Chapter

13 AC Circuits

Quick Review

Alternating Signal

Voltage that vary with time periodically is called alternating voltage

The current driven by alternating voltage in a circuit is called alternating current

Peak value of an alternating current or voltage is the maximum value in either direction

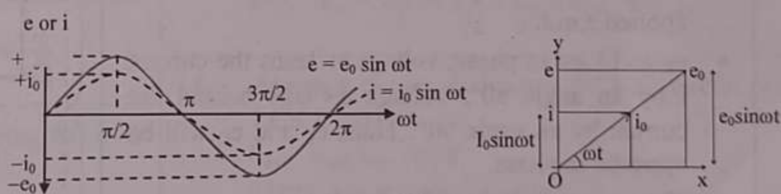
70.7% of peak value is called root mean square value or **RMS value**

Caution

Ammeters and voltmeters always read the r.m.s. values.

Purely resistive circuit

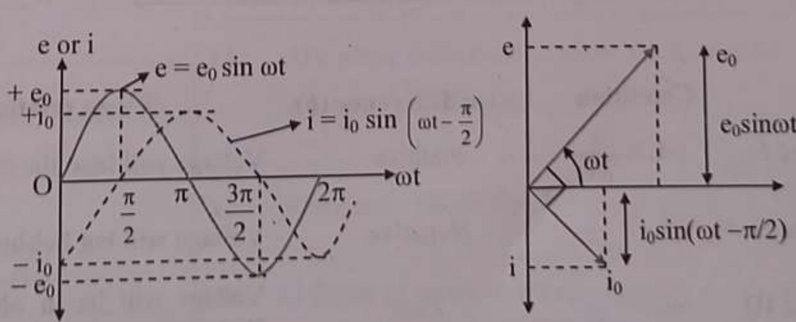
- Current and voltage are in phase
- It opposes A.C. as well as D.C.



Phasor Diagrams

Purely Inductive circuit

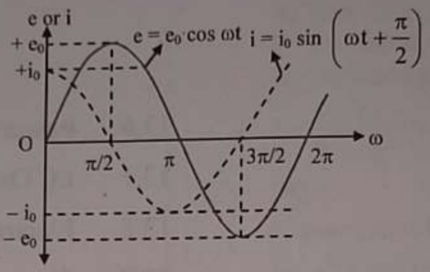
- Current lags voltage by a phase of 90°.
- The effective resistance offered by the capacitor to the alternating current is called inductive reactance
- Average power consumed over one full cycle is zero.

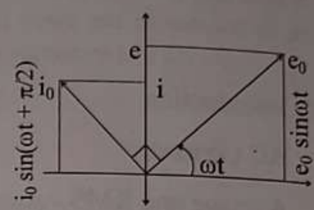


Phasor Diagrams

Purely Capacitive circuit

- Current leads the voltage by a phase of 90°.
- The effective resistance offered by the capacitor to the alternating current is called capacitive reactance
- Average power consumed over one full cycle is zero.





Phasor Diagrams

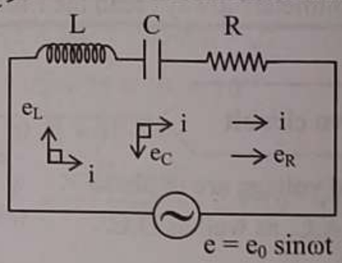
➤ **Resistance and reactance:**

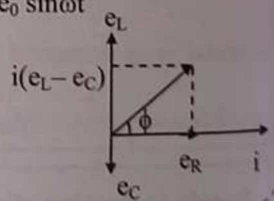
Sr. No.	Resistance	Reactance
i.	Equally effective for AC and DC.	Current is affected (reduced) but energy is not consumed (heat is not generated). The energy consumption by a coil is due to its resistive component.
ii.	Its value is independent of frequency of the AC	Inductive reactance (X_L) is directly proportional and capacitive reactance (X_C) is inversely proportional to the frequency of the AC.
iii.	Current opposed by a resistor is in phase with the voltage.	Current opposed by a pure inductor lags in phase while that opposed by a pure capacitor leads is phase by π^c over the voltage.

➤ **LCR Series circuit:**

- In series combination, the current I remains the same while the vector sum of potential difference across these components will be equal to the applied e.m.f.
- e_R and i are in phase, voltage e_L leads the current i by an angle 90°, voltage e_C lags behind the current by an angle 90°. Thus, e_L and e_C will be opposite in phase.
- The impedance (Z) of the circuit is,

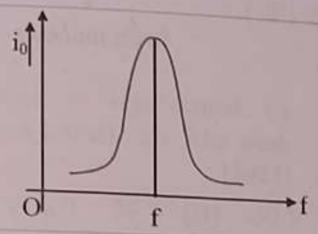
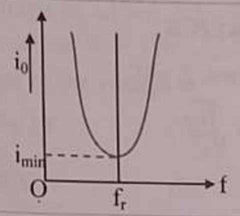
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$





	Condition	Phase difference (ϕ)	Phase relation between voltage and current
Case I	$\omega L > \frac{1}{\omega C}$	Positive	Voltage will **lead** the current
Case II	$\omega L < \frac{1}{\omega C}$	Negative	Voltage will **lag behind** the current
Case III	$\omega L = \frac{1}{\omega C}$	Zero	Voltage will be **in phase** with the current. This state is called **Resonance.**

Electric Resonance:

	Series resonance	Parallel resonance
Resonance occurs when	$X_L = X_C$	$X_L = X_C$
Resonant frequency	$f = \frac{1}{2\pi\sqrt{LC}}$	$f = \frac{1}{2\pi\sqrt{LC}}$
Impedance	Minimum	Maximum
Current	Maximum	Minimum
Graph		
	When a number of frequencies are fed to it, it accepts only one frequency and rejects the other frequencies. The current is maximum for this frequency. Hence it is called **acceptor circuit**.	When alternating current of different frequencies are sent through parallel resonant circuit, it offers a very high impedance to the current of the resonant frequency and rejects it but allows the current of the other frequencies to pass through it, hence called a **rejector circuit**.

Formulae

1. Alternating e.m.f.: $e = e_0 \sin \omega t$
2. Alternating Current: $i = i_0 \sin (\omega t + \alpha)$
3. Average value of current: $i_{av} = 0.637\ i_0$
4. Average value of e.m.f.: $e_{av} = 0.637\ e_0$
5. R.M.S value of induced e.m.f: $e_{rms} = \frac{e_0}{\sqrt{2}}$
6. R.M.S value of induced current: $i_{rms} = \frac{i_0}{\sqrt{2}}$
7. Heat produced by a sinusoidally varying AC over a complete cycle: $H = R(i_{rms})^2 . \frac{2\pi}{\omega}$
8. For a pure inductor circuit:
   $i = i_0 \sin\left[\omega t - \frac{\pi}{2}\right]$, where, $i_0 = \frac{V_0}{\omega L}$
9. Inductive reactance: $X_L = \frac{e_{rms}}{i_{rms}} = \omega L = 2\pi f L$
10. For a pure capacitor circuit:
    $i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$, where, $i_0 = \frac{V_0}{1/\omega C}$
11. Capacitive reactance: $X_C = \frac{e_{rms}}{i_{rms}} = \frac{1}{\omega C} = \frac{1}{2\pi f C}$
12. Impedance:
    i. In LCR circuit,
    $$Z = \frac{e_{rms}}{i_{rms}} = \sqrt{R^2 + (X_L - X_C)^2}$$
    ii. In LR circuit, $Z = \sqrt{R^2 + X_L^2}$
    iii. In CR circuit, $Z = \sqrt{R^2 + X_C^2}$
13. Average power in AC circuit with resistance only: $P_{av} = \frac{e_0 i_0}{2} = \frac{e_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} = e_{r.m.s} \times i_{r.m.s}$
14. Average power dissipated in A.C circuit with LCR in series:
    $P_{av} = e_{rms}\, i_{rms} \cos \phi$
    where, $\cos \phi$ = power factor
15. Power factor: $\cos \phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{R}{Z}$
16. The phase difference between X_L, X_C and R:
    $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$
17. Resonance frequency: $f_r = \frac{1}{2\pi\sqrt{LC}}$
18. Q factor of series resonant circuit:
    $Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{2\Delta\omega} = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$

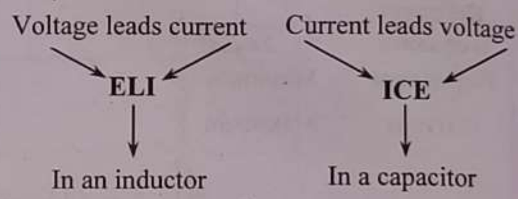
Shortcuts

1. Average value of $\sin^2\omega t$ over complete cycle is $\frac{1}{2}$.

2. Mnemonics

i. In inductive ('L') circuits, voltage ('E') leads current ('I').
Mnemonic: *ELI*

ii. In capacitive ('C') circuits, current ('I') leads voltage ('E').
Mnemonic: *ICE*



3. At resonance, $Z = R$

$$X_L = X_C = \sqrt{\frac{L}{C}}$$

$$I = \frac{V}{R}$$

4. The quality factor of series resonant circuit is also defined as the ratio of voltage drop across capacitor or inductor to the voltage drop across the resistor.

i.e., $Q = \frac{\omega_r L}{R}$ where, ω_r = resonant frequency