HODGKIN-HUXLEY MODEL

- `This model tells us how neurol generate and propogate electircal singal known action potential ( also spikes)`

  Figure 1: Membrane potential and gating variable dynamics

$$
\begin{aligned}
C_m \frac{dV}{dt} & \text{¿} \acute{g}_{Na} m^3 h (E_{Na} - V) + \acute{g}_K n^4 (E_K - V) + g_L (E_L - V) \\
\frac{dm}{dt} & \text{¿} \alpha_m(V)(1-m) - \beta_m(V) m \\
\frac{dh}{dt} & \text{¿} \alpha_h(V)(1-h) - \beta_h(V) h \\
\frac{dn}{dt} & \text{¿} \alpha_n(V)(1-n) - \beta_n(V) n
\end{aligned}
$$

Figure 2: Hodgkin–Huxley model system of differential equations

⚙ Final Two Differential Equations

(Standard 2-variable Hodgkin–Huxley reduction)

Assume $m \approx m_\infty(V)$ and $h \approx h_\infty(V)$. Then the reduced system becomes:

$$C_m \frac{dV}{dt} = -g_{Na} m_\infty(V)^3 h_\infty(V)(V - E_{Na}) - g_K n^4 (V - E_K) - g_L (V - E_L) + I_{\text{ext}}$$

$$\frac{dn}{dt} = \frac{n_\infty(V) - n}{\tau_n(V)}$$

□ Parameter Definitions

- C_m: membrane capacitance
- g_{Na}, g_K, g_L: maximum conductances
- E_{Na}, E_K, E_L: reversal potentials

And the voltage-dependent steady-state functions:

$$m_\infty(V) = \frac{\alpha_m(V)}{\alpha_m(V) + \beta_m(V)}, h_\infty(V) = \frac{\alpha_h(V)}{\alpha_h(V) + \beta_h(V)}, n_\infty(V) = \frac{\alpha_n(V)}{\alpha_n(V) + \beta_n(V)}$$

Time constant for n:

$$\tau_n(V) = \frac{1}{\alpha_n(V) + \beta_n(V)}$$

Parameter roster

Mission Parameters: Hodgkin-Huxley Squid Giant Axon Model

Parameter	Symbol	Value	Units	Strategic Purpose
Membrane Capacitance	`C_m`	`1.0`	`µF/cm²`	Stores the electrical charge across the membrane.
Max Sodium Conductance	`g_Na`	`120.0`	`mS/cm²`	Determines the maximum possible influx of sodium ions; powers the spike's rising phase.
Max Potassium Conductance	`g_K`	`36.0`	`mS/cm²`	Determines the maximum possible efflux of potassium ions; drives the spike's falling phase.
Leak Conductance	`g_L`	`0.3`	`mS/cm²`	Represents the passive flow of other ions, primarily chloride; stabilizes the resting potential.
Sodium Potential	`E_Na`	`50.0`	`mV`	The Nernst potential for sodium. The 'target' voltage the spike aims for during its upstroke.
Potassium Potential	`E_K`	`-77.0`	`mV`	The Nernst potential for potassium. The

Parameter	Symbol	Value	Units	Strategic Purpose
				'target' voltage for hyperpolarization.
Leak Potential	`E_L`	`-54.387`	`mV`	The combined potential for leak ions, chosen to ensure the resting state is stable.

These constant are measured and pysically grounded

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using DifferentialEquations, Plots
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