Chapter

5 Oscillations

Quick Review

Oscillations

- **Periodic**: A fixed time period
- **Non-Periodic**: No fixed time period

➢ Linear Simple Harmonic Motion:

Linear S.H.M.

In a linear S.H.M., the force is directed towards the mean position and its magnitude is directly proportional to the displacement of the body from mean position.

Differential Equation

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

OR

$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

Expressions for x, v and a

Displacement, $x = A \sin\omega t$

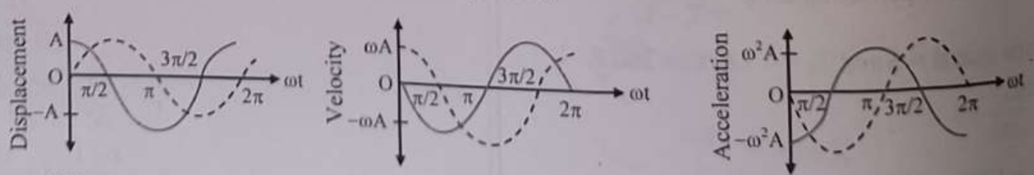
Velocity, $v = \omega\sqrt{A^2 - x^2}$

Acceleration, $a = \omega^2 x$

Graphical representation of S.H.M.

- - - - - Particles starts from mean position

——— Particles starts from extreme position

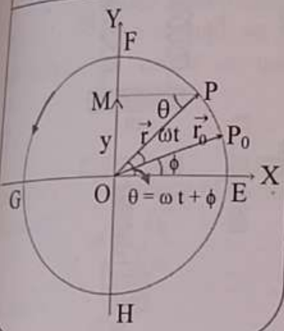


Phase difference between:

Velocity and displacement: $\frac{\pi}{2}$ Velocity and acceleration: $\frac{\pi}{2}$ Displacement and acceleration: π

Linear S.H.M.

Projection of U.C.M.



Phase in S.H.M.

- Phase in S.H.M. is the state of oscillation.
- In S.H.M. the angular displacement θ is used as phase.
- The initial phase of particle i.e., phase at time t = 0 is called **epoch.**

Energy

- Total energy of the particle performing an S.H.M. is the sum of its kinetic and potential energies.
- K.E. is maximum at mean position and minimum at extreme position.
- P.E. is maximum at extreme position and minimum at mean position.

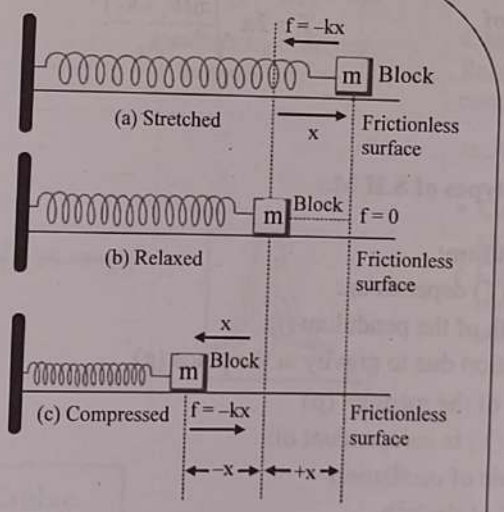
Spring-mass Oscillator:

- The force acting on a particle to bring it back to the original position is known as restoring force.
- When a block is connected to a spring, the spring exerts a restoring force ($f = -kx$) on the block on account of its elastic properties.
- Due to this restoring force, the block performs a linear simple harmonic motion.
- **Conditions for simple harmonic motion:**

i. Oscillation of the particle is always about a fixed point.

ii. The net force or acceleration is always directed towards the fixed point.

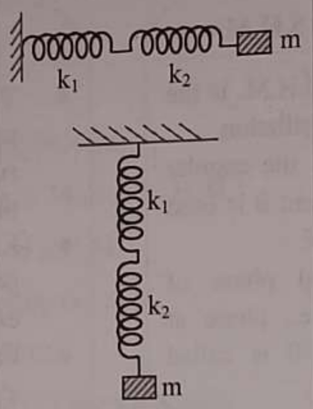
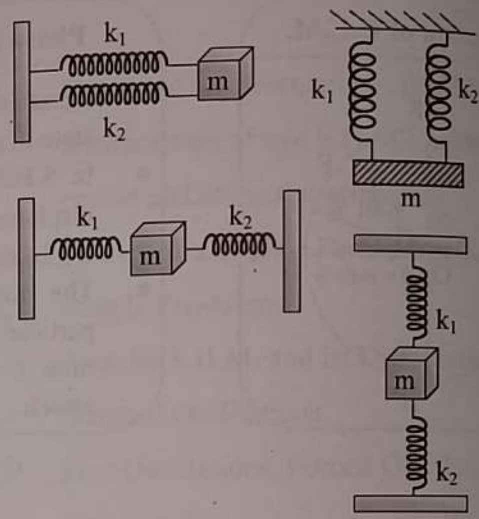
iii. The particle comes back to the fixed point due to restoring force.



Spring mass oscillator

[Students can scan the Q.R. code in *Quill - The Padhai App* to visualize **the Oscillations of Spring mass oscillator.**]

➤ Combination of springs:

Combination	Series Combination	Parallel combination
Diagram		
Effective spring constant for two springs	$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$	$k = k_1 + k_2$
Effective spring constant for n springs of same k	$\frac{k}{2}$	$k(2^n - 1)$
Time period of oscillations	$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$	$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

➤ Various types of S.H.M.:

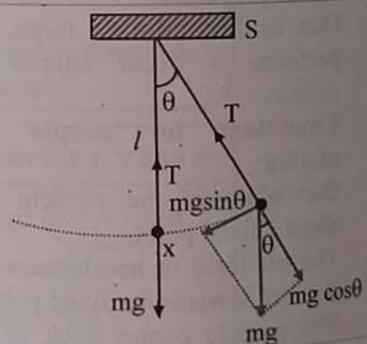
Simple pendulum:

Time period (T) depends on:

i. the length of the pendulum (l)
ii. acceleration due to gravity at the place (g)
iii. Density of the medium (ρ)

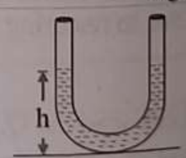
Time period (T) is independent of:

i. amplitude of oscillation
ii. material of the bob



S.H.M of a liquid in U- shaped tube:

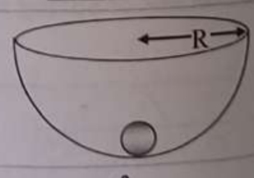
The period of oscillation of liquid is independent of the density ρ of the liquid and the cross-sectional area A of the U –tube in which liquid is placed.



S.H.M of a small ball rolling down in hemispherical bowl :

$T = 2\pi\sqrt{\frac{R - r}{g}}$

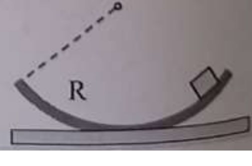
Where, R is Radius of the bowl, r is Radius of the ball.



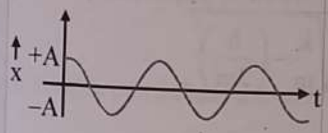
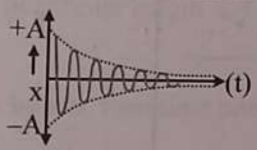
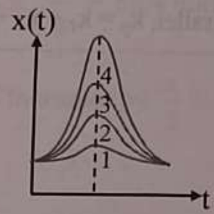
S.H.M. of a block down a circular (concave) track:

$T = 2\pi\sqrt{\frac{R}{g}}$

Where, R = radius of circular track



- **Free, Damped and Forced S.H.M.:**

	Free Oscillation		**Forced Oscillation**	
	Undamped Oscillation	**Damped Oscillation**		
Definition	The oscillations of a body whose amplitude remains same throughout the time are called as undamped oscillations.	The oscillations of a body whose amplitude goes on decreasing with time (due to presence of dissipative forces) are called as damped oscillations.	The oscillations of a particle with fundamental frequency under the influence of restoring force are called as free oscillations.	
Angular frequency	Frequency of oscillation $\omega = \sqrt{\frac{k}{m}}$.	Frequency of oscillation $\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$.	The oscillation has the frequency of driving force and not the natural frequency.	
Equation	$\frac{d^2x}{dt^2} + \omega^2 x = 0$, where ω is angular frequency.	$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$, where b is damping constant.	$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos\omega_d t$, where $F_0 \cos\omega_d t$ is driving force or external periodic force.	
Displacement	$x = A \sin(\omega t + \alpha)$	$x = Ae^{\frac{-bt}{2m}} \sin(\omega' t + \phi)$	$x = A\cos(\omega_d t + \phi)$	
Amplitude	Amplitude remains same	Amplitude decreases continuously with time according to, $x = Ae^{-(b/2m)t}$	Amplitude of the oscillation is given by $A = \frac{F_0}{\left[m^2\left(\omega^2 - \omega_d^2\right)^2 + \omega_d^2 b^2\right]^{1/2}}$ where ω_d is driven frequency.	**Case: I** Small damping: $\omega_d b << m(\omega^2 - \omega_d^2)$ $A = \frac{F_0}{m\left(\omega^2 - \omega_d^2\right)}$ **Case II:** Resonance condition: $\omega_d \approx \omega$ $\Rightarrow A = \frac{F_0}{\omega_d b}$
Diagrams				• Curve 1 shows small damping • Curve 4 shows resonance condition

Formulae

1. Restoring force: $f = -kx = -m\omega^2 x = -\frac{m4\pi^2}{T^2} x$

2. Differential equation of a linear S.H.M:
$\frac{d^2x}{dt^2} + \omega^2 x = 0$

3. Displacement in S.H.M:
General equation: $x = A \sin(\omega t + \alpha)$
$x = A \sin \omega t$ (from mean position, $\alpha = 0$)
$x = A \cos \omega t$ (from extreme position, $\alpha = \frac{\pi}{2}$)

4. Velocity in linear S.H.M:
$v = \pm \omega \sqrt{A^2 - x^2}$

5. Acceleration in linear S.H.M:
$a = -\omega^2 x$

6. Period in S.H.M:
$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

7. Frequency in S.H.M:
$n = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

8. Energy in S.H.M:
i. Potential energy: P.E $=\frac{1}{2}kx^2=\frac{1}{2}m\omega^2x^2$
ii. Kinetic energy:
K.E $=\frac{1}{2}k(A^2-x^2)=\frac{1}{2}m\omega^2(A^2-x^2)$
iii. Total energy:
T.E $=\frac{1}{2}kA^2=\frac{1}{2}m\omega^2A^2=2m\pi^2n^2A^2$

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9. Composition of S.H.M's:
i. Resultant equation of two S.H.M.s
$x_1 = A_1\sin(\omega t+\phi_1)$ and $x_2 = A_2\sin(\omega t+\phi_2)$
is given by, $x = R\sin(\omega t+\phi)$
ii. Resultant amplitude:
$R=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\phi_1-\phi_2)}$
iii. Resultant phase: $\delta=\tan^{-1}\left[\frac{A_1\sin\phi_1+A_2\sin\phi_2}{A_1\cos\phi_1+A_2\cos\phi_2}\right]$
where,
A_1 and A_2 are amplitudes of two S.H.M.s
ϕ_1 and ϕ_2 are initial phases of two S.H.M.s
$(\phi_1-\phi_2)$ = Phase difference between two S.H.M.s

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10. Oscillating spring:
i. Force, $F = mg = -kx$
ii. Period, $T=2\pi\sqrt{\frac{m}{k}}$
iii. When connected in series, $\frac{1}{k_s}=\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}+....$
iv. When connected in parallel, $k_p = k_1 + k_2 + k_3 + ...$

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11. Simple pendulum:
i. Period, $T=2\pi\sqrt{\frac{l}{g}}$
ii. For seconds pendulum, $l=\frac{g}{\pi^2}$
iii. At a given place, $\frac{l_1}{T_1^2}=\frac{l_2}{T_2^2}$

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12. Differential equation for angular S.H.M.:
$I\frac{d^2\theta}{dt^2}+c\theta=0$

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13. Magnet vibrating in uniform magnetic field:
i. Time period, $T=2\pi\sqrt{\frac{I}{\mu B}}$
ii. Angular acceleration, $\alpha=-\left(\frac{\mu B}{I}\right)\theta$

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14. Damped force: $F_d = bv$ (In magnitude)

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15. Equation of damped S.H.M:
$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$

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16. Amplitude of damped oscillation:
$A_d = Ae^{-bt/2m}$

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17. Angular frequency of damped oscillation:
$\omega'=\sqrt{\frac{k}{m}-\left(\frac{b}{2m}\right)^2}$

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18. Time period of damped oscillation:
$T=\frac{2\pi}{\sqrt{\frac{k}{m}-\left(\frac{b}{2m}\right)^2}}$

Shortcuts

1. For a particle executing S.H.M:
i. From mean position in order to travel half of amplitude, time required is given by, $t=\frac{T}{12}$
ii. From extreme position, in order to travel half of amplitude, time required is given by, $t=\frac{T}{6}$ s
2. Displacement of a particle in $\left(\frac{1}{8}\right)^{th}$ of periodic time is 0.7 A i.e. 70% of the amplitude.
3. The work done by simple pendulum in one complete oscillation is zero.
4. A particle, while crossing the mean position with maximum velocity, will lose only 50% of its velocity after travelling the 86% of the amplitude of vibration in one direction.
5. If two masses m_1 and m_2 are connected by a spring, then the time period is given by
$T=2\pi\sqrt{\frac{\mu}{k}}$ (where μ = induced mass $=\frac{m_1m_2}{m_1+m_2}$)

6. If two springs are in parallel, $T = 2\pi\sqrt{\frac{m}{(k_1+k_2)}}$ Where $k_1 + k_2$ is effective spring constant.

7. If two springs are in series, $T = 2\pi\sqrt{m\left(\frac{k_1+k_2}{k_1k_2}\right)}$ where $\frac{k_1k_2}{k_1+k_2}$ is effective spring constant.

8. If the length of spring is made n times, then spring constant will become $\frac{1}{n}$ times. The time period will become $\sqrt{n}$ times.

9. In a spring mass system, if spring is cut into n parts, force constant of each part will be nk.

10. When a spring constant k and of length l is cut into two pieces of length l_1 and l_2 such that $l_2 = nl_1$ where n is a whole number, then spring constant of length l_1 is $\frac{k(n+1)}{n}$ and of length l_2 is $(n + 1)k$.

11. If a spring is not massless and m' is the mass of the spring, then the formula will be,
$T = 2\pi\sqrt{\frac{m+m'/3}{k}}$

12. If a body is connected to a spring and it rolls, then the time period of its S.H.M is given by $T = 2\pi\sqrt{\frac{m}{k}\left(1+\frac{K^2}{r^2}\right)}$ where K is the radius of gyration of the body about an axis passing through its centre of mass.

13. If T_1 and T_2 are the time periods of a body oscillating under the restoring forces F_1 and F_2, then the time period of a body oscillating under the resultant of F_1 and F_2 will be $T = \frac{T_1T_2}{T_1+T_2}$.

14. The time period of a simple pendulum of infinite length is $T = 2\pi\sqrt{\frac{R}{g}} = 86.4$ minutes.

15. If length of simple pendulum increases by x%, then time period increases by $\frac{x}{2}$ %.

16. If g decreases by y% then time period will increase by $\frac{y}{2}$ %.

17. The percentage change in time period of a simple pendulum, when its length changes, is
$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\left(\frac{\Delta l}{l}\right) \times 100\%$

18. The average value of K.E. or P.E. with respect to time is $\frac{1}{4}m\omega^2A^2$.

19. When angular amplitude θ is very large, then $T = 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{\theta^2}{16}\right)$

20. If T_1 and T_2 are the time periods of two bodies starting at the same time, then time t after which they will again be in the same phase is given by $\frac{1}{t} = \frac{1}{T_1} - \frac{1}{T_2}$
where $\frac{1}{T_1} > \frac{1}{T_2}$ i.e. $T_1 < T_2$

21. Work done in producing angular displacement θ to the pendulum from its mean position is given by
$W = E_p = mgl(1 - \cos\theta)$