasoning. The mathematical treatment of logic with similar statement of set theory. (known as axomatic set theory) is now available.

Types of Sentences. In the process of reasoning we explain our ideas with the help of sentences in a particular language. Following types of sentences are generally used in our daily normal life.

Assertive Sentence. A sentence that makes an assertion is called as assertive sentence or a declarative sentence.

e.g., (i) Moon rotates around the Earth.

(ii) Two plus four is 6.

(iii) The sun is a star.

are all declarative or assertive sentences.

Imperative sentence. A sentence that ecpresses a request or a command is called **imperative sentence.**

e.g. (i) Give me a glass of water.

(ii) Do your home work.

(iii) Please do me a favour.

are all imperative sentences.

Exclamatory Sentence. A sentence that expresses some strong feeling is called an **exclamatory sentence.**

e.g. (i) May God help you !

(ii) May you live long !

(iii) How big is the elephant !

Interrogative Sentence. A sentence that asks some question is called an **interrogative sentence.**

e.g. (i) How are you ?

(ii) Where is your pen ?

(iii) Where are you going ?

are all **interrogative sentences.**

2. **Logical Statement or Proposition.**

A logical statement is any sentence which is (i) meaningful (ii) declarative (iii) unambigous. The statement is either true or false or equivalently valid or invalid.

A statement cannot be both true or false at the same time. This fact is known as law of **excluded middle.**

The falseness or truth of a statement is called its truth value.

An open sentence is not a statement.

Mathematical identities are considered to be statements.

Note. A sentence which is both true and false simultaneously is not a statement; it is a **paradox.**

Examples.

(i) **New Delhi is in India is a true statement**

(ii) **Two plus two is four is a true statement.**

(iii) Roses are red, is a true statement.

(iv) ‘Every set is a finite set’ is a false statement.

(v) ‘8 is less then 5, is a false statement.

Thus sentences containing request or wish or command are not statements.

Interrogative sentences, exclamatroy sentences are not statements.

3. **Compound Statements**

If two or more simple statements are combined by the use of words such as “and”, “or”, “not”, “if”, “then”, “if and only if”, then the resulting statement is called **a compound statement**

The words ‘and’ ‘not’, ‘if’, ‘then’, “if and only if” are called **logical connectives.**

In a compound statement, simple statements are called **components.**

Notations : Generally small letters p,q,r,.... denote simple statements.

Examples. (i) p : $\sqrt{2}$ is a real number.

(ii) q : Ram is sleeping.

(i) and (ii) are simple statements.

4. **Truth Tables**

Truth table is that which gives truth values of compound statements.

It has a number of rows and columns.

The number of rows depend upon the number of simple statements.

Note that for n statements, there are 2^n rows.

Truth Tables

(a) Truth Table for single statement p

Number of rows = $2^1 = 2$

P
T
F

(b) **Truth table for two statements p and q :**

Number of rows = $2^2 = 4$

p	q
T	T
T	F
F	T
F	F

(c) **Truth Table for three statements p,q and r :**

Number of rows = $2^3 = 8$

p	q	r
T	T	T
T	F	T
F	T	T
F	F	T
T	T	F
T	F	F
F	T	F
F	F	F

5. **Logical Equivalence**

Two compound statements are said to be logically equivalent or equal if they have identical truth values.

The symbols ‘$\equiv$’ or ‘=’ are used for above.

6. **Conjunction and Disjunction**

(a) Conjunction : If two statements are compound by the connective “and” so as to form a compound statement, then the compound statement called the conjuction of the original statements.

Symbolically : If p, q are the two statements, then their conjuction is denoted by $p \wedge q$ read as “p and q”.

Example : Consider the two statements : “He is hard-working”. “He is intelligent”.

The conjunction is “He is hard-working and intelligent”.

Gold Coins : $p \wedge q$ is true, if p and q are both true.

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(b) Disjunction. If two statements are combined by the connective "or" so as to form a compound statement, then the compound statement is called the disjunction of the original statements.

Symbolically. If p,q are two statements, then their disjunction is denoted by $p \vee q$ and is read is "p or q".

Example . Consider the two statements

"Ram will play", "Sham will play" The disjunction is "Ram or Sham will play".

Gold Coins : $p \vee q$ is false if p and q are both false.

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation. For any statement, we have a statement which is the negation.

Negation is not a contrary statement but is a contradiction.

Symbolically. Negation of p is denoted by ~ p

Example . 1. If p is "Ram is a good boy". Then ~ p is "Ram is not a good boy" or it is not the case that "Ram is a good boy".

2. p : Ram is sad.

~p : Ram is not sad.

(We cannot write Ram is happy)

3. p : Sita is fat.

~p : Sita is not fat.

(Sita is thin does not hold)

Gold Coins : If p is true then ~p is false.

If p is false then ~p is true.

Truth Table

p	~p
T	F
F	T

7. Some properties.

(a) Prove that ~(~P)=P

Proof. Truth Table

p	~p	~(~p)
T	F	T
F	T	F

We observe that first and third columns are identical. Hence

~(~p)=p.

(b) Prove that

$\sim(p \wedge q) = \sim p \vee \sim q$ [De-Morgan's law]

Proof : Truth Table

p	q	~p	~q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
F	T	T	F	F	T	T
T	F	F	T	F	T	T
F	F	T	T	F	T	T

We observe that last two cloumns are identical. Hence

$\sim(p \wedge q) = \sim p \vee \sim q$.

(c) Prove that

$\sim(p \vee q) \equiv \sim p \wedge \sim q$ [De-Morgan's Law]

Proof : Truth Table

p	q	~p	~q	$p\vee q$	$\sim(p\vee q)$	$\sim p\wedge\sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

We observe that last two columns are identical.

Hence $\sim(p\vee q)\equiv\sim p\wedge\sim q$.

(d) Prove that

$(p\wedge q)\wedge r=p\wedge(q\wedge r)$ [Associative Law]

Proof : Truth Table

p	q	r	$p\wedge q$	$q\wedge r$	$(p\wedge q)\wedge r$	$p\wedge(q\wedge r)$
T	T	T	T	T	T	T
T	F	T	F	F	F	F
T	T	F	T	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	F	T	F	F	F	F
F	T	F	F	F	F	F
F	F	F	F	F	F	F

We observe that last two columns are identical.

Hence $(p\wedge q)\wedge r=p\wedge(q\wedge r)$

Example - 1: (a) For each of the following sentences, state whether it is a statement and indicate its truth value if it is a statement.

(i) 4x3+5=17

(ii) Red Fort is in Agra.

(iii) 6 has three prime factors.

(iv) 17 is a prime number.

(v) Where are you going ?

(vi) Listen to me Krishna !

(vii) $\sqrt{2}$ is a rational number.

(b) Write the negation of the following statements :

(i) p : It rained on July 10, 2002

(ii) q : All integers are rational numbers

(iii) r : 2+6=8.

Sol. (a) (i) It is a true statement

(ii) This is a false statement.

(iii) This is a false statement.

(iv) This is a truth statement.

(v) This is not a statement.

(vi) This is imperative and not statement.

(vii) This is a false statement.

(b) (i) ~p : It did not rain on July 10, 2002.

(ii) ~q It is not the case that all integers. are rational numbers.

(iii) ~r : $2+6\neq 8$

Example - 2: You are given the following statements :

p : 4 x 7 = 28

q : Sun is a heavely body

r : Delhi is most populated city in India

s : $\cos 60^0=\frac{1}{2}$

State the truth values of the following :

(i) $p\wedge q, p\wedge r, p\wedge s, q\wedge r, q\wedge s, r\wedge s$

(ii) $p\vee q, p\vee r, p\vee s, q\vee r, q\vee s, r\vee s$

(iii) $\sim(p\vee s)$

(iv) $p\wedge r, r\wedge s, q\wedge r, q\vee s$.

Sol. Here p,q,s are true statements and r is a false statement.

(i) $p\wedge q, p\wedge s, q\wedge s$ are true and $p\wedge r, q\wedge r, r\wedge s$ are false

(ii) All are true.

(iii) False

(iv) False, True, False, True.

Example - 3 : Construct the truth tables for the following : $p \wedge \sim p$

Sol. Truth Table

p	~p	$p \wedge \sim p$
T	F	F

Example - 4 : Construct the truth table for the following : $\sim(p \wedge q)$

Sol. Truth Table

P	q	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Example - 5 : Prove by constructing truth tables that $p \wedge (p \vee q) = p$

Sol. Truth Table

P	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
F	T	T	F
T	F	T	T
F	F	F	T

We observe that first and last column are identical. Hence $p \wedge (p \vee q) = p$

Example - 6 : Prove by constructing truth table that $\sim(\sim p \vee \sim q) = p \wedge q$.

Sol. Truth Table

p	q	~p	~q	$p \wedge q$	$\sim p \vee \sim q$	$\sim(\sim p \vee \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

From columns five and last, we observe that $\sim(\sim p \vee \sim q) = p \wedge q$

Example - 7 : Prove the following : $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$.

Sol. Truth Table

p	q	r	$q \wedge r$	$p \vee q$	$p \vee r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	F	T	F	F
T	T	F	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	F	F	T	F	F	F
F	F	F	F	F	F	F	F

From columns seven and eight, we observe that $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$.

Example - 8 : Prove that following :

(i) $(p \vee q) \vee r = p \vee (q \vee r)$

[Associative law]

(ii) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

[Distributive law]

Sol.(i) Truth Table

p	q	r	$p \vee q$	$q \wedge r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
T	T	F	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	F	T	F	T	T	T
F	T	F	T	T	T	T
F	F	F	F	F	F	F

We observe that last two columns are identical.

Hence $(p \vee q) \vee r = p \vee (q \vee r)$

(iii) Truth Table

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	F	T	T	F	T	T	T
F	T	T	T	F	F	F	F
F	F	T	T	F	F	F	F
T	T	F	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	F	T	F	F	F	F
F	F	F	F	F	F	F	F

We observe that last two columns are identical.

Hence $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$.

8. Conditional and Biconditional Statements

(a) Conditional Statement. If p and q are two statements, then statement of the form "if p then q" is called the conditional statement.

Symbolically. If "p then q" is denoted by $p \Rightarrow q$ and is read as p implies q.

Gold Coins : $p \Rightarrow q$ is true in all cases except when p is true and q is false.

Truth Table

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Properties :

(i) Prove that $p \Rightarrow q = \sim p \vee q$

Proof. Truth Table

p	q	~p	$p \Rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

We observe that the last two statements are identical.

Hence $p \Rightarrow q = \sim p \vee q$

(ii) Prove that $\sim (p \Rightarrow q) = p \wedge \sim q$

Proof. Truth Table

p	q	~q	$p \Rightarrow q$	$\sim (p \Rightarrow q)$	$p \wedge \sim q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

We observe that last two columns are identical.

Hence $\sim (p \Rightarrow q) = p \wedge \sim q$

(b) **Biconditional Statement.** If p and q are two statements, the statement of the form "p if and only if q" is called the biconditional statement.

Symbolicaly. "p if and only if q" is denoted by $p \Leftrightarrow q$ and is read as

(i) p if and only if

(ii) q implies p and p implies q.

(iii) p is necessary and sufficient for q.

(iv) q is necessary and sufficient for p.

(v) p iff q.

(vi) q iff p.

Gold Coins : $p \Leftrightarrow q$ is true if both p and q have the same truth value i.e., either both p and q are true or both are false.

$p \Leftrightarrow q$ is false if p and q have opposite truth values.

Truth Table

p	q	$p \Leftrightarrow q$
T	T	T
F	T	F
T	F	F
F	F	T

Property. Prove that

$$(p \Leftrightarrow q) \Leftrightarrow r = p \Leftrightarrow (q \Leftrightarrow r)$$

Proof. Truth Table

p	q	r	$p \Leftrightarrow q$	$q \Leftrightarrow r$	$(p \Leftrightarrow q) \Leftrightarrow r$	$p \Leftrightarrow (q \Leftrightarrow r)$
T	T	T	T	T	T	T
T	F	T	F	F	F	F
T	T	F	T	F	F	F
T	F	F	F	T	T	T
F	T	T	F	T	F	F
F	F	T	T	F	T	T
F	T	F	F	F	T	T
F	F	F	T	T	F	F

We observe that the last two columns are identical.

Hence $(p \Leftrightarrow q) \Leftrightarrow r = p \Leftrightarrow (q \Leftrightarrow r)$

Example - 9 : Construct a truth table for $\sim p \Rightarrow \sim q$.

Sol. Truth Table

p	q	~p	~q	$\sim p \Rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Example - 10 : Find the truth value of the following : $\sim (p \Rightarrow q)$.

Sol. Truth Table

p	q	$p \Rightarrow q$	$\sim (p \Rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Example - 11 : Prove by constructing truth table that

$$\sim(p \Leftrightarrow q) = \sim p \Leftrightarrow q = p \Leftrightarrow \sim q$$

Sol. Truth Table

p	q	~p	~q	$p \Leftrightarrow q$	$\sim(p \Leftrightarrow q)$	$\sim p \Leftrightarrow q$	$p \Leftrightarrow \sim q$
T	T	F	F	T	F	F	F
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	T	F	F	F

We observe that columns (4), (5) and (6) are identical.

$$\therefore \sim(p \Leftrightarrow q) = \sim p \Leftrightarrow q = p \Leftrightarrow \sim q$$

Example - 12 : Prove Idempotent laws : i.e. prove that $p \wedge p = p, p \vee p = p$

Sol. Truth Table

p	p	$p \wedge p$
T	T	T
F	F	F

We observe that $p \wedge p = p$.

Truth Table

p	p	$p \vee p$
T	T	T
F	F	F

We observe that $p \vee p = p$

Example - 13 : Prove by truth table

$$\sim(p \Rightarrow q) = \sim(\sim p \vee q).$$

Sol. Truth Table

p	q	$p \Rightarrow q$	$\sim(p \Rightarrow q)$	~p	$\sim p \vee q$	$\sim(\sim p \vee q)$
T	T	T	F	F	T	F
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	F	T	T	F

We observe that columns (4) and (7) are identical.

Hence $\sim(p \Rightarrow q) = \sim(\sim p \vee q)$

Example - 14 : Prove by truth table $p \Leftrightarrow q = (p \Rightarrow q) \wedge (q \Rightarrow p)$.

Sol. Truth Table

p	q	$p \Leftrightarrow q$	$p \Rightarrow q$	$q \Leftrightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

We observe that columns (3) and (6) are identical.

Hence $p \Leftrightarrow q = (p \Rightarrow q) \wedge (q \Rightarrow p)$.

9. Tautologies and Fallacies

(Contradictions)

(a) Tautology : This is a statement which is always true for all truth values of its components.

Example. Consider $p \vee \sim p$

Truth Table

p	p	$p \vee \sim p$
T	F	T
F	T	T

We observe that last columns is always true. Hence $p \vee \sim p$ is a tautology.

(b) Fallacy (Contradiction). This is a statement which is always false for all truth values of its components.

Example. Consider $p \wedge \sim p$

Truth Table

p	~p	$p\wedge\sim p$
T	F	F
F	T	F

We observe that last column is always false. Hence $p\wedge\sim p$ is a fallacy (or contradiction).

10. Arguments

An argument is the assertion that a statement called the conclusion follows from other statements called the hypothesis.

Examples. Consider the following three statements :

(i) If he works hard, he will be successful

(ii) He is not successful

(iii) Therefore, he did not work hard.

These three statements taken together constitute an argument. Here first two are hypothesis and last one is conclusion.

Let p : He works hard, q : He is successful. Above argument can be symbolically written as :

$$\left[(p\Rightarrow q)\wedge\sim q\right]\Rightarrow\sim p$$

If the argument is a tautology, it is called valid argument. If the argument is not a tautology, it is invalid argument.

An argument is a statement such that given statements $P_1,P_2,P_3...P_n$ taken together give another statement P.

Symbolically. We wirte as

$P_1,P_2,P_3...P_n$/–p.

The symbol "/-" is read as turnstile.

The statements $P_1,P_2,P_3...,P_{nj}$ are called "premises" or "assumptions". The statement P is called the conclusion.

Valid Argument. The arguments $P_1,P_2,P_3...P_n$/-P is true if P is true, whenever all the premises $P_1,P_2,P_3,...P_n$ are true, otherwise false.

The true argument is called a valid argument and a false argument is called a fallacy.

Remember. For validity

$p_1\wedge p_2\wedge p_3.....\wedge p_n\rightarrow P$ is a tautology.

Example - 15 : Prove by construction of truth tables that $p\vee\sim(p\wedge q)$ is a tauology.

Sol. Truth Table

p	q	$p\wedge q$	$\sim(p\wedge q)$	$p\vee\sim(p\wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Since the column(s) representing the truth values of $p\vee\sim(p\wedge q)$ contain T for all values of p and q.

Hence $p\vee\sim(p\wedge q)$ is a tautology.

Example - 16 : Prove by constructing truth tables that $(p\wedge q)\wedge\sim(p\vee q)$ is a fallacy (contradiction).

Sol. Truth Table

p	q	$p\wedge q$	$p\vee q$	$\sim(p\vee q)$	$(p\wedge q)\wedge\sim(p\vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since there is F for all values of p,q

$\therefore (p\wedge q)\wedge\sim(p\vee q)$ is fallacy.

Example - 17 : Prove by construction of truth tables that $p\Leftrightarrow q\Leftrightarrow\sim q\Leftrightarrow\sim p$, where '~' denotes negation and '$\Leftrightarrow$' denotes if and only if.

Sol. Here we are given that $p \Leftrightarrow q$, we have to prove that $\sim q \Leftrightarrow \sim p$.

The result is established if we show that $[p \Rightarrow q] \Rightarrow [\sim q \Rightarrow \sim p]$ is a tautology.

p	q	$p \Rightarrow q$	~q	~p	$\sim q \Rightarrow \sim p$	$p \Rightarrow q \Rightarrow [\sim q \Rightarrow \sim p]$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

$\therefore$ $[p \Rightarrow q] \Rightarrow [\sim q \Rightarrow \sim p]$ is a tautology.

$\therefore$ If $p \Rightarrow q$, then $\sim q \Rightarrow \sim p$ .... (1)

Now we are given $\sim q \Rightarrow \sim p$, we have to prove that $p \Rightarrow q$.

The result is established if we show that $[\sim q \Rightarrow \sim p] \Rightarrow [p \Rightarrow q]$ is a tautology.

p	q	~q	~p	$\sim q \Rightarrow \sim p$	$p \Rightarrow q$	$\sim q \Rightarrow \sim p \Rightarrow p = q$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

$\therefore$ $[\sim q \Rightarrow \sim p] \Rightarrow [p \Rightarrow q]$ is a tautology

$\therefore$ If $\sim q \Rightarrow \sim p$, then $p \Rightarrow q$ .... (2)

From (1) and (2), $p \Leftrightarrow q \Leftrightarrow \sim q \Rightarrow \sim p$.

Example - 18 : Test the validity of : $p \Rightarrow q$ and $q \Rightarrow r$, then $p \Rightarrow r$

Sol. Truth Table

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$[(p \Rightarrow q) \wedge (q \Rightarrow rq)] \Rightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	F	T	F	T	T	F	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	F	F	T	F	F	T
F	T	F	T	F	T	F	T
F	F	F	T	T	T	T	T

Since the result is tautology.

$\therefore$ argument is valid

Example - 19 : Test the validity of : "If my son stands first in the university, I will give him a cycle. Either he stood first or I was out of station. I did not give my son a cycle this time. Therefore I was out of station".

Sol. Let the symbolic statements be :

p : My son stands first in the university.

q : I give him a cycle.

r : I was out of station.

The argument is $p \Rightarrow q, p \vee r \sim q / -r$

Let us assume that r is false.

Truth Table

p	q	r	$p \Rightarrow q$	$p \vee r$	~q
T	T	F	T	T	F
T	F	F	F	T	T
F	T	F	T	F	F
F	F	F	T	F	T

From the last three columns of the table, we observe that at least one of the premises in each row is false. Hence the argument is valid.

Example - 20 : Prove whether the following statement is a Tautology or Fallacy :

(i) $p \vee q \Rightarrow p \wedge q$ (ii) $p \wedge q \Rightarrow p$.

Sol. (i) Truth Table

p	q	$p \vee q$	$p \wedge q$	$p \vee q \Rightarrow p \wedge q$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

We observe that last column is not always true or false. Hence it is not a tautology.

(ii) Truth Table

p	q	$p \wedge q$	$p \wedge q \Rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

From above, we observe that $p \wedge q \Rightarrow p$. Hence it is tautology

Example - 21 : Prove that the following statement is a tautology :

$$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$$

Sol. Truth Table

p	q	r	$p\vee q$	$p\vee r$	$q\wedge r$	$p\vee(q\wedge r)$	$(p\vee q)\wedge(p\vee r)$	$p\vee(q\wedge r)\Leftrightarrow(p\vee q)\wedge(p\vee r)$
T	T	T	T	T	T	T	T	T
T	F	T	T	T	F	T	T	T
T	T	F	T	T	F	T	T	T
F	T	F	T	F	F	T	T	T
F	F	T	F	T	F	F	F	T
T	F	F	T	T	F	T	T	T
F	T	T	T	T	T	T	T	T
F	F	F	F	F	F	F	F	T

We observe that last column is always true.

Hence $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ is a tautology.

Example - 22 : Prove that the following statement is tautology : $\sim(p \vee q) \Leftrightarrow \sim p \vee \sim q$

Sol. Truth Table

p	q	~p	~q	$\sim p\wedge q$	$\sim(p\wedge q)$	$\sim p\vee\sim q$	$\sim(p\wedge q)\Leftrightarrow\sim p\vee q$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

We observe that last column is always true.

Hence $\sim(p \vee q) \Leftrightarrow \sim p \vee \sim q$ is a tautology.

Example - 23 : Test the validity of : If it rains, then the crops are good, if it does not rain, therefore the crops are not good.

Sol. Let p: It rains, q : crops are good. The argument in symbolic form can be stated as :

$$[p \Rightarrow q \vee \sim p] \Rightarrow q$$

Truth Table

p	q	$p \Rightarrow q$	~p	$p \Rightarrow q \wedge \sim p$	~q	$[(p \Rightarrow q) \wedge \sim p] \Rightarrow \sim q$
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Since $[p \Rightarrow q \vee \sim p] \Rightarrow q$ is not a tautology as shown by column seven, hence the given statement is invalid.

Example - 24 : Prove that $(p \vee q) \wedge (\sim p \wedge \sim q)$ is contradication.

Sol. Truth Table

p	q	$p \vee q$	~p	~q	$\sim p \wedge \sim q$	$(p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	F

We observe that last column is always false.

Hence $(p \wedge q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

Example - 25 : Write the duals of the following statements :

(a) $(p \vee q) \vee r$ $(b)(p \wedge q) \wedge r$

(c) $[\sim (p \vee q)] \wedge [p \vee \{(q \wedge (\sim s))\}]$

(d) $(p \vee q) \wedge (r \vee s)$

(e) $(p \wedge q) \vee r$

Sol. The required duals are given by

(a) $(p \wedge q) \wedge r$ (b) $(p \vee q) \vee r$

(c) $[\sim (p \wedge q)] \vee [p \wedge \{(q \vee (s))\}]$

(d) $(p \wedge q) \vee (r \wedge s)$ (e) $(p \vee q) \wedge r$.

Note. If the compound statement S contains the special variables t (tautology) or c (contradiction), then dual of is obtained by replacing t by c and c by t and $\wedge$ by $\vee$ and $\wedge$ by $\vee$ as usual.

Example - 26 : Write the dual of the following statements :

(a) $(p \wedge q) \vee t$ (b) $(p \vee t) \wedge r$

(c) $(p \vee q) \vee c$ (d) $(p \vee q) \wedge c$

Sol. The required duals are given by

(a) $(p \vee q) \wedge c$ (b) $(p \wedge c) \vee r$

(c) $(p \wedge q) \wedge t$ (d) $(p \wedge q) \vee t$

Gold Coins : The converse of $p \Rightarrow q$ is $q \Rightarrow p$.

The inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$.

The contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$.

A conditional and its contrapositive are logically equivalent.

The converse and the inverse of a conditional are logically equivalent.

$$\sim (p \Rightarrow q) = p \wedge \sim q$$

$$\sim (p \Leftrightarrow q) = (p \wedge \sim q) \vee (\sim p \wedge q)$$

11. Duality.

Two compound statements S_1 and S_2 are said to be duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

The connectives $\wedge$ and $\vee$ are also called duals of each other.

12. Duality Symbol.

Let $S^*(p,q) = p \wedge q$, where $S^*(p,q)$ is the dual statement of S(p,q).

Example - 27 : Prove that

(i) $\sim S(p,q) = S^*(\sim p, \sim q)$

(ii) $\sim S^*(p,q) = S(\sim p, \sim q)$

Sol. (i) $\sim S(p,q) = \sim(p \wedge q)$

$= (\sim p) \vee (\sim q) = S^*(\sim p, \sim q)$

(ii) $\sim S^*(p,q) = \sim(p \vee q)$

$= (\sim p) \wedge (\sim q) = S(\sim p, \sim q)$

Note. The above result of Example 27 can be extended to the compound statement $S(p_1, p_2, ..., p_n)$ having finite number of statements.

$$\sim S(p_1, p_2, ..., p_n) = S^*(\sim p_1, \sim p_2, ..., \sim p_n)$$

$$\sim S^*(p_1, p_2, ..., p_n) = S(\sim p_1, \sim p_2, ..., \sim p_n).$$

where $S^*(p_1, p_2, ..., p_n)$ is the dual of $S(p_1, p_2, ..., p_n)$.

Example - 28 : If $S(p,q,r) = (\sim p) \wedge [\sim (q \vee r)]$, prove that

$$\sim S(p,q,r) = S^*[\sim p, \sim q, \sim r].$$

Sol. Since $S(p,q,r) = (\sim p) \wedge [\sim (q \vee r)]$

$\therefore\ S^*(p,q,r) = (\sim p) \vee [\sim (q \wedge r)]$

$\therefore$

$S^*(\sim p, \sim q, \sim r) = [\sim(\sim p) \vee \sim(\sim q \wedge \sim r)]$

$= p \vee (\sim(\sim(q \vee r)))$

$= p \vee (q \vee r)$

Again

$\sim S(p,q,r) = \sim[(\sim p) \wedge [\sim (q \vee r)]]$

$= \sim(\sim p) \vee \sim[\sim (q \vee r)]$

$= p \vee (q \vee r)$

Hence $\sim S(p,q,r) = S^*(\sim p, \sim q, \sim r)$

Example - 29 : Write the dual of each of the following compound statements :

(i) Ram is healthy or Garima is beautiful.

(ii) Sohan and Sharmila cannot read Urdu.

(iii) I like potato chips and tomato soup.

Sol. (i) Let p : Ram is healthy q : Garima is beautiful.

$\therefore$ given compound statement is $p \vee q$

Its dual $= p \wedge q$

i.e., Ram is healthy and Garima is beautiful.

(ii) Let p : Sohan cannot read Urdu.

q : Sharmila cannot read Urdu.

$\therefore$ given compound statement is $p \wedge q$

Its dual $= p \vee q$

i.e., Sohan or Sharmila cannot read Urdu.

(iii) Let p : I like potato chips.

q : I like tomato soup.

$\therefore$ given compound statement is $p \wedge q$

Its dual is $p \vee q$

i.e., I like potato chips or tomato soup.

13. Algebra of Statements

Statements satisfy many laws, some of which are given below :

(i) Idempotent Laws. If p is any statement, then

(a) $p \vee p \equiv p$

(b) $p \wedge p \equiv p$.

Proof. Follows from the Truth Table given.

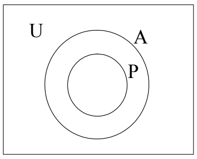
P	$p \vee p$	$p \wedge p$
T	T	T
F	F	F

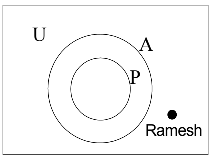
(ii) Associative Laws. If p,q,r are any three statements, then

(a) $p \vee (q \vee r) = (p \vee q) \vee r$

(b) $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

(iii) Commutative Laws. If p,q are any two statements then





Since the dot 'Ramesh' is outside the set A of the absent minded persons. $\therefore$ It is clearly outside the set of professors.

Hence we conclude that Ramesh is not a professor.

Example - 33 : Use Venn Diagrams to examine the validity of the following arguments :

(a) S_1 : All professors are absent-minded.

S_2 : Ramesh is not absent-minded.

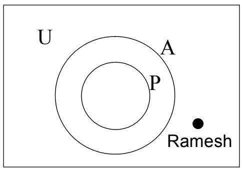
S : Ramesh is not a professor.

(b) S_1 : All professors are absent-minded.

S_2 : Ramesh is not absent-minded

S : Ramesh is a professor.

Sol. (a) Let U denote the universal set.



Clearly the truth of the conclusion S follows directly from the truth of the hypotheses S_1 and S_2. Further it is impossible that the hypotheses S_1 and S_2 are true and the conclusion is false. Hence, the argument in (a) is valid. (b) Consider again the same Venn Diagram as in part (a), the truth of the conclusion so does not follow the truth of the hypotheses S_1 and S_2. Hence the argument in (b) is not valid.

Example - 34 : Use venn Diagram to examine the validity of the following arguments :

(a) S_1 : Natural numbers are integers.

S_2 : x is an integer

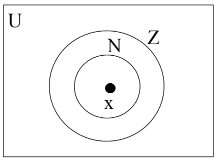
S : x is a natural number.

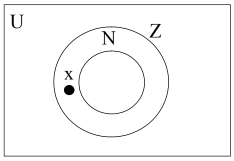
(b) S_1 : Natural number of are integers

S_2 : x is an integer

S_3 : x is not a natural number.

Sol. Let Z denote the set of integers and the N denote the set of natural numbers.





(a)

(b)

The truth of S_1 is represented by placing the set N entirely inside the set Z and the truth of the statement S_2 is represented by placing a dot labelled 'x' inside the set Z. Further, the dot is located somewhere inside the set Z. But w.r.t. the set N, its position cannot the determined. x can lie within N or x can lie outside N but within Z.

$\therefore$ one of the situations shown in the figure is true.

In figure (a), the truth of the conclusion S that x is a natural number follows from the truth of x is a natural number follows from the truth of the hypotheses S_1 and S_2. But according to the figure (b), the truth of the conclusion S does not follow from the truth of the hypotheses S_1 and S_2.

[$\because$ x may be - 3 which is not a natural number]

Since the conclusion S does not necessary follow from the truth of the hypothesis S_1 and S_2.

$\therefore$ arguments in (a) is not valid i.e., is invalid.

(b) Result holds for figure (b) and not for (a) $\therefore$ the argument in (b) is also invalid.

Example - 35 : use Venn Diagram to examine the validity of each of the arguments :

(a) S_1 : All squares are rectangles

(a) $p \vee q = q \vee p$ (b) $p \wedge q = q \wedge p$

(iv) Distributive Laws. If p,q,r are any three statements, then

(a) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

(b) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(v) Identity Laws. If p is any statement, t is tautology and c is a contradiction, then

(a) $p \vee t = t$ (b) $p \wedge t = p$

(c) $p \vee c = p$ (d) $p \wedge c = c$

(vi) Complement Laws. If t is a tautology, c is a contradiction and p is any statement, then

(a) $p \vee (\sim p) = t$ (b) $p \wedge (\sim p) = c$

(c) $\sim t = c$ (d) $\sim c = t$

(vii) Involution Law. If p is any statement, then $\sim (\sim p) = p$.

(viii) De Morgan's Laws. If p and q are two statements, then

(a) $\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$

(b) $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$

14. Use of Venn Diagrams in Logic

An argument is the assertion that statement S follows other statements $S_1, S_2, S_3, \ldots$, etc.

The statement S is called the conclusion and the statements $S_1, S_2, S_3, \ldots$, etc. are called **hypotheses (or promises).**

An argument consisting of the hypotheses $S_1, S_2, \ldots S_n$ and conclusion S is said to be valid if S is true whenever all $S_1, S_2, \ldots S_n$ are true.

e.g., Consider the following statements :

S_1 : Natural numbers are integers

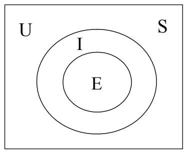
S_2 : x is not an integer

S : x is not a natural number.

Clearly, S follows from S_1 and S_2. Thus S_1, S_2, S taken together constitute an argument in which S_1, S_2 are hypotheses and S is the conclusion.

Example - 30 : Give the Venn Diagram for the truth of the following statement :

Equilateral Triangles are Isosceles Triangles.



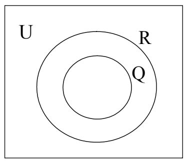
Sol. Let E denote the set of all Equilateral triangles and I denote the set of all Isosceles triangles.

Let U stand for the universal set.

Thus, the required Venn Diagram is as shown $[\because E \subseteq I]$

Example - 31 : Represent the truth of the statement. All rational numbers are real numbers by means of a Venn Diagram.

Sol. Let Q denote the set of all rational numbers and R denote the set of all real numbers.



Let U stand for the universal set.
Then the required Venn Diagram is as shown. $[\because Q \subseteq R]$

Example - 32 : Use Venn Diagrams to find a conclusion that can be inferred from the statements :

S_1 : All professors are absent minded.

S_2 : Ramesh is not absent minded.

Sol. Let P denote the set of professors and let A denote the set of all absent minded persons.

It is true if the set P lies entirely inside the set A. Again the truth of the statement S_2 is represented by placing a dot labelled 'Ramesh' outside the set A.

S_2 : x is not a rectangle

S : x is a square.

(b) S_1 : All squares are rectangles

S_2 : x is not a rectangle

S_0 : x is not a square.

(c) S_1 : All equilateral triangles are isosceles

S_2 : T is an equilateral triangle

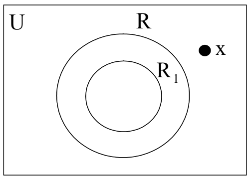
S : T is an isosceles triangle.

(d) S_1 : All equilateral triangles are isosceles.

S_2 : T is an equilateral triangle.

S_0 : T is not an isosceles triangle.

Sol. (a) Let U denote the universal set.



Let R denote the set of all rectangles and R_1 denote the set of all squares. The truth of S_1 is represented by placing the set R_1 entirely inside the set R.

Since x is not a rectangle.

$\therefore$ truth of S_2 is represented by placing a dot.

'x' outside the set R.

Since x lies outside R_1

$\therefore$ x is not a square.

$\therefore$ conclusion is not true.

$\therefore$ arguments in (a) is not valid i.e., in invalid.

(b) Conclusion is true (follows from (a)

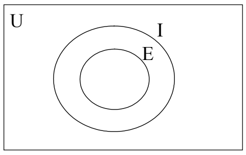
$\therefore$ arguments in (b) is valid.

(c) Let U denote the universal set. Let E denote the set of all equilateral triangle and I denote the set of all isosceles triangles. Truth of S_1 shows that E lies within I.

Since S_2 means E and E lies within I,

$\therefore$ conclusion S is true.

i.e., T is an isosceles triangle.



(d) Since all equilateral triangles are isosceles and T is an equilateral triangle, T is isosceles.

$\therefore$ conclusion here is false.

Hence argument is not valid.

LEVEL - I

1. Which of the following is a statement?

   a) Open the door

   b) Do your homework

   c) Switch on the fan

   d) Two plus two is four.

2. Which of the following is a statement?

   a) May you live long!

   b) May God bless you!

   c) The sun is a star

   d) Hurrah! we have won the match.

3. Which of the following is not a statement?

   a) Roses are red

   b) New Delhi is in India

   c) Every square is a rectangle

   d) Alas! I have failed

4. Which of the following is not a statement?

   a) Every set is a finite set

   b) 8 is less than 6

   c) Where are you going?

   d) The sum of interior angles of a triangle 18^0C

5. Which of the following is not a statement?

   a) Please do me a favour

   b) 2 is an even integer

   c) 2 + 1 = 3

   d) The number 17 is prime

6. Which of the following is not a statement?
   a) Give me a glass of water
   b) Asia is a continent
   c) The earth revolved round the sun
   d) The number 6 has two prime factors 2, 3
7. Which of the following is an open statement?
   a) x is a natural number
   b) Give me a glass of water
   c) Wish you best of luck
   d) Good moring to all
8. Negation of the preposition : If we control population growth, we prosper.
   a) If we do not control population growth, we prosper.
   b) If we control population, we do not prosper.
   c) We control population but we do not prosper.
   d) We do not control population,but we propser.
9. Negation of the conditional : "if it rains, I shall go to school" is
   a) it rains and I shall go to school.
   b) It rains and I shall not to go to school
   c) It does not rain and I shall go to school
   d) None of these
10. Negation of "Paris is in France and London is in England" is
    a) Paris in England and Londonn is in France
    b) pairs is not in France or London is not in England
    c) paris is in England or London is in France
    d) None of these
11. Negation of "2 + 3 = 5 and 8 < 10" is
    a) 2 + 3 $\neq$ 5 and < 10
    b) 2 + 3 = 5 and 8 $\not<$10
    c) 2 + 3 $\neq$ 5 or 8 $\not<$10
    d) None of these
12.. Negation of "Ram is in Class X or Rahim is in Class XII" is
    a) Ram is not in class X but Rahim is in class XII.
    b) Ram is not in class X and Rahim is not in class XII.
    c) Either Ram is not in class X or Rahim is not in Class XII.
    d) None of these
13. Negation of the compound proposition :
    If the examination is difficult, then I shall pass if I study hard
    a) The examination is difficult and I study hard and I shall pass.
    b) The examination is difficult and I study hard but I shall not pass.
    c) The examination is not difficult and I study hard and I shall pass.
    d) None of these.
14. The conditional $(p \wedge q) \Rightarrow p$ is
    a) a tautology
    b) a fallacy i.e. contradiction
    c) neither tautology nor fallacy
    d) None of these
15. Which of the following is a contradiction?
    a) $(p \wedge q) \sim (p \vee q)$
    b) $p \vee (\sim p \wedge q)$
    c) $(p \Rightarrow q) \Rightarrow p$ d) None of these
16. Which of the following is logically equivalent to $\sim(\sim p \Rightarrow q)$?
    a) $p \wedge q$ b) $p \wedge -q$
    c) $\sim p \wedge q$ d) $\sim p \wedge \sim q$
17. $\sim(p \vee q)$ is equal to
    a) $\sim p \vee \sim q$ b) $\sim p \wedge \sim q$
    c) $\sim p \vee q$ d) $p \vee \sim q$

18. $\sim(p \wedge q)$ is equal to

a) $\sim p \vee \sim q$ b) $\sim p \wedge \sim q$

c) $\sim p \wedge q$ d) $p \wedge \sim q$

19. $(\sim(\sim p)) \wedge q$ is equal to

a) $\sim p \wedge q$ b) $p \wedge q$

c) $p \wedge \sim q$ d) $\sim p \wedge \sim q$

20. $\sim[p \vee (\sim q)]$ is equal to

a) $\sim p \vee q$ b) $(\sim p) \wedge q$

c) $\sim p \vee \sim q$ d) $\sim p \wedge \sim q$

21. $\sim[(\sim p) \wedge q]$ is equal to

a) $p \vee (\sim q)$ b) $p \vee q$

c) $p \wedge (\sim q)$ d) $\sim p \wedge \sim q$

22. $\sim(p \Leftrightarrow q)$ is

a) $\sim p \wedge \sim q$ b) $\sim p \vee \sim q$

c) $(p \wedge \sim q) \vee (\sim p \wedge q)$

d) none of these

23. $p \Rightarrow q$ can also be written as

a) $p \Rightarrow \sim q$ b) $\sim p \vee q$

c) $\sim q \Rightarrow \sim p$ d) none of these

24. If p, q, r are simple propositions with truth values T, F, T, then the truth value of $(\sim p \vee q) \wedge \sim r \Rightarrow p$ is

a) true b) false

c) true if r is false d) true if q is true

25. If $(p \wedge \sim r) \Rightarrow (q \vee r)$ is false and q and r are both false, then p is

a) true b) false

c) may be true or false

d) data insufficient.

26. If p and q are simple propositions, then $p \Rightarrow q$ is false when

a) p is true and q is true

b) p is false and q is true

c) p is true and q is false

d) both p and q are false

27. If p and q are simple propositions, then $p \Leftrightarrow \sim q$ is true when

a) p is true and q is true

b) both p and q are false.

c) p is false and q is true.

d) none of these

28. If p, q, r are simple propositions, then $(p \wedge q) \wedge (q \wedge r)$ is true then

a) p, q, r are all false

b) p, q, r are all true

c) p, q are true and r is false

d) p is true and q and r are false

29. $\sim(p \Rightarrow q) \Leftrightarrow \sim p \vee \sim q$ is

a) a tautology b) a contradition

c) neither a tautology nor a contradiction

d) cannot come to any conclusion

30. $(p \wedge \sim q) \wedge (\sim p \vee q)$ is

a) a contradiction b) a tautology

c) either (a) or (b) d) Neither (a) or (b)

31. Which of the following is not logically equivalent to the proposition :

"A real number is either rational or irrational"

a) If a number is neither rational nor irrational then it is not real.

b) If a number is not a rational or not an irrational, then it is not real.

c) If a number is not real, then it is neither rational nor irrational.

d) If a number is real, then it is rational or irrational.

32. If p : It rains today, q : I go to the school, r : I shall meet any friends and s : I shall go for a movie, then which of the following is the proposition.

If it does not rain or if I do not go to school, then I shall meet my friend and go for a movie.

a) $\sim(p \wedge q) \Rightarrow (r \wedge s)$

b) $(\sim p \wedge \sim q) \Rightarrow (r \wedge s)$

c) $\sim(p \vee q) \Rightarrow (r \vee s)$ d) none of these

33. The negation of the compound proposition $p \vee (\sim p \vee q)$ is

a) $(p \wedge \sim q) \wedge \sim p$ b) $(p \wedge \sim q) \vee \sim p$

c) $(p \wedge \sim q) \vee \sim q$ d) none of these

34. The proposition $p \Rightarrow \sim (p \wedge -q)$ is

a) contraction b) a tautology

c) either (a) or (b) d) neither (a) nor (b)

35. Which of the following is true?

a) $p \Rightarrow q \equiv \sim p \Rightarrow \sim q$

b) $\sim(p \Rightarrow \sim q) \equiv \sim p \wedge q$

c) $\sim(\sim p \sim q) \equiv \sim p \wedge q$

d) $\sim(p \Leftrightarrow q) \equiv [\sim(p \Rightarrow q) \wedge (q \Rightarrow p)]$

36. $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to

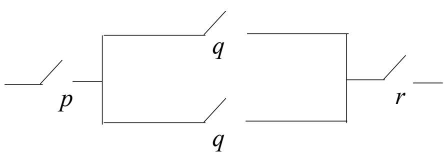
a) $\sim p$ b) p c) q d) $\sim q$

37. The inverse of the proposition $(p \wedge \sim q) \Rightarrow r$ is

1) $\sim r \Rightarrow \sim p \vee q$ 2) $\sim p \vee q \Rightarrow \sim r$

3) $r \Rightarrow p \wedge \sim q$ 4) none of these

38. When does the current flow through the following circuit?



1) p, q, r should be closed

2) p, q, r should be open

3) always

4) none of these

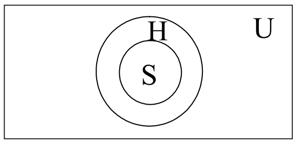
39. Which Venn diagram represent the truth of the statement

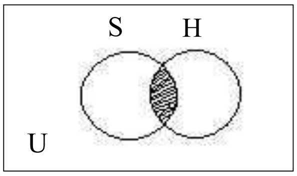
"All students are hard working".

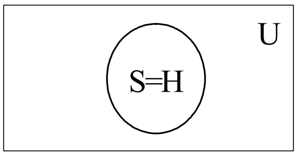
where U = Universal set of human beings

S = Set of all students

H = Set of all hard workers

1) 

2) 

3) 

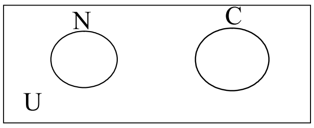
4) none of these

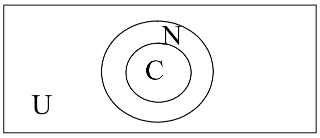
40. Which Venn diagram represent the truth of the statements "No child is naughty"

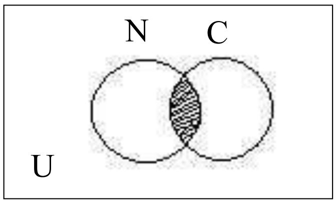
where U = Universal set of human beings

C = Set of children

N = Set of naughty persons

1) 

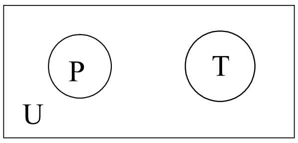
2) 

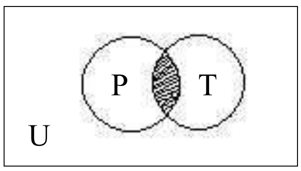
3) 

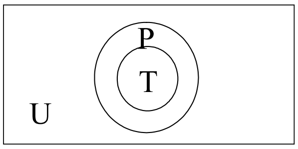
4) none of these

41. Which Venn diagram represent the truth of the statement

"No policeman is a thief"

1) 

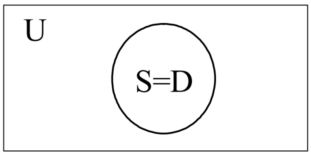
2) 

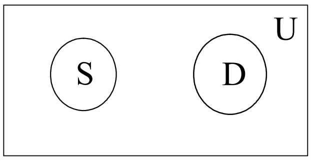
3) 

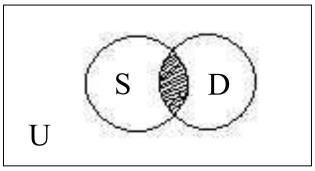
4) none of these

42. Which Venn diagram represent the truth of the statement

“All smokers are drinkers and all drinkers are smokers”

1) 

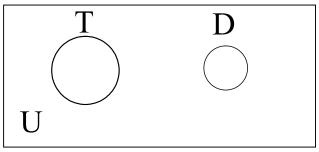
2) 

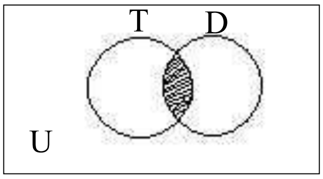
3) 

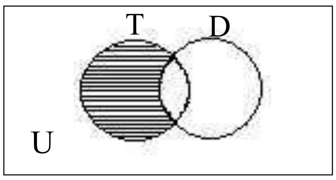
4) none of these

43. Which Venn diagram represent the truth of the statement

“Some teenagers are not dreamers”

1) 

2) 

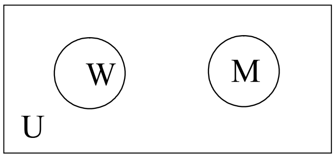
3) 

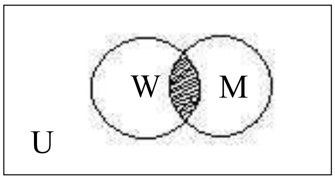
4) none of these

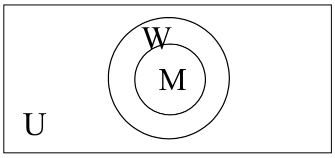
44. Which of the following Venn diagram corresponds to the statement

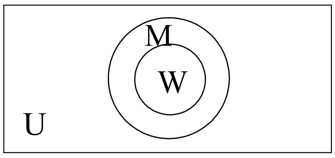
“All mothers are women”

(M is the set of all mother, W is the set of all women)

1) 

2) 

3) 

4) 

KEY

1) 4	2) 3	3) 4	4) 3	5) 1
6) 1	7) 1	8) 3	9) 2	10) 2
11) 3	12) 2	13) 2	14) 1	15) 1
16) 4	17) 2	18) 1	19) 2	20) 2
21) 1	22) 3	23) 2	24) 1	25) 1
26) 3	27) 3	28) 2	29) 3	30) 1
31) 2	32) 1	33) 1	34) 4	35) 3
36) 1	37) 2	38) 1	39) 1	40) 1
41) 1	42) 1	43) 3	44) 3	

HINTS

LEVEL - I

1. “Two plus two is four,” is a statement.
2. “The sun is a star,” is a statement.
3. “Alas ! I have failed,” is not a statement.
4. “Where are you going” is not a statement.
5. “Please do me a favour” is not a statement.
6. “Give me a glass of water” is not a statement.
7. “x is a rational number” is an open statement.
8. p : we control population

   q : we prosper.

   $\therefore$ we have $p \Rightarrow q$

Its negation is $\sim(p \Rightarrow q)$ i.e. $p \wedge \sim q$

i.e. we control population but we do not propser.

9. p : It rains, q : I shall go to school

Thus, we have $p \Rightarrow q$

Its negation is $\sim(p \Rightarrow q)$ i.e. $p \wedge \sim q$

i.e. It rains and I shall not go to school.

10. Let p : Paris is in France

q : London is in England

$\therefore$ we have $p \wedge q$

Its negation is $\sim(p \wedge q) = \sim p \vee \sim q$

i.e. Paris is not in France or London is not in England.

11. Let p : 2 + 3 = 5

q : 8 < 10

Given proposition is $p \wedge q$

Its negation is $\sim(p \wedge q) = \sim p \vee \sim q$

$\therefore$ we have $2 + 3 \neq 5$ or $8 \nless 10$

12. Let p : Ram is in class X

q : Rahim is in class XII

Given proposition is $p \vee q$

Its negation is $\sim(p \vee q) = \sim p \wedge \sim q$

i.e. Ram is not in class X and Rahim is not in class XII.

13. p : examination is difficult

q : I shall pass

r : I study hard

Given result is : $p \Rightarrow (r \Rightarrow q)$

Now $\sim(r \Rightarrow q) = r \wedge \sim q$

$\sim(p \Rightarrow (r \Rightarrow q)) = p \wedge (r \wedge \sim q)$

The examination is difficult and I study hard but I shall not pass.

14.

p	q	$p \wedge q$	$(p \wedge q \Rightarrow p)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$\therefore$ $(p \wedge q) \Rightarrow p$ is a tautology

15.

p	q	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \wedge \sim(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

$\therefore$ $(p \wedge q) \wedge (\sim(p \vee q))$ is a contradiction

16. Since $\sim(p \Rightarrow q) = p \wedge \sim q$

$\sim(\sim p \Rightarrow q) = \sim p \wedge \sim q$

Hence (d) is correct.

17. $\sim(p \vee q) \equiv \sim p \wedge \sim q$

18. $\sim(p \wedge q) \equiv \sim p \vee \sim q$

19. $(\sim(\sim p) \wedge q) = p \wedge q$

20. $\sim(p \vee (\sim q)) \equiv \sim p \wedge \sim(\sim q) \equiv \sim p \wedge q$

21. $\sim((\sim P) \wedge q) \equiv \sim(\sim p) \vee \sim q \equiv \sim p \vee q$

22. $\sim(p \Leftrightarrow q) = (p \wedge -q) \vee (q \wedge \sim p)$

23. $p \Leftrightarrow q \equiv \sim p \vee q$

24. $\sim p \vee q$ means F V F = F, $\sim r$ means F

$(\sim p \vee q) \wedge \sim r$ means F

$\therefore [(\sim p \vee q) \wedge r] \Rightarrow p$ means T.

[$\because$ in $p \Rightarrow q$ we have FTT]

25. Given result means $p \wedge \sim r$ is true, $q \vee r$ is false.

26. $p \Rightarrow q$ is false, when p is true and q is false.

Since q, r are false $\therefore q \vee r$ is false.

Since r is false $\therefore \sim r$ is true.

Since $p \wedge \sim r$ is true $\therefore p$ is true.

27. $p \Leftrightarrow -q$ is ture if p , $\sim q$ are both ture or both false. (c) is true

[$\because q$ true $\Rightarrow \sim q$ false $\therefore P, \sim q$ are both false]

28. $(p\wedge q)\wedge(q\wedge r)$ is true means $p\wedge q, q\wedge r$ are both true. $\Rightarrow$ p,q,r are all true

29.

p	q	$p\Rightarrow q$	$\sim(p\Rightarrow q)$	~p	~q	$\sim p\vee\sim q$	$\sim(p\Rightarrow q)\Leftrightarrow \sim p\vee\sim q$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	T	F	T	F
F	F	T	F	T	T	T	F

Last column shows that result is neither a tautology nor a contradiction.

30.

p	q	~p	~q	$p\wedge\sim q$	$\sim p\vee q$	$(p\wedge\sim q)\wedge(\sim p\vee q)$
T	T	F	F	F	T	F
T	F	T	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Clearly, $(p\wedge\sim q)\wedge(\sim p\vee q)$ is a contradiction i.e. (a) is correct.

31. $\because \sqrt{3}$ is not rational but it is real.

32. Correct result is $(\sim p\vee\sim q)\Rightarrow(r\wedge s)$

$\Rightarrow\sim(p\wedge q)\Rightarrow(r\wedge s)$

33. $\sim[p\vee(\sim p\vee q)]\equiv\vee p\wedge\sim(\sim p\vee q)$

$\equiv\sim p\wedge(\sim(\sim p)\wedge\sim q)\equiv\sim p\wedge(p\wedge\sim q).$

34.

P	q	~q	$p\wedge\sim q$	$\sim(p\wedge\sim q)$	$p\Rightarrow\sim(p\wedge\sim q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

Result is neither tautology nor contradiction.

35. $\sim(p\Rightarrow q)\equiv p\wedge\sim q$

$\therefore$
$\sim(\sim p\Rightarrow\sim q)\equiv\sim p\wedge\sim(\sim q)\equiv\sim p\wedge q$

Thus $\sim(\sim p\Rightarrow\sim q)\equiv\sim p\wedge q$

36. $\sim(p\vee q)\vee(\sim p\vee q)$

$\equiv(\sim p\wedge\sim q)\vee(\sim p\wedge q)$

$\equiv\sim p\wedge(\sim q\vee q)\equiv\sim p$

37. Inverse of $p\Rightarrow q$ is $\sim p\Rightarrow\sim q$

$\therefore$ inverse of $(p\wedge\sim q)\Rightarrow r$ is

$\sim(p\wedge\sim q)\Rightarrow\sim r$ i.e. $(\sim p\vee q)\Rightarrow\vee r$

38. p,q,r should be closed for the current to flow.

39. All students are hard working means $S\subseteq H$

40. "No child is naughty" means $C\cap N=\phi$ i.e. there is no common element between C and N.

41. No policeman is a theif means $P\cap T=\phi$ i.e. there is no common element between P and T

42. All smokers are drinkers and all drinkers are smokers.

$\therefore$ $S\subseteq D$ and $D\subseteq S$
This means S=D

43. Some teenagers are not dreamers means teenagers which are not dreamers.

44. All mothers are women.

$M\subseteq W$ $\therefore$ (c) holds.

PRACTICE EXERCISE

1. The conjunction of 6+3=9,8–3=5 is
   1) $6+3=9$ *and* $8-3=5$
   2) $6+3=9$ *or* $8-3=5$
   3) If $6+3=9$ *then* $8-3=5$
   4) $6+3=9$ *only if* $8-3=5$

2. $p:x$ is odd ; $q:x^2$ is odd. The symbolic form of " x is odd or x^2 is odd", is
   1) $p\vee q$ 2) $p\wedge q$ 3) $(\sim p)\vee q$ 4) $p\vee(\sim q)$

3. $p:x$ is odd ; $q:x^2$ is odd. The symbolic form of "x is odd and x^2 is not odd", is
   1) $p\vee q$ 2) $p\wedge q$

3) $(\sim p)\vee q$ 4) $p\wedge(\sim q)$

4. The truth value of "4+3=7 or $5x4=21$" is
1) T 2) F 3) T or F 4) T and F
5. The truth value of "20+10=2 and 20x10 = 200" is
1) T 2) F 3) T or F 4) T and F
6. The truth value of " 10+2=12 and 10x2 = 20" is
1) T 2) F 3) T or F 4) T and F
7. The truth value of "if 3 + 2 = 5 then 1 x 0 = 0 " is
1) T 2) F 3) T or F 4) T and F
8. The truth value of " if 3 x 6 = 20 then 2+ 7 =9" is
1) T 2) F 3) T or F 4) T and F
9. Which of the following is true ?
1) $4+3=10 \Leftrightarrow 4\times3=12$
2) $4\times3=28 \Leftrightarrow 4+7=1$
3) $5\times 8=40 \Leftrightarrow 8-2=5$
4) $6-3=3 \Leftrightarrow 6\times3=18$
10. The converse of " if two triangles are congruent then they are similar" is
1) If two triangles are similar then they are congrument
2) If two triangles are not congruent then they are not similar
3) If two triangles are not similar then they are not congruent
4) None
11. The inverse of " if two triangle are congruent then they are similar" is
1) If two triangles are similar then they are congrument
2) If two triangles are not congruent then they are not similar
3) If two triangles are not similar then they are not congruent
4) None
12. If inverse of " if $x\in A\cup B$ then $x\in A \ or \ x\in B$ " , is
1) If $x\in A or x\in B$ then $x\in A\cup B$
2) If $x\notin A\cup B \ then x\notin A \ and \ x\notin B$
3)If $x\notin A \ and \ x\notin B then \ x\notin A\cup B$
4) If $x\notin A \ and \ x\notin B then \ x\in A\cup B$
13. p: he is hard working q: he will win. The symbolic form of "if he is hard working then he will win", is
1) $p\vee q$ 2) $p\wedge q$
3) $p\Rightarrow q$ 4) $q\Rightarrow p$
14. p: he is hard working, q: he will win. The symbolic form of "if he will not win then he is not hard working" is
1) $p\Rightarrow q$ 2) $(\sim p)\Rightarrow(\sim q)$
3) $(\sim q)\Rightarrow(\sim p)$ 4) $(\sim q)\Rightarrow p$
15. The truth table of $(\sim p)\wedge q$ is

1)

p	q	$(\sim p)\wedge q$
T	T	F
T	F	F
F	T	T
F	F	F

2)

p	q	$(\sim p)\wedge q$
T	T	F
T	F	T
F	T	F
F	F	F

3)

p	q	$(\sim p)\wedge q$
T	T	F
T	F	F
F	T	F
F	F	F

4)

p	q	$(\sim p)\wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

16. The truth table of $p \wedge (\sim q)$

1)

p	q	$p \wedge (\sim q)$
T	T	F
T	F	F
F	T	T
F	F	F

2)

p	q	$p \wedge (\sim q)$
T	T	F
T	F	T
F	T	F
F	F	F

3)

p	q	$p \wedge (\sim q)$
T	T	F
T	F	F
F	T	F
F	F	T

4)

p	q	$p \wedge (\sim q)$
T	T	T
T	F	F
F	T	F
F	F	F

17. The truth table of $(\sim p) \wedge (\sim q)$

1)

p	q	$(\sim p) \wedge (\sim q)$
T	T	F
T	F	F
F	T	T
F	F	F

2)

p	q	$(\sim p) \wedge (\sim q)$
T	T	F
T	F	T
F	T	F
F	F	F

3)

p	q	$(\sim p) \wedge (\sim q)$
T	T	F
T	F	F
F	T	F
F	F	T

4)

p	q	$(\sim p) \wedge (\sim q)$
T	T	T
T	F	F
F	T	F
F	F	F

18. The truth table of $p \Rightarrow \sim q$ is

1)

p	q	$p \Rightarrow \sim q$
T	T	F
T	F	T
F	T	T
F	F	T

2)

p	q	$p \Rightarrow \sim q$
T	T	T
T	F	T
F	T	T
F	F	F

3)

p	q	$p \Rightarrow \sim q$
T	T	F
T	F	T
F	T	F
F	F	T

4)

p	q	$p \Rightarrow \sim q$
T	T	T
T	F	F
F	T	T
F	F	T

19. The truth table of $(\sim p) \Rightarrow \text{q}$ is

1)

p	q	$(\sim p) \Rightarrow \text{q}$
T	T	F
T	F	T
F	T	T
F	F	T

2)

p	q	$(\sim p) \Rightarrow \text{q}$
T	T	T
T	F	T
F	T	T
F	F	F

3)

p	q	$(\sim p) \Rightarrow \text{q}$
T	T	F
T	F	T
F	T	F
F	F	T

4)

p	q	$(\sim p) \Rightarrow \text{q}$
T	T	T
T	F	F
F	T	T
F	F	T

20. The truth table of $p \wedge (\sim p) \Rightarrow p$

1)

p	q	$p \wedge (\sim p) \Rightarrow p$
T	T	T
T	F	T
F	T	T
F	F	T

2)

p	q	$p \wedge (\sim p) \Rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

3)

p	q	$p \wedge (\sim p) \Rightarrow p$
T	T	T
T	F	T
F	T	T
F	F	F

4)

p	q	$p \wedge (\sim p) \Rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	F

21. $p : x$ is even; q: x^2 is even. The symbolic form of "x is even or x^2 is even", is
1) $p \Leftrightarrow q$ 2) $p \wedge q$
3) $p \vee q$ 4) $p \Rightarrow q$

22. The Truth value of "if $5 \times 7 = 30$ then $2+1=4$" is
1) T 2) F 3) T or F 4) T and F

23. Which of the following is true ?
1) $3+7=10 \Leftrightarrow 1+2=3$
2) $3+7=10 \Leftrightarrow 1+2=2$
3) $3\times 7=10 \Leftrightarrow 1\times 3=3$
4) $3\times 7=8 \Leftrightarrow 1\times 2=2$

24. The contrapositive of "if in a triangle ABC, AB =AC then $\angle B = \angle C$" is
1) If in a triangle ABC, $\angle B = \angle C$ then $AB = AC$
2) If in a triangle ABC, $AB \neq AC, then\ \angle B \neq \angle C$
3) If in a triangle ABC, $\angle B \neq \angle C$ then $AB \neq AC$
4) If in a triangle ABC, $\angle B \neq \angle C$, then $AB = AC$" is

25. The converse of "if in a triangle ABC, AB >AC then $\angle C = \angle B$" is
1) If in a triangle ABC, $\angle C = \angle B$ then $AB > AC$
2) If in a triangle ABC, $AB \not> AC, then\ \angle C \not> \angle B$
3) If in a triangle ABC, $\angle C \not> \angle B$ then $AB \not> AC$
4) If in a triangle ABC, $\angle C \not> \angle B$, then $AB > AC$" is

KEY

1) 1	2) 1	3) 4	4) 1	5) 2
6) 1	7) 1	8) 1	9) 4	10) 1
11) 2	12) 2	13) 3	14) 3	15) 1
16) 3	17) 3	18) 1	19) 2	20) 1
21) 3	22) 1	23) 1	24) 3	25) 1

LEVEL - II

1. Which of the following is equivalent to $\sim(p \leftrightarrow q)$?
1) $\sim p \wedge q$ 2) $p \wedge \sim q$
3) $(p \wedge \sim q) \vee (\sim p \wedge q)$ 2
4) $(p \wedge \sim q) \wedge (\sim p \wedge q)$

2. If each of the statements $p \to q, p \vee -r$ and r is true, then the truth values of p and q are respectively
1) T, T 2) T,F 3) F,T 4) F,F

3. Negation of $p \vee (\sim p \wedge q)$ is equivalent to
1) $\sim(p \wedge q)$ 2) $\sim p \wedge \sim q$
3) $p \wedge \sim q$ 4) $\sim p \wedge q$

4. The statement $p \to (q \vee r)$ is equivalent to
1) $(p \to q) \vee (p \to r)$ 2) $\sim p \to q$
3) $(p \to q) \wedge (p \to r)$ 4) $r \to \sim q$

5. If each of the statements $p \to \sim q, \sim r \to q$ and p is true, then the values of q and r are respectively
1) F,F 2) T,F 3) F,T 4) T,T

6. The statement $p \leftrightarrow q$ is equivalent to
1) $\sim p \wedge q$ 2) $p \wedge \sim q$
3) $\sim p \wedge \sim q$ 4) $(p \wedge q) \vee (\sim p \wedge \sim q)$

7. If each of the statements $p \to q, q \to r$ and $\sim r$ is true,then
1) p is true 2) q is true
3) $p \to r$ is false 4) p is false

8. Contrapositivity of the statement
p: if x and y are integers such that xy is odd, then both x and y are odd.
1) If x and y are integers and if x or y is even, then xy is even.
2) If x and y are integers and if x and y is odd, then xy is odd.
3)If x and y are integers and if xy is odd, then x or y is odd.
4) If x and y are integers and if x and y are even, then xy is even.

9. Converse of the statement p: If n is positive., then n^2 is positive.
1) If n^2 is positive, then n is negative.
2) If n^2 is not positive, then n is not positive.
3) If n^2 is positive, then n is positive.
4) If n is negative, then n^2 is positive

10. If the truth values of p,q and r are respectively, T,T and F, then the truth value of
    1) $p \to (q \to r)$ is T   2) $(p \to q) \to r$ is T
    3) $p \to (q \vee r)$ is T   4) $p \to (q \wedge r)$ is T
11. negation of the statement p: 1/2 is rational and $\sqrt{3}$ is irrational is
    1) $\frac{1}{2}$ is rational or $\sqrt{3}$ is irrational
    2) $\frac{1}{2}$ is not rational or $\sqrt{3}$ is rational
    3) $\frac{1}{2}$ is not rational or $\sqrt{3}$ is irrational
    4) $\frac{1}{2}$ is rational and $\sqrt{3}$ is rational
12. The statement $p \to (q \to r)$ is equivalent to
    1) $(p \to q) \to r$ 2) $p \to (q \vee r)$
    3) $(p \vee q) \to r$ 4) $(p \wedge q) \to r$
13. If $(p \leftrightarrow q) \wedge q$ is true, then the truth values of p and q are respectively
    1) T,F 2) F,T 3) T,T 4) F,F
14. The statement $p \leftrightarrow q$ is equivalent to
    1) $p \wedge \sim q$ 2) $p \leftrightarrow (\sim q)$
    3) $(\sim p) \vee q$ 4) $(\sim q) \leftrightarrow (\sim p)$
15. If each of the statements $p \to q$, $q \to r$ and ~ r is true, then the truth values of q and p are respectively
    1) T,F 2) F,T 3) F,F 4) T,T
16. The statement $p \to q$ is equivalent to
    1) $q \to p$ 2) $\sim(q \to p)$ 3) $\sim q \to \sim p$ 4) $p \wedge \sim q$
17. Consider the statements
    p : if 8 is less than 6, then 2+2=5.
    q: If $\sqrt{2}$ is irrational, then $\sqrt{2}$ +1 is rational.The truth values of p and q are respectively
    1) T,F 2) F,T 3) T,T 4) F,F
18. Let p,q,r and s be statements.
    Asseration (A) :
    $(p \wedge q) \wedge (\sim r \vee \sim s) \wedge (q \to s) \wedge (p \to r) \to (r \to p) \wedge (s \to p)$
    is a tautology
    Reason (R):
    $(p \wedge q) \wedge (\sim r \vee \sim s) \wedge (q \to s) \wedge (p \to r) \Leftrightarrow (r \to p) \wedge (s \to p)$
    Which of the following is true?
    1) A is true; R is true; and R is a correct explanation for A.
    2) A is true; R is true; and R is not a correct explanation for A.
    3) A is true; R is false.
    4) A is false; R is true.
19. Let p,q be Statements.
    Assertion (A) : $p \to (p \vee q)$ is a tautology
    Reason (R) : $p \to (q \to p)$ is a tautology
    Which of the following is true?
    1) A is true; R is true; and R is a correct explanation for A.
    2) A is true; R is true; but R is not a correct explanation for A.
    3) A is true; R is false.
    4) A is false; R is true.
20. Let p,q be statements.
    Assertion (A): $((\sim p \to q) \wedge \sim q) \to p$ is a tautology
    Reason(R) : $((\sim p \to q) \wedge q) \leftrightarrow p$
    Which of the following is true?
    1) A is true; R is true; and R is a correct explanation for A.
    2) A is true; R is true; but R is not a correct explanation for A.
    3) A is true; R is false.
    4) A is false; R is true.
21. the statement $p \to (q \to p)$ is equicalent to
    1) $p \to (p \to q)$ 2) $p \to (p \vee q)$
    3) $p \to (p \wedge q)$ 4) $p \to (p \leftrightarrow q)$
22. Let p be the statement "x is an irrational number",q be the statement "y is a transcendental number", and r be the statement "x is a rational number if y is a transcendental number.
    Statement 1: r is equivalent to either q or p
    Statement 2 : r is equivalent to $\sim (p \leftrightarrow \sim q)$.
    which of the following is correct?
    1) Statement 1 is false : Statement 2 is true
    2) Statement 1 is true : Statement 2 is true Statement 2 is a correct explanation for statement 1.
    3) Statement 1 is true: Statement 2 is true Statement 2 is not a correct explanation for Statement 1
    4) Statement 1 is false; Statement 2 is false.

23. If $p \to (\sim p \vee q)$ is false, then the truth values of p and q are respectively.
1) F,T 2) F, F 3) T, T 4) T,F
24. $p \wedge \sim q$ is equivalent to
1) $p \to q$ 2) $q \to p$
3) $\sim (p \to q)$ 4) $\sim (q \to p)$
25. If $p \to q \wedge r$ is false and q is true, then the truth values of $p \to r$ and $r \to q$ are respectively
1)T,F 2) F,T 3) T,T 4) F,F
26. Contrapositive of $p \to (q \to p)$ is equvalent to
1) $p \to (q \to p)$ 2) $p \to (p \to q)$
3) $p \to (p \leftrightarrow q)$ 4) $p \to p \wedge q$
27. The statement $(p \vee r) \to q$ is equivalent to
1) $(p \to q) \wedge (r \to q)$ 2) $(p \to r) \wedge (r \to q)$
3) $p \to (q \vee r)$ 4) $(p \to q) \vee (r \to q)$
28. If the truth values of $p \leftrightarrow q$ and $q \to r$ are T and F respectively, then the truth value of
1) $p \to (q \wedge r)$ is T 2) $(p \wedge q) \to r$ is T
3) $(p \to q) \to r$ is T 4) $p \to (r \to q)$ is T
29. The statement $p \to (q \to p)$ is equivalent to
1) $\sim p \to (p \to q)$ 2) $p \to (p \leftrightarrow q)$
3) $p \to (p \to q)$ 4) $p \to (p \wedge q)$
30. Converse of $p \to (q \vee r)$ is equivalent to
1) $(q \to p) \wedge (r \to p)$ 2) $(q \to r) \wedge (r \to p)$
3) $q \to (p \wedge r)$ 4) $(q \to p) \vee (r \to p)$
31. Negation of $(p \leftrightarrow q)$ is equivalent to
1) $(p \vee q) \wedge (p \wedge q)$ 2) $(p \leftrightarrow q) \wedge q$
3) $(p \leftrightarrow q) \wedge \sim q$ 4) $(p \vee q) \wedge \sim (p \wedge q)$
32. $\sim (p \leftrightarrow q)$ is equivalent to
1) $\sim p \leftrightarrow q$ 2) $(p \to q) \wedge (q \to p)$
3) $\sim p \to q$ 4) $p \to \sim q$
33. Let M_3 (R) be the set of all 3 x 3 real matrices. Negation of the statement: For every A $\in$ M_3 (R), if rank (A) = 2 then det (A) = 0 is as follows:
1) There exist A $\in$ M_3 (R) Such that rank (A) = 2 and det (A) = 0
2) There exist A $\in M_3$ (R) such that rank (A) $\neq 2$ and det (A) = 0
3) There exist A $\in M_3$ (R) such that rank (A) =2 and det (A) $\neq$ 0
4) There exist A $\in M_3$ (R) such that rank (A) $\neq 2$ and det (A) $\neq$ 0
34. Converse of the statement : If A is a 3x3 real invertible matrix, then A^2 is invetible is as follows
1) If A^2 is not invertible, then A is not invertible
2) If A^2 is invertible, then A is invertible
3) If A is not invertible, then A^2 is invertible
4) If A^2 is not invertible, then A is invertible
35. Let p,q,r be statements. Consider the following
Statement 1: $(P \to r) \wedge (q \to r)$ and $(P \vee q) \to r$ are equivalent
Statement 2 : $(P \to q) \vee (p \to r)$ and $P \to (q \vee r)$ are equivalent
Which of the following is true?
1) Statement 1 and Statement 2 are false
2) Statement 1 and Statement 2 are true
3) Statement 1 is false and Statement 2 is true
4) Statement 1 is true and Statement 2 is false
36. Let p,q,r be statements. Consider the following
Statement 1: $(P \to q) \wedge (p \to r)$ and $P \to (q \wedge r)$ are equivalent
Statement 2 : $(P \wedge q) \to r$ and $(P \to r) \wedge (q \to r)$ are equivalent
Which of the following is true?
1) Statement 1 and Statement 2 are false
2) Statement 1 and Statement 2 are true
3) Statement 1 is false and Statement 2 is true
4) Statement 1 is true and Statement 2 is false.
37. Let p,q,r be statements. Consider the following
Statement 1: $(P \to q) \to r$ and $P \to (q \to r)$ are equivalent
Statement 2 : $(p \wedge q) \to r$ and $(p \to r) \vee (q \to r)$ are equivalent
Which of the following is true?
1) Statement 1 and Statement 2 are false
2) Statement 1 and Statement 2 are true
3) Statement 1 is false and Statement 2 is true
4) Statement 1 is true and Statement 2 is false.

KEY

1) 3	2) 1	3) 2	4) 1	5) 3
6) 4	7) 4	8) 1	9) 3	10) 3

11) 2	12) 4	13) 3	14) 4	15) 3
16) 3	17) 1	18) 3	19) 1	20) 3
21) 2	22) 4	23) 4	24) 3	25) 2
26) 1	27) 1	28) 4	29) 1	30) 1
31) 4	32) 1	33) 3	34) 2	35) 2
36) 3	37) 3			

PREVIOUS COMPETATIVE QUESTIONS

1. Consider the following statements
P : Suman is brilliant
Q : Suman is rich
R : Suman is honest
The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as : **[AIEEE 2011]**
1) $\sim(P\wedge\sim R)\leftrightarrow Q$ 2) $\sim P\wedge(Q\leftrightarrow\sim R)$
3) $\sim(Q\leftrightarrow(P\wedge\sim R))$ 4) $\sim Q\leftrightarrow\sim P\wedge R$

2. Statement I $\sim(p\leftrightarrow\sim q)$ is equivalent to $p\leftrightarrow q$.
Statement II $\sim(p\leftrightarrow\sim q)$ is a tautology.
[AIEEE 2009]

3. The statement $p\rightarrow(q\rightarrow p)$ is equivalent to **[AIEEE 2008]**
1) $p\rightarrow(p\rightarrow q)$ 2) $p\rightarrow(p\vee q)$
3) $p\rightarrow(p\wedge q)$ 4) $p\rightarrow(p\leftrightarrow q)$

4. **Consider :**
Statement -I : $(p\wedge -q)\wedge(\sim p\wedge q)$ is a fallacy.
Statement - II : $(p\rightarrow q)\leftrightarrow(\sim q\rightarrow\sim p)$ is a tautology [JEE MAINS-2013]
1) Statement-I is true; Statement-II is true; Statement -II is not a correct explanation for statement -I
2) Statement-I is true and Statement-II is false.
3) Statement-I is false and Statement-II is true
4) Statement-I is true; Statement-II is true; Statement -II is a correct explanation for statement -I

5. Statement - I : The statement $A\rightarrow(B\rightarrow A)$ is equivalent to $A\rightarrow A(A\vee B)$.
Statement - II : The statement $\sim[(A\wedge B)\rightarrow(\sim A\vee B)]$ is a tautology.
[JEE MAINSONLINE - 2013]
1) statement - I is true, Statement - II is True, statement - II is not a correct explanation for statement - II
2) statement I is true, statement II is false
3) Statement I is false, statement II is true
4) statement I is true, statement II is true, statement II is a correct explanation for statement I

6. **The statement $p\rightarrow(q\rightarrow p)$ is equivalent to : [JEE MAINSONLINE - 2013]**
1) $p\rightarrow q$ 2) $p\rightarrow(p\rightarrow q)$
3) $p\rightarrow(p\vee q)$ 4) $p\rightarrow(p\wedge q)$

7. **For integers m and n, both greater than 1 consider the following three statements :**
P : m divides n Q : m divides n^2
R : m is prime, then
[JEE MAINSONLINE - 2013]
1) $Q\wedge R\rightarrow P$ 2) $P\wedge Q\rightarrow R$
3) $Q\rightarrow R$ 4) $Q\rightarrow P$

8. **Let p and q be any two logical statements and $r: p\rightarrow(\sim p\vee q)$. If r has a truth value, F, then the truth values of p and q are respectively**
[JEE MAINSONLINE - 2013]
1) F,F 2) T,T 3) T,F 4) F,T

9. **Let p and q be two statements. Amongst the following the statement that is equivalent to $p\rightarrow q$ is**
1) $p\wedge\sim q$ 2) $\sim p\wedge q$
3) $\sim p\vee q$ 4) $p\vee\sim q$

10. **The statement that is TRUE among the following is**
1) $p\Rightarrow q$ is equivalent to $p\wedge\sim q$
2) $p\vee q$ and $p\wedge q$ have the same truth value

3) The converse of $\tan x = 0 \Rightarrow x = 0$ is $x \neq 0$ $\tan x = 0$.

4) The contra positive of $3x + 2 = 8 \Rightarrow x = 2$ is $x \neq 2 \Rightarrow 3x + 2 \neq 8$

11. **The logically equivalent proposition of $p \Leftrightarrow q$ is**

1) $p \wedge q$ 2) $(p \Rightarrow q) \wedge (q \Rightarrow p)$

3) $(p \wedge q) \vee (q \Rightarrow p)$ 4) $(p \wedge q) \Rightarrow (q \vee p)$

12. **Let p and q denote the following statements**
p: The sun is shining
q : I shall play tenses on the afternoon the negation of statement "If the sun is shining then I shall play tennis in the afternoon", is :

1) $p \wedge \sim q$ 2) $q \Rightarrow \sim p$

3) $\sim q \Rightarrow \sim p$ 4) $q \wedge \sim p$

13 **The statement $\sim (p \leftrightarrow \sim q)$ is**
(JEE MAINS 2014)

1) a tautology 2) a fallacy

3) equivalent to $p \leftrightarrow q$

4) equivalent to $\sim p \leftrightarrow q$

14. **The contrapositive of the statement " I go to school if it does not rain" is:**
[J.M.O.L-2014]

1) If it rains, I do not go to school

2) If I do not go to school, it rains

3) If it rains, I go to school

4) If I go to school, it rains.

15. **The proposition $\sim (pv \sim q)v \sim (pvq)$ is logically equivalent to [J.M.O.L-2014]**

1) p 2)q 3) $\sim p$ 4) $\sim q$

16. **The contrapositive of the statement 'If I am not feeling well, then I will go to the doctor' is: [J.M.O.L-2014]**

1) if I am feeling well, then I will not go to the doctor

2) If I will go to the doctor, then I am feeling well

3) If I will not go to the doctor, then I am feeling well

4) If I will go to the doctor, then I am not feeling well.

17. **Let p,q,r denote arbitrary statements. Then the logically equaivalent of the satement $p \Rightarrow (q \vee r)$ is**
[J.M.O.L-2014]

1) $(p \vee q) \Rightarrow r$ 2) $(p \Rightarrow q) \vee (p \Rightarrow r)$

3) $(p \Rightarrow -q) \wedge (p \Rightarrow r)$ 4) $(p \Rightarrow q) \wedge (p \Rightarrow -r)$

.KEY

1) 3	2) 3	3) 2	4) 1	5) 2
6) 3	7) 1	8) 3	9) 3	10) 4
11) 2	12) 3	13) 3	14) 2	15) 3
16) 3	17) 2			

HINTS

3.

q	p	$q \to p$	$p \to (q \to p)$	$(p \vee q)$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	F	T

$\therefore$ Statement $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$.

4. s1:

p	q	~p	~q	p^~q	~p^q	(p^~q)^(~p^q)
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

S II:

p	q	~p	~q	$p \Rightarrow q$	$\sim p \Rightarrow \sim q$	$(p \Rightarrow q)$^$(\sim p \Rightarrow \sim q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

S II is not an explanation of S I

5.

statement-1

A	B	B→A	A→(B→A)	AVB	A→(AvB)
T	T	T	T	T	T
T	F	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	T

statement-2

A	B	$\sim A$	$A \wedge B$	$(\sim A \vee B)$	$A \wedge B \to (\sim A \vee B)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	F	T	F	T	T

$\sim\left[(A \wedge B) \rightarrow (\sim A \vee B)\right]$ is not a tautology.

6. $P \rightarrow (q \rightarrow P) \equiv P \rightarrow (P \vee q)$

   Both are tautologies.

7. If m divides n and m divides n^2 $\Rightarrow$ m need not be a prime

   But if m divides n^2 and m is a prime $\Rightarrow$ m divides n i.e. $Q \wedge R \rightarrow P$

8. r is false only when p is true and $(\sim p \vee q)$ is false i.e. p is true , q false

9.

p	q	$p \rightarrow q$	$\sim p$	$\sim pvq$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

10. Contrapositve of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$
11. Definition of biconditional
13.

p	q	$\sim q$	$(p \leftrightarrow \sim q)$	$\sim(p \leftrightarrow \sim q)$	$(p \leftrightarrow q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

14. Contra positive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

15.

p	q	$\sim p$	$p \vee q$	$\sim(p \vee q)$	$p \vee \sim q$	$\sim(p \vee \sim q)$	$\sim(p \vee \sim q) \vee \sim(p \vee q)$	$\sim$p
T	T	F	T	F	T	F	F	F
T	F	T	T	F	T	F	F	F
F	T	F	T	F	F	T	T	T
F	F	T	F	T	T	F	T	T

$\Rightarrow \sim p(pv \sim q)v \sim (pvq) \cong \sim p$

16. Contra positive is " If I will not go to the doctor, then I am feeling well"
17. Logically equivalent of the statement $p \Rightarrow (q \vee r)$ is $(p \Rightarrow q) \vee (p \Rightarrow r)$

***