ASSIGNMENT - 1 INTERMEDIATE DISCRETE MATHEMATICS

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1. Let $\Sigma = \{v, w, x, y\}$ and $A = \bigcup_{n=1}^{4} \Sigma^{n}$. How many strings in A have xy as a proper strings?

$\Sigma = \{v, w, x, y\}$
$|\Sigma| = 4$

xy could be called as proper prefix if the complete set of strings has at least length ≥ 3.

Suppose, such a string has length i, then the remaining $(i-2)$ places of the string can be filled.

Total characters $= n = 4 = |\Sigma|$.
Length of each substring $= i$.

No. of substrings that can be formed, with xy as proper prefix

$\Rightarrow |\Sigma|^{(i-2)} = 4^{(i-2)}$.

$$\text{Total substrings} = \sum_{i=3}^{4} 4^{(i-2)}$$

$$= 20.$$

2. Let Σ be an alphabet. Let $x_i \in \Sigma$ for $1 \leq i \leq 100$ where $x_i \neq x_j$ for all $1 \leq j < i \leq 100$. How many non-empty substrings are there for the strings $s = x_1 x_2 \dots x_{100}$?

W,

i = length of the substrings.

n = length of the string.

If we have a string of length n, then there are $(n-i+1)$ ways to start an i-length substring within it.

Hence, there could be $(n-i+1)$ substrings of length n.

Thus, the total no. of non-empty strings would be :-

$$\Rightarrow \sum_{i=1}^{n} (n-i+1) \quad | \; n = 100$$

$$\Rightarrow \sum_{i=1}^{100} (101-i)$$

$$\Rightarrow 5050$$

3. For $\Sigma = \{0,1\}$ determine whether the string 00010 is in each of the following languages (taken from Σ^*)

ⓐ $\{0,1\}^*$

$00010 \in \{0,1\}^*$ as it is a concatenation of 0,0,0,1,0 . [YES]

ⓑ $\{000, 101\}, \{10, 11\}$

$000 \in \{000, 10\}$

$10 \in \{10, 11\}$

Hence,

$00010 \in \{000, 101\}.\{10, 11\}$, it is a concatenation of 000, 10 [YES]

(c) $\{00\}\{0\}^*\{10\}$

$00010 \in \{00\}\{0\}^*\{10\}$ as its a concatenation of
$00, 0, 10$. [YES]

(d) $\{000\}^*\{1\}^*\{0\}$

$00010 \in \{000\}^*\{1\}^*\{0\}$ as its a concatenation of
$000, 1, 0$. [YES]

(e) $\{00\}^*\{10\}^*$

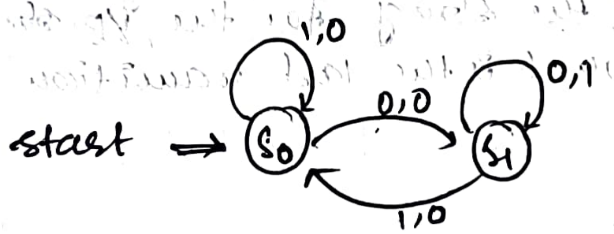
$00010 \notin \{00\},\{10\}$ since there is no way to obtain odd no. of starting zeroes in either of the languages.

(f) $\{0\}^*\{1\}^*\{0\}$

$00010 \in \{0\}^*\{1\}^*\{0\}$ as $\{000\} \in \{0\}^*$
$\{1\} \in \{1\}^*$
$\{0\} \in \{0\}^*$

Hence its a concatenation of $000, 1, 0$.

(4). Machine M has $I = \{0,1\} = O$ and is determined by the state diagram



(a) Describe in words what this finite state machine does?

It outputs 1 whenever two consecutive zeroes appear in input.

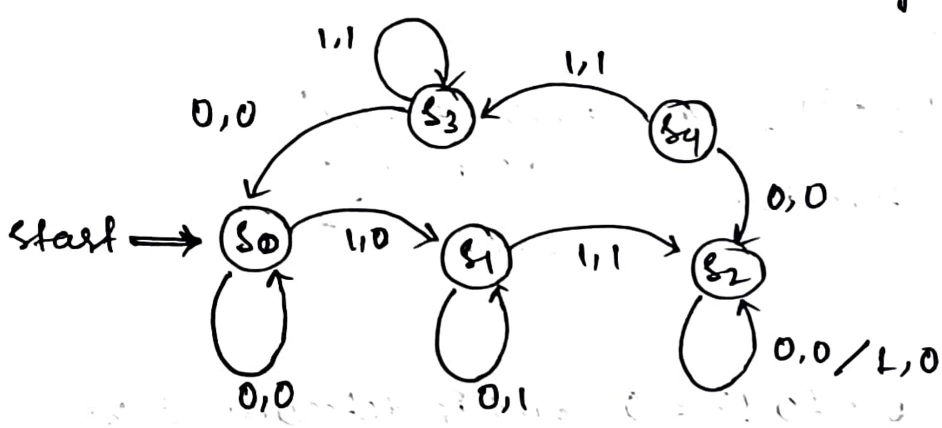
what must state s_1 remember?
The state s_1 remembers if the last input was 0.

(c) find 2 languages $A, B \subseteq \Sigma^*$ such that for every $x \in A, B$, $\omega(s_0, x)$ has 1 as a suffix.

consider $B = \{00\}$ now the last digit of output will certainly be 1. therefore, we can pick any language A.

Hence, $A = \Sigma^*$
$B = \{00\}$.

(5). A finite state machine $M = \{S, I, O, v, \omega\}$ has $I = O = \{0,1\}$ and is determined by the state diag.



(a) Determine the output of the string for the i/p string 110111 starting at s_0. what is the last transition state?

i/p	o/p	state.
1	0	s_1
1	1	s_2
0	0	s_2
1	0	s_2
1	0	s_2
1	0	s_2

output string = 010000

final state = s_2.

(b) Answer part (a) for the same string but with s_1 as the starting state. What about s_2 and s_3 as starting state.

I/p	O/p	state
1	1	s_2
1	0	s_2
0	0	s_2
1	0	s_2
1	0	s_2
1	0	s_2

110111

Beginning from state s_1.

final state = s_2.

I/p	O/p	state.
1	0	s_2
1	0	s_2
0	0	s_2
1	0	s_2
1	0	s_2
1	0	s_2

Beginning from state s_2.

final state s_2

I/p	O/p	state.
1	1	s_3
1	1	s_3
0	0	s_0
1	0	s_1
1	1	s_2
1	0	s_2

Beginning from state s_3

final state = s_2

(c) find the state table for this Machine.

PLEASE

TURN

OVER

	v		w	
	0	1	0	1
s_0	s_0	s_1	0	0
s_1	s_1	s_2	1	1
s_2	s_2	s_2	0	0
s_3	s_0	s_3	0	1
s_4	s_2	s_3	0	1

(d) In which state should we start so that the input string produces the output 10000?

As per the observation, Once you reach s_2 rest of the outputs would be 0s only.

and for getting 1 at the beginning of the o/p string we can start from s_1.

Hence starting from s_1 results the o/p string as 10000.

(e) Determine the i/p string $x \in I^*$ of minimal length such that $v(s_4, x) = s_1$. Is x unique?

The only way to get to state s_1, starting from s_4 is via s_3 and s_0, Hence length of i/p string is at least 3.

$$s_4 \xrightarrow[i/p_1]{1,1} s_3 \xrightarrow[i/p_2]{0,0} s_0 \xrightarrow[i/p_3]{1,0} s_1$$

Required i/p = 1, 0, 1.

Also, x is unique i/p string.