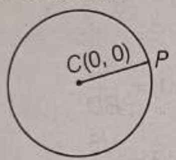
CHAPTER 03

Circle

A **circle** is the set of all points in a plane that are equidistant from a fixed point in that plane.

Or

A circle is defined as the locus of a point in a plane, which moves in a plane such that its distance from a fixed point in that plane is always constant.

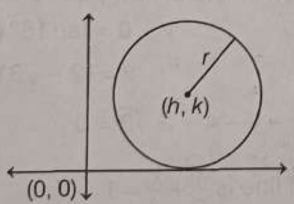


Centre The fixed point C is called the centre of the circle.

Radius The constant distance CP from the centre C to a point P on the circle, is called radius r.

Different forms of Equations of a Circle

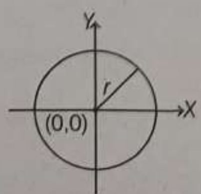
(i) **Centre-radius Form** Circle having centre (h, k) and radius r



Equation of the circle is $(x-h)^2+(y-k)^2=r^2$.

It is also known as standard equation and centre-radius form.

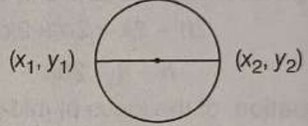
(ii) **Standard Form** Circle having centre (0, 0) and radius r



Equation of the circle is $x^2+y^2=r^2$.

(iii) **Diameter Form** If (x_1, y_1) and (x_2, y_2) are the end points of one of the diameter, then the equation of the circle is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$



The diametric form of a circle can also be written as

$$x^2+y^2-x(x_1+x_2)-y(y_1+y_2)+x_1x_2+y_1y_2=0$$

or x^2+y^2-x (sum of abscissae)
$-\,y$ (sum of ordinates)
+ product of abscissae
+ product of ordinates $=0$

General Form of Equation of a Circle

The general form of the equation of a circle is

$$x^2+y^2+2gx+2fy+c=0 \quad \text{...(i)}$$

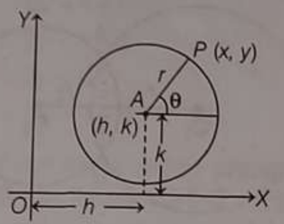
Its centre is $(-g, -f)$ and radius is $\sqrt{g^2+f^2-c}$.

Important Terms

- If $g^2+f^2-c>0$, the equation (i) represents a circle in the xy-plane.
- If $g^2+f^2-c=0$, then the equation (i) represents a point which is true degenerate conic and is the limiting position (radius is 0).
- If $g^2+f^2-c<0$, then the equation (i) does not represent any point in the xy-plane.
- The equation of a circle is a second degree equation in x and y. It contains no term of xy and coefficient of x^2 = Coefficient of y^2

Parametric Form of a Circle

Consider the circle $(x-h)^2+(y-k)^2=r^2$, centred at $A \equiv (h, k)$ and radius r.



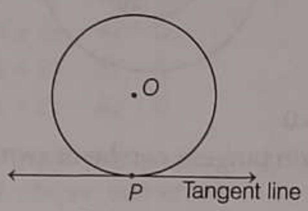
Let $P \equiv (x, y)$, the coordinates of P can be expressed as

$$x = h + r\cos\theta \text{ and } y = k + r\sin\theta$$

These equations represent the coordinates of any point on the circle in terms of the parameter θ.

Tangent

A tangent to a circle is a line that intersects or touches the circle at only one point. The common point of the tangent and the circle is called the **point of contact**.



Equation of Tangent

- Equation of tangent at the point $P(x_1, y_1)$ to a circle $x^2+y^2+2gx+2fy+c=0$ is
$$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$$
- Equation of tangent to circle $x^2+y^2=a^2$ at (x_1, y_1) is given by
$$xx_1+yy_1=a^2$$

Note *To find equation of tangent to the curve at (x_1, y_1) replace x^2 by xx_1, $2x$ by $(x+x_1)$, y^2 by yy_1, $2y$ by $(y+y_1)$.*

Equation of Tangent in Parametric Form

The equation of tangent to the circle $x^2+y^2=r^2$ at the point $(r\cos\theta, r\sin\theta)$ is

$$x\cdot r\cos\theta + y\cdot r\sin\theta = r^2$$

i.e.
$$x\cos\theta + y\sin\theta = r$$

Condition of Tangency

A line $y = mx + c$ is a tangent to the circle $x^2+y^2=a^2$, if $c^2 = a^2m^2 + a^2$ i.e. $c = \pm\sqrt{a^2m^2+a^2}$

and the point of contact is $\left(\frac{-a^2m}{c}, \frac{a^2}{c}\right)$.

Thus, there are two tangents with the same slope, m, $y = mx + \sqrt{a^2(1+m^2)}$ and $y = mx - \sqrt{a^2(1+m^2)}$.

Note *To check the tangency of a straight line to a circle, it is enough to show that the perpendicular from the center to the line is equal to the radius.*

Tangents from a Point to the Circle

Let $P(x_1, y_1)$ be a point in the plane, outside the circle. If a tangent from P to the circle has slope m, the equation of the tangent is $y - y_1 = m(x - x_1)$

i.e.
$$mx - y_1 - mx_1 + y_1 = 0$$

The condition that this is tangent to the circle is

$$\left|\frac{y_1 - mx_1}{\sqrt{1+m^2}}\right| = a, \text{ (the radius).}$$

$\therefore (x_1^2-a^2)m^2 - 2x_1y_1m + (y_1^2-a^2) = 0$ is quadratic equation in m.

It has two roots say m_1 and m_2, which are the slopes of two tangents.

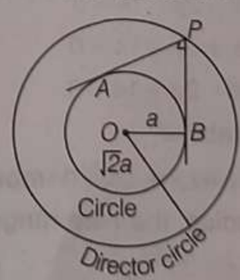
Sum of the roots $(m_1+m_2) = \frac{-(-2x_1y_1)}{(x_1^2-a^2)} = \frac{2x_1y_1}{x_1^2-a^2}$

Product of the roots $(m_1m_2) = \frac{(y_1^2-a^2)}{(x_1^2-a^2)}$

Director Circle

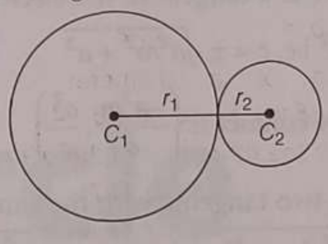
The locus of the point of intersection of two perpendicular tangents to a given circle is known as director circle.

If the equation of circle is $x^2+y^2=a^2$, then the equation of the director circle to this circle is $x^2+y^2=2a^2$.



Extra Information

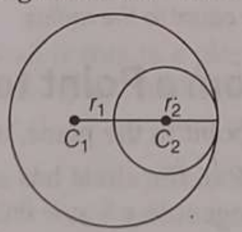
(i) Circles touching each other externally.



If $d(c_1c_2) = r_1 + r_2$.

Exactly three common tangents can be drawn.

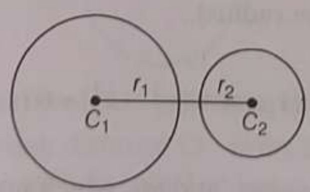
(ii) Circles touching each other internally.



If $d(c_1c_2) = |r_1 - r_2|$

Exactly one common tangent can be drawn.

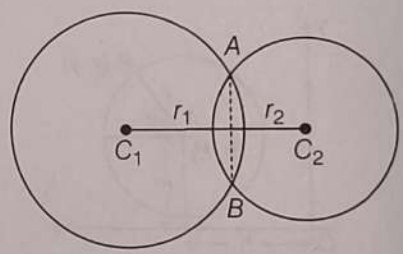
(iii) Disjoint circles.



If $|r_1 + r_2| < d(c_1c_2)$

Exactly four common tangents can be drawn.

(iv) Circles intersecting each other.

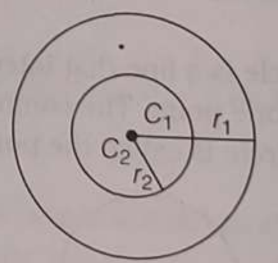


If $d(c_1c_2) < r_1 + r_2$

Line joining the point of intersection is the common chord also called as the radical axis.

Exactly two common tangent can be drawn.

(v) Concentric circles



If $d(c_1c_2) = 0$

No common tangent can be drawn.