Chapter

15 Structure of Atoms and Nuclei

Quick Review

Atom

Consists of positively charged nucleus (protons and neutrons) and negatively charged electrons.

Described using

Atomic models

Dalton's atomic model

- Matter is made up of indestructible particles, called atoms.
- Atoms of a given element are identical.
- Atoms can combine with other atoms to form new substances.

Thomson's model (plum-pudding model)

- An atom is a sphere having a uniform positive charge in which electrons are embedded.
- It is electrically neutral
- Only the negatively charged electrons can be emitted from it.
- **Failure:** Distribution of charges was experimentally found to be different than propose by Thomson's model.

Bohr's model

- Niels Bohr modified Rutherford's model by applying ideas of quantum physics.
- It is based upon the Bohr's three postulates.
- **Failure:**
  i. Unable to explain fine structure of spectral lines.
  ii. valid only for single electron atom.

Rutherford's model

- An atom consists of a very small central positively charged core called the nucleus.
- Electrons revolve around nucleus in circular orbits.
- Atom is electrically neutral.
- **Failure:** Could not explain the stability of the atom and atomic spectra.

Bohr's three postulates are

i. In a hydrogen atom, the electron revolves round the nucleus in a fixed circular orbit with constant speed.
ii. The radius of the orbit of an electron can only take certain fixed values such that the angular momentum of the electron in these orbits is an integral multiple of $\frac{h}{2\pi}$, h being the Planck's constant.
iii. An electron can make a transition from one of its orbits to another orbit having lower energy. In doing so, it emits a photon of energy equal to the difference in its energies in the two orbits.

➤ Hydrogen Spectrum:

Hydrogen spectrum consists of different spectral lines obtained due to transition of an electron in hydrogen atom from upper energy levels to lower energy levels.

- For different series in hydrogen spectrum:

Spectral series	*n*	*m*	*Wavelength*	λ_{max} ($m = n + 1$)	λ_{min} ($m = \infty$)	$\frac{\lambda_{max}}{\lambda_{min}}$	*Region*	*Range*
Lyman	1	2,3,4,...	$\frac{1}{\lambda_{Ly}} = R\left(\frac{1}{1^2} - \frac{1}{m^2}\right)$	$\frac{4}{3R}$	$\frac{1}{R}$	$\frac{4}{3}$	*Ultra-violet*	*911.6 Å to 1216 Å*
Balmer	2	3,4,5,...	$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{m^2}\right)$	$\frac{36}{5R}$	$\frac{4}{R}$	$\frac{9}{5}$	*Visible*	*3646 Å to 6563 Å*
Paschen	3	4,5,6,...	$\frac{1}{\lambda_P} = R\left(\frac{1}{3^2} - \frac{1}{m^2}\right)$	$\frac{144}{7R}$	$\frac{9}{R}$	$\frac{16}{7}$	*Near infra-red*	*8204 Å to 18753 Å*
Brackett	4	5,6,7,...	$\frac{1}{\lambda_{Br}} = R\left(\frac{1}{4^2} - \frac{1}{m^2}\right)$	$\frac{400}{9R}$	$\frac{16}{R}$	$\frac{25}{9}$	*Middle infra-red*	*14585 Å to 40515 Å*
Pfund	5	6,7,8,...	$\frac{1}{\lambda_{Pf}} = R\left(\frac{1}{5^2} - \frac{1}{m^2}\right)$	$\frac{900}{11R}$	$\frac{25}{R}$	$\frac{36}{11}$	*Far infra-red*	*22790 Å to 74583 Å*

The first two lines of minimum energy for any series are called the α-, β- lines of that series.

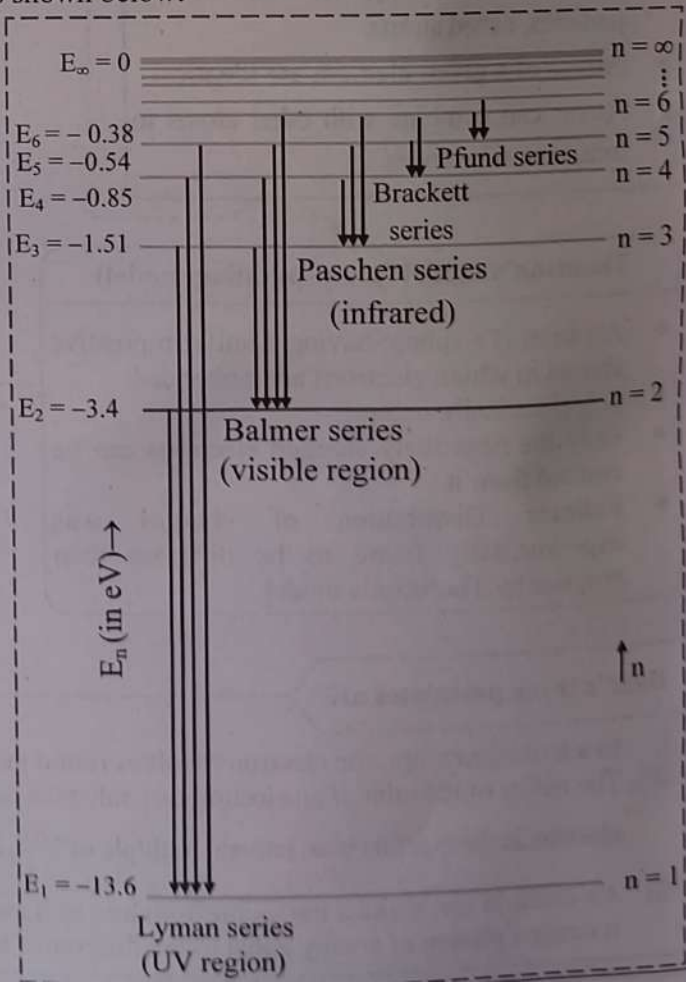
➤ Energy levels:

The definite amount of energies associated with the electrons in different orbits of an atom are called the energy levels (of that atom).

- For H-atom (Z = 1), the different energy levels are as shown below:

Principal quantum number	**Orbit**	**Excited state**	**Energy for H_1 atom**
$n = \infty$	Infinite	Infinite	0 eV
n = 5	Fifth	Fourth	– 0.54 eV
n = 4	Fourth	Third	– 0.85 eV
n = 3	Third	Second	– 1.51 eV
n = 2	Second	First	– 3.40 eV
n = 1	First	Ground	– 13.6 eV

- The negative sign in the energy value indicates that electron has minimum (maximum negative) energy in the first orbit while zero (maximum) energy at infinity from the nucleus.



Important terms related to energy of electron

i. **Excitation energy:** The energy required to take an electron from the ground state to an excited state is called the excitation energy of the electron in that state.

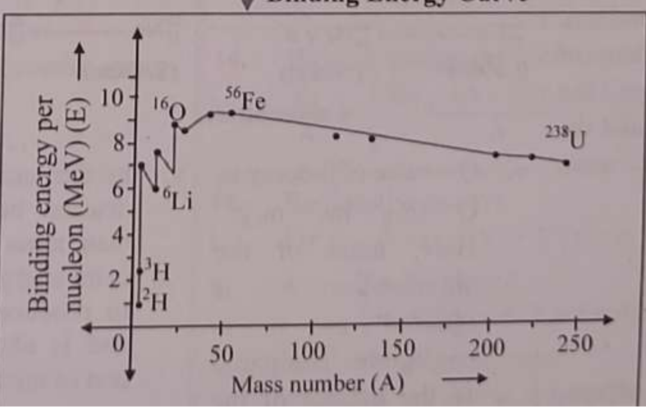
ii. **Binding energy:** Binding energy of an electron is the minimum energy required to make it free from the nucleus.

iii. **Ionization energy:** The ionization energy of an atom is the minimum amount of energy required to be given to an electron in the ground state of that atom to set the electron free.

Nuclei

- **Composition**
  - Atomic number (Z) → Same → Isotopes → Examples: a. ^1_1H, ^2_1H, ^3_1H b. ^3_2He, ^4_2He
  - Mass number (A) → Same → Isobars → Examples: a. ^3_1H, ^3_2He, b. $^{14}_6C$, $^{14}_7N$
  - Neutron number (N) → Same → Isotones → Examples: a. ^9_4Be, $^{10}_5B$, b. ^3_1H, ^4_2He
- **Size** → Radius $R = R_0 A^{1/3}$
- **Mass**
  - **Mass defect (ΔM)**: The difference between the sum of the masses of the nucleons composing a nucleus and the rest mass of the nucleus is known as mass defect.
  - Einstein's mass-energy relation (ΔMc^2)
  - Atomic Mass unit (u) $1 \text{ u} = 1.66 \times 10^{-27}$ kg
  - **Binding Energy (E_B)**: The energy equivalent to that of mass defect is called the binding energy of the nucleus.
  - Binding Energy per nucleon (E_B/A)
  - Graph of E_B/A against A
    - FISSION → **Nuclear Fission**
      - Breaking of heavy nuclei into lighter one
      - Successive fissions form chain reactions
    - FUSION → **Nuclear Fusion**
      - Combining of lighter nuclei
      - Requires high temperature and pressure
    - **Binding Energy Curve**
- **Density**: Constant for all nuclei

$$\rho = \frac{3\text{ m}}{4\pi R_0^3}$$



➢ Q – value

Q – value

i. The difference in the energy equivalent of the mass of the parent atom and that of the sum of masses of the products is called the Q–value or Q, of the decay.

ii. It is equal to the kinetic energy of the products.
In general, $Q = [M_{parent} - M_{products}]c^2$

➢ Various decay processes:

Parent nucleus → Decay processes → Daughter nucleus

Types:

α-decay	β-decay	γ-decay
${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$	${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- +$ antineutrino.	$X_{excited} \rightarrow X_{relaxed} + \gamma$
	β-plus decay: ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ +$ neutrino	

Comparison between various decay processes:

α-decay	β-decay	β-plus decay	γ-decay
i. In α decay the parent nucleus loses helium nucleus (i.e., two protons and two neutrons). ii. The decay can be expressed as ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$ iii. All nuclei with $A > 210$ undergo alpha decay. iv. **Example:** ${}^{212}_{83}Bi \longrightarrow {}^{208}_{81}Tl + \alpha$ (Bismuth) (Thallium) v. Q - value of α-decay is $Q = [m_X - m_Y - m_{He}]c^2$, where m_X, m_Y and m_{He} are atomic masses of the parent atom, the daughter atom and the helium atom respectively.	i. In β decay, the nucleus emits an electron produced by converting a neutron in the nucleus into a proton. i.e., $n \rightarrow p + e^-$ + antineutrino ii. During beta decay, the number of nucleons i.e., the mass number of the nucleus remains unchanged. iii. β decay can be written as, ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^-$ + antineutrino. iv. **Example:** ${}^{60}_{27}Co \longrightarrow {}^{60}_{28}Ni + e^-$ + antineutrino (Cobalt) (Nickel) v. Q – value of β-decay is, $Q = [m_X - m_Y - m_e]c^2$ Here, mass of the antineutrino is ignored as it is negligible compared to the masses of the nuclei.	i. In beta plus decay, a proton gets converted to a neutron by emitting a positron and a neutrino. i.e., $p \rightarrow n + e^+$ + neutrino ii. During the decay the mass number remains unchanged but Z decreases by one and N increases by one. iii. The decay can be written as, ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+$ + neutrino iv. **Example:** ${}^{22}_{11}Na \longrightarrow {}^{22}_{10}Ne + e^+$ + neutrino (Sodium) (Neon) v. In this case, mass of neutron being higher than mass of proton, extra energy is needed to produce a neutron and is obtained from rest of nucleus.	i. In gamma (γ) decay, gamma rays (high energy photons) are emitted by the parent nucleus. ii. The daughter nucleus is same as the parent nucleus as no other particle is emitted, but it has less energy as some energy goes out in the form of the emitted gamma ray. iii. It can be written as, $X_{excited} \rightarrow X_{relaxed} + \gamma$ where, $X_{excited}$ is excited nucleus, $X_{relaxed}$ is same nucleus at lower energy states and γ is released gamma ray photon. iv. Energy released (Q value) in γ-decay is difference in energy levels through which nucleus is transiting. v. Gamma decays usually occur after parent nucleus has undergone α or β decay.

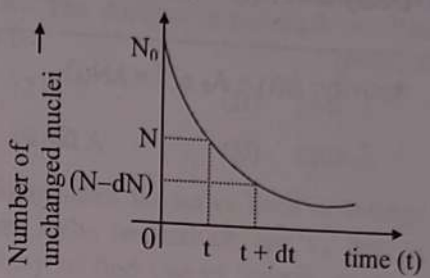
Law of radioactive decay

The law states that the number of nuclei disintegrating per unit time is proportional to the number of nuclei present (not disintegrated) at that instant, i.e., $N(t) = N_0 e^{-\lambda t}$

Radioactive Decay

The nuclear phenomenon in which an unstable nucleus undergoes decay is called radioactive decay.

Decay Curve



Important terms related to radioactive decay

i. **Activity:** The rate of decay, i.e., the number of decays per unit time $-\frac{dN(t)}{dt}$, is called as activity A(t).

ii. **Half life:** Half life period of a radioactive substance is defined as the time in which the half substance is disintegrated.

iii. **Average life:** The arithmetic average of the lives of all the nuclei present initially is called average life of a radioactive element.

Formulae

1. Wave number:
$$\bar{v} = \frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = R_H Z^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$
where, $R_H = \frac{m_e e^4}{8c\varepsilon_0 h^3}$

2. Angular momentum: $L = m_e vr = \frac{nh}{2\pi}$

3. Radius of n^{th} Bohr orbit:
$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m_e Z e^2} \quad \text{i.e., } r_n \propto n^2$$
$$\frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2$$

4. Velocity of electron in n^{th} orbit:
$$v_n = \frac{Ze^2}{2\varepsilon_0 nh} \quad \text{i.e., } v_n \propto \frac{1}{n}$$

5. Energy difference between two successive energy level:
$$\Delta E = E_2 - E_1 = h\nu$$

6. Energy of n^{th} Bohr orbit:
$$E_n = -\frac{m_e Z e^4}{8\varepsilon_0^2 n^2 h^2} \quad \text{i.e., } E_n \propto \frac{1}{n^2}$$
$$\frac{E_1}{E_2} = \left(\frac{n_2}{n_1}\right)^2$$
$$E_n = \frac{-13.6 Z^2}{n^2}\ \text{eV}$$

7. K.E of n^{th} orbit: $K.E = -\frac{m_e Z^2 e^4}{8\varepsilon_0^2 n^2 h^2}$

8. Potential energy in n^{th} orbit:
$$P.E = -\frac{m_e Z^2 e^4}{4\varepsilon_0^2 n^2 h^2} \quad \text{i.e., } P.E \propto \frac{1}{n^2}$$

9. Relation between K.E, P.E and T.E:
   i. $P.E = -2\ K.E$ ii. $P.E = 2\ T.E$
   iii. $T.E = P.E + K.E$

10. Radius of a nucleus: $R = R_0 A^{1/3}$

11. Nuclear density: $\rho = \frac{3m}{4\pi R_0^3}$

12. Mass defect: $\Delta M = [Zm_p + (A - Z)\, m_n] - M$

13. Binding energy: $E_B = \Delta Mc^2\ J = \frac{\Delta Mc^2}{e}\ eV$

14. Binding energy per nucleon:
$$\frac{E_B}{A} = \left[\frac{Zm_p + (A-Z)m_n - M}{A}\right]c^2\ \text{J/nucleon}$$

15. Radioactive decay:
   i. For α – decay: ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$
   ii. a. For β – decay:
   ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- +$ antineutrino.
   b. For β –plus decay:
   ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ +$ neutrino
   iii. For γ – decay: $X_{excited} \rightarrow X_{relaxed} + \gamma$

16. Decay law: $N(t) = N_0e^{-\lambda t}$

17. Activity: $A(t) = A_0\,e^{-\lambda t} = \lambda N(t)$

18. Half life period: $T_{1/2} = \dfrac{0.693}{\lambda}$

19. Average life: $\tau = \dfrac{1}{\lambda} = \dfrac{T_{1/2}}{0.693}$

Shortcuts

1. If mass of electron becomes n times the present value, then the Rydberg constant will also become n times the present value.
2. Maximum number of spectral lines obtained on account of transition of electron present in n^{th} orbit to various lower orbits is given by $\dfrac{n(n-1)}{2}$.
3. Wavelength will be minimum for $n = \infty$ and it will be maximum for the nearest number which is 1^{st} member of series.
4. Number of α and β particles emitted in a decay process is given as,

$$n_\alpha = \frac{A-A'}{4} \text{ and } n_\beta = (2n_\alpha - Z + Z')$$