Super-Resolution from Image Sequences - A Review

Sean Borman, Robert L. Stevenson
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556, USA
sborman@nd.edu

Abstract

Growing interest in super-resolution (SR) restoration of video sequences and the closely related problem of construction of SR still images from image sequences has led to the emergence of several competing methodologies. We review the state of the art of SR techniques using a taxonomy of existing techniques. We critique these methods and identify areas which promise performance improvements.

1. Introduction

The problem of spatial resolution enhancement of video sequences has been an area of active research since the seminal work by Tsai and Huang [20] which considers the problem of resolution enhanced stills from a sequence of low-resolution (LR) images of a translated scene. Whereas in the traditional single image restoration problem only a single input image is available, the task of obtaining a super-resolved image from an undersampled and degraded image sequence can take advantage of the additional spatio-temporal data available in the image sequence. In particular, camera and scene motion lead to frames in the video sequence containing similar, but not identical information. This additional information content, as well as the inclusion of *a-priori* constraints, enables reconstruction of a super-resolved image with wider bandwidth than that of any of the individual LR frames.

Much of the SR literature addresses the problem of producing SR still images from a video sequence – several LR frames are combined to produce a single SR frame. These techniques may be applied to video restoration by computing successive SR frames from a "sliding window" of LR frames.

SR reconstruction is an example of an *ill-posed* inverse problem – a multiplicity of solutions exist for a given set of observation images. Such problems may be tackled by constraining the solution space according to *a-priori* knowledge of the form of the solution (smoothness, positivity etc.) Inclusion of such constraints is critical to achieving high quality SR reconstructions.

We categorize SR reconstruction methods into two main divisions – frequency domain (Section 2) and spatial domain (Section 3). We review the state of the art and identify promising directions for future research (Sections 4 and 5 respectively).

2. Frequency Domain Methods

A major class of SR methods utilize a frequency domain formulation of the SR problem. Frequency domain methods are based on three fundamental principles: **i)** the shifting property of the Fourier transform (FT), **ii)** the aliasing relationship between the continuous Fourier transform (CFT) and the discrete Fourier transform (DFT), **iii)** the original scene is band-limited. These properties allow the formulation of a system of equations relating the aliased DFT coefficients of the observed images to samples of the CFT of the unknown scene. These equations are solved yielding the frequency domain coefficients of the original scene, which may then be recovered by inverse DFT. Formulation of the system of equations requires knowledge of the translational motion between frames to sub-pixel accuracy. Each observation image must contribute *independent* equations, which places restrictions on the inter-frame motion that contributes useful data.

Denote the continuous scene by $f(x, y)$. Global translations yield R shifted images, $f_r(x, y) = f(x + \Delta x_r, y + \Delta y_r),\ r = 1, 2, \ldots, R$. The CFT of the scene is given by $\mathcal{F}(u, v)$ and that of the translations by $\mathcal{F}_r(u, v)$. The shifted images are impulse sampled to yield observed images $y_r[m, n] = f(mT_x + \Delta x_r, nT_y + \Delta y_r)$ with $m = 0, 1, \ldots, M-1$ and $n = 0, 1, \ldots, N-1$. The R corresponding 2D DFT's are denoted $\mathcal{Y}_r[k, l]$. The CFT of the scene and the DFT's of the shifted and sampled images are related via aliasing,

$$\mathcal{Y}_r[k,l]=\alpha\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\mathcal{F}_r\left(\frac{k}{MT_x}+pf_{s_x},\frac{l}{NT_y}+qf_{s_y}\right) \quad (1)$$

where $f_{s_x}=1/T_x$ and $f_{s_y}=1/T_y$ are the sampling rates in the x and y dimensions respectively and $\alpha = 1/(T_xT_y)$. The shifting property of the CFT relates spatial domain translation to the frequency domain as phase shifting as,

$$\mathcal{F}_r(u,v) = e^{j2\pi(\Delta x_r u+\Delta y_r v)}\mathcal{F}(u,v). \quad (2)$$

If $f(x,y)$ is band-limited, $\exists\ L_u, L_v$ s.t. $\mathcal{F}(u,v)\rightarrow 0$ for $|u|\geq L_u f_{s_x}$ and $|v|\geq L_v f_{s_y}$. Assuming $f(x,y)$ is band-limited, we may use (2) to rewrite the alias relationship (1) in matrix form as,

$$\mathbf{Y}=\mathbf{\Phi}\mathbf{F}. \quad (3)$$

$\mathbf{Y}$ is a $R\times 1$ column vector with the r^{th} element being the DFT coefficients $\mathcal{Y}_r[k,l]$ of the observed image $y_r[m,n]$. $\mathbf{\Phi}$ is a matrix which relates the DFT of the observation data to samples of the unknown CFT of $f(x,y)$ contained in the $4L_uL_v\times 1$ vector $\mathbf{F}$. SR reconstruction thus requires finding the DFT's of the R observed images, determining $\mathbf{\Phi}$ (motion estimation), solving the system of equations (3) for $\mathbf{F}$ and applying the inverse DFT to obtain the reconstructed image. Several extensions to the basic Tsai-Huang method have been proposed. A LSI blur PSF is included in [17] and the equivalent of (3) is solved using a least squares approach to mitigate the effects of observation noise and insufficient observation data. A computationally efficient recursive least squares (RLS) solution for (3) is proposed in [10] and extended with a Tikhonov regularization solution method in [11] in an attempt to address the ill-posedness of SR inverse problem. Robustness to errors in observations as well as $\mathbf{\Phi}$ motivated the use of a total least squares (TLS) approach in [1] which is implemented using a recursive algorithm for computational efficiency.

Techniques based on the multichannel sampling theorem [2] have also been considered [21]. Though implemented in the spatial domain, the technique is fundamentally a frequency domain method relying on the shift property of the Fourier transform to model the translation of the source imagery.

Frequency domain SR methods provide the advantages of theoretical simplicity, low computational complexity, are highly amenable to parallel implementation due to decoupling of the frequency domain equations (3) and exhibit an intuitive de-aliasing SR mechanism. Disadvantages include the limitation to global translational motion and space invariant degradation models (necessitated by the requirement for a Fourier domain analog of the spatial domain motion and degradation model) and limited ability for inclusion of spatial domain *a-priori* knowledge for regularization.

3. Spatial Domain Methods

In this class of SR reconstruction methods, the observation model is formulated, and reconstruction is effected in the *spatial domain*. The linear spatial domain observation model can accommodate global and non-global motion, optical blur, motion blur, spatially varying PSF, non-ideal sampling, compression artifacts and more. Spatial domain reconstruction allows natural inclusion of (possibly nonlinear) spatial domain *a-priori* constraints (e.g. Markov random fields or convex sets) which result in bandwidth extrapolation in reconstruction.

Consider estimating a SR image $\mathbf{z}$ from multiple LR images $\mathbf{y_r},\ r\in\{1,2,\ldots,R\}$. Images are written as lexicographically ordered vectors. $\mathbf{y_r}$ and $\mathbf{z}$ are related as $\mathbf{y_r} = \mathbf{H_r z}$. The matrix $\mathbf{H_r}$, which must be estimated, incorporates motion compensation, degradation effects and subsampling. The observation equation may be generalized to $\mathbf{Y} = \mathbf{Hz}+\mathbf{N}$ where $\mathbf{Y} = \left[\mathbf{y_1}^T\cdots\mathbf{y_R}^T\right]^T$ and $\mathbf{H} = \left[\mathbf{H_1}^T\cdots\mathbf{H_R}^T\right]^T$ with $\mathbf{N}$ representing observation noise.

3.1. Interpolation of Non-Uniformly Spaced Samples

Registering a set of LR images using motion compensation results in a single, dense composite image of non-uniformly spaced samples. A SR image may be reconstructed from this composite using techniques for reconstruction from non-uniformly spaced samples. Restoration techniques are sometimes applied to compensate for degradations [17]. Iterative reconstruction techniques, based on the Landweber iteration, have also been applied [12]. Such interpolation methods are unfortunately overly simplistic. Since the observed data result from severely undersampled, spatially averaged areas, the reconstruction step (which typically assumes impulse sampling) is incapable of reconstructing significantly more frequency content than is present in a single LR frame. Degradation models are limited, and no *a-priori* constraints are used. There is also question as to the optimality of separate merging and restoration steps.

3.2. Iterated Backprojection

Given a SR estimate $\hat{\mathbf{z}}$ and the imaging model $\mathbf{H}$, it is possible to *simulate* the LR images $\hat{\mathbf{Y}}$ as $\hat{\mathbf{Y}}=\mathbf{H}\hat{\mathbf{z}}$. Iterated backprojection (IBP) procedures update the estimate of the SR reconstruction by *backprojecting* the error between the j^{th} simulated LR images $\hat{\mathbf{Y}}^{(j)}$ and the observed LR images $\mathbf{Y}$ via the backprojection operator $\mathbf{H}^{BP}$ which apportions "blame" to pixels in the SR estimate $\hat{\mathbf{z}}^{(j)}$. Typically $\mathbf{H}^{BP}$

approximates $\mathbf{H}^{-1}$. Algebraically,

$$\begin{aligned}\hat{\mathbf{z}}^{(j+1)} &= \hat{\mathbf{z}}^{(j)} + \mathbf{H}^{BP}\left(\mathbf{Y} - \hat{\mathbf{Y}}^{(j)}\right) \\ &= \hat{\mathbf{z}}^{(j)} + \mathbf{H}^{BP}\left(\mathbf{Y} - \mathbf{H}\hat{\mathbf{z}}^{(j)}\right).\end{aligned} \tag{4}$$

Equation (4) is iterated until some error criterion dependent on $\mathbf{Y}, \hat{\mathbf{Y}}^{(j)}$ is minimized. Application of the IBP method may be found in [9]. IBP enforces that the SR reconstruction match (via the observation equation) the observed data. Unfortunately the SR reconstruction is not unique since SR is an ill-posed inverse problem. Inclusion of *a-priori* constraints is not easily achieved in the IBP method.

3.3. Stochastic SR Reconstruction Methods

Stochastic methods (Bayesian in particular) which treat SR reconstruction as a statistical estimation problem have rapidly gained prominence since they provide a powerful theoretical framework for the inclusion of *a-priori* constraints necessary for satisfactory solution of the ill-posed SR inverse problem. The observed data $\mathbf{Y}$, noise $\mathbf{N}$ and SR image $\mathbf{z}$ are assumed stochastic. Consider now the *stochastic* observation equation $\mathbf{Y} = \mathbf{H}\mathbf{z} + \mathbf{N}$. The *Maximum A-Posteriori* (MAP) approach to estimating $\mathbf{z}$ seeks the estimate $\hat{\mathbf{z}}_{\mathrm{MAP}}$ for which the *a-posteriori* probability, $\Pr\{\mathbf{z}|\mathbf{Y}\}$ is a maximum. Formally, we seek $\hat{\mathbf{z}}_{\mathrm{MAP}}$ as,

$$\begin{aligned}\hat{\mathbf{z}}_{\mathrm{MAP}} &= \arg\max_{\mathbf{z}}\left[\Pr\{\mathbf{z}|\mathbf{Y}\}\right] \\ &= \arg\max_{\mathbf{z}}\left[\log\Pr\{\mathbf{Y}|\mathbf{z}\} + \log\Pr\{\mathbf{z}\}\right].\end{aligned} \tag{5}$$

The second line is found by applying Bayes' rule, recognizing that $\hat{\mathbf{z}}_{\mathrm{MAP}}$ is independent of $\Pr\{\mathbf{Y}\}$ and taking logarithms. The term $\log\Pr\{\mathbf{Y}|\mathbf{z}\}$ is the *log-likelihood function* and $\Pr\{\mathbf{z}\}$ is the *a-priori density* of $\mathbf{z}$. Since $\mathbf{Y} = \mathbf{H}\mathbf{z} + \mathbf{N}$, the likelihood function is determined by the PDF of the noise as $\Pr\{\mathbf{Y}|\mathbf{z}\} = f_{\mathbf{N}}(\mathbf{Y} - \mathbf{H}\mathbf{z})$. It is common to utilize Markov random field (MRF) image models as the prior term $\Pr\{\mathbf{z}\}$. Under typical assumptions of Gaussian noise the prior may be chosen to ensure a convex optimization in (5) enabling the use of descent optimization procedures. Examples of the application of Bayesian methods to SR reconstruction may be found in [15] using a Huber MRF and [3, 7] with a Gaussian MRF.

Maximum likelihood (ML) estimation has also been applied to SR reconstruction [18]. ML estimation is a special case of MAP estimation (no prior term). Since the inclusion of *a-priori* information is essential for the solution of ill-posed inverse problems, MAP estimation should be used in preference to ML.

A major advantage of the Bayesian framework is the direct inclusion of *a-priori* constraints on the solution, often as MRF priors which provide a powerful method for image modeling using (possibly non-linear) local neighbor interaction. MAP estimation with convex priors implies a globally convex optimization, ensuring solution existence and uniqueness allowing the application of efficient descent optimization methods. Simultaneous motion estimation and restoration is also possible [7]. The rich area of statistical estimation theory is directly applicable to stochastic SR reconstruction methods.

3.4. Set Theoretic Reconstruction Methods

Set theoretic methods, especially the method of projection onto convex sets (POCS), are popular as they are simple, utilize the powerful spatial domain observation model, and allow convenient inclusion of *a priori* information. In set theoretic methods, the space of SR solution images is intersected with a set of (typically convex) constraint sets representing desirable SR image characteristics such as positivity, bounded energy, fidelity to data, smoothness etc., to yield a reduced solution space. POCS refers to an iterative procedure which, given any point in the space SR images, locates a point which satisfies all the convex constraint sets.

Convex sets which represent constraints on the solution space of $\mathbf{z}$ are defined. Data consistency is typically represented by a set $\{\mathbf{z} : |\mathbf{Y} - \mathbf{H}\mathbf{z}| < \delta_0\}$, positivity by $\{\mathbf{z} : z_i > 0\ \forall i\}$, bounded energy by $\{\mathbf{z} : \|\mathbf{z}\| \leq E\}$, compact support $\{\mathbf{z} : z_i = 0, i \in \mathcal{A}\}$ and so on. For each convex constraint set so defined, a *projection operator* is determined. The projection operator $\mathcal{P}_\alpha$ associated with the constraint set $\mathcal{C}_\alpha$ projects a point in the space of $\mathbf{z}$ onto the closest point on the surface of $\mathcal{C}_\alpha$. It can be shown that repeated application of the iteration, $\mathbf{z}^{(n+1)} = \mathcal{P}_1\mathcal{P}_2\mathcal{P}_3\cdots\mathcal{P}_K\mathbf{z}^{(n)}$ will result in convergence to a solution on the surface of the intersection of the K convex constraints sets. Note that this point is in general non-unique and is dependent on the initial guess. POCS reconstruction methods have been successfully applied to sophisticated observation and degradation models [13, 6].

An alternate set theoretic SR reconstruction method [19] uses an ellipsoid to bound the constraint sets. The centroid of this ellipsoid is taken as the SR estimate. Since direct computation of this point is infeasible, an iterative solution method is used.

The advantages of set theoretic SR reconstruction techniques were discussed at the beginning of this section. These methods have the disadvantages of non-uniqueness of solution, dependence of the solution on the initial guess, slow convergence and high computational cost. Though the bounding ellipsoid method ensures a unique solution, this solution is has no claim to optimality.

3.5. Hybrid ML/MAP/POCS Methods

Work has been undertaken on combined ML/MAP/POCS based approaches to SR reconstruction [15, 5]. The desirable characteristics of stochastic estimation and POCS are combined in a hybrid optimization method. The *a-posteriori* density or likelihood function is maximized subject to containment of the solution in the intersection of the convex constraint sets.

3.6. Optimal and Adaptive Filtering

Inverse filtering approaches to SR reconstruction have been proposed, however these techniques are limited in terms of inclusion of *a-priori* constraints as compared with POCS or Bayesian methods and are mentioned only for completeness. Techniques based on adaptive filtering, have also seen application in SR reconstruction [14, 4]. These methods are in effect LMMSE estimators and do not include non-linear *a-priori* constraints.

3.7. Tikhonov-Arsenin Regularization

Due the the ill-posedness of SR reconstruction, Tikhonov-Arsenin regularized SR reconstruction methods have been examined [8]. The regularizing functionals characteristic of this approach are typically special cases of MRF priors in the Bayesian framework.

4. Summary and Comparison

A general comparison of frequency and spatial domain SR reconstructions methods is presented in Table 1.

	Freq. Domain	**Spat. Domain**
Observation model	Frequency domain	Spatial domain
Motion models	Global translation	Almost unlimited
Degradation model	Limited, LSI	LSI or LSV
Noise model	Limited, SI	Very Flexible
SR Mechanism	De-aliasing	De-aliasing *A-priori* info
Computation req.	Low	High
A-priori info	Limited	Almost unlimited
Regularization	Limited	Excellent
Extensibility	Poor	Excellent
Applicability	Limited	Wide
App. performance	Good	Good

Table 1. Frequency vs. spatial domain SR

Spatial domain SR reconstruction methods, though computationally more expensive, and more complex than their frequency domain counterparts, offer important advantages in terms of flexibility. Two powerful classes of spatial domain methods; the Bayesian (MAP) approach and the set theoretic POCS methods are compared in Table 2.

	Bayesian (MAP)	**POCS**
Applicable theory	Vast	Limited
A-priori info	Prior PDF Easy to incorporate No hard constraints	Convex Sets Easy to incorporate Powerful yet simple
SR solution	Unique MAP estimate	Non-unique $\cap$ of constraint sets
Optimization	Iterative	Iterative
Convergence	Good	Possibly slow
Computation req.	High	High
Complications	Optimization under non-convex priors	Defn. of projection operators

Table 2. MAP vs. POCS SR

5. Directions for Future Research

Three research areas promise improved SR methods:

Motion Estimation: SR enhancement of arbitrary scenes containing global, multiple independent motion, occlusions, transparency etc. is a focus of SR research. Achieving this is critically dependent on robust, model based, sub-pixel accuracy motion estimation and segmentation techniques – presently an open research problem. Motion is typically estimated from the observed *undersampled* data – the reliability of these estimates should be investigated. Simultaneous multi-frame motion estimation should provide performance and reliability improvements over common two frame techniques. For non-parametric motion models, constrained motion estimation methods which ensure consistency in motion maps should be used. Regularized motion estimation methods should be utilized to resolve the ill-posedness of the motion estimation problem. Sparse motion maps should be considered. Sparse maps typically provide accurate motion estimates in areas of high spatial variance – exactly where SR techniques deliver greatest enhancement. Reliability measures associated with motion estimates should enable weighted reconstruction. Global and local motion models, combined with iterative motion estimation, identification and segmentation provide a framework for general scene SR enhancement. Independent model based motion predictors and estimators should be utilized for independently moving objects. Simultaneous motion estimation and SR reconstruction approaches should yield improvements in both motion estimates and SR reconstruction.

Degradation Models: Accurate degradation (observation) models promise improved SR reconstructions. Several SR application areas may benefit from improved degradation modeling. Only recently has color SR reconstruction been addressed [16]. Improved motion estimates and reconstructions are possible by utilizing correlated information in color bands. Degradation models for lossy compression schemes (color subsampling and quantization effects) promise improved reconstruction of compressed video. Similarly, considering degradations inherent in magnetic media recording and playback are expected to improve SR reconstructions from low cost camcorder data. The response of typical commercial CCD arrays departs considerably from the simple integrate and sample model prevalent in much of the literature. Modeling of sensor geometry, spatio-temporal integration characteristics, noise and readout effects promise more realistic observation models which are expected to result in SR reconstruction performance improvements.

Restoration Algorithms: MAP and POCS based algorithms are very successful and to a degree, complimentary. Hybrid MAP/POCS restoration techniques will combine the mathematical rigor and uniqueness of solution of MAP estimation with the convenient *a-priori* constraints of POCS. The hybrid method is MAP based but with constraint projections inserted into the iterative maximization of the *a-posteriori* density in a generalization of the constrained MAP optimization of [15]. Simultaneous motion estimation and restoration yields improved reconstructions since motion estimation and reconstruction are interrelated. Separate motion estimation and restoration, as is commonly done, is sub-optimal as a result of this interdependence. Simultaneous multi-frame SR restoration is expected to achieve higher performance since additional spatio-temporal constraints on the SR image ensemble may be included. This technique has seen limited application in SR reconstruction.

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