Formula Sheet

Differentiation.

• Derivative of some standard functions:

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
c (constant)	0
x^n	nx^{n-1}
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
e^x	e^x
a^x	$a^x \log a$
$\log x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \log a}$
$\log e$	1

• Rules of differentiation:

1) $y = u \pm v$ then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

2) $y = uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

3) $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

- Composite function:

$$f[g(x)] = f\circ g \qquad g[f(x)]\ g\circ f.$$

$y = [f(u)] \qquad u = g(x) \qquad \therefore y = f(g(x))$

i.e. $y \to u \to x \qquad \therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ — Chain rule.

- Derivative of composite function:

$$y = f[g(x)] \qquad \therefore \frac{dy}{dx} = f'[g(x)]\frac{d}{dx}g(x)$$

$$= f'[g(x)]\cdot g'(x)$$

- Geometrical meaning of derivative:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

- Derivative of inverse function:

$f(x) = y \qquad \therefore f'(x) = \frac{dy}{dx}$

$\therefore x = f^{-1}y = \frac{dx}{dy} = (f^{-1})'\,y \qquad \therefore \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}},\ \frac{dx}{dy} \neq 0.$

- Derivative of standard inverse function:

$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$	$\lvert x\rvert < 1$	$\cot^{-1}x$	$-\frac{1}{1+x^2}$	$x \in R$		
$\cos^{-1}x$	$-\frac{1}{\sqrt{1-x^2}}$	$\lvert x\rvert < 1$	$\sec^{-1}x$	$\frac{1}{x\sqrt{x^2-1}}$	$x > 1$	$-\frac{1}{x\sqrt{x^2-1}}$	$x < 1$
$\tan^{-1}x$	$\frac{1}{1+x^2}$	$x \in R$	$\operatorname{cosec}^{-1}x$	$-\frac{1}{x\sqrt{x^2-1}}$	$x > 1$	$\frac{1}{x\sqrt{x^2-1}}$	$x < 1$

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- Some important formulae for inverse trigo funtution:

1. $\sin^{-1}(\sin\theta)=\theta$, $\sin(\sin^{-1}x)=x$	4. $\text{cosec}^{-1}(\text{cosec}\,\theta)=\theta$, $\text{cosec}(\text{cosec}^{-1}x)=x$
2. $\cos^{-1}(\cos\theta)=\theta$, $\cos(\cos^{-1}x)=x$	5. $\sec^{-1}(\sec\theta)=\theta$, $\sec(\sec^{-1}x)=x$
3. $\tan^{-1}(\tan\theta)=\theta$, $\tan(\tan^{-1}x)=x$	6. $\cot^{-1}(\cot\theta)=\theta$, $\cot(\cot^{-1}x)=x$.

1. $\sin^{-1}(\cos\theta)=\sin^{-1}\left[\sin\left(\frac{\pi}{2}-\theta\right)\right]=\frac{\pi}{2}-\theta$.

2. $\cos^{-1}(\sin\theta)=\cos^{-1}\left[\cos\left(\frac{\pi}{2}-\theta\right)\right]=\frac{\pi}{2}-\theta$

3. $\tan^{-1}(\cot\theta)=\tan^{-1}\left[\tan\left(\frac{\pi}{2}-\theta\right)\right]=\frac{\pi}{2}-\theta$

4. $\cot^{-1}(\tan\theta)=\cot^{-1}\left[\cot\left(\frac{\pi}{2}-\theta\right)\right]=\frac{\pi}{2}-\theta$

5. $\text{cosec}^{-1}(\sec\theta)=\text{cosec}^{-1}\left[\text{cosec}\left(\frac{\pi}{2}-\theta\right)\right]=\frac{\pi}{2}-\theta$

6. $\sec^{-1}(\text{cosec}\,\theta)=\sec^{-1}\left[\sec\left(\frac{\pi}{2}-\theta\right)\right]=\frac{\pi}{2}-\theta$.

1. $\sin^{-1}(x)=\text{cosec}^{-1}\left(\frac{1}{x}\right)$
2. $\text{cosec}^{-1}(x)=\sin^{-1}\left(\frac{1}{x}\right)$
3. $\cos^{-1}(x)=\sec^{-1}\left(\frac{1}{x}\right)$
4. $\sec^{-1}(x)=\cos^{-1}\left(\frac{1}{x}\right)$
5. $\tan^{-1}(x)=\cot^{-1}\left(\frac{1}{x}\right)$
6. $\cot^{-1}(x)=\tan^{-1}\left(\frac{1}{x}\right)$

1. $\sin^{-1}x+\cos^{-1}x=\pi/2$
2. $\tan^{-1}x+\cot^{-1}x=\pi/2$
3. $\text{cosec}^{-1}x+\sec^{-1}x=\pi/2$.

- Some important substitution:

Expression	Substitution.
$\sqrt{1-x^2}$	$x=\sin\theta$ or $x=\cos\theta$.
$\sqrt{1+x^2}$	$x=\tan\theta$ or $x=\cot\theta$
$\sqrt{x^2-1}$	$x=\sec\theta$ or $x=\operatorname{cosec}\theta$.
$\sqrt{\frac{a+x}{a-x}}$ or $\sqrt{\frac{a-x}{a+x}}$	$x=a\cos 2\theta$ or $x=a\cos\theta$
$\sqrt{\frac{1+x}{1-x}}$ or $\sqrt{\frac{1-x}{1+x}}$	$x=\cos 2\theta$ or $x=\cos\theta$
$\sqrt{\frac{a+x^2}{a-x^2}}$ or $\sqrt{\frac{a-x^2}{a+x^2}}$	$x^2=a\cos 2\theta$ or $x^2=a\cos\theta$
$\frac{2x}{1+x^2}$	$x=\tan\theta$
$\frac{1-x^2}{1+x^2}$	$x=\tan\theta$
$3x-4x^3$ or $1-2x^2$	$x=\sin\theta$
$4x^3-3x$ or $2x^2-1$	$x=\cos\theta$
$\frac{3x-x^3}{1-3x^2}$	$x=\tan\theta$
$\frac{2f(x)}{1+f(x)^2}$ or $\frac{1-f(x)^2}{1+f(x)^2}$	$f(x)=\tan\theta$.

- $\tan^{-1}\left(\frac{x+y}{1-xy}\right)=\tan^{-1}x+\tan^{-1}y$

$\tan^{-1}\left(\frac{x-y}{1+xy}\right)=\tan^{-1}x-\tan^{-1}y$

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- Important Results:

1) $\frac{d}{dx}(x) = 1 \quad \therefore \frac{d}{dx}(x^\circ) = \frac{\pi}{180}$

2) $x^x = x^x[1+\log x]$

3) $\frac{d}{dx}(\underset{LHS}{xy} = \underset{RHS}{constant}) \quad \therefore \frac{dy}{dx} = \frac{-y}{x}$

4) $x^m y^n = (x+y)^{m+n} = \frac{y}{x}$

5) $\frac{y}{x} = const \quad \therefore \frac{dy}{dx} = \frac{y}{x}$

6] $\frac{d}{dx} \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x)} \ldots}}$

$= \frac{f'(x)}{2y-1}$.

- Parametric Function:

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

- Diff. one funt^n w.r.t. another funt^n:

$u = f(x) \qquad v = g(x)$

$\frac{du}{dx} = f'(x) \qquad \frac{dv}{dx} = g'(x) \qquad \therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{f'(x)}{g'(x)}$

- Higher order derivative:

$y = f(x)$

$\therefore \frac{dy}{dx} = f'(x) \qquad \therefore \frac{d^2y}{dx^2} = f''(x)$.

- Succesive (n^{th} order) diff:

  Find 1^{st}, 2^{nd}, 3^{rd} derivative.

  Observe changes in coeff, angles, power, sign.

  Find n^{th} order w.r.t. changes.