Implementation and Comparison of Advanced Process Control(APC) to Stirred Tank Mixing Process Project Report

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1.1.1 Process Description

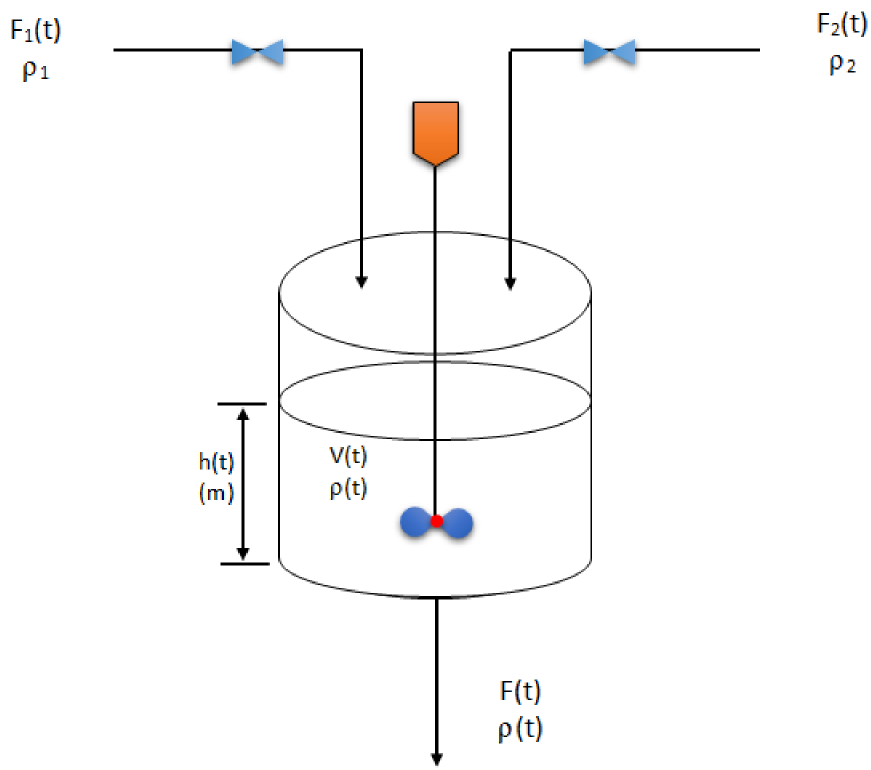


Figure 1: Process Flow Diagram: Stirred Tank Mixing Process.

The Continuously stirred mixing tank in figure(1) has two time-varying inlets F1(t) and F2(t) with different density. The density of both the inlets is constant and is given by ρ_1 and ρ_2. It is assumed that the tank is well mixed so that outlet F(t) has the same density as the density in the tank i.e. $\rho(t)$. The volume of the tank occupied by the liquid is V(t) and the corresponding height is h(t) with the surface area S. The outlet flow F(t) has the following relationship with the head of the liquid in the tank.

$$F(t) = k\sqrt{h(t)} \tag{1}$$

Where, k= Proportionality Constant. The aim is to control and maintain the density of the outlet and the volume of liquid inside the mixing tank under influence of any disturbances in the inlet at any given set point.

1.1.2 Material Balance

The material balance equations of this process are as follows.

$$In - Out = Accumulation \tag{2}$$

$$\frac{dV}{dt} = F_1 + F_2 - F(t) \tag{3}$$

$$\frac{d(\rho(t)V(t))}{dt} = \rho_1 F_1(t) + \rho_2 F_2(t) - \rho F(t) = \rho_1 F_1(t) + \rho_2 F_2(t) - \rho k \sqrt{\frac{V(t)}{S}} \tag{4}$$

In this case,
The states variables are:

$$x = \begin{bmatrix} V \\ \rho \end{bmatrix} \tag{5}$$

The manipulated input variables are:

$$u = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{6}$$

1.2 Optimal Control Problem

$$J = \frac{1}{2} x^T(t_f) P_f x(t_f) + \frac{1}{2} \int_{t_0}^{t^f} [x^T(t) Q x(t) + u^T(t) R u(t)] dt$$

(7)

Where, Q(t) and R(t) = Weighting matrix of state and input variables.

2 Introduction

2.1 Objective

The objective of the present study is to demonstrate the application of Optimal Control strategies to Stirred tank mixing process to achive the following control objectives.

- control the density, ρ
- volume,V of the liquid inside the tank

These objectives are achieved by manipulating the inlet F_1 and F_2. The work presented here discuses three methods to that end.

2.2 Methods

The Linear Quadratic Regulator is used with the linearized system equation around study state, and with non linear system equation. The response of the system with some initial condition and disturbance is presented. The minimum principle is used with non linear equation. MPC is discussed with two approaches and the system response for two different cases is shown.

3 Initial Steps

3.1 Steady state Equation

The material balance equations of the system are:

$$\frac{dV}{dt} = F_1 + F_2 - F(t) = F_1(t) + F_2(t) - K\sqrt{\frac{V(t)}{S}} \tag{8}$$

because

$$(h = \frac{V(t)}{S}) \tag{9}$$

$$\frac{d(\rho(t)V(t))}{dt} = \rho_1 F_1(t) + \rho_2 F_2(t) - \rho F(t) = \rho_1 F_1(t) + \rho_2 F_2(t) - \rho k\sqrt{\frac{V(t)}{S}} \tag{10}$$

The above two nonlinear equations can be Linearized around a steady state.
At steady state volume and density remains constant. Hence, Their derivative will be zero and above equations can be written as follows.

$$F_{1s} + F_{2s} - K\sqrt{\frac{V_s}{S}} = 0 \tag{11}$$

$$\rho_1 F_{1s} + \rho_2 F_{2s} - \rho_s K\sqrt{\frac{V_s}{S}} = 0 \tag{12}$$

For the given steady state values of F_1 and F_2 the above two equations can be solved to determine the corresponding steady state values of V and ρ.

3.2 Linearization

The linearization of the Non-linear equation is performed around a steady state:
EQUATION 1

$$\frac{dV}{dt} = F_1 + F_2 - K\sqrt{\frac{V}{S}} \tag{13}$$

$$f(x,u) = f(x_s,u_s) + \frac{\partial f}{\partial x}|_{xs,us}(x - x_s) + \frac{\partial f}{\partial u}|_{xs,us}(u - u_s) \tag{14}$$

$$\frac{dV}{dt} = -K\frac{1}{2\sqrt{V_s S}} + (1)(F_1 - F_{1s}) + (1)(F_2 - F_{2s}) \tag{15}$$

Define

$$V^{'} = V - V_s \Rightarrow \frac{dV^{'}}{dt} = \frac{dV}{dt}; \tag{16}$$

$$F_1' = F_1 - F_{1s} and \tag{17}$$

$$F_2' = F_2 - F_{2s} \tag{18}$$

So, Linearized equation (around steady state) is:

$$\frac{dV^{'}}{dt} = F_1^{'} + F_1^{'} - \frac{K}{2\sqrt{V_s S}} V' \tag{19}$$

EQUATION 2

$$\frac{d(\rho V)}{dt} = \rho_1 F_1 + \rho_2 F_2 - \rho K \sqrt{\frac{V}{S}} \tag{20}$$

$$\rho_s \frac{dV}{dt} + V_s \frac{d\rho}{dt} = \rho_1 F_1 + \rho_2 F_2 - \rho K \sqrt{\frac{V}{S}} = f(x,u) \tag{21}$$

$$f(x,u) = f(x_s,u_s) + \frac{\partial f}{\partial x}|_{xs,us}(x - x_s) + \frac{\partial f}{\partial u}|_{xs,us}(u - u_s) \tag{22}$$

$$\rho_s \frac{dV}{dt} + V_s \frac{d\rho}{dt} = -\frac{\rho_s K}{2\sqrt{V_s S}}(V - V_s) - K\sqrt{\frac{V_s}{S}}(\rho - \rho_s) + \rho_1(F_1 - F_s) + \rho_2(F_2 - F_s) \tag{23}$$

Define

$$\rho^{'} = \rho - \rho_s \tag{24}$$

$$\frac{d\rho^{'}}{dt} = \frac{d\rho}{dt} \tag{25}$$

$$\rho_s \frac{dV'}{dt} + V_s \frac{d\rho'}{dt} = -\frac{\rho_s K}{2\sqrt{V_s S}} V' - K\sqrt{\frac{V_s}{S}}\rho' + \rho_1 F_1' + \rho_2 F_2' \tag{26}$$

So,Linearized equation (around steady state) is

$$\rho_s \frac{dV'}{dt} + V_s \frac{d\rho'}{dt} = \rho_1 F_1' + \rho_2 F_2' - \frac{\rho_s K}{2\sqrt{V_s S}} V' - K\sqrt{\frac{V_s}{S}}\rho' \tag{27}$$

Steady state conditions:

$$F_{1s} = 0.01 m^3/s \tag{28}$$

$$F_{2s} = 0.005 m^3/s \tag{29}$$

$$\rho_1 = 823 kg/m^3 \tag{30}$$

$$\rho_2 = 890 kg/m^3 \tag{31}$$

$$S = 0.79 m^2 \tag{32}$$

K= 0.0133

Using these steady state input variables equation (2.4) and (2.5) can be solved to obtain the corresponding steady state state variables.

$$V_s = 1.00 m^3 \tag{33}$$

$$\rho_s = 845.333 kg/m^3 \tag{34}$$

Using these values the Linearized equations (2.12) and (2.20) around this steady state can be written as follows.

$$\frac{dV'}{dt} = F_1' + F_1' - 0.0075V' \tag{35}$$

and

$$\frac{d\rho'}{dt} = -22.333F_1' + 44.667F_2' - 0.015\rho' \tag{36}$$

So, Continuous Time linear system equation is:

$$\begin{bmatrix} \dot{V}' \\ \dot{\rho}' \end{bmatrix} = \begin{bmatrix} -0.0075 & 0 \\ 0 & -0.015 \end{bmatrix} \begin{bmatrix} V' \\ \rho' \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -22.33 & 44.66 \end{bmatrix} \begin{bmatrix} F_1' \\ F_2' \end{bmatrix} \tag{37}$$

It is important to note that the above Linearized equation is only valid near the region of the steady state that is used to derive it.

3.3 Controllability of the system

It is very important that we check the controllability of our system before we move forward with our analysis. The system is said to be controllable if there exists a set of inputs that can transfer the system from it's current state to desirable state in finite time. The controllability of the system can be checked by checking the rank of the controllability matrix.

$$C_{ctrb} = [BAB......A^{n-1}B] \tag{38}$$

The system is controllable if the rank of the matrix is equal to length of A in the state space equation. However, In this study, MATLAB function ***ctrb*** was utilized to check the controllability of the system. It was found that the rank of controllability matrix is 2 , hence, System is controllable.

4 Linear Quadratic Regulator(LQR)

The linear quadratic regulator is a special case of optimal control problem where the cost function is expressed as a linear combination of system states.[2] The LQR is defined only for linear state equation, However, both linear and non-linear cases are discussed here.

4.1 LQR with linear system equation

Here,The infinite duration LQR is considered for which optimal control law is time invariant.The Linearized system equation is

$$\begin{bmatrix} \dot{V}' \\ \dot{\rho}' \end{bmatrix} \begin{bmatrix} -0.0075 & 0 \\ 0 & -0.015 \end{bmatrix} \begin{bmatrix} V' \\ \rho' \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -22.33 & 44.66 \end{bmatrix} \begin{bmatrix} F_1' \\ F_2' \end{bmatrix} \tag{39}$$

The performance measure to be minimize is given as follows.

$$J = \frac{1}{2} \int_0^\infty [x'^T Q x' + u'^T u'] dt \tag{40}$$

Where,

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 0.0025 \end{bmatrix} \tag{41}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{42}$$

$$x' = \begin{bmatrix} V' \\ \rho' \end{bmatrix} \tag{43}$$

$$u' = \begin{bmatrix} F_1' \\ F_2' \end{bmatrix} \tag{44}$$

Note that x' and u' are deviation variable from steady state as give by equation (2.9-2.11) and (2.17). The Optimal Control law is given by

$$u'^*(t) = -R^{-1} B^T p^*(t) x'^*(t) \tag{45}$$

Since, This is an infinite duration LQR the value of $p^*(t)$ would be time invariant and can be obtained by solving the Riccati equation given below.

$$Q + A^T P + PA - PBR^-1 B^T P = 0 \tag{46}$$

So, equation(3.7)becomes

$$u'^*(t) = -R^{-1} B^T P x'^*(t) = -K x'^*(t) \tag{47}$$

Equation (3.9) is an optimal control law which than can be used with the system equation given by equation (3.1) to obtain system state and input trajectories. The MATLAB function ***lqr*** is used here to solve the Riccati equation and the following results were obtained.

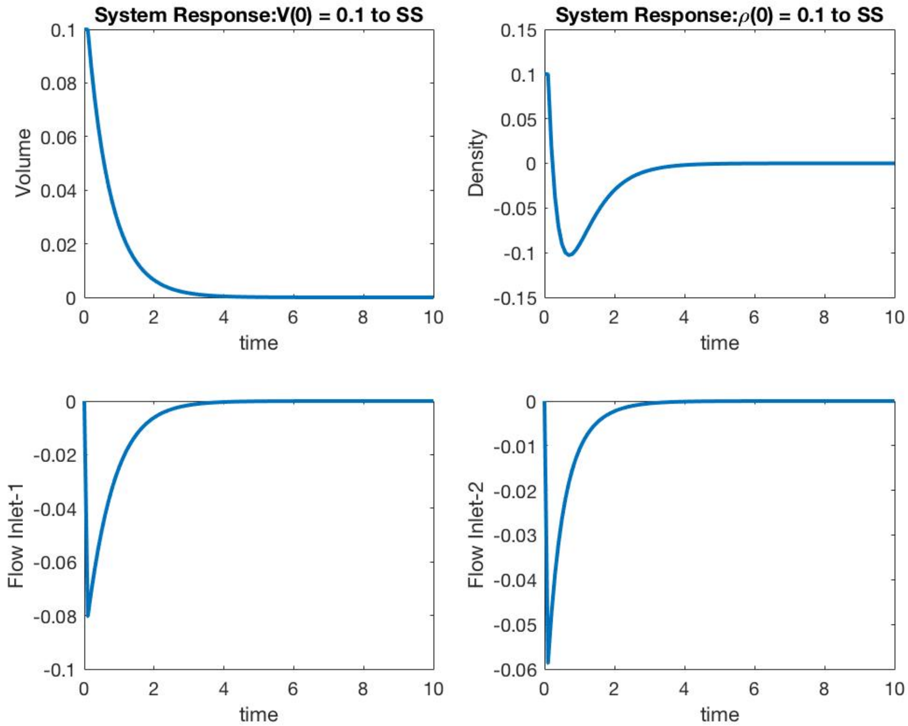
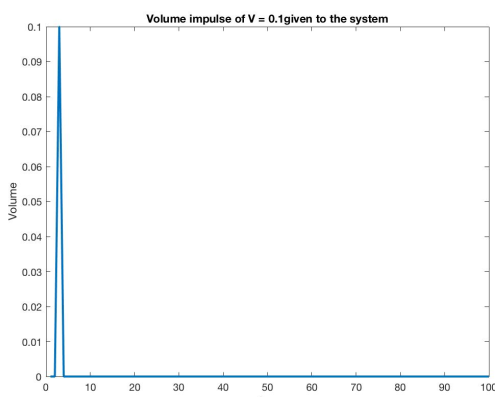
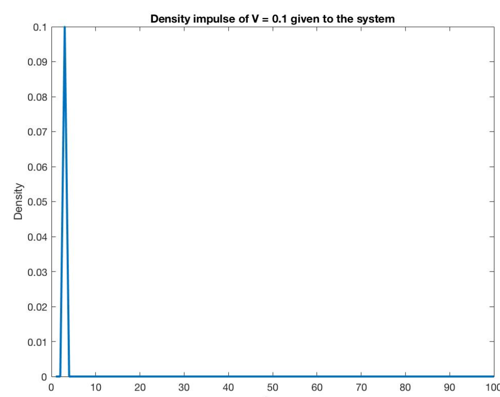


Figure 2: Linear LQR: Initial condition to steady state.



(a) Volume Impulse.



(b) ρ Impulse.

Figure 3: Impulse to the system at time t= 0.3s.

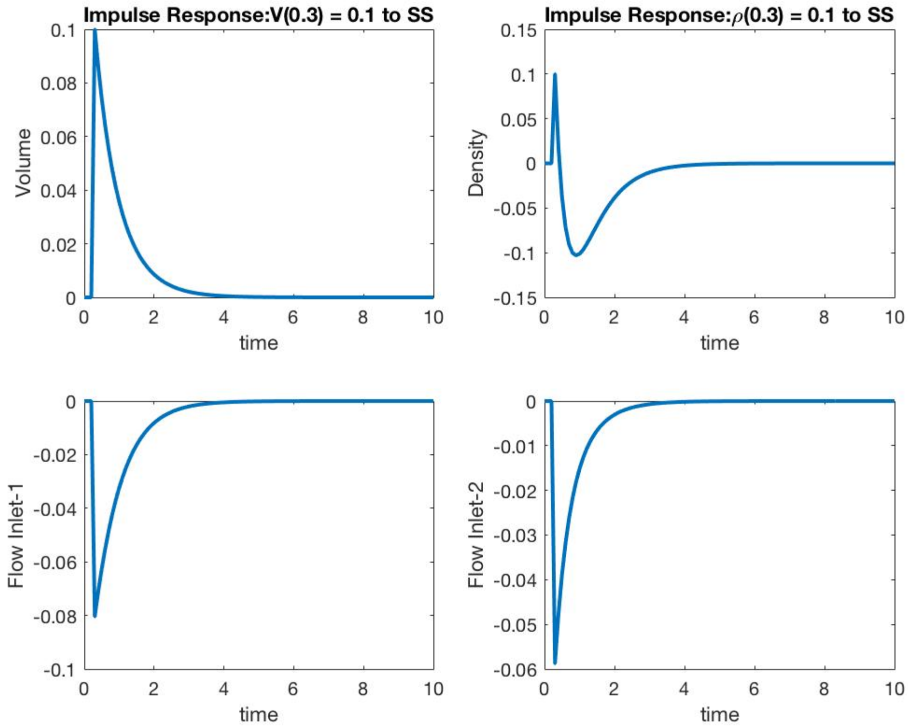


Figure 4: Linear LQR: Impulse Response.

Figure 2 shows the response of the system from initial condition V' = 0.1 m^3 and $\rho' = 0.1$ Kg/m^3 to steady state condition. These value of initial conditions refers to the magnitude of state away from steady state. Similarly, Impulse of magnitude 0.1 is given to the system at time 0.3 s as shown in figure3a and 3b and figure 4 shows the impulse response of the system along with trajectories of F_1 and F_2.

4.2 Non-linear equation

The non-linear equation of system is given by

$$\frac{dV}{dt} = F_1 + F_2 - 0.015\sqrt{V} \tag{48}$$

$$\frac{d\rho}{dt} = \frac{1}{V}((823 - \rho)F_1 + (890 - \rho)F_2 \tag{49}$$

The only difference in this case is that state and input variable are now expressed directly, rather than the deviation variables because we are directly using non linear equation of the system. So, u'(t) of equation (3.9) would become

$$u - u_s = -K(x - x_s) \tag{50}$$

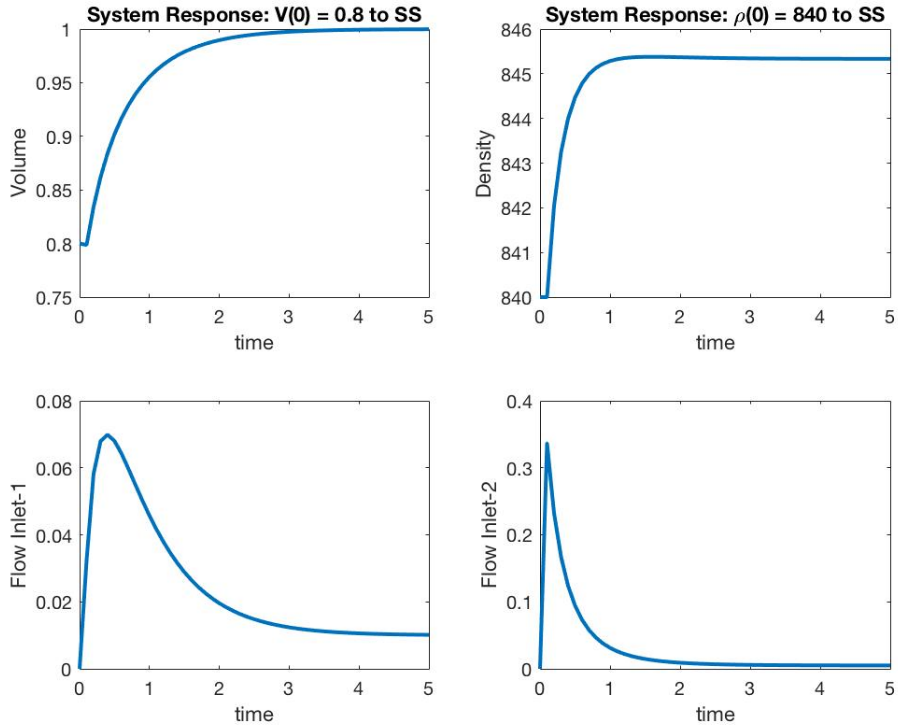
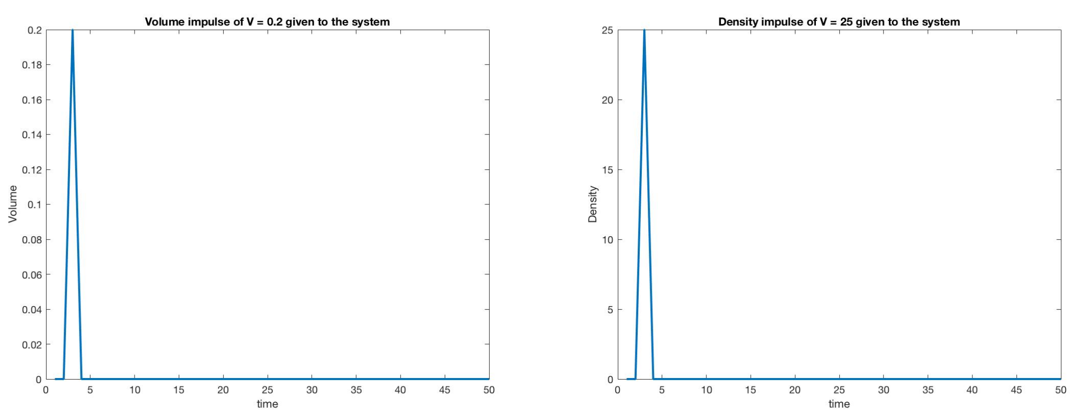


Figure 5: Nonlinear LQR: Initial condition to steady state.



(a) Volume Impulse.

(b) ρ Impulse.

Figure 6: Impulse to the system at time t= 0.3 s.

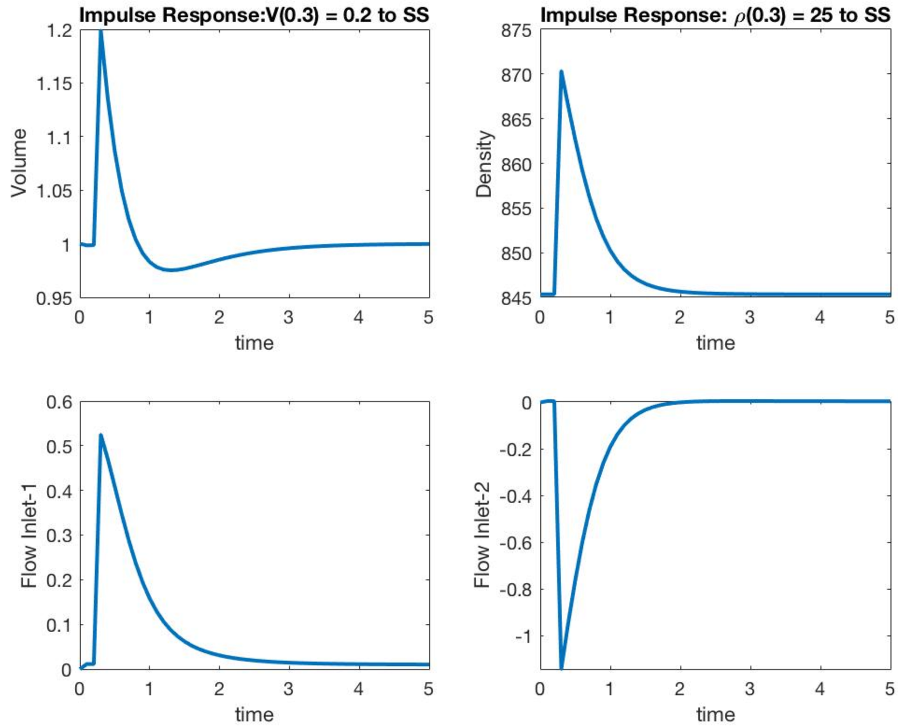


Figure 7: Nonlinear LQR: Impulse Response.

So,

$$u = u_s - K(x - x_s) \tag{51}$$

Figure 5 shows the response of the system from initial condition V = 0.8 m^3 and $\rho' = 840$ Kg/m^3 to steady state condition. Similar to case 1, An impulse of magnitude V(0.3) = 0.2 m^3 and $\rho(0.3) = 25Kg/m^3$ is given to the system at time 0.3 S as shown in figure 6a and 6 and figure 7 shows the impulse responce of the system along with trajectories of F_1 and F_2.

For both linear and non linear cases, after introducing the disturbance, system is going back to steady states which indicates that LQR is working fairly well for the current system.

4.3 Choice of Q and R

The choice of Q and R matrix is arbitrary and depends on the designer. In this case, The Q was selected by assuming that the maximum deviation that ρ and V would be subjected to is 20 kg/m^3 and 1 m^3 from it's steady state.
Hence,

$$q_{22} = \left(\frac{20}{1}\right)^2 q_{11} \tag{52}$$

So,

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 0.0025 \end{bmatrix} \tag{53}$$

As for R, both inlet flow F_1 and F_2 were given equal importance. So,

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{54}$$

For the sake of generality, All the cases henceforth in this study are discussed with the same values of Q and R.

5 Minimum Principle

As we have seen, the LQR does not handle any state and input constraints. However, in reality, this is not the case as every system has some physical limitation. The inclusion of these constraints will effect the necessary conditions which are explained in Pontryagin's Minimum Principle. In the present analysis, however, we are not considering any constraints for simplicity of the calculation. Furthermore, in the present study, unlike LQR, we can directly use non-linear system equation to design the controller. In addition, the terminal time t_f is taken 8 to make this a fixed time problem. The choice of t_f is made based on the behavior of system observed in the previous section.

The necessary conditions are as follows:

$$\dot{x} = f(x^*(t), u^*(t)) \tag{55}$$

$$\dot{p}(t) = -\frac{\partial H}{\partial x}(x^*(t), u^*(t), t) \tag{56}$$

$$\frac{\partial H}{\partial u} = 0 \tag{57}$$

By taking K=0.0133 and S=0.79 m^2 the non linear equation of system can be written as:

$$\frac{dV}{dt} = F_1 + F_2 - 0.015\sqrt{V} \tag{58}$$

$$\frac{d\rho}{dt} = \frac{1}{V}((823 - \rho)F_1 + (890 - \rho)F_2 \tag{59}$$

The performance measure to be minimize is given by

$$J = (x_f - x_s)^T Q(x_f - x_s) + \int_0^{18} (x(t) - x_s)^T Q(x(t) - X_s) + (u(t) - u_s)^T R(u(t) - u_s) \tag{60}$$

It is assumed that the system will reach it's steady state value by $t_f = 8$ to avoid the complicated mathematical equation in the upcoming steps. As a result, the terminal cost in the above equation would be zero. The consequence of this assumption is explained with the results.

$$J = \int_0^8 \begin{bmatrix} x_1 - 1 & x_2 - 845.333 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.0025 \end{bmatrix} \begin{bmatrix} x_1 - 1 \\ x_2 - 845.33 \end{bmatrix} + \tag{61}$$

$$\begin{bmatrix} u_1 - 0.01 & u_2 - 0.005 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 - 0.001 \\ u_2 - 0.005 \end{bmatrix} \tag{62}$$

Where,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} V \\ \rho \end{bmatrix} \tag{63}$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{64}$$

Now, The Hamiltonian is defined as,

$$H = g + p^T f(x,u)$$

So,

$$\begin{aligned} H = (x_1^2 - 2x_1 - 4.22x_2 + 0.0025x_2^2 + 1787.5 + u_1^2 - 0.02u_1 - 0.02u_2 + u_2^2 + 0.000125) \\ +p_1u_1 + p_1u_2 - 0.015p_1\sqrt{x_1} + \tfrac{p_2}{\sqrt{x_1}}(823 - x_2)u_1 + \tfrac{p_2}{\sqrt{x_1}}(890 - x_2)u_2 \end{aligned}$$

Necessary Conditions

$$\dot{p_1} = -\frac{\partial h}{\partial x_1} \tag{65}$$

$$\dot{p_1} = -[2x_1 - 2 - 0.00075\frac{p_1}{\sqrt{x_1}} - \frac{p_2}{2x_1^{\frac{3}{2}}}(823 - x_2)u_1 + \frac{p_2}{2x_1^{\frac{3}{2}}}(890 - x_2)u_2] \tag{66}$$

similarly,

$$\dot{p_2} = -\frac{\partial h}{\partial x_2} \tag{67}$$

$$\dot{p_2} = -[-4.22 + 0.005x_2 - \frac{p_2}{\sqrt{x_1}}u_1 - \frac{p_2}{\sqrt{x_1}}u_2] \tag{68}$$

Since,there are no input constraints,

$$\frac{\partial H}{\partial u_1} = 2u_1 - 0.02 + \frac{p_2}{\sqrt{x_1}}(823 - x_2) = 0 \tag{69}$$

$$u_1 = \frac{1}{2}[0.02 - p_1 + \frac{p_2}{\sqrt{x_1}}(823 - x_2) \tag{70}$$

Similarly,

$$\frac{\partial H}{\partial u_2} = 2u_2 - 0.02 + \frac{p_2}{\sqrt{x_1}}(890 - x_2) = 0 \tag{71}$$

$$u_1 = \frac{1}{2}[0.02 - p_1 + \frac{p_2}{\sqrt{x_1}}(890 - x_2)] \tag{72}$$

Boundry Condition

Since, there are no input constraints and t_f is fixed,

$$[\frac{\partial h}{\partial x} - p^*](t_f)\delta x_f = 0 \tag{73}$$

$$p^*(t_f) = 0 \tag{74}$$

Hence,

$$\begin{bmatrix} p_1^*(t_f) \\ p_2^*(t_f) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{75}$$

The initial condition of the system is as follows.

Initial Volume in the tank V(0) = 0.8 m^3
Initial Density of the liquid in tank $\rho(0)$ = 840 Kg/m^3

This is a two point boundary value problem.The state and co state equations of the system are as follows.

$$\dot{V} = F_1 + F_2 - 0.015\sqrt{V} \tag{76}$$

$$\dot{\rho} = \frac{1}{V}((823 - \rho)F_1 + (890 - \rho)F_2 \tag{77}$$

$$\dot{p_1} = -[2x_1 - 2 - 0.00075\frac{p_1}{\sqrt{x_1}} - \frac{p_2}{2x_1^{\frac{3}{2}}}(823 - x_2)u_1 + \frac{p_2}{2x_1^{\frac{3}{2}}}(890 - x_2)u_2] \tag{78}$$

$$\dot{p_2} = -[-4.22 + 0.005x_2 - \frac{p_2}{\sqrt{x_1}}u_1 - \frac{p_2}{\sqrt{x_1}}u_2] \tag{79}$$

Where, u_1 and u_2 are as given by equation (3.14) and (3.16) and boundry conditions are:

$$\begin{bmatrix} x_1^*(0) \\ x_2^*(0) \end{bmatrix} = \begin{bmatrix} 0.8 \\ 840 \end{bmatrix} \tag{80}$$

$$\begin{bmatrix} p_1^*(8) \\ p_2^*(8) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{81}$$

The above TPBVP was solved using MATLAB function **bvp4c** (with code provided in eclass) with initial guess of states as 1 and 845.333 and the following results were obtained.

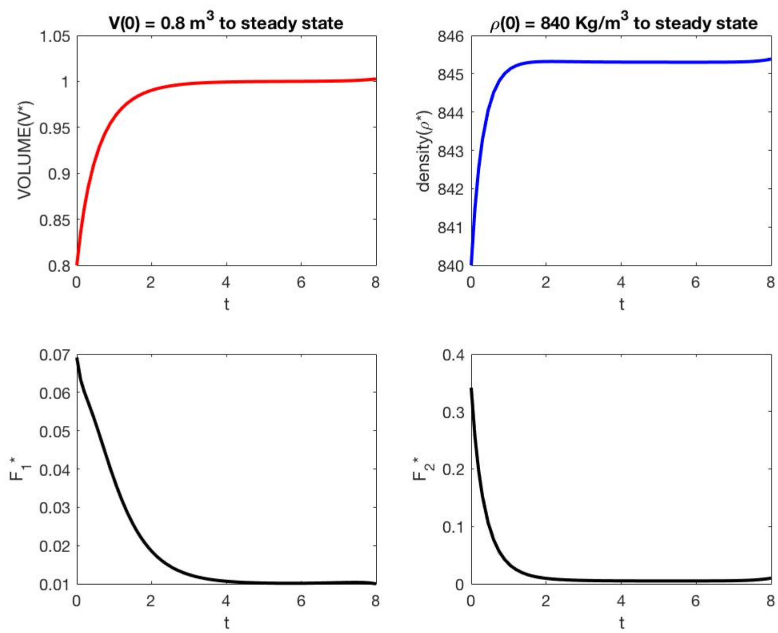


Figure 8: Minimum Principle: Initial condition to steady state.

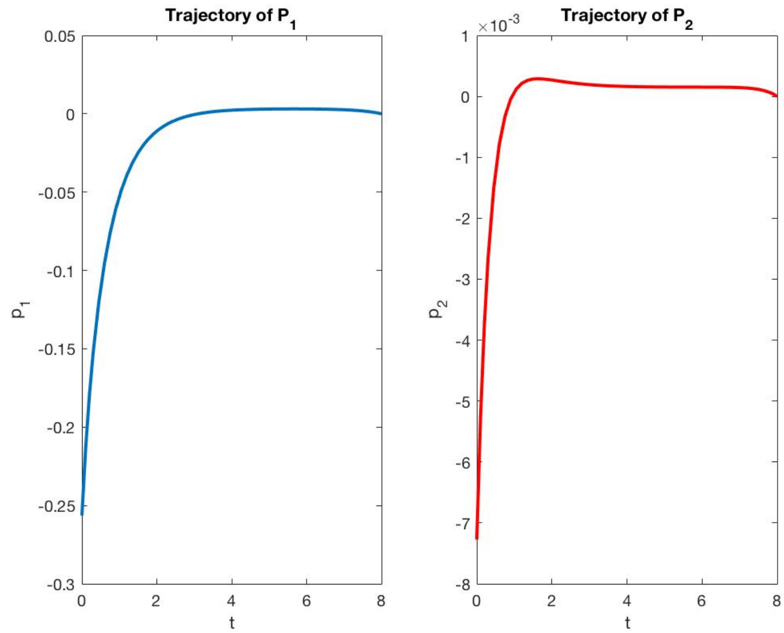


Figure 9: Minimum Principle: Trajectories of p_1^* and p_2^*.

Figure 8 shows the response of the system from initial condition to steady state and the corresponding trajectory of input flow F_1 and F_2. The trajectories of the state variables(i.e.$V and \rho$) shows an unusual small rise from the steady state value near the terminal time t_f. This rise is the consequence of the zero terminal cost assumption that was used at the beginning of calculation. The trajectories of p_1^* and p_2^* are shown in Figure 9.

6 Model Predictive Control(MPC)

The "Model Predictive Control", as the name suggests, predicts the set of control actions for a finite time by solving an optimal control problem using the current state of the system as an initial state. In other words, it generates the series of control action that would take the system from its current state to the desired state with minimum control action. Out of this sequence, the first control action is applied to the system and the response of the system to this control action is measured. The same process is repeated at each sampling time.[1]

In MPC, the control law is updated every time instant based on the present state of the system. This is how MPC differs from the conventional control where there is a predefined control law(U=-kX) which is applied to the current state of the system to generate the corresponding control action. In addition, constraints are also incorporated in the control action generated by MPC which is not the case in LQR. In this case, the input constraints imposed on the controller were that flow rates F_1 and F_2 should be within 0 and 0.03 m^3/s

The finite time for which MPC generates the control actions every sampling time is called "Prediction Horizon". Ideally, the length of prediction horizon should be infinite to ensure the stability. However, it is not possible to handle infinite time numerically. The present analysis discusses two approach to handle infinite time.

6.1 Using Sufficiently Long Prediction Horizon

In this approach, the prediction horizon(N) is chosen long enough so that it covers all the transient period of the system. However, very large prediction horizon would increase the computational cost. In the present case, two N were considered

i) Initial Condition to Steady state condition: N = 6

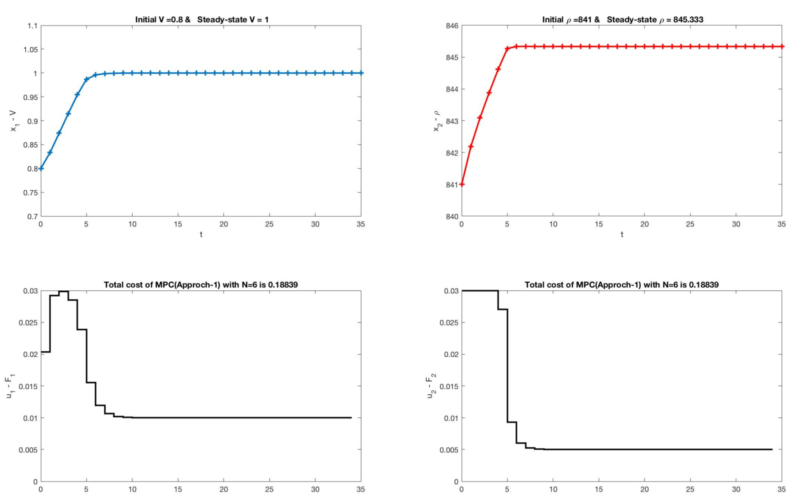


Figure 10: MPC Approach 1:Initial Condition to steady state with N=6 .

Initial Condition:

- V=0.8m^3
- $\rho = 841 Kg/m^3$
  Steady State:
- V=1 m^3
- $\rho = 845.33 Kg/m^3$

  Figure 10 shows the trajectories of V andρ to drive the system from initial condition to steady state along with the corresponding trajectories of F_1 and F_2. The total cost J = 0.18839. These results are obtained using MATLAB Function ***fmincon***.

ii)Initial Condition to Set Point: N = 14

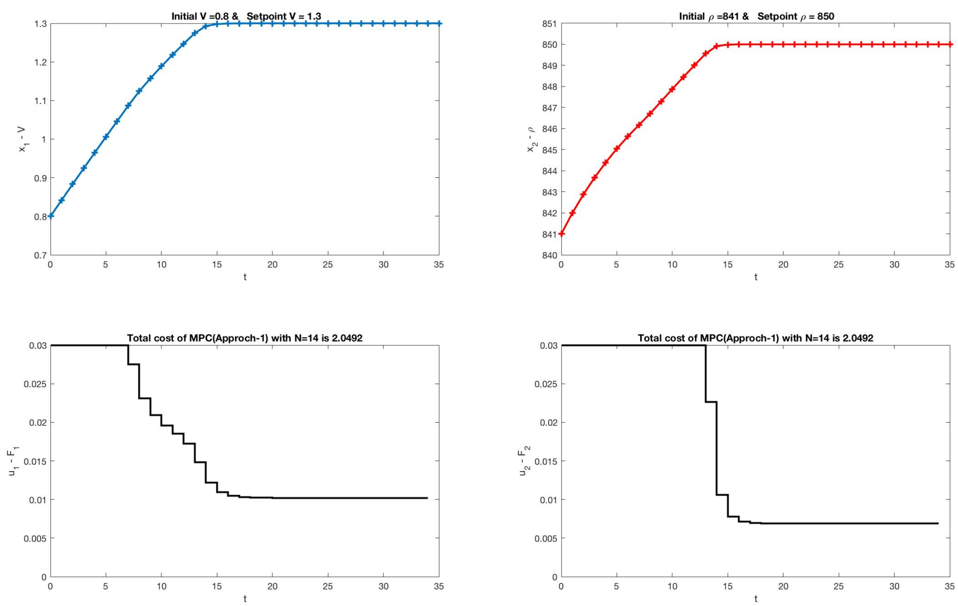


Figure 11: MPC Approach 1:Initial Condition to set point with N=14.

Initial Condition:

- V=0.8m^3
- $\rho = 841 Kg/m^3$

Set point:

– V=1.3m^3

– $\rho = 850Kg/m^3$

Figure 11 shows the trajectories of V andρ to drive the system from initial condition to set-point along with the corresponding trajectories of F_1 and F_2. The total cost J = 2.0492.

The reason behind choosing two different N was to demonstrate that the change of set point also changes the length of prediction horizon required along with the cost. For a complex system this approach could reduce computational burden.

6.2 Using a terminal point constraint

In this approach, One additional constraint is imposed on the controller, which is to drive the system to desired state within the chosen prediction horizon for each sampling time. This approach may present some feasibility issue if the initial state is far away from the final state and prediction horizon is not long enough. In this approach,Same two cases were considered as in approach 1.

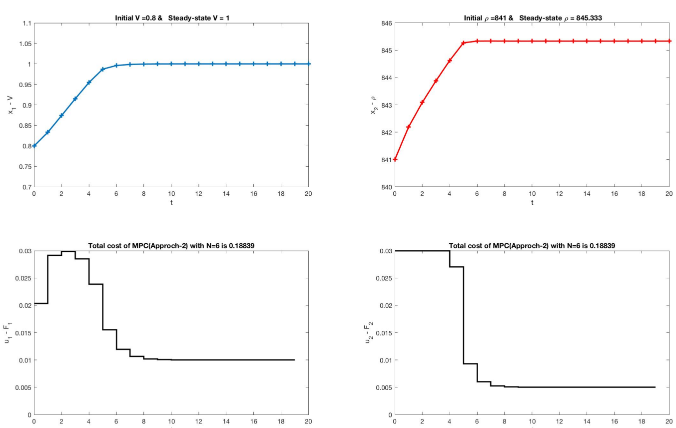


Figure 12: MPC Approach 2:Initial Condition to steady state with N=6.

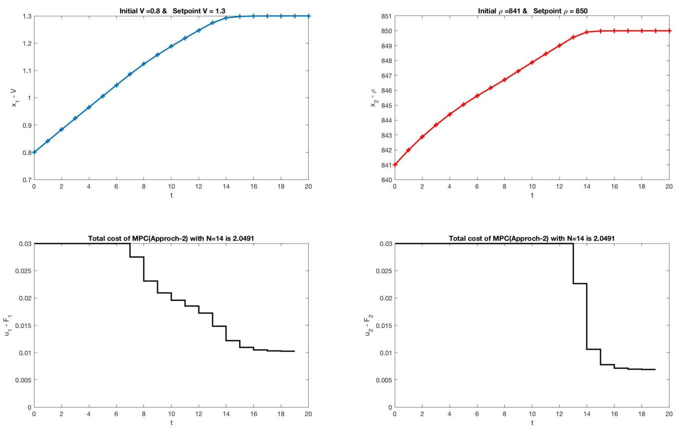


Figure 13: MPC Approach 2:Initial Condition to set point with N=14.

Figure 12 and figure 13 (Refer to figure14 and 15if the figure is not legible) shows the response of a system for Initial condition to steady state and set point respectively. The prediction horizon was same as it was in case 1. It is clear that the total cost (J) is nearly same for approach 1 and approach 2. However, if the N is reduced for second approach and the set point is not feasible within that N than approach 2 would give slightly higher cost than that of approach 1.

The value of Q and R also has a significant effect on the controller performance. The large values of Q over R would drive the system quickly to the set-point at the expanse of high control action. Similarly, The large value of R over Q would reduce the control action with slow transition of system to set point.[1]

	1	**2**	**3**
Q	1 - 0 0 - 1795	1 - 0 0 - 1	1 - 0 0 - 0.00005
R	1 - 0 0 - 1	1 - 0 0 - 1	1 - 0 0 1
t_{vs}	offset	12	6
$t_{\rho s}$	4	4	11

The above table contains a comparison of different Q values and it's effect on the time that system takes to reach the steady state. Notice that when forth component of q matrix is large rho is reaching to set point faster and it takes longest time when forth component is below zero. In addition, giving more weight to forth component more weight ensured that the corresponding state(i.e.ρ) reaches the steady state even at the expanse of offset(i.e.case 1) in the other state(i.e.V).

6.3 MATLAB

To implement the MPC with MATLAB to current system the nonlinear equation were first discretized and following form was obtained.

$$V(t+1) = V(t) + [F_1 + F_2 - 0.015\sqrt{V}]t_s \tag{82}$$

$$\rho(t+1) = \rho(t) + [\frac{1}{V}((823-\rho)F_1 + (890-\rho)F_2)]t_s \tag{83}$$

The above equation was integrated with forward Euler's method with sampling time = 0.1 s in a function ***diff1***.The cost of MPC would take following form.

$$J = (x_f - x_s)^T Q(x_f - x_s) + \Sigma_0^{N-1}(x(t) - x_s)^T Q(x(t) - X_s) + (u(t) - u_s)^T R(u(t) - u_s) \tag{84}$$

To generate the above mentioned results the MATLAB code provided in eclass was utilized with some modification.

7 Summary

The present study discusses the application of LQR, Minimum Principle and MPC to the Stirred tank mixing process to control the density and volume of the tank. Both, LQR and Minimum principle, have shown very similar results. However, in both aforementioned methods no constraints were imposed on the controller. On the other hand, MPC took fairly long time, when compare to the LQR and Minimum Principle, to reach the steady state owing to the imposed input constraint of 0 to $0.3m^3$ for inlet flow rate. It was observed that the values of Q and R has significant effect on the performance of controller. As mentioned earlier, the selection of these tuning parameter is a trade off between control effort and response time that solely depends on the mindset of designer.

8 Reference

1. J. B. Rawlings and D. Q. Mayne, Model Predictive Control: Teory and Design, Nob Hill Publishing, 2009.
2. D. E. Kirk, Optimal Control Teory: An Introduction, Dover Publications, 2004.
3. Poyen, C Eng Faruk, Mukherjee, Deep, Banerjee, Dibyendu and Guin, Santu Implementation Of Linear Quadratic Regulator For CSTR Tank.

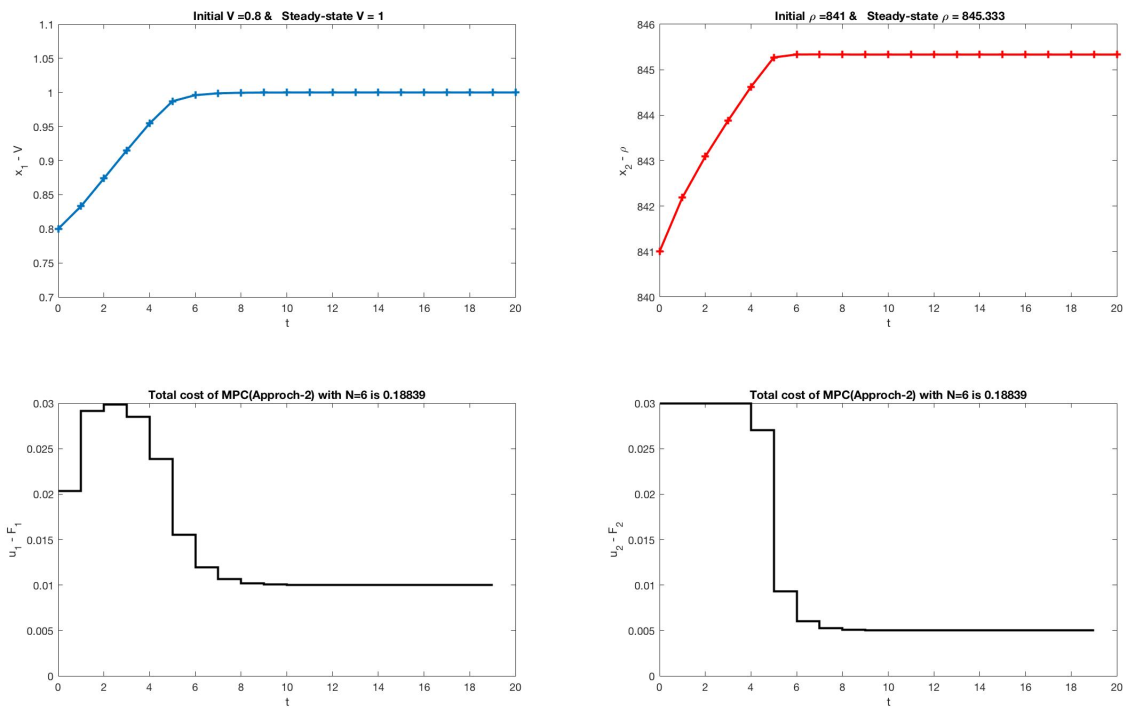


Figure 14: MPC Approach 2:Initial Condition to steady state with N=6.

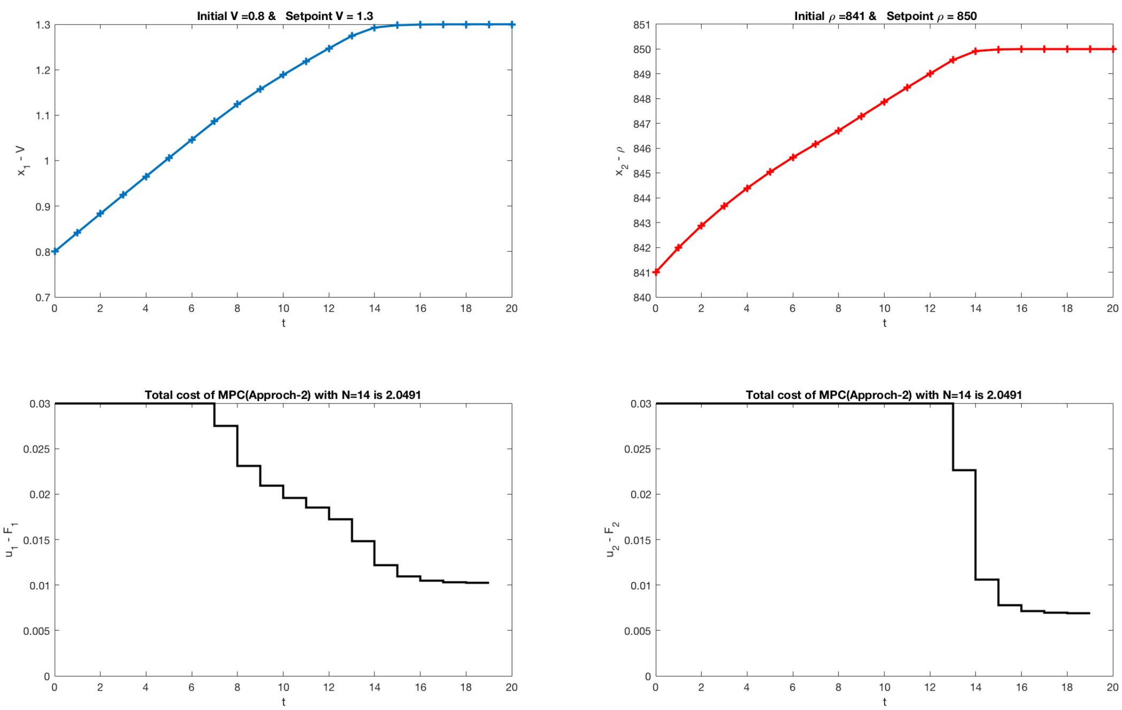


Figure 15: MPC Approach 2:Initial Condition to set point with N=14.