Conic Section

Parabola

Equation of chord:

$$y(t_1+t_2) = 2x + 2at_1t_2$$

Diff. forms of tangent:

$\Rightarrow y = mx + \frac{a}{m}$ → for standard

$\Rightarrow y = \frac{x}{t} + at$

$\Rightarrow x^2 \to xx_1, \quad y^2 \to yy_1, \quad x \to \frac{x+x_1}{2}, \quad y \to \frac{y+y_1}{2}, \quad xy = \frac{xy_1+yx_1}{2}$

Normal of Parabola:

$\Rightarrow y = -tx + 2at + at^3$

$\Rightarrow y = mx - 2am - am^3$

Chord of contact:

$\Rightarrow T = 0, \quad yy_1 - 4a\left(\frac{x+x_1}{2}\right) = 0$

COC with given mid point:

$\Rightarrow yy_1 - 4a\left(\frac{x+x_1}{2}\right) = y_1^2 - 4ax_1$

Pair of tangents:

$$SS_1 = T^2$$

$S = y^2 - 4ax$

$S_1 = y_1^2 - 4ax_1$

$T = yy_1 - 4a\left(\frac{x+x_1}{2}\right)$

Ellipse

Equation of chord:

$$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$$

Diff. forms of tangent:

$\Rightarrow y = mx \pm \sqrt{a^2m^2+b^2}$ → for standard

$\Rightarrow \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

$\Rightarrow \frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

Normal of Ellipse:

$\Rightarrow ax\sec\theta - by\csc\theta = a^2 - b^2 = a^2e^2$

$\Rightarrow \frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 - b^2 = a^2e^2$

Director Circle:

locus of P.O.I of $\perp^r$ tangents.

$$x^2 + y^2 = a^2 + b^2$$

Note:

⇒ if ~~focal~~ chord passes through $(d, 0)$,

$$\frac{d-a}{d+a} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}$$

⇒ P.O.I of tangents at α, β

$$x = \frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} \qquad y = \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$$

Hyperbola

Eqn of chord:

$$\frac{x}{a}\cos\left(\frac{\alpha-\beta}{2}\right) - \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha+\beta}{2}\right)$$

Diff. forms of tangent:

$\Rightarrow y = mx \pm \sqrt{a^2m^2-b^2}$

$\Rightarrow \frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

$\Rightarrow \frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$

Normal of Hyperbola

$\Rightarrow \frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2 = a^2e^2$

$\Rightarrow \frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2 = a^2e^2$

Director circle:

$$x^2 + y^2 = a^2 - b^2$$

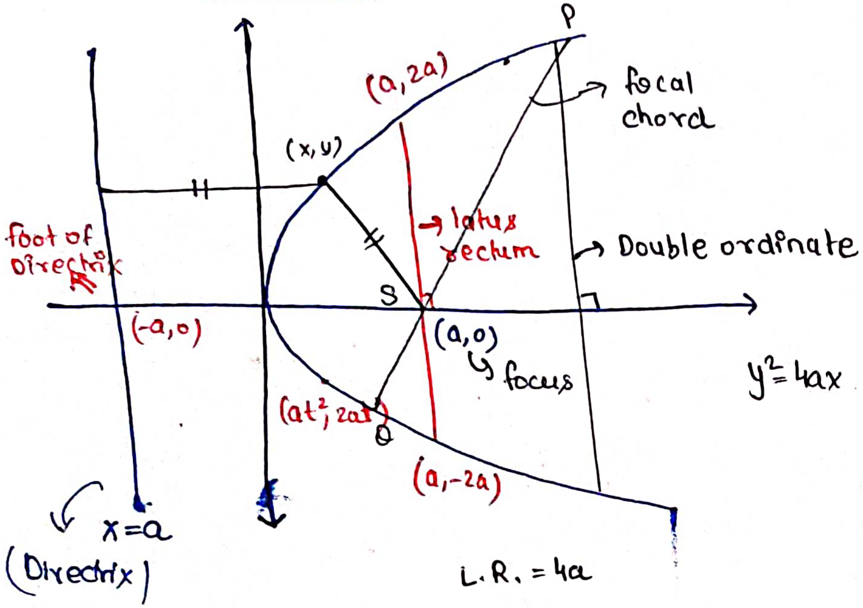
Asymptotes:

$$y = \pm\frac{b}{a}x$$

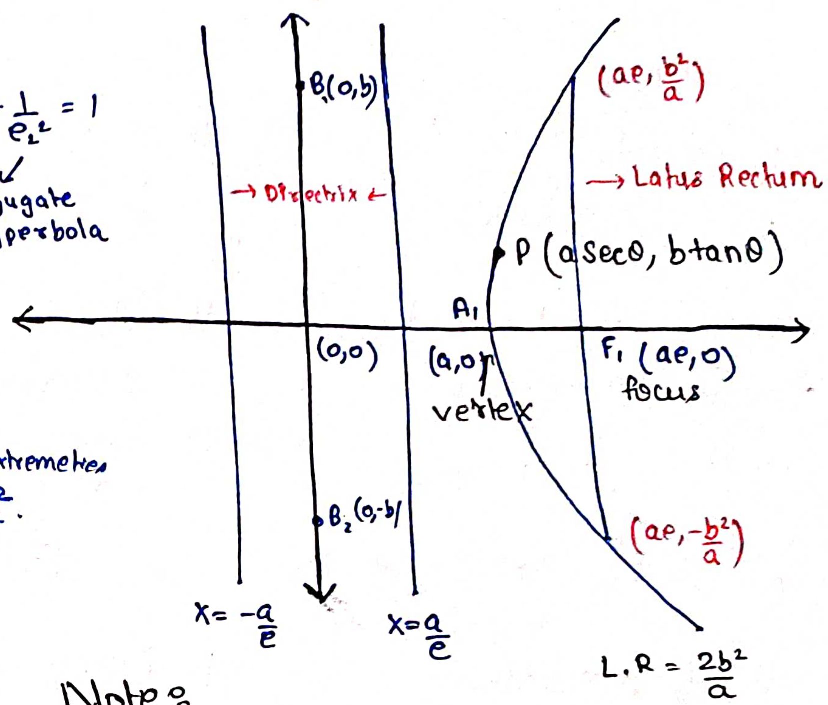
combined eqn of assymtodes:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

Parabola



Hyperbola



$\frac{1}{e_1^2}+\frac{1}{e_2^2}=1$ → conjugate Hyperbola

* area of rhombus formed by tangents at extremeties of L.R. of ellipse is $\frac{2a^2}{e}$.

Note:

* if line joining t_1 & t_2 passes through $(c,0)$, $\boxed{t_1t_2 = -\frac{c}{a} = \text{const.}}$
* if line joining t_1 & t_2 is focal chord, $\boxed{t_1t_2=-1}$
* if chord of parabola subtends 90° at centre, $\boxed{t_1t_2=-4}$
* H.M. of SP & SQ is semi major latus rectum '2a'.
* Length of focal chord : $\boxed{a\left(t+\frac{1}{t}\right)^2}$ or $\boxed{4a\,\text{Cosec}^2\theta}$
  min length = 4a = latus rectum.
* P.O.I of tangents at t_1 & t_2 : $[at_1t_2,\ a(t_1+t_2)]$
* if Normal at t_1 int. parabola again at t_2 : $\boxed{t_2=-t_1-\frac{2}{t_1}}$
* if Normal at t_1 & t_2 int. at t_3, $\boxed{\begin{matrix} t_1t_2=2 \\ t_1+t_2+t_3=0\end{matrix}}$ ⇒ line joining t_1, t_2 passes through $(-2a,0)$

Note:

* quadrilateral formed by joing the focus of conjugate Hyperbolas is $\boxed{\text{square.}}$
* $\boxed{e=\sqrt{2}}$ for rectangular Hyperbola.
* if chord passes through $(d,0)$ then, $\boxed{\frac{a-d}{a+d}=\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}}$
* Rectangular Hyperbola : $xy=c^2$ $a^2=b^2$
  focus : $\pm(\sqrt{2}c, \sqrt{2}c)$
  vertex : $\pm(c,c)$
  T.A : $y=x$
  C.A : $y=-x$
  Director circle : $x^2+y^2=0$
  auxilary circle : $x^2+y^2=(\sqrt{2}c)^2$

L.R. $=\frac{2b^2}{a}=2a=2\sqrt{2}c$