Formula sheet

Matrices

- Inverse of matrices:
  - If $|A| \neq 0$, then A^{-1} exits.
  - If inverse exists, then it is unique.
  - Inverse of 2x2, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$|A| = ad - bc \qquad \therefore A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

• diagonal interchang
• non-diagonal sign change

  - Inverse of 3x3,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad A^{-1} = \begin{array}{c|cccc} a & b & c & a & b \\ \hline d & e & f & d & e \\ g & h & i & g & h \\ a & b & c & a & b \\ d & e & f & d & e \end{array} = \begin{bmatrix} ei-fh & hc-ib & bf-ce \\ fg-id & ia-gc & cd-af \\ dh-eg & gb-ha & ae-bd \end{bmatrix}$$

- Elementry Transformation:

i) Row Transformation

consider $AA^{-1} = I$

convert A into $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

⋮

$A^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$

ii) Coloumn Transformation

consider $A^{-1}A = I$

convert A into $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

⋮

$A^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$

- Adjoint of matrix:

cofactor matrix:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$\therefore$ adjoint = Transpose of cofactor matrix

$$= \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

- Application of matrix:

i) Method of inversion

$x+2y=5 \quad 2x-y=3$

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$A^{-1}(AX) = A^{-1}B$$

$$IX = A^{-1}B$$

$$\therefore X = A^{-1}B$$

ii) Method of Reduction.

Reduce to (convert)

i) diagonal

ii) Upper or lower triangular

- Important result and short tricks:

1. $(A')' = A$
2. $(kA)' = kA'$
3. $(AB)' = B'A'$
4. $(A^{-1})^{-1} = A$
5. $(A')^{-1} = (A^{-1})'$
6. $(AB)^{-1} = B^{-1}A^{-1}$
7. $(kA)^{-1} = 1/k \ A^{-1}$
8. $AA^{-1} = A^{-1}A = I$
9. $adj(kA) = k^{n-1} adj A$.
10. $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = |A|$
11. $a_{11}a_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$
12. $A(adj A) = (adj A)A = |A|I_n$
13. $|adj A| = |A|^{n-1}$
14. $adj(adj A) = |A|^{n-1}A$
15. $|adj(adj A)| = |A|^{(n-1)^2}$
16. Three type of elementry trasf.
    i) $R_i \leftrightarrow R_j \quad C_i \leftrightarrow C_j$
    ii) $R_i \rightarrow kR_j \quad C_i \rightarrow kC_j$
    iii) $R_i \rightarrow R_i + kR_j \quad C_i \rightarrow C_i + kC_j$
17.

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \quad a^u = \begin{bmatrix} a^u & 0 & 0 \\ 0 & b^u & 0 \\ 0 & 0 & c^u \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$$