The Energy spectrum of Kaon from lattice QCD

S Mittal
UG student
IISER Mohali

H Saxena
Masters student
JNCASR

S Gupta
UG Student
IISER Kolkata

A Mandal
UG student
CU & IIT M

Other Contributors
P Jana, A Bora, R Rana, MN Pandey, RR Das,
C Pancharia, R Dhumane, T Bhowmik

August 12, 2024

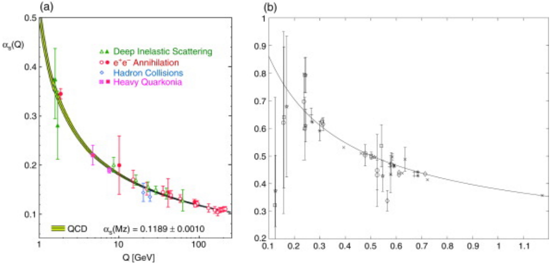
Quantum Chromodynamics (QCD)

- What are the contents of protons (or neutrons)?- quarks and gluons
- **Hadrons** is a class of **strongly interacting particles**. There are broadly two kinds of hadrons-
  - **Baryons**- 3 quarks (For e.g. protons and neutrons)
  - **Mesons**- quark and anti-quark pair (For e.g. kaons and pions)
- The theory which explains the strong interactions is known as **Quantum Chromodynamics**.
- The name comes from the Greek word '*chroma*' meaning 'color' to account for color charges in strong interactions.
- Unlike photons in QED, gluons can self-interact.

QCD (contd.)

QCD (unlike QED) cannot be solved analytically because:

- Infinte number of degrees of freedom in the lagrangian. This manifests itself physically in **hot-vacuum**
- At high energies → Weak coupling → use of Perturabtion theory
- At low energies → Strong coupling → **Lattice QCD**



Theory: Lattice QCD

Step-by-step solution starting with the quark and gluon fields:

1. Taking time as another dimension (Wick's rotation).
2. Take patches in the field space containing an infinite number of points and pixelate them.
3. Performing Monte-Carlo simulations to choose random configurations of QCD vacuum.

Now it looks like a 4D classical grid!
We can simulate it using principles of statistical mechanics.

Theory: Lattice QCD (cont.)

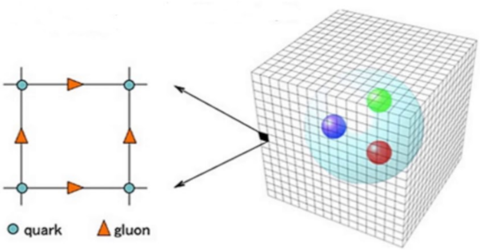


Figure: Schematic representation of lattice QCD for a Baryon (Source: YouTube)

$$\text{Lattice spacing} \longrightarrow a$$
$$\text{Lattice size} \longrightarrow L \sim Na$$

- We place quarks or antiquarks on the lattice points and gluons on the links.
- The values of observables change upon changing lattice size but these values have a simple relation with lattice size.

Theory: Kaon and 2-point function

The two-point correlation function and Effective energy.

- The goal of our analysis is to extract the Energy of the Kaon as it evolves.

From theory

$$C_M^{2pt}(t, \vec{p}) = \sum_{\vec{x}} J_M(t, \vec{x}) J_M^{\dagger}(t_i, \vec{x_i}) e^{-\iota \vec{p} \cdot \vec{x}}$$

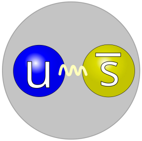


Figure: Quark structure of K^+

- Energy of the particle with spatial momentum $\vec{p}$.

$$E_{eff}(t_j) = \ln \left[\frac{C^{2pt}(t_j)}{C^{2pt}(t_{j+1})} \right]$$

Theoretical framework: Jackknife Resampling

- Systematic way of determining standard error in a set of random measurement.

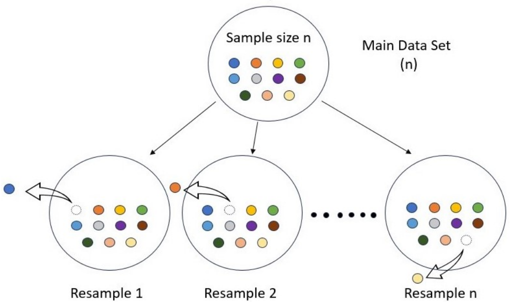


Figure: Schematic representation of Jackkninfe resampling (Source: Wikipedia)

$$f_m = \sum_{m \neq n} \frac{f(x_n)}{N-1} \qquad \langle f \rangle = \sum_m \frac{f_m}{N_{bin}} \quad , \quad N_{bin} = N$$

$$\delta f = \sqrt{\frac{N-1}{N} \sum_m (f_m - \langle f \rangle)^2}$$

Procedure

The procedure to find dispersion relation for the kaon from the simulation data involves the following steps broadly:

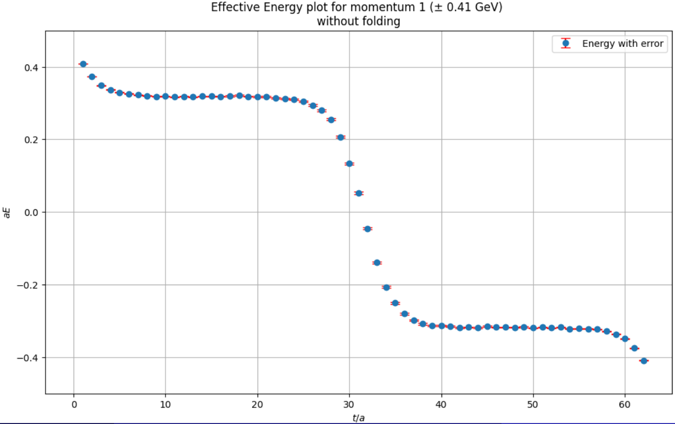
1. Extract the real part of the 2pt function values from the given data for all the time steps from 0 to 63. This step is done for all the N configurations.

P_3 [GeV]	0	±0.41	±0.83	±1.24	±1.66	±2.07
N_{confs}	1,198	1,198	1,198	1,198	1,198	1,198
N_{src}	1	8	8	8	24	200
N_{tot}	1,198	9,584	9,584	9,584	28,752	239,600

Table: Statistics for the kaon matrix elements are shown for different momenta boosts. N_{confs}, N_{src}, N_{total} are the number of configurations, source positions per configuration, and total statistics, respectively.

Procedure(contd.)

2. For cases where the momentum boost of the particle is non-zero in z-direction, average of 2pt function values for both positive and negative values of p_z is taken.
3. The average for the values of 2pt function at the i^{th} time step and $(64 - i)^{th}$ time step is also taken for all $i \in 1, 2, ..., 31$.



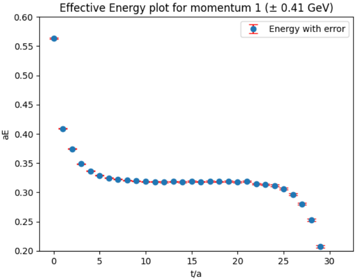
Procedure(contd.)

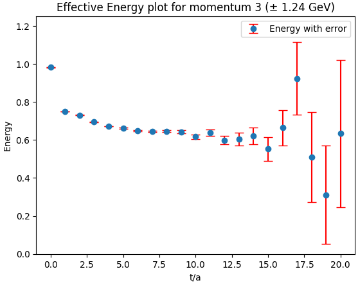
4. Jackknife resampling is carried out for each time step from 0 to 32 using the N configurations to obtain N bins.
5. The effective energy values for each jackknife bin is obtained by taking natural logarithm of the ratio of 2pt function at t and t+1 for all $t \in 0, 1, 2, ..., 31$. Then the energy values are averaged for all bins and jackknife error is found out for each time step.
6. These mean energy values are plotted on a graph along with the jackknife errors and a plateau region is is figured out on the graph by visual analysis. a plateau fit is done for each energy bin, which gives us a value for the effective energy. Then the effective energies from all the bins is average and the jackknife error is calculated.
7. These steps are repeated for different values of momentum to obtain corresponding values of energy. This data is then plotted on a energy-momentum graph, which is then compared with the theoretical dispersion relation.

Results

Initially we have considered the case where the momentum along X and Y axis is 0, i.e. $p_x = p_y = 0$. So the momentum is only along the Z axis.

- After doing all the procedure we end up with plots like this from every momentum value:

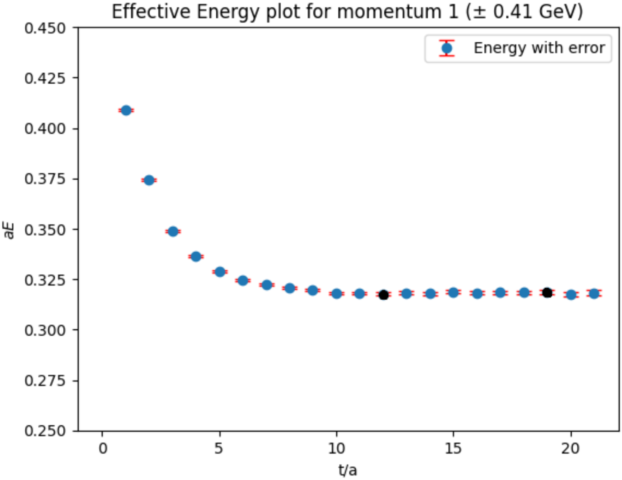




The errors become larger if the momentum of the particle is higher

Results

Effective Energy plot for momentum 1 along Z axis



The plateau region has been identified

Results: Analytical Expression for Plateau fitting

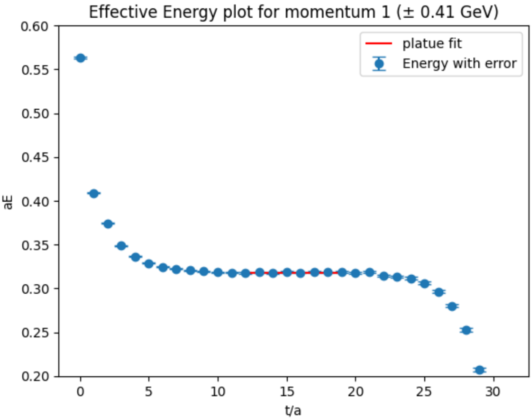
Plateau fitting

$$E_{plateau} = \frac{\sum_{t_{low}}^{t_{high}} E_{eff}(t) \left(\frac{1}{\delta E_{eff}(t)}\right)^2}{\sum_{t_{low}}^{t_{high}} \left(\frac{1}{\delta E_{eff}(t)}\right)^2}$$

Instead of using the mean we are using plateau fit so that we are giving weights to the points and the weight is inversely proportional to the square of the error associate with the energy value.

Results: Plateau fitting

Here is the plot for Effective Energy plot for momentum 1 along Z axis



$$E_{plateau-fit} = 0.31814708 \pm 2.9280 * 10^{-7}$$

Dispersion Relation

- From Einstein's Special Theory of relativity we know that

$$E = \sqrt{m^2c^4 + p^2c^2}$$

- In natural units we take

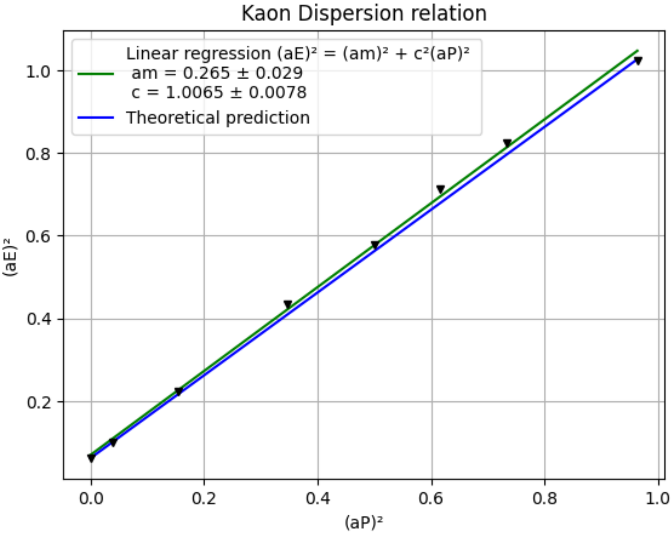
$$c = \hbar = 1$$

- So finally the equation becomes

$$a^2E^2 = a^2m^2 + a^2(p_x^2 + p_y^2 + p_z^2)$$

where is a is in lattice units

Lattice data for various momenta are shown here:



Conclusion

- Lattice data are validated by theory even for high momenta.
- No theoretical input has been made to the Lattice Data.

Acknowledgements

- Dr. Martha Constantinou
- REYES Program
- NERSC allocation for Perlmutter Supercomputer

Thank You for Your Attention