Report on Application of Linear Algebra In Eigenvalues and Eigenvectors.

Submitted by

18h052-nirav madani

18h060-vedant parekh

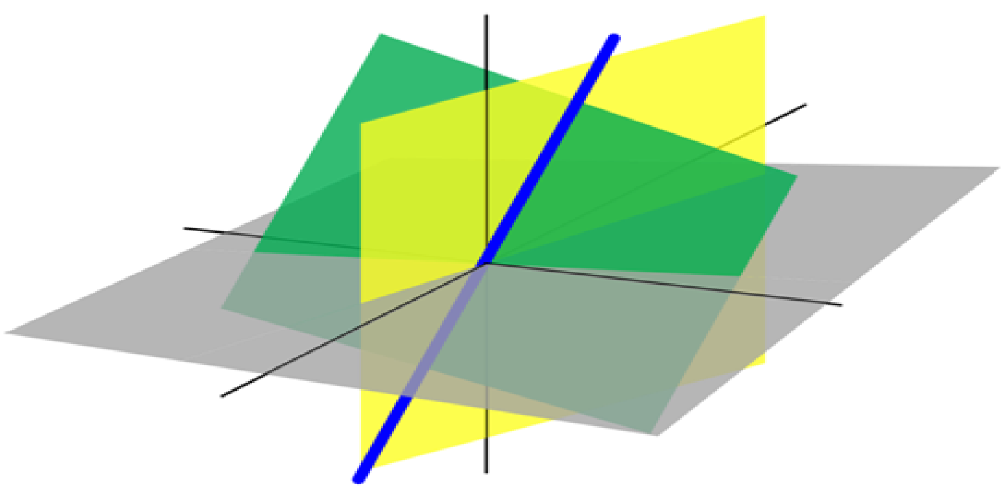
Submitted to

Prof. Amit Mishra

Department of Mathematics and Humanity

ITNU

Contents



Why linear algebra is required in real world?

1.To understand geometrical meaning instead of numerical meaning to the system.

2.Here,the system means traffic solution,machine learning,optimizations etc..

Real life applications of linear algebra.

1.Machine learning.
2.Data sciences.
3.Traffic analysis.
4.Quantum mechanics.

Eigenvalues and eigenvectors.

What are EigenVectors?

Let A be any vector, almost all the vectors change their direction when they are multiplied by vector A.But there are certain exceptional vectors that do not change their direction when multiplied by A, these are called Eigenvectors of A.

Mathematically $A X = \lambda X$, where X is eigenvector of A and λ is its EigenValue.λ can be positive, negative or 0!

Introduction to Quantum mechanics.

- The wave function for a given physical system contains the measurable information about the system. To obtain specific values for physical parameters, for example energy, you operate on the wave function with the quantum mechanical operator associated with that parameter. The operator associated with energy is the ***Hamiltonian***, and the operation on the wavefunction is the Schrodinger equation. Solutions exist for the time independent Schrodinger equation only for certain values of energy, and these values are called "eigenvalues*" of energy.

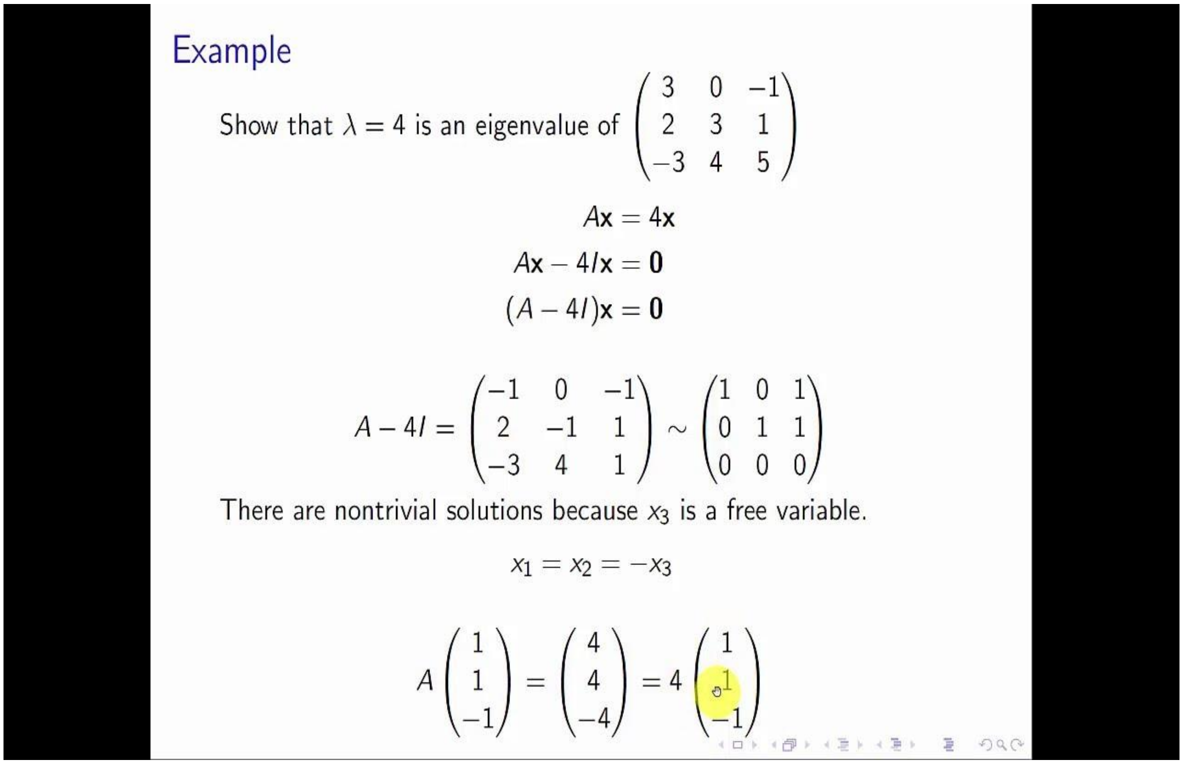
- Corresponding to each eigenvalue is an "eigenfunction*". The solution to the Schrodinger equation for a given energy Ei involves also finding the specific function $\boldsymbol{\Psi}$i which describes that energy state. The solution of the time independent Schrodinger equation takes the form.

- $H_{op}\boldsymbol{\Psi}_i = E_i\boldsymbol{\Psi}_i$

- The eigenvalue concept is not limited to energy. When applied to a general operator Q, it can take the form

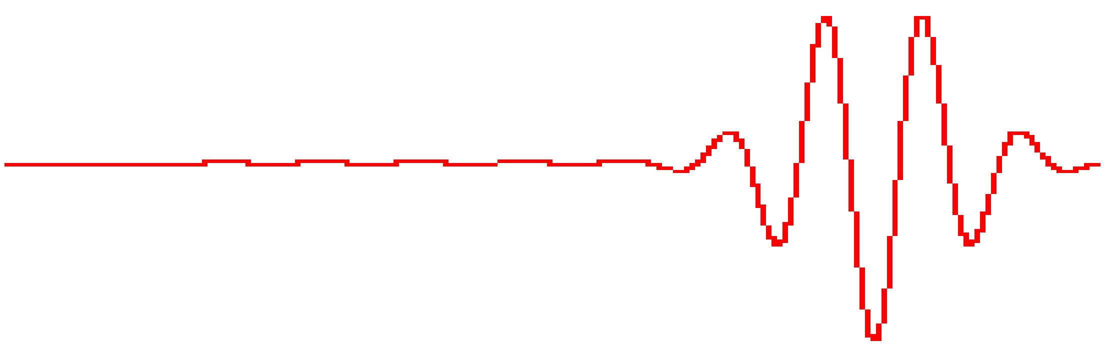
- $Q_{op}\boldsymbol{\Psi}_i = q_i\boldsymbol{\Psi}_i$

- Fun Fact :"Eigenvalue" comes from the German "Eigenwert" which means ***proper or characteristic*** value. "Eigenfunction" is from "Eigenfunktion" meaning "proper or characteristic function".



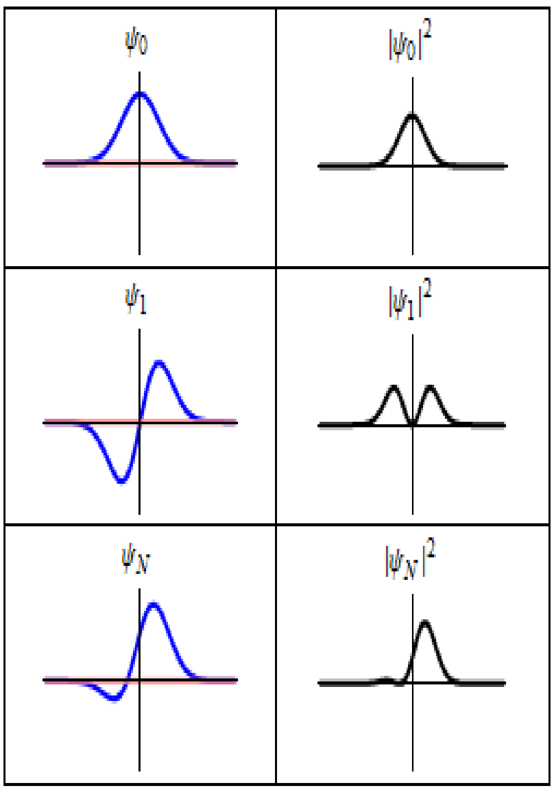
Visualizing with EigenFunctions

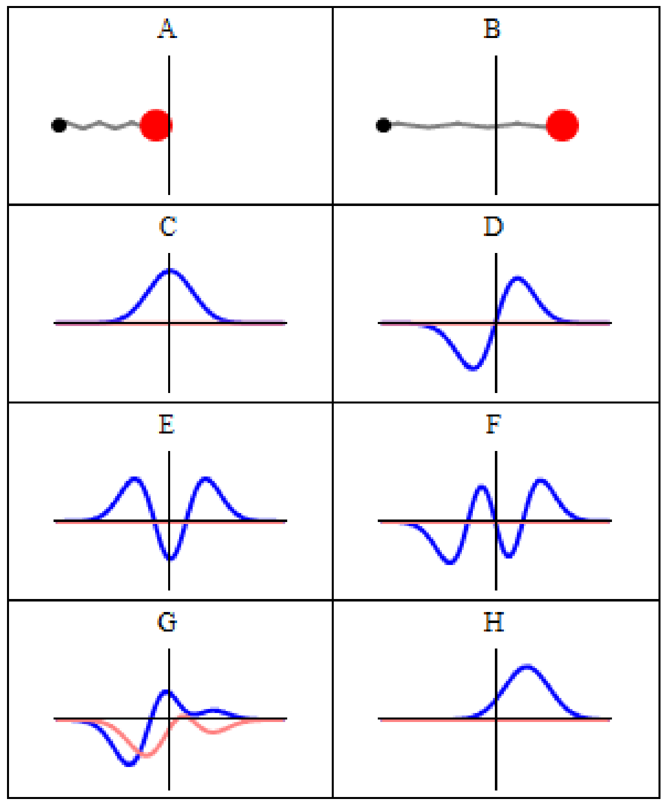
Time-dependent Schrödinger equation :A wave function that satisfies the nonrelativistic Schrödinger equation with V = 0. In other words, this corresponds to a particle traveling freely through empty space. The real part of the wave function is plotted here.



Three wavefunction solutions to the Time-Dependent

Schrödinger equation for a harmonic oscillator. Left: The real part (blue) and imaginary part (red) of the wavefunction. Right: The probability of finding the particle at a certain position. The top two rows are the lowest two energy eigenstates, and the bottom is the superposition state , which is not an energy eigenstate. The right column illustrates why energy eigenstatesare also called "stationary states".





A harmonic oscillator in classical mechanics (A-B) and quantum mechanics (C-H). In (A-B), a ball, attached to a spring (gray line), oscillates back and forth. In (C-H), wavefunction solutions to the Time-Dependent Schrödinger Equation are shown for the same potential. The horizontal axis is position, the vertical axis is the real part (blue) or imaginary part (red) of the wavefunction. (C,D,E,F) are stationary states (energy eigenstates), which come from solutions to the Time-Independent Schrodinger Equation. (G-H) are non-stationary states, solutions to the Time-Dependent but not Time-Independent Schrödinger Equation. (G) is a randomly-generated superposition of the four states (E-F). H is a "coherent state" ("Glauber state")

Conclusion.

From this we have studied about

Geometrically, an eigenvector, corresponding to a real nonzero eigenvalue, points in a direction that is stretched by the transformation and the eigenvalue is the factor by which it is stretched. If the eigenvalue is negative, the direction is reversed.

The wavefunction for a given physical system contains the measurable information about the system. To obtain specific values for physical parameters, for example energy, you operate on the wavefunction with the **quantum** mechanical operator associated with that parameter.

List of refrences

1.https://www.youtube.com/watch?v=5sHCGsEYtLE

2.http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/eigen.html

3.https://physics.stackexchange.com/questions/233922/eigenvalue-physical-meaning