Chapter

3 Motion in a Plane

Quick Review

Motion

Displacement

Difference between the position vectors of the object in that time interval.

$\Delta \vec{x} = \vec{x}_2 - \vec{x}_1$

Path length

Actual distance travelled by the particle during its motion.

Average velocity

Displacement $\Delta \vec{x}$ of the object during the time interval Δt over which average velocity is being calculate, divided by that time interval.

Instantaneous velocity

The limiting value of the average velocity of the object over a small time interval 'Δt' around t when the value of time interval goes to zero.

Average speed

The total path length (distance) travelled by the object during the time interval over which average speed is being calculated, divided by that time interval.

Instantaneous speed

The limiting value of the average speed of the object over a small time interval 'Δt' around t when the value of time interval goes to zero.

Average acceleration

The change in velocity divided by the total time required for the change.

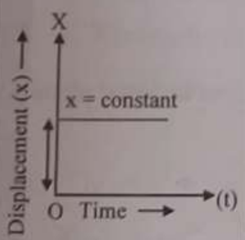
Instantaneous acceleration

The limiting value of the average acceleration of the object over a small time interval 'Δt' around t when the value of time interval goes to zero.

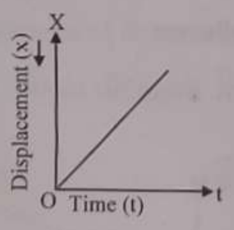
➤ **Position-time and velocity-time graph:**

Motion

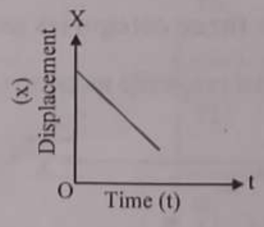
Position-time graph



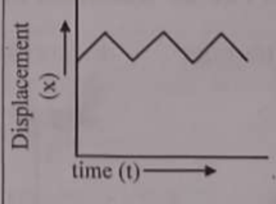
Object at rest



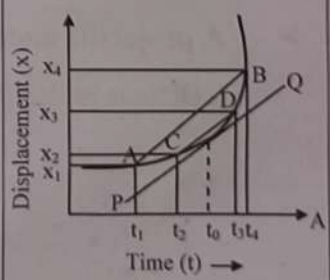
Object moving with uniform velocity along positive x-axis



Object moving with uniform velocity along negative x- axis



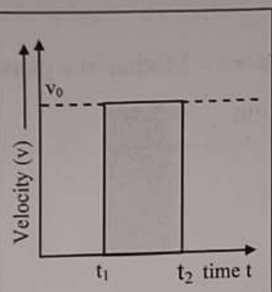
Object performing oscillatory motion with constant speed



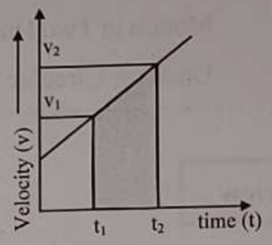
Particle moving with non-uniform velocity

Motion

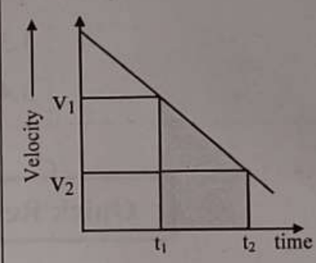
Velocity-time graph



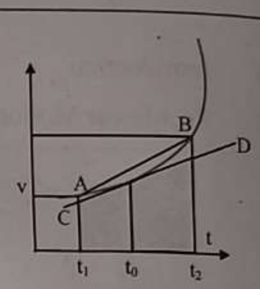
Object is moving with zero acceleration



Object is moving with constant positive acceleration



Object is moving with constant negative acceleration



Object is moving with non-uniform acceleration

Relative velocity

Relative velocity is defined as the time rate of change of relative position of one object with respect to another.

Relative velocity of A w.r.t. B is given by, $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Relative velocity of B w.r.t. A is given by, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

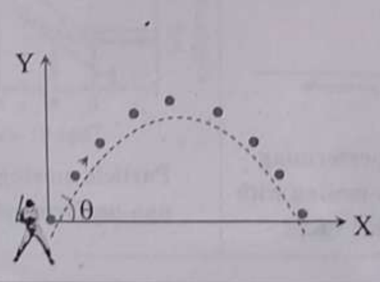
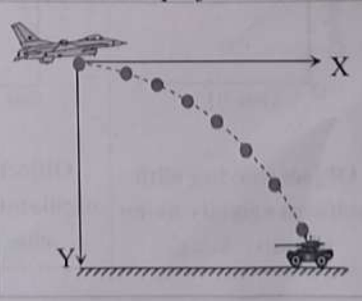
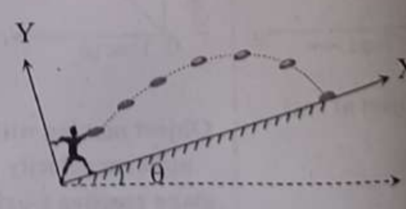
Projectile Motion

An object in flight after being thrown with some velocity is called a projectile and its motion is called projectile motion.

Equation: $\left[y = (\tan\theta)x - \frac{1}{2}\left(\frac{g}{u^2 \cos^2 x}\right)x^2 \right]$

Horizontal Range (R)	Maximum height (H)	Time of Flight (t)
• The maximum horizontal distance travelled by the projectile. • It is maximum when angle of projection (θ) is 45°.	The maximum height H reached by the projectile is the distance travelled along the vertical (y) direction in time t_A.	• It is the total time taken by the projectile to go up and come down to the same level from which it was projected. • The time taken to reach the maximum height is called time of ascent (t_A). • The time taken by projectile to travel to the ground from the maximum height, is called time of descent (t_D).

➢ **A projectile motion can be classified into three categories as follows:**

i. Oblique projectile motion:	ii. Horizontal projectile motion:	iii. Projectile motion on an inclined plane:
		

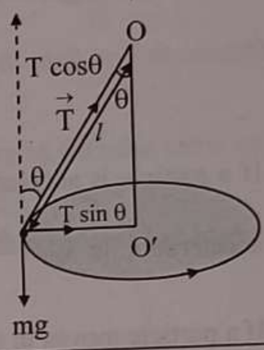
Uniform Circular Motion (UCM)	• **Time Period:** The time taken by a particle performing uniform circular motion to complete one revolution. • **Centripetal Force:** A force in the case of circular motion which is directed towards the centre along the radius is called centripetal (radial) force.

Conical Pendulum

- A simple pendulum, which is given such a motion that the bob describes a horizontal circle and the string making a constant angle with the vertical describes a cone.
- Time period of a conical pendulum depends on the following factors:

i. Length of pendulum (l)
ii. Acceleration due to gravity (g)
iii. Angle of inclination (θ)

i.e., $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$



Formulae

1. Basic kinematical formulae:

i. Average speed $= \frac{\text{Total path length}}{\text{Total time interval}}$

$= \frac{\text{Total distance}}{\text{Total time}} = \frac{x}{t}$

ii. Instantaneous speed: $\vec{v} = \lim_{\Delta t \to 0}\frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$

iii. Average velocity:

$\vec{v}_{avg} = \frac{\text{Displacement}}{\text{Time interval}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta \vec{x}}{\Delta t}$

iv. Acceleration:

$a = \frac{\text{Change in velocity}}{\text{Time}} = \frac{d\vec{v}}{dt}$

v. Average acceleration: $\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$

vi. Instantaneous acceleration: $\vec{a}_{inst} = \lim_{\Delta t \to 0}\frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

2. Kinematic Equations of linear motion:

i. $\vec{v} = \vec{u} + \vec{a}t$
ii. $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
iii. $v^2 = u^2 + 2\vec{a}.\vec{s}$
iv. $\vec{s} = \frac{(\vec{v}+\vec{u})}{2}t$

3. Relative velocity of a body A with respect to B:

i. $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$;
ii. $v = \sqrt{v_x^2 + v_y^2}$ (in magnitude)

4. Velocity of projectile:

i. $u_x = u\cos\theta$ (along horizontal)
ii. $u_y = u\sin\theta$ (along vertical)

5. Horizontal distance covered by projectile:

$x = (u\cos\theta)\,t$

6. Vertical distance of projectile:

$y = (u\sin\theta)\,t - \frac{1}{2}gt^2$

7. Equation of trajectory:

$y = x(\tan\theta) - \frac{gx^2}{2u^2\cos^2\theta}$

8. Maximum height: $H = \frac{u^2\sin^2\theta}{2g}$

9. Time of flight: $T = \frac{2u\sin\theta}{g}$

10. Time of ascent = Time of descent $= \frac{u\sin\theta}{g}$

11. Horizontal range: $R = \frac{u^2\sin 2\theta}{g}$

12. Maximum horizontal range: $R_{max} = \frac{u^2}{g}$

13. Time period in uniform circular motion:

$T = \frac{2\pi r}{v}$

14. Angular speed in uniform circular motion:

$$\omega = \frac{v}{r}$$

15. Centripetal acceleration:

i. $\vec{a} = -\omega^2 \vec{r}$

ii. $a = \omega^2 r$ (in magnitude)

16. Centripetal force:

$$F = m\omega^2 r = \frac{mv^2}{r} = m\omega v$$

17. Time period of a conical pendulum:

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}$$

Shortcuts

1. If a particle is accelerated for time t_1 with acceleration a_1 and for time t_2 with acceleration a_2 then average acceleration is $\vec{a}_{av} = \dfrac{\vec{a}_1 t_1 + \vec{a}_2 t_2}{t_1 + t_2}$

2. If a particle moves in two equal intervals of time at different speed v_1 and v_2 respectively, then $v_{av} = \dfrac{v_1 + v_2}{2}$

3. When a body starts from rest and moves with uniform acceleration, distance covered by the body in n^{th} second is directly proportional to $(2n - 1)$ i.e., ratio of the distances covered in 1^{st} s, 2^{nd} s and 3^{rd} s is $[2(1) - 1] : [2(2) - 1] : [2(3) - 1] = 1:3:5$.

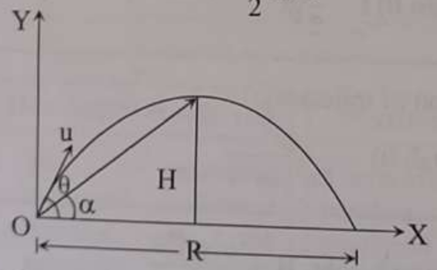
4. Horizontal range of projectile is same when angles of projection are (Complimentary)

i. θ and $90° - \theta$ or

ii. $(45° + \theta)$ and $(45° - \theta)$

5. A ball is dropped from a building of height h and it reaches after t seconds on earth. From the same building if two balls are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively then $t = \sqrt{t_1 t_2}$

6. The angle of elevation α of the highest point of the projectile and the angle of projection θ are related to each other as $\tan\alpha = \dfrac{1}{2}\tan\theta$



7. When a projectile is projected at an angle 45°, the range is maximum and the height attained by the projectile is $H = \dfrac{u^2}{4g} = \dfrac{R_{max}}{4}$

8. The height attained by a projectile is maximum, when $\theta = 90°$.

$$H_{max} = \frac{u^2}{4g}$$

9. When the range of the projectile is maximum, the time of flight is $T = 2t = \dfrac{\sqrt{2}u}{g}$

10. The time of flight of the projectile is also largest for $\theta = 90°$.

$$T_{max} = \frac{2u}{g}$$

11. In U.C.M., if central angle or angular displacement is given, then simply apply $dv = 2v\sin\dfrac{\theta}{2}$ to determine change in velocity.