07

Most of the electric power generated and used in the world is in the form of alternating current. This is because

(i) alternating voltages can be easily and efficiently converted from one value to the other by means of transformers.

(ii) the alternating current energy can be transmitted and distributed over long distances economically without much loss of energy.

ALTERNATING CURRENT

In this chapter, we will study about some alternating current system that transfers energy efficiently and we will also discuss some of the devices that make use of alternating current.

CHAPTER CHECKLIST

- Introduction to Alternating Current
- AC Circuits
- AC Devices

|TOPIC 1|
Introduction to Alternating Current

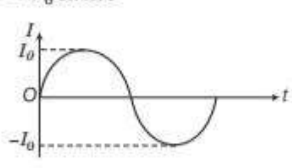
Alternating Current (AC)

If the direction of current changes alternatively (periodically) and its magnitude changes continuously with respect to time, then the current is called **alternating current**. It is sinusoidal (i.e. represented by sine or cosine angles) in nature.

Alternating current can be defined as the current whose magnitude and direction changes with time and attains the same magnitude and direction after a definite time interval It changes continuously between zero and a maximum value and flows in one direction in the first half cycle and in the opposite direction in the next half cycle.

The instantaneous value of AC is given by

$$I = I_0 \sin\omega t \qquad \left[\because \omega = \frac{2\pi}{T} = 2\pi\nu\right]$$



Current *vs* time graph of an AC

where, I = current at any instant t, I_0 = maximum/peak value of AC,

ν = frequency and ω = angular frequency.

Note Current whose direction does not change with time through a load is known as direct current (DC).

Advantages of AC over DC

(i) AC generation is easy and economical.
(ii) It can be easily converted into DC with the help of rectifier.
(iii) In AC, energy loss is minimum, so it can be transmitted over large distances.

Disadvantages of AC over DC

(i) AC shock is attractive, while DC shock is repulsive so, 220V AC is more dangerous than 220V DC.
(ii) AC cannot be used in electroplating process because here constant current with constant polarity is needed which is given by DC.

Alternating emf or Voltage

It can be defined as the voltage whose magnitude and direction changes with time and attains the same magnitude and direction after a definite time interval. The instantaneous value of alternating emf or voltage is given by

$$E = E_0 \sin \omega t$$

where, E = voltage at any time t, E_0 = maximum/peak value of alternating voltage and ω = angular frequency.

Note Alternating current, alternating emf, flux, etc., all are sinusoidal waves.

MEAN OR AVERAGE VALUE OF AC

It is defined as the value of AC (Alternating Current) which would send same amount of charge through a circuit in half cycle (i.e. $T/2$) that is sent by steady current in the same time. It is denoted by I_m or I_{av}.

Let the instantaneous value of alternating current is represented by

$$I = I_0 \sin \omega t \quad \text{... (i)}$$

The AC changes continuously with time. Suppose current is kept constant for small time (dt). Then, small amount of charge (dq) in small time (dt) is given by

$$dq = Idt = I_0 \sin \omega t \, dt \quad \text{[from Eq. (i)]}$$

To calculate total charge send by AC over half cycle is given by

$$\int dq = \int_0^{T/2} I_0 \sin \omega t \, dt$$

or

$$q_s = I_0 \int_0^{T/2} \sin \omega t \, dt$$

Here, q_s is steady charge over half cycle.

$$\Rightarrow \quad q_s = I_0 \left[\frac{-\cos \omega t}{\omega}\right]_0^{T/2} = \frac{-I_0}{\omega}[\cos \omega t]_0^{T/2}$$

$$= \frac{-I_0}{\omega}\left[\cos \frac{\omega T}{2} - \cos 0°\right]$$

$$= \frac{-I_0}{\omega}\left[\cos \frac{\omega T}{2} - 1\right] = \frac{-I_0}{\omega}\left[\cos \frac{2\pi}{2} - 1\right]$$

$$\left[\because \omega = \frac{2\pi}{T} \Rightarrow \omega T = 2\pi\right]$$

$$= \frac{-I_0}{\omega}[\cos \pi - 1] = \frac{-I_0}{\omega}[-1-1] \quad [\because \cos \pi = -1]$$

$$\Rightarrow \quad q_s = \frac{2I_0}{\omega}$$

Also, the charge sent by AC in positive half cycle is

$$q_{AC} = I_m \times \frac{T}{2}$$

where, I_m is mean value of AC over half cycle.

According to the definition,

$$q_s = q_{AC} \quad \text{[over any half cycle]}$$

$$\Rightarrow \quad \frac{2I_0}{\omega} = I_m \times \frac{T}{2}$$

$$\Rightarrow \quad I_m = \frac{4I_0}{\omega T} = \frac{4I_0}{2\pi} \quad [\because \omega T = 2\pi]$$

$$\Rightarrow \quad I_m = \frac{2I_0}{\pi} = 0.637 I_0$$

$$\therefore \quad \boxed{I_m = 0.637 I_0}$$

Mean value of AC (I_m) is 63.7% of the peak value of AC (I_0) over positive half cycle. For negative half cycle, the mean value of AC will be $-2I_0/\pi$. Therefore, in a complete cycle, the mean value of AC will be zero.

In the same way, mean value of alternating emf (E_m) is

$$\boxed{E_m = \frac{2E_0}{\pi} = 0.637 E_0}$$

Root Mean Square (RMS) Value of AC

It is defined as that value of Alternating Current (AC) over a complete cycle which would generate same amount of heat in a given resistor that is generated by steady current in the same resistor and in the same time during a complete cycle.

It is also called **virtual value** or **effective value** of AC.

It is represented by I_{rms} or I_{eff} or I_V. Suppose I is the current which flows in the resistor having resistance (R) in time (T) produces heat (H).

Instantaneous value of AC,

$$I = I_0 \sin \omega t$$

If dH is small amount of heat produced in time dt in resistor R, then

$$dH = I^2 R\, dt \qquad [\because H = I^2 RT] \ldots \text{(i)}$$

In complete cycle $(0 \rightarrow T)$, the total heat produced is H.

After integrating Eq. (i), we get

$$\int dH = \int_0^T I^2 R\, dt \quad \Rightarrow \quad H = \int_0^T I^2 R\, dt$$

Put the value of I in the above equation, we get

$$H = \int_0^T (I_0 \sin \omega t)^2 R\, dt$$

$$= I_0^2 R \int_0^T \sin^2 \omega t\, dt = I_0^2 R \int_0^T \left[\frac{1 - \cos 2\omega t}{2}\right] dt$$

$$\left[\because \sin^2 \omega t = \frac{1 - \cos 2\,\omega t}{2}\right]$$

$$= \frac{I_0^2 R}{2} \int_0^T (1 - \cos 2\omega t)\, dt$$

$$= \frac{I_0^2 R}{2} \left[\int_0^T dt - \int_0^T \cos 2\omega t\, dt\right]$$

$$= \frac{I_0^2 R}{2} \left[|t|_0^T - \left|\frac{\sin 2\omega t}{2\omega}\right|_0^T \right]$$

$$= \frac{I_0^2 R}{2} \left[(T - 0) - \frac{1}{2\omega} |\sin 2\omega T - \sin 0°|\right]$$

$$= \frac{I_0^2 R}{2} \left[T - \frac{1}{2\omega} |\sin 2 \times 2\pi - 0|\right] [\because \omega T = 2\pi]$$

$$= \frac{I_0^2 R}{2} \left[T - \frac{1}{2\omega} |0 - 0|\right] \qquad [\because \sin 4\pi = 0]$$

$$= \frac{I_0^2 RT}{2} \qquad \ldots \text{(ii)}$$

If I_{rms} is rms value of alternating current and H is the heat produced by rms current (I_{rms}), then

$$H = I_{\text{rms}}^2 RT \qquad \ldots \text{(iii)}$$

On comparing Eqs. (ii) and (iii), we get

$$I_{\text{rms}}^2 RT = \frac{I_0^2 RT}{2} \Rightarrow \quad I_{\text{rms}}^2 = \frac{I_0^2}{2}$$

$$\Rightarrow \qquad \boxed{I_{\text{rms}} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0}$$

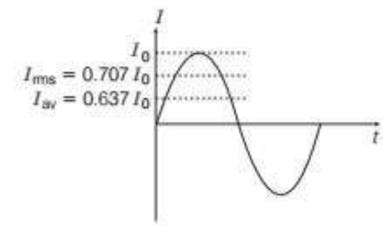
$$\Rightarrow \qquad \boxed{I_{\text{rms}} = 70.7\% \text{ of } I_0}$$

From the above equation, we conclude that rms value of current is 70.7% of the peak value of current.

In the same way, the rms value of alternating emf (E_{rms} or E_{eff} or E_V) is

$$\boxed{E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = 0.707 E_0 = 70.7\% \text{ of } E_0}$$

The different values I_0, I_{av} and I_{rms} are shown in figure given below.



RMS and Average value of current on the same graph

AC Ammeter and Voltmeter

AC ammeter and voltmeter always measure the virtual value of AC or alternating emf.

They are also called hot wire instruments because deflection in the needle depends upon the heat produced in any coil.

$$H \propto I_{\text{rms}}^2$$

If we connect ordinary DC ammeter or voltmeter to AC circuit, they read zero because average value of alternating current/voltage over a full cycle is zero.

EXAMPLE |1| The instantaneous current from an AC source is given by $I = 5 \sin 314 t$. What is the rms value of the current?

Sol. Given, $\quad I = 5 \sin 314\, t \qquad$ ...(i)

We know that, $I = I_0 \sin \omega t \qquad$ ...(ii)

On comparing Eqs. (i) and (ii), we have

$$I_0 = 5 \text{ A} \quad \text{and} \quad \omega = 314$$

$$\therefore \quad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.54 \text{ A}$$

EXAMPLE |2| Calculate the instantaneous voltage for AC supply of 220 V and 50Hz.

Sol. Given, $\quad E_V = 220\text{V}$, $\nu = 50$ Hz and $E = ?$

Since, we know that for calculating the peak value of alternating voltage E_0, we can use the relation

$$E_0 = \sqrt{2} E_V = 1.414 \times 220 = 311 \text{ V}$$

Therefore, instantaneous voltage, $E = E_0 \sin \omega t$

$$E = E_0 \sin (2\pi\nu) t$$

$$= 311 \sin (2\pi \times 50) t$$

$$= 311 \sin 100\pi t$$

EXAMPLE |3| In an AC circuit, the rms voltage is $100\sqrt{2}$ V. Determine the peak value of voltage and its mean value during a positive half cycle.

Sol. Given, $E_V = 100\sqrt{2}$ V

Peak value of voltage, $E_0 = ?$

Mean value of voltage, $E_m = ?$

$\because \quad E_0 = \sqrt{2}E_V = \sqrt{2}(100\sqrt{2}) = 200$ V

$\therefore$ During positive half cycle $(0 \to T/2)$,

$$E_m = \frac{2E_0}{\pi} = \frac{2\times 200}{3.14} = 127.4 \text{ V}$$

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. The peak voltage in a 220 V, AC source is

(a) 220 V (b) about 160 V
(c) about 310 V (d) 440 V

2. If the rms current in a 50 Hz AC circuit is 5 A, the value of the current 1/300 s after its value becomes zero is **NCERT Exemplar**

(a) $5\sqrt{2}$ A (b) $5\sqrt{3/2}$ A
(c) $5/6$ A (d) $5/\sqrt{2}$ A

3. If the reading of AC mains voltage by a voltmeter is 200 V, then the root mean square value of this voltage will be

(a) $200\sqrt{2}$ V (b) $100\sqrt{2}$ V
(c) 200 V (d) $400/\pi$ V

4. The reading of an ammeter in an alternating circuit is 4 A. The peak (maximum) value of current in the circuit is

(a) 4 A (b) 8 A
(c) $4\sqrt{2}$ A (d) $\frac{2}{\sqrt{2}}$ A

5. When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. This means **NCERT Examplar**

(a) input voltage cannot be AC voltage, but a DC voltage
(b) maximum input voltage is 220 V
(c) the meter reads not V but $< V^2 >$ and is calibrated to read $\sqrt{< V^2 >}$
(d) the pointer of the meter is stuck by some mechanical defect

VERY SHORT ANSWER Type Questions

6. Define the term rms value of the current. How is it related to the peak value? **All India 2010C**

7. The peak value of emf in AC is E_0. Write its (i) rms (ii) average value over a complete cycle. **Foreign 2011**

8. In many European homes and offices, the rms voltage available from a wall socket is 240 V. What is the maximum voltage in this case?

9. An AC current $I = I_0 \sin \omega t$ produces certain heat H in a resistor R over a time $T = 2\pi/\omega$. Write the value of the DC current that would produce the same heat in the same resistor in the same time. **All India 2009C**

10. An alternating current is given by $I = I_1 \cos\omega t + I_2 \sin\omega t$. Determine the rms value of current through the circuit.

11. The current through an AC circuit is $I_t = I_0(t/\tau)$ for sometime. Determine the rms current through the circuit over time interval $t = 0$ to $t = \tau$.

12. Can the instantaneous power output of an AC source ever be negative? Can the average power output be negative? **NCERT Exemplar**

SHORT ANSWER Type Questions

13. Establish an expression for the average voltage of AC voltage $V = V_0 \sin \omega t$ over the time interval $t = 0$ and $t = \pi/\omega$.

14. Which of the following 120 V AC devices cost more to operate (i) one that draws an rms current of 10 A or (ii) one that draws a peak current of 12 A? Explain the reason for your answer.

15. Show that heat produced in a cycle of AC is same as the heat produced by DC with $I = I_{rms}$.

16. Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current?

NCERT Exemplar

NUMERICAL PROBLEMS

17. The instantaneous value of current in an AC circuit is $I = 2 \sin (100\,\pi t + \pi/3)$ A. At what first time, the current will be maximum ?

18. An alternating current in a circuit is given by $I = 20\sin(100\pi t + 0.05\pi)$ A. What is the rms value of current?

19. (i) The peak voltage of an AC supply is 300 V. What is its rms voltage?

(ii) The rms value of current in an AC circuit is 10 A. What is the peak current? **NCERT**

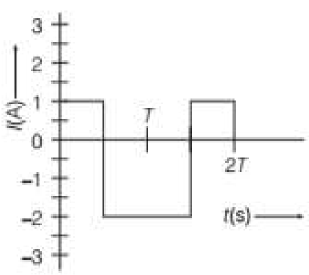
20. A light bulb is rated 100 W for 220 V AC supply of 50 Hz. Calculate

(i) resistance of the bulb.

(ii) the rms current through the bulb.

All India 2012

21. The alternating current in a circuit is described by the graph shown in the figure. Find the rms current in this graph. **NCERT Exemplar**



HINTS AND SOLUTIONS

1. (c) Given, $E_V = 220$ V

Peak voltage, $E_0 = \sqrt{2}E_V = \sqrt{2} \times 220 = 310$V

Thus, option (c) is correct.

2. (b) Given, $\nu = 50$ Hz, $I_{rms} = 5$A

$$t = \frac{1}{300}\text{ s}$$

We have to find $I(t)$

$$I_0 = \text{Peak value}$$
$$= \sqrt{2}\, I_{rms} = \sqrt{2} \times 5$$
$$= 5\sqrt{2}\text{ A}$$
$$I = I_0 \sin \omega t = 5\sqrt{2} \sin 2\pi \nu t$$
$$= 5\sqrt{2} \sin 2\pi \times 50 \times \frac{1}{300}$$
$$= 5\sqrt{2} \sin \frac{\pi}{3}$$
$$= 5\sqrt{2} \times \frac{\sqrt{3}}{2} = 5\sqrt{3/2}\text{ A}$$

3. (c) $E_V = 200\text{ V} = E_{rms}$

Root mean square value of this voltage is the effective value of voltage i.e., equal to the voltage indicated in voltmeter.

4. (c) Given, $I_{rms} = 4$ A

The peak value of current, $I_0 = I_{rms}\sqrt{2} = 4\sqrt{2}$ A

5. (c) The voltmeter connected to AC mains reads mean value ($<V^2>$) and is calibrated in such a way that it gives value of $<V^2>$, which is multiplied by form factor to give rms value.

6. It is defined as the value of Alternating Current (AC) over a complete cycle which would generate same amount of heat in a given resistor that is generated by steady current in the same resistor and in the same time during a complete cycle. It is also called virtual value or effective value of AC.

Let the peak value of the current be I_0.

$$\therefore \qquad I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$\Rightarrow \qquad I_{rms} = \frac{I_0}{\sqrt{2}}$$

where, I_0 = peak value of AC.

7. E_0 = peak value of emf in a complete cycle,

(i) rms value $[E_{rms}] = \dfrac{E_0}{\sqrt{2}}$

(ii) average value $[E]$ = zero

8. As we know, $E_{rms} = \dfrac{E_0}{\sqrt{2}}$

$\therefore E_0$ (Maximum voltage)

$$= E_{rms} \times 1.414 = 339.36\text{ V}$$

9. An AC current $I = I_0 \sin\omega t$ produces certain heat H in a resistor R over a time $T = 2 \times 3.14/\omega$, is given by $H = \left(\dfrac{I_0}{\sqrt{2}}\right)^2 RT$ and the same amount of heat produced by DC current in same time is given by $H = I^2RT$.

As, these heats are equal, then $I^2RT = \left(\dfrac{I_0}{\sqrt{2}}\right)^2 RT$

So, $I = \dfrac{I_0}{\sqrt{2}}$, where I stands for DC and I_0 is the peak value of AC current.

10. As, $I_{rms_1} = \dfrac{I_1}{\sqrt{2}}$

and $I_{rms_2} = \dfrac{I_2}{\sqrt{2}}$

Hence, the resultant of these two currents,

$$I_{rms} = \sqrt{\left(\frac{I_1}{\sqrt{2}}\right)^2 + \left(\frac{I_2}{\sqrt{2}}\right)^2} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

11. The mean square current is

$$\overline{I^2} = \frac{1}{\tau}\int_0^{\tau} I_0^2 (t/\tau)^2\, dt$$

$$= \frac{I_0^2}{\tau^3}\int_0^{\tau} t^2 dt = \frac{I_0^2}{\tau^3}\left[\frac{t^3}{3}\right]_0^{\tau} = \frac{I_0^2}{3} \qquad [\because t = \tau]$$

$$\therefore I_{rms} = \sqrt{\overline{I^2}} = \sqrt{\left(\frac{I_0^2}{3}\right)} = \frac{I_0}{\sqrt{3}}$$

12. Yes, the instantaneous power can be negative as,
$P_{instantaneous} = I_{in} \times V_{in} = I_0 \sin \omega t \times V_0 \cos \omega t$
No, because it is average, so it will be positive.

13. Average voltage, V_m or $V_{av} = \frac{2V_0}{\pi} = 0.637 V_0$

Refer to the text on page 290.

14. Alternating currents and voltage are generaly measured in the terms of their rms values.

Since, the electric cost is calculated on the power used,

i.e. $\quad P \propto I_{rms}^2$

$\therefore$ For $\quad I_{rms} = 10$ A

$$P_1 \propto (10)^2 \qquad \text{...(i)}$$

and for $\quad I_0 = 12$ A,

$$\Rightarrow \quad I_{rms} = \frac{12}{1.414} = 8.48 \text{ A} \qquad \left[\therefore I_{rms} = \frac{I_0}{\sqrt{2}}\right]$$

$$\Rightarrow \quad P_2 \propto (8.48)^2 \qquad \text{...(ii)}$$

So, from Eqs. (i) and (ii), the device that draws a rms current of 10 A costs more to operate.

15. For an AC, $I_t = I_0 \sin \omega t$

Heat produced in a resistance in small time dt,

$$dU = I_t^2 R\, dt = (I_0 \sin \omega t)^2 R\, dt$$

$\therefore$ Heat produced during a full cycle of AC,

$$U = \int dU = I_0^2 R \int_0^T \sin^2 \omega t\, dt$$

$$= \frac{I_0^2}{2} R\,[T] \qquad [\because \omega T = 2\pi]$$

$$\Rightarrow \quad U = I_{rms}^2 RT \qquad \left[\because I_{rms} = \frac{I_0}{\sqrt{2}}\right]$$

Thus, we see that AC produces same heating effect as DC of value $I = I_{rms}$.

16. As we know that, the AC current changes its direction with time. So, AC ampere must be defined in terms of some property which is independent of direction of current. Thus, Joule's heating effect is the property, which defines rms value of AC.

17. Here, $\quad I = 2 \sin (100\,\pi t + \pi/3)$ A $\qquad$ ...(i)

Since, the relation between current and time gives us

$$\frac{2\pi t}{T} = 100\,\pi t$$

$$\therefore \quad T = \frac{2\pi}{100\pi} = \frac{1}{50} \text{ s} \qquad \text{...(ii)}$$

$$\Rightarrow \quad t = \frac{T}{4} = \frac{1}{50 \times 4} = \frac{1}{200} \text{ s}$$

18. $I_{rms} = 10\sqrt{2}$ A

Refer to the Example 1 on page 290.

19. (i) $E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212.1$ V $\qquad [\because E_0 = \text{peak voltage}]$

(ii) $I_{rms} = 10$ A

$$\Rightarrow I_{rms} = \frac{I_0}{\sqrt{2}} \qquad [\because I_0 = \text{peak current}]$$

$$\Rightarrow \quad I_0 = \sqrt{2} I_{rms} = 10 \times \sqrt{2} = 14.14 \text{ A}$$

20. (i) Power, $P = EI \;\Rightarrow\; P = E \times \frac{E}{R}$ $\qquad \left[\because I = \frac{E}{R}\right]$

$$\Rightarrow \quad R = \frac{E^2}{P} = \frac{(220)^2}{100}$$

$$= \frac{48400}{100} = 484\ \Omega$$

(ii) The peak voltage of the source is $E_{rms} = \frac{E_0}{\sqrt{2}}$

$$\Rightarrow \quad E_0 = E_{rms} \times \sqrt{2}$$

$$= 220\sqrt{2} = 311.13 \text{ V}$$

$$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}} = \frac{311.13}{484\sqrt{2}}$$

$$= \frac{311.13}{684.479} = 0.45 \text{ A}$$

21. From the graph, $I_1 = 1$ A, $I_2 = -2$A and $I_3 = 1$A

$$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2}{3}} = \sqrt{\frac{1^2 + (-2)^2 + 1^2}{3}}$$

$$= \sqrt{\frac{6}{3}} = \sqrt{2} = 1.414 \text{ A}$$

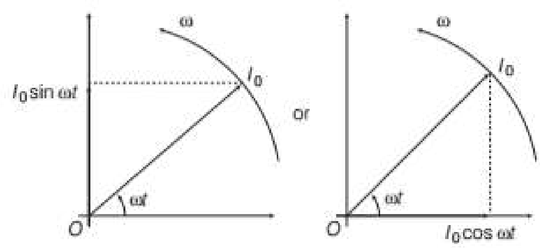
|TOPIC 2|
AC Circuits

Phasor Diagrams

The study of AC circuit is much simplified, if we represent alternating current, alternating emf as rotating vector, with the angle between the vectors equal to the phase difference between the current and the emf. These rotating vectors representing current and alternating emf are called **phasors**.

A diagram representing alternating current and alternating emf (of same frequency) as rotating vectors (phasors) with the phase angle between them is called **phasor diagram**.

The length of the vector represents the maximum or peak value, i.e. I_0 and E_0. The projection of the vector on fixed axis gives the instantaneous value of alternating current and alternating emf. In sine form, ($I = I_0 \sin \omega t$ and $E = E_0 \sin \omega t$), projection is taken on Y-axis. In cosine form, ($I = I_0 \cos \omega t$ and $E = E_0 \cos \omega t$), projection is taken on X-axis.



Phasor diagram

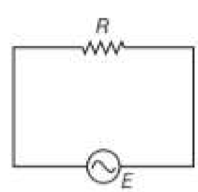
DIFFERENT TYPES OF AC CIRCUIT

In this section, we will derive voltage current relations for individual as well as combined circuit elements carrying a sinusoidal current. Here, we will only consider resistors, inductors and capacitors.

AC through Resistor

Suppose a resistor of resistance R is connected to an AC source of emf with instantaneous value (E), which is given by

$$E = E_0 \sin \omega t \qquad \ldots(\text{i})$$



An AC voltage applied to a resistor

Let E be the potential drop across resistance (R), then

$$E = IR \qquad \ldots(\text{ii})$$

$\because$ Instantaneous emf = Instantaneous value of potential drop

From Eqs. (i) and (ii), we have

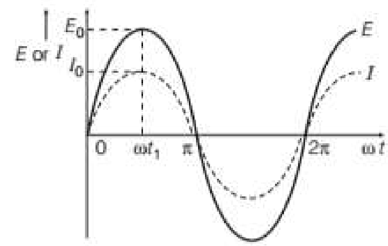
$$IR = E = E_0 \sin \omega t$$

$$\Rightarrow \quad I = \frac{E}{R} = \frac{E_0 \sin \omega t}{R}$$

$$\Rightarrow \quad \boxed{I = I_0 \sin \omega t} \quad \left[\because I_0 = \frac{E_0}{R}\right] \qquad \ldots(\text{iii})$$

Comparing $I_0 = E_0/R$ with Ohm's law, we find that resistors work equally well for both AC and DC voltages.

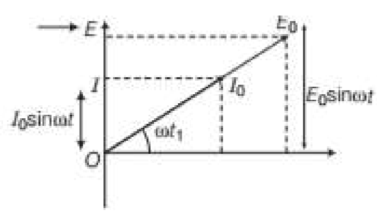
From Eqs. (i) and (iii), we get that for resistor **there is zero phase difference between instantaneous alternating current and instantaneous alternating emf (i.e. they are in same phase)**.



Graph of E and I *versus* ωt

Phasor Diagram

Here, peak values E_0 and I_0 are represented by vectors rotating with angular velocity ω with respect to horizontal reference. Their projections on vertical axis give their instantaneous values.



Phasor diagram for a purely resistive circuit

EXAMPLE |1| A resistance of 20 Ω is connected to a source of alternating current rated 110 V, 50 Hz. Find the

(i) rms current.

(ii) maximum instantaneous current in the resistor.

(iii) time taken by the current to change from its maximum value to the rms value.

Sol. Given, resistance, $R = 20\Omega$

The rms value of voltage, $E_{rms} = 110\,V$

Frequency, $\nu = 50\,Hz$

(i) $I_{rms} = \frac{E_{rms}}{R} = \frac{110}{20} = 5.5\text{ A}$

(ii) $I_0 = \sqrt{2}\,I_{rms} = 1.414 \times 5.5 = 7.8\text{A}$

(iii) Let the AC be represented by $I = I_0 \cos \omega t$

At $t = 0, I = I_0 \cos 0 = I_0 (\text{max})$

At $t = t$, let $I = I_V = \frac{I_0}{\sqrt{2}} = I_0 \cos \omega t$

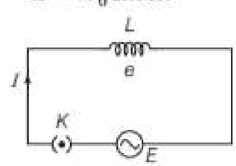
$\because \cos \omega t = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} \Rightarrow \omega t = \frac{\pi}{4} \Rightarrow 2\pi\nu t = \frac{\pi}{4}$

$\therefore \quad t = \frac{1}{8\nu} = \frac{1}{8 \times 50} = 2.5 \times 10^{-3}\text{s}$

AC through Inductor

Suppose an inductor with self-inductance (L) is connected to an AC source with instantaneous emf (E), which is given by

$$E = E_0 \sin \omega t \qquad \ldots\text{(i)}$$



An AC source connected to an inductor

When key K is closed, then current I begins to grow because magnetic flux linked with it changes and induced emf produces which opposes the applied emf.

According to Lenz's law,

$$e = -L\frac{dI}{dt} \qquad \ldots\text{(i)}$$

where, e is induced emf and $\frac{dI}{dt}$ is the rate of change of current.

To maintain the flow of current in the circuit, applied voltage must be equal and opposite to the induced emf i.e.

$$E = -e$$

$$\therefore \quad E = -\left(-\frac{LdI}{dt}\right) = \frac{LdI}{dt} \quad \text{or} \quad dI = \frac{E}{L}dt \quad \text{[from Eq. (i)]}$$

Integrating the above equation on both sides, we get

$$\int dI = \int \frac{E}{L}dt \Rightarrow I = \int \frac{E_0 \sin \omega t}{L} dt \quad [\because E = E_0 \sin \omega t]$$

$$\Rightarrow \quad I = \frac{E_0}{L}\left[\frac{-\cos \omega t}{\omega}\right]$$

$$\Rightarrow \quad I = -\frac{E_0}{\omega L}\sin(\pi/2 - \omega t) \quad \left[\because \sin\left(\frac{\pi}{2} - \omega t\right) = \cos \omega t\right]$$

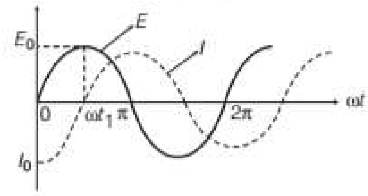
$$\Rightarrow \quad I = \frac{E_0}{\omega L}\sin(\omega t - \pi/2) \qquad \ldots\text{(ii)}$$

If $\sin(\omega t - \pi/2) = \text{maximum} = 1$, then $I = I_0$

where, peak value of current, $I_0 = \frac{E_0}{\omega L}$

$$\therefore \quad \boxed{I = I_0 \sin(\omega t - \pi/2)} \qquad \ldots\text{(iii)}$$

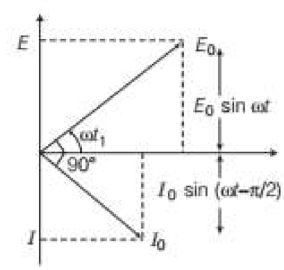
From Eqs. (i) and (iii), it is clear that **in a pure inductor, the current lags behind the voltage by a phase angle of $\pi/2$ radians (90°) or the voltage leads the current by a phase angle of $\pi/2$ radians (90°).**



Graph of *E* and *I versus* ω*t*

Phasor Diagram

The phasor representing peak emf E_0 makes an angle ωt_1 in anti-clockwise direction from horizontal axis. As current lags behind the voltage by 90°, so the phasor representing I_0 is turned 90° clockwise with the direction of E_0.



Phasor digram for purely inductive circuit

Inductive Reactance (X_L)

The opposing nature of inductor to the flow of alternating current is called **inductive reactance**.

As, $\quad I = \frac{E_0}{\omega L}\sin(\omega t - \pi/2)$ or $I_0 = E_0/\omega L$

Comparing the above with Ohm's law, $I_0 = \frac{E_0}{R}$. The quantity ωL is analogous to the resistance and is denoted by X_L.

So, $\quad \boxed{X_L = \omega L}$

where, X_L is called inductive reactance.

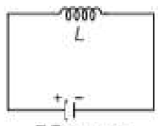
If f is the frequency of AC source, then

$$X_L = \omega L = 2\pi\nu L \qquad \ldots(\text{i}) \quad \left[\because \omega = \frac{2\pi}{T} = 2\pi\nu\right]$$

The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω). The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit. It is also directly proportional to the inductance and to the frequency of the AC current. Thus, if the frequency of AC increases, its inductive reactance also increases.

If inductor is connected to DC source

$$\nu = 0 \qquad \left[\because \nu = \frac{1}{T}\right]$$



DC source

Here, ν is frequency. So, from Eq. (i)
$X_L = 0$.
Therefore, inductor passes DC and blocks AC of very high frequency.

EXAMPLE |2| Alternating emf of $E = 220 \sin 100\pi t$ is applied to a circuit containing an inductance of $(1/\pi)$ H. Write equation for instantaneous current through the circuit. What will be the reading of AC galvanometer connected in the circuit?

Sol. Given, $E = 220 \sin 100\pi t$

$$E_0 = 220\,\text{V}, \omega = 100\pi, L = (1/\pi)\,\text{H}$$

Since, inductive reactance, $X_L = \omega L$

$$X_L = 100\pi \times \frac{1}{\pi} = 100\,\Omega$$

$$\therefore \quad I_0 = \frac{E_0}{X_L} = \frac{220}{100} = 2.2\,\text{A}$$

As current lags behind the emf by a phase angle of $\frac{\pi}{2}$.

$$\therefore \quad I = I_0 \sin(\omega t - \pi/2) = 2.2 \sin(100\pi t - \pi/2)$$

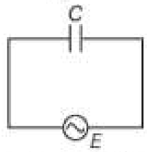
$\therefore$ Reading of AC galvanometer, $I_V = \frac{I_0}{\sqrt{2}} = \frac{2.2}{\sqrt{2}}$

$$= \frac{2.2}{\sqrt{2}} = 1.55\,\text{A}$$

AC through Capacitor

Let us consider a capacitor with capacitance C be connected to an AC source with an emf having instantaneous value,

$$E = E_0 \sin \omega t \qquad \ldots(\text{i})$$



An AC source connected to a capacitor

Due to this emf, charge will be produced and it will charge the plates of capacitor with positive and negative charges. If potential difference across the plates of capacitor is V, then

$$V = \frac{q}{C} \text{ or } q = CV$$

The instantaneous value of current in the circuit,

$$I = \frac{dq}{dt} = \frac{d}{dt}(CE) \qquad [\because V = E]$$

$$= \frac{d}{dt}(CE_0 \sin \omega t) \qquad [\because E = E_0 \sin \omega t]$$

$$= CE_0 \cos \omega t \times \omega$$

$$= \frac{E_0}{1/\omega C} \cos \omega t$$

$$\Rightarrow \quad I = \frac{E_0}{1/\omega C} \sin(\omega t + \pi/2) \qquad \ldots(\text{ii})$$

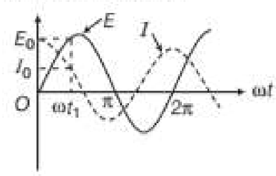
$$[\because \cos \omega t = \sin(\pi/2 + \omega t)]$$

I will be maximum when $\sin(\omega t + \pi/2) = 1$, so that $I = I_0$

where, peak value of current is, $I_0 = \frac{E_0}{1/\omega C}$

$$\therefore \quad \boxed{I = I_0 \sin(\omega t + \pi/2)} \qquad \ldots(\text{iii})$$

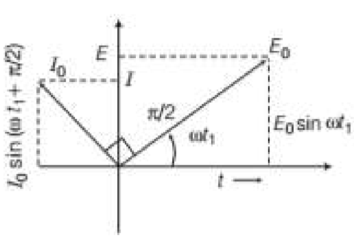
From Eqs. (i) and (iii), it is clear that **in a perfect capacitor, the current leads the voltage by a phase angle of $\pi/2$ radians (90°) or the voltage lags behind the current by a phase angle of $\pi/2$ radians (90°).**



Graph of E and I *versus* ωt

Phasor Diagram

The phasor representing peak emf E_0 makes an angle ωt_1 in anti-clockwise direction with respect to horizontal axis. As current leads the voltage by 90°, the phasor representing I_0 is turned 90° anti-clockwise with the phasor representing E_0. The projections of these phasors on the vertical axis give instantaneous values of E and I.



Phasor digram for purely capacitive circuit

Capacitive Reactance (X_C)

The instantaneous value of alternating current through a capacitor is given by

$$I = \frac{E_0}{1/\omega C} \sin(\omega t + \pi/2) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Comparing the above with Ohm's law we get, $I_0 = \dfrac{E_0}{1/\omega C}$

$$\therefore \qquad \boxed{X_C = \frac{1}{\omega C}}$$

where, X_C is called capacitive reactance.

The opposing nature of capacitor to the flow of alternating current is called **capacitive reactance**.

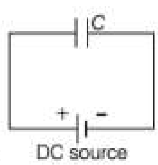
If ν is the frequency of the alternating current, then

$$\boxed{X_C = \frac{1}{2\pi \nu C}} \qquad \left[\because \omega = \frac{2\pi}{T} = 2\pi\nu\right]$$

The dimension of capacitive reactance is same as that of resistance and its SI unit is ohm (Ω). The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit. It is inversely proportional to the capacitance and frequency of the current.

Thus, if frequency of AC increases, then its capacitive reactance decreases.

When capacitor is connected to DC source,



$$X_C = \frac{1}{\omega C} = \frac{1}{0} = \infty$$

$[\because$ for DC, $\omega = 2\pi\nu = 0$, as $\nu = 0]$

Thus, capacitor blocks DC and acts as open circuit while it passes AC of high frequency.

EXAMPLE |3| A capacitor of $10\mu F$ is connected to an AC source of emf $E = 220 \sin 100\pi t$. Write the equation of instantaneous current through the circuit. What will be the reading of AC ammeter connected in the circuit?

Sol. Given, capacitance, $C = 10\mu F = 10 \times 10^{-6}$ F,

emf, $E = 220 \sin 100\pi t = E_0 \sin \omega t$

$\therefore \quad E_0 = 220$ V, $\omega = 2\pi\nu = 100\pi \Rightarrow \nu = 50$ Hz

Since, capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 10^{-5}} = 318.5\,\Omega$$

$$I_0 = \frac{E_0}{X_C} = \frac{220}{318.5} = 0.691 \text{ A}$$

So, reading of AC ammeter,

$$I_V = \frac{I_0}{\sqrt{2}} = \frac{0.691}{1.414} = 0.489 \text{ A}$$

AC THROUGH *L-C-R* CIRCUIT

Suppose that an inductor (L), a capacitor (C) and a resistor (R) are connected in series to an AC source. I is the current passing through this circuit. As R, L and C are in series, therefore at any instant through the three elements, AC has the same amplitude and phase. Let it be represented by

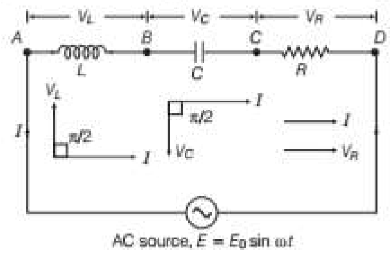
$$I = I_0 \sin \omega t$$

However, voltage across each element bears a different phase relationship with the current.

$V_L = I_0 X_L$ [V_L is maximum voltage across L]

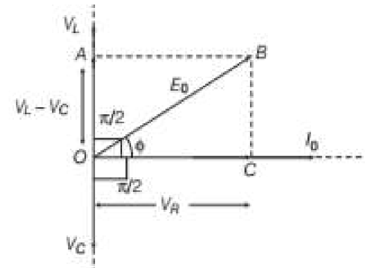
$V_C = I_0 X_C$ [V_C is maximum voltage across C]

$V_R = I_0 R$ [V_R is maximum voltage across R]



An AC source connected to *L-C-R* circuit

Inside the above figure for a *L-C-R* circuit, phasor diagrams of each L, C and R are given. To form phasor diagram for series *L-C-R* circuit, combine all these phasor diagrams.



Phasor diagram for a series *L-C-R* circuit

Since, voltage (V_L) is in upward direction and voltage (V_C) in downward direction, so net voltage upto point A is $V_L - V_C$ (assuming $V_L > V_C$) and net maximum voltage is V_0.

From phasor diagram,

$$OB = \sqrt{(OC)^2 + (CB)^2} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow \qquad E_0 = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2} \qquad [\because OB = E_0]$$

$$\Rightarrow \qquad E_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \qquad Z = \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}$$

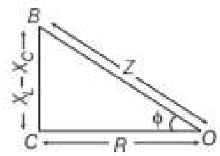
Here, Z is called **impedance**.

Impedance

It is the total resistance of a circuit applied in the path of alternating current. It is given by

$$\boxed{Z=\sqrt{R^2+(X_L-X_C)^2}} \quad \ldots(i)$$

From phasor diagram, it is clear that voltage leads the current by an angle ϕ.



Impedance diagram of L - C - R circuit

$\therefore$ From ΔOCB,

$$\tan\phi=\frac{CB}{OC}=\frac{V_L-V_C}{V_R}=\frac{I_0X_L-I_0X_C}{I_0R}$$

$$\Rightarrow \quad \tan\phi=\frac{X_L-X_C}{R} \quad \ldots(ii)$$

So, the alternating emf in the series L-C-R circuit would be represented by $E=E_0\sin(\omega t+\phi)$.

Eqs. (i) and (ii) are graphically shown in the above shown graph. This is called impedance diagram, which is a right angled triangle with Z as its hypotenuse.

The amplitude and phase of current for an L-C-R series circuit is obtained by using the technique of phasors. But this method of analysing AC circuits have certain disadvantages. Firstly, the phasor diagram does not signify anything about initial condition. One can take any arbitrary value of t and draw different phasors which shows the relative angle between different phasors. The solution so obtained is called the steady state solution.

Special Cases

(i) When $X_L=X_C$, then $Z=R$ and $\tan\phi=0$ $\quad [\because \phi=0°]$

Hence, voltage and current are in the same phase. Therefore, the AC circuit is non-inductive.

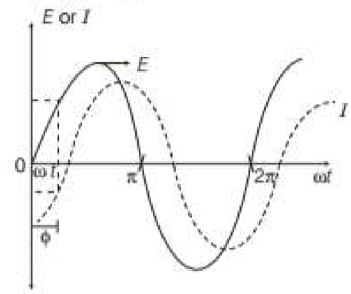
(ii) When $X_L>X_C$, then $\tan\phi$ is positive.

Hence, voltage leads the current by a phase angle ϕ. Therefore, the AC circuit is inductance dominated circuit.

(iii) When $X_C>X_L$, then $\tan\phi$ is negative.

Hence, voltage lags behind the current by a phase angle ϕ. Therefore the AC circuit is capacitance dominated circuit.

A graph (given below) is showing variation of E and I with ωt for the case, $X_L>X_C$.



Graph of E and I *versus* ωt for series L-C-R circuit when $X_C<X_L$

EXAMPLE |4| A capacitor of 100μF and a coil of resistance 50Ω and inductance 0.5 H are connected in series with a 110 V -50 Hz source. Calculate the rms value of current in the circuit.

Sol. Given, capacitance, $C=100\,\mu F=100\times10^{-6}\,F=10^{-4}\,F$

Resistance, $R=50\,\Omega$

Inductance, $L=0.5\,H$

Rms value of voltage, $E_V=110\,V$

Frequency, $\nu=50\,Hz$

Since, capacitive reactance,

$$X_C=\frac{1}{\omega C}=\frac{1}{2\pi\nu C}=\frac{1}{2\times3.14\times50\times10^{-4}}$$

$$X_C=31.85\,\Omega$$

and inductive reactance,

$$X_L=\omega L=2\pi\nu L=2\times3.14\times50\times0.5=157\,\Omega$$

$\because$ Impedance, $Z=\sqrt{R^2+(X_L-X_C)^2}$

$$=\sqrt{(50)^2+(157-31.85)^2}=134.77\,\Omega$$

$$\Rightarrow \quad I_V=\frac{E_V}{Z}=\frac{110}{134.77}=0.816\,A$$

EXAMPLE |5| A coil of 0.01H inductance and 1Ω resistance is connected to 200 V, 50 Hz AC supply. Find the impedance of the circuit and time lag between maximum alternating voltage and current. **NCERT Exemplar**

Sol. Given, inductance, $L=0.01\,H$

Resistance, $R=1\,\Omega$

Voltage, $V=200\,V$

Frequency, $\nu=50\,Hz$

Impedance of the circuit, $Z=\sqrt{R^2+X_L^2}$

$$=\sqrt{R^2+(2\pi\nu L)^2}=\sqrt{1^2+(2\times3.14\times50\times0.01)^2}$$

$$=\sqrt{10.86}=3.3\,\Omega$$

$$\tan \phi = \frac{\omega L}{R} = \frac{2\pi \nu L}{R} = \frac{2 \times 3.14 \times 50 \times 0.01}{1} = 3.14$$

$$\Rightarrow \quad \phi = \tan^{-1}(3.14) = 72^\circ$$

Phase difference, $\phi = \frac{72 \times \pi}{180}$ rad

Time lag between maximum alternating voltage and current,

$$\Delta t = \frac{\phi}{\omega} = \frac{72\pi}{180 \times 2\pi \times 50} = \frac{1}{250}\text{ s}$$

Resonance

In a series L-C-R circuit, when phase (ϕ) between current and voltage is zero, then the circuit is said to be a **resonant circuit**.

As applied frequency increases, then

$X_L = \omega L$, X_L increases and $X_C = \frac{1}{\omega C}$, X_C decreases.

At some angular frequency (ω_r), $X_L = X_C$

where, $\quad X_L = \omega_r L,\ X_C = \frac{1}{\omega_r C}$

The frequency at which X_C and X_L become equal, is called **resonant frequency**.

$$\Rightarrow \quad \omega_r L = \frac{1}{\omega_r C} \text{ or } \omega_r^2 = \frac{1}{LC} \text{ or } (2\pi\nu_r)^2 = \frac{1}{LC}$$

$[\because \omega_r = 2\pi\nu_r$, where ν_r is resonating frequency$]$

$$2\pi\nu_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad \boxed{\nu_r = \frac{1}{2\pi\sqrt{LC}}}$$

At resonating frequency,

$$Z = R = \text{Minimum}$$

$$\therefore \quad I = \frac{E}{Z} = \text{Maximum}$$

Since, Z is minimum, therefore I will be maximum.

EXAMPLE |6| A 2 μF capacitor, 100 Ω resistor and 8 H inductor are connected in series with an AC source. What should be the frequency of source for which the current drawn in the circuit is maximum? If peak value of emf of source is 200 V, find the maximum current, inductive reactance, capacitive reactance, total impedance, peak value of current in the circuit. What is the phase relation between the voltages across inductor and resistor? Also, give the phase relation between voltages across inductor and capacitor.

Sol. Given, capacitance, $C = 2\,\mu\text{F} = 2 \times 10^{-6}$ F

Resistance, $R = 100\ \Omega$

Inductance, $L = 8$ H

Peak value of voltage, $E_0 = 200$ V

When frequency of AC source is equal to resonant frequency,

then current drawn in the circuit is maximum.

$$\therefore \quad \nu = \nu_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14 \times \sqrt{8 \times 2 \times 10^{-6}}}$$

$$= \frac{1000}{8 \times 3.14} = 39.8\text{ Hz}$$

Peak value of current, $I_0 = \frac{E_0}{R} = \frac{200}{100} = 2$ A

$$\because \quad X_C = X_L = \omega L = 2\pi\nu L$$

$$= 2 \times 3.14 \times 39.8 \times 8 = 2000\,\Omega \Rightarrow Z = R = 100\,\Omega$$

The voltages across inductor and resistor differ in phase by 90° and the voltages across inductor and capacitor differ in phase by 180°.

Quality Factor (Q-Factor)

It is the measure of sharpness of the resonance of an L-C-R circuit. It is defined as the ratio of voltage developed across the inductance or capacitance at resonance to the impressed voltage, which is the **voltage applied across R**.

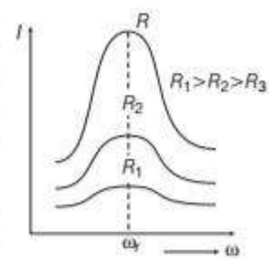
$$Q\text{-factor} = \frac{\text{Voltage across } L \text{ (or } C)}{\text{Voltage across } R}$$

$$Q\text{-factor} = \frac{V_L \text{ or } V_C}{V_R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\boxed{Q\text{-factor} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}}$$

Q is just a number having no dimensions, it can also be called **voltage multiplication factor** of the circuit.

The electronic circuit with high Q values would respond to a very narrow range of frequencies and *vice-versa*. Higher the value of Q, the narrower and sharper is the resonance.



I versus ω graph of an *L-C-R* circuit

Q-factor can also be defined as the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency such that at each frequency, the current amplitude becomes $\frac{1}{\sqrt{2}}$ times the value at resonant frequency.

Mathematically,

$$Q\text{-factor or } Q = \frac{\omega_r}{\omega_1 - \omega_2}$$

where, ω_1 and ω_2 are frequencies when current decreases to 0.707 $(1/\sqrt{2})$ times the peak value of current.

We can also write,
$$\omega_1 = \omega_r + \Delta\omega$$
$$\omega_2 = \omega_r - \Delta\omega$$

The difference $\omega_1 - \omega_2 = 2\Delta\omega$ is often called the bandwidth of the circuit.

Thus, from the above, Q-factor can also be defined as the ratio of resonant angular frequency to bandwidth of the circuit.

The smaller the bandwidth $(\Delta\omega)$, the sharper and narrower is the resonance.

Significance of Q-Factor

- Q-factor denotes the sharpness of tuning.
- High Q-factor indicates lower rate of energy loss.
- Higher value of Q-factor indicates sharper peak in the current.
- For $R = 0$, Q-factor = infinity

AVERAGE POWER ASSOCIATED IN AC CIRCUIT

Power is defined as the rate of doing work.

$$P = \frac{dW}{dt} \quad \dots\text{(i)}$$

or

Power is defined as the product of voltage and current.

In AC circuit, both emf and current change continuously with respect to time. So in it we have to calculate average power in complete cycle $(0 \to T)$.

Instantaneous power, $P = EI$ ...(ii)

$$[\because E = E_0 \sin \omega t,\ I = I_0 \sin(\omega t + \phi)]$$

Here, E and I are instantaneous voltage and current, respectively. If the instantaneous power remains constant for a small time dt, then small amount of work done in maintaining the current for a small time dt is

$$\frac{dW}{dt} = EI$$

$$\Rightarrow \quad dW = EI\, dt \quad \dots\text{(iii)}$$

Integrating Eq. (iii) on both sides, we get

$$\int dW = \int_0^T EI\, dt$$

Total work done or energy spent in maintaining current over one full cycle,

$$W = \int_0^T E_0 \sin \omega t \cdot I_0 \sin(\omega t + \phi)\, dt$$

$$= E_0 I_0 \int_0^T \sin \omega t(\sin \omega t \cos\phi + \cos \omega t \sin \phi)\, dt$$

$$= E_0 I_0 \left[\cos\phi \int_0^T \sin^2 \omega t\, dt + \sin\phi \int_0^T \sin \omega t \cos \omega t\, dt\right]$$

$$= E_0 I_0 \left[\cos\phi \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt + \frac{\sin\phi}{2} \int_0^T 2 \sin \omega t \cos \omega t\, dt\right]$$

$$= \frac{E_0 I_0}{2}\left[\cos\phi \left(\int_0^T dt - \int_0^T \cos 2\omega t\, dt\right) + \sin\phi \int_0^T \sin 2\omega t\, dt\right]$$

$$= \frac{E_0 I_0}{2}\left[\left(\cos\phi [t]_0^T - \int_0^T \cos 2\omega t\, dt\right) + \sin\phi \int_0^T \sin 2\omega t\, dt\right]$$

But $\int_0^T \cos 2\omega t\, dt = 0$ or and $\int_0^T \sin 2\omega t\, dt = 0$

$$\therefore \quad W = \frac{E_0 I_0 T}{2} \cos\phi$$

Average power associated in AC circuit,

$$P_{av} = \frac{W}{T} = \frac{E_0 I_0 T \cos\phi}{2T} = \frac{E_0 I_0}{2} \cos\phi$$

$$\boxed{P_{av} = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos\phi}$$

or
$$P_{av} = E_{rms} I_{rms} \cos\phi$$
$$= E_V I_V \cos\phi$$

Here, $\cos\phi$ is power factor, which is defined as the cosine of the angle of lag or lead.

If P_{av} is true power or average power, then power factor is given by,

$$\cos\phi = \frac{P_{av}}{E_{rms} I_{rms}} = \frac{\text{True power}}{\text{Apparent power}} \quad \cos\phi = \frac{R}{Z}$$

Here, ϕ is the phase difference between I_{rms} and E_{rms}.

Special Cases

(i) **AC circuit containing R**

When $\phi = 0°$, then $P_{av} = E_V I_V \cos 0°$

$$P_{av} = E_V I_V$$

So, average power in R is maximum.

(ii) **AC circuit containing L**

When $\phi = \frac{\pi}{2}$, then $P_{av} = E_V I_V \cos\frac{\pi}{2}$

$$P_{av} = 0$$

So, average power in L is zero.

(iii) **AC circuit containing C**

When $\phi = \frac{\pi}{2}$, then $P_{av} = E_V I_V \cos\frac{\pi}{2}$

$$P_{av} = 0$$

So, average power in C is zero.

(iv) **AC circuit containing L and R**

When $\tan\phi = \frac{\omega L}{R} \Rightarrow \cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$,

then $\quad P_{av} = E_V I_V \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$

(v) **AC circuit containing C and R**

When $\tan\phi = \frac{1/\omega C}{R} \Rightarrow \cos\phi = \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$

then $\quad P_{av} = E_V I_V \cdot \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$

(vi) **AC circuit containing L, C and R**

When $\tan\phi = \frac{\omega L - 1/\omega L}{R}$

$\Rightarrow \quad \cos\phi = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$

then $\quad P_{av} = E_V I_V \cdot \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

EXAMPLE |7| A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series L-C-R circuit in which $R = 3\,\Omega$, $L = 25.48$ mH and $C = 796\,\mu$F. Find

(i) the impedance of the circuit.

(ii) phase difference between the voltage across the source and current.

(iii) the power dissipated in the circuit.

(iv) the power factor. **NCERT**

Sol. Given, $E_0 = 283\,\text{V}$, $\nu = 50\,\text{Hz}$, $R = 3\,\Omega$,

$$L = 25.48\,\text{mH} = 25.48 \times 10^{-3}\,\text{H}$$

and $\quad C = 796\,\mu\text{F} = 796 \times 10^{-6}\,\text{F}$

(i) Since, inductive reactance, $X_L = \omega L$

$\Rightarrow \quad X_L = 2\pi\nu L$

$$= 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3} = 8\,\Omega$$

Since, capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}}$$

$$X_C = 4\,\Omega$$

$\therefore$ Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{3^2 + (8-4)^2} = 5\,\Omega$$

(ii) Phase difference,

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{8-4}{3}\right) = 53.1^\circ$$

It means that the current in the circuit lags behind the voltage by 53.1°

(iii) Power dissipated in the circuit, $P = I_V^2 R$

$\because \quad I_V = \frac{I_0}{\sqrt{2}Z} = \frac{283}{1.414 \times 5} = 40\,\text{A}$

$\therefore \quad P = I_V^2 R = (40)^2 \times 3 = 4800\,\text{W}$

(iv) Power factor, $\cos\phi = \cos 53.1^\circ = 0.60^\circ$

EXAMPLE |8| Suppose the frequency of the source in the above example can be varied.

(i) What is the frequency of the source at which resonance occurs ?

(ii) Calculate the impedance, the current and power dissipated of resonant condition. **NCERT**

Sol. (i) Resonant frequency, $\nu = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2 \times 3.14 \times \sqrt{25.48 \times 10^{-3} \times 796 \times 10^{-6}}}$$

$$= 35.4\,\text{Hz}$$

(ii) At resonance, $Z = R = 3\,\Omega$

$\Rightarrow \quad I_V = \frac{E_V}{Z} = \frac{283}{\sqrt{2} \times 3} = 66.7\,\text{A} \quad \left[\because E_V = \frac{E_0}{\sqrt{2}}\right]$

$\therefore$ Power dissipated, $P = I_V^2 R$

$$= (66.7)^2 \times 3 = 13350\,\text{V}$$

WATTLESS CURRENT

The current which consumes no power for its maintenance in the circuit is called **wattless current** or **idle current**.

or

If the resistance in an AC circuit is zero, although current flows in the circuit, then the average power remains zero, i.e. there is no energy dissipation in the circuit, such a circuit is called wattless circuit and the current flowing is called wattless current.

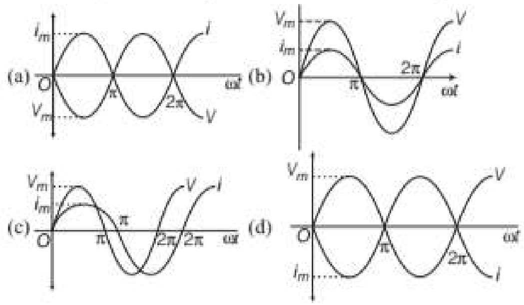
If the circuit contains either inductance or capacitance only, then phase difference between current and voltage is 90°, i.e. $\phi = 90^\circ$. The average power in such a circuit is

$$P_{av} = V_{rms} \times I_{rms} \times \cos\phi = V_{rms} \times I_{rms} \times \cos 90^\circ = 0$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. Which of the following graphs shows, in a pure resistor, the voltage and current are in phase?



2. Voltage and current in an AC circuit are given by

$V = 5\sin(100\,\pi t - \pi/6)$

and $I = 4\sin(100\,\pi t + \pi/6)$

(a) voltage leads the current by 30°
(b) current leads the voltage by 30°
(c) current leads the voltage by 60°
(d) voltage leads the current by 60°

3. A resistance of 20 Ω is connected to a source of an alternating potential, $V = 220\sin(100\,\pi\, t)$. The time taken by current to change from its peak value to rms value is

(a) 0.2 s (b) 0.25 s
(c) 25×10^{-3} s (d) 2.5×10^{-3} s

4. The inductive reactance is directly proportional to the

(a) inductance
(b) frequency of the current
(c) Both (a) and (b)
(d) amplitude of current

5. A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance if the frequency of the source is 50 Hz.

(a) 785 Ω (b) 6.50 Ω
(c) 7.85 Ω (d) 8.75 Ω

6. Current I across the capacitor in a purely capacitive AC circuit is

(a) $i_m \sin(\omega t + \pi/4)$
(b) $i_m \sin(\omega t + \pi/2)$
(c) $i_m \cos(\omega t + \pi/4)$
(d) $i_m \cos(\omega t + \pi/2)$

7. The amplitude of the oscillating current in the a pure capacitive AC circuit is, if $V = V_m \sin\omega t$ and capacitance $= C$.

(a) $\omega C V_m$ (b) $2\omega C V_m$ (c) $\dfrac{\omega C V_m}{4}$ (d) $\dfrac{3\,\omega C V_m}{2}$

8. A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. The capacitive reactance is

(a) 220 Ω (b) 215 Ω (c) 212 Ω (d) 204 Ω

9. L, C and R represents self inductance, capacitance and resistance respectively. Which of the following dimensional formula is not of frequency?

(a) $\dfrac{1}{RC}$ (b) $\dfrac{R}{L}$ (c) $\dfrac{1}{\sqrt{LC}}$ (d) $\dfrac{C}{L}$

10. To reduce the resonant frequency in an L-C-R series circuit with a generator **NCERT Exemplar**

(a) the generator frequency should be reduced
(b) another capacitor should be added in parallel to the first
(c) the iron core of the inductor should be removed
(d) dielectric in the capacitor should be removed

11. In a series L-C-R circuit, the capacitance C is changed to 4C. To keep the resonant frequency same, the inductance must be changed by

(a) $2L$ (b) $L/2$
(c) $4L$ (d) $L/4$

12. Which of the following combinations should be selected for better tuning of an L-C-R circuit used for communication? **NCERT Examplar**

(a) $R = 20\,\Omega$, $L = 1.5$ H, $C = 35\,\mu$ F
(b) $R = 25\,\Omega$, $L = 2.5$ H, $C = 45\,\mu$F
(c) $R = 15\,\Omega$, $L = 3.5$ H, $C = 30\,\mu$F
(d) $R = 25\,\Omega$, $L = 1.5$ H, $C = 45\,\mu$F

VERY SHORT ANSWER Type Questions

13. An electric lamp is connected in series with a capacitor and an AC source is glowing with a certain brightness. How does the brightness of the lamp change on increasing the capacitance?

14. Explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a DC circuit after the steady state. **NCERT**

15. How does the sign of the phase angle ϕ, by which the supply voltage leads the current in an L-C-R series circuit, change as the supply frequency is gradually increased from very low to very high values. **NCERT Exemplar**

16. Define 'quality factor' of resonance in series L-C-R circuit. What is its SI unit? **Delhi 2016**

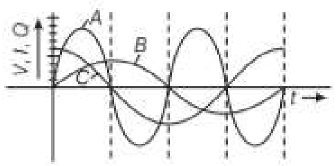
17. How can you improve the quality factor of a series resonance circuit?

18. Mention the significance of quality factor. **Foreign 2012**

19. A device X is connected to an AC source $V = V_0 \sin\omega t$. The variation of voltage, current and power in one complete cycle is shown in the following figure.

(i) Which curve shows power consumption over a full cycle?

(ii) Identify the device X.

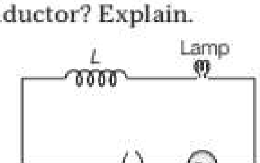


SHORT ANSWER Type Questions

20. Explain, why the reactance offered by an inductor increases with increasing frequency of an alternating voltage? **NCERT Exemplar**

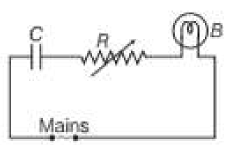
21. (i) When an AC source is connected to an ideal inductor, show that the average power supplied by the source over a complete cycle is zero.

(ii) A lamp is connected in series with an inductor and an AC source. What happens to the brightness of the lamp when the key is plugged in and an iron rod is inserted inside the inductor? Explain.



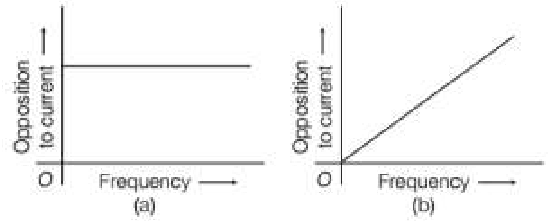
All India 2016

22. A capacitor C, a variable resistance R and a bulb B are connected in series to the AC mains in circuit as shown in the figure. The bulb glows with some brightness. How will the glow of the bulb change, if (i) a dielectric slab is introduced between the plates of the capacitor, keeping resistance R to be same;



(ii) the resistance R is increased keeping the same capacitance? **Delhi 2014**

23. (i) The graphs (a) and (b) represent the variation of the opposition offered by the circuit element to the flow of alternating current with frequency of the applied emf. Identify the circuit elements corresponding to each graph.



(ii) Write the expression for the impedance offered by the series combination of the above two elements connected across the AC source. Which will be ahead in phase in this circuit, voltage or current? **All India 2011**

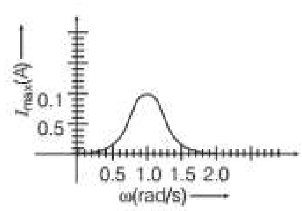
24. (i) Draw a graph showing variation of amplitude of circuit current with changing frequency of applied voltage in a series L-C-R circuit for two different values of resistance R_1 and $R_2 (R_1 > R_2)$.

(ii) Define the term 'Sharpness of Resonance'. Under what condition, does a circuit become more selective? **Foreign 2016**

25. Prove that an ideal capacitor in an AC circuit does not dissipate power. **All India 2017 C**

26. In the analogy between series L-C-R circuit and a mass on a spring, the mass is analogous to the inductance and the spring constant is analogous to the inverse of the capacitance. Explain giving reason.

27. In series L-C-R circuit, the plot of I_{max} *versus* ω is shown in the figure. Find the bandwidth and mark in the figure.



NCERT Exemplar

LONG ANSWER Type I Questions

28. An inductor L of inductance X_L is connected in series with a bulb B and an AC source. How would brightness of the bulb change when

(i) number of turns in the inductor is reduced?

(ii) an iron rod is inserted in the inductor?

(iii) a capacitor of reactance $X_C = X_L$ is inserted in series in the circuit? Justify your answer in each case. **All India 2015**

29. (i) When an AC source is connected to an ideal capacitor. Show that the average power supplied by the source over a complete cycle is zero.

(ii) A lamp is connected in series with a capacitor. Predict your observations when the system is connected first across a DC and then an AC source. What happens in each case, if the capacitance of the capacitor is reduced? **Delhi 2013 C**

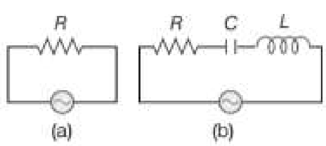
30. Answer the following questions.

(i) What is the minimum value of the power factor of a circuit? Under what circumstances can it occur?

(ii) State the maximum value of the power factor? Under what circumstances can this occur?

31. An AC voltage $V = V_m \sin \omega t$ is applied across an inductor of inductance L. Find the instantaneous power P_i supplied to the inductor. Show graphically the variation of P_i with ωt.

32. Study the circuits (a) and (b) shown in the figure and answer the following questions:



(i) Under which conditions would the rms currents in the two circuits be the same?

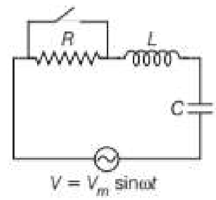
(i) Can the rms current in circuit (b) be larger than that in (a)? **NCERT Exemplar**

33. In the L-C-R circuit, shown in the figure, the AC driving voltage is $V = V_m \sin \omega t$.

(i) Write down the equation of motion for $q(t)$.

(ii) At $t = t_0$, the voltage source stops and R is short circuited. Now, write down how much energy is stored in each of L and C.

(iii) Describe subsequent motion of charges. **NCERT Exemplar**



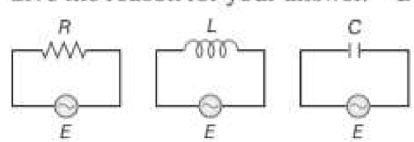
LONG ANSWER Type II Questions

34. An AC source of voltage $V = V_0 \sin \omega t$ is connected to a series combination of L, C and R. Use the phasor diagram to obtain expressions for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit in the condition called? **Delhi 2016**

35. (i) What do you understand by sharpness of resonance in a series L-C-R circuit? Derive an expression for Q-factor of the circuit.

(ii) Three electrical circuits having AC sources of variable frequency are shown in the figures. Initially, the current flowing in each of these is same. If the frequency of the applied AC source is increased, how will the current flowing in these circuits be affected?

Give the reason for your answer. **Delhi 2011**



36. Derive an expression for the impedance of a series L-C-R circuit connected to an AC supply of variable frequency. Plot a graph showing variation of current with the frequency of the applied voltage. Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set? **Delhi 2011**

37. (i) Show that a series L-C-R circuit at resonance behaves as a purely resistive circuit. Compare the phase relation between current and voltage in series L-C-R circuit for

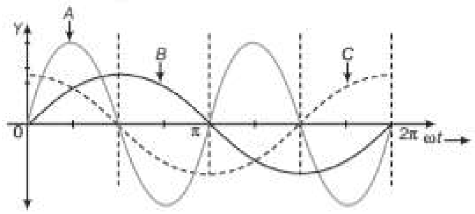
(a) $X_L > X_C$

(b) $X_L = X_C$ using phasor diagrams.

(ii) What is an acceptor circuit and where it is used?

38. In a series, L-C-R circuit connected to an AC source of variable frequency and voltage $V = V_m \sin \omega t$, draw a plot showing the variation of current I with angular frequency ω, for two different values of resistance R_1 and $R_2 (R_1 > R_2)$. Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced? Define Q-factor of the circuit and give its significance.
All India 2013

39. A device X is connected to an AC source, $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph.



(i) Identify the device X.
(ii) Which of the curves A, B and C represent the voltage, current and the power consumed in the circuit? Justify the answer.
(iii) How does its impedance vary with frequency of the AC source? Show graphically.
(iv) Obtain an expression for the current in the circuit and its phase relation with AC voltage. **All India 2017**

40. (i) A voltage $V = V_0 \sin\omega t$ applied to a series L-C-R circuit derives a current $I = I_0 \sin\omega t$ in the circuit. Deduce the expression for the average power dissipated in the circuit.
(ii) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.
(iii) Define the term wattless current. **Delhi 2012**

41. A device X is connected across an AC source of voltage $V = V_0 \sin\omega t$. The current through X is given as $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.

(a) Identify the device X and write the expression for its reactance.
(b) Draw graphs showing variation of voltage and current with time over one cycle of AC, for X.
(c) How does the reactance of the device X vary with frequency of the AC? Show this variation graphically.
(d) Draw the phasor diagram for the device X.
CBSE 2018

NUMERICAL PROBLEMS

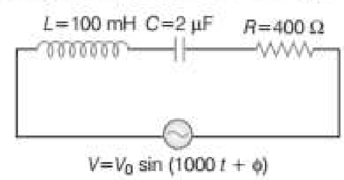
42. An alternating voltage given by $E = 140 \sin 314t$ is connected across a pure resistor of 50 Ω. Find
(i) the frequency of the source.
(ii) the rms current through the resistor.
All India 2012

43. A coil of inductance 0.5 H and resistance 100 Ω is connected to a 240 V, 50 Hz AC supply.
(i) What is the maximum current in the coil?
(ii) What is the time lag between the voltage maximum and current maximum?
NCERT

44. A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.
(i) What is the maximum current in the circuit?
(ii) What is the time lag between the current maximum and the voltage maximum?
NCERT

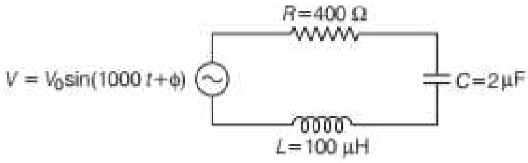
45. A resistor of 400 Ω, an inductor of $5/\pi$ H and a capacitor of $\frac{50}{\pi}\mu F$ are connected in series across a source of alternating voltage of $140 \sin 100\pi t$ V. Find the voltage (rms) across the resistor, the inductor and the capacitor. Is the algebraic sum of these voltage more than the source voltage? If yes, resolve the paradox. **Foreign 2010**

46. (i) Find the value of the phase difference between the current and the voltage in the series L-C-R circuit shown below. Which one leads in phase, current or voltage?



(ii) Without making any other change, find the value of the additional capacitor C', to be connected in parallel with the capacitor C, in order to make the power factor of the circuit unity. **Delhi 2017**

47. Determine the value of phase difference between the current and the voltage in the given series L-C-R circuit. **Delhi 2015**



48. A 10 V, 650 Hz source is connected to a series combination of $R = 100\ \Omega$, $C = 10\mu F$ and $L = 0.15$ H. Find out the time in which resistance will get heated by 10 °C, if thermal capacity of the material = 2 J/°C.

49. Calculate the quality factor of a series L-C-R circuit with $L = 2.0$ H, $C = 2\mu F$ and $R = 10\ \Omega$. **Foreign 2012**

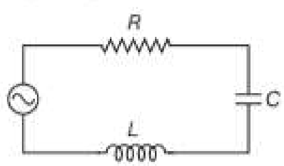
50. Resonance frequency of a circuit is ν. If the capacitance is made 4 times the initial value, find the change in the resonance frequency.

51. A $2\mu F$ capacitor, $100\ \Omega$ resistor and 8 H inductor are connected in series with an AC source.

(i) What should be the frequency of the source such that current drawn in the circuit is maximum? What is this frequency called?

(ii) If the peak value of emf of the source is 200 V, find the maximum current. **Foreign 2016**

52. The figure shows a series L-C-R circuit with $L = 10.0$ H, $C = 40\ \mu F$, $R = 60\ \Omega$ connected to variable frequency 240 V source. Calculate



(i) the angular frequency of the source which drives the circuit at resonance.

(ii) the current at the resonating frequency.

(iii) the rms potential drop across the inductor at resonance. **Delhi 2012**

53. Obtain the resonant frequency (ω_r) of a series L-C-R circuit with $L = 2.0$ H, $C = 32\mu F$ and $R = 10\ \Omega$. What is the Q-value of this circuit? **NCERT**

54. An inductor of 200 mH, capacitor of $400\ \mu F$ and a resistor of $10\ \Omega$ are connected in series to AC source of 50V of variable frequency. Calculate

(i) the angular frequency at which maximum power dissipation occurs on the circuit and the corresponding value of effective current, and

(ii) the value of Q-factor on the circuit. **All India 2017 C**

55. Obtain the resonant frequency and Q-factor of a series L-C-R circuit with $L = 3.0$ H, $C = 27\ \mu F$ and $R = 7.4\ \Omega$. It is designed to improve the sharpness of resonance of the circuit by reducing its full width at half maximum by a factor of 2. Suggest a suitable way. **NCERT**

56. A $100\ \Omega$ resistor is connected to a 220 V, 50 Hz supply.

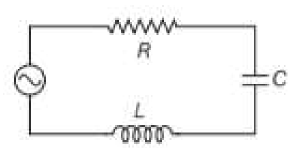
(i) What is the rms value of current in the circuit?

(ii) What is the net power consumed over a full cycle? **NCERT**

57. A 44 mH inductor is connected to 220 V, 50 Hz AC supply. Determine the rms value of the current in the circuit. What is the net power absorbed over a complete cycle? Explain. **NCERT**

58. A $60\ \mu F$ capacitor is connected to a 110 V, 60 Hz AC supply. Determine the rms value of current in the circuit. What is the net power absorbed over a complete cycle? Explain. **NCERT**

59. A series L-C-R circuit connected to a variable frequency 230 V source has $L = 5.0$ H, $C = 80\ \mu F$, $R = 40\ \Omega$, as shown in the figure.



(i) Determine the source frequency which drives the circuit in resonance.

(ii) Obtain the impedance of the circuit and amplitude of current at the resonant frequency.

(iii) Determine the rms potential drop across the three elements of the circuit. Show that the potential drop across the L-C combination is zero at the resonating frequency. **NCERT**

60. A circuit containing 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance in the circuit is negligible.

(i) Obtain the current amplitude and rms value.

(ii) Obtain the rms value of potential drop across each element.

(iii) What is the average power transferred to inductor?

(iv) What is the average power transferred to capacitor?

(v) What is the total average power absorbed by the circuit? **NCERT**

61. A series L-C-R circuit with $L = 0.12$ H, $C = 480$ nF, $R = 23\ \Omega$ is connected to a 230 V variable frequency supply.

(i) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.

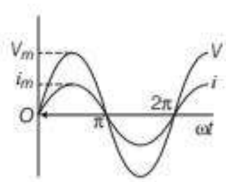
(ii) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of maximum power.

(iii) For which frequency of the source is the power transferred to the circuit half the power at resonant? What is the current amplitude at these frequencies?

(iv) What is the Q-factor of the given circuit? **NCERT**

HINTS AND SOLUTIONS

1. (b)



In a pure resistor, the voltage and current are in phase. The minima zero and maxima occur at the same respective times.

2. (c) Phase difference $\Delta\phi = \phi_2 - \phi_1 = \pi/6 - (-\pi/6) = \pi/3$

So, current leads the voltage by $\pi/3$.

3. (d) Current in at peak value so its equation is

$$i = i_0 \sin(100\pi t + \pi/2)$$

Peak value to rms value means current becomes $1/\sqrt{2}$ times.

So, from $\quad i = i_0 \sin(100\pi t + \pi/2)$

$$\frac{i_0}{\sqrt{2}} = i_0 \sin(100\pi t + \pi/2)$$

$$\sin 3\pi/4 = \sin(100\pi t + \pi/2)$$

$$\Rightarrow \quad t = \frac{1}{400}\text{ s}$$

Time taken by current to change from its peak value to rms value,

i.e., $\quad t = \frac{1}{400}\text{ s} = 2.5 \times 10^{-3}$ s

4. (c) Inductive reactance, $X_L = \omega L = 2\pi f L$

5. (c) The inductive reactance,

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 25 \times 10^{-3} = 7.85\ \Omega$$

6. (b) Current I across the capacitor is $i_m \sin(\omega t + \pi/2)$.

7. (a) The amplitude of the oscillating current is

$$I_m = V_m / X_c = \omega C V_m$$

8. (c) The capacitive reactance is

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\pi\ (50\text{ Hz})\ (15.0 \times 10^{-6}\text{ F})} = 212\ \Omega$$

9. (d) $\frac{C}{L}$ is not the dimensional formula of frequency because $\frac{C}{L} = \frac{[M^{-1}L^{-2}T^4A^2]}{[ML^2T^{-2}A^{-2}]}$ but dimensional formula of frequency is $[T^{-1}]$.

10. (b) We know that resonant frequency in an L-C-R circuit is given by

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

Now to reduce ν_0 either we can increase L or we can increase C.

To increase capacitance, we must connect another capacitor parallel to the first.

11. (d) The resonant frequency, $f = \frac{1}{\sqrt{4\pi^2 LC}}$

Again, $\quad f = \frac{1}{\sqrt{4\pi^2 (L/4) \times 4C}}$

$$\Rightarrow \quad f = \frac{1}{\sqrt{4\pi^2 LC}}$$

If the value of L is changed to $L/4$, then the resonant frequency will remain unchanged.

12. (c) Quality factor (Q) of an L-C-R circuit is given by

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

where R is resistance, L is inductance and C is capacitance of the circuit. To make Q high,

R should be low, L should be high and C should be low.

These conditions are best satisfied by the values given in option (c).

13. Capacitive reactance is given by,

$$X_C = \frac{1}{\omega C} \Rightarrow X_C \propto \frac{1}{C}$$

This means, with the increase in the capacitance, the capacitive reactance decreases. So, if an electric lamp is connected in a series with a capacitor and an AC source is glowing with certain brightness, then with the increase in the capacitance, the brightness of the lamp increases.

14. By comparison, at very high frequency, the resistance due to capacitor is negligible and hence it works like a pure conductor of negligible capacitive reactance.

In DC circuits, $\omega = 0$ (at steady state)

$$\Rightarrow \quad X_C = \frac{1}{\omega C} = \infty$$

So, it behaves like an open circuit.

15. The phase angle (ϕ) by which voltage leads the current in L-C-R series circuit is given by

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{2\pi\nu L - \frac{1}{2\pi\nu C}}{R}$$

If $\tan\phi < 0$ (for $\nu < \nu_0$), then circuit is capacitive.

If $\tan\phi > 0$ (for $\nu > \nu_0$), then circuit is inductive.

At resonance, $\tan\phi = 0$ $\quad \left[\text{for } \nu = \nu_0 = \frac{1}{2\pi\sqrt{LC}}\right]$

16. The quality factor (Q) of resonance in series L-C-R circuit is defined as the ratio of voltage drop across inductor (or capacitor) to the applied voltage,

i.e. $Q = \frac{V_L \text{ or } V_C}{V_R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$

It is an indicator of sharpness of the resonance. Quality factor has no unit.

17. To improve quality factor, ohmic resistance should be made as small as possible.

18. Refer to text on pages 299 and 300.

19. (i) Curve A shows power consumption over a full cycle.

(ii) Device X is a capacitor. As in a perfect capacitor, the current (curve C) leads the voltage (curve B) by a phase angle of $\frac{\pi}{2}$.

20. Refer to text on pages 295 and 296.

21. (i) As, $P_{av} = V_{rms} I_{rms} \cos\phi$

In ideal inductor, current I_{rms} lags behind applied voltage V_{rms} by $\pi/2$.

$\therefore \phi = \pi/2$

Thus, $P_{av} = V_{rms} I_{rms} \cos \pi/2$
$= V_{rms} I_{rms} \times 0$
$= 0$

(ii) Brightness of the lamp decreases. It is because when iron rod is inserted inside the inductor, its inductance L increases, thereby increasing its inductive reactance X_L and hence impedance Z of the circuit. As $I_{rms} = \frac{V_{rms}}{Z}$, so this decreases the current I_{rms} in the circuit and hence the brightness of lamp.

22. (i) As the dielectric slab is introduced between the plates of the capacitor, its capacitance will increase. Hence, the potential drop across the capacitor will decrease, i.e. $V = \frac{Q}{C}$.

As a result, the potential drop across the bulb will increase as they are connected in series. Thus, its brightness will increase.

(ii) As the resistance R is increased, the potential drop across the resistor will increase. As a result, the potential drop across the bulb will decrease as they are connected in series. Thus, its brightness will decrease.

23. (i) From graph (a), it is clear that resistance (opposition to current) is not changing with frequency, i.e. resistance does not depend on frequency of applied voltage, so the circuit element here is pure resistive (R). From graph (b), it is clear that resistance increases linearly with frequency, so the circuit element here is inductive in nature.

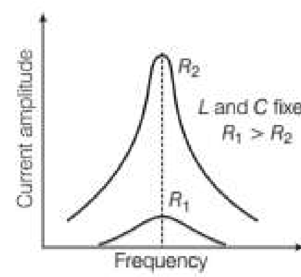
Inductive resistance, $X_L = 2\pi\nu L \Rightarrow X_L \propto \nu$

(ii) Impedance offered by the series combination of resistance (R) and inductor (L).

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi\nu L)^2}$$

In L-R circuit, the applied voltage leads the current by phase ϕ, where $\tan\phi = \frac{X_L}{R}$

24. (i) Graph showing the variation of amplitude of circuit current with changing frequency is given below.



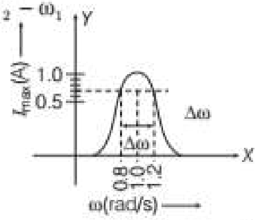
(ii) **Sharpness of resonance** Refer to text on pages 299 and 300.

Circuit become more selective if the resonance is more sharp, maximum current is more, the circuit is close to resonance for smaller range of ($2\Delta\omega$) of frequencies. Thus, the tuning of the circuit will be good.

25. Refer to text on page 301.

26. Refer to text on page 302.

27. Consider the diagram,
Bandwidth $= \omega_2 - \omega_1$



where, ω_1 and ω_2 correspond to frequencies at which magnitude of current is $\frac{1}{\sqrt{2}}$ times of maximum value.

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.7 \text{ A}$$

Clearly, from the diagram, the corresponding frequencies are 0.8 rad/s and 1.2 rad/s.

$$\Delta\omega = \text{Bandwidth} = 1.2 - 0.8 = 0.4 \text{ rad/s}$$

28. (i) We know that if the number of turns in the inductor decreases, then inductance L decreases. So, the net resistance of the circuit decreases. Hence, the current through the circuit increases, increasing the brightness of the bulb.

(ii) As the current increases and brightness of bulb increases, because L increases.

(iii) If the capacitor of reactance $X_C = X_L$ is connected in series with the circuit, then

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\Rightarrow \quad Z = R \qquad [\because X_L = X_C]$$

This is a case of resonance. In this case, the maximum current will flow through the circuit. Hence, the brightness of the bulb will increase.

29. (i) Refer to text on pages 296 and 297.

(ii) When DC source is connected, the condenser is charged but no current flows in the circuit. Therefore, the lamp does not glow. No change occurs even when capacitance of capacitor is reduced.

When AC source is connected, the capacitor offers capacitive reactance $X_C = \frac{1}{\omega C}$. The current flows in the circuit and the lamp glows. On reducing C, X_C increases. Therefore, glow of the bulb reduces.

30. Refer to the text on pages 300 and 301.

31. In an inductor, the current lags the voltage by 90°. If the source voltage is sinusoidal, then the current is also sinusoidal, but shifted in phase. The instantaneous power defined as the product of the instantaneous voltage and current can also be seen to be sinusoidal in time. However, in contrast to the resistive load, the instantaneous power in the inductor goes negative for part of the cycle of the source driving it.

As, $V(t) = V_m \sin \omega t$

$\therefore \quad I(t) = -I_m \cos \omega t$

Instantaneous power, $P_i = V(t) \cdot I(t)$

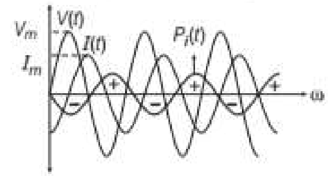
$$= V_m \sin \omega t \times (-I_m \cos \omega t)$$

$$= -\frac{V_m I_m}{2} \times 2 \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} [\sin 2\omega t + \sin 0]$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

The variation of P_i with ωt is as given in the figure.



The instantaneous power alternates positive and negative at twice the frequency of source supplying it.

32. Let $(I_{rms})_a$ = rms current in circuit (a)

$(I_{rms})_b$ = rms current in circuit (b)

$$(I_{rms})_a = \frac{V_{rms}}{R} = \frac{V}{R}$$

$$(I_{rms})_b = \frac{V_{rms}}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

(i) When $(I_{rms})_a = (I_{rms})_b$

$$R = \sqrt{R^2 + (X_L - X_C)^2}$$

$\Rightarrow \quad X_L = X_C$ in resonance condition

(ii) As, $Z \geq R$

$$\Rightarrow \quad \frac{(I_{rms})_a}{(I_{rms})_b} = \frac{\sqrt{R^2 + (X_L - X_C)^2}}{R} = \frac{Z}{R} \geq 1$$

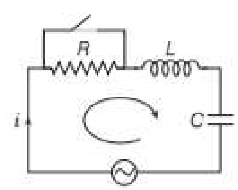
$\Rightarrow \quad (I_{rms})_a \geq (I_{rms})_b$

No, the rms current in circuit (b) cannot be larger than that in (a).

33. (i) Consider the R-L-C circuit as shown in the figure.

Given, $\quad V = V_m \sin \omega t$

Let current at any instant be i.



Note We have to apply KVL, write the equations in the form of current and charge, double differentiate the equation with respect to time and find the required relations.

Applying KVL in the given circuit,

$$iR + L\frac{di}{dt} + \frac{q}{C} - V_m \sin \omega t = 0 \qquad \text{...(i)}$$

Now, we can write, $\quad i = \frac{dq}{dt} \Rightarrow \frac{di}{dt} = \frac{d^2q}{dt^2}$

From Eq. (i), we get

$$\frac{dq}{dt}R + L\frac{d^2q}{dt^2} + \frac{q}{C} = V_m \sin \omega t$$

$$\Rightarrow \quad L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_m \sin \omega t$$

This is the required equation of variation motion of charge.

(ii) Let $q = q_m \sin(\omega t + \phi) = -q_m \cos(\omega t + \phi)$

$$i = i_m \sin(\omega t + \phi) = q_m \omega \sin(\omega t + \phi)$$

$$i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

and $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

When R is short circuited at $t = t_0$, energy is stored in L and C.

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}L\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \sin^2(\omega t + \phi)$$

and $U_C = \frac{1}{2} \times \frac{q^2}{C} = \frac{1}{2C} \times \left(\frac{i_m}{\omega}\right)^2 \cos^2(\omega t_0 + \phi)$

$$= \frac{i_m^2}{2C\omega^2} \cos^2(\omega t_0 + \phi) \qquad [\because i_m = q_m \omega]$$

$$= \frac{1}{2C}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \frac{\cos^2(\omega t_0 + \phi)}{\omega^2}$$

$$= \frac{1}{2C\omega^2}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \cos^2(\omega t_0 + \phi)$$

(iii) When R is short circuited, it becomes an L-C oscillator. The capacitor will go on discharging and all energy will go to L.

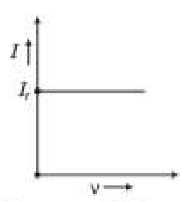
Hence, there is an oscillation of energy from electrostatic to magnetic and again to electrostatic.

34. Refer to text on page 298.

35. (i) Refer to text on page 299.

(ii) Let initially, I_r be current flowing in all the three circuits. If frequency of applied AC source is increased, then the change in current will occur in following manner.

(a) **AC circuit containing resistance only** where, ν_i = initial frequency of AC source.



Frequency of AC source

There is no effect on current with the increase in frequency.

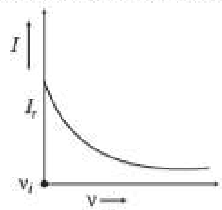
(b) **AC circuit containing inductance only** with the increase of frequency of AC source, inductive reactance increases as,

$$I = \frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi\nu L}$$

$$\Rightarrow \quad X_L = 2\pi\nu L$$

For given circuit, $I \propto \frac{1}{\nu}$

Current decreases with the increase in frequency.



Frequency of AC source

(c) **AC circuit containing capacitor only**

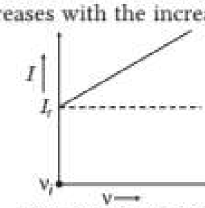
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$\text{Current, } I = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{\left(\frac{1}{2\pi\nu C}\right)}$$

$$I = 2\pi\nu C V_{rms}$$

For given circuit, $I \propto \nu$

Current increases with the increase in frequency.



Frequency of AC source

36. Refer to text on page 298.

For graph showing variation of current with frequency Refer to text on page 299.

The receiving antenna picks up the frequencies transmitted by different stations and a number of voltage appears in L-C-R circuit corresponding to different frequencies. But maximum current flows in circuit for that AC voltage which have got the frequency is equal to resonant frequency of circuit

i.e. $$\nu = \frac{1}{2\pi\sqrt{LC}}$$

For higher quality factor resonance, the signal received from other stations becomes weak due to sharpness of resonance. Thus, signal of desired frequency or program is tunned in.

37. (i) Refer to text on pages 297, 298 and 299.

(ii) Acceptor circuit is series L-C-R circuit.

38. **For graph** refer to text on page 299 and for conditions and Q-factor refer to text on pages 299 and 300.

39. (i) Device X is a capacitor.

As, the current is leading voltage by $\frac{\pi}{2}$ radians.

(ii) As, $E(t) = E_0 \sin \omega t$

Current, $I(t) = I_0 \cos \omega t$

As, in the case of capacitor,

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{[current is leading voltage]}$$

Average power, $P = E(t) I(t) = E_0 I_0 \cos \phi / 2$

where, ϕ = phase difference

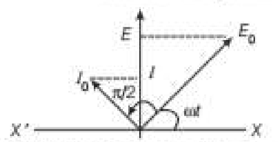
Hence, curve A represents power, curve B represents voltage and curve C represents current.

(iii) As, X_C = capacitive reactance $= \frac{1}{C\omega}$

where, ω is angular frequency.

So, reactance or impedance decreases with increase in frequency. Graph of X_C *versus* ω is shown below,

Phasor diagram



Current leads emf by $\pi/2$ radians

(iv) Refer to text on page 296.

40. (i) Refer to text on page 300.

(ii) Average power delivered by an AC circuit is

$P_{av} = V_{rms} I_{rms} \cos \phi$

where, $\cos \phi$ is known as the power factor for the circuit.

If $\cos \phi$ is minimum, the power delivered is minimum and hence, power dissipated will be maximum for the circuit.

(iii) Refer to text on page 301.

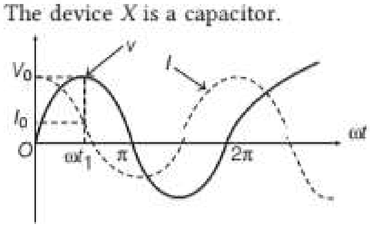
41. (a) Given, $V = V_0 \sin \omega t$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

As it is clear that, the current leads the voltage by a phase angle $\frac{\pi}{2}$.

$\therefore$ The device X is a capacitor.

(b)



(c) The reactance of the capacitance is given as

$$X_C = \frac{1}{\omega C}$$

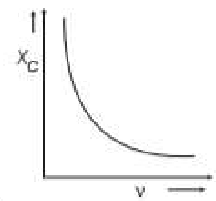
where, ω = angular frequency

and C = capacitance of capacitor.

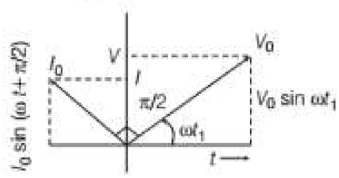
$$\therefore \quad X_C = \frac{1}{2\pi \nu C}$$

where, ν = frequency of AC or $X_C \propto \frac{1}{\nu}$

$\therefore$ The graphical representation between reactance of capacitance and frequency is given as



(d) **Phasor diagram**



42. (i) As given, $E = 140 \sin 314 t$

On comparing with $E = E_0 \sin \omega t$, we have

$\omega = 314$, $E_0 = 140$ V

$\because \quad \omega = 2\pi\nu$

$\Rightarrow \quad \nu = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14} = 50$ Hz

(ii) $E_0 = 140$ V

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{140}{\sqrt{2}} = 99.29 \text{ V}$$

$$\therefore \; I_{rms} = \frac{E_{rms}}{R} = \frac{99.29}{50} = 1.98 \text{ A}$$

43. Given, $L = 0.5$ H, $R = 100\ \Omega$,

$\nu = 50$ Hz, $V_{rms} = 240$ V

(i) $$I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}} = \frac{\sqrt{2} \times 240}{\sqrt{(100)^2 + (100 \times \pi \times 0.5)^2}}$$

$[\because \omega = 2\pi\nu = 100\,\pi]$

$= 1.82$ A

(ii) 3.19×10^{-3} s; refer to Example 5 of on pages 298 and 299.

44. (i) Impedance, $Z = \sqrt{R^2 + X_C^2}$

$$= \sqrt{R^2 + \left(\frac{1}{2\pi\nu C}\right)^2}$$

$$= \sqrt{(40)^2 + \left(\frac{1}{2 \times 3.14 \times 60 \times 100 \times 10^{-6}}\right)^2} = 48\,\Omega$$

As, $\quad I_V = \frac{E_V}{Z} \Rightarrow I_V = \frac{110 \text{ V}}{48\,\Omega}$

and $\quad I_0 = \sqrt{2}\, I_V = 1.414 \times \frac{110}{48} = 3.24$ A

(ii) As, $\tan\phi = \frac{X_C}{R} = \frac{1}{2\pi\nu CR}$

$$= \frac{1}{2\times 3.14\times 60\times 10^{-4}\times 40} = 0.6628$$

$$\Rightarrow \quad \phi = \tan^{-1}(0.6628) = 33.5° = \frac{33.5\pi}{180}\text{ rad}$$

$\therefore$ Time lag, $t = \frac{\phi}{\omega} = \frac{33.5\pi}{180}\times\frac{1}{2\pi\times 60}$

$$= 1.55\times 10^{-3}\text{s}$$

45. Given, applied voltage, $V = 140\sin 100\pi t$ V

$$C = \frac{50}{\pi}\mu\text{F} = \frac{50}{\pi}\times 10^{-6}\text{ F},$$

$$L = \frac{5}{\pi}\text{ H}, R = 400\ \Omega$$

Comparing with $V = V_0\sin\omega t$, we get

$$V_0 = 140\text{ V and }\omega = 100\pi$$

Inductive reactance, $X_L = \omega L = 100\pi\times\frac{5}{\pi} = 500\ \Omega$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{100\pi\times\frac{50}{\pi}\times 10^{-6}}$

$$= 200\ \Omega$$

Impedance of the circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{(400)^2 + (500-200)^2}$$

$$= \sqrt{1600+900} = 500\ \Omega$$

Maximum current in the circuit,

$$I_0 = \frac{V_0}{Z} = \frac{140}{500}$$

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{140}{500\times\sqrt{2}} = 0.2\text{ A}$$

V_{rms} across resistor, $V_R = I_{\text{rms}}R$

$$= 0.2\times 400 = 80\text{ V}$$

V_{rms} across inductor, $V_L = I_{\text{rms}}X_L$

$$= 0.2\times 500 = 100\text{ V}$$

V_{rms} across capacitor, $V_C = I_{\text{rms}}X_C$

$$= 0.2\times 200 = 40\text{ V}$$

Now, $\quad V \neq V_R + V_L + V_C$

Because V_C, V_L and V_R are not in same phase, instead

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{80^2 + (100-40)^2} = 100\text{ V}$$

which is same as that of applied rms voltage.

46. Refer to Example 7 on page 301.

$$\phi = 135°$$

Since, $\quad \omega L < \frac{1}{\omega C}$ or $X_L < X_C$

Therefore, current is leading in phase by a phase angle 135°.

(ii) For unit power factor, $\cos\phi = 1$

$$\Rightarrow \quad \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C'}\right)^2}} = 1$$

where, C' is the total capacitance.

$$\Rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C'}\right)^2 = R^2$$

$$\Rightarrow \quad \omega L = \frac{1}{\omega C'}$$

$$\Rightarrow \quad \omega L = 100 = \frac{1}{\omega C} = \frac{1}{1000}C'$$

$$\Rightarrow \quad C' = \frac{1}{10^5} = 10^{-5}\text{ F} = 10\ \mu\text{F}$$

Additional capacitance C' required in parallel

$$= C' - C = 10\ \mu\text{F} - 2\ \mu\text{F} = 8\ \mu\text{F}$$

47. Refer to Example 7 on page 301.

Phase difference, $\quad \phi = \frac{-\pi}{4}$

48. Here, $E_V = 10$ V, $\nu = 650$ Hz, $R = 100\ \Omega$,

$C = 10\ \mu\text{F} = 10\times 10^{-6}$ F, $L = 0.15$ H,

$\Delta\theta = 10°$C, $ms = 2$ J/°C

As, $\quad X_L = 2\pi\nu L = 2\times\frac{22}{7}\times 650\times 0.15 = 612.86\ \Omega$

and $X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\times\frac{22}{7}\times 650\times 10\times 10^{-6}} = 24.48\ \Omega$

$$\Rightarrow \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(100)^2 + (612.86 - 24.48)^2} = 596.82\ \Omega$$

$$\Rightarrow \quad I_V = \frac{E_V}{Z} = \frac{10}{596.82} = 0.0168$$

As, $\quad I_V^2 Rt = (ms)\,\Delta\theta$

$$\therefore \quad t = \frac{(ms)\,\Delta\theta}{I_V^2 R} = \frac{2\times 10}{(0.0168)^2\times 100} = 708.6\text{ s}$$

49. Given, $L = 2.0$ H, $C = 2\ \mu\text{F} = 2\times 10^{-6}$ F,

$R = 10\ \Omega$

Now, Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{2\times 10^{-6}}} = \frac{1}{10\times 10^{-3}}$

$$= \frac{1}{10^{-2}} = 100$$

50. As, resonance frequency, $\nu = \frac{1}{2\pi\sqrt{LC}}$

i.e. $\quad \nu \propto \frac{1}{\sqrt{C}}$

$$\therefore \quad \nu' \propto \frac{1}{\sqrt{C'}} = \frac{1}{\sqrt{4C}} = \frac{1}{2\sqrt{C}} = \frac{1}{2}\nu$$

51. (i) Refer to Example 6 on page 317.

$\nu = 39.80$ Hz

(ii) $\therefore I_0 = \frac{E_0}{R} = \frac{200}{100} = 2\text{A}$

52. Given, $L = 10$ H, $C = 40\,\mu$F, $R = 60\,\Omega$, $V_{rms} = 240$ V

(i) Refer to the Q. 54 on page 320.

$\omega_r = 50$ rad/s

(ii) Current at resonating frequency,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} \qquad [\because \text{ at resonance, } Z = R]$$

$$= \frac{240}{60} = 4\text{ A}$$

(iii) Inductive reactance, $X_L = \omega L$

At resonance, $X_L = \omega_r L = 50 \times 10 = 500\,\Omega$

Potential drop across inductor,

$$V_{rms} = I_{rms} \times X_L = 4 \times 500 = 2000\text{ V}$$

53. Given, $L = 2.0$ H, $C = 32 \times 10^{-6}$ F and $R = 10\,\Omega$

$$\therefore \quad \omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.0 \times 32 \times 10^{-6}}}$$

$$= \frac{10^3}{8} = 125\text{ rad/s}$$

and $$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}}$$

$$= \frac{1}{10 \times 4 \times 10^{-3}} = 25$$

54. Refer to Q. 51 and Q. 52 on page 306.

[**Ans.** 0.112×10^3 rad/s, 5A; 2.23]

55. 111.1 rad/s and 45.04; Refer to Q. 52 on page 306.

Now, to reduce the full width of half maximum by a factor of 2 without changing ω_r, we have to take

$$R' = \frac{R}{2} = \frac{7.4}{2} = 3.7\,\Omega$$

56. Here, $R = 100\,\Omega$, $E_V = 220$ V, $\nu = 50$ Hz

(i) $I_V = \frac{E_V}{R} = \frac{220}{100} = 2.2\text{ A}$

(ii) Net power consumed,

$$P_{av} = E_V I_V = 220 \times 2.2 = 484\text{ W}$$

57. Given, inductance, $L = 44$ mH $= 44 \times 10^{-3}$H, $V_{rms} = 220$ V

Frequency of inductor, $\nu = 50$ Hz

Inductive reactance, $X_L = 2\pi\nu L$

$= 2 \times 3.14 \times 50 \times 44 \times 10^{-3} = 13.82\,\Omega$

The rms value of current in the circuit,

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220}{13.82} = 15.9\text{ A}$$

Power absorbed, $P = V_{rms}\, I_{rms} \cos\phi$

For pure inductive circuit, $\phi = 90°$

$\therefore \qquad P = 0$

Thus, power spent in one half cycle is retrieved in the other half cycle.

58. Given, $C = 60\,\mu\text{F} = 60 \times 10^{-6}$ F, $V_{rms} = 110$ V

and $\nu = 60$ Hz

$$\therefore \quad I_{rms} = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{\frac{1}{2\pi\nu C}}$$

$$\Rightarrow \quad I_{rms} = V_{rms}\, 2\pi\nu C$$

$$= 110 \times 2 \times 3.14 \times 60 \times 60 \times 10^{-6} = 2.5\text{ A}$$

Power absorbed, $P = V_{rms} I_{rms} \cos\phi$

For pure capacitive circuit, $\phi = 90°$

$\therefore \qquad P = 0$

Thus, power spent in one half cycle is retrieved in the other half cycle.

59. Given, $L = 5$ H, $C = 80\,\mu\text{F} = 80 \times 10^{-6}$F,

$R = 40\,\Omega$, $V_{rms} = 230$ V

(i) Resonance angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50\text{ rad/s}$$

(ii) Impedance, $$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

At resonance, $\omega L = \frac{1}{\omega C}$

$\therefore \qquad Z_r = R = 40\,\Omega$

Amplitude of current at resonance frequency,

$$I_0 = \frac{V_0}{Z_r} = \frac{\sqrt{2} \times 230}{40} = 8.13\text{ A}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{8.13}{\sqrt{2}} = 5.75\text{ A}$$

(iii) Potential difference across L,

$$V_L = I_{rms} \times (\omega_r \times L)$$

$$= 5.75 \times 50 \times 5 = 1437.5\text{ V}$$

Potential difference across C,

$$V_C = I_{rms} \times \frac{1}{\omega_r C} = \frac{5.75}{50 \times 80 \times 10^{-6}} = 1437.5\text{ V}$$

$\therefore$ Potential difference across L and C combination,

$$V_{LC} = I_{rms}\left(\omega_r L - \frac{1}{\omega_r C}\right) = 0$$

$\therefore$ Potential difference across R,

$$V_R = I_{rms} R = 5.75 \times 40 = 230\text{ V}$$

60. Given, $L = 80\text{ mH} = 80\times10^{-3}\text{H}$, $R = 0$, $\nu = 50\text{ Hz}$

$$C = 60\mu\text{F} = 60\times10^{-6}\text{ F},$$
$$\omega = 2\pi\nu = 100\pi\text{ rad/s}$$
$$V_{rms} = 230\text{ V},$$
and $$V_0 = \sqrt{2}V_{rms} = \sqrt{2}\times230\text{ V}$$

(i) $I_0 = ?$ and $I_{rms} = ?$

$$\Rightarrow I_0 = \frac{V_0}{Z} = \frac{V_0}{\left(\omega L - \frac{1}{\omega C}\right)}$$
$$= \frac{230\sqrt{2}}{\left(100\pi\times80\times10^{-3} - \frac{1}{100\pi\times60\times10^{-6}}\right)}$$
$$= \frac{230\sqrt{2}}{\left(8\pi - \frac{1000}{6\pi}\right)} = -11.63\text{ A}$$
$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{-11.63}{\sqrt{2}} = -8.23\text{ A}$$

Hence, negative sign indicates that emf lags behind the current by 90°.

(ii) For L, $V_L = I_{rms}\omega L$
$$= 8.23\times100\pi\times80\times10^{-3}$$
$$= 206.84\text{ V}$$

For C, $V_C = I_{rms}\frac{1}{\omega C} = 8.23\times\frac{1}{100\pi\times60\times10^{-6}}$
$$= 436.84\text{ V}$$

Since, voltage across L and C are 180° out of phase, therefore they are subtracted.

Thus, applied rms voltage $= 436.84 - 206.84$
$$= 230.0\text{ V}$$

(iii) Average power transferred per cycle by source to inductor is always zero because of phase difference of $\pi/2$ between voltage and current through L.

(iv) Average power transferred per cycle by the source to capacitor is always zero because of phase difference of $\pi/2$ between voltage and current through C.

(v) $\therefore$ Total average power absorbed by the circuit is also zero.

61. Given, $L = 0.12\text{ H}$, $C = 480\text{ nF} = 480\times10^{-9}\text{ F}$

$$R = 23\ \Omega,\ V_{rms} = 230\text{ V},\quad V_0 = 230\sqrt{2}\text{ V}$$

(i) $$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

I_0 would be maximum, if

$$\omega_r = \omega = \frac{1}{\sqrt{LC}}$$
$$= \frac{1}{\sqrt{0.12\times480\times10^{-9}}}$$
$$= 4166.7\text{ rad/s}$$

$\therefore$ Source frequency, $\nu_r = \frac{\omega_r}{2\pi}$
$$= \frac{4166.7}{2\pi} = 663.14\text{ Hz}$$
$$\Rightarrow \quad I_0 = \frac{V_0}{R} = \frac{\sqrt{2}\times230}{23}$$
$$= 14.14\text{ A}$$

(ii) Average power absorbed by the circuit is maximum, if
$$I = I_0 \text{ at } \omega = \omega_r$$
$$P_{av} = \frac{1}{2}I_0^2R = \frac{1}{2}(14.14)^2\times23$$
$$= 2299.3\text{ W} = 2300\text{ W}$$

(iii) Power transferred to circuit is half the power of resonant frequency, when
$$\Delta\omega = \frac{R}{2L} = \frac{23}{2\times0.12} = 95.83\text{ rad/s}$$
$$\Delta\nu = \frac{\Delta\omega}{2\pi} = \frac{95.83}{2\pi} = 15.2\text{ Hz}$$

$\therefore$ Frequency when power transferred is half
$$= \nu_r \pm \Delta\nu = 663.14 \pm 15.2$$
$$= 678.34 \text{ and } 647.94\text{ Hz}$$

$\therefore$ Current amplitude at these frequencies
$$= \frac{I_0}{\sqrt{2}} = \frac{14.14}{1.414} = 10\text{ A}$$

(iv) $$Q = \frac{\omega_r L}{R} = \frac{4166.7\times0.12}{23} = 21.74$$

|TOPIC 3|
AC Devices

CHOKE COIL

Choke coil is an electrical device used for controlling current in an AC circuit without wasting electrical energy in the form of heat.

(i) To reduce low frequency alternating currents, choke coils with laminated soft iron cores are used. These are called ***af* choke coils**.

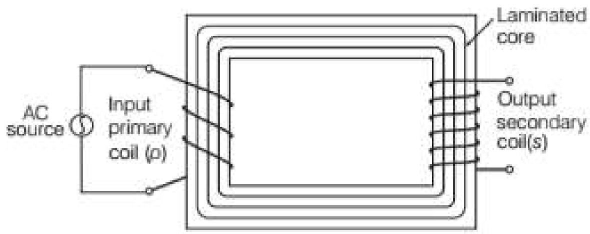
(ii) To reduce high frequency alternating currents, choke coils with air cores are used. These are called ***rf* choke coils**.

TRANSFORMER

It is a device, which is used to increase or decrease the alternating voltage.

The transformers are of the following types

1. Step-up transformer
2. Step-down transformer



Step-down transformer

Principle

Transformer is based upon the principle of mutual induction.

Construction

It consists of two coils, primary coil (p) and secondary coil (s), insulated from each other wounded on soft iron core. Often the primary coil is the input coil and secondary coil is the output coil. These soft iron cores are laminated to minimise eddy current loss.

Working and Theory

The value of the emf induced in secondary coil due to alternating voltage applied to primary coil depends on the number of turns in the secondary coil. We consider an **ideal transformer** in which the primary coil has **negligible resistance** and all the flux in the core links both primary and secondary windings. Let ϕ be the flux in each turn in the core at time t due to current in the primary when a voltage V_p is applied to it.

Then, the induced emf or voltage E_s, in the secondary with N_s turns is

$$E_s = -N_s \frac{d\phi}{dt} \quad \ldots(\text{i})$$

The alternating flux ϕ also induces an emf, called back emf in the primary. This is

$$E_p = -N_p \frac{d\phi}{dt} \quad \ldots(\text{ii})$$

But $E_p = V_p$. If this was not, so the primary current would be infinite, since the primary has zero resistance (as considered). If the secondary is an open circuit or the current taken from it is small, then to a good approximation.

$$E_s = V_s$$

where, V_s is the voltage across the secondary.

Therefore, Eqs. (i) and (ii) can be written as

$$V_s = -N_s \frac{d\phi}{dt} \quad \ldots(\text{iii})$$

and

$$V_p = -N_p \frac{d\phi}{dt} \quad \ldots(\text{iv})$$

From Eqs. (iii) and (iv), we have

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \ldots(\text{v})$$

The above relation has been obtained using three assumptions.

(i) The primary resistance and current are small.

(ii) The same flux links both the primary and the secondary as very little flux escapes from the core.

(iii) The secondary current is small.

If the transformer is assumed to be 100% efficient (no energy losses), the power input is equal to the power output. Since $P = IV$, we get

$$I_p V_p = I_s V_s \quad \ldots(\text{vi})$$

Although, some energy is always lost, still this is a good approximation, since a well designed transformer may have an efficiency of more than 95%.

Combining Eqs. (v) and (vi), we have

$$\boxed{\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p}} \quad \dots\text{(vii)}$$

Since, I and V both oscillate with the same frequency as the AC source, Eq. (vii) also gives the ratio of the amplitudes or rms values of corresponding quantities.

Now, we can observe how a transformer affects the voltage and current, we have

$$V_s = \left(\frac{N_s}{N_p}\right) V_p \text{ and } I_s = \left(\frac{N_p}{N_s}\right) I_p \quad \dots\text{(viii)}$$

That is, if the secondary coil has a greater number of turns than the primary (i.e. $N_s > N_p$), the voltage is stepped up ($V_s > V_p$). This type of arrangement is called a **step-up transformer**. However, in this arrangement, there is less current in the secondary than in the primary (i.e. $N_p / N_s < 1$ and $I_s < I_p$).

If the secondary coil has less number of turns than the primary (i.e. $N_s < N_p$), we have a **step-down transformer**.

In this case, $V_s < V_p$ and $I_s > I_p$. That is, the voltage is stepped-down, (or reduced) and the current is increased. The equations obtained above apply to ideal transformers (without any energy losses).

Energy Loss in Transformers

In actual transformers, small energy losses do occur due to the following reasons.

(i) **Flux leakage** There is always some leakage of flux that is not all of the flux due to primary passes through the secondary. This is due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

(ii) **Resistance of the windings** The wire used for the windings has some resistance and so, energy is also lost due to heat produced in the wire (I^2R). In high current, low voltage windings, energy losses are minimised by using thick wire.

(iii) **Eddy currents** The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by having a laminated core.

(iv) **Hysteresis** The magnetisation of the core is repeatedly reversed by an alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.

Uses of Transformers

Transformers are used in almost all AC operations. Some of the following are given below.

(i) In the induction furnaces.

(ii) In voltage regulators for TV, computer, refrigerator, etc.

(iii) A step-down transformer is used for the purpose of weldings.

EXAMPLE |1| How much current is drawn by the primary coil of a transformer which steps down 220 V to 22 V to operate device with an impedance of 220 Ω?

Sol. Given, $E_p = 220\text{ V}$, $E_s = 22\text{ V}$ and $R_s = 220\ \Omega$

Since, $I_s = \frac{E_s}{R_s} = \frac{22}{220} = 0.1\text{A}$

In an ideal transformer, $\frac{I_p}{I_s} = \frac{E_s}{E_p}$

$\therefore$ $I_p = \frac{E_s}{E_p} \times I_s$

$= \frac{22 \times 0.1}{220} = 10^{-2}\text{ A}$

EXAMPLE |2| A step-down transformer converts a voltage of 2200 V into 220 V in the transmission line. Number of turns in primary coil is 5000. Efficiency of transformer is 90% and its output power is 8 kW. Determine

(i) number of turns in the secondary coil.

(ii) input power.

Sol. Given, $E_p = 2200\text{ V}$, $E_s = 220\text{ V}$, $N_p = 5000$

Efficiency, $\eta = 90\%$

Output power, $P_o = 8\text{ kW}$

Since, efficiency,

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{P_o}{P_i}$$

$\Rightarrow$ $P_i = \frac{P_o}{\eta} = \frac{8}{90/100} = 8.9\text{ kW}$

Also, $\frac{N_s}{N_p} = \frac{E_s}{E_p} \Rightarrow N_s = 500$

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?
(a) 600 (b) 550 (c) 400 (d) 375

2. The output of a step-down transformer is measured to be 24 V when connected to a 12 W light bulb. The value of the peak current is **NCERT Exemplar**
(a) $1/\sqrt{2}$ A (b) $\sqrt{2}$ A
(c) 2 A (d) $2\sqrt{2}$ A

3. What is not possible in a transformer?
(a) Eddy current (b) Direct current
(c) Alternating current (d) Induced current

4. The large scale transmission and distribution of electrical energy over long distances is done with the use of
(a) dynamo (b) transformers
(c) generator (d) capacitor

5. A 60 W load is connected to the secondary of a transformer whose primary draws line voltage of 220 V. If a current of 0.54 A flows in the load, then what is the current in the primary coil?
(a) 2.7 A (b) 0.27 A
(c) 1.65 A (d) 2.85 A

VERY SHORT ANSWER Type Questions

6. Can we control direct current without much loss of energy? Can a choke coil do so?

7. Write the name of quantities which do not change during transformer operation.

8. Mention the two characteristic properties of the material suitable for making core of a transformer. **All India 2012**

9. A transformer is used to step-down AC voltage. What device do you use to step-down DC voltage?

10. A transformer has 150 turns in its primary and 1000 in secondary. If the primary is connected to 440 V DC supply, what will be the induced voltage in the secondary side?

11. What would happen if the primary winding of a transformer is connected to a battery?

SHORT ANSWER Type Questions

12. A 100% efficient transformer has n_1 turns in its primary and n_2 turns in its secondary. If the power input to the transformer is W (watt), what is the power output?

13. Answer the following questions.
(i) A choke coil in series with a lamp is connected to a DC line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations, if the connection is to an AC line.
(ii) Why is choke coil needed in the use of fluorescent tubes with AC mains? Why we cannot use an ordinary resistor instead of the choke coil? **NCERT**

14. When a DC voltage is applied to a transformer, the primary coil sometimes will overheat and eventually burn. Explain, why?

LONG ANSWER Type I Questions

15. Write the function of a transformer. State its principle of working with the help of a diagram. Mention various energy losses in this device. **Delhi 2016**

16. Transformer A has a primary voltage E_p and a secondary voltage E_s. Transformer B has twice the number of turns on both its primary and secondary coils compared with transformer A. If the primary voltage on transformer B is $2E_p$, what is its secondary voltage? Explain briefly.

17. At a hydroelectric power plant, the water pressure head is at height of 300 m and the water flow available is 100 m^3/s. If the turbine generator efficiency is 60%, estimate the electric power available from the plant. (Take, $g = 9.8 \text{ m/s}^2$)

18. 1 MW power is to be delivered from a power station to a town 10 km away. One uses a pair of Cu wires of radius 0.5 cm for this purpose. Calculate the fraction of ohmic losses to power transmitted, if

(i) power is transmitted at 220 V. Comment on the feasibility of doing this.

(ii) a step-up transformer is used to boost the voltage at 11000 V, power transmitted, then a step-up transformer is used to bring voltage is 220 V. (Take, $\rho_{Cu} = 1.7 \times 10^{-8}$ SI unit)

NCERT Exemplar

LONG ANSWER Type II Questions

19. (i) Draw a labelled diagram of a step-down transformer. State the principle of its working.

(ii) Express the turn ratio in terms of voltages.

(iii) Find the ratio of primary and secondary currents in terms of turn ratio in an ideal transformer.

(iv) Define choke coil. **All India 2016**

20. Draw a schematic diagram of a step-up transformer. Using its working principle, deduce the expression for the secondary to the primary voltage in terms of number of turns in the two coils? In an ideal transformer, how is this ratio related to the currents in the two coils? How is this transformer used in large scale transmission and distribution of electrical energy over large distances?

NUMERICAL PROBLEMS

21. How much current is drawn by the primary of a transformer connected to 220 V supply when it delivers power to a 110 V-550 W refrigerator? **All India 2016**

22. A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary winding to get the power output at 220 V. **Delhi 2017**

23. 1 kW power is supplied to a 200 turns primary of the transformer at 500 mA. The secondary gives 220 V. Find the number of turns in the primary.

24. The primary coil of an ideal step up transformer has 100 turns and transformation ratio is also 100. The input voltage and power are respectively 220 V and 1100 W. Calculate

(i) the number of turns in secondary.

(ii) the current in primary.

(iii) the voltage across secondary.

(iv) the current in secondary.

(v) the power in secondary. **Delhi 2016**

25. A 60 W load is connected to the secondary of transformer whose primary draws line voltage. If current of 0.54 A flows in the load, what is the current in the primary coil? Comment on the type of transformer being used. **NCERT Exemplar**

26. A step-up transformer is operated on a 2.5 kV line. It supplies a load with 20 A. The ratio of the primary winding to the secondary is 10 : 1. If the transformer is 90% efficient, calculate

(i) the power output (ii) the voltage and

(iii) the current in the secondary.

27. A step-down transformer is used at 220 V to provide a current of 0.5 A to a 15 W bulb. If the secondary has 20 turns, find the number of turns in the primary coil and the current that flows in the primary coil.

28. A step-up transformer operates on a 220 V line and supplies a load of 2 A. The ratio of the primary to the secondary windings is 1:5. Determine the secondary voltage, primary current and power output. Assume efficiency to be 100%.

29. A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wires line carrying power is 0.5 Ω/km. The town gets power from the line through a 4000-220 V step-down transformer at a sub-station in the town.

(i) Estimate the line power of loss in the form of heat.

(ii) How much power of the plant supply, assuming there is negligible power loss due to leakage?

(iii) Characterise the step-up transformer at the plant. **NCERT**

30. Do the same question as above with the replacement of the earlier transformer by a 40000-220 V step-down transformer (neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred? **NCERT**

HINTS AND SOLUTIONS

1. (c) Here, $\varepsilon_p = 2300$ V, $N_p = 4000$, $\varepsilon_s = 230$ V

Let N_s be the required number of turns in the secondary

As, $\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$, $N_s = N_p\left(\frac{\varepsilon_s}{\varepsilon_p}\right)$

$$= 4000\left(\frac{230\text{ V}}{2300\text{ V}}\right) = 400$$

2. (a) Secondary voltage, $V_S = 24$V

Power associated with secondary,

$$P_S = 12\text{ W}$$

$$I_S = \frac{P_S}{V_S} = \frac{12}{24}$$

$$= \frac{1}{2}\text{ A} = 0.5\text{ A}$$

Peak value of the current in the secondary,

$$I_0 = I_S\sqrt{2}$$

$$= (0.5)(1.414) = 0.707 = \frac{1}{\sqrt{2}}\text{ A}$$

3. (b) Transformer is used to convert the value of AC voltage. It works on the principle of electromagnetic induction. So, direct current is not possible in it.

4. (b) Large scale distribution and transmission of electrical energy over long distances is done with the help of transformer.

5. (b) $P = 60$ W, $\varepsilon_p = 220$ V, $i_s = 0.54$ A

As, $P = \varepsilon_s i$

$\Rightarrow$ $\varepsilon_s = \frac{60\text{ W}}{0.54\text{ A}} = 110\text{ V}$

As, $\varepsilon_p i_p = \varepsilon_s i_s$

$$i_p = \left(\frac{\varepsilon_s}{\varepsilon_p}\right) i_s = \left(\frac{110\text{ V}}{2200\text{ V}}\right)(0.54\text{ A}) = 0.27\text{ A}$$

6. No, there is no such device that can control DC without any energy loss. Even a choke coil cannot control DC.

7. Power and frequency.

8. (i) Low retentivity or coercivity.

(ii) Low hysteresis loss or high permeability and susceptibility.

9. An ohmic resistance can be used to step-down DC voltage, such as in potential dividing arrangement.

10. Zero, as transformer works only in AC and in case of DC supply, there is no induced emf in secondary because there is no change in flux through the transformer circuit.

11. Transformer works only in AC. When primary is connected to DC, there is no induced emf in secondary coil as there is no change in flux leakage.

12. For 100% efficient transformer, $P_i = P_o$

$\therefore$ The power output is W (watt).

13. (i) A choke has no impedance, if it is connected to DC line. Therefore, lamp shines brightly and has no effect on inserting iron core in the choke.

But choke offers impedance, if it is connected to AC line. So the bulb lights dimly. When an iron core is inserted in the choke, the impedance to AC increases. Hence, the brightness of the bulb decreases.

(ii) We use the choke coil instead of resistance because the power loss across resistor is maximum, while the power loss across choke is zero.

For resistor, $\phi = 0°$,

$$P = I_{\text{rms}}V_{\text{rms}}\cos 0°$$
$$= I_{\text{rms}} \cdot V_{\text{rms}} = \text{maximum}$$

For inductor, (choke coil)

$$\phi = 90°,$$
$$P = I_{\text{rms}}V_{\text{rms}}\cos 90° = 0$$

14. If in a case, the transformer primary winding would be connected to a DC supply, the inductive reactance of the winding would be zero as DC has no frequency. So, the effective impedance of the winding will therefore be very low and equal only to the resistance of the copper used. Thus, winding will draw a very high current from the DC supply causing it to overheat and eventually burn out, because as we know $I = V/R$.

15. Refer to text on pages 315 and 316.

16. Given, $N_{pB} = 2N_{pA}$, $N_{sB} = 2N_{sA}$, $E_{pB} = 2E_{pA}$

As we know, $\frac{N_s}{N_p} = \frac{E_s}{E_p}$

For transformer B, $\frac{N_{sB}}{N_{pB}} = \frac{2N_{sA}}{2N_{pA}} = \frac{E_{sB}}{E_{pB}}$

$$\Rightarrow \frac{E_{sB}}{E_{pB}} = \frac{E_{sB}}{2E_{pA}} = \frac{E_{sA}}{E_{pA}} \Rightarrow E_{sB} = 2E_{sA}$$

$\therefore$ Secondary voltage on transformer B is equal to the twice of secondary voltage on transformer A.

17. Given, height, $h = 300$ m, $V = \frac{\text{volume}}{\text{second}} = 100\text{ m}^3/\text{s}$,

$$\eta = 60\%,\ g = 9.8\text{ m/s}^2$$

Electric power = ?

$$\text{Hydroelectric power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \times \text{Displacement}}{\text{Time}}$$
$$= \text{Force} \times \text{Velocity}$$
$$= \text{Pressure} \times \text{Area} \times \text{Velocity}$$
$$= \text{Pressure} \times \text{Volume} = p \times V$$

$$\therefore \text{Power available} = \frac{60}{100}pV = \frac{3}{5} \times h \times \rho \times g \times V$$

$$= \frac{3}{5} \times 300 \times 10^3 \times 9.8 \times 100$$

$[\because$ density of water $= 10^3 \text{ kg/m}^3]$

$$= 1.764 \times 10^8 = 176.4 \text{ MW}$$

18. (i) The town is 10 km away, length of pair of Cu wires used, $l = 20$ km $= 20000$ m.

Resistance of Cu wires,

$$R = \rho\frac{l}{A} = \rho\frac{l}{\pi(r)^2} = \frac{1.7 \times 10^{-8} \times 20000}{3.14\,(0.5 \times 10^{-2})^2} = 4\ \Omega$$

I at 220 V, $VI = 10^6$ W; $I = \frac{10^6}{220} = 0.45 \times 10^4$ A

$RI^2 =$ power loss $= 4 \times (0.45)^2 \times 10^8 > 10^6$ W

Therefore, this method cannot be used for transmission.

(ii) When power, $P = 10^6$ W is transmitted at 11000 V.

$VI' = 10^6 \text{ W} = 11000\, I'$

Current drawn, $I' = \frac{1}{1.1} \times 10^2$

Power loss $= RI^2 = \frac{1}{121} \times 4 \times 10^6 = 3.3 \times 10^4$ W

$\therefore$ Fraction of power loss $= \frac{3.3 \times 10^4}{10^6} = 3.3\%$

19. Refer to text on pages 315 and 316.

20. Refer to text on pages 315 and 316.

21. (iv) $P_{\text{in}} = P_{\text{out}} = 550 \text{ W} \Rightarrow \varepsilon_p I_p = 550$

$$220 \times I_p = 550 \Rightarrow I_p = \frac{550}{220} = \frac{5}{2} = 2.5 \text{ A}$$

22. Given, input voltage, $V_p = 2200$ V

Number of turns, $n_1 = 3000$

Output voltage, $V_s = 220$ V

As, $\quad \frac{V_s}{V_p} = \frac{n_2}{n_1}$

$\Rightarrow \quad \frac{220}{2200} = \frac{n_2}{3000}$

$\Rightarrow \quad n_2 = \frac{220}{2200} \times 3000$

$\therefore$ Number of turns in the secondary winding,

$$n_2 = 300 \text{ turns.}$$

23. $N_p = 22$; refer to Q. 22 on page 318.

24. Here, $N_p = 100, \frac{N_s}{N_p} = 100$

$$\varepsilon_i = \varepsilon_p = 220 \text{ V},\ P_I = 1100 \text{ W}$$

(i) $N_p = 100$

$\therefore N_s = 10000$

(ii) $I_p = \frac{P_I}{\varepsilon_p} = \frac{1100}{220} = 5$ A

(iii) $\varepsilon_s = \frac{N_s}{N_p} \times \varepsilon_p = 100 \times 220 = 22000$ V

(iv) $I_s = \frac{P_o}{\varepsilon_s} = \frac{1100}{22000} = \frac{1}{20}$ A $\qquad [\because P_O = P_I]$

(v) $P_s = P_o = P_I = 1100$ W

25. Here, power, $P_L = 60$ W

Current, $I_L = 0.54$ A

Voltage, $V_L = \frac{P_L}{I_L} = \frac{60}{0.54}$

$$= 111.11 \text{ V} = 111 \text{ V}$$

On average, the input current is half a load current.

$$I_p = \frac{I_L}{2} = \frac{0.54}{2} = 0.27 \text{ A}$$

The transformer is step-down.

26. Given, input voltage, $V_p = 2.5 \times 10^3$ V

Input current, $I_p = 20$ A

Also, $\quad \frac{N_p}{N_s} = \frac{10}{1} \Rightarrow \frac{N_s}{N_p} = \frac{1}{10} \quad$ ...(i)

Percentage efficiency $= \frac{\text{Output power}}{\text{Input power}} \times 100$

$\Rightarrow \quad \frac{90}{100} = \frac{\text{Output power}}{V_p I_p}$

(i) Output power $= \frac{90}{100} \times (V_p I_p)$

$$= \frac{90}{100} \times (2.5 \times 10^3 \text{ V}) \times (20 \text{ A})$$

$$= 4.5 \times 10^4 \text{ W}$$

(ii) $\because \quad \frac{V_s}{V_p} = \frac{N_s}{N_p}$

$\Rightarrow \quad V_s = \frac{N_s}{N_p} \times V_p$

Voltage, $\quad V_s = \frac{1}{10} \times 2.5 \times 10^3 \text{ V} = 250$ V

(iii) $\quad V_s I_s = 4.5 \times 10^4$ W

Current, $\quad I_s = \frac{4.5 \times 10^4}{V_s} = \frac{4.5 \times 10^4}{250} = 180$ A

27. Approx 147 turns, 0.0682 A; refer to Q. 25 on page 318.

28. 1000 V, 10 A, 2000 W; refer to Q. 26 on page 318.

29. Generating power of electric plant = 800 kW at $V = 220$ V, resistance/length $= 0.5\Omega$/km

Distance = 15 km, generating voltage = 440 V,

Primary voltage, $V_p = 4000$ V

Secondary voltage, $V_s = 220$ V

(i) Power $= I_p \cdot V_p$

$\Rightarrow 800 \times 1000 = I_p \times 4000$

$\Rightarrow \quad I_p = 200$ A

Line power loss in form of heat

$$= (I_p)^2 \times R \text{ (2 lines)}$$

$$= (I_p)^2 \times 0.5 \times 15 \times 2$$

$$= (200)^2 \times 0.5 \times 15 \times 2$$

$$= 60 \times 10^4 \text{ W} = 600 \text{ kW}$$

(ii) If there is no power loss due to leakage, then the power supply by plant $= 800 + 600 = 1400$ kW

(iii) Voltage drop across the line $= I_p \cdot R$ (2 lines)

$$= 200 \times 0.5 \times 15 \times 2 = 3000 \text{ V}$$

Voltage from transmission $= 3000 + 4000 = 7000$ V

As, it is given that the power is generated at 440 V. So, the step-up transformer needed at the plant is 440 V-7000 V.

30. Given, primary voltage, $V_p = 40000$ V

Let the current in primary be I_p.

$\therefore \quad V_p \cdot I_p = P$

$800 \times 1000 = 40000 \times I_p$

$I_p = 20$ A

(i) Line power loss $= I_p^2 \times R$ (2 lines)

$$= (20)^2 \times 2 \times 0.5 \times 15$$

$$= 6000 \text{ W} = 6 \text{ kW}$$

(ii) Power supply by plant $= 800 + 6 = 806$ kW

Voltage drop on line $= I_p \cdot R$ (2 lines)

$$= 20 \times 2 \times 0.5 \times 15$$

$$= 300 \text{ V}$$

Voltage for transmission $= 40000 + 300 = 40300$ V

Step-up transformer needed at the plant

$= 440$ V-40300 V

Power loss at higher voltage

$$= \frac{6}{800} \times 100 = 0.74\%$$

Power loss at lower voltage

$$= \frac{600}{1400} \times 100 = 42.8\%$$

Hence, the power loss is minimum at higher voltage. So, the high voltage transmission is preferred.

SUMMARY

- **Alternating Current** If the direction of current changes alternatively and its magnitude change continuously with respect to time is called Alternating current. It is sinusoidal in nature.
- The instantaneous value of AC is given by $I = I_0 \sin \omega t$ and instantaneous value of alternating emf is given by $E = E_0 \sin \omega t$.
- Mean value of AC is that value which send same change through a cicruit in half cycle which is sent by steady current in same time.

  $\therefore \quad I_m = 0.637 I_0$ and $E_m = 0.637 E_0$

  where, I_0 and E_0 are the peak values of current and voltage, respectively.
- **Root mean square** (RMS) value of AC is that value over a complete cycle that generates same amount of heat in the given resistor that is generated by steady current in the same resistor.

  $\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}}$ and $E_{rms} = \frac{I_0}{\sqrt{2}}$
- A diagram representing alternating current and alternating emf (of same frequency) as rotating vectors (phasors) with the phase angle between them is called **phasor diagram**.
- **AC through Resistor Only** In this case, there is zero phase difference between instantaneous alternating current and instantaneous alternating emf. So, they are in same phase.
- **AC through Capacitor Only** In this case, the current leads the voltage by a phase angle of $\frac{\pi}{2}$ or the voltage lags behind the current by the phase angle of $\frac{\pi}{2}$.

  Capacitive reactance, $X_c = \frac{1}{2\pi fC}$
- **AC through Inductor Only** In this case, the current lags behind the voltage by phase angle of $\frac{\pi}{2}$ or the voltage leads the current by phase angle of $\frac{\pi}{2}$. Inductive reactance, $X_L = 2\pi fL$.
- **AC through *L-C-R* Series Circuit** In this case, Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

and $$\phi = \frac{X_L - X_C}{R}$$

- When $X_L > X_C$, then the AC circuit is inductance dominated circuit .
- When $X_C > X_L$, then the AC circuit is capacitance dominated circuit.
- In series *L-C-R* circuit, if phase (ϕ) between current and voltage is zero, then the circuit is said to be **resonant circuit.** Resonating frequency is given by,

$$\nu_r = \frac{1}{2\pi\sqrt{LC}}$$

- **Quality Factor** (Q-Factor) determines the sharpness of the resonance.

$$Q\text{-fact or } = \frac{1}{R}\sqrt{\frac{L}{C}}$$

- **Average Power Associated in AC Circuit,**

$$P_{av} = I_{rms}\, E_{rms} \cos\theta$$

$$\Rightarrow \quad P_{av} = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos\theta$$

- **Wattless Current** is the current which consumes no power for its maintenance in the circuit.
- **Transformer** is used to increase or decrease the altrenating voltage.

  It is of the two types

  (i) **Step-up Transformer** $N_s > N_p$

  $V_s > V_p$

  $I_s < I_p$

  (ii) **Step-down Transformer** $N_s < N_p$

  $V_s < V_p$

  $I_s > I_p$

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. If an AC main supply is given to be 220 V. What would be the average emf during a positive half-cycle?

(a) 198 V (b) 386 V
(c) 256 V (d) None of these

2. If an alternating voltage is represented as $E = 141 \sin(628t)$, then the rms value of the voltage and the frequency are respectively

(a) 141 V, 628 Hz
(b) 100 V, 50 Hz
(c) 100 V, 100 Hz
(d) 141 V, 100 Hz

3. In a purely inductive AC circuit, $L = 30.0$ mH and the rms voltage is 150 V, frequency $\nu = 50$ Hz. The inductive reactance is

(a) $15.9\,\Omega$ (b) $9.42\,\Omega$ (c) $10\,\Omega$ (d) $8.85\,\Omega$

4. In an AC circuit, the power factor

(a) is zero when the circuit contains an ideal resistance only
(b) is unity when the circuit contains an ideal resistance only
(c) is unity when the circuit contains a capacitance only
(d) is unity when the circuit contains an ideal inductance only

5. If in an alternating circuit, the voltage is V and current is I, then the value of power dissipated in the circuit is

(a) VI
(b) $VI/2$
(c) $VI/\sqrt{2}$
(d) depends upon the angle between V and I

6. In an AC circuit, the instantaneous values of emf and current are $e = 200 \sin(314)\,t$ V and $I = \sin(314t + \pi/3)$ A. The average power consumed is

(a) 200 W (b) 100 W
(c) 50 W (d) 25 W

7. A coil of resistance $50\,\Omega$ and inductance 10 H is connected with a battery of 50 V. The energy stored in the coil is

(a) 125 J (b) 62.5 J (c) 250 J (d) 500 J

8. The value of power factor is maximum in an alternating circuit, when circuit consists

(a) only inductive (b) only capacitive
(c) only L-C (d) only resistive

9. In R-L-C series circuit with $C = 1.00$ nF two values of R are
(i) $R = 100\,\Omega$ and
(ii) $R = 200\,\Omega$. For the source applied with $V_m = 100$ V. Resonant frequency is

(a) 1×10^3 rad/s (b) 1×10^6 rad/s
(c) 1.56×10^6 rad/s (d) 1.75×10^3 rad/s

10. The L-C-R circuit is connected to source of an alternating current. At the resonance, the phase difference between current flowing in the circuit and potential difference will be

(a) zero (b) $\pi/4$ (c) $\pi/2$ (d) π

11. The phenomenon of resonance is common among systems that have a tendency

(a) to oscillate at a particular frequency
(b) to get maximum amplitude
(c) Both (a) and (b)
(d) Neither (a) nor (b)

12. The value of emf in the secondary coil of transformer depends on

(a) the number of turns (b) material used
(c) voltage (d) induced flux

ASSERTION AND REASON

Directions (Q. Nos. 13-21) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

13. **Assertion** Today, most of the electrical devices use/require AC voltage.

Reason Most of the electrical energy sold by power companies is transmitted and distributed as alternating current.

14. **Assertion** Phasors V and I for the case of a resistor are in the same direction.

Reason The phase angle between the voltage and the current is zero.

15. **Assertion** When the capacitor is connected to an AC source, it limits or regulates the current, but does not completely prevent the flow of charge.

Reason The capacitor is alternately charged and discharged as the current reverses each half-cycle.

16. **Assertion** Capacitor serves as a barrier for DC and offers an easy path to AC.

Reason Capacitor reactance is inversely proportional to frequency.

17. **Assertion** If $X_C > X_L$, ϕ is positive and the circuit is predominantly capacitive. The current in the circuit leads the source voltage.

Reason If $X_C < X_L$, ϕ is negative and the circuit is predominantly inductive, the current in the circuit lags the source voltage.

18. **Assertion** In a series R-L-C circuit, the voltages across resistor, inductor and capacitor are 8V, 16V and 10V, respectively. The resultant emf in the circuit is 10 V.

Reason Resultant emf of the circuit is given by the relation.

$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

19. **Assertion** Resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit.

Reason Voltage across L and C cancel each other and the current amplitude is V_m/R, the total source voltage appearing across R causes resonance.

20. **Assertion** In series L-C-R circuit resonance can take place.

Reason Resonance takes place if inductance and capacitive reactances are equal and opposite.

21. **Assertion** The wire used for the windings of transformer has some resistance.

Reason Energy is lost due to heat produced in the wire (I^2R).

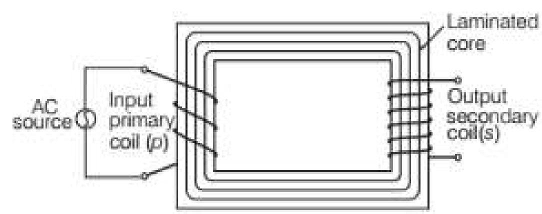
CASE BASED QUESTIONS

Directions (Q.Nos. 22-23) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

22. **The Transformer**

Transformer is a device, which is used to increase or decrease the alternating voltage. The transformers are of the following types

1. Step-up transformer 2. Step-down transformer



Transformer is based upon the principle of mutual induction. It consists of two coils, primary coil (p) and secondary coil (s), insulated from each other wounded on soft iron core. Often the primary coil is the input coil and secondary coil is the output coil. These soft iron cores are laminated to minimise eddy current loss.

(i) What is not possible in a transfomer?
- (a) Eddy current
- (b) Direct current
- (c) Alternating current
- (d) Induced current

(ii) Which quantities do not change during transformer operation?
- (a) Power
- (b) Frequency
- (c) Voltage
- (d) Both (a) and (b)

(iii) A transformer has 150 turns in its primary and 1000 in secondary. If the primary is connected to 440 V DC supply, what will be the induced voltage in the secondary side?
(a) 10 V (b) 3 V (c) 5 V (d) Zero

(iv) The ratio of secondary to primary turns in an ideal transformer is 4 : 5. If power input is P, then the ratio of power output to power input is

(a) 4 : 9 (b) 9 : 4

(c) 5 : 4 (d) 1 : 1

(v) A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?

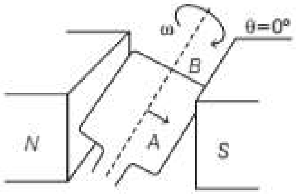
(a) 600 (b) 550

(c) 400 (d) 375

23. AC Generator

An AC generator produces electrical energy from mechanical work, just the opposite of what a motor does. In it, a shaft is rotated by some mechanical means, such as an engine or a turbine starts working and an emf is induced in the coil.

It is based on the phenomenon of electromagnetic induction which states that whenever magnetic flux linked with a conductor (or coil) changes, an emf is induced in the coil.



(i) Which method is used to induce an emf or current in a loop in AC generator?

(a) A change in the loop's orientation

(b) A change in its effective area

(c) Both (a) and (b)

(d) Neither (a) nor (b)

(ii) When the coil is rotated with a constant angular speed ω, the angle θ between the magnetic field vector B and the area vector A of the coil at any instant t, is

(a) $\theta = AB$ (b) $\theta = At$

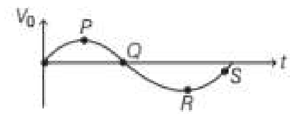
(c) $\theta = \omega t$ (d) $\theta = Bt$

(iii) The change of flux is greatest at θ is equal to (given, $\phi_B = NBA\cos\omega t$)

(a) 90°, 270° (b) 90°, 45°

(c) 60°, 90° (d) 180°, 90°

(iv) The graph below shows the voltage output plotted against time. Which point on the graph shows that the coil is in a vertical position?



(a) P (b) Q (c) R (d) S

(v) An AC generator consists of a coil of 1000 turns and cross-sectional area of 100 cm^2, rotating at an angular speed of 100 rpm in a uniform magnetic field of 3.6×10^{-2} T. The maximum emf produced in the coil is

(a) 1.77 V (b) 2.77 V

(c) 3.77 V (d) 4.77 V

VERY SHORT ANSWER Type Questions

24. Prove mathematically that the average value of alternating current over one complete cycle is zero.

25. Draw the graph showing the variation of reactance of (i) a capacitor (ii) an inductor with the angular frequency of an AC circuit.

26. Distinguish between resistance, reactance and impedance for AC circuit.

27. The total impedance of a circuit decreases, when a capacitor is added in series with L and R. Explain why?

SHORT ANSWER Type Questions

28. Show mathematically that an ideal inductor does not consume any power in an AC circuit.

29. Discuss the use of transformer for long distance transmission of electrical energy.

30. What are the factors which reduces the efficiency of the transformer?

31. What is iron loss in a transformer and how it can be reduced?

LONG ANSWER Type I Questions

32. Draw a circuit diagram showing a series L-C-R circuit and derive an equation for its resonant frequency.

33. Explain the principle, construction and working of a step-down transformer. Can it be used with a DC circuit?

34. Find the expression for the true power and apparent power in an AC circuit. Determine the condition so that current in the circuit may be wattless.

35. An AC source of voltage, $V = V_m \sin \omega t$, is applied across a series L-C-R circuit. Draw the phasor diagram for this circuit when the,

(i) capacitive impedance exceeds the inductive impedance.

(ii) inductive impedance exceeds the capacitive impedance.

36. Answer the following questions.

(i) In any AC circuit, is the applied instantaneous voltage equal to the algebraic sum of instantaneous voltage across the series elements of the circuit? Is the same true for rms voltage?

(ii) A capacitor is used in the primary circuit of an induction coil. Why?

(iii) An applied voltage signal consists of superposition of a DC voltage and an AC voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the DC signal will appear across C and the AC signal across L. **NCERT**

LONG ANSWER Type II Question

37. (i) Obtain the expression for the average power consumed in a series L-C-R circuit connected to AC source for which the phase difference between the voltage and the current in the circuit is ϕ.

(ii) Define the quality factor in an AC circuit. Why should the quality factor have high value in receiving circuits? Name the factors on which it depends.

NUMERICAL PROBLEMS

38. The electric mains in a house are marked 220 V, 50 Hz. Write down the equation for instantaneous voltage.

39. What is the power dissipation in an AC circuit in which voltage and current are given by $E = 300\sin(\omega t + \pi/2)$ and $I = 5\sin\omega t$?

40. How much current is drawn by the primary coil of a transformer, which step-down 220 V-44 V to operate a device with an impedance of 440 Ω?

ANSWERS

1. (*a*)	**2.** (*c*)	**3.** (*b*)	**4.** (*b*)	**5.** (*d*)
6. (*c*)	**7.** (*a*)	**8.** (*d*)	**9.** (*a*)	**10.** (*a*)
11. (*a*)	**12.** (*a*)	**13.** (*a*)	**14.** (*a*)	**15.** (*a*)
16. (*a*)	**17.** (*b*)			

18. (*a*) The resultant emf in the L-C-R circuit is given by

$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow \quad E = \sqrt{(8)^2 + (16-10)^2}$$

$$\Rightarrow \quad E = \sqrt{64+36} \Rightarrow E = 10 \text{ V}$$

19. (*a*) It is important to note that resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit. Only then do the voltage across L and C cancel each other (both being out of phase) and the current amplitude is V_m/R, the total source voltage appearing across R. This means that we cannot have resonance in a R-L or R-C circuit.

20. (*a*)

21. (*b*)

22. (i) (*b*) Transformer is used to convert the value of AC voltage. It works on the principle of electromagnetic induction, so direct current is not possible in it.

(ii) (*d*) The power and frequency do not change in a transformer operation. It changes voltage in a circuit.

(iii) (*d*) As transformer works only in AC and in case of DC supply, there is no induced emf or voltage in secondary because there is no change in flux through the transformer circuit.

(iv) (*d*) In an ideal transformer, there is no energy loss and flux is completely confined with the magnetic core, i.e. perfectly coupled.

So, $$\frac{P_{out}}{P_{in}} = 1$$

(v) (*c*) Here, $\varepsilon_p = 2300$ V, $N_p = 4000$, $\varepsilon_s = 230$ V

Let N_s be the required number of turns in the secondary.

As, $$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$$

$$\Rightarrow \quad N_s = N_p\left(\frac{\varepsilon_s}{\varepsilon_p}\right) = 4000\left(\frac{230 \text{ V}}{2300 \text{ V}}\right) = 400$$

23. (i) (*c*) One method to induce an emf or current in a loop is through a change in the loop's orientation or a change in its effective area.

As the coil rotates in a magnetic field **B**, the effective area of the loop (the face perpendicular to the field) is $A\cos\theta$, where θ is the angle between **A** and **B**.

(ii) (c) When the coil is rotated with a constant angular speed ω, the angle θ between the magnetic field vector **B** and the area vector **A** of the coil at any instant t is $\theta = \omega t$ (assuming $\theta = 0°$ at $t = 0$).

(iii) (a) $\frac{d\phi_B}{dt} = -NBA\omega \sin\omega t$, change of flux is greatest for $\omega t = \theta = 90°, 270°, \theta = 90°, 270°$.

(iv) (b) When the coil passes through its vertical position, its side is moving parallel to the magnetic flux between the magnetic poles, so no change of flux occurs. Hence, no emf is induced in it and output voltage is zero, i.e. at point Q.

(v) (c) Given, $N = 1000$, $A = 100\ \text{cm}^2 = 10^{-2}\ \text{m}^2$,

$$\nu = 100\ \text{rpm} = \frac{100}{60}\ \text{rps}$$

and $B = 3.6 \times 10^{-2}$ T

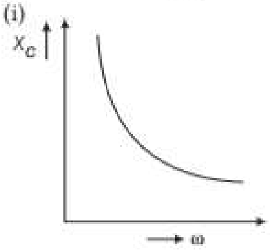
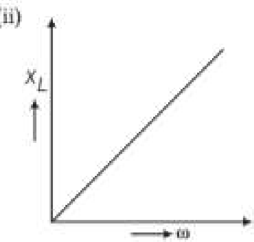
$\therefore$ Maximum emf produced in the coil is

$$e_0 = NBA\,\omega = NBA\,(2\pi\nu)$$

$$= 1000 \times 3.6 \times 10^{-2} \times 10^{-2} \times 2 \times \frac{22}{7} \times \frac{100}{60}$$

$$= 3.77\ \text{V}$$

24. Refer to text on page 300.

25. (i)  (ii) 

26. The basic function of the three is the same i.e. to oppose the flow of current through the circuit. But the difference lie in their expressions. As,

Resistance, $R = V / I$

Capacitive reactance, $X_C = \frac{1}{\omega C}$

Inductive reactance, $X_L = \omega L$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

27. Impedance of an L-R circuit is

$$Z = \sqrt{R^2 + X_L^{\ 2}}$$

But with the introduction of a capacitor in series with a L and R, the new impedance will be

$$Z' = \sqrt{R^2 + (X_T - X_C)^2}$$

Hence, the total impedance decreases.

28. Refer to text on pages 295 and 296.

29. Refer to text on pages 316.

30. Refer to text on page 316.

31. Refer to text on page 316.

32. Refer to text on pages 297 and 298.

33. Refer to text on pages 315 and 316.

34. Refer to text on pages 300 and 301.

35. Refer to text on pages 297 and 298.

36. (i) Yes, it is true for instantaneous voltage.
No, it is not true for rms voltage because voltages across various elements may not be in same phase.

(ii) Because when the circuit is broken, then the large amount of induced voltage is used up in charging the capacitor. Thus, sparking is avoided.

(iii) $X_L = \omega L = 2\pi\nu L$

and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

***Case* I** For DC, If $\nu = 0$, then $X_C = \infty$
Thus, capacitor blocks DC

***Case* II** For AC of higher frequency, X_L is also higher, thus the inductor blocks the current. Hence, AC signal appears across L.

37. (i) Refer to text on page 300.
(ii) Refer to text on pages 299 and 300.

38. Refer to text on page 301. [**Ans.** $220\sqrt{2}\sin 100\pi t$]

39. 720 W, $P_{av} = E_0 / \sqrt{2}_x I_0 / \sqrt{2} \cos\theta$, [here, $\theta = 0°$]

40. Refer to Example 1 on page 316. [**Ans.** 0.02A]