Moving Horizon Estimation

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1 Model

The original discrete time state space model is of the form:

$$x_{k+1} = f(x(k), u(k)) + w(k) \tag{1}$$

$$y(k) = Cx(k) + v(k) \tag{2}$$

where,
w(k) = Process noise with co-variance **Q**.
v(k) = Measurement noise with co-variance **R**.

2 Cost Function

The full information problem becomes computationally intensive with time, and hence, in MHE, truncated sequence($x_k{}_{k=T-N}^{k=T}$) of finite length N is evaluated at every step. The cost function is as follows,

$$J = \sum_{k=T-N}^{T-1} [(y_k - Cx_k)^2_{R^{-1}} + (x_{k+1} - f(x_k, u_k))^2_{Q^{-1}}] + \Gamma_{T-N} \tag{3}$$

Where, Γ_{T-N} is an approximate of the arrival cost obtained by first-order Taylor series approximation (similar to EKF).

Arrival Cost (Γ_{T-N}):

$$\Gamma_k = (x_k - \hat{x_k})^2_{P_k^{-1}} \tag{4}$$

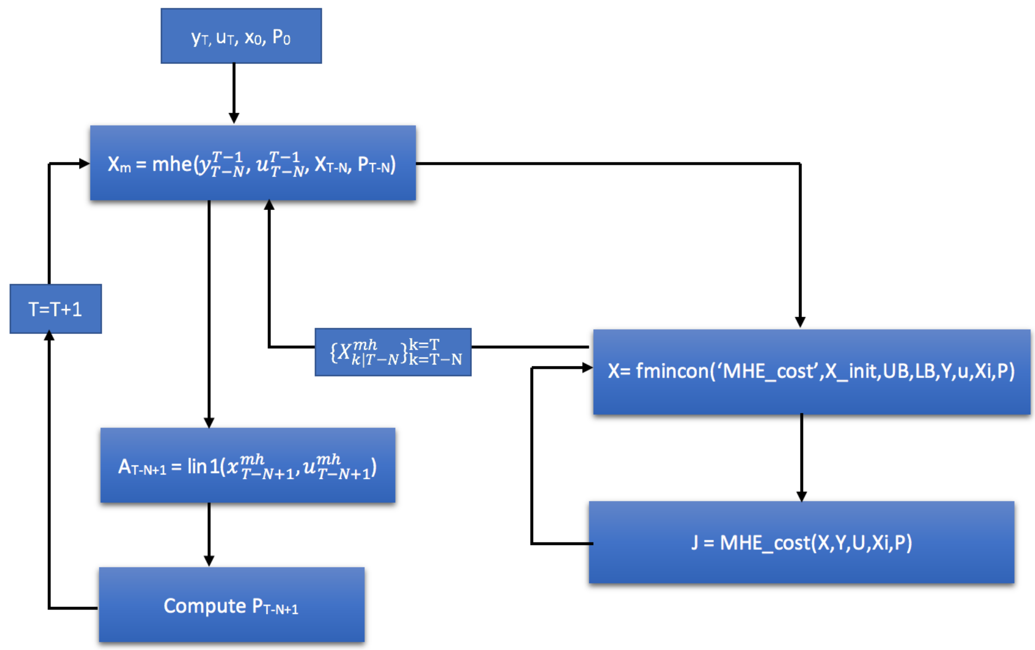
Where, matrix P is obtained by solving the Riccati equation

$$P_{k+1} = A_k P_k A_k' - A_k P_k C'(CP_k C' + R)^{-1} CP_k A_k' + Q \tag{5}$$

Where,

$$A_k = \frac{\partial f(x_k, u_k)}{\partial x}|_{(x_k^{mh}, u_k)} \tag{6}$$

3 Matlab Flow



The figure describes the MATLAB flow for MHE.

- step 1: The function ***mhe*** takes Output measurements ($[y_k]_{k=T-N}^{k=T-1}$), Input($[u_k]_{k=T-N}^{k=T-1}$), Initial guess of states(x_0) and P_0 as an input and gives states estimate ($[x_k]_{k=T-N}^{k=T}$) as an output.

- step 2: ***mhe*** uses MATLAB function ***fmincon*** to perform constrained non-linear optimization for state evaluation. UB and LB incorporates the lower and upper bounds imposed on the evaluated stats.

- step 3: The function fmincon uses ***MHE cost*** to evaluate the cost for given states as per equation (3).

- step 4: Once the states are evaluated, function ***lin1*** linearizes the system equation around x_{T-N+1}^{mh} and returns value of A as per equation (6).

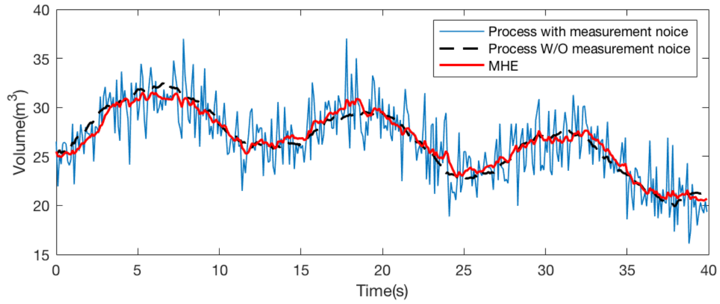
- step 5: P_{T-N+1} is calculated using equation(5) and A obtained in previous step.

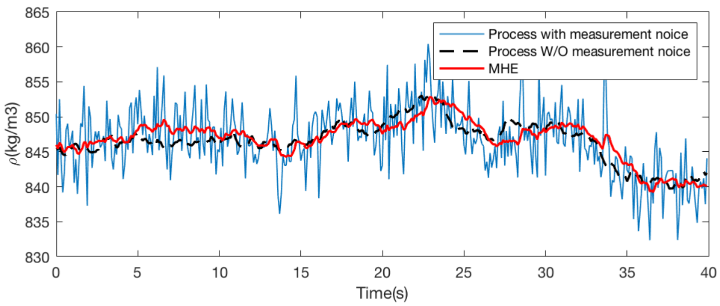
- step 6: T = T+1 and go back to step 1.

4 Results

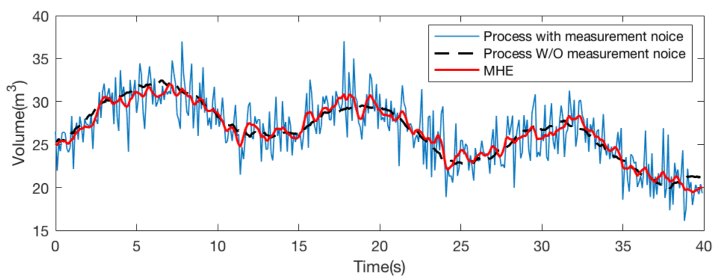
The results with different window length are shown below.

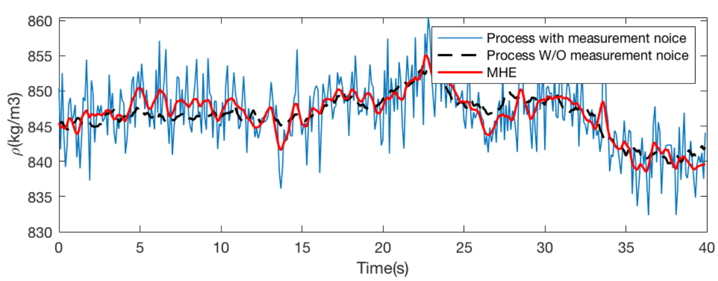
- N = 1


Process with measurement noice
Process W/O measurement noice
MHE
Volume(m^3)
Time(s)

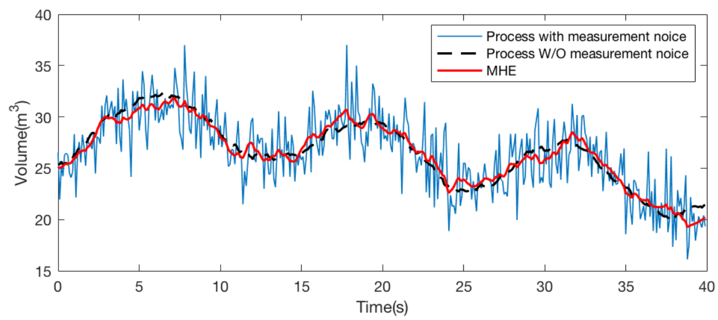

Process with measurement noice
Process W/O measurement noice
MHE
ρ(kg/m3)
Time(s)

- N = 5

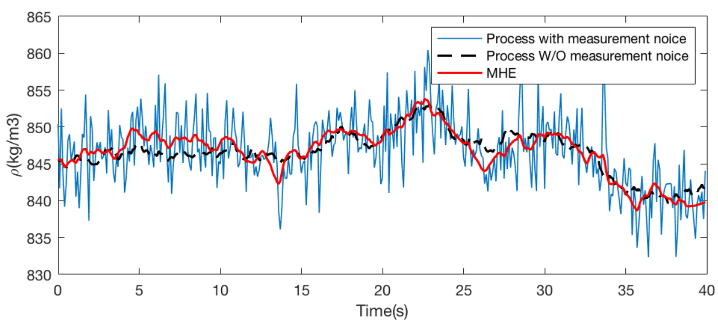

Process with measurement noice
Process W/O measurement noice
MHE
Volume(m^3)
Time(s)


Process with measurement noice
Process W/O measurement noice
MHE
ρ(kg/m3)
Time(s)

- N = 10

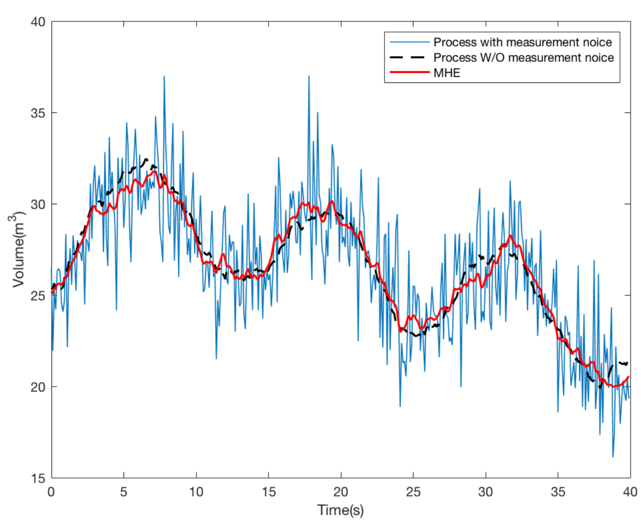


Process with measurement noice
Process W/O measurement noice
MHE
Volume(m^3)
Time(s)

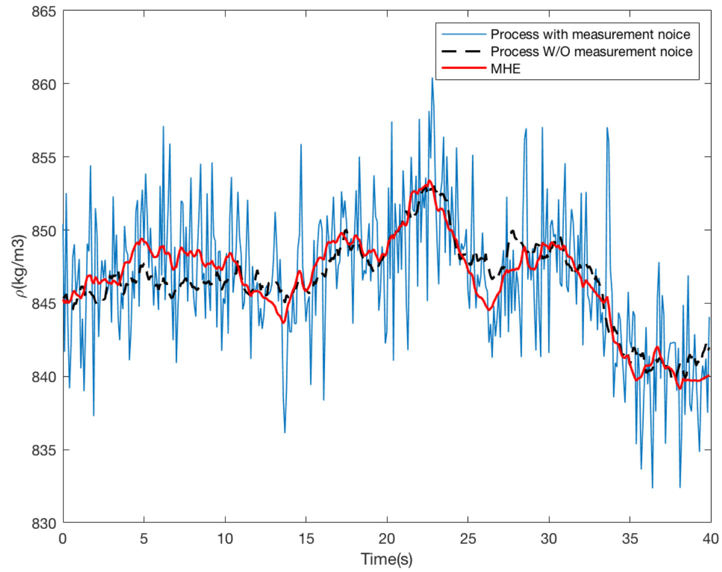


Process with measurement noice
Process W/O measurement noice
MHE
ρ(kg/m3)
Time(s)

- N = 15



Process with measurement noice
Process W/O measurement noice
MHE
Volume(m^3)
Time(s)



As it can be seen that increasing the window length is not bringing any significant improvement in the results.

The above results are obtained using following parameters,

- Sampling time, $T_s = 0.1$s.
- Window Size, N = 15.
- $$C = \begin{bmatrix} 1.268 & 0 \\ 0 & 1 \end{bmatrix}$$
- $$P_0 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1.5 \end{bmatrix}$$
- $$Q = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.1 \end{bmatrix}$$
- $$R = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}$$
- Process Model:

$$V_{k+1} = V_k + T_s(-0.015\sqrt{V_k} + F_{1k} + F_{2k}) \tag{7}$$

$$\rho_{k+1} = \rho_k + \frac{T_s}{V}[(823 - \rho)F_{1k} + (890 - \rho)F_{2k})] \tag{8}$$

5 References

[**1**] C. V. Rao, J. B. Rawlings and D. Q. Mayne, "Constrained state estimation for nonlinear discrete-time systems: stability and moving horizon approximations," in IEEE Transactions on Automatic Control, vol. 48, no. 2, pp. 246-258, Feb. 2003.doi:10.1109/TAC.2002.808470

[**2**] Yi Zhang, On the Fundamentals of Moving Horizon Estimation.

[**3**] H. Sandberg, Moving Horizon Estimation. Lacture Notes, California Institute of Technology. Available at http://www.cds.caltech.edu/ murray/wiki/images/b/b3/Stateestim.pdf.