PROBABILITY DISTRIBUTION FUNCTIONS

In [21]:
```
import numpy as np
import scipy as sp
import scipy.special as spsp
import matplotlib.pyplot as plt
```

Binomial Distribution

The probability density function (PDF) of the Binomial distribution is:

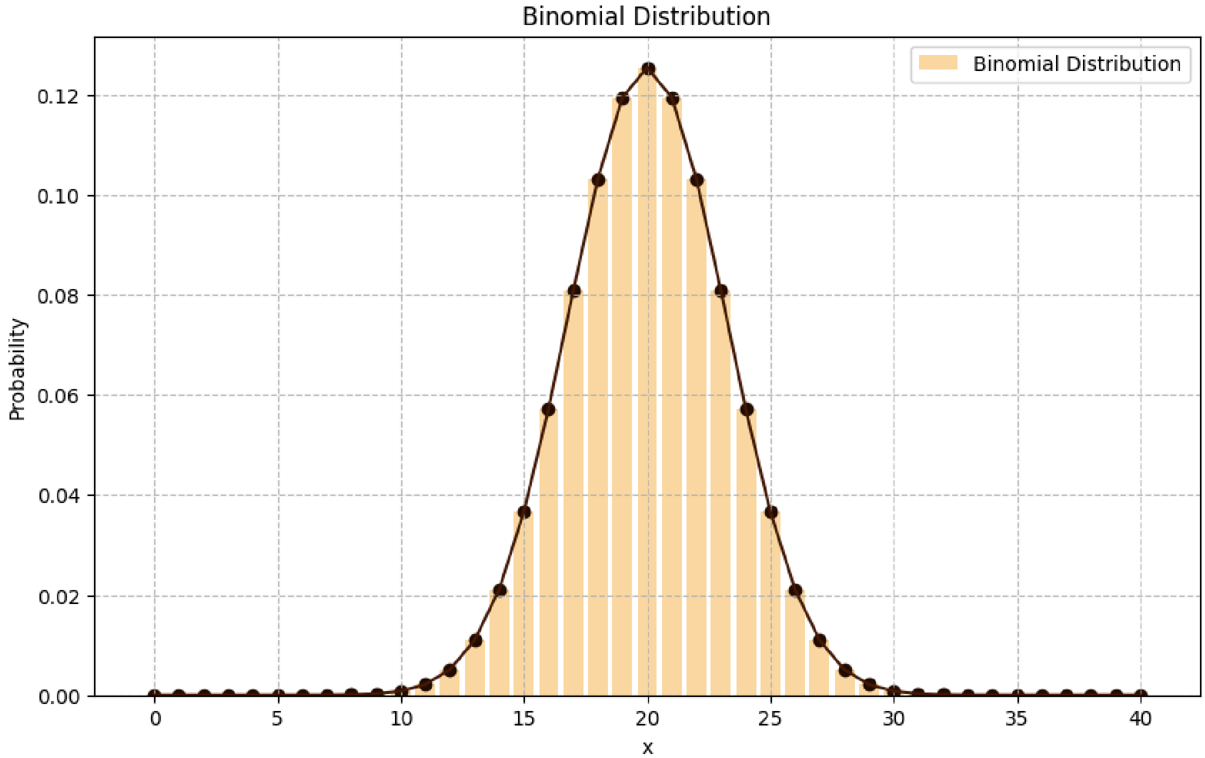
$$f(x) = P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Where:

- (n) is the number of trials,
- (k) is the number of successes,
- (p) is the probability of success in a single trial,
- ($\binom{n}{k}$) is the binomial coefficient, calculated as:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

In [22]:
```
n = 40
p = 0.5
q = 1 - p

BinomialPDF = []
x = []

for k in range(n+1):
    y = spsp.comb(n, k) * p**k * q**(n-k)
    BinomialPDF.append(y)
    x.append(k)

plt.figure(figsize=(10, 6))
plt.bar(x, BinomialPDF, label="Binomial Distribution", color="#fadaa0")
plt.plot(x, BinomialPDF, color="#451a08")
plt.scatter(x, BinomialPDF, color="#260e03")
plt.title("Binomial Distribution")
plt.legend()
plt.xlabel("x")
plt.ylabel("Probability")
plt.grid(linestyle='--', alpha=0.8)
plt.show()
```



Normal Distribution

The probability density function (PDF) of the normal distribution is:

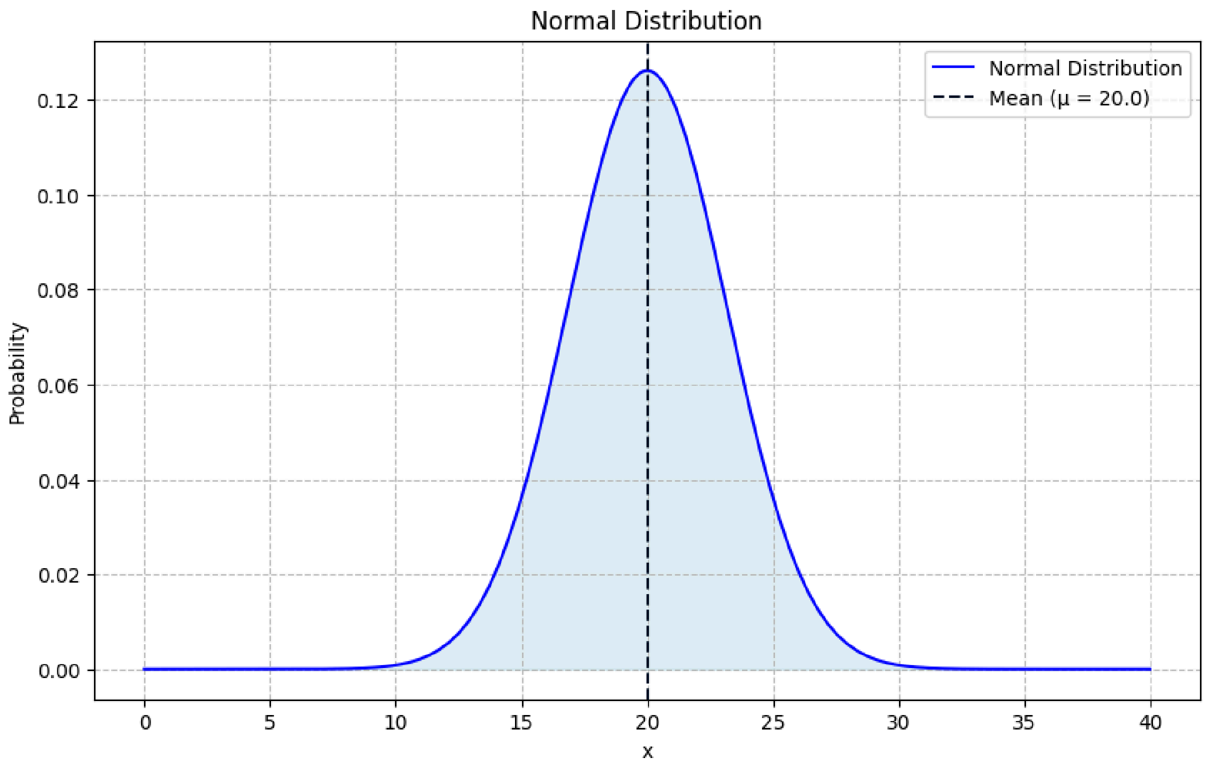
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Where:

- μ is the mean,
- σ is the standard deviation,
- x is the variable.

In [23]:
```
n = 40
p = 0.5
q = 1 - p

sigma = np.sqrt(n*p*q)
mean = n*p

NormalPDF = []
X = np.arange(0, n, 0.01)

for x in X:
    y = (1/ (sigma*np.sqrt(2*np.pi))) * np.exp((-1/2) * ((x-mean)/sigma)**2)
    NormalPDF.append(y)

plt.figure(figsize=(10, 6))
plt.plot(X, NormalPDF, label="Normal Distribution", c="blue")
```

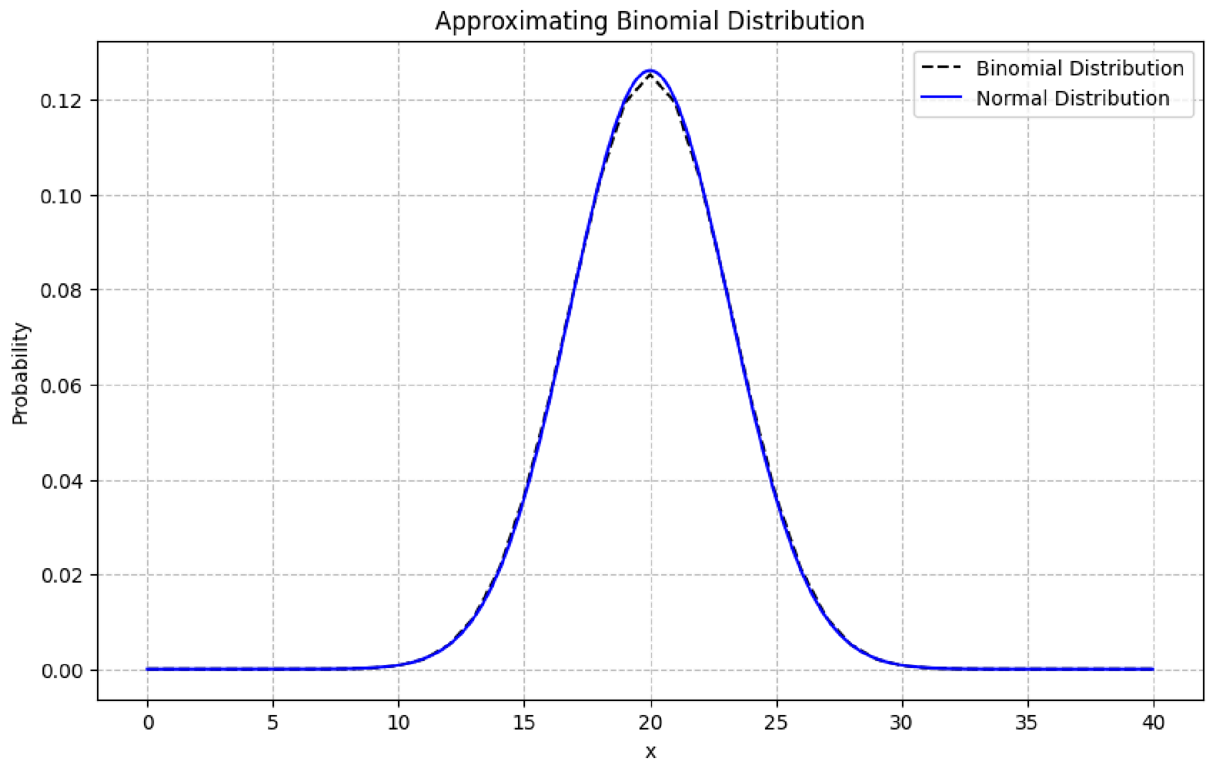
```
plt.fill_between(X, NormalPDF, where=(X>0), color="lightblue", alpha=0.4)
plt.axvline(mean, color="#031126", linestyle="dashed", label=f"Mean (μ = {me
plt.title("Normal Distribution")
plt.legend()
plt.xlabel("x")
plt.ylabel("Probability")
plt.grid(linestyle='--', alpha=0.8)
plt.show()
```



Checking Approximation

In [24]:

```
plt.figure(figsize=(10, 6))
plt.plot(BinomialPDF, label="Binomial Distribution", c="black", linestyle="c
plt.plot(X, NormalPDF, label="Normal Distribution", c="blue")
plt.title("Approximating Binomial Distribution")
plt.legend()
plt.xlabel("x")
plt.ylabel("Probability")
plt.grid(linestyle='--', alpha=0.8)
plt.show()
```

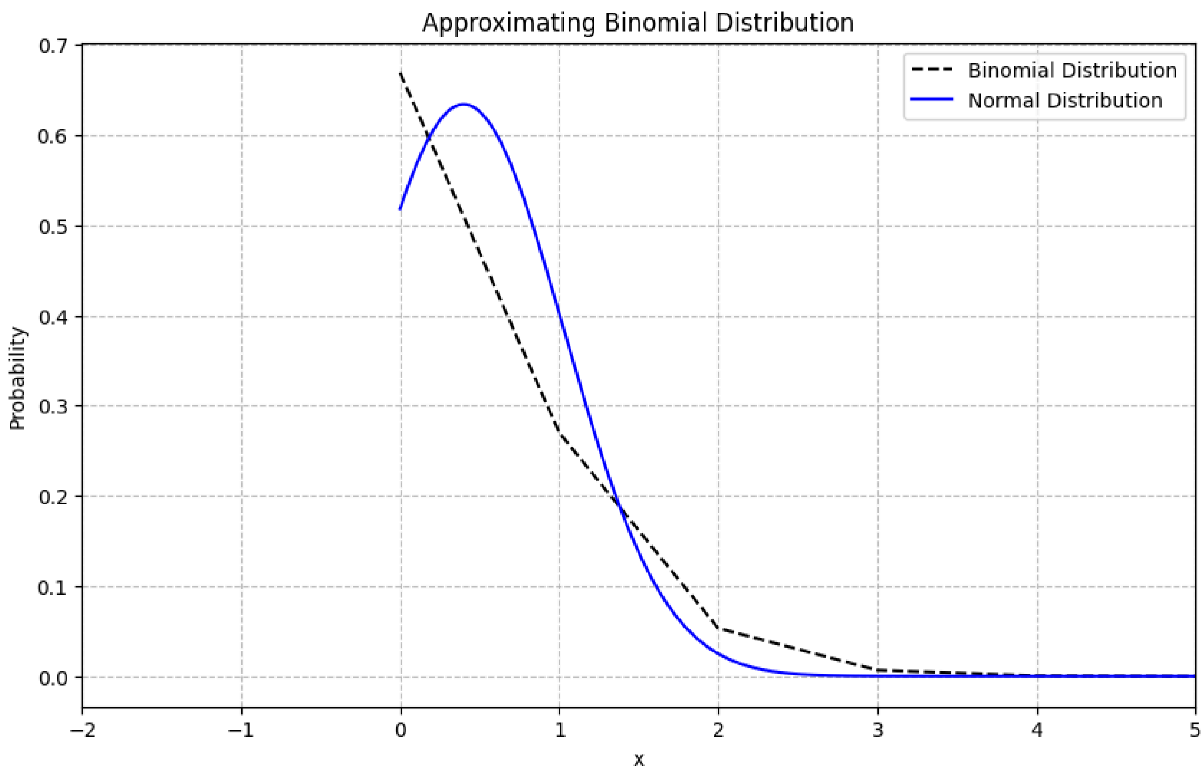


Limits of Normal Distribution

- small number of trails (n)
- small probability of success (p)

In [25]:
```
n = 40
p = 0.01
q = 1 - p


BinomialPDF = []
x = []

for k in range(n+1):
    y = spsp.comb(n, k) * p**k * q**(n-k)
    BinomialPDF.append(y)
    x.append(k)

sigma = np.sqrt(n*p*q)
mean = n*p

NormalPDF = []
X = np.arange(0, n, 0.01)

for x in X:
    y = (1/ (sigma*np.sqrt(2*np.pi))) * np.exp((-1/2) * ((x-mean)/sigma)**2)
    NormalPDF.append(y)

plt.figure(figsize=(10, 6))
plt.plot(BinomialPDF, label="Binomial Distribution", c="black", linestyle="d
```

```
plt.plot(X, NormalPDF, label="Normal Distribution", c="blue")
plt.title("Approximating Binomial Distribution")
plt.legend()
plt.xlim(-2, 5)
plt.xlabel("x")
plt.ylabel("Probability")
plt.grid(linestyle='--', alpha=0.8)
plt.show()
```



Poisson Distribution

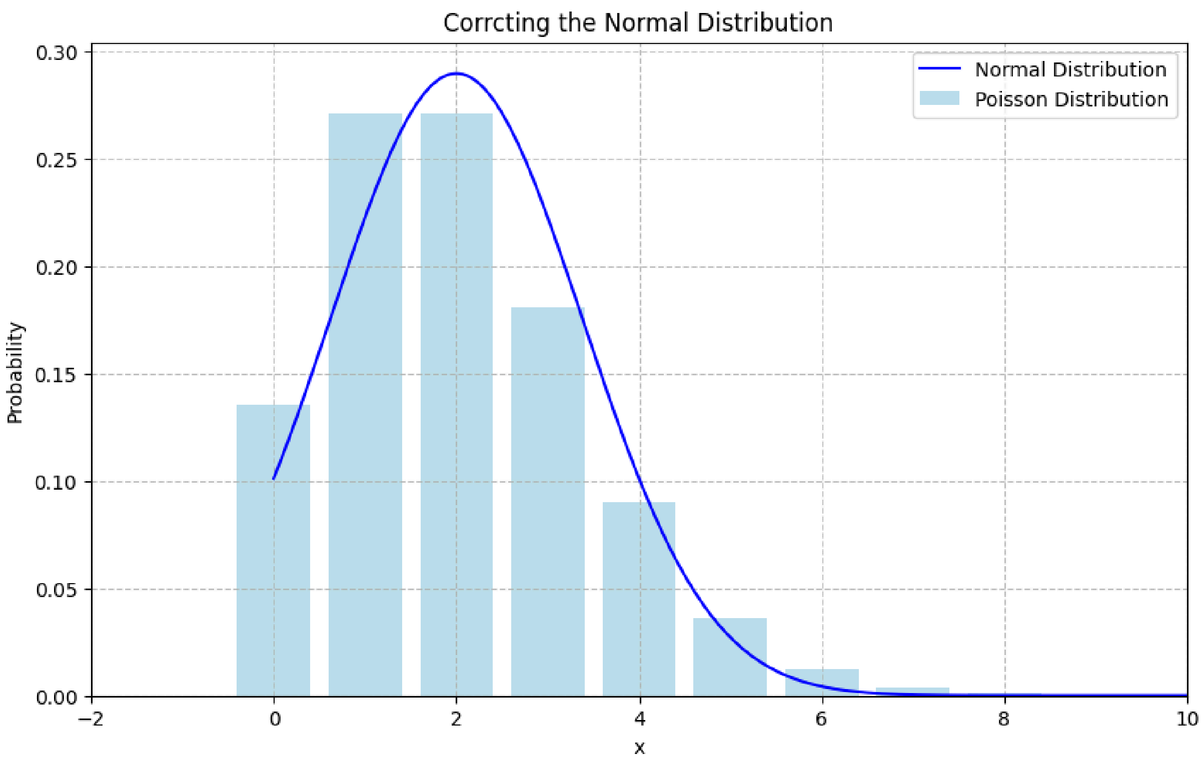
$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where,

- λ is (np) , (n) is the number of trials and (p) is the probability of success

In [26]:
```
n = 40
p = 0.05
q = 1 - p

l = n * p

X_poisson = np.arange(0, n)

PoissonPDF = []
for x in X_poisson:
    y = (l**x) * (np.exp(-l)) / spsp.factorial(x)
    PoissonPDF.append(y)
```

```
sigma = np.sqrt(n * p * q)
mean = n * p

X_normal = np.arange(0, n, 0.01)

NormalPDF = (1 / (sigma * np.sqrt(2 * np.pi))) * np.exp(-0.5 * ((X_normal -

# Plot
plt.figure(figsize=(10, 6))
plt.bar(X_poisson, PoissonPDF, label="Poisson Distribution", color="lightblu
plt.plot(X_normal, NormalPDF, label="Normal Distribution", c="blue")
plt.title("Corrcting the Normal Distribution")
plt.legend()
plt.xlim(-2, 10)
plt.grid(linestyle='--', alpha=0.8)
plt.xlabel("x")
plt.ylabel("Probability")
plt.show()
```



In [ ]: