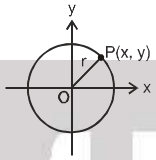
CIRCLE

Four circles to the kissing come, The smaller are the benter. The bend is just the inverse of The distance from the centre. Through their intrigue left Euclid dumb There's now no need for rule of thumb. Since zero bend's a dead straight line And concave bends have minus sign, The sum of squares of all four bends Is half the square of their sum. .... Soddy, Frederick

A circle is a locus of a point in a plane whose distance from a fixed point (called centre) is always constant (called radius).

Equation of a circle in various forms :

(a) The circle with centre as origin & radius 'r' has the equation; $x^2 + y^2 = r^2$.



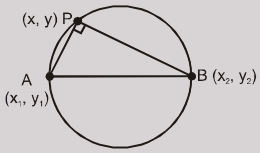
(b) The circle with centre (h, k) & radius 'r' has the equation; $(x - h)^2 + (y - k)^2 = r^2$.

(c) The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$
with centre as (–g, –f) & radius = $\sqrt{g^2 + f^2 - c}$.

This can be obtained from the equation $(x - h)^2 + (y - k)^2 = r^2$
$\Rightarrow$ $x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0$
Take $-h = g$, $-k = f$ and $h^2 + k^2 - r^2 = c$
Condition to define circle :-
$g^2 + f^2 - c > 0$ $\Rightarrow$ real circle.
$g^2 + f^2 - c = 0$ $\Rightarrow$ point circle.
$g^2 + f^2 - c < 0$ $\Rightarrow$ imaginary circle, with real centre, that is (– g, – f)

Note : That every second degree equation in x & y, in which coefficient of x^2 is equal to coefficient of y^2 & the coefficient of xy is zero, always represents a circle.

(d) The equation of circle with (x_1, y_1) & (x_2, y_2) as extremeties of its diameter is:
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.



This is obtained by the fact that angle in a semicircle is a right angle.
$\therefore$ (Slope of PA) (Slope of PB) = – 1

$\Rightarrow \frac{y - y_1}{x - x_1} \cdot \frac{y - y_2}{x - x_2} = -1 \Rightarrow (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Note that this will be the circle of least radius passing through (x_1, y_1) & (x_2, y_2).

Example # 1 Find the equation of the circle whose centre is (0, 3) and radius is 3.
Solution. The equation of the circle is $(x - 0)^2 + (y - 3)^2 = 3^2$
$\Rightarrow$ $x^2 + y^2 - 6y = 0$

Example # 2 Find the equation of the circle which passes through(1, –1) and two of its dimeter are $x + 2y - 5 = 0$ and $x - y + 1 = 0$

Solution. Let P be the point of intersection of the lines
$x + 2y - 5 = 0$ ....(i)
and $x - y + 1 = 0$ ....(ii)

Solving (i) and (ii), we get x = 1, y = 2. So, coordinates of centre are (1, 2). since circle passes through (1,–1) so

radius = $\sqrt{(1-1)^2+(2+1)^2}$ $\Rightarrow$ radius = 3

Hence the equation of the required circle is $(x - 1)^2 + (y + 2)^2 = 9$

Example # 3 If the equation $ax^2 + (b - 3)xy + 3y^2 + 6ax + 2by - 3 = 0$ represents the equation of a circle then find a, b

Solution. $ax^2 + (b - 3)xy + 3y^2 + 6ax + 2by - 3 = 0$

above equation will represent a circle if

coefficient of x^2 = coefficient of y^2

a = 3

coefficient of xy = 0

b = 3

Example # 4 Find the equation of a circle whose diametric end points are (x_1, y_1) and (x_2, y_2) where x_1, x_2 are the roots of $x^2 - ax + b = 0$ and y_1, y_2 are the roots of $y^2 - by + a = 0$.

Solution. We know that the equation of the circle described on the line segment joining (x_1, y_1) and (x_2, y_2) as a diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

$x^2 + y^2 - (x_1 + x_2)x - (y_1 + y_2)y + x_1x_2 + y_1 y_2 = 0$

Here, $x_1 + x_2 = a$, $x_1x_2 = b$

$y_1 + y_2 = b$, $y_1y_2 = a$

So, the equation of the required circle is

$x^2 + y^2 - ax - by + a + b = 0$

Self practice problems :

(1) Find the equation of the circle passing through the point of intersection of the lines $x + 3y = 0$and $2x - 7y = 0$ and whose centre is the point of intersection of the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0$.

(2) Find the equation of the circle whose centre is (1, 2) and which passes through the point (4, 6)

(3) Find the equation of a circle whose radius is 6 and the centre is at the origin.

Answers :

(1) $x^2 + y^2 + 4x - 2y = 0$ (2) $x^2 + y^2 - 2x - 4y - 20 = 0$ (3) $x^2 + y^2 = 36$.

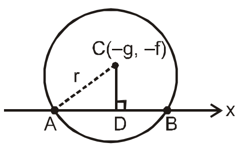
Intercepts made by a circle on the axes:

The intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the co–ordinate axes are $2\sqrt{g^2 - c}$ (on x-axis) & $2\sqrt{f^2 - c}$ (on y-axis) respectively.

If $g^2 > c$ $\Rightarrow$ circle cuts the x axis at two distinct points.

$g^2 = c$ $\Rightarrow$ circle touches the x–axis.

$g^2 < c$ $\Rightarrow$ circle lies completely above or below the x–axis.



$AB = 2AD = 2\sqrt{r^2 - CD^2} = 2\sqrt{r^2 - f^2} = 2\sqrt{g^2 + f^2 - c - f^2} = 2\sqrt{g^2 - c}$

Example # 5 Find the locus of the centre of the circle whose x and y intercepts are a and b respectively.

Solution. Equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

x intercept = a

$2\sqrt{g^2 - c} = a$ $\qquad g^2 - c = \dfrac{a^2}{4}$ ...... (i)

y intercept = b

$2\sqrt{f^2 - c} = b$ $\qquad f^2 - c = \dfrac{b^2}{4}$ ...... (ii)

subtracting equation (i) and (ii)

$g^2 - f^2 = \dfrac{a^2 - b^2}{4}$

hence locus of centre is $x^2 - y^2 = \dfrac{a^2 - b^2}{4}$

Self practice problems :

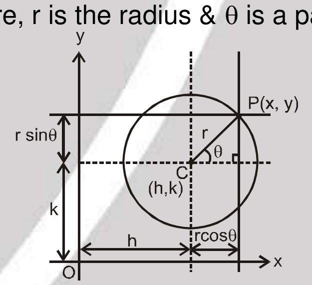
(4) Find the equation of a circle which touches the positive axis of y at a distance 3 from the origin and intercepts a distance 6 on the axis of x.

(5) Find the equation of a circle which touches positive y-axis at a distance of 2 units from the origin and cuts an intercept of 3 units with the positive direction of x-axis.

Answers : (4) $x^2 + y^2 \pm 6\sqrt{2}\ x - 6y + 9 = 0$ $\qquad$ (5) $x^2 + y^2 - 5x - 4y + 4 = 0$

Parametric equations of a circle:

The parametric equations of $(x - h)^2 + (y - k)^2 = r^2$ are: $x = h + r\cos\theta$; $y = k + r\sin\theta$; $-\pi < \theta \le \pi$ where (h, k) is the centre, r is the radius & θ is a parameter.



Example # 6 Find the parametric equations of the circle $x^2 + y^2 + 4x + 6y + 9 = 0$

Solution. We have : $x^2 + y^2 + 4x + 6y + 9 = 0$

$\Rightarrow \quad (x + 2)^2 + (y + 3)^2 = 2^2$

So, the parametric equations of this circle are

$x = -2 + 2\cos\theta$, $y = -3 + 2\sin\theta$.

Example # 7 Find the equation of the following curve in cartesian form

$x + y = \cos\theta$, $x - y = \sin\theta$ where θ is the parameter.

Solution. We have : $x + y = \cos\theta$ .... (i)

$x - y = \sin\theta$ .... (ii)

$(i)^2 + (ii)^2$

$\Rightarrow \quad (x + y)^2 + (x - y)^2 = 1$

$x^2 + y^2 = \dfrac{1}{2}$

Clearly, it is a circle with centre at (0, 0) and radius $\dfrac{1}{\sqrt{2}}$.

Self practice problems :

(6) Find the parametric equations of circle $x^2 + y^2 - 6x + 4y - 12 = 0$

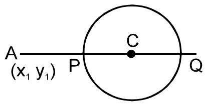
(7) Find the cartesian equations of the curve $x = 1 + \sqrt{2}\cos\theta$, $y = 2 - \sqrt{2}\sin\theta$

Answers : (6) $x = 3 + 5\cos\theta$, $y = -2 + 5\sin\theta$ (7) $(x - 1)^2 + (y - 2)^2 = 2$

Position of a point with respect to a circle:

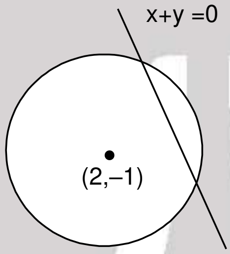
The point (x_1, y_1) is inside, on or outside the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$.
according as $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c <, = \text{ or } > 0$.

Note : The greatest & the least distance of a point A (lies outside the circle) from a circle with centre C & radius r is AC + r & AC − r respectively.



Example # 8 Check whether the point (1, 2) lies in smaller or larger region made by circle $x^2 + y^2 - 4x + 2y - 11 = 0$ and the line $x + y = 0$

Solution : We have $x^2 + y^2 - 4x + 2y - 11 = 0$ or $S = 0$,



where $S = x^2 + y^2 - 4x + 2y - 11$.
For the point (1, 2), we have $S_1 = 1^2 + 2^2 - 4 \times 1 + 2 \times 2 - 11 < 0$
Hence, the point (1, 2) lies inside the circle
Points (1,2) and (2, −1) lie on same side of the line $x + y = 0$
Hence the point (1,2) lies in the larger region.

Self practice problem :

(8) How are the points (0, 1) (3, 1) and (1, 3) situated with respect to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$?

Answer : (8) (0, 1) lies on the circle ; (3, 1) lies outside the circle ; (1, 3) lies inside the circle.

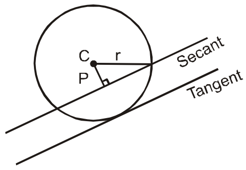
Line and a circle:

Let L = 0 be a line & S = 0 be a circle. If r is the radius of the circle & p is the length of the perpendicular from the centre on the line, then:

(i) $p > r \Leftrightarrow$ the line does not meet the circle i. e. passes out side the circle.
(ii) $p = r \Leftrightarrow$ the line touches the circle. (It is tangent to the circle)
(iii) $p < r \Leftrightarrow$ the line is a secant of the circle.
(iv) $p = 0 \Rightarrow$ the line is a diameter of the circle.

Also, if $y = mx + c$ is line and $x^2 + y^2 = a^2$ is circle then

(i) $c^2 < a^2 (1 + m^2) \Leftrightarrow$ the line is a secant of the circle.
(ii) $c^2 = a^2 (1 + m^2) \Leftrightarrow$ the line touches the circle. (It is tangent to the circle)
(iii) $c^2 > a^2 (1 + m^2) \Leftrightarrow$ the line does not meet the circle i. e. passes out side the circle.



These conditions can also be obtained by solving $y = mx + c$ with $x^2 + y^2 = a^2$ and making the discriminant of the quadratic greater than zero for secant, equal to zero for tangent and less the zero for the last case.

Example # 9 For what value of λ, does the line $x + y = \lambda$ touch the circle $x^2 + y^2 - 2x - 2y = 0$

Solution. We have : $x + y = \lambda$ ......(i) and $x^2 + y^2 - 2x - 2y = 0$....... (ii)

If the line (i) touches the circle (ii), then

length of the $\perp$ from the centre (1, 1) = radius of circle (ii)

$$\Rightarrow \left|\frac{1+1-\lambda}{\sqrt{1^2+1^2}}\right| = \sqrt{2} \quad \Rightarrow \quad |2-\lambda| = 2 \quad \Rightarrow \quad \lambda = 0 \text{ or } 4$$

Hence, the line (i) touches the circle (ii) for $\lambda = 0$ or 4

Self practice problem :

(9) Find the range of values of m for which the line $y = mx + 2$ cuts the circle $x^2 + y^2 = 1$ at distinct points

Answers : (9) $m \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

Slope form of tangent :

$y = mx + c$ is always a tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$. Hence, of tangent is $y = mx \pm a\sqrt{1+m^2}$ and the point of contact is $\left(-\frac{a^2m}{c}, \frac{a^2}{c}\right)$.

Point form of tangent :

(i) The equation of the tangent to the circle $x^2 + y^2 = a^2$ at its point (x_1, y_1) is, $x x_1 + y y_1 = a^2$.

(ii) The equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at its point (x_1, y_1) is : $xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$.

Note : In general the equation of tangent to any second degree curve at point (x_1, y_1) on it can be obtained by replacing x^2 by $x x_1$, y^2 by yy_1, x by $\frac{x + x_1}{2}$, y by $\frac{y + y_1}{2}$, xy by $\frac{x_1y + xy_1}{2}$ and c remains as c.

Parametric form of tangent :

The equation of a tangent to circle $x^2 + y^2 = a^2$ at $(a\cos\alpha, a\sin\alpha)$ is $x\cos\alpha + y\sin\alpha = a$.

NOTE : The point of intersection of the tangents at the points $P(\alpha)$ & $Q(\beta)$ is $\left(\frac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \frac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}\right)$

Example # 10 Find the equation of the tangent to the circle $x^2 + y^2 - 2x - 2y - 11 = 0$ at (3, 4).

Solution. Equation of tangent is

$$3x + 4y - 2\left(\frac{x+3}{2}\right) - 2\left(\frac{y+4}{2}\right) - 11 = 0$$

or $2x + 3y - 18 = 0$

Hence, the required equation of the tangent is $2x + 3y - 18 = 0$

Example # 11 Find the equation of tangents to the circle $x^2 + y^2 - 4x + 2y = 0$ which are perpendicular to the line $x + 2y + 4 = 0$

Solution. Given circle is $x^2 + y^2 - 4x + 2y = 0$ ....(i)

and given line is $x + 2y + 4 = 0$ ....(ii)

Centre of circle (i) is (2,–1)) and its radius $\sqrt{5}$ is Equation of any line

$2x - y + k = 0$ perpendicular to the line (ii) ....(iii)

If line (iii) is tangent to circle (i) then

$$\frac{|4+1+k|}{\sqrt{5}} = \sqrt{5} \quad \text{or} \quad |k + 5| = 5 \quad \text{or} \quad k = 0, -10$$

Hence equation of required tangents are $2x - y = 0$ and $2x - y - 10 = 0$

Self practice problem :

(10) Find the equation of the tangents to the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ which are

(i) parallel,

(ii) perpendicular to the line $3x - 4y - 1 = 0$

Answer.

(10) (i) $3x - 4y + 20 = 0$ and $3x - 4y - 10 = 0$ (ii) $4x + 3y + 5 = 0$ and $4x + 3y - 25 = 0$

Normal :

If a line is normal / orthogonal to a circle, then it must pass through the centre of the circle. Using this fact normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is; $y - y_1 = \dfrac{y_1 + f}{x_1 + g}(x - x_1)$.

Example # 12 Two normals of a circle are $2x + 3y = 5$ and $3x - 4y + 1 = 0$. Find its equation having radius 2

Solution. Since point of intersection of normals is the centre of the circle

point of intersection of lines $2x + 3y = 5$ and $3x - 4y + 1 = 0$ is (1,1)

equation of circle having centre (1,1) and radius 2 is

$(x - 1)^2 + (y - 1)^2 = 4$

Self practice problem :

(11) Find the equation of the normal to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at the point (2, 3).

Answer : (11) $x - y + 1 = 0$

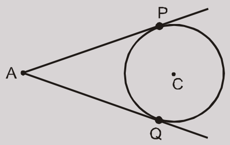
Pair of tangents from a point :

The equation of a pair of tangents drawn from the point A (x_1, y_1) to the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ is : $SS_1 = T^2$.

Where $S \equiv x^2 + y^2 + 2gx + 2fy + c$; $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$.



Example # 13 Find the equation of the pair of tangents drawn to the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ from thepoint (2, 1)

Solution. Given circle is $S = x^2 + y^2 + 4x - 6y + 9 = 0$

Let $P \equiv (2, 1)$

For point P, $S_1 = 16$

Clearly P lies outside the circle

and $T \equiv 2x + y + 2(x + 2) - 3(y + 1) + 9 = 0$

i.e $T \equiv 2(2x - y + 5)$

Now equation of pair of tangents from P(2, 1) to circle (1) is $SS_1 = T^2$

or $16(x^2 + y^2 + 4x - 6y + 9) = 4(2x - y + 5)^2$ or $12y^2 - 16x - 56y + 16xy + 44 = 0$

or $3y^2 - 4x - 14y + 4xy + 11 = 0$

Self practice problems :

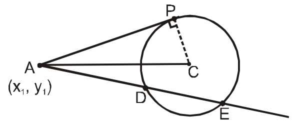
(12) Find the joint equation of the tangents through (7, 1) to the circle $x^2 + y^2 = 25$.

Answer : (12) $12x^2 - 12y^2 + 7xy - 175x - 25y + 625 = 0$

Length of a tangent and power of a point :

The length of a tangent from an external point (x_1, y_1) to the circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is given by $L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2f_1y + c} = \sqrt{S_1}$.



AP = length of tangent

$AP^2 = AD . AE$

Square of length of the tangent from the point A is also called the power of point w.r.t. a circle.
Power of a point w.r.t. a circle remains constant.
Power of a point P is positive, negative or zero according as the point 'A' is outside, inside or on the circle respectively.

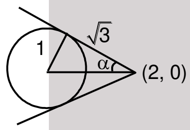
Example # 14 Find the angle between the tangents drawn from the point (2, 0) to the circle $x^2 + y^2 = 1$

Solution. Given circle is $x^2 + y^2 = 1$ ....(i)

Given point is (2, 0).

Now length of the tangent from (2, 0) to circle (i) = $\sqrt{2^2 + 0^2 - 1} = \sqrt{3}$

$\tan\alpha = \frac{1}{\sqrt{3}}$



$\alpha = \frac{\pi}{6}$

so angle between tangents = $2\alpha = \frac{\pi}{3}$

Self practice problems :

(13) The lenght of tangents from P(1, –1) & Q(3, 3) to a circle are $\sqrt{2}$ and $\sqrt{6}$ respectively. Then find the lenght of tangent from R (– 1, – 5) to the same circle.

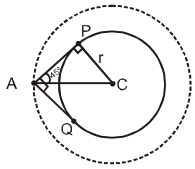
(14) Find the lenght of tangent drawn from any point on circle $x^2 + y^2 + 4x + 6y - 3 = 0$ to the circle $x^2 + y^2 + 4x + 6y + 4 = 0$.

Answer. (13) $\sqrt{38}$ (14) $\sqrt{7}$

Director circle :

The locus of the point of intersection of two perpendicular tangents is called the director circle of the given circle. The director circle of a circle is the concentric circle having radius equal to times the original circle.

Proof :



AC = r cosec 45º = $r\sqrt{2}$

Example # 15 Find the equation of director circle of the circle $x^2 + y^2 + 6x + 8y - 2 = 0$

Solution : Centre & radius of given circle are (–3, –4) & $\sqrt{27}$ respectively.

Centre and radius of the director circle will be (–3, –4) & $\sqrt{27}.\sqrt{2} = \sqrt{54}$ respectively.

$\therefore$ equation of director circle is $(x + 3)^2 + (y + 4)^2 = 54$

$\Rightarrow$ $x^2 + y^2 + 6x + 8y - 29 = 0$

Self practice problems :

(15) Find the angle between the tangents drawn from $(5, \sqrt{7})$ to the circle $x^2+y^2 = 16$

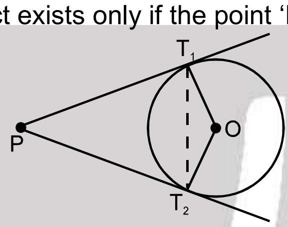
Answer (15) $\frac{\pi}{2}$

Chord of contact :

If two tangents PT_1 & PT_2 are drawn from the point $P(x_1, y_1)$ to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact T_1T_2 is:
$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.$

Note : Here R = radius; L = length of tangent.

(a) Chord of contact exists only if the point 'P' is not inside.



(b) Length of chord of contact $T_1T_2 = \frac{2LR}{\sqrt{R^2 + L^2}}$.

(c) Area of the triangle formed by the pair of the tangents & its chord of contact $= \frac{RL^3}{R^2 + L^2}$

(d) Tangent of the angle between the pair of tangents from $(x_1, y_1) = \left(\frac{2RL}{L^2 - R^2}\right)$

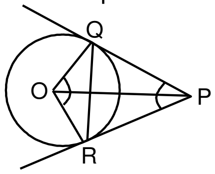
(e) Equation of the circle circumscribing the triangle PT_1T_2 is:
$(x - x_1)(x + g) + (y - y_1)(y + f) = 0.$

Example # 16 Find the equation of the chord of contact of the tangents drawn from (0, 1) to the circle $x^2 + y^2 - 2x + 4y = 0$

Solution. Given circle is $x^2 + y^2 - 2x + 4y = 0$ ....(i)
Let P = (0, 1)
For point P (0, 1), $T = x \cdot 0 + y \cdot 1 - (x + 0) + 2(y + 1)$
i.e. $T = x - 3y - 2$
Now equation of the chord of contact of point P(0, 1) w.r.t. circle (i) will be
$x - 3y - 2 = 0$

Example # 17 If the chord of contact of the tangents drawn from (α, β) to the circle $x^2 + y^2 = a^2$ subtends right angle at the centre then prove that $\alpha^2 + \beta^2 = 2a^2$.

Solution. $\angle QOR = \angle QPR = \frac{\pi}{2}$
so OQPR is a square



$OQ^2 = PQ^2$
$a^2 = \alpha^2 + \beta^2 - a^2$
$\alpha^2 + \beta^2 = 2a^2$

Self practice problems :

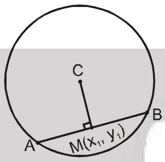
(16) Find the co-ordinates of the point of intersection of tangents at the points where the line $x - 2y + 1 = 0$ meets the circle $x^2 + y^2 = 25$

(17) If the chord of contact of the tangetns drawn from a point on circle $x^2 + y^2 = a^2$ to another circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$ then prove that a,b,c are in G.P.

Answers : (16) (–25, 50) (17) $\frac{405\sqrt{3}}{52}$ sq. unit ; $4x + 6y - 25 = 0$

Equation of the chord with a given middle point:

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point $M(x_1, y_1)$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$.



Notes : (i) The shortest chord of a circle passing through a point 'M' inside the circle, is one chord whose middle point is M.

(ii) The chord passing through a point ' M ' inside the circle and which is at a maximum distance from the centre is a chord with middle point M.

Example # 18 Find the equation of the chord of the circle $x^2 + y^2 + 2x - 2y - 4 = 0$, whose middle point is (0, 0)

Solution. Equation of given circle is $S \equiv x^2 + y^2 + 2x - 2y - 4 = 0$
Let $L \equiv (0, 0)$
For point L(0, 0), $S_1 = -4$ and
$T \equiv x.0 + y(0) + (x + 0) - (y + 0) - 4$ i.e. $T \equiv x - y - 4$
Now equation of the chord of circle (i) whose middle point is L(0, 0) is
$T = S_1$ or $x - y = 0$
Second Method : Let C be the centre of the given circle, then $C \equiv (-1, 1)$. $L \equiv (0, 0)$ slope of CL = –1
$\therefore$ Equation of chord of circle whose middle point is L, is $y - 0 = 1(x - 0)$
($\because$ chord is perpendicular to CL) or $x - y = 0$

Self practice problems :

(18) Find the equation of that chord of the circle $x^2 + y^2 = 15$, which is bisected at (3, 2)

(19) A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. Find the locus of the centre of the circle drawn on this chord as diameter.

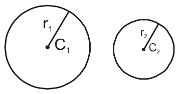
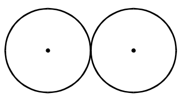
Answers : (18) $3x + 2y - 13 = 0$ (19) $x^2 + y^2 - ax = 0$

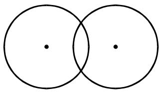
Equation of the chord joining two points of circle :

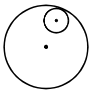
The equation of chord PQ to the circle $x^2 + y^2 = a^2$ joining two points $P(\alpha)$ and $Q(\beta)$ on it is given by the equation of a straight line joining two point α & β on the circle $x^2 + y^2 = a^2$ is

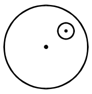
$$x \cos\frac{\alpha+\beta}{2} + y \sin\frac{\alpha+\beta}{2} = a \cos\frac{\alpha-\beta}{2}.$$

Common tangents to two circles:

	Case	Number of Tangents	Condition
(i)		4 common tangents (2 direct and 2 transverse)	$r_1 + r_2 < c_1 c_2$.
(ii)		3 common tangents.	$r_1 + r_2 = c_1 c_2$.

(iii)  2 common tangents. $|r_1 - r_2| < c_1 c_2 < r_1 + r_2$

(iv)  1 common tangent. $|r_1 - r_2| = c_1 c_2$.

(v)  No common tangent. $c_1 c_2 < |r_1 - r_2|$.

(Here C_1C_2 is distance between centres of two circles.)

Notes : (i) The direct common tangents meet at a point which divides the line joining centre of circles externally in the ratio of their radii.
Transverse common tangents meet at a point which divides the line joining centre of circles internally in the ratio of their radii.

(ii) Length of an external (or direct) common tangent & internal (or transverse) common tangent to the two circles are given by:
$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2}$ & $L_{int} = \sqrt{d^2 - (r_1 + r_2)^2}$, where d = distance between the centres of the two circles and r_1, r_2 are the radii of the two circles. Note that length of internal common tangent is always less than the length of the external or direct common tangent.

Example # 19 Examine if the two circles $x^2 + y^2 - 4x - 6y + 9 = 0$ and $x^2 + y^2 - 10x - 6y + 18 = 0$ intersect or not

Solution. Given circles are $x^2 + y^2 - 4x - 6y + 9 = 0$ ....(i)
and $x^2 + y^2 - 10x - 6y + 18 = 0$ ....(ii)
Let A and B be the centres and r_1 and r_2 the radii of circles (i) and (ii) respectively, then
$A \equiv (2, 3)$, $B \equiv (5, 3)$, $r_1 = 2$, $r_2 = 4$
Now AB = 3 and $r_1 + r_2 = 6$, $|r_1 - r_2| = 2$
Thus $|r_1 - r_2| < AB < r_1 + r_2$, hence the two circles intersect.

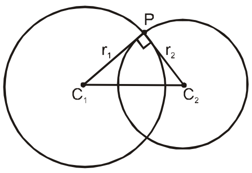
Self practice problems :

(20) Find the position of the circles $x^2 + y^2 - 10x + 4y - 20 = 0$ and $x^2 + y^2 + 14x - 6y + 22 = 0$ with respect to each other.

Answer : (20) touch externally

Orthogonality of two circles:

Two circles $S_1 = 0$ & $S_2 = 0$ are said to be orthogonal or said to intersect orthogonally if the tangents at their point of intersection include a right angle. The condition for two circles to be orthogonal is:
$2 g_1 g_2 + 2 f_1 f_2 = c_1 + c_2$.
Proof :
$(C_1C_2)^2 = (C_1P)^2 + (C_2P)^2$
$\Rightarrow \quad (g_1 - g_2)^2 + (f_1 - f_2)^2 = g_1^2 + f_1^2 - c_1 + g_2^2 + f_2^2 - c_2$
$\Rightarrow \quad 2g_1g_2 + 2f_1f_2 = c_1 + c_2$



Notes :
(a) The centre of a variable circle orthogonal to two fixed circles lies on the radical axis of two circles.
(b) If two circles are orthogonal, then the polar of a point 'P' on first circle w.r.t. the second circle passes through the point Q which is the other end of the diameter through P. Hence locus of a point which moves such that its polars w.r.t. the circles $S_1 = 0$, $S_2 = 0$ & $S_3 = 0$ are concurrent in a circle which is orthogonal to all the three circles.

(c) The centre of a circle which is orthogonal to three given circles is the radical centre provided the radical centre lies outside all the three circles.

Example # 20 If the circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $2x^2 + 2y^2 + 2g_2x + 2f_2y + c_2 = 0$ are orthogonal to each other then prove that $g_1g_2 + f_1f_2 = c_1 + \frac{c_2}{2}$

Solution. Given circles are $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ ....(i)

and $2x^2 + 2y^2 + 2g_2x + 2f_2y + c_2 = 0$

or $x^2 + y^2 + g_2x + f_2y + \frac{c_2}{2} = 0$ ....(ii)

Since circles (i) and (ii) cut orthogonally

$\therefore \quad 2g_1\left(\frac{g_2}{2}\right) + 2f_1\left(\frac{f_2}{2}\right) = c_1 + \frac{c_2}{2}$

$g_1g_2 + f_1f_2 = c_1 + \frac{c_2}{2}$

Self practice problems :

(21) For what value of λ the circles $x^2 + y^2 + 8x + 3y + 9 = 0$ and $x^2 + y^2 + 2x - y - \lambda = 0$ cut orthogonally.

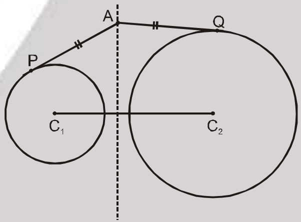
(22) Find the equation to the circle which passes through the origin and has its centre on the line $x - y = 0$ and cuts the circle $x^2 + y^2 - 4x - 6y + 10 = 0$ orthogonally.

Answer : (21) $\frac{5}{2}$ (22) $x^2 + y^2 - 2x - 2y = 0$

Radical axis and radical centre:

The radical axis of two circles is the locus of points whose powers w.r.t. the two circles are equal. The equation of radical axis of the two circles $S_1 = 0$ & $S_2 = 0$ is given by

$S_1 - S_2 = 0$ i.e. $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$.



The common point of intersection of the radical axes of three circles taken two at a time is called the radical centre of three circles. Note that the length of tangents from radical centre to the three circles are equal.

Notes :

(a) If two circles intersect, then the radical axis is the common chord of the two circles.

(b) If two circles touch each other, then the radical axis is the common tangent of the two circlesat the common point of contact.

(c) Radical axis is always perpendicular to the line joining the centres of the two circles.

(d) Radical axis will pass through the mid point of the line joining the centres of the two circles only if the two circles have equal radii.

(e) Radical axis bisects a common tangent between the two circles.

(f) A system of circles, every two which have the same radical axis, is called a coaxial system.

(g) Pairs of circles which do not have radical axis are concentric.

Example # 21 Find the co-ordinates of the point from which the lengths of the tangents to the following three circles be equal.

$x^2 + y^2 = 1$
$x^2 + y^2 - 8x + 15 = 0$
$x^2 + y^2 + 10y + 24 = 0$

Solution : Here we have to find the radical centre of the three circles. First reduce them to standard form in which coefficients of x^2 and y^2 be each unity. Subtracting in pairs the three radical axes are

$x = 2$; $8x + 10y + 9 = 0$
$10y + 25 = 0$

solving any two, we get the point $\left(2, -\dfrac{5}{2}\right)$ which satisfies the third also. This point is called the radical centre and by definition the length of the tangents from it to the three circles are equal.

Self practice problem :

(23) Find the point from which the tangents to the three circles $x^2 + y^2 - 4x + 7 = 0$, $2x^2 + 2y^2 - 3x + 5y + 9 = 0$ and $x^2 + y^2 + y = 0$ are equal in length. Find also this length.

Answer : (23) (2, – 1) ; 2.

Family of Circles:

This article is aimed at obtaining the equation of a group of circles having a specific characteristic. For example, the equation $x^2 + y^2 + 4x + 2y + \lambda = 0$ where λ is arbitrary, represents a family of circles with fixed centre (–2, –1) but variable radius. We have the following results for some other families of circles.

(a) The equation of the family of circles passing through the points of intersection of two circles $S_1 = 0$ & $S_2 = 0$ is : $S_1 + K S_2 = 0$
($K \neq -1$, provided the co–efficient of x^2 & y^2 in S_1 & S_2 are same)

(b) The equation of the family of circles passing through the point of intersection of a circle S = 0 & a line L = 0 is given by S + KL = 0.

(c) The equation of a family of circles passing through two given points (x_1, y_1) & (x_2, y_2) can be written in the form:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + K \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0, \text{ where K is a parameter.}$$

(d) The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point (x_1, y_1) is $(x - x_1)^2 + (y - y_1)^2 + K(y - y_1 - m(x - x_1)) = 0$, where K is a parameter.

(e) Family of circles circumscribing a triangle whose sides are given by $L_1 = 0$, $L_2 = 0$ and $L_3 = 0$ is given by; $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided co–efficient of xy = 0 and co–efficient of x^2 = co–efficient of y^2.

(f) Equation of circle circumscribing a quadrilateral whose side in order are represented by the lines $L_1 = 0$, $L_2 = 0$, $L_3 = 0$ & $L_4 = 0$ are $u L_1L_3 + \lambda L_2L_4 = 0$ where values of u & λ can be found out by using condition that co–efficient of x^2 = co–efficient of y^2 and co–efficient of xy = 0.

Example # 22 Find the equation of the circle passing through the point (1, 1) and points of intersection of the circles $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$.

Solution. Any circle through the intersection of given circles is $S_1 + \lambda S_2 = 0$

or $x^2 + y^2 + 13x - 3y + \lambda(x^2 + y^2 + 2x - 7y/2 - 25/2) = 0$

This circle passes through (1, 1)

$1 + 1 + 13 - 3 + \lambda(1 + 1 + 2 - 7/2 - 25/2) = 0$

$\lambda = 1$

Putting the value of λ in (i) the required circle is $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

Example # 23 Find the equations of smallest circle which passes through the points of intersection of the line x + y = 1 and the circle $x^2 + y^2 = 9$.

Solution. The required circle by $S + \lambda L = 0$ is

$x^2 + y^2 - 9 + \lambda(x + y - 1) = 0$ ....(i) centre (– g, – f) = $\left(-\frac{\lambda}{2}, -\frac{\lambda}{2}\right)$

centre lies on the line x + y = 1

$-\frac{\lambda}{2} - \frac{\lambda}{2} = 1$

$\lambda = -1$

Putting the value of λ in (i) the required circle is

$x^2 + y^2 - x - y - 8 = 0$

Example # 24 Find the equation of circle passing through the points A(1, 1) & B(0, 3) and whose radius is $\sqrt{\frac{5}{2}}$.

Solution. Equation of AB is 2x + y – 3 = 0

$\therefore$ equation of circle is

$(x - 1)(x) + (y - 1)(y - 3) + \lambda(2x + y - 3)$

$= 0$ or $x^2 + y^2 + (2\lambda - 1)x + (\lambda - 4)y + 3 - 3\lambda = 0$

$$\sqrt{\left(\frac{2\lambda-1}{2}\right)^2 + \left(\frac{\lambda-4}{2}\right) + 3\lambda - 3} = \sqrt{\frac{5}{2}}$$

$\lambda = 1$

$\therefore$ equation of circle is $x^2 + y^2 + x - 3y = 0$

Example # 25 A variable circle always touhces x + y = 2 at (1, 1), cuts the circle $x^2 + y^2 + 4x + 5y - 6 = 0$. Prove that all common chords pass through a fixed point. Also find the point.

Solution : Equation of circle is $(x - 1)^2 + (y - 1)^2 + \lambda(x + y - 2) = 0$

$x^2 + y^2 + x(\lambda - 2) + y(\lambda - 2) + 2 - 2\lambda = 0$

common chord of this circle with $x^2 + y^2 + 4x + 5y - 6 = 0$ is

$(\lambda - 6)x + (\lambda - 7)y + 8 - 2\lambda = 0$

$\lambda(x + y - 2) + (-6x - 7y + 8) = 0$

this chord passes through the point of intersection of the lines x + y – 2 = 0 and –6x – 7y + 8 = 0 which is (6, –4)

Example # 26 Find the equation of circle circumcscribing the triangle whose sides are 3x – y – 12 = 0, 5x – 3y – 28 = 0 & x + y – 4 = 0.

Solution : $L_1L_2 + \lambda L_2L_3 + \mu L_1L_3 = 0$

$(3x - y - 12)(5x - 3y - 28) + \lambda(5x - 3y - 28)(x + y - 4) + \mu(3x - y - 12)(x + y - 4) = 0$

coefficient of x^2 = coefficient of y^2

$\Rightarrow$ $5\lambda + 3\mu + 15 = 3 - 3\lambda - \mu$

$2\lambda + \mu + 3 = 0$ ....(ii)

coefficient of xy = 0

$\Rightarrow$ $\lambda + \mu - 7 = 0$ ....(iii)

Solving (ii) and (iii), we have

$\lambda = -10, \mu = 17$

Puting these values of λ & μ in equation (i), we get $2x^2 + 2y^2 - 9x + 11y + 4 = 0$

Self practice problems :

(24) Find the equation of the circle passing through the points of intersection of the circles $x^2 + y^2 - 6x + 2y + 4 = 0$ and $x^2 + y^2 + 2x - 4y - 6 = 0$ and with its centre on the line y = x.

(25) Find the equation of circle circumscribing the quadrilateral whose sides are x + y = 10, x – 7y + 50 = 0, 22x – 4y + 125 = 0 and 2x – 4y – 5

Answers : (24) $7x^2 + 7y^2 - 10x - 10y - 12 = 0$

(25) $x^2 + y^2 = \frac{125}{2}$

Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Equation of circle, parametric equation, position of a point

A-1. Find the equation of the circle that passes through the points (1, 0), (– 1, 0) and (0, 1).

A-2. ABCD is a square in first quadrant whose side is a, taking AB and AD as axes, prove that the equation to the circle circumscribing the square is $x^2 + y^2 = a(x + y)$.

A-3. Find the equation to the circle which passes through the origin and cuts off intercepts equal to 3 and 4 from the positive axes.

A-4. Find equation of circle which touches x & y axis & perpendicular distance of centre of circle from $3x + 4y + 11 = 0$ is 5. Given that circle lies in I[st] quadrant.

A-5.✍ Find the equation to the circle which touches the axis of x at a distance 3 from the origin and intercepts a distance 6 on the axis of y.

A-6. Find equation of circle whose cartesian equation are $x = -3 + 2 \sin \theta$, $y = 4 + 2 \cos \theta$

A-7. Find the values of p for which the power of a point (2, 5) is negative with respect to a circle $x^2 + y^2 - 8x - 12y + p = 0$ which neither touches the axes nor cuts them.

Section (B) : Line and circle, tangent, pair of tangent

B-1. If radii of the largest and smallest circle passing through the point (1, –1) and touching the circle $x^2 + y^2 + 2\sqrt{2}\, y - 2 = 0$ are r_1 and r_2 respectively, then find the sum of r_1 and r_2 .

B-2. Find the points of intersection of the line $x - y + 2 = 0$ and the circle $3x^2 + 3y^2 - 29x - 19y + 56 = 0$. Also determine the length of the chord intercepted.

B-3. Show that the line $7y - x = 5$ touches the circle $x^2 + y^2 - 5x + 5y = 0$ and find the equation of the other parallel tangent.

B-4. Find the equation of the tangents to the circle $x^2 + y^2 = 4$ which make an angle of 60º with the positive x-axis in anticlockwise direction.

B-5. Show that two tangents can be drawn from the point (9, 0) to the circle $x^2 + y^2 = 16$; also find the equation of the pair of tangents and the angle between them.

B-6. If the length of the tangent from (f, g) to the circle $x^2 + y^2 = 6$ be twice the length of the tangent from (f, g) to the circle $x^2 + y^2 + 3x + 3y = 0$, then will $f^2 + g^2 + 4f + 4g + 2 = 0$?

Section (C) : Normal, Director circle, chord of contact, chord with mid point

C-1. Find the equation of the normal to the circle $x^2 + y^2 = 5$ at the point (1, 2)

C-2. Find the equation of the normal to the circle $x^2 + y^2 = 2x$, which is parallel to the line $x + 2y = 3$.

C-3. Find the equation of director circle of the circle $(x + 4)^2 + y^2 = 8$

C-4.✍ Tangents are drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$; prove that the area of the triangle formed by them and the straight line joining their points of contact is $\dfrac{a(h^2+k^2-a^2)^{3/2}}{h^2+k^2}$ c.

C-5. Find the equation of the chord of the circle $x^2 + y^2 + 6x + 8y + 9 = 0$ whose middle point is (– 2, – 3).

C-6.✍ Tangents are drawn to the circle $x^2 + y^2 = 12$ at the points where it is met by the circle $x^2 + y^2 - 5x + 3y - 2 = 0$; find the point of intersection of these tangents.

Section (D) : Position of two circle, Orthogonality, Radical axis and radical centre

D-1. Find the equations to the common tangents of the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$

D-2. Show that the circles $x^2 + y^2 - 2x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 6 = 0$ cut each other orthogonally.

D-3. Find the equation of the circle passing through the origin and cutting the circles $x^2 + y^2 - 4x + 6y + 10 = 0$ and $x^2 + y^2 + 12y + 6 = 0$ at right angles.

D-4. Given the three circles $x^2 + y^2 - 16x + 60 = 0$, $3x^2 + 3y^2 - 36x + 81 = 0$ and $x^2 + y^2 - 16x - 12y + 84 = 0$, find (1) the point from which the tangents to them are equal in length and (2) this length.

Section (E) : Family of circles , Locus, Miscellaneous

E-1. If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$, find the equation of a circle with this chord as diameter.

E-2. Find the equation of a circle which touches the line $2x - y = 4$ at the point $(1, -2)$ and
(i) Passes through (3, 4)
(ii) Radius = 5

E-3. Show that the equation $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ represents for different values of λ a system of circles passing through two fixed points A and B on the x-axis, and also find the equation of that circle of the system the tangent to which at A and B meet on the line $x + 2y + 5 = 0$.

E-4. Consider a family of circles passing through two fixed points A (3, 7) and B (6, 5). Show that the chords in which the circles $x^2 + y^2 - 4x - 3 = 0$ cuts the members of the family are concurrent at a point. Also find the co-ordinates of this point.

E-5. Find the equation of the circle circumscribing the triangle formed by the lines $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$.

E-6. Prove that the circle $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touches each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Equation of circle, parametric equation, position of a point

A-1. The radius of the circle passing through the points (1, 2), (5, 2) & (5, – 2) is:

(A) $5\sqrt{2}$ (B) $2\sqrt{5}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$

A-2. The centres of the circles $x^2 + y^2 - 6x - 8y - 7 = 0$ and $x^2 + y^2 - 4x - 10y - 3 = 0$ are the ends of the diameter of the circle
(A) $x^2 + y^2 - 5x - 9y + 26 = 0$ (B) $x^2 + y^2 + 5x - 9y + 14 = 0$
(C) $x^2 + y^2 + 5x - y - 14 = 0$ (D) $x^2 + y^2 + 5x + y + 14 = 0$

A-3. The circle described on the line joining the points (0, 1), (a, b) as diameter cuts the x–axis in points whose abscissa are roots of the equation:
(A) $x^2 + ax + b = 0$ (B) $x^2 - ax + b = 0$ (C) $x^2 + ax - b = 0$ (D) $x^2 - ax - b = 0$

A-4. The intercepts made by the circle $x^2 + y^2 - 5x - 13y - 14 = 0$ on the x-axis and y-axis are respectively
(A) 9, 13 (B) 5, 13 (C) 9, 15 (D) none

A-5. Equation of line passing through mid point of intercepts made by circle $x^2 + y^2 - 4x - 6y = 0$ on co-ordinate axes is
(A) $3x + 2y - 12 = 0$ (B) $3x + y - 6 = 0$ (C) $3x + 4y - 12 = 0$ (D) $3x + 2y - 6 = 0$

A-6. Two thin rods AB & CD of lengths 2a & 2b move along OX & OY respectively, when 'O' is the origin. The equation of the locus of the centre of the circle passing through the extremities of the two rods is:
(A) $x^2 + y^2 = a^2 + b^2$ (B) $x^2 - y^2 = a^2 - b^2$ (C) $x^2 + y^2 = a^2 - b^2$ (D) $x^2 - y^2 = a^2 + b^2$

A-7. Let A and B be two fixed points then the locus of a point C which moves so that (tan∠BAC) (tan ∠ABC)=1, $0 < \angle BAC < \frac{\pi}{2}$, $0 < \angle ABC < \frac{\pi}{2}$ is
(A) Circle (B) pair of straight line (C) A point (D) Straight line

A-8. **STATEMENT-1 :** The length of intercept made by the circle $x^2 + y^2 - 2x - 2y = 0$ on the x-axis is 2.
STATEMENT-2 : $x^2 + y^2 - \alpha x - \beta y = 0$ is a circle which passes through origin with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ and radius $\sqrt{\frac{\alpha^2 + \beta^2}{2}}$
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true

Section (B) : Line and circle, tangent, pair of tangent

B-1. Find the co-ordinates of a point p on line $x + y = -13$, nearest to the circle $x^2 + y^2 + 4x + 6y - 5 = 0$
(A) (– 6, – 7) (B) (– 15, 2) (C) (– 5, – 6) (D) (– 7, – 6)

B-2. The number of tangents that can be drawn from the point (8, 6) to the circle $x^2 + y^2 - 100 = 0$ is
(A) 0 (B) 1 (C) 2 (D) none

B-3. Two lines through (2, 3) from which the circle $x^2 + y^2 = 25$ intercepts chords of length 8 units have equations
(A) $2x + 3y = 13$, $x + 5y = 17$ (B) $y = 3$, $12x + 5y = 39$
(C) $x = 2$, $9x - 11y = 51$ (D) $y = 0$, $12x + 5y = 39$

B-4. The line $3x + 5y + 9 = 0$ w.r.t. the circle $x^2 + y^2 - 4x + 6y + 5 = 0$ is
(A) chord dividing circumference in 1 : 3 ratio (B) diameter
(C) tangent (D) outside line

B-5. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is
(A) 3 (B) 2 (C) 3/2 (D) 1

B-6. The tangent lines to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line $4x + 3y + 5 = 0$ are given by:
(A) $4x + 3y - 7 = 0$, $4x + 3y + 15 = 0$ (B) $4x + 3y - 31 = 0$, $4x + 3y + 19 = 0$
(C) $4x + 3y - 17 = 0$, $4x + 3y + 13 = 0$ (D) $4x + 3y - 31 = 0$, $4x + 3y - 19 = 0$

B-7. The condition so that the line $(x + g)\cos\theta + (y + f)\sin\theta = k$ is a tangent to $x^2 + y^2 + 2gx + 2fy + c = 0$ is
(A) $g^2 + f^2 = c + k^2$ (B) $g^2 + f^2 = c^2 + k$ (C) $g^2 + f^2 = c^2 + k^2$ (D) $g^2 + f^2 = c + k$

B-8. The tangent to the circle $x^2 + y^2 = 5$ at the point (1, –2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ at
(A) (–2, 1) (B) (–3, 0) (C) (–1, –1) (D) (3, –1)

B-9. The angle between the two tangents from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ equals
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

B-10. A point A(2, 1) is outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ & AP, AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is :
(A) $(x + g)(x - 2) + (y + f)(y - 1) = 0$ (B) $(x + g)(x - 2) - (y + f)(y - 1) = 0$
(C) $(x - g)(x + 2) + (y - f)(y + 1) = 0$ (D) $(x - g)(x - 2) + (y - f)(y - 1) = 0$

B-11. A line segment through a point P cuts a given circle in 2 points A & B, such that PA = 16 & PB = 9, find the length of tangent from points to the circle
(A) 7 (B) 25 (C) 12 (D) 8

B-12. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + q = 0$ is:
(A) $\sqrt{q - p}$ (B) $\sqrt{p - q}$ (C) $\sqrt{q + p}$ (D) $\sqrt{2q + p}$

B-13. The equation of the diameter of the circle $(x - 2)^2 + (y + 1)^2 = 16$ which bisects the chord cut off by the circle on the line $x - 2y - 3 = 0$ is
(A) $x + 2y = 0$ (B) $2x + y - 3 = 0$ (C) $3x + 2y - 4 = 0$ (D) $3x - 2y - 4 = 0$

B-14. The locus of the point of intersection of the tangents to the circle $x^2 + y^2 = a^2$ at points whose parametric angles differ by $\frac{\pi}{3}$ is
(A) $x^2 + y^2 = \frac{4a^2}{3}$ (B) $x^2 + y^2 = \frac{2a^2}{3}$ (C) $x^2 + y^2 = \frac{a^2}{3}$ (D) $x^2 + y^2 = \frac{a^2}{9}$

Section (C) : Normal, Director circle, chord of contact, chord with mid point

C-1. The equation of normal to the circle $x^2 + y^2 - 4x + 4y - 17 = 0$ which passes through (1, 1) is
(A) $3x + y - 4 = 0$ (B) $x - y = 0$ (C) $x + y = 0$ (D) $3x - y - 4 = 0$

C-2. The normal at the point (3, 4) on a circle cuts the circle at the point (–1, –2). Then the equation of the circle is
(A) $x^2 + y^2 + 2x - 2y - 13 = 0$ (B) $x^2 + y^2 - 2x - 2y - 11 = 0$
(C) $x^2 + y^2 - 2x + 2y + 12 = 0$ (D) $x^2 + y^2 - 2x - 2y + 14 = 0$

C-3. The co-ordinates of the middle point of the chord cut off on $2x - 5y + 18 = 0$ by the circle $x^2 + y^2 - 6x + 2y - 54 = 0$ are
(A) (1, 4) (B) (2, 4) (C) (4, 1) (D) (1, 1)

C-4. The locus of the mid point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is:
(A) $x + y = 2$ (B) $x^2 + y^2 = 1$ (C) $x^2 + y^2 = 2$ (D) $x + y = 1$

C-5. The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through the point
(A) (1, 2) (B) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (C) (2, 4) (D) (4, 4)

C-6. The locus of the centers of the circles such that the point (2, 3) is the mid point of the chord $5x + 2y = 16$ is:
(A) $2x - 5y + 11 = 0$ (B) $2x + 5y - 11 = 0$ (C) $2x + 5y + 11 = 0$ (D) $2x - 5y - 11 = 0$

C-7. Find the locus of the mid point of the chord of a circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 - 2x - 2y = 0$ subtends a right angle at the origin.
(A) $x^2 + y^2 - 2x - 2y = 0$ (B) $x^2 + y^2 + 2x - 2y = 0$
(C) $x^2 + y^2 + 2x + 2y = 0$ (D) $x^2 + y^2 - 2x + 2y = 0$

Section (D) : Position of two circle, Orthogonality, Radical axis and radical centre

D-1. Number of common tangents of the circles $(x + 2)^2 + (y-2)^2 = 49$ and $(x - 2)^2 + (y + 1)^2 = 4$ is:
(A) 0 (B) 1 (C) 2 (D) 3

D-2. The equation of the common tangent to the circle $x^2 + y^2 - 4x - 6y - 12=0$ and $x^2 + y^2 + 6x+18y + 26 = 0$ at their point of contact is
(A) $12x + 5y + 19 = 0$ (B) $5x + 12y + 19 = 0$ (C) $5x - 12y + 19 = 0$ (D) $12x - 5y + 19 = 0$

D-3. Equation of the circle cutting orthogonally the three circles $x^2 + y^2 - 2x + 3y - 7 = 0$, $x^2 + y^2 + 5x - 5y + 9 = 0$ and $x^2 + y^2 + 7x - 9y + 29 = 0$ is
(A) $x^2 + y^2 - 16x - 18y - 4 = 0$ (B) $x^2 + y^2 - 7x + 11y + 6 = 0$
(C) $x^2 + y^2 + 2x - 8y + 9 = 0$ (D) $x^2 + y^2 + 16x - 18y - 4 = 0$

D-4. If the length of a common internal tangent to two circles is 7, and that of a common external tangent is 11, then the product of the radii of the two circles is:
(A) 18 (B) 20 (C) 16 (D) 12

Section (E) : Family of circles , Locus, Miscellaneous

E-1. The locus of the centre of the circle which bisects the circumferences of the circles $x^2 + y^2 = 4$ & $x^2 + y^2 - 2x + 6y + 1 = 0$ is:
(A) a straight line (B) a circle (C) a parabola (D) pair of straight line

E-2. Equation of a circle drawn on the chord $x \cos \alpha + y \sin \alpha = p$ of the circle $x^2 + y^2 = a^2$ as its diameter, is
(A) $(x^2 + y^2 - a^2) - 2p (x\sin\alpha + y\cos\alpha - p) = 0$ (B) $(x^2 + y^2 - a^2) - 2p (x\cos\alpha + y\sin\alpha - p) = 0$
(C) $(x^2 + y^2 - a^2) + 2p (x\cos\alpha + y\sin\alpha - p) = 0$ (D) $(x^2 + y^2 - a^2) - p (x\cos\alpha + y\sin\alpha - p) = 0$

E-3. Find the equation of the circle which passes through the point (1, 1) & which touches the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ at the point (2, 3) on it.
(A) $x^2 + y^2 + x - 6y + 3 = 0$ (B) $x^2 + y^2 + x - 6y - 3 = 0$
(C) $x^2 + y^2 + x + 6y + 3 = 0$ (D) $x^2 + y^2 + x - 3y + 3 = 0$

E-4. Find the equation of circle touching the line $2x + 3y + 1 = 0$ at (1, – 1) and cutting orthogonally the circle having line segment joining (0, 3) and (– 2, – 1) as diameter.
(A) $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ (B) $2x^2 + 2y^2 - 10x + 5y + 1 = 0$
(C) $2x^2 + 2y^2 - 10x - 5y - 1 = 0$ (D) $2x^2 + 2y^2 + 10x - 5y + 1 = 0$

E-5. Equation of the circle which passes through the point (–1, 2) & touches the circle $x^2 + y^2 - 8x + 6y = 0$ at origin, is -
(A) $x^2 + y^2 - 2x - \frac{3}{2}y = 0$ (B) $x^2 + y^2 + x - 2y = 0$
(C) $x^2 + y^2 + 2x + \frac{3}{2}y = 0$ (D) $x^2 + y^2 + 2x - \frac{3}{2}y = 0$

E-6. Two circles are drawn through the point (a, 5a) and (4a, a) to touch the axis of 'y'. They intersect at an angle of θ then $\tan\theta$ is -
(A) $\frac{40}{9}$ (B) $\frac{9}{40}$ (C) $\frac{1}{9}$ (D) $\frac{1}{\sqrt{3}}$

PART - III : MATCH THE COLUMN

1.

	Column – I		Column – II
(A)	Number of values of a for which the common chord of the circles $x^2 + y^2 = 8$ and $(x - a)^2 + y^2 = 8$ subtends a right angle at the origin is	(p)	0
(B)	The number of circles touching all the three lines $3x + 7y = 2$, $21x + 49y = 5$ and $9x + 21y = 0$ are	(q)	2
(C)	The length of common chord of circles $x^2 + y^2 - x - 11y + 18 = 0$ and $x^2 + y^2 - 9x - 5y + 14 = 0$ is	(r)	5
(D)	Number of common tangents of the circles $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 + 6x - 6y + 2 = 0$ is	(s)	3

2.

	Column – I		Column – II
(A)	If director circle of two given circles C_1 and C_2 of equal radii touches each other, then ratio of length of internal common tangent of C_1 and C_2 to their radii equals to	(p)	13
(B)	Let two circles having radii r_1 and r_2 are orthogonal to each other. If length of their common chord is k times the square root of hormonic mean between squares of their radii, then k^4 equals to	(q)	7
(C)	The axes are translated so that the new equation of the circle $x^2 + y^2 - 5x + 2y - 5 = 0$ has no first degree terms and the new equation $x^2 + y^2 = \frac{\lambda^2}{4}$, then the value of λ is	(r)	4
(D)	The number of integral points which lie on or inside the circle $x^2 + y^2 = 4$ is	(s)	2

Exercise-2

PART - I : ONLY ONE OPTION CORRECT TYPE

1. If $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ & $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units, then abcd is equal to:

(A) 4 (B) 16 (C) 1 (D) 2

2. From the point A (0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$ a chord AB is drawn & extended to a point M such that AM = 2 AB. The equation of the locus of M is :

(A) $x^2 + 8x + y^2 = 0$ (B) $x^2 + 8x + (y - 3)^2 = 0$

(C) $(x - 3)^2 + 8x + y^2 = 0$ (D) $x^2 + 8x + 8y^2 = 0$

3. If tangent at (1, 2) to the circle c_1: $x^2 + y^2 = 5$ intersects the circle c_2: $x^2 + y^2 = 9$ at A & B and tangents at A & B to the second circle meet at point C, then the co–ordinates of C is

(A) (4, 5) (B) $\left(\frac{9}{15}, \frac{18}{5}\right)$ (C) (4, – 5) (D) $\left(\frac{9}{5}, \frac{18}{5}\right)$

4. A circle passes through point $\left(3, \sqrt{\frac{7}{2}}\right)$ touches the line pair $x^2 - y^2 - 2x + 1 = 0$. Centre of circle lies inside the circle $x^2 + y^2 - 8x + 10y + 15 = 0$. Co-ordinate of centre of circle is

(A) (4, 0) (B) (5, 0) (C) (6, 0) (D) (0, 4)

5. The length of the tangents from any point on the circle $15x^2 + 15y^2 - 48x + 64y = 0$ to the two circles $5x^2 + 5y^2 - 24x + 32y + 75 = 0$ and $5x^2 + 5y^2 - 48x + 64y + 300 = 0$ are in the ratio

(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 2 : 1

6. The distance between the chords of contact of tangents to the circle; $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin & the point (g, f) is:

(A) $\sqrt{g^2 + f^2}$ (B) $\frac{\sqrt{g^2 + f^2 - c}}{2}$ (C) $\frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}$ (D) $\frac{\sqrt{g^2 + f^2 + c}}{2\sqrt{g^2 + f^2}}$

7. If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$, then the angle between the tangents is:

(A) α (B) 2α (C) $\frac{\alpha}{2}$ (D) $\frac{\alpha}{3}$

8. The locus of the mid points of the chords of the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ which subtend an angle of $\frac{\pi}{3}$ radians at its circumference is:

(A) $(x-2)^2 + (y+3)^2 = 6.25$ (B) $(x+2)^2 + (y-3)^2 = 6.25$
(C) $(x+2)^2 + (y-3)^2 = 18.75$ (D) $(x+2)^2 + (y+3)^2 = 18.75$

9. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ & $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touch each other then :

(A) $f_1g_1 = f_2g_2$ (B) $\frac{f_1}{g_1} = \frac{f_2}{g_2}$ (C) $f_1f_2 = g_1g_2$ (D) $f_1 + f_2 = g_1 + g_2$

10. A circle touches a straight line $\ell x + my + n = 0$ & cuts the circle $x^2 + y^2 = 9$ orthogonally. The locus of centres of such circles is:

(A) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$ (B) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$
(C) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 + 9)$ (D) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$

11. The locus of the point at which two given unequal circles subtend equal angles is:
(A) a straight line (B) a circle (C) a parabola (D) an ellipse

12. A circle is given by $x^2 + (y-1)^2 = 1$. Another circle C touches it externally and also the x-axis, then the locus of its centre is
(A) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$ (B) $\{(x, y) : x^2 + (y-1)^2 = 4\} \cup \{(x, y) : y \le 0\}$
(C) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$ (D) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

13. The locus of the centre of a circle touching the circle $x^2 + y^2 - 4y - 2x = 4$ internally and tangent on which from (1, 2) is making a 60° angle with each other.
(A) $(x-1)^2 + (y-2)^2 = 2$ (B) $(x-1)^2 + (y-2)^2 = 4$
(C) $(x+1)^2 + (y-2)^2 = 4$ (D) $(x+1)^2 + (y+2)^2 = 4$

14. **STATEMENT-1 :** If three circles which are such that their centres are non-collinear, then exactly one circle exists which cuts the three circles orthogonally.
STATEMENT-2 : Radical axis for two intersecting circles is the common chord.

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true

15. The centre of family of circles cutting the family of circles $x^2 + y^2 + 4x\left(\lambda - \frac{3}{2}\right) + 3y\left(\lambda - \frac{4}{3}\right) - 6(\lambda + 2) = 0$ orthogonally, lies on

(A) $x - y - 1 = 0$(B) $4x + 3y - 6 = 0$ (C) $4x + 3y + 7 = 0$ (D) $3x - 4y - 1 = 0$

16. The circle $x^2 + y^2 = 4$ cuts the circle $x^2 + y^2 + 2x + 3y - 5 = 0$ in A & B. Then the equation of the circle on AB as a diameter is:
(A) $13(x^2 + y^2) - 4x - 6y - 50 = 0$ (B) $9(x^2 + y^2) + 8x - 4y + 25 = 0$
(C) $x^2 + y^2 - 5x + 2y + 72 = 0$ (D) $13(x^2 + y^2) - 4x - 6y + 50 = 0$

PART-II: NUMERICAL VALUE QUESTIONS

INSTRUCTION :

- The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

1. Find maximum number of points having integer coordinates (both x, y integer) which can lie on a circle with centre at $\left(\sqrt{2}, \sqrt{3}\right)$ is (are)

2. If equation of smallest circle touching the circles $x^2 + y^2 - 2y - 3 = 0$ and $x^2 + y^2 - 8x - 18y + 93 = 0$ is $x^2 + y^2 - 4x - fy + c = 0$ then value of f + c is

3. A line meets the co–ordinate axes in A and B. A circle is circumscribed about the triangle OAB. If d_1 and d_2 are the distances of the tangent to the circle at the origin O from the points A and B respectively and diameter of the circle is $\lambda_1 d_1 + \lambda_2 d_2$, then find the value of $\lambda_1 + \lambda_2$.

4. A circle is inscribed (i.e. touches all four sides) into a rhombous ABCD with one angle 60º. The distance from the centre of the circle to the nearest vertex is equal to 1. If P is any point of the circle, then $|PA|^2 + |PB|^2 + |PC|^2 + |PD|^2$ is equal to :

5. Let x & y be the real numbers satisfying the equation $x^2 - 4x + y^2 + 3 = 0$. If the maximum and minimum values of $x^2 + y^2$ are M & m respectively, then find the numerical value of (M + m).

6. Find absolute value of 'c' for which the set,
$\{(x, y) | x^2 + y^2 + 2x \le 1\} \cap \{(x, y) | 5x - 12y + c \ge 0\}$ contains only one point is common.

7. A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centres of the circles then area of the rhombus is

8. If (α, β) is a point on the circle whose centre is on the x-axis and which touches the line x + y = 0 at (2, –2), then find the greatest value of 'α' is

9. Two circles whose radii are equal to 4 and 8 intersect at right angles, then length of their common chord is

10. A variable circle passes through the point A (a, b) & touches the x–axis and the locus of the other end of the diameter through A is $(x - a)^2 = \lambda by$, then find the value of λ

11. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points B (1, 7) & D (4, – 2) on the circle meet at the point C. Find the area of the quadrilateral ABCD.

12. If the complete set of values of a for which the point (2a, a + 1) is an interior point of the larger segment of the circle $x^2 + y^2 - 2x - 2y - 8 = 0$ made by the chord whose equation is $3x - 4y + 5 = 0$ is (p,q) then value of p + q is

13. The circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q, then find the number of values of 'a' for which the line $5x + by - a = 0$ passes through P and Q.

14. The circumference of the circle $x^2 + y^2 - 2x + 8y - q = 0$ is bisected by the circle $x^2 + y^2 + 4x + 12y + p = 0$, then find p + q

15. A circle touches the line y = x at a point P such that OP = $4\sqrt{2}$ where O is the origin. The circle contains the point (–10, 2) in its interior and the length of its chord on the line x + y = 0 is $6\sqrt{2}$. If the equation of the circle $x^2 + y^2 + 2g x + 2fy + 3c = 0$, then value of g + f + c is

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. The equation of circles passing through (3, –6) touching both the axes is
(A) $x^2 + y^2 - 6x + 6y + 9 = 0$ (B) $x^2 + y^2 + 6x - 6y + 9 = 0$
(C) $x^2 + y^2 + 30x - 30y + 225 = 0$ (D) $x^2 + y^2 - 30x + 30y + 225 = 0$

2. Equations of circles which pass through the points (1, –2) and (3, – 4) and touch the x-axis is
(A) $x^2 + y^2 + 6x + 2y + 9 = 0$ (B) $x^2 + y^2 + 10x + 20y + 25 = 0$
(C) $x^2 + y^2 - 6x + 4y + 9 = 0$ (D) $x^2 + y^2 + 10x + 20y - 25 = 0$

3. The centre of a circle passing through the points (0, 0), (1, 0) & touching the circle $x^2 + y^2 = 9$ is :
(A) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \sqrt{2}\right)$ (C) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{1}{2}, -\sqrt{2}\right)$

4. The equation of the circle which touches both the axes and the line $\frac{x}{3} + \frac{y}{4} = 1$ and lies in the first quadrant is $(x - c)^2 + (y - c)^2 = c^2$ where c is
(A) 1 (B) 2 (C) 4 (D) 6

5. Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0$, $7x + y = 50$ & divide the circumference of the circle $x^2 + y^2 = 100$ into two arcs whose lengths are in the ratio 2 : 1.
(A) $3x - 4y - 25 = 0$ (B) $4x + 3y - 25 = 0$ (C) $4x - 3y - 25 = 0$ (D) $3x + 4y - 25 = 0$

6. Tangents are drawn to the circle $x^2 + y^2 = 50$ from a point 'P' lying on the x-axis. These tangents meet the y-axis at points 'P_1' and 'P_2'. Possible coordinates of 'P' so that area of triangle PP_1P_2 is minimum, is/are
(A) (10, 0) (B) $(10\sqrt{2}, 0)$ (C) (–10, 0) (D) $(-10\sqrt{2}, 0)$

7. If (a, 0) is a point on a diameter segment of the circle $x^2 + y^2 = 4$, then $x^2 - 4x - a^2 = 0$ has
(A) exactly one real root in (– 1, 0] (B) Exactly one real root in [2, 5]
(C) distinct roots greater than-1 (D) Distinct roots less than 5

8. The tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are perpendicular if
(A) h = r (B) h = – r (C) $r^2 + h^2 = 1$ (D) $r^2 = h^2$

9. The equation (s) of the tangent at the point (0, 0) to the circle where circle makes intercepts of length 2a and 2b units on the coordinate axes, is (are) -
(A) ax + by = 0 (B) ax – by = 0 (C) x = y (D) bx + ay = ab

10. Consider two circles $C_1 : x^2 + y^2 - 1 = 0$ and $C_2 : x^2 + y^2 - 2 = 0$. Let A(1,0) be a fixed point on the circle C_1 and B be any variable point on the circle C_2. The line BA meets the curve C_2 again at C. Which of the following alternative(s) is/are correct ?
(A) $OA^2 + OB^2 + BC^2 \in [7, 11]$, where O is the origin.
(B) $OA^2 + OB^2 + BC^2 \in [4, 7]$, where O is the origin.
(C) Locus of midpoint of AB is a circle of radius $\frac{1}{\sqrt{2}}$.
(D) Locus of midpoint of AB is a circle of area $\frac{\pi}{2}$.

11. One of the diameter of the circle circumscribing the rectangle ABCD is $x - 3y + 1 = 0$.
If two verticles of rectangle are the points (– 2, 5) and (6, 5) respectively, then which of the following hold(s) good?
(A) Area of rectangle ABCD is 64 square units.
(B) Centre of circle is (2, 1)
(C) The other two vertices of the rectangle are (– 2, – 3) and (6, – 3)
(D) Equation of sides are x = – 2, y = – 3, x = 5 and y = 6.

12. Three concentric circles of which the biggest is $x^2 + y^2 = 1$, have their radii in A.P. If the line y = x + 1 cuts all the circles in real and distinct points. The permissible values of common difference of A.P. is/are
(A) 0.4 (B) 0.6 (C) 0.01 (D) 0.1

13. If $4\ell^2 - 5m^2 + 6\ell + 1 = 0$. Prove that $\ell x + my + 1 = 0$ touches a definite circle, then which of the following is/are true.
(A) Centre (0, 3) (B) centre (3, 0) (C) Radius $\sqrt{5}$ (D) Radius 5

14. If the circle C_1: $x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the co–ordinates of the centre of C_2 are:
(A) $\left(\frac{9}{5}, \frac{12}{5}\right)$ (B) $\left(\frac{9}{5}, \frac{-12}{5}\right)$ (C) $\left(\frac{-9}{5}, \frac{-12}{5}\right)$ (D) $\left(\frac{-9}{5}, \frac{+12}{5}\right)$

15. For the circles $x^2 + y^2 - 10x + 16y + 89 - r^2 = 0$ and $x^2 + y^2 + 6x - 14y + 42 = 0$ which of the following is/are true.
(A) Number of integral values of r are 14 for which circles are intersecting.
(B) Number of integral values of r are 9 for which circles are intersecting.
(C) For r equal to 13 number of common tangents are 3.
(D) For r equal to 21 number of common tangents are 2.

16. Which of the following statement(s) is/are correct with respect to the circles $S_1 \equiv x^2 + y^2 - 4 = 0$ and $S_2 \equiv x^2 + y^2 - 2x - 4y + 4 = 0$?
(A) S_1 and S_2 intersect at an angle of 90°.
(B) The point of intersection of the two circle are (2, 0) and $\left(\frac{6}{5}, \frac{8}{5}\right)$.
(C) Length of the common of chord of S_1 and S_2 is $\frac{4}{\sqrt{5}}$.
(D) The point (2, 3) lies outside the circles S_1 and S_2.

17. Two circles, each of radius 5 units, touch each other at (1, 2). If the equation of their common tangent is $4x + 3y = 10$. The equations of the circles are
(A) $x^2 + y^2 + 6x + 2y - 15 = 0$ (B) $x^2 + y^2 - 10x - 10y + 25 = 0$
(C) $x^2 + y^2 - 6x + 2y - 15 = 0$ (D) $x^2 + y^2 - 10x + 10y + 25 = 0$

18. $x^2 + y^2 = a^2$ and $(x - 2a)^2 + y^2 = a^2$ are two equal circles touching each other. Find the equation of circle (or circles) of the same radius touching both the circles.
(A) $x^2 + y^2 + 2ax + 2\sqrt{3}\,ay + 3a^2 = 0$ (B) $x^2 + y^2 - 2ax + 2\sqrt{3}\,ay + 3a^2 = 0$
(C) $x^2 + y^2 + 2ax - 2\sqrt{3}\,ay + 3a^2 = 0$ (D) $x^2 + y^2 - 2ax - 2\sqrt{3}\,ay + 3a^2 = 0$

19. The circle $x^2 + y^2 - 2x - 3ky - 2 = 0$ passes through two fixed points, (k is the parameter)
(A) $(1 + \sqrt{3}, 0)$ (B) $(-1 + \sqrt{3}, 0)$ (C) $(-\sqrt{3} - 1, 0)$ (D) $(1 - \sqrt{3}, 0)$

20. Curves $ax^2 + 2hxy + by^2 - 2gx - 2fy + c = 0$ and $a'x^2 - 2hxy + (a' + a - b)y^2 - 2g'x - 2f'y + c = 0$ intersect at four concyclic point A, B, C and D. If P is the point $\left(\frac{g'+g}{a'+a}, \frac{f'+f}{a'+a}\right)$, then which of the following is/are true
(A) P is also concyclic with points A, B, C, D (B) PA, PB, PC in G.P.
(C) $PA^2 + PB^2 + PC^2 = 3PD^2$ (D) PA, PB, PC in A.P.

PART - IV : COMPREHENSION

Comprehension # 1 (Q. No. 1 to 3)

Let S_1, S_2, S_3 be the circles $x^2 + y^2 + 3x + 2y + 1 = 0$, $x^2 + y^2 - x + 6y + 5 = 0$ and $x^2 + y^2 + 5x - 8y + 15 = 0$, then

1. Point from which length of tangents to these three circles is same is
(A) (1, 0) (B) (3, 2) (C) (10, 5) (D) (– 2, 1)

2. Equation of circle S_4 which cut orthogonally to all given circle is
(A) $x^2 + y^2 - 6x + 4y - 14 = 0$ (B) $x^2 + y^2 + 6x + 4y - 14 = 0$
(C) $x^2 + y^2 - 6x - 4y + 14 = 0$ (D) $x^2 + y^2 - 6x - 4y - 14 = 0$

3. Radical centre of circles S_1, S_2, & S_4 is

(A) $\left(-\frac{3}{5},-\frac{8}{5}\right)$ (B) (3, 2) (C) (1, 0) (D) $\left(-\frac{4}{5},-\frac{3}{2}\right)$

Comprehension # 2 (Q. No. 4 to 6)

Two circles are $S_1 \equiv (x + 3)^2 + y^2 = 9$

$S_2 \equiv (x - 5)^2 + y^2 = 16$

with centres C_1 & C_2

4. A direct common tangent is drawn from a point P (on x-axis) which touches S_1 & S_2 at Q & R, respectively. Find the ratio of area of ΔPQC_1 & ΔPRC_2.

(A) 3 : 4 (B) 9 : 16 (C) 16 : 9 (D) 4 : 3

5. From point 'A' on S_2 which is nearest to C_1, a variable chord is drawn to S_1. The locus of mid point of the chord.

(A) circle (B) Diameter of s_1

(C) Arc of a circle (D) chord of s_1 but not diameter

6. Locus obtained in question 5 cuts the circle S_1 at B & C, then line segment BC subtends an angle on the major arc of circle S_1 is

(A) $\cos^{-1}\frac{3}{4}$ (B) $\frac{\pi}{2}-\tan^{-1}\frac{4}{3}$ (C) $\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{3}{4}$ (D) $\frac{\pi}{2}\cot^{-1}\left(\frac{4}{3}\right)$

Exercise-3

Marked questions are recommended for Revision.
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is
[**Note** : [k] denotes the largest integer less than or equal to k] **[IIT-JEE - 2010, Paper-2, (3, 0), 79]**

2. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point **[IIT-JEE 2011, Paper-2, (3, –1), 80]**

(A) $\left(-\frac{3}{2},0\right)$ (B) $\left(-\frac{5}{2},2\right)$ (C) $\left(-\frac{3}{2},\frac{5}{2}\right)$ (D) (–4, 0)

3. The straight line 2x – 3y = 1 divides the circular region $x^2 + y^2 \le 6$ into two parts.
If $S = \left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$, **[IIT-JEE 2011, Paper-2, (4, 0), 80]**
then the number of point(s) in S lying inside the smaller part is

4. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line 4x – 5y = 20 to the circle $x^2 + y^2 = 9$ is **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(A) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) + 20x - 45y = 0$

Paragraph for Question Nos. 5 to 6

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2 + y^2 = 1$. **[IIT-JEE 2012, Paper-2, (3, –1), 66]**

5. A common tangent of the two circles is
(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}y = 4$ (D) $x + 2\sqrt{3}y = 6$

6. A possible equation of L is
(A) $x - \sqrt{3}y = 1$ (B) $x + \sqrt{3}y = 1$ (C) $x - \sqrt{3}y = -1$ (D) $x + \sqrt{3}y = 5$

7*. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are) **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**
(A) $x^2 + y^2 - 6x + 8y + 9 = 0$ (B) $x^2 + y^2 - 6x + 7y + 9 = 0$
(C) $x^2 + y^2 - 6x - 8y + 9 = 0$ (D) $x^2 + y^2 - 6x - 7y + 9 = 0$

8*. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**
(A) radius of S is 8 (B) radius of S is 7
(C) centre of S is (–7, 1) (D) centre of S is (–8, 1)

9*. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then **[JEE (Advanced) 2016, Paper-1, (4, –2)/62]**
(A) $Q_2Q_3 = 12$ (B) $R_2R_3 = 4\sqrt{6}$
(C) area of the triangle OR_2R_3 is $6\sqrt{2}$ (D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

10*. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) **[JEE (Advanced) 2016, Paper-1, (4, –2)/62]**
(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

11. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points? **[JEE(Advanced) 2017, Paper-1,(3, 0)/61]**

PARAGRAPH "X" **[JEE(Advanced) 2018, Paper-1, (3, –1)/60]**
Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.
(***There are two questions based on PARAGRAPH "X", the question given below is one of them***)

12. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope –1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, then, the points E_3, F_3, and G_3 lie on the curve
(A) $x + y = 4$ (B) $(x - 4)^2 + (y - 4)^2 = 16$ (C) $(x - 4)(y - 4) = 4$ (D) $xy = 4$

13. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(A) $(x + y)^2 = 3xy$ (B) $x^{2/3} + y^{2/3} = 2^{4/3}$ (C) $x^2 + y^2 = 2xy$ (D) $x^2 + y^2 = x^2y^2$

14*. Let T be the line passing through the points P(–2, 7) and Q(2, –5). Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R(1, 1) be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE **[JEE(Advanced) 2018, Paper-2, (4, –2)/60]**

(A) The point (–2, 7) lies in E_1 (B) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie in E_2

(C) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2 (D) The point $\left(0, \frac{3}{2}\right)$ does **NOT** lie in E_1

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[AIEEE 2010, (4, –1), 144]**

(1) $-35 < m < 15$ (2) $15 < m < 65$ (3) $35 < m < 85$ (4) $-85 < m < -35$

2. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if : **[AIEEE-2011, I, (4, –1), 120]**

(1) $2|a| = c$ (2) $|a| = c$ (3) $a = 2c$ (4) $|a| = 2c$

3. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is - **[AIEEE-2011, II, (4, –1), 120]**

(1) $x^2 + y^2 - 2x - 2y + 1 = 0$ (2) $x^2 + y^2 - x - y = 0$
(3) $x^2 + y^2 + 2x + 2y - 7 = 0$ (4) $x^2 + y^2 + x + y - 2 = 0$

4. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is : **[AIEEE- 2012, (4, –1), 120]**

(1) $\frac{10}{3}$ (2) $\frac{3}{5}$ (3) $\frac{6}{5}$ (4) $\frac{5}{3}$

5. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point **[AIEEE - 2013, (4, –1),120]**

(1) (–5, 2) (2) (2, – 5) (3) (5, – 2) (4) (–2, 5)

6. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{\sqrt{3}}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

7. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k (x - 2y + 3) = 0$, $k \in R$, is a **[JEE(Main) 2015, (4, – 1), 120]**

(1) straight line parallel to x-axis (2) straight line parallel to y-axis
(3) circle of radius $\sqrt{2}$ (4) circle of radius $\sqrt{3}$

8. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is **[JEE(Main) 2015, (4, – 1), 120]**

(1) 1 (2) 2 (3) 3 (4) 4

9. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on : **[JEE(Main) 2016, (4, – 1), 120]**

(1) an ellipse which is not a circle (2) a hyperbola
(3) a parabola (4) a circle

10. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (– 3, 2), then the radius of S is : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $5\sqrt{3}$ (2) 5 (3) 10 (4) $5\sqrt{2}$

11. Let the orthocenter and centroid of a triangle be A (–3, 5) and B(3,3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segement AC as diameter , is : **[JEE(Main) 2018, (4, – 1), 120]**

(1) $3\sqrt{\frac{5}{2}}$ (2) $\frac{3\sqrt{5}}{2}$ (3) $\sqrt{10}$ (4) $2\sqrt{10}$

12. Three circles of radii, a, b, c (a < b < c) touch each other externally, If they have x-axis as a common tangent, then : **[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]**

(1) a, b, c are in A.P. (2) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$ (3) $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$ are in A.P. (4) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

13. If a circle C passing through the point (4,0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is: **[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]**

(1) $2\sqrt{5}$ (2) $\sqrt{57}$ (3) 4 (4) 5

14. If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval:

[JEE(Main) 2019, Online (12-01-19),P-1 (4, – 1), 120]

(1) (2, 17) (2) [12, 21] (3) [13, 23] (4) (23,31)

15. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is -

[JEE(Main) 2019, Online (12-01-19),P-2 (4, – 1), 120]

(1) $(x^2+y^2)(x + y) = R^2xy$ (2) $(x^2 + y^2)^3 = 4R^2x^2y^2$

(3) $(x^2 + y^2)^2 = 4Rx^2y^2$ (4) $(x^2 + y^2)^2 = 4R^2x^2y^2$

16. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is :

[JEE(Main) 2019, Online (08-04-19),P-1 (4, – 1), 120]

(1) 105 (2) 210 (3) 320 (4) 160

17. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is : **[JEE(Main) 2019, Online (09-04-19),P-1 (4, – 1), 120]**

(1) $x^2 + y^2 - 4x^2y^2 = 0$ (2) $x^2 + y^2 - 16x^2y^2 = 0$ (3) $x^2 + y^2 - 2x^2y^2 = 0$ (4) $x^2 + y^2 - 2xy = 0$

18. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is : **[JEE(Main) 2019, Online (10-04-19),P-2 (4, – 1), 120]**

(1) $x = \sqrt{1+4y}, y \geq 0$ (2) $y = \sqrt{1+4x}, x \geq 0$ (3) $x = \sqrt{1+2y}, y \geq 0$ (4) $y = \sqrt{1+2x}, x \geq 0$

19. If a line, $y = mx + c$ is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then :

[JEE(Main) 2020, Online (08-01-20),P-2 (4, –1), 120]

(1) $c^2 + 7c + 6 = 0$ (2) $c^2 + 6c + 7 = 0$ (3) $c^2 - 6c + 7 = 0$ (4) $c^2 - 7c + 6 = 0$

20. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, (k > 0) touch each other at a point, then the largest value of k is ________ **[JEE(Main) 2020, Online (09-01-20),P-2 (4, 0), 120]**

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. $x^2 + y^2 = 1$

A-3. $x^2 + y^2 - 3x - 4y = 0$

A-4. $x^2 + y^2 - 4x - 4y + 4 = 0$

A-5. $x^2 + y^2 \pm 6\sqrt{2}y \pm 6x + 9 = 0$

A-6. $(x + 3)^2 + (y - 4)^2 = 4$

A-7. (36, 47)

Section (B) :

B-1. 2

B-2. (1, 3), (5, 7), $4\sqrt{2}$

B-3. $x - 7y - 45 = 0$

B-4. $\sqrt{3}x - y \pm 4 = 0$

B-5. $16x^2 - 65y^2 - 288x + 1296 = 0$, $\tan^{-1}\left(\frac{8\sqrt{65}}{49}\right)$

B-6. Yes

Section (C) :

C-1. $2x - y = 0$

C-2. $x + 2y - 1 = 0$

C-3. $(x + 4)^2 + y^2 = 16$

C-5. $x + y + 5 = 0$

C-6. $\left(6, -\frac{18}{5}\right)$

Section (D) :

D-1. $x = 0$, $3x + 4y = 10$, $y = 4$ and $3y = 4x$.

D-3. $2(x^2 + y^2) - 7x + 2y = 0$

D-4. $\left(\frac{33}{4}, 2\right)$; $\frac{1}{4}$

Section (E) :

E-1. $x^2 + y^2 - 2x - 4y = 0$.

E-2. (i) $(x - 1)^2 + (y + 2)^2 + 20\,(2x - y - 4) = 0$
(ii) $(x - 1)^2 + (y + 2)^2 \pm \sqrt{20}\,(2x - y - 4) = 0$

E-4. $\left(\frac{52}{3}, -\frac{23}{9}\right)$

E-5. $x^2 + y^2 - 17x - 19y + 50 = 0$

PART - II

Section (A) :

A-1.	(D)	**A-2.**	(A)	**A-3.**	(B)	**A-4.**	(C)	**A-5.**	(D)	**A-6.**	(B)	**A-7.**	(A)
A-8.	(C)												

Section (B) :

B-1.	(A)	**B-2.**	(B)	**B-3.**	(B)	**B-4.**	(B)	**B-5.**	(A)	**B-6.**	(B)	**B-7.**	(A)
B-8.	(D)	**B-9.**	(C)	**B-10.**	(A)	**B-11.**	(C)	**B-12.**	(A)	**B-13.**	(B)	**B-14.**	(A)

Section (C) :

C-1.	(A)	**C-2.**	(B)	**C-3.**	(A)	**C-4.**	(C)	**C-5.**	(B)	**C-6.**	(A)	**C-7.**	(A)

Section (D) :

D-1.	(B)	**D-2.**	(B)	**D-3.**	(A)	**D-4.**	(A)

Section (E) :

E-1.	(A)	**E-2.**	(B)	**E-3.**	(A)	**E-4.**	(A)	**E-5.**	(D)	**E-6.**	(A)

PART - III

1. (A) $\to$ (q),(B) $\to$ (p), (C) $\to$ (r), (D) $\to$ (s)

2. (A) $\to$ (s), (B) $\to$ (r), (C) $\to$ (q), (D) $\to$ (p)

EXERCISE - 2

PART - I

1.	(C)	**2.**	(B)	**3.**	(D)	**4.**	(A)	**5.**	(A)	**6.**	(C)	**7.**	(B)
8.	(B)	**9.**	(B)	**10.**	(A)	**11.**	(B)	**12.**	(D)	**13.**	(B)	**14.**	(D)
15.	(B)	**16.**	(A)										

PART - II

1.	01.00	**2.**	32.88 or 32.89	**3.**	02.00	**4.**	11.00	**5.**	10.00	**6.**	13.38
7.	13.85 or 13.86			**8.**	06.82 or 06.83			**9.**	07.15	**10.**	04.00
11.	75.00	**12.**	01.30	**13.**	00.00	**14.**	10.00	**15.**	18.66 or 18.67		

PART - III

1.	(AD)	**2.**	(BC)	**3.**	(BD)	**4.**	(AD)	**5.**	(CD)	**6.**	(AC)
7.	(ABCD)	**8.**	(ABD)	**9.**	(AB)	**10.**	(ACD)	**11.**	(ABC)	**12.**	(CD)
13.	(BC)	**14.**	(BD)	**15.**	(AC)	**16.**	(ACD)	**17.**	(AB)	**18.**	(BD)
19.	(AD)	**20.**	(BCD)								

PART - IV

1.	(B)	**2.**	(D)	**3.**	(A)	**4.**	(B)	**5.**	(C)	**6.**	(A)

EXERCISE - 3

PART - I

1.	3	**2.**	(D)	**3.**	2	**4.**	(A)	**5.**	(D)	**6.**	(A)	**7.**	(AC)
8.	(BC)	**9.**	(ABC)	**10.**	(A,C)	**11.**	(2)	**12.**	(A)	**13.**	(D)	**14.**	(BD)

PART - II

1.	(1)	**2.**	(2)	**3.**	(2)	**4.**	(1)	**5.**	(3)	**6.**	(2)	**7.**	(3)
8.	(3)	**9.**	(3)	**10.**	(1)	**11.**	(1)	**12.**	(2)	**13.**	(4)	**14.**	(2)
15.	(2)	**16.**	(2)	**17.**	(1)	**18.**	(4)	**19.**	(2)	**20.**	36		

High Level Problems (HLP)

✍ **Marked Questions may have for Revision Questions.**

SUBJECTIVE QUESTIONS

1. Find the equation of the circle passing through the points A(4, 3), B(2, 5) and touching the axis of y. Also find the point P on the y–axis such that the angle APB has largest magnitude.

2. Let a circle be given by $2x(x - a) + y(2y - b) = 0$, $(a \neq 0, b \neq 0)$. Find the condition on a & b if two chords, each bisected by the x–axis, can be drawn to the circle from $\left(a, \frac{b}{2}\right)$

3. A circle is described to pass through the origin and to touch the lines x = 1, x + y = 2. Prove that the radius of the circle is a root of the equation $\left(3 - 2\sqrt{2}\right) t^2 - 2\sqrt{2}\, t + 2 = 0$.

4. If (a, α) lies inside the circle $x^2 + y^2 = 9 : x^2 - 4x - a^2 = 0$ has exactly one root in $(-1, 0)$, then find the area of the region in which (a, α) lies.

5. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a given circle. Find the locus of the foot of the perpendicular drawn from the origin upon any chord of S which subtends right angle at the origin.

6. A ball moving around the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ in anti–clockwise direction leaves it tangentially at the point P(–2, –2). After getting reflected from a straight line it passes through the centre of the circle. Find the equation of this straight line if its perpendicular distance from P is $\frac{5}{2}$. You can assume that the angle of incidence is equal to the angle of reflection.

7. The lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6 unit. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts of intercepts of length 8 on these lines.

8. The chord of contact of tangents drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Show that a, b, c are in G.P.

9. Find the locus of the middle points of chords of a given circle $x^2 + y^2 = a^2$ which subtend a right angle at the fixed point (p, q).

10. Let $a\ell^2 - bm^2 + 2d\ell + 1 = 0$, where a, b, d are fixed real numbers such that $a + b = d^2$. If the line $\ell x + my + 1 = 0$ touches a fixed circle then find the equation of circle

11. The centre of the circle S = 0 lies on the line $2x - 2y + 9 = 0$ and S = 0 cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle S = 0 passes through two fixed points and also find their co–ordinates.

12. Prove that the two circles which pass through the points (0, a), (0, – a) and touch the straight line y = m x + c will cut orthogonaly if $c^2 = a^2(2 + m^2)$.

13. Consider points A $(\sqrt{13}, 0)$ and B $(2\sqrt{13}, 0)$ lying on x-axis. These points are rotated in an-anticlockwise direction about the origin through an angle of $\tan^{-1}\left(\frac{2}{3}\right)$. Let the new position of A and B be A′ and B′ respectively. With A′ as centre and radius $\frac{2\sqrt{13}}{3}$ a circle C_1 is drawn and with B′ as a centre and radius $\frac{\sqrt{13}}{3}$ circle C_2 is drawn. Find radical axis of C_1 and C_2.

14. P(a, b) is a point in the first quadrant. If the two circles which pass through P and touch both the co–ordinate axes cut at right angles, then find condition in a and b.

15. Prove that the square of the tangent that can be drawn from any point on one circle to another circle is equal to twice the product of perpendicular distance of the point from the radical axis of two circles and distance between their centres.

16. Find the equation of the circle which cuts each of the circles, $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 10 = 0$ & $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter.

17. Show that if one of the circle $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2g_1x + c = 0$ lies within the other, then gg_1 and c are both positive.

18. Let ABCD is a rectangle. Incircle of ΔABD touches BD at E. Incircle of ΔCBD toches BD at F.

If AB = 8 units, and BC = 6 units, then find length of EF.

19. Let circles S_1 and S_2 of radii r_1 and r_2 respectively ($r_1 > r_2$) touches each other externally. Circle S radii r touches S_1 and S_2 externally and also their direct common tangent. Prove that the triangle formed by joining centre of S_1, S_2 and S is obtuse angled triangle.

20. Circles are drawn passing through the origin O to intersect the coordinate axes at point P and Q such that m. OP + n. OQ is a constant. Show that the circles pass through a fixed point.

21. A triangle has two of its sides along the axes, its third side touches the circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0$. Find the equation of the locus of the circumcentre of the triangle.

22. Let S_1 be a circle passing through A(0, 1), B(–2, 2) and S_2 is a circle of radius $\sqrt{10}$ units such that AB is common chord of S_1 and S_2. Find the equation of S_2.

23. The curves whose equations are
$S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
$S' = a'x^2 + 2h'xy + b'y^2 + 2g'x + 2f'y + c' = 0$
intersect in four concyclic points then find relation in a, b, h, a′, b′, h′.

24. A circle of constant radius 'r' passes through origin O and cuts the axes of coordinates in points P and Q, then find the equation of the locus of the foot of perpendicular from O to PQ.

25. The ends A , B of a fixed straight line of length 'a' and ends A′ and B′ of another fixed straight line of length 'b' slide upon the axis of X & the axis of Y (one end on axis of X & the other on axis of Y). Find the locus of the centre of the circle passing through A, B, A′ and B′.

Answers

1. $x^2 + y^2 - 4x - 6y + 9 = 0$ OR $x^2 + y^2 - 20x - 22y + 121 = 0$, P(0, 3), $\theta = 45°$

2. $(a^2 > 2b^2)$

4. $4\left\{\sqrt{5} + \frac{9}{2}\cot^{-1}\left(\frac{2}{\sqrt{5}}\right)\right\}$

5. $x^2 + y^2 + gx + fy + \frac{c}{2} = 0$

6. $(4\sqrt{3} - 3)x - (4 + 3\sqrt{3})y - (39 - 2\sqrt{3}) = 0$

7. $x^2 + y^2 - 10x - 4y + 4 = 0$

9. $2x^2 + 2y^2 - 2px - 2qy + p^2 + q^2 - a^2 = 0$

10. $x^2 + y^2 - 2dx + d^2 - b = 0$

11. $(-4, 4)$; $\left(-\frac{1}{2}, \frac{1}{2}\right)$

13. $9x + 6y = 65$

14. $a^2 - 4ab + b^2 = 0$

16. $x^2 + y^2 - 4x - 6y - 4 = 0$

18. 2

21. $2(x + y) - a = \frac{2xy}{a}$

22. $x^2 + y^2 + 2x - 3y + 2 \pm \sqrt{7}(x + 2y - 2) = 0$

23. $\frac{a-b}{h} = \frac{a'-b'}{h'}$

24. $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = 4r^2$

25. $(2ax - 2by)^2 + (2bx - 2ay)^2 = (a^2 - b^2)^2$