14 DIFFERENTIATION

Synopsis

11.1 DIFFERENTIABILITY :

1. Differentiability of a function at a point :

Definition :

Let $f : (a, b) \rightarrow R$ and $c \in (a, b)$ be a function.

Then, $f(x)$ is said to be derivable or differentiable at $x = c$ iff $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists finitely.

Then value of this limit is called the derivative or differential coefficient of the function f at $x = c$. It is denoted by $f'(c)$ or $Df'(c)$ or $\left\{\frac{d}{dx}(f(x))\right\}_{x=c}$

Thus, $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

Right hand derivative :

If $y = f(x)$ is a real valued function and a is any real number, then $\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$, if it exists, is called the right hand derivative of $f(x)$ at $x = a$ and is denoted by $Rf'(a)$ or $f'(a^+)$.

Left hand derivative :

If $y = f(x)$ is a real valued function and a is any real number, then $\lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$, if it exists, is called the left hand derivative of $f(x)$ at $x = a$ and is denoted by $Lf'(a)$ or $f'(a^+)$.

2. Differentiability in a set or in an interval :

   i. A function f is said to be derivable (or differentiable) in the open interval (a, b) iff f is derivable at each point of (a, b).

   ii. A function f is said to be derivable (or differentiable) in the closed interval $[a, b]$ iff f is derivable at each point of $[a, b]$.

3. Differentiable function :

A function is said to be differentiable function if it is differentiable at every point of its domain.

IMPORTANT NOTES :

i. The sum, difference, product and quotient of two differentiable function is differentiable.

ii. The composition of differentiable function is a differentiable function.

4. Relation between differentiability and continuity :

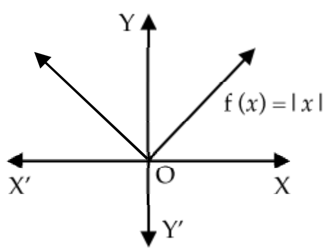
   i. Differentiability implies continuity always. i.e., if a function f is derivable at a point $x = a$, then its must be continuous at that point.

   ii. Continuity does not necessarily imply differentiability i.e., if a function is continuous at a point, it may not be derivable at the point.

eg.

The function $f(x) = |x|$ is continuous at $x = 0$ but it is not differentiable at $x = 0$,

As shown in the figure there is sharp edge at $x = 0$. Hence, function is not differentiable but continuous at $x = 0$.



5. Derivative :

The rate change of a qvantity y with respect to another qvantity x is called the derivative or differential coefficient of y with respect to x.

It is denoted by $\frac{dy}{dx}$ or y' or y_1 or D.

6. Derivatives of some standard function :

i. Differentiation of a algebraic function :

a. $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

b. $\frac{d}{dx}\left(ax+b\right)^n = n\left(ax+b\right)^{n-1}.a$

c. $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$

d. $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$

ii. Differentiation of trigonometric function :

a. $\frac{d}{dx}(\sin x) = \cos x$

b. $\frac{d}{dx}(\cos x) = -\sec x$

c. $\frac{d}{dx}(\tan x) = \sec^2 x$

d. $\frac{d}{dx}(\sec x) = \sec x \tan x$

e. $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$

f. $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

iii. Differentiation of inverse trigonometric function :

a. $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}, \text{for } -1 < x < 1$

b. $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}, \text{for } -1 < x < 1$

c. $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}, \text{for } x \in R$

d. $\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}, \text{for } x \in R$

e. $\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}, \text{for } |x| > 1$

f. $\frac{d}{dx}\left(\text{cosec}^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}, \text{for } |x| > 1$

iv. Differentiation of logarithmic and exponential function :

a. $\frac{d}{dx}\left(\log x\right) = \frac{1}{x}, \text{for } x > 0$

b. $\frac{d}{dx}\left(\log_a x\right) = \frac{1}{x \log a}, \text{for } x > 0, a > 0, a \neq 1.$

c. $\frac{d}{dx}\left(e^x\right) = e^x$

d. $\frac{d}{dx}\left(a^x\right) = a^x \log a, \text{for } a > 0$

IMPORTANT NOTES :

i. $\frac{d}{dx}(\text{constant}) = 0$

ii. $\frac{d}{dx}|x| = \frac{x}{|x|}\left(x \neq 0\right)$

iii. $\frac{d}{dx}\log|x| = \frac{1}{x}\left(x \neq 0\right)$

iv. $\frac{d}{dx}\left([x]\right) = 0, x \notin \text{I}$ and $\frac{d}{dx}\left([x]\right)$ does not exist at any integer.

11.2 USEFUL RESULTS :

1. $\sin^{-1}(-x) = -\sin^{-1} x$
2. $\cos^{-1}(-x) = \pi - \cos^{-1} x$
3. $\tan^{-1}(-x) = -\tan^{-1} x$
4. $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x$
5. $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$
6. $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x$
7. $\cot^{-1}\left(\frac{1}{x}\right) = \tan^{-1} x$
8. $\sec^{-1}\left(\frac{1}{x}\right) = \cos^{-1} x$
9. $\operatorname{cosec}^{-1}\left(\frac{1}{x}\right) = \sin^{-1} x$
10. $\sin^{-1}\sqrt{1-x^2} = \cos^{-1} x$
11. $\cos^{-1}\sqrt{1-x^2} = \sin^{-1} x$
12. $\tan^{-1}\sqrt{x^2-1} = \sec^{-1} x$
13. $\sec^{-1}\sqrt{1+x^2} = \tan^{-1} x$
14. $\operatorname{cosec}^{-1}\sqrt{1+x^2} = \cot^{-1} x$
15. $\cot^{-1}\sqrt{x^2-1} = \operatorname{cosec}^{-1} x$
16. $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1} x$
17. $\sin^{-1}\left(3x - 4x^3\right) = 3\sin^{-1} x$
18. $\cos^{-1}\left(2x^2 - 1\right) = 2\cos^{-1} x$
19. $\cos^{-1}\left(1 - 2x^2\right) = 2\sin^{-1} x$
20. $\cos^{-1}\left(4x^3 - 3x\right) = 3\cos^{-1} x$
21. $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\cdot\tan^{-1} x$
22. $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = 3\tan^{-1} x$
23. $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ If $x, y > 0$ and $xy < 1$.

11.3. HIGHER DERIVATIVES (SUCCESSIVE DIFFERENTIATION) :

Let $y = f(x)$ be a differentiable function of x and let its derivative be called the first derivative of the function denoted by $\frac{dy}{dx}$, y' or $f'(x)$. The symbols $\frac{d^2y}{dx^2}, y''$ or $f''(x)$ is second dirivative of function. The derivative of the second derivative is called the third derivative of the function and is denoted by one aof the symbols $\frac{d^3y}{dx^3}, y'''$, or $f'''(x)$,... The n-th derivative is denoted by $\frac{d^ny}{dx^n}$.

NOTE :

i. The successive differential coefficients are also denoted by
   i. $y_1, y_2, y_3, \ldots, y_n$
   ii. $Dy, D^2y, \ldots, D^ny$

ii. The derivative of a given order at a point can exist only when the function and all derivatives of lower order are differentiable at the point.

11.4. DIFFERENTIATION OF INVERSE TRIGONOMETRIC FUNCTION USING TRIGONOMETRICAL TRANSFORMATIONS :

1. $\sin x = 2\sin\frac{x}{2}.\cos\frac{x}{2} = \frac{2\tan x/2}{1+\tan^2 x/2}$

2. $\sin 2x = 2\sin x.\cos x$

3. $\sin 3x = 3\sin x - 4\sin^3 x$

4. $\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$

5. $\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

6. $\tan x = \frac{2\tan x/2}{1-\tan^2 x/2}$

7. $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

8. $\cos 3x = 4\cos^3 x - 3\cos x$

9. $\tan 3x \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

10. $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1} x$

11. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1} x$

12. $\tan^{-1}\sqrt{\frac{1-x}{1+x}} = \frac{1}{2}\cos^{-1} x$

13. $\sin^{-1} x + \cot^{-1} x = \pi/2, \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

    $\sec^{-1} x + \cos ec^{-1} x = \frac{\pi}{2}$

14. $\tan\left(\frac{\pi}{4} \pm x\right) = \frac{1 \pm \tan x}{1 \mp \tan x}$

15. $\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1+x^2}\right)$

    Provided $x, y, \geq 0$ and $x^2 + y^2 \leq 1$.

16. $$\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1+x^2}\right)$$

    If $x, y, > 0$ and $x^2 + y^2 \leq 1$.

17. $$\cos^{-1} x \pm \cos^{-1} y = \pi - \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

    (Provided $x, y, \geq 0$ and $x^2 + y^2 > 1$).

18. $$\cos^{-1} x \pm \cos^{-1} y = \pi - \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

    (If $x, y, > 0$ and $x^2 + y^2 \leq 1$).

19. $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

    (If $x, y, > 0$ and $xy > 1$)

20. $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

    (If $x, y > 0$)

Some Useful Substitutions :

Expression	Substitition	Formula	Result
$3x - 4x^3$	$x = \sin\theta$	$3\sin\theta - 4\sin^3\theta$	$\sin 3\theta$
$4x^3 - 3x$	$x = \cos\theta$	$4\cos^3\theta - 3\cos\theta$	$\cos 3\theta$
$\dfrac{3x - x^3}{1 - 3x^2}$	$x = \tan\theta$	$\dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$	$\tan 3\theta$
$\dfrac{2x}{1 + x^2}$	$x = \tan\theta$	$\dfrac{2\tan\theta}{1 + \tan^2\theta}$	$\tan 2\theta$
$\dfrac{2x}{1 - x^2}$	$x = \tan\theta$	$\dfrac{2\tan\theta}{1 - \tan^2\theta}$	$\tan 2\theta$
$1 - 2x^2$	$x = \sin\theta$	$1 - 2\sin^2\theta$	$\cos 2\theta$
$2x^2 - 1$	$x = \cos\theta$	$2\cos^2\theta - 1$	$\cos 2\theta$
$1 - x^2$	$x = \sin\theta$ $x = \cos\theta$	$1 - \sin^2\theta$ $1 - \cos^2\theta$	$\cos^2\theta$ $\sin^2\theta$
$x^2 - 1$	$x = \sec\theta$ $x = \cos ec\,\theta$	$\sec^2\theta - 1$ $\mathrm{cosec}^2\theta - 1$	$\tan^2\theta$ $\cot^2\theta$
$1 + x^2$	$x = \tan\theta$ $x = \cot\theta$	$1 + \tan^2\theta$ $1 + \cot^2\theta$	$\sec^2\theta$ $\mathrm{cosec}^2\theta$

Multiple Choice Questions

CLASSWORK

(1) If $f(x) = \begin{cases} x - 5 & \text{for } x \le 1 \\ 4x^2 - 9 & \text{for } 1 < x < 2 \\ 3x + 4 & \text{for } x \ge 2 \end{cases}$

then the right hand derivative of (x) at $x = 2$ is

(a) 4 (b) 0 (c) 2 (d) 3

(2) It is given that f′(a) exists, then $\lim\limits_{x\to a} \dfrac{x f(a) - a f(x)}{x - a}$ is equal to

(a) f(a) + af′(a) (b) f(a) – af′(a)

(c) f′(a) (d) – f′(a)

(3) If $f(x)$ is differentiable at $x = a$, then

$\lim\limits_{x\to a} \dfrac{x^2 f(a) - a^2 f(x)}{x - a}$ is equal to

(a) $2a\,f(a) - a^2 f'(a)$ (b) $a^2 f(a) - 2af'(a)$

(c) $2a\,f(a) + a^2 f'(a)$ (d) none of these

(4) If $g(x) = (x^2 + 2x + 3) f(x)$ and $f(0) = 5$ and

$\lim\limits_{x\to a} \dfrac{f(x) - 5}{x} = 4$, then $g'(0) =$

(a) 12 (b) 20 (c) 14 (d) 18

(5) If f : R → is defined by

$f(x) = \begin{cases} \dfrac{x-2}{x^2 - 3x + 2} & , \text{if } x \in R - \{1, 2\} \\ 2 & , \text{if } x = 1 \\ 1 & , \text{if } x = 2 \end{cases}$

then $\lim\limits_{x\to 2} \dfrac{f(x) - f(2)}{x - 2} =$

(a) $-\dfrac{1}{2}$ (b) 0 (c) –1 (d) 1

(6) If f(4) = 4, f′(4) = 1, then $\lim\limits_{x\to 4} \dfrac{2 - \sqrt{f(x)}}{2 - \sqrt{x}}$ is equal to

(a) – 2 (b) – 1 (c) 1 (d) 2

(7) If f(x) is differentiable function f′′(0) = a, then

$\lim\limits_{x\to 0} \dfrac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is equal to

(a) 4a (b) 3a (c) 2a (d) 5a

(8) The left-hand derivative of $f(x) = [x] \sin \pi x$ at $x = k$, k being an integer, is

(a) $(-1)^{k-1} k\pi$ (b) $(-1)^k (k - 1)\pi$

(c) $(-1)^k k\pi$ (d) $(-1)^{k-1} k\pi$

(9) which of the following functions is differentiable at $x = 0$?

(a) $\sin(|x|) - |x|$ (b) $\cos(|x|) + |x|$

(c) $\cos(|x|) - |x|$ (d) $\sin(|x|) + |x|$

(10) Suppose f(x) is differentiable for all x and

$\lim\limits_{h\to 0} \dfrac{1}{h} f(1+h) = 5$, then f′(l) equals

(a) 3 (b) 6 (c) 5 (d) 4

(11) If $f(x) = \begin{cases} \sqrt{x^2 + a^2} & x \ne 0 \\ 0, & x = 0 \end{cases}$, then

(a) $f'(x)$ exists in (–2, 2)

(b) f(x) is discontinuous everywhere

(c) $f(x)$ is continuous everywhere

(d) $f'(x)$ exists in (–1, 1)

(12) $\lim\limits_{h\to 0} \dfrac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)}$ given that f′(2) = 6 and f′(1) = 4

(a) is equal 3 (b) does not exists

(c) is equal to $-\dfrac{3}{2}$ (d) is equal to $\dfrac{3}{2}$

(13) Let f : R → R be a function defined by f(x) = min{x + 1, |x| + 1}. Then, which of the following is true ?

(a) f(x) is not differentiable at x = 0

(b) f(x) = 1 for all x ≥ R

(c) f(x) is not differentiable at x = 1

(d) f(x) is differentiable everywhere

(14) $\frac{d}{dx} e^{x+3\log x} =$

(a) $e^x (x+3)$ (b) $e^x . x^2 (x+3)$

(c) $e^x . x (x + 3)$ (d) $e^x + \frac{3}{x}$

(15) If f is a differentiable function of x,

then $\lim_{h\to 0} \frac{[f(x+h)]^2 - [f(x)]^2}{2h} =$

(a) $\frac{1}{2}$ {[$f'(x)$]2 – [$f'(x)$]^2}

(b) $[f'(x)]^2$

(c) $f(x) . f'(x)$

(d) $\frac{1}{2} [f'(x)]^2$

(16) If f is a derivable function of x, then

$\lim_{h\to 0} \frac{f(x+h) - f(x-h)}{h} =$

(a) $f'(0)$ (b) $2f'(x)$

(c) $f'(x)$ (d) $(1/2)f'(x)$

(17) If f(9) and f′(9) = 4, then $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3} =$

(a) 1 (b) –4 (c) 4 (d) 2

(18) If f is an odd function such that f′(3) = – 2, then the value of f′(–3) is equal to

(a) –3 (b) 3 (c) 2 (d) –2

(19) If $y = \tan^{-1} (\sec x - \tan x)$ then $\frac{dy}{dx}$ = ?

(a) –1/2 (b) 1 (c) 2 (d) 1/2

(20) If $y = (\sin x)^{\log x}$ then $\frac{dy}{dx}$ is equal to

(a) $\sin x^{\log x} \left\{ \frac{\log \cos x}{x} - \sin x \cdot \log x \right\}$

(b) $\sin x^{\log x} \left\{ \frac{\log \sin x}{x} + \cot x \cdot \log x \right\}$

(c) $\sin x^{\log x} \left\{ \frac{\log \sin x}{x} - \cot x \cdot \log x \right\}$

(d) $\sin x^{\log x} \left\{ \frac{\log \cos x}{x} + \sin x \cdot \log x \right\}$

(21) If $f(x) = \cot^{-1} \left[\frac{x^x - x^{-x}}{2} \right]$, then $f'(l)$ is equal to

(a) –3 (b) 1 (c) –1 (d) 2

(22) If $y = \cot^{-1} \left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right]$, then = ?

(a) $-\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $\frac{1}{3}$

(23) If $y =$

$\sqrt{\log x + \sqrt{\log x + \sqrt{\log x + ...\infty}}}$, then ,then $\frac{dy}{dx} = ?$

(a) $\frac{1}{y(2x-1)}$ (b) $\frac{1}{2y(x-1)}$

(c) $\frac{1}{2x(y-1)}$ (d) $\frac{1}{x(2y-1)}$

(24) If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$

then $x^3 y \frac{dy}{dx}$ = ?

(a) 2 (b) 0 (c) 1 (d) 2

(25) If $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ then $\frac{dy}{dx}$ is equal

(a) $\pi/2$ (b) 0 (c) 1 (d) π

(26) If $y = \sin^{-1} x$, then $(1-x^2)\cdot\frac{dy^2}{dx^2} - x\cdot\frac{dy}{dx}$ is equal to

(a) –1 (b) 0 (c) 1 (d) None

(27) If $y = \sin x^{\sin x^{\sin x \ldots \text{to} \infty}}$, then $\frac{dy}{dx}$ is equal to -

(a) $\frac{y^2 \cdot \cot x}{1 - y \cdot \log(\sin x)}$ (b) $\frac{y^2 \cot x}{1 + y \cdot \log(\sin x)}$

(c) $\frac{y^2 \cdot \tan x}{1 - y \cdot \log(\sin x)}$ (d) None of these

(28) If $y = \log(e^{mx} - e^{-mx})$, then $\frac{dy}{dx} =$

(a) $m\left[\frac{e^{mx}+e^{-mx}}{e^{mx}-e^{-mx}}\right]$ (b) $m\left[\frac{e^{-mx}-e^{mx}}{e^{mx}+e^{-mx}}\right]$

(c) $m\left[\frac{e^{mx}-e^{-mx}}{e^{-mx}+e^{-mx}}\right]$ (d) $m\left[\frac{e^{mx}-m.e^{x}}{me^{x}+e^{-mx}}\right]$

(29) If $x^y = 2^{x-y}$, then $\frac{dy}{dx} =$

(a) $\frac{x\log 2 - y}{x(\log\cdot x + \log 2)}$ (b) $\frac{x\log 2 - x}{y\log 2y}$

(c) $\frac{x\log 2 - y}{2x\cdot\log x}$ (d) $\frac{x\log 2 + x}{y\log 2y}$

(30) If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, then $\frac{dy}{dx} =$

(a) $\frac{(ax + hy - 9)}{(hx + by + f)}$ (b) $-\frac{(ax + hy + 9)}{(hx + by + f)}$

(c) $-\frac{(ax - hy - 9)}{(hx - by + f)}$ (d) $\frac{(ax - hy - \delta)}{(hx + by + f)}$

(31) If $e^x = x^y$, then $\frac{dy}{dx} =$

(a) $\frac{x-y}{y\log y}$ (b) $\frac{x+y}{x\log x}$

(c) $\frac{x+y}{y\log y}$ (d) $\frac{x-y}{x\log x}$

(32) If $y^y = x.\sin y$, then $\frac{dy}{dx} =$

(a) $\frac{1}{x(1-\log y+\sin y)}$ (b) $\frac{1}{x(1+\log y-\cot y)}$

(c) $\frac{1}{x(1+\log y+\sin y)}$ (d) $\frac{1}{x(1-\log y+\cos y)}$

(33) If $y = \sec^{-1}\left(\frac{1+e^{2\sqrt{x}}}{2e^{\sqrt{x}}}\right)$ then $\frac{dy}{dx} =$

(a) $\frac{e^x}{x(1-e^x)}$ (b) $\frac{e^{\sqrt{x}}}{\sqrt{x}(1+e^{\sqrt{x}})}$

(c) $\frac{e^{\sqrt{x}}}{\sqrt{x}(1-e^{\sqrt{x}})}$ (d) $\frac{e^{\sqrt{x}}}{\sqrt{x}(1+e^{\sqrt{x}})}$

(34) If $y = \sin^{-1}\frac{2x}{1+x^2} + \sec^{-1}\frac{1+x^2}{1-x^2}$ then $\frac{dy}{dx} =$

(a) $\frac{-4}{1+x^2}$ (b) $\frac{1}{x}$

(c) $\frac{1}{1+x^2}$ (d) $\frac{4}{1+x^2}$

(35) If $y = \sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$, then $\frac{dy}{dx} =$

(a) $\frac{2x}{\sqrt{1-x^2}}$ (b) $\frac{-2x}{\sqrt{1-x^2}}$

(c) $\dfrac{1}{3\sqrt{1-x^2}}$ (d) $\dfrac{-x}{\sqrt{1-x^2}}$

(36) If $y^x + x^y = a^b$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{yx^{y-1}+y^x}{xy^{x-1}+x^y}$ (b) $-\dfrac{yx^{y-1}+y^x\log y}{xy^{x-1}+x^y\log x}$

(c) $\dfrac{yx^{y-1}+y^x\log y}{xy^{x-1}+x^y\log x}$ (d) $-\dfrac{yx^{y-1}+y^x}{xy^{x-1}+x^y}$

(37) If $y = \log_{10}x + \log_x 10 + \log_x x + \log_{10}10$ then $\dfrac{dy}{dx} =$

(a) $\dfrac{1}{x\log_e 10} - \dfrac{1}{x(\log_e x)^2}$

(b) $\dfrac{1}{x\log_e 10} - \dfrac{\log_e 10}{x(\log_e x)^2}$

(c) $\dfrac{1}{x\log_e 10} - \dfrac{1}{x\log_{10} e}$

(d) None of these

(38) If $y = \dfrac{x}{2}\sqrt{a^2+x^2} + \dfrac{a^2}{2}\log(x+\sqrt{x^2+a^2})$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{1}{\sqrt{x^2+a^2}}$ (b) $\sqrt{x^2+a^2}$

(c) $\dfrac{1}{\sqrt{x^2+a^2}}$ (d) $2\sqrt{x^2+a^2}$

(39) If $x^p y^q = (x+y)^{p+q}$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{x}{y}$ (b) $\dfrac{y}{x}$ (c) $-\dfrac{x}{y}$ (d) $-\dfrac{y}{x}$

(40) If $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{y^2-1}{1-x^2}$ (b) $\sqrt{\dfrac{x^2-1}{1-y^2}}$

(c) $\sqrt{\dfrac{1-y^2}{1-x^2}}$ (d) $\sqrt{\dfrac{1-x^2}{1-y^2}}$

(41) If $\sin y = x\sin(a+y)$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{\sin^2(a+y)}{\cos a}$ (b) $\dfrac{\sin^2(a+y)}{\sin(a+2y)}$

(c) $\dfrac{\sin^2(a+y)}{\cos(a+2y)}$ (d) $\dfrac{\sin^2(a+y)}{\sin a}$

(42) $\dfrac{d}{dx}e^{x+3\log x}$

(a) $-\cos ec^2x$

(b) $e^x \cdot x^2(x+3)$

(c) $e^x + \dfrac{3}{x}$

(d) None of these

(43) $\dfrac{d}{dx}[\log_e x)(\log_a x)] =$

(a) $\dfrac{2\log_a x}{x}$ (b) $\dfrac{\log_e x}{x}$

(c) $\dfrac{\log_a x}{x}$ (d) $\dfrac{2\log x}{x}$

(44) If $\cos x = \dfrac{1}{\sqrt{1+t^2}}$ and $\sin y = \dfrac{1}{\sqrt{1+t^2}}$, then $\dfrac{dy}{dx} =$

(a) 1 (b) $\dfrac{1-t}{\sqrt{1+t^2}}$

(c) $\dfrac{1}{\sqrt{1+t^2}}$ (d) -1

(45) If $x^y \cdot y^x = 1$, then $\frac{dy}{dx}$ =

(a) $-\frac{y(y+x\log y)}{x(y+x\log x)}$ (b) $\frac{y(y+x\log y)}{x(y\log x+y)}$

(c) $\frac{y(y+x\log y)}{x(x+y\log x)}$ (d) None of these

(46) The derivative of the function $\cot^{-1}[(\cos 2x)^{1/2}]$ at $x = \frac{\pi}{6}$ is

(a) $6^{1/2}$ (b) $(2/3)^{1/2}$

(c) $(1/3)^{1/2}$ (d) $3^{1/2}$

(47) If $y = e^{(1+\log_e x)}$,then the value of $\frac{dy}{dx}$ =

(a) $\log_e x e^{\log_e ex}$ (b) e

(c) 1 (d) 0

(48) If $y = \cos^{-1}\left(\frac{2\cos x - 3\sin x}{\sqrt{3}}\right)$,then $\frac{dy}{dx}$ =

(a) constant $\neq$ 1 (b) 0

(c) constant = 1 (d) None of these

(49) If the function $y = \sin(\log x) + \cos(\log x)$ satisfies the equation, then

(a) $x^2y'' + xy + 3y = 0$

(b) $x^2y'' + xy + y = 0$

(c) $x^2y'' + xy + 2y = 0$

(d) None of these

(50) If y = $\sin^{-1}\left(\frac{\log(x^2)}{1+(\log x)^2}\right)$, $\frac{dy}{dx}$ =

(a) $\frac{2}{f(1-\log x^2)}$ (b) $\frac{1}{x}$ $[1 - (\log x)^2]$

(c) $\frac{1}{x}$ $[1 + (\log x)^2]$ (d) $\frac{2}{x}$ $[1 + (\log x)^2]$

(51) The derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x = \pi/4$, Where $f(1) = 2$ and $g'\left(\sqrt{2}\right) = 4$ is

(a) $-\sqrt{2}$ (b) 2 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

(52) If $y = \log\left(e^{3x}\left(\frac{x-4}{x+3}\right)^{3/4}\right)\frac{dy}{dx}$ =

(a) $\frac{3}{4}\left[3+\frac{1}{x-4}+\frac{1}{x-3}\right]$

(b) $\frac{3}{4}\left[4+\frac{1}{x-4}-\frac{1}{x+3}\right]$

(c) $\frac{3}{4}\left[4+\frac{1}{x-4}+\frac{1}{x-3}\right]$

(d) $\frac{3}{4}\left[3+\frac{1}{x-4}-\frac{1}{x-3}\right]$

(53) If $y = x^x + x^a + a^x + a^a$ then $\frac{dy}{dx}$ =

(a) $x^x(1 - \log x) - ax^{a+1} + a^x . \log a$

(b) $x^x(1 + \log x) + ax^{a-1} + a^x . \log a$

(c) $x^x(1 - \log x) + ax^{a+1} + a^x . \log a$

(d) $x^x(1 + \log x) - ax^{a-1} + a^x . \log a$

(54) If $y = \tan^{-1}\left(\frac{a\sin x + b\cos x}{a\cos x - b\sin x}\right)$ then $\frac{dy}{dx}$ =

(a) 4 (b) 1 (c) 2 (d) 3

(55) If $y = \left(x+\sqrt{1+x^2}\right)^n$ then $(1 + x^2) . \frac{d^2y}{dx^2} + x.\frac{dy}{dx}$ is equal to -

(a) $-n^2y$ (b) $-y$ (c) ny (d) n^2y

(56) If $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3(x^3 - y^3)$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{y^2}{x^2}\sqrt{\frac{1-x^6}{1-y^6}}$ (b) $\frac{x^2}{y^2}\sqrt{\frac{1-x^6}{1-y^6}}$

(c) $\frac{y^2}{x^2}\sqrt{\frac{1-y^6}{1-x^6}}$ (d) $\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

(57) If $\sin^{-1}\left[\sqrt{x-ax} - \sqrt{a-ax}\right]$, then $\frac{dy}{dx} =$

(a) $\frac{1}{2\sqrt{x}.\sqrt{1-x}}$ (b) $\sin\sqrt{x}$

(c) $\sin\sqrt{a}$ (d) $\frac{1}{2\sqrt{x}}$

(58) If $y = \frac{a - b\cdot\cos x,}{a + b\cdot\cos x}$, than $(a + b\cdot\cos x)^2 y_1 =$

(a) $-2ab\sin x$ (b) $ab\sin x$

(c) $2ab\sin x$ (d) $-ab\sin x$

(59) Derivative of Log [x + $\sqrt{x^2 + a^2}$] w.r.t.x

(a) $\frac{-1}{\sqrt{x^2+a^2}}$ (b) $\frac{1}{\sqrt{x^2+a^2}}$

(c) $\frac{2x}{\sqrt{x^2+a^2}}$ (d) $\frac{1}{\sqrt{x^2+a^2}}$

(60) Derivative of $\tan 2x \tan 6x \tan 8x$ w.r.t.x is

(a) $\sec^2 8x - \sec^2 6x - \sec^2 2x$

(b) $8\sec^2 8x + 6\sec^2 6x + 2\sec^2 2x$

(c) $\sec^2 8x + \sec^2 6x + \sec^2 2x$

(d) $8\sec^2 8x - 6\sec^2 6x - 2\sec^2 2x$

(61) If $A + B = \frac{\pi}{4}$, then

$\frac{d}{dx}\{[1 + \tan(A - x)][1 + \tan(B + x)]\} =$

(a) 0 (b) –1

(c) 1 (d) tan A tan B

(62) If $u = \sec x + \tan x - 1$, $v = \sec x - \tan x + 1$, then $u'v + v'u =$

(a) $-2\sec^2 x$ (b) $\sec^2 x$

(c) $-\sec^2 x$ (d) $2\sec^2 x$

(63) $\frac{d}{dx}\left[\cot^{-1}\left(\frac{1+\sqrt{1-x^2}}{x}\right)\right] =$

(a) $\frac{-2}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{-1}{\sqrt{1-x^2}}$ (d) $\frac{1}{2\sqrt{1-x^2}}$

(64) $\frac{d}{dx}\left[\cot^{-1}\left(\frac{1-x}{1+x}\right)\right] =$

(a) $\frac{2}{1+x^2}$ (b) $\frac{-1}{1+x^2}$

(c) $\frac{1}{1-x^2}$ (d) $\frac{1}{1+x^2}$

(65) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{x-\sqrt{a^2-x^2}}{x+\sqrt{a^2-x^2}}\right)\right] =$

(a) $\frac{x}{\sqrt{a^2-x^2}}$ (b) $\frac{-1}{\sqrt{a^2-x^2}}$

(c) $\frac{1}{\sqrt{a^2-x^2}}$ (d) $\sqrt{a^2-x^2}$

(66) $\dfrac{d}{dx}\left[\sin\left(2\tan^{-1}\sqrt{\dfrac{1+x}{1-x}}\right)\right]=$

(a) $\dfrac{-x}{\sqrt{x^2-1}}$ (b) $\dfrac{x}{\sqrt{1-x^2}}$

(c) $\dfrac{x}{\sqrt{x^2-1}}$ (d) $\dfrac{-x}{\sqrt{1-x^2}}$

(67) $\dfrac{d}{dx}\left[\sin^{-1}\left(\dfrac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)\right]=$

(a) $\dfrac{-1}{2\sqrt{1-x^2}}$ (b) $\dfrac{1}{\sqrt{1-x^2}}$

(c) $\dfrac{1}{2\sqrt{x^2-1}}$ (d) $\dfrac{1}{2\sqrt{1-x^2}}$

(68) $\dfrac{d}{dx}\left[\tan^{-1}\left(\dfrac{12x-64x^3}{1-48x^2}\right)\right]=$

(a) $\dfrac{-12}{1+16x^2}$ (b) $\dfrac{1}{1+16x^2}$

(c) $\dfrac{4}{1+16x^2}$ (d) $\dfrac{12}{1+16x^2}$

(69) $\dfrac{d}{dx}\left[\sin^{-1}\left(x-\dfrac{4x^3}{27}\right)\right]=$

(a) $\dfrac{-2}{\sqrt{4-x^2}}$ (b) $\dfrac{-3}{\sqrt{9-x^2}}$

(c) $\dfrac{3}{\sqrt{9-x^2}}$ (d) $\dfrac{2}{\sqrt{4-x^2}}$

(70) If $f(x) = \dfrac{x^2-a^2}{x^2+a^2}$ and $f'(1) = 1$, then $a =$

(a) 2 (b) –1 (c) 1 (d) ±1

(71) If $y = \tan^{-1}\left[\dfrac{1-2\log x}{1+2\log x}\right] + \tan^{-1}\left[\dfrac{3+2\log x}{1-6\log x}\right]$, then

$\left(\dfrac{dy}{dx}\right)_{x=2} + \left(\dfrac{dy}{dx}\right)_{x=3} =$

(a) –2 (b) –6 (c) 0 (d) 2

(72) $\dfrac{d}{dx}\left[\sin^{-1}\left(\dfrac{3x-x^3}{2}\right)\right]=$

(a) $\dfrac{3}{\sqrt{4-x^2}}$ (b) $\dfrac{-3}{\sqrt{1-x^2}}$

(c) $\dfrac{-3}{\sqrt{4-x^2}}$ (d) $\dfrac{3}{\sqrt{x^2-4}}$

(73) If $ax^2 + 2hxy + by^2 = 0$, then $\dfrac{dy}{dx} =$

(a) $-\dfrac{y}{x}$ (b) $\dfrac{x}{y}$ (c) $\dfrac{y}{x}$ (d) $-\dfrac{x}{y}$

(74) If $x^2 y^2 = (x + y)^{n+2}$ and $\dfrac{dy}{dx} = \dfrac{y}{x}$, then n =

(a) 4 (b) 2 (c) 3 (d) 6

(75) If $\dfrac{d}{dx}\left(\tan x + \dfrac{2}{3}\tan^3 x + \dfrac{1}{5}\tan^5 x\right) = \sec^n x$, then n =

(a) 5 (b) 7 (c) 6 (d) 4

(76) $\dfrac{d}{dx}\left[\lim\limits_{h\to 0}\dfrac{(x+h)^4-x^4}{h}\right] =$

(a) $24x$ (b) $12x^2$ (c) $36x^2$ (d) $72x$

(77) $\dfrac{d}{dx}\left[\tan^{-1}\left(\dfrac{\log(ex)}{\log(e/x)}\right)\right] = \dfrac{1}{x\,f(x)}$, then $f(x)$

(a) $1 + 2\log x$ (b) $\sqrt{1-(\log x)^2}$

(c) $1 + (\log x)^2$ (d) 1

(78) If $y = 4^{\log_2 (\sin x)} + 9^{\log_3 (\cos x)}$, then $(\log 2)(\log 3)\, y_1 =$

(a) 1 (b) 0 (c) $\sin 2x$ (d) $\log 5$

(79) Given $f(1) = -1$ and $f'(1) = 2$ If $h(x) = [f(x)]^2$, then : $h'(1) =$

(a) 8 (b) $\frac{5}{9}$ (c) $\frac{10}{9}$ (d) -4

(80) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{1}{2x}-\frac{x}{2}\right)\right] =$

(a) 1 (b) 0 (c) $\frac{-2}{1+x^2}$ (d) $\frac{2}{x^2-1}$

(81) If $x = t^2 + \frac{1}{t^2}$, $y = t - \frac{1}{t}$, then $\frac{dy}{dx} =$

(a) $\frac{1}{2y}$ (b) $\frac{1}{1+t^2}$

(c) $\frac{1}{t^2-1}$ (d) $\frac{1}{1-t^2}$

(82) If $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$, then $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$

(a) a^2t^2 (b) $-t^2$ (c) t^2 (d) $-a^2t^2$

(83) If $x = \cos^{-1} t, y = \log(1-t^2)$, then $\left(\frac{dy}{dt}\right)$ at $t = \frac{1}{2}$ is

(a) $-\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) $-\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

(84) If y = cos (3 $\cos^{-1} x$), then $\frac{d^3y}{dx^3} =$

(a) $24x$ (b) 0 (c) 12 (d) 24

(85) If $y = f(x)$ is an invertible and a *twice differentiable* function of x, then $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 \frac{d^2x}{dy^2} =$

(a) $\frac{d^3y}{dx^3}$ (b) $\frac{d^4y}{dx^4}+\left(\frac{dy}{dx}\right)^3$

(c) $1+\left(\frac{dy}{dx}\right)^3\cdot\frac{d^4x}{dy^4}$ (d) 0

(86) Derivative of $(f'0g)(x)$ w.r.t.g (x) is

(a) $(f0g')(x)$ (b) $(f'0g)(x)$

(c) $(f'0g)(x)\ 0\ g'(x)$ (d) $(f'0g)(x) : g'(x)$

(87) If $x = f(t)$, $y = g(t)$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{f''(t)}{g''(t)}$

(b) $\frac{g''(t)}{f''(t)}$

(c) $\frac{g''(t)f'(t)-g'(t)f''(t)}{[f'(t)]^3}$

(d) $\frac{g''(t)f'(t)-g'(t)f''(t)}{[f'(t)]^2}$

(88) If $y = a\cos mx + b\sin mx$, then $y_2 =$

(a) my (b) $-m^2y$ (c) m^2y (d) $-my$

(89) If $y = \left(x+\sqrt{1+x^2}\right)^m$ then $(1 + x^2)\, y2 + xy_1 =$

(a) $-m$ (b) m^2y (c) $-m^2y$ (d) 0

(90) If $y = (\tan^{-1}x)^2$, then $(1 + x^2)^2 y_2 + 2x(1 + x^2)y_1 =$

(a) -1 (b) 0 (c) 1 (d) 2

(91) If $p(x)$ is a polynomial of degree 2 and $p(3) = 0$, $p'(0) = 1$, $p''(2) = 2$, then $p(x) =$

(a) $x^2 + x - 12$ (b) $x^2 + x + 12$

(c) $x^2 - x + 12$ (d) $-x^2 + x + 12$

(92) If $exp(m) = e^m$ and $y = exp\left[\sqrt{\frac{x-1}{x+1}}\right]$ then $(x^2 - 1)\, y_1 =$

(a) $y\log y$ (b) y

(c) $\log y$ (d) $-y\log y$

(93) If f and g are derivable functions of x such that $g'(a) \neq 0$, $g(a) = b$ and $f(g(x)) = x$ then $f'(b) =$

(a) $\dfrac{1}{f(a)}$ (b) $\dfrac{1}{g(a)}$ (c) $\dfrac{1}{g'(a)}$ (d) $\dfrac{1}{g'(b)}$

(94) If $y = \dfrac{ax^2}{(x-a)(x-b)(x-c)} + \dfrac{bx}{(x-b)(x-c)} + \dfrac{c}{x-c} + 1$,

then : $\dfrac{1}{y}\dfrac{dy}{dx} =$

(a) $\log(x - a) + \log(x - b) + \log(x - c)$

(b) $\dfrac{1}{x}\left(\dfrac{a}{a-x} + \dfrac{b}{b-x} + \dfrac{c}{c-x}\right)$

(c) $\dfrac{a}{a-x} + \dfrac{b}{b-x} + \dfrac{c}{c-x}$

(d) $\dfrac{1}{x}\left(\dfrac{a}{x-a} + \dfrac{b}{x-b} + \dfrac{c}{x-c}\right)$

(95) If $\dfrac{d}{dx}\left(\dfrac{1+x^4+x^8}{1+x^2+x^4}\right) = ax3 + bx$, then $(a, b) \equiv$

(a) $(-2, -4)$ (b) $(4, 2)$

(c) $(4, -2)$ (d) $(-2, 4)$

(96) If $x = a\sin\theta$, $y = b\cos\theta$, than $\dfrac{d^2y}{dx^2} =$

(a) $\dfrac{b}{a^2}\sec^3\theta$ (b) $\dfrac{a}{b^2}\sec^2\theta$

(c) $\dfrac{-b}{a^2}\sec^2\theta$ (d) $\dfrac{-b}{a^2}\sec^3\theta$

(97) f and g are differentiable functions of x such that $f(g(x)) = x$. If $g'(a) \neq 0$ and $g(a) = b$, then $f'(b) =$

(a) $\dfrac{1}{g'(a)}$ (b) $g(a)$

(c) $\dfrac{1}{g(a)}$ (d) $-g'(a)$

(98) If r is the radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, then ; $(y + f)^3 \dfrac{d^2y}{dx^2} =$

(a) r^3 (b) $-r^2$ (c) r^2 (d) $g^2 + f^2 - r$

(99) Derivative of $\sin(3\sin^{-1}x) + \cos(3\cos^{-1}x)$ w.r.t. x is equal to

(a) 1 (b) $\dfrac{3}{\sqrt{1-x^2}}$

(c) $\dfrac{-3}{\sqrt{1-x^2}}$ (d) itself

(100) If $f(x) = \log_2[\log_3(\log_5 x)]$, then $f'(125) =$

(a) $\dfrac{1}{250\log 2\log 3\log 5}$

(b) $\dfrac{1}{125\log 2\log 3\log 5}$

(c) $\dfrac{1}{375\log 2\log 3\log 5}$

(d) $\dfrac{1}{500\log 2\log 3\log 5}$

(101) Derivative of $\cos^{-1}\left(\dfrac{1}{\sqrt{t^2+1}}\right)$ w.r.t. $\sin^{-1}\left(\dfrac{1}{\sqrt{t^2+1}}\right)$ is

(a) $\dfrac{2t^2}{(t^2+1)^{3/2}}$ (b) 0

(c) 1 (d) $\dfrac{t}{\sqrt{t^2+1}}$

(102) If $x = t + \dfrac{1}{t}$ and $y = t - \dfrac{1}{t}$, where $t \neq 0$, then : $\dfrac{d^2y}{dx^2} =$

(a) $-4t^2(t^2 - 1)^{-2}$ (b) $-4t(t^2 - 1)^{-2}$

(c) $-4t^3(t^2 - 1)^{-3}$ (d) $(t^2 + 1)(t^2 - 1)^{-2}$

(103) If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$ and $g'(a) = 2$, then :

$\lim_{x\to a} \dfrac{g(x)f(a) - g(a)f(x)}{x-a}$

(a) 5 (b) –5 (c) 0 (d) $\dfrac{1}{5}$

(104) If $y = \dfrac{1}{1+x^{b-a}+x^{c-a}} + \dfrac{1}{1+x^{a-b}+x^{c-b}} + \dfrac{1}{1+x^{a-c}+x^{b-c}}$

then : $\dfrac{dy}{dx} =$

(a) –1 (b) 0

(c) 1 (d) $(a+b+c)\,x^{a+b+c+-1}$

(105) If $y = \sum_{r=1}^{x} \tan^{-1}\left(\dfrac{1}{1+r+r^2}\right)$, then : $\dfrac{dy}{dx} =$

(a) 1 (b) $\dfrac{1}{1+x^2}$

(c) $\dfrac{1}{1+(1+x)^2}$ (d) 0

(106) If $y = \tan\left[\dfrac{1}{2}\cos^{-1}\left(\dfrac{1-u^2}{1+u^2}\right) + \dfrac{1}{2}\sin^{-1}\left(\dfrac{2u}{1+u^2}\right)\right]$

and $x = \dfrac{2u}{1-u^2}$, then : $\dfrac{dy}{dx} =$

(a) 2 (b) –1 (c) 0 (d) 1

(107) If $f'(1) = 2$ and $g'\left(\sqrt{2}\right) = 4$, then the derivative of $f(\tan x)$ w.r.t. $g(\sec x)$, at $x = \dfrac{\pi}{4}$, is

(a) –1 (b) $\dfrac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 1

(108) If $\sqrt{y-\sqrt{y-\sqrt{y-...\text{to}\infty}}} = \sqrt{x+\sqrt{x+\sqrt{x+...\text{to}\infty}}}$

then : $\dfrac{dy}{dx} =$

(a) 1 (b) $\dfrac{x+y+1}{x-y+1}$

(c) $\dfrac{y-x+1}{y-x-1}$ (d) $\dfrac{y-x-1}{y-x+1}$

(109) If $x = \dfrac{e^t + e^{-t}}{2}$, $y = \dfrac{e^t - e^{-t}}{2}$, then : $\dfrac{dy}{dx} =$

(a) $-\dfrac{y}{x}$ (b) $\dfrac{x}{y}$ (c) $\dfrac{y}{x}$ (d) $-\dfrac{x}{y}$

(110) $x = \theta - \dfrac{1}{\theta}$; $y = \theta + \dfrac{1}{\theta}$ them : $\dfrac{d^2y}{dx^2} =$

(a) $\dfrac{4}{y^3}$ (b) $\dfrac{y}{x}$

(c) $\dfrac{x^2-y^2}{y^3}$ (d) $\dfrac{y^2-x^2}{x^3}$

(111) If $y = x + \dfrac{1}{x} + x^2 + \dfrac{1}{x^2}$ and $\dfrac{d^2y}{dx^2} = 1 + c,\ \dfrac{2}{x^3} + \dfrac{6}{x^4} = 0$ then : $c =$

(a) 2 (b) 0 (c) 1 (d) –1

(112) Given $f(x)$ is a polynomial of degree two such that $f(1) = f(-1)$.

If a_1, a_2, a_3 are in A.P., then

$f'(a_1)$, $f'(a_2)$, $f'(a_3)$ are in

(a) H.P. (b) A.P

(c) G.P. (d) none of these

(113) If : $3f(x) - 2f\left(\dfrac{1}{x}\right) = x$, then : $f'(2)$

(a) 7/2 (b) 2/7 (c) 1/2 (d) 2

(114) If : $f(1) = 3$ and $f'(1) = 2$, then : $\dfrac{d}{dx}\{\log[f(e^x + 2x]\}$ at $x = 0$ is

(a) 0 (b) 2/3 (c) 3/2 (d) 2

(115) $\dfrac{d^2x}{dy^2} =$

(a) $-\dfrac{(d^2y/dx^2)}{(dy/dx)^2}$ (b) $\dfrac{1}{(dy/dx)^2}$

(c) $\dfrac{(d^2y/dx^2)}{(dy/dx)^2}$ (d) $\dfrac{1}{(d^2y/dx^2)}$

(116) Let $f(x)$ and $g(x)$ be two functions having finite non-zero third order derivatives.

If : $f(x) \cdot g(x) = 1$, for all $x \in$ R,

then : $\dfrac{f'''}{f'} - \dfrac{g'''}{g'} =$

(a) $3\left(\dfrac{f''}{f} - \dfrac{g''}{g}\right)$ (b) $\dfrac{f''}{f} - \dfrac{g''}{g}$

(c) $2\left(\dfrac{f''}{f} - \dfrac{g''}{g}\right)$ (d) None of these

(117) If $y = \sin x$

$\left[\dfrac{1}{\cos x \cdot \cos 2x} + \dfrac{1}{\cos 2x \cdot \cos 3x} + \dfrac{1}{\cos 3x \cdot \cos 4x}\right]$,then

: $\dfrac{dy}{dx}$ at $x =$ is

(a) 4 (b) 0 (c) 1 (d) 2

(118) If : $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

where p is a constant, then : $\dfrac{d^3}{dx^3}$ $[f(x)]$ at $x = 0$ is

(a) independent of p (b) p

(c) $p + p^2$ (d) $p + p^3$

(119) If : $f\left(\dfrac{x^2+1}{x}\right) = \dfrac{x^4+1}{x^2}$, where $x \neq 0$, then : $f'(3) =$

(a) 9 (b) 6 (c) 0 (d) 3

HOME WORK

Multiple Choice Questions

11.1 Rules of differentiation and some standard differentiation

(1) $\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 =$

(a) $1-\frac{1}{2x}$ (b) $1-\frac{1}{x^2}$

(c) $1+\frac{1}{x^2}$ (d) None of these

(2) Derivation of $x^6 + 6x$ with respect to x is

(a) $6x^5 + x6^{x-1}$ (b) $12x$

(c) $x + 4$ (d) $6x^5 + 6^x \log 6$

(3) The derivative of $\tan x - x$ with respect to x is

(a) $\tan^2 x$ (b) $1 - \tan^2 x$

(c) $\tan x$ (d) $-\tan^2 x$

(4) If $y = e^{(1+\log_e x)}$, then the value of $\frac{dy}{dx} =$

(a) $\log_e x \; e^{1+\log_e ex}$ (b) e

(c) 1 (d) 0

(5) If $y = \frac{(1-x)^2}{x^2}$, then $\frac{dy}{dx}$ is

(a) $-\frac{2}{x^3}+\frac{2}{x^2}$ (b) $\frac{2}{x^2}+\frac{2}{x^3}$

(c) $-\frac{2}{x^2}+\frac{2}{x^3}$ (d) $-\frac{2}{x^2}-\frac{2}{x^3}$

(6) $\frac{d}{dx}\left[\log_{\sqrt{x}}\left(\frac{1}{x}\right)\right]$ is equtal to

(a) 0 (b) $-\frac{1}{2\sqrt{x}}$

(c) -2 (d) $-\frac{1}{x^2\sqrt{x}}$

(7) If pv = 81, then $\frac{dp}{dv}$ at v = 9 is equal to

(a) 2 (b) 1

(c) –1 (d) None of these

(8) The values of x, at which the first derviative of the function $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$ w.r.t.x is $\frac{3}{4}$, are

(a) $\pm\frac{2}{\sqrt{3}}$ (b) ± 2 (c) $\pm\frac{1}{2}$ (d) $\pm\frac{\sqrt{3}}{2}$

(9) If $y = e^x \log x$, then $\frac{dy}{dx}$ is

(a) $\frac{e^x}{\log x}$ (b) $\frac{e^x}{x}$

(c) $e^x\left(\frac{1}{x}+x\log x\right)$ (d) $e^x\left(\frac{1}{x}+\log x\right)$

(10) If $y = (1 + x^2)\tan^{-1}x - x$, then $\frac{dy}{dx} =$

(a) $\frac{2x}{\tan^{-1}x}$ (b) $\tan^{-1}x$

(c) $2x\tan^{-1}x$ (d) $2x\tan^{-1}x - 1$

(11) If $y = x \sin x$, then

(a) $\frac{1}{y}.\frac{dy}{dx}=\frac{1}{x}-\cot x$ (b) $\frac{1}{y}.\frac{dy}{dx}=\frac{1}{x}+\cot x$

(c) $\frac{dy}{dx}=\frac{1}{x}+\cot x$ (d) None of these

(12) $\frac{d}{dx}\left(e^{x+3\log x}\right) =$

(a) $e^x(x + 3)$ (b) $e^x.x^2(x + 3)$

(c) $e^x.x(x + 3)$ (d) $e^x + \frac{3}{x}$

(13) If $y = x^2 \log x + \frac{2}{\sqrt{x}}$, then $\frac{dy}{dx}$ =

(a) $x + 2x \log x - \frac{2}{x^{\frac{3}{2}}}$ (b) $x + 2x \log x - \frac{1}{\sqrt{x}}$

(c) $x + 2x \log x - \frac{1}{x^{\frac{3}{2}}}$ (d) none of these

(14) If $y = x\left[\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)+\sin x\right] + \frac{1}{2\sqrt{x}}$, then $\frac{dy}{dx}$ =

(a) $(1 + x) \cos x + (1 + x) \sin x - \frac{1}{4x\sqrt{x}}$

(b) $(1 + x) \cos x + (1 - x) \sin x - \frac{1}{4x\sqrt{x}}$

(c) $(1 - x) \cos x + (1 + x) \sin x + \frac{1}{4x\sqrt{x}}$

(d) None of these

(15) If $f(x) = e^x g(x)$, $g(0) = 2$, $g'(0) = 1$, then $f'(0)$ is

(a) 0 (b) 1 (c) 3 (d) 2

(16) If $f(x) = x \tan^{-1}x$, then $f'(1)$ =

(a) 2 (b) $1 + \frac{\pi}{4}$

(c) $\frac{1}{2}+\frac{\pi}{4}$ (d) $\frac{1}{2}-\frac{\pi}{4}$

(17) If $y = 1 + x + \frac{x^2}{2!}+\frac{x^3}{3!}+......\infty$, then $\frac{dy}{dx}$ =

(a) $y + 1$ (b) y

(c) $y - 1$ (d) None of these

(18) If $y = \frac{1}{a-z}$, then $\frac{dz}{dy}$ =

(a) $-(z + a)^2$ (b) $(a - z)^2$

(c) $-(z - a)^2$ (d) $(z + a)^2$

(19) If $y = x + \frac{1}{x}$, then

(a) $x^2\frac{dy}{dx} - xy + 2 = 0$ (b) $x^2\frac{dy}{dx} + xy = 0$

(c) $x^2\frac{dy}{dx} + xy + 2 = 0$ (d) None of these

(20) If $y = a \sin x + b \cos x$, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is a

(a) constant (b) function of x

(c) function of y (d) function of x and y

(21) $\frac{d}{dx}\left(\cos^{-1}\sqrt{\frac{1+\cos x}{2}}\right) =$

(a) $\frac{1}{3}$ (b) 1

(c) $\frac{1}{2}$ (d) None of these

(22) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right] =$

(a) $\frac{-1}{\sqrt{1-\left(\frac{a-x}{1+a}\right)^2}}$ (b) $-\frac{1}{1+x^2}$

(c) $\frac{1}{1+a^2}-\frac{1}{1+x^2}$ (d) $\frac{1}{1+\left(\frac{a-x}{1+ax}\right)^2}$

(23) $\frac{d}{dx}\left[\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)\right] =$

(a) 1 (b) $-\frac{1}{2}$ (c) 0 (d) $\frac{1}{2}$

(24) $\frac{d}{dx}\left(\tan^{-1}\sqrt{\frac{1+\cos\frac{x}{2}}{1-\cos\frac{x}{2}}}\right)$ is equal to

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(25) If $y = \tan^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$, then $\frac{dy}{dx}$ is equal to

(a) 1 (b) 0 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

(26) $\frac{d}{dx}\left(\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}\right) =$

(a) $\sec^2\left(\frac{\pi}{4}-x\right)$ (b) $\sec^2 x$

(c) $-\sec^2\left(\frac{\pi}{4}-x\right)$ (d) $\sec^2\left(\frac{\pi}{4}+x\right)$

(27) If $y = \tan^{-1}\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)$, then $\frac{dy}{dx}$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\pi}{4}$ (d) 0

(28) If $y = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, then $\frac{dy}{dx} =$

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 3

(29) If $f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$, then $f'\left(\frac{\pi}{3}\right) =$

(a) $\frac{1}{4}$ (b) $\frac{1}{2(1+\cos x)}$

(c) $\frac{1}{2}$ (d) None of these

(30) $\frac{d}{dx}\left(\sqrt{\frac{1+\cos 2x}{1-\cos 2x}}\right) =$

(a) $-2\ \text{cosec}^2\frac{x}{2}$ (b) $\sec^2 x$

(c) $-\text{cosec}^2 x$ (d) $2\sec^2\frac{x}{2}$

(31) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right] =$

(a) 1 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) -1

(32) $\frac{d}{dx}[\tan^{-1}(\sec x + \tan x] =$

(a) $\sec x$ (b) 1 (c) $\frac{1}{2}$ (d) $\cos x$

(33) If $y = \sin^{-1}\left(\frac{19x}{20}\right) + \cos^{-1}\left(\frac{19x}{20}\right)$, then $\frac{dy}{dx} =$

(a) -1 (b) 0

(c) 1 (d) None of these

(34) If $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then $\frac{dy}{dx} =$

(a) 1 (b) 0

(c) $\frac{1}{\sqrt{x}+1}$ (d) None of these

(35) If $y = \sin[\cos^{-1}\{\sin(\cos^{-1}x)\}]$, then $\frac{dy}{dx}$ at $x = \frac{1}{2}$ is equal to

(a) 1 (b) 0 (c) -1 (d) $\frac{2}{\sqrt{3}}$

(36) $\frac{d}{dx}\left(\frac{e^x}{1+x^2}\right) =$

(a) $\frac{e^x(1-x^2)}{1+x^2}$ (b) $\frac{e^x(1+x^2)}{(1+x^2)^2}$

(c) $\frac{e^x(1-x^2)}{(1+x^2)^2}$ (d) $\frac{e^x(1+x)^2}{1+x^2}$

(37) $\frac{d}{dx}\left(\frac{\log x}{\sin x}\right) =$

a) $\dfrac{\frac{\sin x}{x} - x\log x.\cos x}{\sin x}$

(b) $\dfrac{\frac{\sin x}{x} - x\log x.\cos x}{\sin^2 x}$

(c) $\dfrac{\sin x - \log x.\cos x}{\sin^2 x}$

(d) $\dfrac{\frac{\sin x}{x} - \log x}{\sin^2 x}$

(38) $\frac{d}{dx}\left(\frac{\sec x + \tan x}{\sec x - \tan x}\right) =$

(a) $\frac{2\cos x}{1 - \sin x}$ (b) $\frac{2\cos x}{(1 - \sin x)^2}$

(c) $\frac{\cos x}{(1 - \sin x)^2}$ (d) None of these

(39) $\frac{d}{dx}\left(\frac{1}{x^4 \sec x}\right) =$

(a) $\frac{4\cos x - x\sin x}{x^5}$ (b) $\frac{x\sin x + 4\cos x}{x^5}$

(c) $\frac{-(x\sin x + 4\cos x)}{x^5}$ (d) None of these

(40) $\frac{d}{dx}(x^2 e^x \sin x) =$

(a) $xe^x (2 \sin x + x \sin x + \cos x)$

(b) $xe^x (2 \sin x + x \sin x + x \cos x)$

(c) $xe^x (2 \sin x + x \sin x - \cos x)$

(d) None of these

(41) If $y = (1 + x^{\frac{1}{4}})(1 + x^{\frac{1}{2}})(1 - x^{\frac{1}{4}})$, then $\frac{dy}{dx} =$

(a) $\sqrt{x}$ (b) 1 (c) –1 (d) x

(42) The derivative of $f(x) = x\,|x|$ is

(a) $2\,|x|$ (b) $2x$ (c) $-2x$ (d) $2x^2$

(43) $\frac{d}{dx}\left[\left(\frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x \tan^2 x}\right)\cot 3x\right] =$

(a) $\sec x \tan x$ (b) $\tan 2x \tan x$

(c) $\tan 3x \tan x$ (d) $\sec^2 x$

(44) If $u(x, y) = y \log x + x \log y$, then

$u_x u_y - u_x \log x - u_y \log y + \log x \log y =$

(a) 2 (b) 0 (c) –1 (d) 1

(45) If $f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$, then $f'(a) =$

(a) a (b) –1 (c) 1 (d) 0

(46) If $y = \dfrac{a + bx^{\frac{3}{2}}}{x^{\frac{5}{4}}}$ and $y' = 0$ at $x = 5$, the ratio $a : b$ is equal to

(a) 1 : 2 (b) $\sqrt{5}$: 1 (c) 5 : 2 (d) 3 : 5

(47) If $y = e^{\sqrt{x}}$, then $\frac{dy}{dx}$ equals

(a) $\frac{2\sqrt{x}}{e^{\sqrt{x}}}$ (b) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$ (c) $\frac{\sqrt{x}}{e^{\sqrt{x}}}$ (d) $\frac{x}{e^{\sqrt{x}}}$

(48) $\frac{d}{dx}\left(e^{x^3}\right)$ is equal to

(a) $2x^2 e^{x^3}$ (b) $3x e^{x^3}$ (c) $3x^2 e^{x^3}$ (d) $3x\left(e^{x^3}\right)^2$

(49) If $y = 3^{x^2}$, then $\frac{dy}{dx}$ is equal to

(a) $(x^2 - 1).3$ (b) $(x^2)3^{x^2-1}$

(c) $3x^2.2x$ (d) $3^{x^2}.2x.\log 3$

(50) $\frac{d}{dx}\left[\log\left(x+\frac{1}{x}\right)\right] =$

(a) $1 + \frac{1}{x}$ (b) $x + \frac{1}{x}$

(c) $\frac{1+\frac{1}{x^2}}{1+\frac{1}{x}}$ (d) $\frac{1-\frac{1}{x^2}}{x+\frac{1}{x}}$

(51) The derivative of $\sqrt{\sqrt{x}+1}$ is

(a) $\frac{1}{4\sqrt{x\left(\sqrt{x}+1\right)}}$ (b) $\frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}$

(c) $\frac{1}{\sqrt{x}\sqrt{x+1}}$ (d) $\frac{4}{\sqrt{x\left(\sqrt{x}+1\right)}}$

(52) If $y = \sin^{-1}\sqrt{x}$, then $\frac{dy}{dx} =$

(a) $\frac{1}{\sqrt{1-x}}$ (b) $\frac{2}{\sqrt{x}\sqrt{1-x}}$

(c) $\frac{-2}{\sqrt{x}\sqrt{1-x}}$ (d) $\frac{1}{2\sqrt{x}\sqrt{1-x}}$

(53) $\frac{d}{dx}[\log(\sec x + \tan x)] =$

(a) $\cot x$ (b) $\cos x$ (c) $\sec x$ (d) $\tan x$

(54) $\frac{d}{dx}(\sin 2x^2)$ equals

(a) $4x \sin(x^2)\cos(x^2)$ (b) $4x \cos(2x^2)$

(c) $2\sin(x^2)\cos(x^2)$ (d) $4x \sin(x^2)$

(55) $\frac{d}{dx}(x^2 + \cos x)^4 =$

(a) $4(x^2 + \cos x)^3 (2x + \sin x)$

(b) $4(x^2 + \cos x)^3 (2x - \sin x)$

(c) $4(x^2 - \cos x)^3 (2x - \sin x)$

(d) $4(x^2 + \cos x)^3 (2x - \sin x)$

(56) If $y = \cot^{-1}(x^2)$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{-2x}{\sqrt{1+x^2}}$ (b) $\frac{2x}{1+x^4}$

(c) $\frac{2x}{\sqrt{1+4x}}$ (d) $\frac{-2x}{1+x^4}$

(57) $\frac{d}{dx}[\log(\tan x)] =$

(a) $\operatorname{cosec} 2x$ (b) $2\sec 2x$

(c) $2\operatorname{cosec} 2x$ (d) $\sec 2x$

(58) $\frac{d}{dx}[\cos(\sin x^2)] =$

(a) $-\sin(\sin x^2).\cos^2 x.2x$

(b) $\sin(\sin x^2).\cos x^2.2x$

(c) $-\sin(\sin x^2).\cos x^2.\ 2x$

(d) None of these

(59) If $y = (\cos x^2)^2$, then $\frac{dy}{dx}$ is equal to

(a) $-x \cos 2x^2$ (b) $-4x \sin 2x^2$

(c) $-x \sin x^2$ (d) $-2x \sin 2x^2$

(60) If $y = \log\left(\tan\sqrt{x}\right)$, then the value of $\frac{dy}{dx}$ is

(a) $\frac{\sec^2\sqrt{x}}{2\sqrt{x}\tan\sqrt{x}}$ (b) $\frac{1}{2\sqrt{x}}$

(c) $\frac{\sec^2\sqrt{x}}{\sqrt{x}\tan x}$ (d) $2\sec^2\sqrt{x}$

(61) $\frac{d}{dx}[\cos(1-x^2)^2] =$

(a) $-2(1-x^2)\sin(1-x^2)^2$

(b) $-2x(1-x^2)\sin(1-x^2)^2$

(c) $-4x(1-x^2)\sin(1-x^2)^2$

(d) $4x(1-x^2)\sin(1-x^2)^2$

(62) If $y = \sec x°$, then $\frac{dy}{dx}$ =

(a) $\frac{180}{\pi}$ sec $x°$ tan $x°$ (b) sec x tan x

(c) sec $x°$ tan $x°$ (d) $\frac{\pi}{180}$ sec $x°$ tan $x°$

(63) $10^{-x \tan x}\left[\frac{d}{dx}(10^{x \tan x})\right]$ is equal to

(a) $x \tan x \log 10$

(b) $\tan x + x \sec^2 x$

(c) $\log 10 (\tan x + x \sec^2 x)$

(d) $\log 10\left(\tan x + \frac{x}{\cos^2 x} + \tan x \sec x\right)$

(64) The differential coefficient of $f[\log(x)]$, when $f(x) = \log x$ is

(a) $\frac{\log x}{x}$ (b) $x \log x$ (c) $\frac{x}{\log x}$ (d) $\frac{1}{x \log x}$

(65) $\frac{d}{dx}(xe^{x^2})$ =

(a) $e^x.2x^2 + e^{x^2}$ (b) $2x^2 e^{x^2} + e^{x^2}$

(c) $x^2 e^{x^2} + e^{x^2}$ (d) None of these

(66) $\frac{d}{dx}\left(x^3 \tan^2 \frac{x}{2}\right)$ =

(a) $x^3 \tan^2 \frac{x}{2} .\sec^2 \frac{x}{2} + 3x^2 \tan^2 \frac{x}{2}$

(b) $x^3 \tan \frac{x}{2} .\sec^2 \frac{x}{2} + 3x \tan^2 \frac{x}{2}$

(c) $x^3 \tan \frac{x}{2} .\sec^2 \frac{x}{2} + 3x^2 \tan^2 \frac{x}{2}$

(d) None of these

(67) $\frac{d}{dx}$ $(e^x \log \sin 2x)$ =

(a) $e^x (\log \cos 2x + \cot 2x)$

(b) $e^x (\log \sin 2x + 2 \cot 2x)$

(c) $e^x (\log \cos 2x + 2 \cot 2x)$

(d) None of these

(68) If $y = \frac{\tan x + \cot x}{\tan x - \cot x}$, then $\frac{dy}{dx}$ =

(a) $-2 \tan 2x \sec 2x$ (b) $2 \tan 2x \sec 2x$

(c) $\tan 2x \sec 2x$ (d) $-\tan 2x \sec 2x$

(69) $\frac{d}{dx}\left(a^{\log_{10} \text{cosec}^{-1} x}\right)$ =

(a) $-a^{\log_{10} \text{cosec}^{-1} x} . \frac{1}{\text{cosec}^{-1} x} . \frac{1}{x\sqrt{x^2 - 1}} . \log_{10} a$

(b) $a^{\log_{10} \text{cosec}^{-1} x} . \frac{1}{\text{cosec}^{-1} x} . \frac{1}{x\sqrt{x^2 - 1}} . \log_{10} a$

(c) $-a^{\log_{10} \text{cosec}^{-1} x} . \frac{1}{\text{cosec}^{-1} x} . \frac{1}{|x|\sqrt{x^2 - 1}} . \log_{10} a$

(d) $a^{\log_{10} \text{cosec}^{-1} x} . \frac{1}{\text{cosec}^{-1} x} . \frac{1}{|x|\sqrt{x^2 - 1}} . \log_{10} a$

(70) $\frac{d}{dx}\left(e^{\sqrt{1-x^2}} . \tan x\right)$ =

(a) $e^{\sqrt{1-x^2}}\left[\sec^2 x + \frac{\tan x}{\sqrt{1-x^2}}\right]$

(b) $e^{\sqrt{1-x^2}}\left[\sec^2 x + \frac{x \tan x}{\sqrt{1-x^2}}\right]$

(c) $e^{\sqrt{1-x^2}}\left[\sec^2 x - \frac{x \tan x}{\sqrt{1-x^2}}\right]$

(d) None of these

(71) If $y = \sin\left(\sqrt{\sin x + \cos x}\right)$, then $\dfrac{dy}{dx}$ =

(a) $\dfrac{\cos\sqrt{\sin x + \cos x}}{2\sqrt{\sin x + \cos x}}$. $(\cos x - \sin x)$

(b) $\dfrac{\cos\sqrt{\sin x + \cos x}}{2\sqrt{\sin x + \cos x}}$

(c) $\dfrac{\cos\sqrt{\sin x + \cos x}}{\sqrt{\sin x + \cos x}}$

(d) None of these

(72) If $y = \sqrt{\dfrac{1+\tan x}{1-\tan x}}$, then $\dfrac{dy}{dx}$ =

(a) $\dfrac{1}{2}\sqrt{\dfrac{1-\tan x}{1+\tan x}}.\sec\left(\dfrac{\pi}{4}+x\right)$

(b) $\dfrac{1}{2}\sqrt{\dfrac{1-\tan x}{1+\tan x}}.\sec^2\left(\dfrac{\pi}{4}+x\right)$

(c) $\sqrt{\dfrac{1-\tan x}{1+\tan x}}.\sec^2\left(\dfrac{\pi}{4}+x\right)$

(d) None of these

(73) $\dfrac{d}{dx}\left[\log\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)\right]$ =

(a) $-\sec x$ (b) $\text{cosec } x$

(c) $-\text{cosec } x$ (d) $\sec x$

(74) $\dfrac{d}{dx}\left(\sqrt{\sec^2 x + \text{cosec}^2 x}\right)$ =

(a) $-4 \text{ cosec } x.\cot 2x$ (b) $4 \text{ cosec } 2x.\cot 2x$

(c) $-4 \text{ cosec } 2x.\cot 2x$ (d) None of these

(75) If $y = \tan^{-1}\left(\dfrac{\sqrt{a}-\sqrt{x}}{1+\sqrt{ax}}\right)$, then $\dfrac{dy}{dx}$ =

(a) $-\dfrac{1}{2(1+x)\sqrt{x}}$ (b) $\dfrac{1}{2(1+x)\sqrt{x}}$

(c) $\dfrac{1}{(1+x)\sqrt{x}}$ (d) None of these

(76) If $y = \tan^{-1}\left(\dfrac{x^{\frac{1}{3}}+a^{\frac{1}{3}}}{1-x^{\frac{1}{3}}a^{\frac{1}{3}}}\right)$, then $\dfrac{dy}{dx}$ =

(a) $-\dfrac{a}{3x^{2/3}\left(1+x^{2/3}\right)}$ (b) $\dfrac{1}{3x^{2/3}\left(1+x^{2/3}\right)}$

(c) $\dfrac{1}{3x^{2/3}\left(1+x^{2/3}\right)}$ (d) $-\dfrac{1}{3x^{2/3}\left(1+x^{2/3}\right)}$

(77) If $y = \sin\left(\dfrac{1+x^2}{1-x^2}\right)$, then $\dfrac{dy}{dx}$ =

(a) $\dfrac{4x}{(1-x^2)}.\cos\left(\dfrac{1+x^2}{1-x^2}\right)$

(b) $\dfrac{4x}{1-x^2}.\cos\left(\dfrac{1+x^2}{1-x^2}\right)$

(c) $\dfrac{x}{(1-x^2)^2}.\cos\left(\dfrac{1+x^2}{1-x^2}\right)$

(d) $\dfrac{x}{(1-x^2)}.\cos\left(\dfrac{1+x^2}{1-x^2}\right)$

(78) If $y = \cot^{-1}\left(\dfrac{1+x}{1-x}\right)$, then $\dfrac{dy}{dx}$ =

(a) $-\dfrac{2}{1+x^2}$ (b) $\dfrac{1}{1+x^2}$

(c) $-\dfrac{1}{1+x^2}$ (d) $\dfrac{2}{1+x^2}$

(79) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then $(1 - x^2)\frac{dy}{dx}$ is equal to

(a) $xy - 2$ (b) $x + y$

(c) $1 + xy$ (d) $1 - xy$

(80) $\frac{d}{dx}\left(\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right) =$

(a) $\sec\frac{x}{2}$ (b) $\sec x$

(c) $\operatorname{cosec} x$ (d) $\operatorname{cosec}\frac{x}{2}$

(81) $\frac{d}{dx}\left[\frac{e^{ax}}{\sin(bx+c)}\right] =$

(a) $\frac{e^{ax}[a\sin(bx+c) - b\cos(bx+c)]}{\sin^2(bx+c)}$

(b) $\frac{e^{ax}[a\sin(bx+c) + b\cos(bx+c)]}{\sin^2(bx+c)}$

(c) $\frac{e^{ax}[a\sin(bx+c) - b\cos(bx+c)]}{\sin(bx+c)}$

(d) None of these

(82) $\frac{d}{dx}[e^x \log(1 + x^2)] =$

(a) $e^x\left[\log(1+x^2) - \frac{x}{1+x^2}\right]$

(b) $e^x\left[\log(1+x^2) + \frac{2x}{1+x^2}\right]$

(c) $e^x\left[\log(1+x^2) - \frac{2x}{1+x^2}\right]$

(d) $e^x\left[\log(1+x^2) + \frac{x}{1+x^2}\right]$

(83) $\frac{d}{dx}[e^x \cos(bx + c)] =$

(a) $e^{ax}[\cos(bx + c) - \sin(bx + c)]$

(b) $e^{ax}[a\cos(bx + c) - b\sin(bx + c)]$

(c) $e^{ax}[a\sin(bx + c) - b\cos(bx + c)]$

(d) None of these

(84) $\frac{d}{dx}\{e^{-ax^2}\log(\sin x)\} =$

(a) $e^{-ax^2}[\cot x - 2ax\log(\sin x)]$

(b) $e^{-ax^2}[\cot x + 2ax\log(\sin x)]$

(c) $e^{-ax^2}[\cot x + ax\log(\sin x)]$

(d) None of these

(85) If $y = \log x . e^{(\tan x + x^2)}$, then $\frac{dy}{dx} =$

(a) $e^{(\tan x+x^2)}\left[\frac{1}{x} + (\sec^2 x - 2x)\log x\right]$

(b) $e^{(\tan x+x^2)}\left[\frac{1}{x} + (\sec^2 x + x)\log x\right]$

(c) $e^{(\tan x+x^2)}\left[\frac{1}{x} + (\sec^2 x - x)\log x\right]$

(d) $e^{(\tan x+x^2)}\left[\frac{1}{x} + (\sec^2 x + 2x)\log x\right]$

(86) If $y = \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$, then $\frac{dy}{dx} =$

(a) $\frac{4}{(e^{2x} - e^{-2x})^2}$ (b) $\frac{-8}{(e^{2x} - e^{-2x})^2}$

(c) $\frac{8}{(e^{2x} - e^{-2x})^2}$ (d) $\frac{-4}{(e^{2x} - e^{-2x})^2}$

(87) If $y = \sqrt{\sin\sqrt{x}}$, then $\frac{dy}{dx}$ =

(a) $\frac{1}{2\sqrt{\sin x}}$ (b) $\frac{1}{2\sqrt{\cos\sqrt{x}}}$

(c) $\frac{\sqrt{\cos\sqrt{x}}}{2x}$ (d) $\frac{\cos\sqrt{x}}{4\sqrt{x}\sqrt{\sin\sqrt{x}}}$

(88) Differential coefficient of $\sqrt{\sec\sqrt{x}}$ is

(a) $\frac{1}{2}\sqrt{x}\sec\sqrt{x}\sin\sqrt{x}$ (b) $\frac{1}{4\sqrt{x}}(\sec\sqrt{x})^{\frac{3}{2}}\sin\sqrt{x}$

(c) $\frac{1}{4\sqrt{x}}\sec\sqrt{x}\sin\sqrt{x}$ (d) $\frac{1}{2}\sqrt{x}(\sec\sqrt{x})^{\frac{3}{2}}\sin\sqrt{x}$

(89) $\frac{d}{dx}\left[\log\left(\sqrt{\sin\sqrt{e^x}}\right)\right]$ =

(a) $\frac{1}{2}e^{\frac{x}{2}}\cot\left(e^{\frac{x}{2}}\right)$ (b) $\frac{1}{4}e^{\frac{x}{2}}\cot\left(e^{\frac{x}{2}}\right)$

(c) $e^{\frac{x}{2}}\cot\left(e^{\frac{x}{2}}\right)$ (d) $\frac{1}{4}e^{x}\cot(e^{x})$

(90) If $f(x) = \frac{1}{\sqrt{x^2+a^2}+\sqrt{x^2+b^2}}$, then $f'(x)$ is equal to

(a) $(a^2+b^2)\left[\frac{1}{\sqrt{x^2+a^2}} - \frac{2}{\sqrt{x^2+b^2}}\right]$

(b) $\frac{x}{(a^2-b^2)}\left[\frac{1}{\sqrt{x^2+a^2}} - \frac{1}{\sqrt{x^2+b^2}}\right]$

(c) $\frac{x}{(a^2+b^2)}\left[\frac{1}{\sqrt{x^2+a^2}} - \frac{2}{\sqrt{x^2+b^2}}\right]$

(d) $\frac{x}{(a^2-b^2)}\left[\frac{1}{\sqrt{x^2+a^2}} + \frac{1}{\sqrt{x^2+b^2}}\right]$

(91) If $y = \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$, then $\frac{dy}{dx}$ =

(a) $\frac{ay}{x\sqrt{x^2-a^2}}$ (b) $\frac{ay}{x\sqrt{a^2-x^2}}$

(c) $\frac{ay}{\sqrt{a^2-x^2}}$ (d) None of these

(92) If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ is

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{x}{\sqrt{1+x^2}}$

(c) $\frac{-x}{\sqrt{1+x^2}}$ (d) None of these

(93) If $x = \exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$, then $\frac{dy}{dx}$ equals

(a) $2x\,[1 + \tan(\log x)] + \sec^2(\log x)$

(b) $2x\,[1 + \tan(\log x)] + x\sec^2(\log x)$

(c) $x\,[1 + \tan(\log x)] + \sec^2(\log x)$

(d) $2x\,[1 + \tan(\log x)] + x^2\sec^2(\log x)$

(94) Derivative of the function $f(x) = \log_5(\log_7 x)$, $x > 7$, is

(a) $\frac{1}{x(\log_e x)}$

(b) $\frac{1}{x(\log_e 5)(\log_e 7)(\log_7 x)}$

(c) $\frac{1}{x(\log_e 5)(\log_e 7)}$

(d) None of these

(95) If $y = \frac{1}{4}u^4$, $u = \frac{2}{3}x^3 + 5$, then $\frac{dy}{dx}$ =

(a) $\frac{2x^2}{27}(2x^3 + 15)^3$ (b) $\frac{x^2}{27}(2x^3 + 15)^3$

(c) $\frac{2x}{27}(2x^3 + 5)^3$ (d) None of these

(96) If $f(x) = \frac{1}{1-x}$, then the derivative of the composite function f[f {f (x)}] is equal to

(a) 2 (b) 0 (c) $\frac{1}{2}$ (d) 1

(97) Let $f(x) = e^x$, $g(x) = \sin^{-1} x$ and $h(x) = f(g(x))$, then $\frac{h'(x)}{h(x)}$ =

(a) $\frac{1}{1-x^2}$ (b) $e^{\sin^{-1}x}$

(c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\sin^{-1}x$

(98) If $y = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)$, then $\frac{dy}{dx}$ =

(a) $\frac{2}{\sqrt{x^2+a^2}}$ (b) $\sqrt{x^2+a^2}$

(c) $\frac{1}{\sqrt{x^2+a^2}}$ (d) $2\sqrt{x^2+a^2}$

(99) If $y = f\left(\frac{5x+1}{10x^2-3}\right)$ and $f'(x) = \cos x$, then $\frac{dy}{dx}$ =

(a) $\cos\left(\frac{5x+1}{10x^2-3}\right)$

(b) $\cos\left(\frac{5x+1}{10x^2-3}\right)\frac{d}{dx}\left(\frac{5x+1}{10x^2-3}\right)$

(c) $\frac{5x+1}{10x^2-3}\cos\left(\frac{5x+1}{10x^2-3}\right)$

(d) None of these

(100) If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, then $\frac{dy}{dx}$ =

(a) $\frac{-2x^2+2x+2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

(b) $\frac{6x^2-2x+2}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

(c) $\frac{6x^2-2x+2}{(x^2+1)^2}\sin^2\left(\frac{2x-1}{x^2+1}\right)$

(d) $\frac{-2x^2+2x+2}{(x^2+1)^2}\sin^2\left(\frac{2x-1}{x^2+1}\right)$

(101) If $y = \tan^{-1}(\sec x - \tan x)$, then $\frac{dy}{dx}$ =

(a) $\frac{-1}{2}$ (b) 2 (c) -2 (d) $\frac{1}{2}$

(102) If $y = \frac{5x}{\sqrt[3]{(1-x)^2}} + \cos^2(2x+1)$, then $\frac{dy}{dx} =$

(a) $\frac{5(3-x)}{3(1-x)^{\frac{2}{3}}} - 2\sin(2x+1)$

(b) $\frac{5(3-x)}{3(1-x)^{\frac{5}{3}}} - 2\sin(4x+2)$

(c) $\frac{5(3-x)}{3(1-x)^{\frac{2}{3}}} - 2\sin(4x+4)$

(d) None of these

(103) If $y = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{\frac{3}{2}}}\right)$, then $y'(1)$ is

(a) $-\frac{1}{4}$ (b) 0 (c) $\frac{1}{2}$ (d) –1

(104) If $f(x) = \log_x(\log x)$, then $f'(x)$ at $x = e$ is

(a) 1 (b) e

(c) $\frac{1}{e}$ (d) None of these

(105) If $y = \cot^{-1}(\cot 2x)^{\frac{1}{2}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ will be

(a) $(6)^{\frac{1}{2}}$ (b) $\left(\frac{2}{3}\right)^{\frac{1}{2}}$ (c) $\left(\frac{1}{3}\right)^{\frac{1}{2}}$ (d) $(3)^{\frac{1}{2}}$

(106) If $f(1) = 3$, $f'(1) = 2$, then $\frac{d}{dx}[\log f(e^x + 2x)]$ at $x = 0$ is

(a) 0 (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 2

(107) If $f(x) = \cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$, then the value of $f'(e) =$

(a) $\frac{2}{e^2}$ (b) 1 (c) $\frac{1}{e}$ (d) $\frac{2}{e}$

(108) If $f(x) = \sqrt{1+\cos^2(x^2)}$, then $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is

(a) $\frac{\pi}{\sqrt{6}}$ (b) $\frac{\sqrt{\pi}}{6}$ (c) $-\sqrt{\frac{\pi}{6}}$ (d) $\frac{1}{\sqrt{6}}$

(109) If $x = a(t + \sin t)$ and $y = a(1 - \cos t)$, then $\frac{dy}{dx}$ equals

(a) $\tan t$ (b) $\tan\frac{t}{2}$ (c) $\cot\frac{t}{2}$ (d) $\tan 2t$

(110) If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, then $\frac{dy}{dx} =$

(a) $\operatorname{cosec}\theta$ (b) $\cos\theta$

(c) $\tan\theta$ (d) $\sec\theta$

(111) If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2at}{1+t^2}$, then $\frac{dy}{dx} =$

(a) $\frac{a(t^2-1)}{t}$ (b) $\frac{a(1-t^2)}{2t}$

(c) $\frac{a(t^2-1)}{2t}$ (d) $\frac{a(t^2+1)}{2t}$

(112) If $x = \frac{2t}{1+t^2}$, $y = \frac{1-t^2}{1+t^2}$, then $\frac{dy}{dx}$ equals

(a) $\frac{2t}{1-t^2}$ (b) $\frac{2t}{t^2+1}$

(c) $\frac{2t}{t^2-1}$ (d) None of these

(113) If $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$, then $\frac{dy}{dx} =$

(a) $-\cot t$ (b) $\tan t$

(c) $-\tan t$ (d) $\cot t$

(114) If $x = \sin^{-1}(3t - 4t^3)$ and $y = \cos^{-1}\sqrt{1-t^2}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{5}$ (d) $\frac{3}{2}$

(115) If $\cos x = \frac{1}{\sqrt{1+t^2}}$ and $\sin y = \frac{t}{\sqrt{1+t^2}}$, then $\frac{dy}{dx} =$

(a) 1 (b) -1 (c) $\frac{1-t}{1+t^2}$ (d) $\frac{1}{1+t^2}$

(116) If $y = \sin(2\sin^{-1} x)$, then $\frac{dy}{dx} =$

(a) $\frac{2+4x^2}{\sqrt{1+x^2}}$ (b) $\frac{2-4x^2}{\sqrt{1-x^2}}$

(c) $\frac{2+4x^2}{\sqrt{1-x^2}}$ (d) $\frac{2-4x^2}{\sqrt{1+x^2}}$

(117) If $x = a \sin 2\theta(1 + \cos 2\theta)$,

$y = b \cos 2\theta(1 - \cos 2\theta)$, then $\frac{dy}{dx} =$

(a) $\frac{b}{a\tan\theta}$ (b) $\frac{b\tan\theta}{a}$

(c) $\frac{a\tan\theta}{b}$ (d) $\frac{a}{b\tan\theta}$

(118) If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx} =$

(a) $\frac{x}{y}$ (b) $\frac{-y}{x}$ (c) $\frac{y}{x}$ (d) $\frac{-x}{y}$

(119) If $x = a\cos^3\theta$, $y = a\sin^3\theta$, then $\sqrt{1+\left(\frac{dy}{dx}\right)^2} =$

(a) $|\sec\theta|$ (b) $\tan^2\theta$

(c) $\sec^2\theta$ (d) $\tan\theta$

(120) If $y = f(x^3)$, $z = g(x^2)$, $f'(x) = \cos x$ and $g'(x) = \sin x$, then $\frac{dy}{dx}$ is

(a) $\tan x$ (b) $\frac{3x}{2}\cos x^3 \operatorname{cosec} x^2$

(c) $\frac{2}{3}\sin x^3 \sec x^2$ (d) None of these

(121) If $x = a\cos^4\theta$, $y = a\sin^4\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{3\pi}{4}$ is

(a) a^2 (b) -1 (c) 1 (d) $-a^2$

(122) If $x = \sin t \cos 2t$ and $y = \cos t \sin 2t$, then at $t = \frac{\pi}{4}$, the value of $\frac{dy}{dx}$ is equal to

(a) $-\frac{1}{2}$ (b) -2 (c) 1 (d) $\frac{1}{2}$

(123) If $x = 2\cos t - \cos 2t$, $y = 2\sin t - \sin 2t$, then at $t = \frac{\pi}{4}$, $\frac{dy}{dx} =$

(a) $\frac{\sqrt{2+1}}{2}$ (b) $\sqrt{2}+1$

(c) $\sqrt{2+1}$ (d) None of these

(124) The differential coefficient of x^6 with respect to x^3 is

(a) $2x^3$ (b) $5x^2$ (c) $3x^3$ (d) $5x^5$

(125) The derivative of $\sin^2 x$ with respect to $\cos^2 x$ is

(a) $-\tan x$ (b) $\tan^2 x$

(c) $\tan x$ (d) None of these

(126) The differential of e^{x^3} with respect to $\log x$ is

(a) $3x^2 e^{x^3} + 3x^2$ (b) e^{x^3}

(c) $3x^2 e^{x^3}$ (d) $3x^3 e^{x^3}$

(127) The derivative of $\log_{10} x$ with respect to x^2 is

(a) $\frac{1}{2x^2}\log_{10} e$ (b) $\frac{1}{2x^2}\log_e 10$

(c) $\frac{2}{x^2}\log_{10} e$ (d) None of these

(128) Differential coefficient of $\cos^{-1}(\sqrt{x})$ with respect to $\sqrt{1-x}$ is

(a) $-\frac{1}{\sqrt{x}}$ (b) $\sqrt{x}$ (c) $-\sqrt{x}$ (d) $\frac{1}{\sqrt{x}}$

(129) Differential coefficient of $\sin^{-1}\left(\frac{1-x}{1+x}\right)$ w.r.t. $\sqrt{x}$ is

(a) 1 (b) $\frac{1}{2\sqrt{x}}$

(c) $\frac{\sqrt{x}}{\sqrt{1-x}}$ (d) None of these

(130) Differential coefficient of $\sin^{-1} x$ w.r.t. $\cos^{-1}\sqrt{1-x^2}$ is

(a) 2 (b) 1

(c) $\frac{1}{1+x^2}$ (d) None of these

(131) The differential coefficient of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is

(a) 0 (b) 1

(c) –1 (d) None of these

(132) The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ w.r.t. $\cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$ is

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

(133) Differential coefficient of $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1} x$ is

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

(134) The differential coefficient of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1} x$ is

(a) 1 (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) None of these

(135) Differential coefficient of $\tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$ w.r.t. $\cos^{-1}(x^2)$ is

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

(136) Differential coefficient of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is

(a) 1 (b) 2 (c) 4 (d) 6

(137) Derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1+3x}$ at $x=-\frac{1}{3}$ is

(a) $\frac{1}{3}$ (b) 0

(c) $\frac{1}{2}$ (d) None of these

(138) The derivative of f (tan x) with respect to g (sec x) at $x = \frac{\pi}{4}$, where f' (1) = 2 and g' ($\sqrt{2}$) = 4, is

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 1

(139) If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$ then

(a) $(x^2+4)\left(\frac{dy}{dx}\right)^2 = (y^2+4)$

(b) $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$

(c) $(x^2+4)\left(\frac{dy}{dx}\right)^2 = x^2(y^2+4)$

(d) None of these

(140) If $x^3+y^3-3axy=0$ then $\frac{dy}{dx}$ equals

(a) $\frac{x^2+ay}{ax-y^2}$ (b) $\frac{ay-x^2}{y^2-ax}$

(c) $\frac{ay-x^2}{ay-y^2}$ (d) $\frac{x^2+ay}{x^2+ax}$

(141) If $\sin^2 x + 2\cos y + xy = 0$ then $\frac{dy}{dx} =$

(a) $\frac{y+2\sin x}{\sin y + x}$ (b) $\frac{y+2\sin x}{2\sin y + x}$

(c) $\frac{y+\sin 2x}{2\sin y - x}$ (d) None of these

(142) If $x^3+8xy+y^3 = 64$ then $\frac{dy}{dx} =$

(a) $\frac{3x+8y^2}{8x^2+3y}$ (b) $-\frac{3x^2+8y}{8x+3y^2}$

(c) $\frac{3x^2+8y}{8x+3y^2}$ (d) None of these

(143) If $y\sec x + \tan x + x^2y = 0$ then $\frac{dy}{dx} =$

(a) $-\frac{2xy+\sec^2 x+y\sec x\tan x}{x^2+\sec x}$

(b) $\frac{2xy+\sec^2 x+y\sec x\tan x}{x^2+\sec x}$

(c) $-\frac{2xy+\sec^2 x+\sec x\tan x}{x^2+\sec x}$

(d) None of these

(144) If $\cos(x+y) = y\sin x$ then $\frac{dy}{dx} =$

(a) $\frac{y\cos x - \sin(x+y)}{\sin x - \sin(x+y)}$

(b) $-\frac{\sin(x+y) + y\cos x}{\sin x + \sin(x+y)}$

(c) $\frac{\sin(x+y) + y\cos x}{\sin x + \sin(x+y)}$

(d) $\frac{y\cos x - \sin(x+y)}{\sin x - \sin(x+y)}$

(145) If $\tan(x+y) + \tan(x-y) = 1$ then $\frac{dy}{dx} =$

(a) $\frac{\sec^2(x+y) - \sec^2(x-y)}{\sec^2(x+y) + \sec^2(x-y)}$

(b) $\frac{\sec^2(x+y) + \sec^2(x-y)}{\sec^2(x+y) - \sec^2(x-y)}$

(c) $\frac{\sec^2(x+y) + \sec^2(x-y)}{\sec^2(x-y) - \sec^2(x+y)}$

(d) None of these

(146) If $\sin(x+y) = \log(x+y)$, then $\frac{dy}{dx} =$

(a) -1 (b) 2 (c) -2 (d) 1

(147) If $\sin y = x\sin(a+y)$, then $\frac{dy}{dx} =$

(a) $\frac{\sin^2(a+y)}{\cos a}$ (b) $\frac{\sin^2(a+y)}{\sin(a+2y)}$

(c) $\frac{\sin^2(a+y)}{\cos(a+2y)}$ (d) $\frac{\sin^2(a+y)}{\sin a}$

(148) If $\sin(xy) + \frac{x}{y} = x^2 - y$, then $\frac{dy}{dx} =$

(a) $-\frac{y[2xy - y^2\cos(xy) - 1]}{xy^2\cos(xy) + y^2 - x}$

(b) $\frac{y[2xy - y^2\cos(xy) - 1]}{xy^2\cos(xy) + y^2 - x}$

(c) $\frac{2xy - y^2\cos(xy) - 1}{xy^2\cos(xy) + y^2 - x}$

(d) None of these

(149) If $3\sin(xy) + 4\cos(xy) = 5$, then $\frac{dy}{dx} =$

(a) $\frac{3\cos(xy) + 4\sin(xy)}{4\cos(xy) - 3\sin(xy)}$

(b) $-\frac{y}{x}$

(c) $\frac{3\sin(xy) + 4\cos(xy)}{3\cos(xy) - 4\sin(xy)}$

(d) None of these

(150) If $2x^2 - 3xy + y^2 + x + 2y - 8 = 0$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{3y - 4x + 1}{2y + 3x + 2}$ (b) $\frac{3y - 4x - 1}{2y - 3x + 2}$

(c) $\frac{3y + 4x + 1}{2y + 3x + 2}$ (d) $\frac{3y - 4x - 1}{2y - 3x - 2}$

(151) If $x^2 e^y + 2xye^x + 13 = 0$, then $\frac{dy}{dx} =$

(a) $-\frac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$

(b) $\frac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$

(c) $\frac{2xe^{x-y}+2y(x+1)}{x(xe^{y-x}+2)}$

(d) None of these

(152) If $x = a\left(t-\frac{1}{t}\right)$, $y = a\left(t+\frac{1}{t}\right)$, then $\frac{dy}{dx} =$

(a) $\frac{-x}{y}$ (b) $\frac{y}{x}$ (c) $\frac{-y}{x}$ (d) $\frac{x}{y}$

(153) If $x = y\sqrt{1-y^2}$, then $\frac{dy}{dx} =$

(a) $\frac{\sqrt{1-y^2}}{1+2y^2}$ (b) 0

(c) x (d) $\frac{\sqrt{1-y^2}}{1-2y^2}$

(154) $\frac{d}{dx}\left(\sqrt{x\sin x}\right) =$

(a) $\frac{x\sin x+\cos x}{2\sqrt{x\sin x}}$ (b) $\frac{\sin x+x\cos x}{2\sqrt{x\sin x}}$

(c) $\frac{\sin x+x\cos x}{\sqrt{x\sin x}}$ (d) $\frac{x\sin x+\cos x}{\sqrt{2\sin x}}$

(155) If $y = \sqrt{\frac{1+e^x}{1-e^x}}$, then $\frac{dy}{dx} =$

(a) $\frac{e^x}{(1-e^x)\sqrt{1+e^x}}$ (b) $\frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$

(c) $\frac{e^x}{(1-e^x)\sqrt{1-e^x}}$ (d) $\frac{e^x}{(1-e^x)\sqrt{1+e^{2x}}}$

(156) If $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1}-x\right)$,

then $(x^2+1)\frac{dy}{dx} + xy + 1 =$

(a) 2 (b) 0

(c) 1 (d) None of these

(157) If $y = \frac{1}{\sqrt{a^2-b^2}}\cos^{-1}\left[\frac{a\cos(x-\alpha)+b}{\theta}\right]$, where

$\theta = a + b\cos(x-\alpha)$, then $\frac{dy}{dx} =$

(a) $\frac{2}{\theta^2}$ (b) $\frac{1}{\theta}$ (c) $\frac{2}{\theta}$ (d) $\frac{1}{\theta^2}$

(158) If $\log(x+y) = 2xy$, then $y'(0) =$

(a) 1 (b) -1 (c) 2 (d) 0

(159) If $xe^{xy} = y + \sin^2 x$, then at $x = 0$, $\frac{dy}{dx} =$

(a) -1 (b) -2 (c) 1 (d) 2

(160) If $2^x + 2^y = 2^{x+y}$, then the valie of $\frac{dy}{dx}$ at $x = y = 1$ is

(a) 0 (b) -1 (c) 1 (d) 2

(161) Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then, $y'(1)$ equals

(a) -1 (b) 1 (c) $\log 2$ (d) $-\log 2$

(162) If $\sin y - e^{-x\cos y} = e$, then $\frac{dy}{dx}$ at $(1, \pi)$ is

(a) $\sin y$ (b) $-x\cos y$

(c) e (d) $\sin y - x\cos y$

(163) $\frac{d}{dx}(e^{x\sin x}) =$

(a) $e^{x\sin x}(x\cos x + \sin x)$

(b) $e^{x\sin x}(\cos x + x\sin x)$

(c) $e^{x\sin x}(x\cos + \sin x)$

(d) None of these

(164) If $y = x^{\sqrt{x}}$, then $\frac{dy}{dx} =$

(a) $x^{\sqrt{x}}\left(\frac{2+\log x}{2\sqrt{x}}\right)$ (b) $\sqrt{x}\left(\frac{2+\log x}{2\sqrt{x}}\right)$

(c) $\frac{2+\log x}{2\sqrt{x}}$ (d) None of these

(165) If $y = x^{\sin x}$, then $\frac{dy}{dx} =$

(a) $x^{\sin x}.\left(\frac{x\cos x.\log x+\sin x}{x}\right)$

(b) $\frac{y\left(x\cos x.\log x+\cos x\right)}{x}$

(c) $y(x\sin x . \log x + \cos x)$

(d) None of these

(166) $\frac{dy}{dx}(x^{\log_e x-1})$

(a) $2x^{(\log_e x-1)} . \log_e x$ (b) $x^{(\log_e x-1)}$

(c) $\frac{2}{x}\log_e x$ (d) $x^{(\log_e x-1)}.\log_e x$

(167) If $y = x^x$, than $\frac{dy}{dx} =$

(a) $x^x \log ex$ (b) $x^x\left(1+\frac{1}{x}\right)$

(c) $1 + \log x$ (d) $x^x \log x$

(168) If $x^y = e^{x-y}$, then $\frac{dy}{dx} =$

(a) $\log x . [\log(ex)]^{-2}$ (b) $\log x . [\log(ex)]^2$

(c) $\log x . (\log x)^2$ (d) None of these

(169) If $x^y = y^x$, then $\frac{dy}{dx} =$

(a) $\frac{y\left(x\log_e y+y\right)}{x\left(y\log_e x+x\right)}$ (b) $\frac{y\left(x\log_e y-y\right)}{x\left(y\log_e x-x\right)}$

(c) $\frac{x\left(x\log_e y-y\right)}{y\left(y\log_e x-x\right)}$ (d) $\frac{x\left(x\log_e y+y\right)}{y\left(y\log_e x+x\right)}$

(170) If $y = x^{(x^x)}$, then $\frac{dy}{dx} =$

(a) $y\,[x^x(\log ex).\log x + x^x]$

(b) $y\,[x^x(\log ex).\log x + x]$

(c) $y\,[x^x(\log ex).\log x + x^{x-1}]$

(d) $y\,[x^x(\log x).\log x + x^{x-1}]$

(171) If $x^y . y^x = 100$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{y\left(y+x\log y\right)}{x\left(y\log x+x\right)}$ (b) $-\frac{y\left(x+y\right)\log x}{x\left(x\log y+y\right)}$

(c) $-\frac{y}{x}$ (d) $-\frac{x}{y}$

(172) If $x^{p+q} = (x+y)^{p+q}$, then $\frac{dy}{dx} =$

(a) $\frac{y}{x}$ (b) $-\frac{y}{x}$ (c) $\frac{x}{y}$ (d) $-\frac{x}{y}$

(173) If $y = (1+x)^x$, then $\frac{dy}{dx} =$

(a) $(1+x)^x\left[\frac{x}{1+x}+\log ex\right]$

(b) $\frac{x}{1+x}+\log(1+x)$

(c) $(1+x)^x\left[\frac{x}{1+x}+\log(1+x)\right]$

(d) None of these

(174) If $y = \left(1+\frac{1}{x}\right)^x$, then $\frac{dy}{dx} =$

(a) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)-\frac{1}{1+x}\right]$

(b) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)\right]$

(c) $\left(x+\frac{1}{x}\right)^x\left[\log(x-1)-\frac{x}{x+1}\right]$

(d) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)+\frac{1}{1+x}\right]$

(175) If $y = \sqrt{\frac{1+x}{1-x}}$, then $\frac{dz}{dy} =$

(a) $\frac{2}{(1+x)^{\frac{1}{2}}(1-x)^{\frac{3}{2}}}$ (b) $\frac{1}{(1+x)^{\frac{1}{2}}(1-x)^{\frac{3}{2}}}$

(c) $\frac{1}{2(1+x)^{\frac{1}{2}}(1-x)^{\frac{3}{2}}}$ (d) $\frac{1}{(1+x)^{\frac{3}{2}}(1-x)^{\frac{1}{2}}}$

(176) If $y = \frac{\sqrt{x}(2x+3)^2}{\sqrt{x+1}}$, then $\frac{dz}{dy} =$

(a) $y\left[\frac{1}{2x}+\frac{4}{2x+3}-\frac{1}{2(x+1)}\right]$

(b) $y\left[\frac{1}{3x}+\frac{4}{2x+3}+\frac{1}{2(x+1)}\right]$

(c) $y\left[\frac{1}{3x}+\frac{4}{2x+3}+\frac{1}{x+1}\right]$

(d) None of these

(177) If $y = \frac{2(x-\sin x)^{\frac{3}{2}}}{\sqrt{x}}$, then $\frac{dy}{dx} =$

(a) $\frac{2(x-\sin x)^{\frac{3}{2}}}{\sqrt{x}}\left(\frac{3}{2}.\frac{1-\cos x}{1-\sin x}-\frac{1}{2x}\right)$

(b) $\frac{2(x-\sin x)^{\frac{3}{2}}}{\sqrt{x}}\left(\frac{3}{2}.\frac{1-\cos x}{x-\sin x}-\frac{1}{2x}\right)$

(c) $\frac{2(x-\sin x)^{\frac{1}{2}}}{\sqrt{x}}\left(\frac{3}{2}.\frac{1-\cos x}{x-\sin x}-\frac{1}{2x}\right)$

(d) None of these

(178) If $y = \frac{e^x \log x}{x^2}$, then $\frac{dy}{dx} =$

(a) $\frac{e^x\left[1+(x+2)\log x\right]}{x^3}$ (b) $\frac{e^x\left[1-(x-2)\log x\right]}{x^4}$

(c) $\frac{e^x\left[1-(x-2)\log x\right]}{x^3}$ (d) $\frac{e^x\left[1+(x-2)\log x\right]}{x^3}$

(179) If $y = \dfrac{e^{2x}\cos x}{x\sin x}$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{e^{2x}\left[\left(2x-1\right)\cot x - x\operatorname{cosec}^2 x\right]}{x^2}$

(b) $\dfrac{e^{2x}\left[\left(2x+1\right)\cot x - x\operatorname{cosec}^2 x\right]}{x^2}$

(c) $\dfrac{e^{2x}\left[\left(2x-1\right)\cot x + x\operatorname{cosec}^2 x\right]}{x^2}$

(d) None of these

(180) $\dfrac{d}{dx}\left\{\left(\sin x\right)^x\right\} =$

(a) $\dfrac{x\cos x + \sin x\log\left(\sin x\right)}{\sin x}$

(b) $\left(\sin x\right)^x\left[\dfrac{x\cos x + \sin x\log\left(\sin x\right)}{\sin x}\right]$

(c) $\left(\sin x\right)^x\left[\dfrac{x\sin x + \sin x\log\left(\sin x\right)}{\sin x}\right]$

(d) None of these

(181) $\dfrac{d}{dx}\left\{\left(\sin x\right)^{\log x}\right\} =$

(a) $\left(\sin x\right)^{\log x}\left[\dfrac{1}{x}\log\left(\sin x\right) + \cot x\right]$

(b) $\left(\sin x\right)^{\log x}\left[\dfrac{1}{x}\log\left(\sin x\right) + \cot x\log x\right]$

(c) $\left(\sin x\right)^{\log x}\left[\dfrac{1}{x}\log\left(\sin x\right) + \log x\right]$

(d) None of these

(182) If $y = (\tan x)^{\cot x}$, then $\dfrac{dy}{dx} =$

(a) $y \operatorname{cosec}^2 x\ (1 - \log \tan x)$

(b) $y \operatorname{cosec}^2 x\ (1 + \log \tan x)$

(c) $y \operatorname{cosec}^2 x\ (\log \tan x)$

(d) None of these

(183) If $y = (\sin x)^{\tan x}$, then $\dfrac{dy}{dx}$ is equal to

(a) $(\sin x)^{\tan x} . (1 + \sec^2 x . \log \sin x)$

(b) $\tan x . (\sin x)^{\tan x - 1} . \cos x$

(c) $(\sin x)^{\tan x} . \sec^2 x . \log \sin x$

(d) $\tan x . (\sin x)^{\tan x - 1}$

(184) If $y = \left\{f\left(x\right)\right\}^{\phi(x)}$, then $\dfrac{dy}{dx}$ is

(a) $e^{\phi \log f}\left\{\dfrac{\phi}{f}.\dfrac{df}{dx} + \log f.\dfrac{d\phi}{dx}\right\}$

(b) $\dfrac{\phi}{f}\left(\dfrac{df}{dx}\right) + \dfrac{d\phi}{dx}\log f$

(c) $e^{\phi \log f}\left\{\phi\dfrac{f'}{f}.\dfrac{df}{dx} + \phi' \log f'\right\}$

(d) None of these

(185) If $y = (x \log x)^{\log(\log x)}$, then $\dfrac{dy}{dx} =$

(a) $(x \log x)^{\log(\log x)}\left\{\dfrac{1}{x\log x}\left[\log x + \log\left(\log x\right)\right] + \log\left(\log x\right)\left(\dfrac{1}{x} + \dfrac{1}{x\log x}\right)\right\}$

(b) $(x \log x)^{\log(\log x)} \log (\log x)\left[\dfrac{2}{\log x} + \dfrac{1}{x}\right]$

(c) $(x \log x)^{\log(\log x)}\dfrac{\log\left(\log x\right)}{x}\left[\dfrac{1}{\log x} + 1\right]$

(d) None of these

(186) The first derivative of the function $\left[\cos^{-1}\left(\sin\sqrt{\frac{1+x}{2}}\right)+x^x\right]$ with respect to x at $x = 1$ is

(a) $\frac{3}{4}$ (b) 0 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(187) If $y^x - x^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(a) $2(1 - \log 2)$ (b) $2(1 + \log 2)$

(c) $2 - \log 2$ (d) $1 + \log 2$

(188) If $y = [(\tan x)^{\tan x}]^{\tan x}$, than at $x = \frac{\pi}{4}$, the value of $\frac{dy}{dx}$ =

(a) 0 (b) 1

(c) 2 (d) None of these

(189) The value of $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$, where y is given by $y = x^{\sin x} + \sqrt{x}$, is

(a) $1+\frac{1}{\sqrt{2x}}$ (b) 1 (c) $\frac{1}{\sqrt{2x}}$ (d) $-\frac{1}{\sqrt{2x}}$

(190) If $y = e^{x+e^{x+e^{x+\dots\infty}}}$, then $\frac{dy}{dx}$ =

(a) $\frac{y}{1-y}$ (b) $\frac{1}{1-y}$ (c) $\frac{y}{1+y}$ (d) $\frac{y}{y-1}$

(191) If $y = x^{x^{x\dots\infty}}$, then $\frac{dy}{dx}$ =

(a) $\frac{y^2}{x(1+y\log x)}$ (b) $\frac{y^2}{x(1-y\log x)}$

(c) $\frac{y}{x(1+y\log x)}$ (d) $\frac{y}{x(1-y\log x)}$

(192) If $y = (\sin x)^{(\sin x)^{(\sin x)\dots\infty}}$, then $\frac{dy}{dx}$ =

(a) $\frac{y^2\cot x}{1-y\log(\sin x)}$ (b) $\frac{y^2\cot x}{1+y\log(\sin x)}$

(c) $\frac{y\cot x}{1-y\log(\sin x)}$ (d) $\frac{y\cot x}{1+y\log(\sin x)}$

(193) The differential equation satisfied by the function $y = \sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\dots\infty}}}$ is

(a) $(2y-1)\frac{dy}{dx}-\sin x=0$

(b) $(2y-1)\cos x+\frac{dy}{dx}=0$

(c) $(2y-1)\cos x-\frac{dy}{dx}=0$

(d) $(2y-1)\frac{dy}{dx}-\cos x=0$

(194) If $y = \sqrt{\log x+\sqrt{\log x+\sqrt{\log x+\dots\infty}}}$, then $\frac{dy}{dx}$ =

(a) $\frac{x}{2y-1}$ (b) $\frac{x}{2y+1}$

(c) $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

(195) If $y = x^2+\cfrac{1}{x^2+\cfrac{1}{x^2+\cfrac{1}{x^2+\dots\infty}}}$, then $\frac{dy}{dx}$ =

(a) $\frac{2xy}{2y-x^2}$ (b) $\frac{xy}{y+x^2}$

(c) $\frac{xy}{y-x^2}$ (d) $\frac{2xy}{2+\frac{x^2}{y}}$

(196) If $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}\ldots\infty}}$, then $\frac{dy}{dx} =$

(a) $\frac{y^2}{2x - 2y\log x}$ (b) $\frac{y^2}{2x + \log x}$

(c) $\frac{y^2}{2x + 2y\log x}$ (d) $\frac{y^2}{x(2 - y\log x)}$

(197) If $y = \tan^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$, then $\frac{dy}{dx} =$

(a) $-\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{x}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\frac{\sqrt{1-x^2}}{x}$

(198) If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ equals

(a) $\frac{2}{1-x^2}$ (b) $\frac{1}{1+x^2}$

(c) $\frac{1}{1-x^2}$ (d) $-\frac{2}{1+x^2}$

(199) $\frac{d}{dx}\left[\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] =$

(a) $\frac{1}{1+x^2}$ (b) $-\frac{1}{1+x^2}$

(c) $-\frac{2}{1+x^2}$ (d) $\frac{2}{1+x^2}$

(200) $\frac{d}{dx}\left[\cos^{-1}\left(\frac{x-x^{-1}}{x+x^{-1}}\right)\right] =$

(a) $\frac{1}{1+x^2}$ (b) $\frac{-1}{1+x^2}$

(c) $\frac{2}{1+x^2}$ (d) $\frac{-2}{1+x^2}$

(201) If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$, then $\frac{dy}{dx}$

(a) $\frac{4}{1-x^2}$ (b) $\frac{1}{1+x^2}$ (c) $\frac{4}{1+x^2}$ (d) $\frac{-4}{1+x^2}$

(202) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)\right] =$

(a) $\frac{a}{a^2+x^2}$ (b) $\frac{-a}{a^2+x^2}$

(c) $\frac{1}{a\sqrt{a^2+x^2}}$ (d) $\frac{1}{\sqrt{a^2+x^2}}$

(203) The differential coefficient of $\cos^{-1}\left(\sqrt{\frac{1+x}{2}}\right)$ with respect to x is

(a) $-\frac{1}{2\sqrt{1-x^2}}$ (b) $\frac{1}{2\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1-x}}$ (d) $\sin^{-1}\left(\sqrt{\frac{1+x}{2}}\right)$

(204) $\frac{d}{dx}\left[\cos^{-1}\left(\sqrt{\frac{1+x^2}{2}}\right)\right]=$

(a) $\frac{-1}{2\sqrt{1-x^4}}$ (b) $\frac{1}{2\sqrt{1-x^4}}$

(c) $\frac{-x}{\sqrt{1-x^4}}$ (d) $\frac{x}{\sqrt{1-x^4}}$

(205) If $y=\sin^{-1}\left(x\sqrt{1-x}+\sqrt{x}\sqrt{1-x^2}\right)$, then $\frac{dy}{dx}=$

(a) $\frac{-2x}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$ (b) $\frac{-1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$

(c) $\frac{1}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{x-x^2}}$ (d) None of these

(206) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right]=$

(a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{3}{(1+x)\sqrt{x}}$

(c) $\frac{2}{(1+x)\sqrt{x}}$ (d) $\frac{3}{2(1+x)\sqrt{x}}$

(207) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)\right]$ is equal to

(a) $\frac{1}{1+x^2}$ (b) $\frac{1}{2(1+x^2)}$

(c) $\frac{x^2}{2\sqrt{1+x^2}\left(\sqrt{1+x^2}-1\right)}$ (d) $\frac{2}{1+x^2}$

(208) The differential coefficient of $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ is

(a) $\sqrt{1-x^2}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{1}{2\sqrt{1-x^2}}$ (d) x

(209) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)\right]=$

(a) $\frac{-x}{\sqrt{1-x^4}}$ (b) $\frac{x}{\sqrt{1-x^4}}$

(c) $\frac{-1}{2\sqrt{1-x^4}}$ (d) $\frac{1}{2\sqrt{1-x^4}}$

(210) If $\sqrt{1-x^2}+\sqrt{1-x^2}=a(x-y)$, then $\frac{dy}{dx}$

(a) $\sqrt{\frac{1-x^2}{1-y^2}}$ (b) $\sqrt{\frac{1-y^2}{1-x^2}}$

(c) $\sqrt{\frac{x^2-1}{1-y^2}}$ (d) $\sqrt{\frac{y^2-1}{1-x^2}}$

(211) If $\sqrt{1-x^6}+\sqrt{1-y^6}=a^3\left(x^3-y^3\right)$, then $\frac{dy}{dx}$

(a) $\frac{x^2}{y^2}\sqrt{\frac{1-x^6}{1-y^6}}$ (b) $\frac{y^2}{x^2}\sqrt{\frac{1-y^6}{1-x^6}}$

(c) $\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$ (d) None of these

(212) If $f(x) = \cot^{-1}\left(\frac{x^x - x^x}{2}\right)$, then $f'(1)$ is equal to

(a) -1 (b) 1 (c) $\log 2$ (d) $-\log 2$

(213) If $x = \log p$ and $y = \frac{1}{p}$, then

(a) $\frac{d^2y}{dx^2} - 2p = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$

(c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (d) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

(214) If $y = a + bx^2$, where a, b and arbitrary constants, then

(a) $\frac{d^2y}{dx^2} = 2xy$ (b) $x\frac{d^2y}{dx^2} = \frac{dy}{dx}$

(c) $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0$ (d) $x\frac{d^2y}{dx^2} = 2xy$

(215) If $y = A\cos(nx) + B\sin(nx)$, then $\frac{dy}{dx} =$

(a) n^2y (b) $-y$

(c) $-n^2y$ (d) None of these

(216) If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2} =$

(a) $-16x$ (b) $16x$ (c) x (d) $-x$

(217) If $y = \cos^{-1}\left(\frac{3x}{2}\right) - \sin^{-1}\left(\frac{3x}{2}\right)$, then $\frac{d^2y}{dx^2}$ is

(a) $-3\sqrt{1-y^2}$ (b) $9y$

(c) $-9y$ (d) $3\sqrt{1-y^2}$

(218) If $y = ae^{mx} + be^{-mx}$, then $\frac{d^2y}{dx^2} - m^2y =$

(a) $m^2(ae^{mx} - be^{-mx})$ (b) 1

(c) 0 (d) None of these

(219) If $y = ax^{n+1} + bx^{-n}$, then $x^2\frac{d^2y}{dx^2} =$

(a) $n(n-1)y$ (b) $n(n+1)y$

(c) ny (d) n^2y

(220) If $y = a^x \cdot b^{2x-1}$, then $\frac{d^2y}{dx^2}$ is

(a) $y^2 \cdot \log ab^2$ (b) $y \cdot \log ab^2$

(c) $y \cdot (\log ab^2)^2$ (d) $y \cdot (\log a^2b)^2$

(221) If $y = a\cos(\log x) + b\sin(\log x)$, where a, b are parameters, then $x^2y'' + xy' =$

(a) y (b) $-y$ (c) $2y$ (d) $-2y$

(222) If $x^py^q = (x+y)^{p+q}$, then $\frac{d^2y}{dx^2} =$

(a) 0 (b) 1

(c) 2 (d) None of these

(223) If $x = t^2$, $y = t^3$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3t}{2}$

(224) Let $y = t^{10} + 1$ and $t^8 + 1$, then $\frac{d^2y}{dx^2}$ is

(a) $\frac{5t}{4}$ (b) $20t^8$

(c) $\frac{5}{16t^6}$ (d) None of these

(225) If $x = a\sin\theta$ and $y = b\cos\theta$, then $\frac{d^2y}{dx^2}$ is

(a) $\frac{a}{b^2}\sec^2\theta$ (b) $\frac{-b}{a}\sec^2\theta$

(c) $\frac{-b}{a^2}\sec^3\theta$ (d) $\frac{-b}{a}\sec^3\theta$

(226) The 2nd decrivative of a $\sin^3 t$ with respect to a $\cos^3 t$ at $t = \frac{\pi}{4}$ is

(a) $\frac{4\sqrt{2}}{3a}$ (b) 2

(c) $\frac{1}{12a}$ (d) None of these

(227) If $e \tan^{-1} x$, then $\left(1+x^2\right)\frac{d^2y}{dx^2} =$

(a) $\left(1-x^2\right)\frac{dy}{dx}$ (b) $-2x\frac{dy}{dx}$

(c) $-x\frac{dy}{dx}$ (d) 0

(228) If $y = x^2 e^{mx}$, where m is a constatn,then $\frac{d^3y}{dx^3} =$

(a) $me^{mx}(m^2x^2 + 6mx + 6)$

(b) $2m^3 xe^{mx}$

(c) $me^{mx}(m^2x^2 + 2mx + 2)$

(d) None of these

(229) Let f be a polynomial, then the second derivative of $f(e^x)$ is

(a) $f'(e^x)$ (b) $f''(e^x)\, e^x + f'(e^x)$

(c) $f''(e^x)\, e^{2x} + f''(e^x)$ (d) $f''(e^x)\, e^{2x} + f'(e^x)\, e^x$

(230) If $y = ae^x + be^{-x} + c$, where a, b, c are parameters, they $y''' =$

(a) y (b) y' (c) 0 (d) y''

(231) If $f(x) = be^{ax} + ae^{bx}$, then $f''(0)$ is equal to

(a) 0 (b) $2ab$

(c) $ab\,(a + b)$ (d) ab

(232) If $u = x^2 + y^2$ and x = s + 3t, y = 2s –t, then $\frac{d^2u}{ds^2} =$

(a) 12 (b) 32 (c) 36 (d) 10

(233) $\frac{d^n}{dx^n}\left(\log x\right) =$

(a) $\frac{\left(n-1\right)!}{x^n}$ (b) $\frac{n!}{x^n}$

(c) $\frac{\left(n-2\right)!}{x^n}$ (d) $\left(-1\right)^{n-1}\frac{\left(n-1\right)!}{x^n}$

(234) The nth derivative of xe^x vanishes when

(a) $x = 0$ (b) $x = -1$

(c) $x = -n$ (d) $x = n$

(235) $\frac{d^2}{dx^2}(2 \cos x \cos 3x)$

(a) $2^2(\cos 2x + 2^2 \cos 4x)$

(b) $2^2(\cos 2x - 2^2 \cos 4x)$

(c) $2^2(-\cos 2x + 2^2 \cos 4x)$

(d) $-2^2(\cos 2x + 2^2 \cos 4x)$

(236) If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \ldots\ldots.$, then $\frac{d^2y}{dx^2} =$

(a) x (b) $-x$ (c) $-y$ (d) y

(237) If y2 = ax2 + bx + c, then $y^2\frac{d^2y}{dx^2}$ is

(a) a constant (b) a function of x only

(c) a function of y only (d) a function of x and y

(238) If $y = x + e^x$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{1}{\left(1+e^x\right)^2}$ (b) $\frac{e^x}{\left(1+e^x\right)^2}$

(c) $-\frac{e^x}{\left(1+e^x\right)^2}$ (d) e^x

(239) If $x^2 + y^2 =$, then

(a) $yy'' - 2(y')^2 + 1 = 0$ (b) $yy'' + (y')^2 + 1 = 0$

(c) $yy'' + (y')^2 - 1 = 0$ (d) $yy'' + 2(y')^2 + 1 = 0$

(240) If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant.

Then $\frac{d^3}{dx^3}$ $\{f(x)\}$ at $x = 0$ is

(a) p (b) $p + p^2$

(c) $p + p^3$ (d) independent of p

(241) $f(x) = \begin{vmatrix} x^3 & x^2 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant.

Then $\frac{d^3 f(x)}{dx^3}$ is

(a) Proportional to x^2 (b) Proportional to x

(c) Proportional to x^3 (d) a constant

(242) If $f(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$, than the value of

$\frac{d^n}{dx^n}$ $\{f(x)\}$ at $x = 0$ for $n = 2m + 1$ is

(a) –1 (b) 0

(c) 1 (d) independent of a

(243) $\frac{d^2y}{dx^2}$ equals

(a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (b) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(244) If $y = e^{\tan x}$, then $\cos^2 x \frac{d^2y}{dx^2}$ is equal to

(a) $(1 - \sin 2x)\frac{dy}{dx}$ (b) $-(1 + \sin 2x)\frac{dy}{dx}$

(c) $(1 + \sin 2x)\frac{dy}{dx}$ (d) None of these

(245) If $y = \sin(m \sin^{-1} x)$, then $(1 - x|)\, y'' - xy'$ is equal to

(a) m^2y (b) my

(c) $-m^2y$ (d) None of thse

(246) If $2 = e^{ax} \sin bx$, then $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + a^2y$ is equal to

(a) 0 (b) 1 (c) $-b^2y$ (d) $-by$

(247) If $y = \cos(m \sin^{-1} x)$, which of the following is true?

(a) $(1 - x^2)\, y_2 + xy_1 - m^2y = 0$

(b) $(1 - x^2)\, y_2 - xy_1 + m^2y = 0$

(c) $(1 + x^2)\, y_2 + xy_1 + m^2y = 0$

(d) $(1 + x^2)\, y_2 - xy_1 + m^2y = 0$

(248) If $f(x) = e^x \sin x$, then $f^6(x)$ is equal to

(a) $e^{6x} \sin 6x$ (b) $-8e^x \cos x$

(c) $8e^x \sin x$ (d) $8e^x \cos x$

(249) If $f(x) = x^n$, then the value of

$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \frac{f'''(1)}{3!} + \ldots\ldots.. + \frac{(-1)^n f^n(1)}{n!}$ is

(a) 2^n (b) 2^{n-1} (c) 0 (d) 1

(250) If $y = \sin x + e^x$, then $\frac{d^2y}{dx^2}$ =

(a) $(-\sin x + e^x)^{-1}$ (b) $\frac{\sin x - e^x}{(\cos x + e^x)^2}$

(c) $\frac{\sin x - e^x}{(\cos x + e^x)^3}$ (d) $\frac{\sin x + e^x}{(\cos x + e^x)^3}$

(251) If $y = \left(x + \sqrt{1+x^2}\right)^n$, then $\left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$

(252) If $y = x^3 \log \log_e (1 + x)$, then $y''(0)$ equals

(a) 0 (b) –1 (c) $6 \log_e 2$ (d) 6

(253) If $y = \sin(\sin x)$ and $\frac{d^2y}{dx^2} + \frac{dy}{dx} \tan x + f(x) = 0$, then $f(x)$ equals

(a) $\sin^2 x \sin (\cos x)$ (b) $\sin^2 x \cos (\sin x)$

(c) $\cos^2 x \sin (\cos x)$ (d) $\sin^2 x \sin (\sin x)$

(254) If $y = x \log \left(\frac{x}{a+bx}\right)$, then $x^3 \frac{d^2y}{dx^2} =$

(a) $x\frac{dy}{dx} - y$ (b) $\left(x\frac{dy}{dx} - y\right)^2$

(c) $y\frac{dy}{dx} - x$ (d) $\left(y\frac{dy}{dx} - x\right)^2$

(255) If $e^y + xy = e$, then the value of $\frac{d^2y}{dx^2}$ for $x = 0$, is

(a) $\frac{1}{e}$ (b) $\frac{1}{e^2}$

(c) $\frac{1}{e^3}$ (d) None of these

(256) If $f(x) = \tan^{-1}\left[\frac{\log\left(\frac{e}{x^2}\right)}{\log\left(ex^2\right)}\right] + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\tan^{-1} [(\log x)^n]$ (b) 0

(c) $\frac{1}{2}$ (d) None of these

(257) If $x = \sin t$ and $y = \sin pt$, then the value of $\left(1-x^2\right)\frac{d^2y}{dx^2} - \frac{dy}{dx} + p^3y$ is equal to

(a) 0 (b) 1 (c) –1 (d) $\sqrt{2}$

(258) If $x = \cos \theta$, $y = \sin 5\theta$, then $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$ is equal to

(a) $-5y$ (b) $5y$ (c) $25y$ (d) $-25y$

(259) If $x = e^t \sin t$, $y = e^t \cos t$, t is a parameter, then $\frac{d^2y}{dx^2}$ at (1, 1) is equal to

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$ (c) 0 (d) $\frac{1}{2}$

CLASS WORK - ANSWER KEY

1 **d**	2 **b**	3 **a**	4 **c**	5 **c**	6 **c**	7 **b**	8 **b**	9 **a**	10 **c**
11 **c**	12 **a**	13 **d**	14 **b**	15 **b**	16 **b**	17 **c**	18 **d**	19 **a**	20 **b**
21 **c**	22 **b**	23 **d**	24 **b**	25 **b**	26 **b**	27 **a**	28 **a**	29 **a**	30 **b**
31 **d**	32 **b**	33 **d**	34 **c**	35 **d**	36 **b**	37 **d**	38 **b**	39 **b**	40 **c**
41 **d**	42 **b**	43 **a**	44 **a**	45 **c**	46 **b**	47 **b**	48 **c**	49 **b**	50 **c**
51 **d**	52 **b**	53 **b**	54 **b**	55 **d**	56 **d**	57 **d**	58 **c**	59 **d**	60 **d**
61 **a**	62 **d**	63 **d**	64 **d**	65 **c**	66 **d**	67 **a**	68 **c**	69 **d**	70 **d**
71 **c**	72 **a**	73 **c**	74 **c**	75 **c**	76 **b**	77 **c**	78 **b**	79 **d**	80 **c**
81 **a**	82 **a**	83 **b**	84 **d**	85 **d**	86 **b**	87 **c**	88 **b**	89 **b**	90 **d**
91 **a**	92 **a**	93 **c**	94 **b**	95 **c**	96 **c**	97 **d**	98 **b**	99 **d**	100 **c**
101 **c**	102 **c**	103 **a**	104 **b**	105 **c**	106 **d**	107 **b**	108 **c**	109 **b**	110 **a**
111 **c**	112 **b**	113 **c**	114 **d**	115 **a**	116 **c**	117 **d**	118 **a**	119 **b**	

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HOME WORK - ANSWER KEY

1 **b**	2 **d**	3 **a**	4 **b**	5 **a**	6 **a**	7 **c**	8 **b**	9 **d**	10 **c**
11 **b**	12 **b**	13 **c**	14 **b**	15 **c**	16 **c**	17 **b**	18 **b**	19 **a**	20 **a**
21 **c**	22 **b**	23 **d**	24 **b**	25 **c**	26 **c**	27 **a**	28 **b**	29 **c**	30 **c**
31 **b**	32 **c**	33 **b**	34 **b**	35 **a**	36 **c**	37 **c**	38 **b**	39 **c**	40 **b**
41 **c**	42 **a**	43 **d**	44 **d**	45 **d**	46 **b**	47 **b**	48 **c**	49 **d**	50 **d**
51 **a**	52 **d**	53 **c**	54 **b**	55 **d**	56 **d**	57 **c**	58 **c**	59 **d**	60 **a**
61 **d**	62 **d**	63 **c**	64 **d**	65 **b**	66 **c**	67 **b**	68 **a**	69 **c**	70 **c**
71 **a**	72 **b**	73 **d**	74 **c**	75 **a**	76 **b**	77 **a**	78 **c**	79 **c**	80 **c**
81 **a**	82 **b**	83 **b**	84 **a**	85 **d**	86 **b**	87 **d**	88 **b**	89 **b**	90 **b**
91 **b**	92 **b**	93 **b**	94 **b**	95 **a**	96 **d**	97 **c**	98 **b**	99 **b**	100 **a**
101 **a**	102 **b**	103 **a**	104 **c**	105 **b**	106 **d**	107 **c**	108 **c**	109 **b**	110 **c**
111 **c**	112 **c**	113 **b**	114 **a**	115 **a**	116 **b**	117 **b**	118 **d**	119 **a**	120 **b**
121 **b**	122 **d**	123 **b**	124 **a**	125 **d**	126 **d**	127 **a**	128 **d**	129 **d**	130 **b**
131 **b**	132 **d**	133 **b**	134 **b**	135 **b**	136 **c**	137 **b**	138 **b**	139 **b**	140 **b**
141 **c**	142 **b**	143 **a**	144 **b**	145 **c**	146 **a**	147 **d**	148 **b**	149 **b**	150 **b**
151 **a**	152 **d**	153 **d**	154 **b**	155 **b**	156 **b**	157 **b**	158 **a**	159 **c**	160 **b**
161 **a**	162 **c**	163 **a**	164 **a**	165 **a**	166 **a**	167 **a**	168 **a**	169 **b**	170 **c**
171 **a**	172 **a**	173 **c**	174 **a**	175 **b**	176 **a**	177 **b**	178 **d**	179 **a**	180 **b**
181 **b**	182 **a**	183 **a**	184 **a**	185 **a**	186 **a**	187 **a**	188 **c**	189 **a**	190 **a**
191 **b**	192 **a**	193 **d**	194 **c**	195 **a**	196 **d**	197 **c**	198 **d**	199 **d**	200 **d**
201 **c**	202 **d**	203 **a**	204 **c**	205 **c**	206 **d**	207 **b**	208 **c**	209 **a**	210 **d**
211 **c**	212 **a**	213 **c**	214 **b**	215 **c**	216 **a**	217 **c**	218 **c**	219 **b**	220 **c**
221 **b**	222 **a**	223 **b**	224 **c**	225 **c**	226 **a**	227 **a**	228 **a**	229 **d**	230 **b**
231 **c**	232 **d**	233 **d**	234 **c**	235 **d**	236 **d**	237 **a**	238 **c**	239 **b**	240 **d**
241 **d**	242 **b**	243 **d**	244 **c**	245 **c**	246 **c**	247 **d**	248 **b**	249 **c**	250 **c**
251 **a**	252 **a**	253 **d**	254 **b**	255 **b**	256 **b**	257 **a**	258 **d**	259 **a**	

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