Assignment 01

ME 670: Advanced Computational Fluid Dynamics

Lid Driven Cavity

Using Finite Volume Method with SIMPLE Algorithm

Nirmal S.
[234103107]
Computational Mechanics
MTech. Mechanical Engineering
IIT-Guwahati

Table of Contents

Problem Statement:

Consider the lid-drive cavity model problem shown in fig. 1. The square cavity is formed by three stationary walls (sides and bottom) and one moving (top) wall of length L. The cavity is very long along the z-direction. It is filled with a Newtonian fluid of density ρ and viscosity μ. The top lid/wall moves with a velocity $(u, v) = (U, 0)\, m/s$. The flow can be assumed two-dimensional, incompressible, steady, and isothermal. Solve the non-dimensional steady Navier-Stokes equations using the finite volume method on a staggered grid and SIMPLE scheme.

Use a 129×129 uniform and Cartesian finite volume grid. Use the Hybrid differencing scheme for solving the momentum equations. For convergence, use the $\|L\infty\|<10{-}5$ criterion. For the solution of the discretized equations, you can use the point Gauss-Seidel (GS) or ADI method.

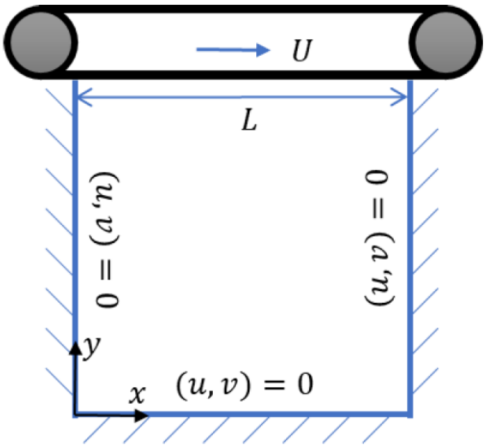


Figure 1: A Schematic of the lid-driven cavity problem

For Re $(=\rho U/L)$ = 100, 400 and 1000

1. Show contours for velocity magnitude and vorticity (ω). Also show streamline pattern by plotting the contours of stream function (ψ).
2. Plot the x-velocity profile at $x{=}L/2$, and x-velocity profile at $y{=}L/2$. Compare your results by plotting the values from Tables I and II in the paper by Ghia et al.
3. Compare the following results for the primary vortex given in Table V of Ghia et al. with your results: $\psi_{\min}$, location (x, y) of $\psi_{\min}$ and the value of ω at the location of $\psi_{\min}$.

Grid Details

A uniform, cartesian, staggered grid of size 129×129 is used to solve the problem. The control volume of the problem is given by the double red line.

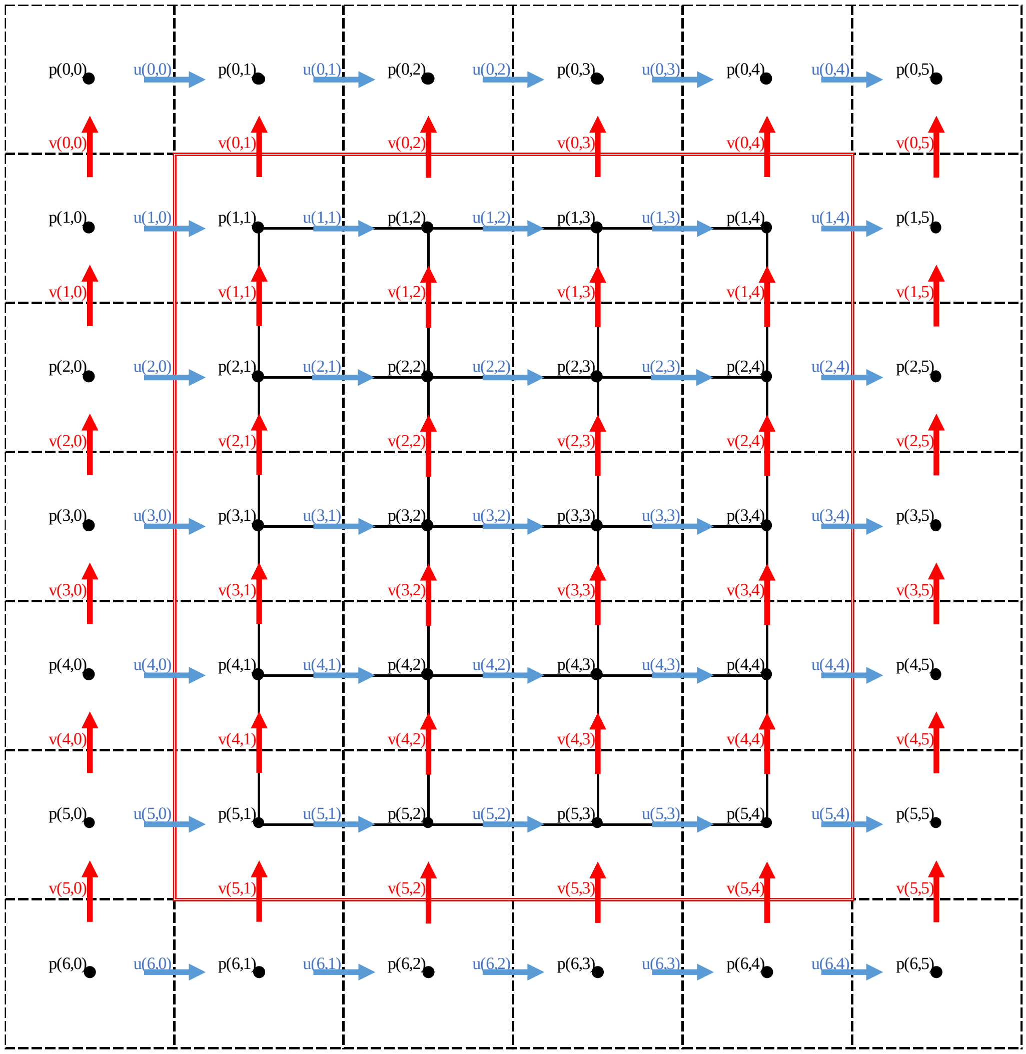


Figure 2: Staggered grid type used in problem. The grid here is shown for a 6 × 5 collocated grid with one extra layer of control volume cell around it.

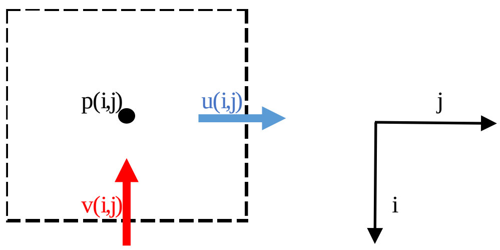


Figure 3: Control volume cell and the (i,j) notation of pressure, u-velocity and v-velocity. The direction of i and j is shown as well.

Discretised Equations

Navier Stokes Equation:

Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \rho \mathbf{u} = 0$$

Incompressibility condition:

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} \rho = 0 \quad \text{or} \quad \boldsymbol{\nabla} \cdot \mathbf{u} = 0$$

Momentum Equation:

$$\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{uu} = -\frac{1}{\rho} \boldsymbol{\nabla} \mathrm{p} + \nu \nabla^2 \mathbf{u} + \boldsymbol{g}$$

Here, $\rho = \textit{density of fluid}$
$\mathbf{u} = \text{velocity vector of fluid}$
$\boldsymbol{g} = \text{gravity vector}$

Getting the momentum equation to non-dimensional form:

$$\frac{U^2}{L}\left[\frac{\partial\left(\frac{\mathbf{u}}{U}\right)}{\partial\left(\frac{t}{L/U}\right)} + L\boldsymbol{\nabla} \cdot \frac{\mathbf{u}}{U}\frac{\mathbf{u}}{U}\right] = \frac{U^2}{L}\left[-L\boldsymbol{\nabla}\frac{\mathrm{p}}{\rho U^2} + \frac{\nu}{\mathrm{UL}}(L\nabla)^2\frac{\mathbf{u}}{U} + \frac{\boldsymbol{g}L}{U^2}\right]$$

$$\frac{\partial \mathbf{u}^*}{\partial t^*} + \boldsymbol{\nabla}^* \cdot \mathbf{u}^*\mathbf{u}^* = -\boldsymbol{\nabla}^*\mathrm{p}^* + \frac{1}{Re}\nabla^{*2}\mathbf{u}^* + \frac{1}{Fr^2}\widehat{\boldsymbol{g}}$$

Here, $x^* = \frac{x}{L}$, $\boldsymbol{\nabla}^* = L\boldsymbol{\nabla}$, $\mathbf{u}^* = \frac{\mathbf{u}}{U}$, $t^* = \frac{t}{L/U}$,
$p^* = \frac{\mathrm{p}}{\rho U^2}$, $Fr = \sqrt{\frac{U^2}{gL}}$, $Re = \frac{UL}{\nu}$

In indicial notation it can be simplified to:

$$\frac{\partial \mathbf{u}_i^*}{\partial t^*} + \frac{\partial}{\partial x_j^*}\left(\mathbf{u}_i^*\mathbf{u}_j^*\right) = -\frac{\partial p^*}{\partial x_i^*} + \frac{1}{Re}\frac{\partial^2 \mathbf{u}_i^*}{\partial x_i^* \partial x_j^*} - \frac{1}{Fr^2}\widehat{\boldsymbol{g}_i}$$

Considering ***steady state*** and ***without influence of gravity***, the equation becomes:

$$\frac{\partial}{\partial x_j^*}\left(\mathbf{u}_i^*\mathbf{u}_j^*\right) = -\frac{\partial p^*}{\partial x_i^*} + \frac{1}{Re}\frac{\partial^2 \mathbf{u}_i^*}{\partial x_i^* \partial x_j^*}$$

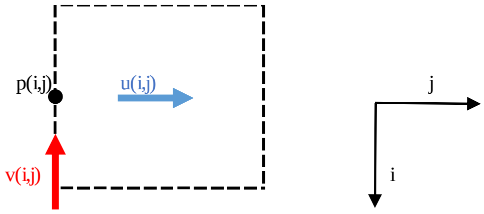
For **simplicity**, let us rewrite [ ]* as [ ] making the above equation as:

$$\frac{\partial}{\partial x_j}\left(\mathbf{u}_i\mathbf{u}_j\right) = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \mathbf{u}_i}{\partial x_i \partial x_j}$$

For the finite volume we consider, the above equation can be integrated as

$$\iiint\limits_V \boldsymbol{\nabla} \cdot \mathbf{uu}\, dV = -\iiint\limits_V \boldsymbol{\nabla}\mathrm{p}\, dV + \frac{1}{Re}\iiint\limits_V \nabla^2 \mathbf{u}\, dV$$

X-Momentum Equation in 2D for u-control volume:



Integrating each term,

$$\int_s^n \int_w^e u\frac{\partial u}{\partial x} dx\, dy = F_e u_e - F_w u_w$$

$$\int_s^n \int_w^e v\frac{\partial u}{\partial y} dx\, dy = F_n u_n - F_s u_s$$

$$\int_s^n \int_w^e -\frac{\partial p}{\partial x} dx\, dy = -(p_e - p_w)A_e$$

$$\int_s^n \int_w^e \frac{1}{Re}\frac{\partial^2 u}{\partial x^2}\, dx\, dy = \frac{1}{Re}\left(\frac{\partial u}{\partial x}\Big|_e - \frac{\partial u}{\partial x}\Big|_w\right)A_e$$

$$\int_s^n \int_w^e \frac{1}{Re}\frac{\partial^2 u}{\partial y^2}\, dx\, dy = \frac{1}{Re}\left(\frac{\partial u}{\partial y}\Big|_n - \frac{\partial u}{\partial y}\Big|_s\right)A_n$$

Where:

$$F_e = \left[\frac{u_i^{j+1} + u_i^j}{2}\right]\Delta y \times 1; \quad F_w = \left[\frac{u_i^j + u_i^{j-1}}{2}\right]\Delta y \times 1$$

$$F_n = \left[\frac{v_{i-1}^{j+1} + v_{i-1}^j}{2}\right]\Delta x \times 1; \quad F_s = \left[\frac{v_i^{j+1} + v_i^j}{2}\right]\Delta x \times 1$$

$$D_e = D_w = \frac{1}{Re}\frac{\Delta y \times 1}{\Delta x}; \quad D_n = D_s = \frac{1}{Re}\frac{\Delta x \times 1}{\Delta y}$$

We get the x-momentum equation in form of:

$$a_P u_P = \sum_{nb} a_{nb} u_{nb} - (p_e - p_w)A_e$$

From Hybrid Scheme, the coefficients are calculated as:

$$a_E = \max\left[-F_e, D_e - \frac{F_e}{2}, o\right]; \quad a_W = \max\left[F_w, D_w + \frac{F_w}{2}, o\right];$$

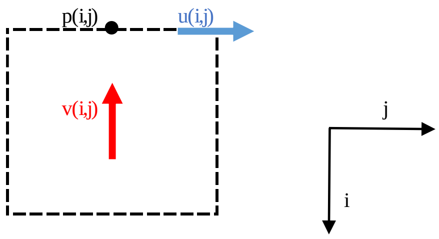
$$a_N = \max\left[-F_n, D_n - \frac{F_n}{2}, o\right]; \quad a_S = \max\left[F_s, D_s + \frac{F_s}{2}, o\right];$$

$$a_P = a_E + a_W + a_N + a_S + (F_e - F_w + F_n - F_s);$$

$$A_e = \Delta y \times 1; \quad A_n = \Delta x \times 1$$

$$p_e = p_i^{j+1}; \quad p_w = p_i^j$$

Y-Momentum Equation in 2D for v-control volume:



Integrating each term,

$$\int_s^n \int_w^e u\frac{\partial v}{\partial x} dx\, dy = F_e v_e - F_w v_w$$

$$\int_s^n\int_w^e v\frac{\partial v}{\partial y}dx\,dy = F_n v_n - F_s v_s$$
$$\int_s^n\int_w^e -\frac{\partial p}{\partial y}dx\,dy = -(p_n - p_s)A_n$$
$$\int_s^n\int_w^e \frac{1}{Re}\frac{\partial^2 v}{\partial x^2}\,dx\,dy = \frac{1}{Re}\left(\frac{\partial v}{\partial x}|_e - \frac{\partial v}{\partial x}|_w\right)A_e$$
$$\int_s^n\int_w^e \frac{1}{Re}\frac{\partial^2 v}{\partial y^2}\,dx\,dy = \frac{1}{Re}\left(\frac{\partial v}{\partial y}|_n - \frac{\partial v}{\partial y}|_s\right)A_n$$

Where:

$$F_e = \left[\frac{u_{i+1}^j + u_i^j}{2}\right]\Delta y\times 1;\quad F_w = \left[\frac{u_{i+1}^{j-1} + u_i^{j-1}}{2}\right]\Delta y\times 1$$
$$F_n = \left[\frac{v_i^j + v_{i-1}^j}{2}\right]\Delta x\times 1;\quad F_s = \left[\frac{v_{i+1}^j + v_i^j}{2}\right]\Delta x\times 1$$
$$D_e = D_w = \frac{1}{Re}\frac{\Delta y\times 1}{\Delta x};\quad D_n = D_s = \frac{1}{Re}\frac{\Delta x\times 1}{\Delta y}$$

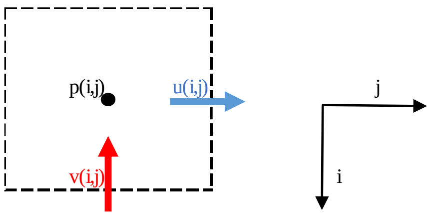
We get the y-momentum equation in form of:

$$a_P v_P = \sum\nolimits_{nb} a_{nb} v_{nb} - (p_n - p_s)A_n$$

From Hybrid Scheme, the coefficients are calculated as:

$$a_E = \max\left[-F_e, D_e - \frac{F_e}{2}, o\right];\quad a_W = \max\left[F_w, D_w + \frac{F_w}{2}, o\right];$$
$$a_N = \max\left[-F_n, D_n - \frac{F_n}{2}, o\right];\quad a_S = \max\left[F_s, D_s + \frac{F_s}{2}, o\right];$$
$$a_P = a_E + a_W + a_N + a_S + (F_e - F_w + F_n - F_s);$$
$$A_e = \Delta y\times 1\quad A_n = \Delta x\times 1$$

Continuity Equation in 2D for p-control volume:



$$\nabla\cdot\mathbf{u} = 0$$
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$\int_s^n\int_w^e \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dx\,dy = (u_e A_e - u_w A_w) + (v_n A_n - v_s A_s)$$

Where:

$$u_e = u_i^j;\quad u_w = u_i^{j-1};\quad v_n = v_{i-1}^j;\quad v_s = v_i^j$$
$$A_e = \Delta y\times 1;\quad A_n = \Delta x\times 1$$

C-Code: SIMPLE Algorithm and Boundary Conditions

For simplicity, let us consider a grid of 5×4 (as shows in *Figure 2*) to show the working of the code. A layer of control volume cell is considered outside the boundary to solve the equation.

Grid Creation:

```
// *** Calculations and 2D array creation ***
double dx = x_length/x_no_divisions; // Division length
double dy = y_length/y_no_divisions;
int n = x_no_divisions + 1;  // No. of points
int m = y_no_divisions + 1;
```

For this example, we take `x_no_divisions = 4` and `y_no_divisions = 5`. Hence the values for `m` and `n` becomes: `m = 6; n = 5.`

We define the final collocated grid with the dimension as required $m \times n = 6 \times 5$ here.
The u-velocity staggered grids are defined for $(m+1) \times n = 7 \times 5$ here; from $u_{i=0}^{j=0} \rightarrow u_6^4$
The v-velocity staggered grids are defined for $m \times (n+1) = 6 \times 6$ here; from $v_{i=0}^{j=0} \rightarrow v_5^5$
The pressure staggered grids are defined for $(m+1) \times (n+1) = 7 \times 6$ here; from $p_{i=0}^{j=0} \rightarrow p_6^5$

```
// Final Collocated Variables
double u_final[m][n], v_final[m][n], p_final[m][n], stream_final[m][n],
       vorticity_final[m][n];

// Staggered Grid
double u[m+1][n], u_star[m+1][n], d_e[m+1][n],
       v[m][n+1], v_star[m][n+1], d_n[m][n+1],
       p[m+1][n+1], p_star[m+1][n+1],
       pc[m+1][n+1], b[m+1][n+1];
```

`u_final[m][n]:` Final u values on collocated grid
`u[m+1][n]:` u values on staggered grid
`u_star[m+1][n]:` Intermediate values of u during SIMPLE algorithm
`d_e[m+1][n]:` Value of A_e/a_P stored for each point
Similar variables for v_final, v, v_star and d_n[m][n+1]
`p_final[m][n]:` Final p values on collocated grid
`p[m+1][n+1]:` p values on staggered grid
`p_star[m+1][n+1]:` Intermediate values of p during SIMPLE algorithm
`p_c[m+1][n+1]:` Pressure Correction values
`b[m+1][n+1]:` To store the continuity equation error at each point

```
// *** Initialisation ***
initialise_Values(m, n, u, u_star, d_e, v, v_star, d_n, p, p_star,
                  pc, b, u_initialise, v_initialise, p_initialise);
```

Initialise all values of the grid to a predetermined value based on the input provided.
Ex: Here we have kept all values to be initialised as zero.

Boundary Conditions:

Boundary condition of u-velocity:

```
void u_Boundary_Conditions(int m, int n, double u[m+1][n], double u_left_value, double u_right_value,
                           double u_top_value, double u_bottom_value){
    for(int i=0; i<m+1; i++){
            u[i][0] = u_left_value;
            u[i][n-1] = u_right_value;
            }
    for(int j=0; j<n; j++){
            u[0][j] = u_top_value*2 - u[1][j];
            u[m][j] = u_bottom_value*2 - u[m-1][j];
            }
    return;
    }
```

Since the left and right most values of staggered u lie on the actual control volume boundary, we can maintain the u values of left and right as:

$$u_i^0 = u_{left}, \quad u_i^{n-1} = u_{right} \quad \forall\, i \in [0, m];$$

For the top and bottom, since the control volume boundary lie between two of the points, we consider the average of the two as the boundary value:

$$\frac{(u_0^j + u_1^j)}{2} = u_{top} \quad \Rightarrow u_0^j = 2 \times u_{top} - u_1^j \quad \forall\, j \in [0, n-1];$$

$$\frac{(u_m^j + u_{m-1}^j)}{2} = u_{bottom} \quad \Rightarrow u_m^j = 2 \times u_{bottom} - u_{m-1}^j \quad \forall\, j \in [0, n-1];$$

Boundary condition of v-velocity:

```
void v_Boundary_Conditions(int m, int n, double v[m][n+1], double v_left_value, double v_right_value,
                           double v_top_value, double v_bottom_value){
    for(int i=0; i<m; i++){
            v[i][0] = v_left_value*2 - v[i][1];
            v[i][n] = v_right_value*2 - v[i][n-1];
    }
    for(int j=0; j<n+1; j++){
            v[0][j] = v_top_value;
            v[m-1][j] = v_bottom_value;
    }
    return;
}
```

Since the top and bottom most values of staggered v lie on the actual control volume boundary, we can maintain the v values of top and bottom as:

$$v_0^j = v_{top}, \quad v_{m-1}^j = v_{bottom} \quad \forall\, j \in [0, n];$$

For the top and bottom, since the control volume boundary lie between two of the points, we consider the average of the two as the boundary value:

$$\frac{(v_i^0 + v_i^1)}{2} = v_{left} \quad \Rightarrow v_i^0 = 2 \times v_{left} - v_i^1 \quad \forall\, i \in [0, m-1];$$

$$\frac{(v_i^n + v_i^{n-1})}{2} = v_{right} \quad \Rightarrow v_i^n = 2 \times v_{right} - v_i^{n-1} \quad \forall\, i \in [0, m-1];$$

Boundary condition of pressure:

```
void p_Boundary_Conditions(int m, int n, double p[m+1][n+1]){
    for(int i=0; i<m+1; i++){
            p[i][0] = p[i][1];
            p[i][n] = p[i][n-1];
    }
    for(int j=0; j<n+1; j++){
            p[0][j] = p[1][j];
            p[m][j] = p[m-1][j];
    }
    return;
}
```

For pressure, we consider the first derivative of pressure along x-axis is zero on left and right boundaries and first derivate of it along y-axis is zero on the top and bottom.

$$p_i^0 = p_i^1, \quad p_i^n = p_i^{n-1} \quad \forall\, i \,\epsilon\, [0, m];$$

$$p_0^j = p_1^j, \quad p_m^j = p_{m-1}^j \quad \forall\, j \,\epsilon\, [0, n];$$

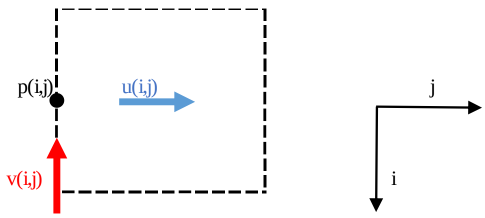
SIMPLE Algorithm:

The algorithm is applied until the L_2 norm of error in u-velocity and v-velocity is less than $\varepsilon = 10^{-5}$

X - Momentum Equation: on the u-control volume cell

```
// 1.1 X-Momentum Equation Interior:
for(int i=1; i<m; i++){
    for(int j=1; j<n-1; j++){
            Fe = (u[i][j+1] + u[i][j])/2.0*dy*1;
            Fw = (u[i][j] + u[i][j-1])/2.0*dy*1;
            Fn = (v[i-1][j+1] + v[i-1][j])/2.0*dx*1;
            Fs = (v[i][j+1] + v[i][j])/2.0*dx*1;

            De = (1/Re)*(dy*1/dx);
            Dw = (1/Re)*(dy*1/dx);
            Dn = (1/Re)*(dx*1/dy);
            Ds = (1/Re)*(dx*1/dy);

            aE = max(-Fe, De - Fe/2.0, 0);
            aW = max( Fw, Dw + Fw/2.0, 0);
            aN = max(-Fn, Dn - Fn/2.0, 0);
            aS = max( Fs, Ds + Fs/2.0, 0);

            aP = aE + aW + aN + aS + Fe - Fw + Fn - Fs;

            d_e[i][j] = dy*1/aP;

            if(method =='G') u_star[i][j] = (aE*u_star[i][j+1] + aW*u_star[i][j-1] + aN*u_star[i-1][j] +
aS*u_star[i+1][j])/aP - d_e[i][j]*(p[i][j+1] - p[i][j]);
            if(method =='J') u_star[i][j] = (aE*u[i][j+1] + aW*u[i][j-1] + aN*u[i-1][j] + aS*u[i+1][j])/aP -
d_e[i][j]*(p[i][j+1] - p[i][j]);

    }
}


// 1.2 u-Boundary Conditions
u_Boundary_Conditions(m, n, u_star, u_left_value, u_right_value, u_top_value,
u_bottom_value);
```

For the interior points, we first calculate the coefficients



$$F_e = \left[\frac{u_i^{j+1} + u_i^j}{2}\right] \Delta y \times 1; \quad F_w = \left[\frac{u_i^j + u_i^{j-1}}{2}\right] \Delta y \times 1$$

$$F_n = \left[\frac{v_{i-1}^{j+1} + v_{i-1}^j}{2}\right] \Delta x \times 1; \quad F_s = \left[\frac{v_i^{j+1} + v_i^j}{2}\right] \Delta x \times 1$$

$$D_e = D_w = \frac{1}{Re}\frac{\Delta y \times 1}{\Delta x}; \quad D_n = D_s = \frac{1}{Re}\frac{\Delta x \times 1}{\Delta y}$$

From Hybrid Scheme, the coefficients are calculated as:

$$a_E = \max\left[-F_e, D_e - \frac{F_e}{2}, o\right]; \quad a_W = \max\left[F_w, D_w + \frac{F_w}{2}, o\right];$$

$$a_N = \max\left[-F_n, D_n - \frac{F_n}{2}, o\right]; \quad a_S = \max\left[F_s, D_s + \frac{F_s}{2}, o\right];$$

$$a_P = a_E + a_W + a_N + a_S + (F_e - F_w + F_n - F_s);$$

$$A_e = \Delta y \times 1; \quad A_n = \Delta x \times 1$$

$$de_i^j = A_e / a_{P_i}^{\ j}$$

Based on the selection whether to solve using Gauss-Siedel or Jacobi method, we have two equations:

$$u_i^{*j} = (\textstyle\sum_{nb} a_{nb} u_{nb}^*)/a_P - de(p_i^{j+1} - p_i^j)\ ; \qquad \text{for Gauss Seidel}$$

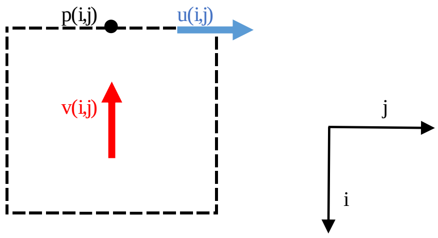
$$u_i^{*j} = (\textstyle\sum_{nb} a_{nb}\, u_{nb})/a_P - de(p_i^{j+1} - p_i^j)\ ; \qquad \text{for Jacobi}$$

Y - Momentum Equation: on the v-control volume cell

```
// 1.3 Y-Momentum Equation Interior:
for(int i=1; i<m-1; i++){
    for(int j=1; j<n; j++){
            Fe = (u[i+1][j] + u[i][j])/2.0*dy*1;
            Fw = (u[i+1][j-1] + u[i][j-1])/2.0*dy*1;
            Fn = (v[i][j] + v[i-1][j])/2.0*dx*1;
            Fs = (v[i+1][j] + v[i][j])/2.0*dx*1;

            De = (1/Re)*(dy*1/dx);
            Dw = (1/Re)*(dy*1/dx);
            Dn = (1/Re)*(dx*1/dy);
            Ds = (1/Re)*(dx*1/dy);

            aE = max(-Fe, De - Fe/2.0, 0);
            aW = max( Fw, Dw + Fw/2.0, 0);
            aN = max(-Fn, Dn - Fn/2.0, 0);
            aS = max( Fs, Ds + Fs/2.0, 0);

            aP = aE + aW + aN + aS + Fe - Fw + Fn - Fs;

            d_n[i][j] = dx*1/aP;

            if(method =='G') v_star[i][j] = (aE*v_star[i][j+1] + aW*v_star[i][j-1] + aN*v_star[i-1][j] +
aS*v_star[i+1][j])/aP - d_n[i][j]*(p[i][j] - p[i+1][j]);
            if(method =='J') v_star[i][j] = (aE*v[i][j+1] + aW*v[i][j-1] + aN*v[i-1][j] + aS*v[i+1][j])/aP -
d_n[i][j]*(p[i][j] - p[i+1][j]);
    }
}
```

```
// 1.4 v-Boundary Conditions
v_Boundary_Conditions(m, n, v_star, v_left_value, v_right_value, v_top_value, v_bottom_value);
```

For the interior points, we first calculate the coefficients



$$F_e = \left[\frac{u_{i+1}^{j} + u_i^{j}}{2}\right]\Delta y \times 1; \quad F_w = \left[\frac{u_{i+1}^{j-1} + u_i^{j-1}}{2}\right]\Delta y \times 1$$

$$F_n = \left[\frac{v_i^{j} + v_{i-1}^{j}}{2}\right]\Delta x \times 1; \quad F_s = \left[\frac{v_{i+1}^{j} + v_i^{j}}{2}\right]\Delta x \times 1$$

$$D_e = D_w = \frac{1}{Re}\frac{\Delta y \times 1}{\Delta x}; \quad D_n = D_s = \frac{1}{Re}\frac{\Delta x \times 1}{\Delta y}$$

From Hybrid Scheme, the coefficients are calculated as:

$$a_E = \max\left[-F_e, D_e - \frac{F_e}{2}, o\right]; \quad a_W = \max\left[F_w, D_w + \frac{F_w}{2}, o\right];$$

$$a_N = \max\left[-F_n, D_n - \frac{F_n}{2}, o\right]; \quad a_S = \max\left[F_s, D_s + \frac{F_s}{2}, o\right];$$

$$a_P = a_E + a_W + a_N + a_S + (F_e - F_w + F_n - F_s);$$

$$A_e = \Delta y \times 1 \quad A_n = \Delta x \times 1$$

$$dn_i^j = A_n / a_{P_i^j}$$

Based on the selection whether to solve using Gauss-Siedel or Jacobi method, we have two equations:

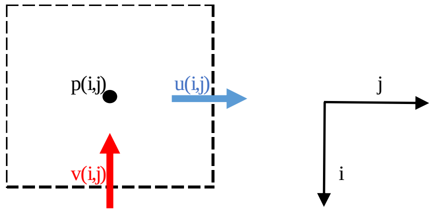
$$v_i^{*j} = \left(\sum\nolimits_{nb} a_{nb} v_{nb}^{*}\right)/a_P - dn\left(p_i^j - p_{i+1}^j\right); \qquad \text{for Gauss Seidel}$$

$$v_i^{*j} = \left(\sum\nolimits_{nb} a_{nb}\, v_{nb}\right)/a_P - dn\left(p_i^j - p_{i+1}^j\right); \qquad \text{for Jacobi}$$

Pressure Correction Equation: on the p-control volume cell

```
// 2.1 Initialising pressure correction array to zero
for(int i=0; i<m+1; i++){
    for(int j=0; j<n+1; j++){
            pc[i][j] = 0.0;
    }
}

// 2.2 Pressure Correction Interior
for(int i=1; i<m; i++){
    for(int j=1; j<n; j++){
            aE = d_e[i][j]*dy*1;
            aW = d_e[i][j-1]*dy*1;
            aN = d_n[i-1][j]*dx*1;
            aS = d_n[i][j]*dx*1;

            aP = aE + aW + aN + aS;

            b[i][j] =  (u_star[i][j] - u_star[i][j-1])*dy*1 + (v_star[i-1][j] - v_star[i][j])*dx*1;

            pc[i][j] = (aE*pc[i][j+1] + aW*pc[i][j-1] + aN*pc[i-1][j] + aS*pc[i+1][j] - b[i][j])/aP;

    }
}
```

From the calculate values of de and dn, we calculate the values of a_{nb}



$$a_E = de_i^j \times A_e; \quad a_W = de_i^{j-1} \times A_w;$$
$$a_N = dn_{i-1}^j \times A_n; \quad a_S = dn_i^j \times A_s;$$
$$a_P = a_E + a_W + a_N + a_S;$$

Residual of continuity equation is calculated as:

$$b_i^j = (u_i^{*j} - u_i^{*j-1})A_e + (v_{i-1}^{*j} - v_i^{*j})A_n$$

The pressure correction term is updated using Gauss Seidel method as:

$$pc_i^j = \left(\textstyle\sum_{nb} a_{nb} pc_{nb} - b_i^j\right)/a_P$$

Correcting Pressure and Velocity values

```
error_u = 0.0;
error_v = 0.0;

// 3.1 Correcting Pressure Field
for(int i=1; i<m; i++){
    for(int j=1; j<n; j++){
            p[i][j] = p[i][j] + pressure_alpha*pc[i][j];
    }
}

// 3.2 p-Boundary Conditions
p_Boundary_Conditions(m, n, p);


// 3.3 Correcting u-velocity
for(int i=1; i<m; i++){
    for(int j=1; j<n-1; j++){
            error_u += pow(u[i][j] - (u_star[i][j] - vel_alpha*d_e[i][j]*(pc[i][j+1] - pc[i][j])), 2);
            u[i][j] = u_star[i][j] - vel_alpha*d_e[i][j]*(pc[i][j+1] - pc[i][j]);
            u_star[i][j] = u[i][j];
    }
}

// 3.4 u-Boundary Conditions
u_Boundary_Conditions(m, n, u, u_left_value, u_right_value, u_top_value, u_bottom_value);
u_Boundary_Conditions(m, n, u_star, u_left_value, u_right_value, u_top_value, u_bottom_value);

// 3.5 Correcting v-velocity
for(int i=1; i<m-1; i++){
    for(int j=1; j<n; j++){
            error_v += pow(v[i][j] - (v_star[i][j] - vel_alpha*d_n[i][j]*(pc[i][j] - pc[i+1][j])), 2);
            v[i][j] = v_star[i][j] - vel_alpha*d_n[i][j]*(pc[i][j] - pc[i+1][j]);
            v_star[i][j] = v[i][j];
    }
}

// 3.6 v-Boundary Conditions
v_Boundary_Conditions(m, n, v, v_left_value, v_right_value, v_top_value, v_bottom_value);
v_Boundary_Conditions(m, n, v_star, v_left_value, v_right_value, v_top_value, v_bottom_value);
```

The values of pressure and velocities are updated with the formula:

$$p_i^j = p_i^j + \alpha_p \, pc$$

$$u_i^j = u_i^{*j} - \alpha_u \, de_i^j \left(pc_i^{j+1} - pc_i^j\right)$$
$$v_i^j = v_i^{*j} - \alpha_v \, dn_i^j \left(pc_i^{j+1} - pc_i^j\right)$$

Once the values have converged, we calculate the collocated grid's values for pressure and velocities with the code:

```
void calculate_Collocated_Grid(int m, int n, double u[m+1][n], double v[m][n+1], double p[m+1][n+1], double
u_final[m][n], double v_final[m][n], double p_final[m][n]){
    for(int i=0; i<m; i++){
            for(int j=0; j<n; j++){
                    u_final[i][j] = (u[i][j] + u[i+1][j])/2.0;
                    v_final[i][j] = (v[i][j] + v[i][j+1])/2.0;
                    p_final[i][j] = (p[i][j] + p[i+1][j] + p[i][j+1] + p[i+1][j+1])/4.0;
            }
    }
}
```

The values of Stream function and Vorticity is calculated from the collocated grid velocities and then the results are shown in the next section.

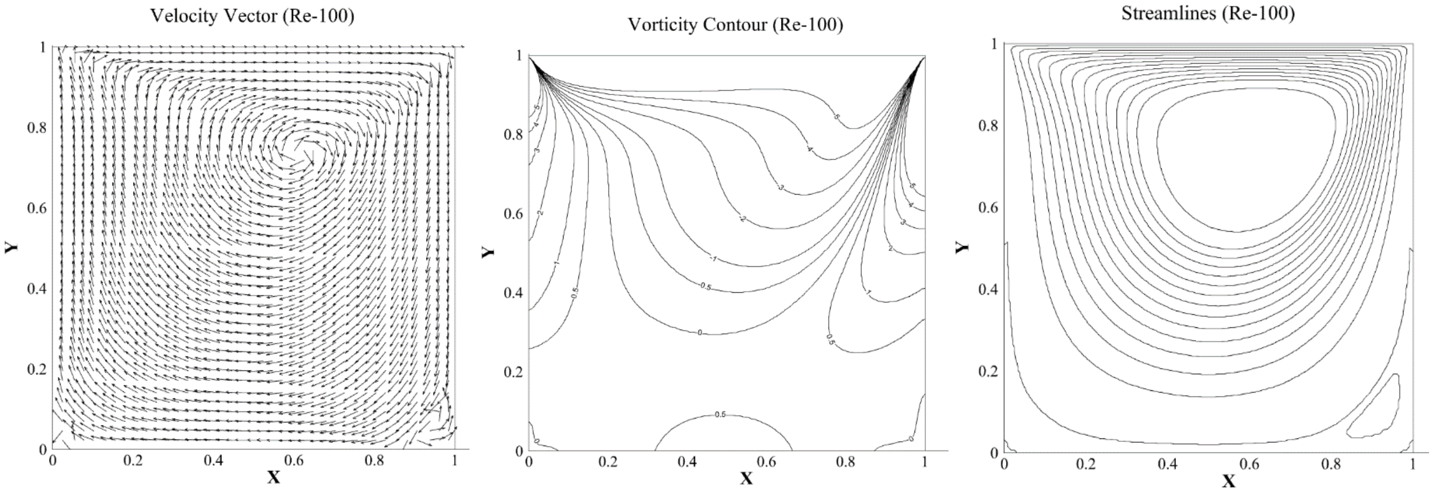
Results

Parameters taken to solve for different Reynold's number

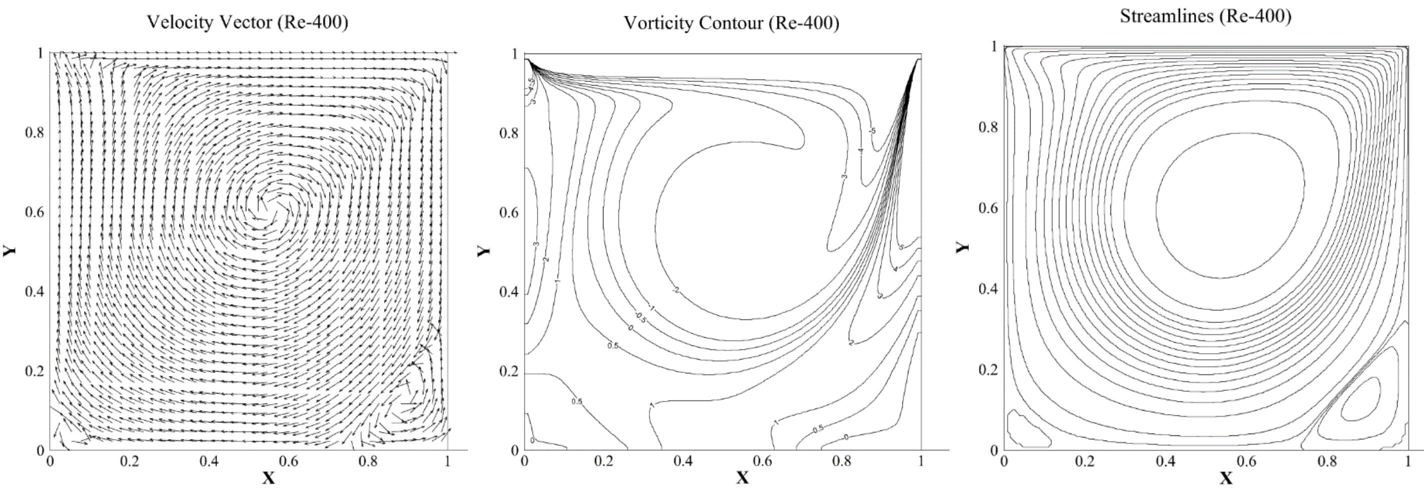
Re	Pressure Relaxation factor (α_p)	Velocity Relaxation factor (α_v)	Solution Method	Converged?
100	0.5	0.5	Gauss Seidel	Yes
400	0.8	0.8	Gauss Seidel	Yes
1000	1.0	1.0	Jacobi	No

1. Contours and Velocity Magnitude plots:

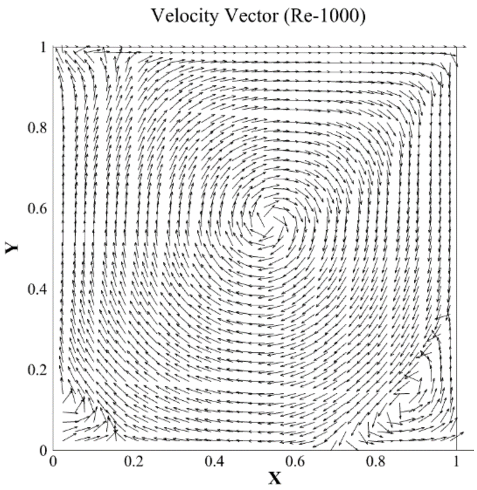
Re 100

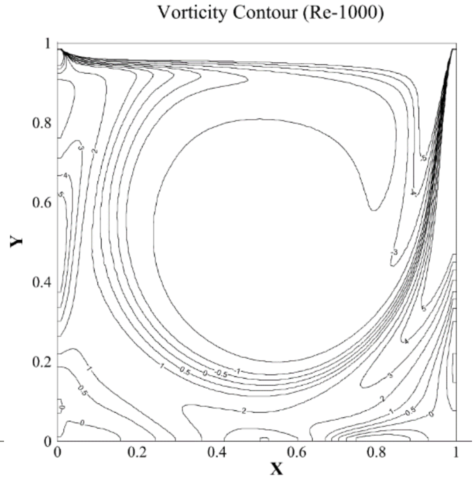


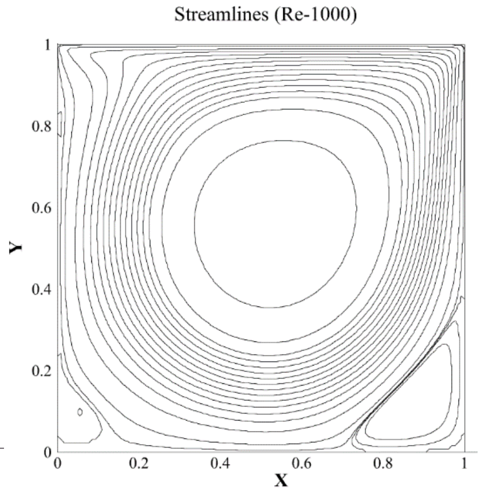
Re 400



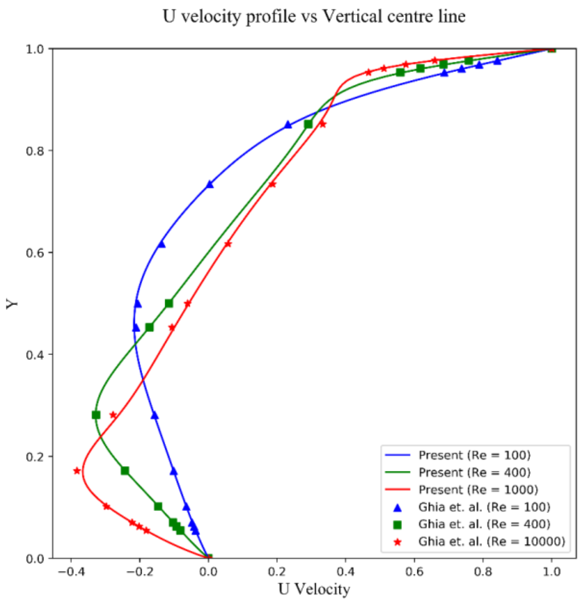
Re 1000

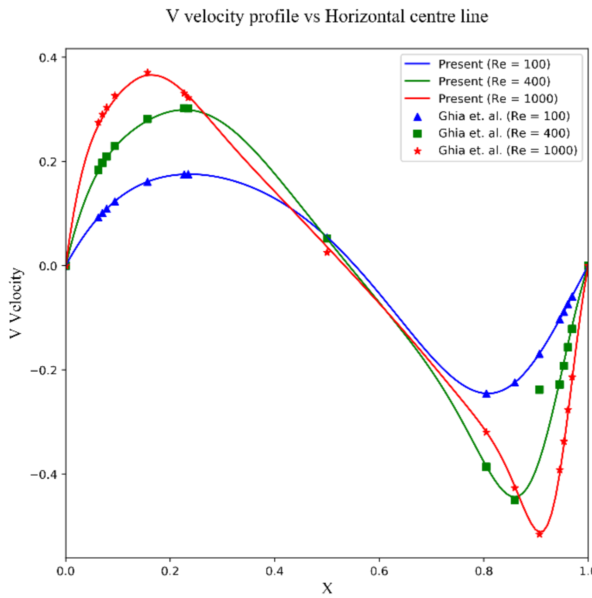






2. U velocity profile at vertical midsection and V velocity profile at horizontal midsection.





3. **Re-100:** Stream: -0.1033, x_val:0.617, y_val:0.742, vorticity: -3.1670
   Re-400: Stream: -0.1132, x_val:0.555, y_val:0.617, vorticity: -2.2867
   Re-1000: Stream: -0.1152, x_val:0.539, y_val:0.570, vorticity: -1.9967