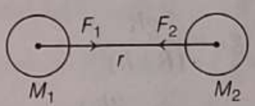
CHAPTER 03

Gravitation

The force by which material objects are attracted towards each other is known as **gravitation** or **gravitational force**. The pair of the two massive bodies always exerts a gravitational force of same magnitude but in mutually opposite direction on each other.

Newton's Law of Gravitation

Newton's law of gravitation states that "Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them."



Gravitational fore between two particles

Thus, the magnitude of the gravitational force F between two particles of masses m_1 and m_2 placed at a distance r is

$$F \propto \frac{m_1 m_2}{r^2}$$

or $$F = G\frac{m_1 m_2}{r^2}$$

Here, G is a universal constant called **gravitational constant** whose magnitude is

$$G = 6.67 \times 10^{-11} \text{ N-m}^2 \text{ kg}^{-2}$$
$$= 6.67 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}$$

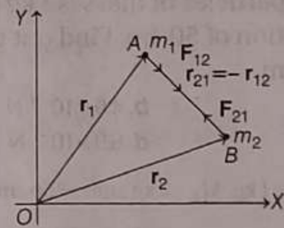
The dimensions of gravitational constant are

$$[G] = [M^{-1}L^3T^{-2}]$$

The direction of the gravitational force F is along the line joining the two particles. It is a vector quantity. Its unit is newton N and dimensions are $[MLT^{-2}]$.

Vector Form of Newton's Law of Gravitation

The vector form of Newton's law of gravitation signifies that the gravitational forces acting between the two particles form action and reaction pair.



Vector form of gravitational force ($\mathbf{F}_{12}$) on m_1 due to m_2

In figure, it can be seen that the two particles of masses m_1 and m_2 are placed at a distance r, therefore according to Newton's law of gravitation, force on m_1 due to m_2,

$$\mathbf{F}_{12} = -\frac{Gm_1m_2}{|\mathbf{r}_{12}|^2} \cdot \hat{\mathbf{r}}_{12} \quad \text{...(i)}$$

$$\mathbf{F}_{12} = -Gm_1m_2 \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} \quad \left(\because \hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|}\right)$$

where, $\hat{\mathbf{r}}_{12}$ is a unit vector pointing from m_2 to m_1.

The negative sign in Eq.(i) indicates the direction of force $\mathbf{F}_{12}$ is opposite to that of $\hat{\mathbf{r}}_{12}$.

Similarly, force on m_2 due to m_1,

$$\mathbf{F}_{21} = -\frac{Gm_1m_2}{|\mathbf{r}_{21}|^2} \cdot \hat{\mathbf{r}}_{21} \quad \text{...(ii)}$$

where, $\hat{\mathbf{r}}_{21}$ is a unit vector pointing from m_1 to m_2.

From Eqs. (i) and (ii), we get

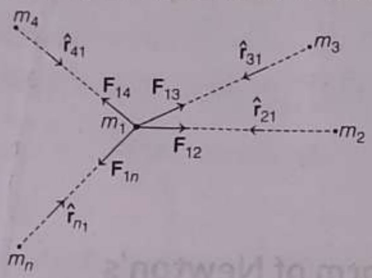
$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

$$\Rightarrow \quad |\mathbf{F}_{12}| = |\mathbf{F}_{21}| \quad \text{...(iii)}$$

As, $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ are directed towards the centres of the two particles, so gravitational force is conservative in nature.

Principle of Superposition of Gravitational Forces

Suppose $F_1, F_2, ..., F_n$ be the individual forces due to the masses $m_1, m_2, ..., m_n$ which are given by the universal law of gravitation, then from the principle of superposition, each of these forces acts independently and uninfluenced by the other bodies as shown in figure.



Superposition of gravitational forces

So, the resultant force F can be expressed in vector addition of various forces, $\mathbf{F} = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + ... + \mathbf{F}_{1n}$

Example 1. Two particles of masses 1 kg and 2 kg are placed at a separation of 50 cm. Find out gravitational force between them.

a. 5.3×10^{-10} N b. 4.9×10^{-9} N
c. 6.5×10^{-8} N d. 6.9×10^{-7} N

Sol. (*a*) Given, $M_1 = 1$ kg, $M_2 = 2$ kg and $r = 50 \text{ cm} = \frac{1}{2}$ m

$\therefore$ Gravitational force,

$$F = \frac{G\,M_1 M_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 2}{\left(\frac{1}{2}\right)^2} = 5.3 \times 10^{-10} \text{ N}$$

Gravity

In Newton's law of gravitation, we consider the gravitation as the force. But in case of two gravitationally interacting masses (or bodies), if one is the earth, then the gravitational force is called as the gravity.

Acceleration due to Gravity

The acceleration produced in the motion of a body under the effect of gravity (earth's gravitation) is called acceleration due to gravity (g).

Acceleration due to gravity, $g = \frac{F}{m} = \frac{\frac{GMm}{R^2}}{m} = \frac{GM}{R^2}$

On the surface of the earth, $g = \frac{GM}{R^2}$

where, mass of the earth, $M = 6 \times 10^{24}$ kg and radius of the earth, $R = 6.4 \times 10^6$ m and gravitational constant, $G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2}.

Substituting the values of G, M and R, we get

$$g = 9.81 \text{ ms}^{-2}$$

- g in term of density and radius of a planet, is given by $g = \frac{4}{3}\pi G R \rho$

  where, ρ is density of the planet.
- Acceleration due to gravity on the surface of moon is $g/6$.

Example 2. If the mass and radius of the earth, each decreases by 50%, the acceleration due to gravity would be

a. remain same b. decrease by 50%
c. decrease by 100% d. increase by 100%

Sol. (*d*) Here, acceleration due to gravity, $g = GM/R^2$ ...(i)

and changed value of acceleration due to gravity due to variation of mass and radius,

$$g' = \frac{G(M/2)}{(R/2)^2} = \frac{2GM}{R^2} = 2g \quad \text{[from Eq. (i)]}$$

$$\therefore \text{ \% increase in } g = \left(\frac{g' - g}{g}\right) \times 100 = \left(\frac{2g - g}{g}\right) \times 100 = 100\%$$

Variation of Acceleration due to Gravity

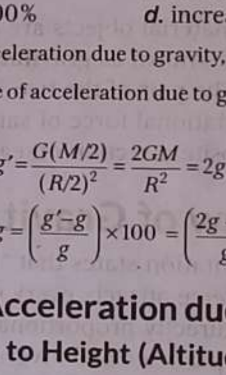
1. Variation due to Height (Altitude)

The value of acceleration due to gravity at a height h from the surface of the earth is given by

$$g_h = \frac{gR^2}{(R+h)^2}$$

If $h << R$, then $g_h = g\left(1 - \frac{2h}{R}\right)$

or $$g - g_h = \Delta g = \frac{2h}{R} g$$



Finding the acceleration due to gravity at height h

Example 3. At what height from the surface of the earth, will the value of g be reduced by 36% from the value at the surface? (Take, $R_e = 6400$ km)

a. 1800 km b. 1600 km c. 2000 km d. 2200 km

Sol. (*b*) Let at height h, the value of g reduces by 36%. It becomes 64% of that at the surface.

Thus, $g_h = 64\% \text{ of } g = g\frac{64}{100}$

$\because \quad g_h = g\dfrac{R_e^{\,2}}{(R_e+h)^2}$

$\Rightarrow \quad \dfrac{64}{100}g = g\dfrac{R_e^{\,2}}{(R_e+h)^2}$

$\Rightarrow \quad \left(\dfrac{8}{10}\right)^2 = \left(\dfrac{R_e}{R_e+h}\right)^2$

$\Rightarrow \quad \dfrac{8}{10} = \dfrac{R_e}{R_e+h} \Rightarrow \dfrac{R_e+h}{R_e} = \dfrac{10}{8}$

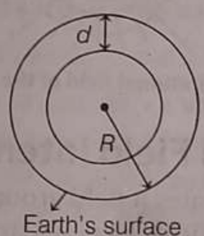
$\Rightarrow \quad \dfrac{h}{R_e}+1 = \dfrac{10}{8} \Rightarrow \dfrac{h}{R_e} = \dfrac{2}{8}$

$\therefore \quad h = \dfrac{R_e}{4} = \dfrac{6400}{4} = 1600\text{ km}$

2. Variation due to Depth

The value of acceleration due to gravity at a depth d from the surface of the earth is given by

$$g_d = g\left(1-\frac{d}{R}\right)$$



Finding the acceleration due to gravity at depth d

- Decrease in the value of g with depth d is, $g - g' = \dfrac{g}{R}d$

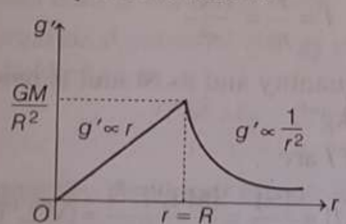
$\therefore$ Frictional decrease in the value of g with depth

$$= \frac{g-g'}{g} = \frac{d}{R}$$

$\therefore$ Percentage decrease in the value of $g = \dfrac{g-g'}{g}\times 100$

We can see from this equation that $g' = 0$ at $d = R$, *i.e.* acceleration due to gravity is zero (minimum value) at the centre of the earth.

- The graphical representation of change in the value of g with height and depth is as follows



Graphical representation for variation in g

For $r \le R$, $\quad g' = g\left(1-\dfrac{d}{R}\right) = \dfrac{gr}{R} \quad$ (as, $r = R-d$)

or $\quad g' \propto r$

For $r > R$, $\quad g' = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2} = \dfrac{gR^2}{r^2} \quad$ (as, $r = R+h$)

or $\quad g' \propto \dfrac{1}{r^2}$

Note *The acceleration due to gravity is maximum (having standard value of 9.8 ms^{-2}) at the earth's surface. It decreases either we go at higher altitudes or we move below the surface.*

Example 4. What would be the acceleration due to gravity at a depth of 2000 km from the earth's surface, assuming that earth has uniform density? (Take, $R = 6400$ km)

a. 4.37 ms^{-2} b. 6.737 ms^{-2}
c. 5.709 ms^{-2} d. 4.751 ms^{-2}

Sol. (*b*) Given, depth, $d = 2000\text{ km} = 2\times10^6$ m

Radius of earth, $R = 6400\text{ km} = 6.4\times10^6$ m

The acceleration due to gravity at a point at a depth d from the earth's surface is given by

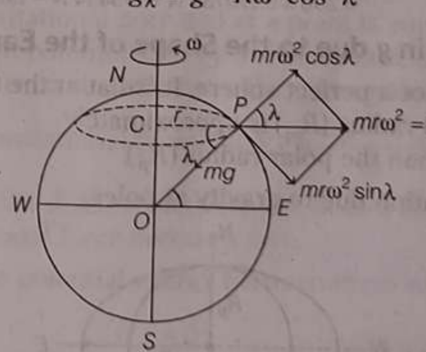
$$g_d = g\left[1-\frac{d}{R}\right]$$

$$\therefore \quad g_d = 9.8\left[1-\frac{2\times10^6}{6.4\times10^6}\right] = \frac{9.8\times4.4}{6.4} = 6.737\text{ ms}^{-2}$$

3. Variation due to Rotation (Motion) of the Earth

Due to rotation of the earth, the value of g decreases as the speed of rotation of the earth increases. The value of acceleration due to gravity at a latitude λ is given by

$$g'_\lambda = g - R\omega^2\cos^2\lambda$$



Variation in g due to rotation of the earth

Following conclusions can be written from the above figure

(i) The effect of centrifugal force due to rotation of the earth reduces the effective value of g.

(ii) The effective value of g is not truly in vertical direction.

(iii) At the equator, $\lambda = 0°$
Therefore, $g' = g - R\omega^2$

(iv) At the poles, $\lambda = 90°$
Therefore, $g' = g$

- If the earth stops rotating about its own axis, then there will be no change in the value of g on the poles, but there will be increases in the value of g by about $0.35\ \text{m/s}^2$ at equator.
- If the earth starts rotating 17 times faster than its present rate, the value of g on the equator will become zero, but it will remain unchanged at the poles.

Example 5. Find the weight of the body of mass 200 kg on the earth at equator and at the latitude of 30°.
(Take, $R = 6400$ km)

a. 1953 and 1955 N **b.** 1960 and 1965 N
c. 1860 and 1800 N **d.** None of these

Sol. (*a*) Given, mass, $m = 200$ kg and radius, $R = 6400$ km

Angular velocity of the earth, $\omega = \dfrac{2\pi}{T}$

$$= \frac{2\pi}{24 \times 60 \times 60} = \frac{2\pi}{86400}$$

$$\Rightarrow \quad \omega = 7.268 \times 10^{-5}\ \text{rads}^{-1}$$

To find the weight of the body, at the equator, $\lambda = 0$

$\therefore$ Acceleration due to gravity at equator, $g = g - R\omega^2 \cos\lambda$

$$= [9.8 - 6.4 \times 10^6 \times (7.268 \times 10^{-5})^2 \times 1]$$
$$= 9.8 - 0.03379 = 9.766\ \text{ms}^{-2}$$

Weight at equator $= mg = 200 \times 9.766 \approx 1953$ N

At latitude, $\lambda = 30°$

$$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$$

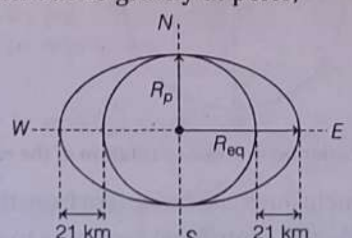
Therefore, at the latitude of 30°, the weight of the body

$$= m(g - R\omega^2 \cos 30°)$$
$$= 200 \times (9.8 - 0.03379 \times 0.866)$$
$$= 200 \times 9.770734\ \text{N} = 1954.14\ \text{N} \simeq 1955\ \text{N}$$

4. Variation in g due to the Shape of the Earth

The earth is not a perfect sphere. It is flat at the two poles. The equitorial radius (R_{eq}) is approximately 21 km more than the polar radius (R_p).

Now, acceleration due to gravity at poles,



Variation in g due to shape of the earth

$$g_p = \frac{GM}{R_{p}^2}$$

Acceleration due to gravity at equator,

$$g_{eq} = \frac{GM}{R_{eq}^2}$$

$$\because \quad R_p < R_{eq}$$

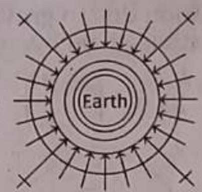
$$\therefore \quad g_p > g_{eq}$$

Difference in g at poles and equator due to shape

$$(\Delta g) = g_p - g_{eq} = 0.02\ \text{ms}^{-2}$$

Gravitational Field

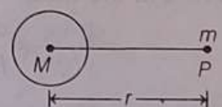
Gravitational field of a given body is the space around it, within which gravitational force due to that body may be experienced.



Gravitational field of the earth

Gravitational Field Intensity

A massive body creates a field around it which attracts other masses. This field is known as **gravitational field.** It is a field of conservative force. The force exerted by this field on a body of unit mass within the field is called gravitational field intensity or gravitational field strength.



Gravitational force on point mass m at point P due to mass M is

$$F = \frac{GMm}{r^2}$$

Intensity of gravitational field of M at the point P is

$$I = \frac{F}{m} = \frac{GM}{r^2}$$

It is a vector quantity and its SI unit is newton/kilogram or Nkg^{-1}.

Dimensions of I are

$$[I] = \frac{[\text{N}]}{[\text{kg}]} = \frac{[\text{M}^1\text{L}^1\text{T}^{-2}]}{[\text{M}^1]} = [\text{M}^0\text{L}^1\text{T}^{-2}]$$

Direction of intensity is same as that of the force.

The total intensity of gravitational field at a point is the vector sum of the intensity of gravitational field due to individual mass bodies present in that field.

- Intensity of gravitational field of uniform solid sphere and spherical shell are given in tabular form below.

Intensity due to Uniform Solid Sphere

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = \frac{GMr}{R^3}$

Intensity due to Spherical Shell

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = 0$

Example 6. At what distance (in metre) from the centre of the earth, the intensity of gravitational field will be zero? (Take, mass of earth and moon are 5.98×10^{24} kg and 7.35×10^{22} kg respectively and the distance between moon and earth is 3.85×10^8 m.)

a. Zero **b.** 3.90×10^7 **c.** 8×10^8 **d.** 3.46×10^8

Sol. (*d*) Given mass of earth, $M_e = 5.98 \times 10^{24}$ kg

Mass of moon, $M_m = 7.35 \times 10^{22}$ kg

Let x be the distance of the point from the centre of earth, where gravitational intensity is zero. Therefore,

$$\frac{GM_e}{x^2} = \frac{GM_m}{(3.85 \times 10^8 - x)^2}$$

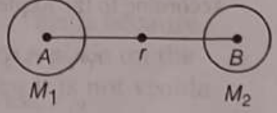
or $$\frac{x}{3.8 \times 10^8 - x} = \sqrt{\frac{M_e}{M_m}} = \sqrt{\frac{5.98 \times 10^{24}}{7.35 \times 10^{22}}} \cong 9$$

$$\Rightarrow \quad \frac{x}{9} + x = 3.85 \times 10^8$$

$$\Rightarrow \quad x = 9 \times 3.85 \times 10^8 / 10 = 3.46 \times 10^8 \text{ m}$$

Gravitational Potential Energy

The gravitational potential energy of a body is the energy associated with it due to its position in the gravitational field of another body and is measured by the amount of work done in bringing a body from infinity to a given point in the gravitational field of the other body.



Thus, the gravitational potential energy of M_2 in the gravitational field of M_1 is

$$U = \frac{-G M_1 M_2}{r}$$

It is a scalar quantity. Its SI unit is joule and dimensions are $[ML^2T^{-2}]$.

- Gravitational potential energy of a body of mass m placed on the surface of earth of radius R and mass M is

$$U = -\frac{GMm}{R} = -mgR \quad [\text{as, } GM = gR^2]$$

- Work done against the gravitational forces in taking a body of mass m from surface of earth to a height h is the change in potential energy of the body.

$$W = \Delta U = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] \quad (\because r_1 = R \text{ and } r_2 = R+h)$$

$$W = \Delta U = \frac{GMmh}{R(R+h)} = mgh\left(\frac{R}{R+h}\right)$$

(i) If $h << R$, then
$$W = mgh\left(\frac{R}{R}\right) = mgh$$

(ii) If $h = R$, then
$$W = \frac{GMm}{2R} = \frac{1}{2}mgh$$

Example 7. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R are 10 m / s^2 and 6400 km, respectively. What is the required energy for this work done?

a. 7.3×10^{10} J **b.** 6.4×10^{10} J
c. 8.3×10^{10} J **d.** 5.4×10^{10} J

Sol. (*b*) Work done, $W = 0 - \left[\frac{-GMm}{R}\right] = \frac{GMm}{R}$

$$= gR^2 \times \frac{m}{R} = mgR$$

$$= 1000 \times 10 \times 6400 \times 10^3$$

$$= 6.4 \times 10^{10} \text{ J}$$

Gravitational Potential

The gravitational potential at a point is equal to the change in potential energy per unit mass, as the mass is brought from the infinity to the given point.

Thus, gravitational potential, $V_A = \frac{U_A - U_\infty}{m}$

$\because$ When the distance between the bodies is infinity, the gravitational force becomes zero.

Thus, the potential energy between them will be zero.

i.e. $$U_\infty = 0$$

$$\therefore \quad V_A = \frac{U_A}{m} \Rightarrow V_A = \frac{\frac{-GMm}{r}}{m}$$

$$\Rightarrow \quad V_A = \frac{-GM}{r}$$

It is a scalar quantity. Its SI unit is J kg^{-1} and dimensions are $[L^2T^{-2}]$.

- At every point inside a spherical shell of radius R, the gravitational potential is same as that on the surface of the shell, *i.e.* $V = -\frac{GM}{R}$

Potential due to Uniform Solid Sphere

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$V = -\frac{GM}{r}$	$V = -\frac{GM}{R}$	$V = \frac{-GM}{R}\left[3 - \frac{r}{R}\right]^2$

Example 8. The radius of the earth is 6.37×10^6 m, its mass 5.98×10^{24} kg. Determine the gravitational potential on the surface of the earth.
(Take, $G = 6.67 \times 10^{-11}$ N-m^2/kg^2)

a. 6.2616×10^7 Jkg^{-1} b. -62.616×10^{-7} Jkg^{-1}
c. -6.2616×10^7 Jkg^{-1} d. 62.619×10^7 J kg^{-1}

Sol. (c) Given, $M = 5.98 \times 10^{24}$ kg and $R = 6.37 \times 10^6$ m

$\therefore$ Gravitational potential, $V = \frac{-GM}{R} = -\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.37 \times 10^6}$

$= -6.2616 \times 10^7$ Jkg^{-1}

Kepler's Laws of Planetary Motion

To explain the motion of the planets, Kepler formulated the following three laws

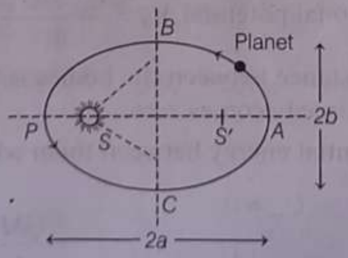
Kepler's First Law (Law of Orbits)

According to this law,

"Every planet revolves around the sun in an elliptical orbit with the sun situated at one of the two foci ".

As shown in the figure, the planet moves around the sun in an elliptical orbit. An ellipse has two foci S and S' and the sun remains at one focus S.

The points P and A on the orbit are called the **perihelion** and the **aphelion** and represents the closest and farthest distances from the sun, respectively.



Kepler's law of orbits

Kepler's Second Law (Law of Areas)

According to this law,

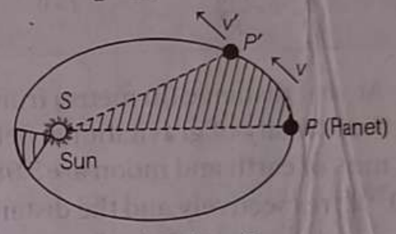
"The radius vector drawn from the sun to a planet, sweeps out equal areas in equal time intervals, *i.e.* its areal velocity (or the area swept out by it per unit time) is constant."

This is referred to as the **law of areas** and gives the relationship between the orbital speed of the planet and its distance from the sun.

Areal velocity, $v_A = \frac{1}{2}r^2\frac{d\theta}{dt}$ = constant

$\Rightarrow \quad \frac{1}{2}rv_L$ = constant

$\left[\because \frac{d\theta}{dt} = \omega \text{ and } \omega r = v_L \text{ (linear velocity)}\right]$



Kepler's law of areas

Hence, planets move faster when they are nearer to the sun and they move slowly when they are further from the sun.

Example 9. A planet is moving along an eliptical orbit is closest to the sun at a distance of 0.5×10^5 m and farthest away at a distance of 1.5×10^{15} m. If the speed of satellite at farthest point is 5 kms^{-1}, then its speed at the closest point will be

a. 20 km s^{-1} b. 15 km s^{-1} c. 10 km s^{-1} d. 16 km s^{-1}

Sol. (b) Let the mass of the planet is m.

$r_1 = 0.5 \times 10^{15}$ m, $v_1 = ?$

$r_2 = 1.5 \times 10^{15}$ m, $v_2 = 5$ km s^{-1}

According to the Kepler's second law, $\frac{1}{2}rv$ = constat

$\Rightarrow \quad r_1v_1 = v_2r_2$

$\therefore \quad v_1 = \frac{v_2r_2}{r_1} = \frac{5 \times 1.5 \times 10^{15}}{0.5 \times 10^{15}}$

$= 15$ km s^{-1}

Kepler's Third Law (Law of Periods)

According to this law,

"The square of the planet's time period is proportional to the cube of the semi-major axis of its elliptical orbit."

This is known as the **harmonic law** and gives the relationship between the size of the orbit of a planet and its time of revolution.

i.e. $\quad T^2 \propto r^3$

or $\quad \frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

Example 10. If the distance between the earth and the sun gets doubled, then what would be the duration of the year?

a. 2.828 yr b. 3.285 yr c. 1.234 yr d. 5.234 yr

Sol (*a*) Given, $r_2 = 2r_1$ and time period, $T_1 = 1$ year

According to Kepler's third law, $T_1^2 \propto r_1^3$ and $T_2^2 \propto r_2^3$

$$\therefore \quad \frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$$

$$\text{or} \quad \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

$$\text{or} \quad \frac{T_2}{T_1} = \left(\frac{2r_1}{r_1}\right)^{3/2} = \sqrt{8} = 2\sqrt{2}$$

$$\Rightarrow \quad T_2 = T_1 \times 2 \times 1.414 = 2.828 \text{ yr}$$

Satellite

A body which revolves in an orbit around the planet is called satellite. There are two types of satellites that are given below

(i) **Natural Satellites** A body revolving around a planet under its gravitational field in the universe is called natural satellite. *e.g.* The moon is natural satellite of the earth. All planets are natural satellites of the sun.

(ii) **Artificial Satellites** The man made body revolving around the planet under the influence of gravitational field is called an artificial satellite. *e.g.* There are many man made satellites, launched in space for different purposes like INSAT-1 B, INSAT-2D, IRS-IC, etc.

Communication (Geostationary) Satellite

When angular velocity of a satellite is different from that of the spinning earth, then it cannot be used for continuous communication for 24 h. This is because electromagnetic wave projected from a place on the earth will not reach the satellite when it is not visible.

A satellite which revolves in the equatorial plane of the earth and has

(i) same period as that of the spinning earth, *i.e.* 24 h.

(ii) same direction of rotation as that of the spinning of earth, *i.e.* from West to East.

So, those satellites which support these features are known as communication satellites. Such types of satellites appears to be stationary from a place on the earth.

Hence, these are also called as **geostationary satellites.** Since, their speed and plane of the orbit is adjusted to be same as that of the earth, they are also called **geo synchronous satellites.**

Uses of Communication Satellite

(i) For the transmission of television and radio wave signals over large areas of the earth's surface.

(ii) For broadcasting telecommunication.

(iii) For military purposes.

(iv) For weather forecasting and meteorological purposes.

(v) For astronomical observations.

(vi) For study of solar and cosmic radiations.

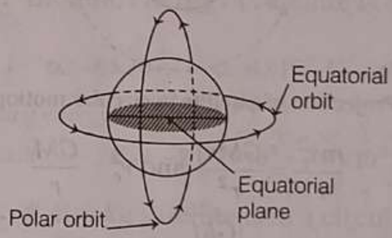
(vii) To relay distress signals from ships.

(viii) To transmit cyclone warnings to coastal villages.

(ix) For Geoposition System (GPS).

Polar or Sun Synchronous Satellite

Polar satellites are low altitude satellites ($h \approx 500$ to 800 km) which circle the globe in a North-South orbit passing over the North and South poles. These satellites revolve in polar orbit around the earth.



Equatorial and polar orbits

The time period of a polar satellite is approximately 100 min. It crosses any altitude many times a day. A strip on the earth's surface (shown shaded in figure) is visible from the satellite during one cycle. For the whole revolution of the satellite, the earth has rotated a little on its axis, so that an adjacent strip becomes visible.

In this way, the satellite can eventually scan the entire surface of the earth strip by strip during the entire day.

The area viewed by the camera of the polar satellite during one revolution of the satellite is narrow but has advantage of viewing with greater resolution. Informations gathered are extremely useful for remote sensing, meterology and environmental studies.

European SPOT and the **IERS** (Indian Earth Resources Satellites) are examples of such satellites. These satellites have been used for various purposes.

Some of these are

(i) used in spying work for military purpose.

(ii) to know the exact shape and dimensions of the earth.

(iii) to take astronomical observations in the absence of atmospheric disturbances.

(iv) to study topography of the moon, the venus and the mars.

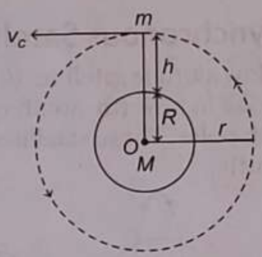
Critical or Orbital Velocity of a Satellite

The horizontal velocity of projection of a satellite for which satellite performs the circular motion around the earth is called critical velocity of a satellite.

Consider a satellite of mass m revolving around the earth at height h above the earth's surface. Let M and R be the mass and radius of the earth, respectively. The satellite is revolving around the earth in a circular orbit of radius $r = (R+h)$ with critical velocity v_c.

The required centripetal force is provided by the gravitational force.

Centripetal force = Gravitational force



Projection of satellite in circular motion

$$\frac{mv_c^2}{r} = \frac{GMm}{r^2} \text{ and } v_c^2 = \frac{GM}{r}$$

Critical velocity, $v_c = \sqrt{\frac{GM}{r}}$

- Further $r = R+h$, where h is the height of the satellite from earth's surface and R is the radius of the earth.

$$v_c = \sqrt{\frac{GM}{R+h}}$$

- If the satellite is orbiting near the earth's surface, then

$$R+h \approx R$$

$$\therefore \quad v_c = \sqrt{\frac{GM}{R}}$$

- On the surface of earth, $GM = gR^2$

$$\therefore \quad v_c = \sqrt{gR}$$

If the orbit of a satellite is close to the surface of the earth. The atmosphere of the earth affects the motion of a satellite due to presence of air resistance.

Time Period of a Satellite (Periodic Time)

It is the time taken by satellite to complete one revolution around the earth.

$$\therefore \quad T = \frac{\text{Circumference of the orbit}}{\text{Orbital velocity}} = \frac{2\pi r}{v_o}$$

$$T = 2\pi r\sqrt{\frac{r}{GM_e}} \qquad \left(\because v_o = \sqrt{\frac{GM_e}{r}}\right)$$

Time period, $T = 2\pi\sqrt{\frac{r^3}{GM_e}}$

Now, $$T = 2\pi\sqrt{\frac{(R_e+h)^3}{GM_e}} \qquad (\because R_e + h = r)$$

$$\Rightarrow \quad T = \frac{2\pi}{R_e}\sqrt{\frac{(R_e+h)^3}{g}} \qquad (\because GM_e = gR_e^2)$$

For a satellite orbiting near to the earth's surface,

i.e. $\quad h << R_e$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6 \text{ min}$$

The period of revolution of the satellite depends upon its height above earth's surface and larger is the height of the satellite, the greater will be its time period of revolution.

Example 11. Assuming the radius of the earth to be 6.4×10^6 m. Calculate the time period T of a satellite for equatorial orbit at 1.4×10^3 km above the surface of the earth and the speed of the satellite in this orbit.

a. 6831 s and 7200 ms^{-1}

b. 6853 s and 7151 ms^{-1}

c. 6832 s and 7190 ms^{-1}

d. 6850 s and 7400 ms^{-1}

Sol (*b*) Given, radius of the earth, $R_e = 6.4\times10^6$ m

Height, $h = 1.4\times10^3$ km

Mass of the earth, $M_e = 5.98\times10^{24}$ kg

Gravitational constant, $G = 6.67\times10^{-11}$ N-m^2 kg^{-2}

$$r = R_e + h$$
$$= (6.4\times10^6 + 1.4\times10^6)\text{ m}$$
$$= 7.8\times10^6 \text{ m}$$

Time period, $$T = \left[\frac{4\pi^2 r^3}{GM_e}\right]^{1/2}$$

$$= \left[\frac{4\times(3.14)^2\times(7.8\times10^6)^3}{6.67\times10^{-11}\times5.98\times10^{24}}\right] = 6853.4 \text{ s}$$

$$\approx 6853 \text{ s}$$

Speed of satellite, $v = \sqrt{\frac{GM_e}{r}}$

$$= \left[\frac{6.67\times10^{-11}\times5.98\times10^{24}}{7.8\times10^6}\right]^{1/2} = 7151 \text{ ms}^{-1}$$

Height of Satellite

As it is known that the time period of satellite,

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \qquad \text{...(i)}$$

By squaring on both sides of Eq. (i), we get

$$T^2 = 4\pi^2 \frac{(R+h)^3}{gR^2}$$

$$\Rightarrow \quad \frac{gR^2T^2}{4\pi^2} = (R+h)^3$$

$$\Rightarrow \quad h = \left(\frac{T^2 gR^2}{4\pi^2}\right)^{1/3} - R$$

Energy of Satellite

The total mechanical energy (E) of a satellite revolving around the sun is the sum of it potential energy and kinetic energy.

Potential Energy of a Satellite If a satellite of mass m is revolving around the earth of mass M and radius R, with orbital velocity v in an orbit of radius r, then potential energy of satellite is

$$\text{PE} = -\frac{GMm}{r} \quad [\text{where}, r = R+h] \quad \text{...(i)}$$

Kinetic Energy of a Satellite The kinetic energy of satellite near the earth's surface,

$$\text{KE} = \frac{1}{2}mv^2 \qquad \left(\because v = \sqrt{\frac{GM}{r}} \text{ for circular orbit}\right)$$

$$= \frac{1}{2}m\frac{GM}{r}$$

$$\text{KE} = \frac{GMm}{2r} \quad \text{...(ii)}$$

Total Mechanical Energy of a Satellite Total mechanical energy $E = \text{PE} + \text{KE}$

$$= -\frac{GMm}{r} + \frac{GMm}{2r} = -\frac{GMm}{2r}$$

$$E = -\frac{GMm}{2r}$$

Thus, the total mechanical energy of a satellite,

$$E = -\frac{GMm}{2r} \quad \text{(Near the earth's surface)}$$

Binding Energy

The minimum amount of energy required to take the body infinitely away from the earth, so that the body becomes completely free from the gravitational field of the earth is called binding energy.

Binding energy for a body stationary on the earth,

$$\text{BE} = \frac{GMm}{R}$$

Binding energy for a body revolving around the earth at a height h,

$$\text{BE} = \frac{GMm}{2(R+h)}$$

Relation between Potential, Kinetic, Total and Binding Energy

The kinetic energy of an orbiting satellite in circular orbit of radius r is $\text{KE} = \frac{1}{2}\frac{GMm}{r}$ ...(i)

Its potential energy is $\text{PE} = -\frac{GMm}{r}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{\text{KE}}{\text{PE}} = -\frac{1}{2}$$

$$\therefore \quad \text{PE} = -2\,\text{KE} \quad \text{...(iii)}$$

Also, $\quad \text{TE} = \text{KE} + \text{PE}$ ...(iv)

From Eqs. (iii) and (iv), we get

$$\text{TE} = \text{KE} - 2\,\text{KE} = -\,\text{KE}$$

Hence, $\quad \text{BE} = -\,\text{TE} = \text{KE}$

Example 12. Binding energy of satellite is 4×10^8 J. Its PE is

a. -4×10^8 J b. -8×10^8 J c. 8×10^8 J d. 4×10^8 J

Sol (*b*) Binding energy, $E_B = 4\times10^8$ J

$\therefore$ PE of satellite $= -2E_B = -2\times4\times10^8 = -8\times10^8$ J

Example 13. A 400 kg satellite is in a circular orbit of radius $2R$ around the earth. How much energy is required to transfer it to a circular orbit of radius $4R$?

a. 3.13×10^9 J b. 3.59×10^9 J c. 4.1×10^9 J d. 5.21×10^9 J

Sol (*a*) Given, mass of satellite, $m = 400$ kg

Initial energy is given by $E_i = \frac{-GMm}{4R}$

Final energy is given by $E_f = \frac{-GMm}{8R}$

$\therefore$ Change in total energy is given by

$$\Delta E = E_f - E_i = \frac{-GMm}{8R} - \left(\frac{-GMm}{4R}\right)$$

or $$\Delta E = \frac{GMm}{8R} = \left(\frac{GM}{R^2}\right)\frac{mR}{8} = \frac{gmR}{8}$$

$$= \frac{9.81\times400\times6.37\times10^6}{8}$$

$$\Rightarrow \quad \Delta E = 3.13\times10^9 \text{ J}$$

Weightlessness Condition in a Satellite

Weight of a body is the force with which it is attracted towards the centre of the earth. When a body is falling freely under gravity, there is no reaction acting on the body. So, it is in a state of weightlessness.

If a body of mass m placed on a surface inside a satellite moving around the earth, then, forces on the body are

Gravitational pull of the earth $= \frac{GMm}{r^2}$

and reaction by the surface $= R$

By Newton's law, $\dfrac{GMm}{r^2} - R = ma$

$$\frac{GMm}{r^2} - R = m\left(\frac{GM}{r^2}\right) \Rightarrow R = 0$$

Thus, the surface does not exert any force on the body and hence, its apparent weight is zero.

- Weightlessness does not depend upon mass of a person and height of the satellite.
- As there is no resultant force acting on a body, a simple pendulum will not oscillate inside the satellite or inside a freely falling lift.
- If the lift is falling down with acceleration a, the apparent weight of a person is $w' = m(g-a)$.
- If the lift is moving upward with acceleration a, the apparent weight of a person is $w'' = m(g+a)$.

Example 14. A person sitting on a chair in a satellite feels weightless because

a. the earth does not attract the objects inside a satellite
b. the normal force by the chair on the person balances the earth's attraction
c. the normal force is zero
d. the person in satellite is not accelerated

Sol (c) We are conscious of our weight due to the normal reaction acting on our body to balance the weight of our body. Since, inside a satellite, there is condition of weightlessness, so normal force is zero.

Escape Velocity

The escape velocity on earth (or any planet) is defined as the minimum velocity with which a body has to be projected vertically upwards from the surface of earth (or any other planet), so that it just crosses the gravitational field of earth (or of that planet) and never returns on its own. If the body is projected with a velocity equal to its escape velocity, then

Kinetic energy of the body = Binding energy of the body

i.e. $$\frac{1}{2}mv_e^2 = \frac{GMm}{R} \Rightarrow v_e = \sqrt{\left(\frac{2GM}{R}\right)} \quad \text{...(i)}$$

Also, we know that, $g = \dfrac{GM}{R^2} \Rightarrow GM = gR^2$

Hence, we have $$v_e = \sqrt{2gR} \quad \text{...(ii)}$$

Eqs. (i) and (ii) give the values of escape velocity of satellite on earth's surface.

Also, $$v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \times \rho} = R\sqrt{\frac{8}{3}\pi G\rho}$$

Escape velocity, $v_e = R\sqrt{\dfrac{8}{3}\pi G\rho}$

Example 15. The escape velocity from the earth's surface is about 11.2 kms^{-1}. The escape velocity from the surface of planet having radius twice and having the same mean density as that of the earth, will be

a. 15.5 kms^{-1} b. 55.5 kms^{-1} c. 11.2 kms^{-1} d. 22.4 kms^{-1}

Sol (d) Escape velocity, $v = R\sqrt{\dfrac{8}{3}\pi G\rho}$

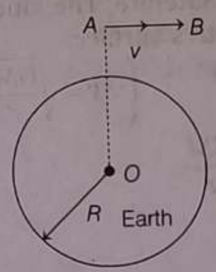
$\therefore \quad v \propto R$

$\because \quad \dfrac{v_1}{v_2} = \dfrac{R_1}{R_2}$ or $\dfrac{11.2}{v_2} = \dfrac{R}{2R} \Rightarrow v_2 = 22.4 \text{ km s}^{-1}$

Projection of a Satellite

Artificial satellites can be put into stable orbits by means of multistage rockets. The satellite is placed on the tip of the rocket which is launched from the earth. When rocket reaches its maximum vertical height h, a special mechanism gives a thrust to the satellite at point A.

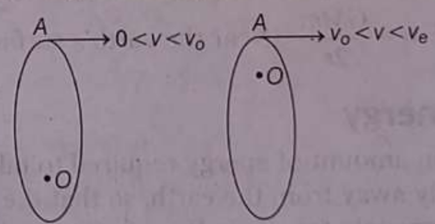
Let a satellite be projected from point A with velocity v in the direction AB. For different values of v, the paths are different.



A satellite projected from *A*

Here, are the possible cases

(i) If $v = 0$, path will be a straight line from A to O.
(ii) If $0 < v < v_o$, path will be an ellipse with centre O of the earth as a focus.
(iii) If $v = v_o$, path will be a circle with O as the centre.
(iv) If $v_o < v < v_e$, path is again an ellipse with O as a focus.



Elliptical orbit with focus at lower side and upper side

(v) If $v = v_e$, satellite will escape from the gravitational pull of the earth and path is a parabola.
(vi) If $v > v_e$, satellite again will escape but now the path is a hyperbola.

Here, v_o = orbital speed $\left(\sqrt{\dfrac{GM}{r}}\right)$ at A

and v_e = escape velocity at A.