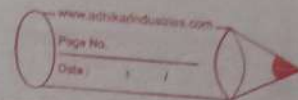


# Formula Sheet.

## Definite Integration.



- Definite integration as limit of Sum:

$$\int_a^b f(x) dx = \lim_{\substack{n \rightarrow \infty \\ h \rightarrow 0}} \sum_{r=1}^n hf(a+rh)$$

$$nh = b - a \quad h = \frac{b-a}{n}$$

- Important Result:

$$1] \sum_{r=1}^n 1 = n$$

$$3] \sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{2}$$

$$2] \sum_{r=1}^n r = \frac{n(n+1)}{2}$$

$$4] \sum_{r=1}^n r^3 = \left[ \frac{n(n+1)}{2} \right]^2 = \frac{n^2(n+1)^2}{4}$$

- Fundamental theorem of integral calculus:

$$\int f(x) dx = g(x) + c$$

$$\therefore \int_a^b f(x) dx = [g(x) + c]_a^b$$

$$= [g(b) + c - g(a) - c]$$

$$\int_a^b f(x) dx = g(b) - g(a)$$

- Properties of definite integral:

$$1] \int_a^a f(x) dx = 0$$

$$2] \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$3] \int_a^b F(x) dx = \int_a^b f(t) dt.$$

$$4] \int_a^b F(x) dx = \int_a^c F(x) dx + \int_c^b F(x) dx$$

where,  $a < c < b$ ,  $c \in [a, b]$

$$5] \int_a^b F(x) dx = \int_a^b F(a+b-x) dx$$

$$6] \int_0^a F(x) dx = \int_0^a F(a-x) dx.$$

• Important Results:-

$$1. \int_0^{\pi/2} \log(\tan x) dx = \int_0^{\pi/2} \log(\cot x) dx = 0$$

$$2. \int_0^{\pi/2} \log(\sin x) dx = \int_0^{\pi/2} \log(\cos x) dx = -\frac{\pi}{2} \log 2$$

$$3. \int_0^{\pi/2} \log(\operatorname{cosec} x) dx = \int_0^{\pi/2} \log(\sec x) dx = \frac{\pi}{2} \log 2.$$

• Properties:-

$$1] \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx, \text{ if } f(x) \text{ even funt}^n$$

$$= 0, \text{ if } f(x) \text{ odd funt}^n$$

$$f(x) \text{ even funt}^n \text{ if } f(-x) = f(x)$$

$$\text{and } f(x) \text{ odd funt}^n \text{ if } f(-x) = -f(x)$$

$$2] \int_a^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$$



• Reduction Formula:-

$$\int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx$$

$$= \frac{(n-1)}{n} \cdot \frac{(n-3)}{(n-2)} \cdot \frac{(n-5)}{(n-4)} \cdots \frac{4}{5} \cdot \frac{2}{3} \cdot (\text{odd})$$

$$= \frac{(n-1)}{n} \cdot \frac{(n-3)}{(n-2)} \cdot \frac{(n-5)}{(n-4)} \cdots \frac{3}{4} \cdot \frac{\pi}{2} \cdot (\text{even})$$

• Short trick:-

$$(x-a)^m \cdot (b-x)^n \, dx = (b-a)^{m+n+1} \frac{m!n!}{(m+n+1)!}$$