ME 543: Computational Fluid Dynamics

COMPUTER ASSIGNMENT – 3B

Study of Backward Facing Step Flows using Finite Difference Method.

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Backward Facing Step Flows

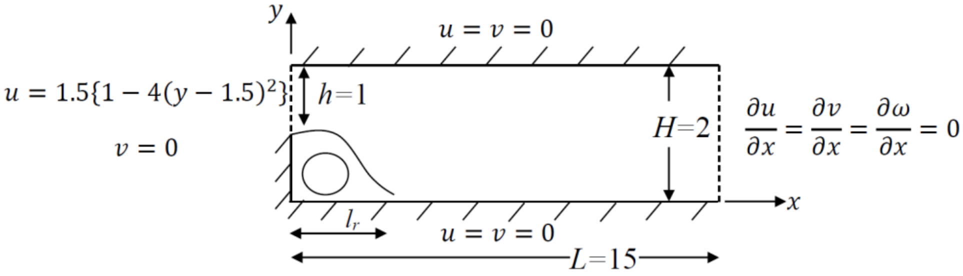

Fig 1: Backward Facing Step Flows

Governing Equations (Differential Equations):

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$

Discretized Equations: $n\text{: current iteration}, \quad n+1\text{: next iteration}$

Vorticity Equation:

$$\frac{\psi_{i,j+1}^{n} - \psi_{i,j-1}^{n+1}}{2\Delta y}\left[\frac{\omega_{i+1,j}^{n} - \omega_{i-1,j}^{n+1}}{2\Delta x}\right] - \frac{\psi_{i+1,j}^{n} - \psi_{i-1,j}^{n+1}}{2\Delta x}\left[\frac{\omega_{i,j+1}^{n} - \omega_{i,j-1}^{n+1}}{2\Delta y}\right]$$
$$= \frac{1}{Re}\left[\frac{\omega_{i+1,j}^{n} - 2\omega_{i,j}^{n+1} + \omega_{i-1,j}^{n+1}}{(\Delta x)^2} + \frac{\omega_{i,j+1}^{n} - 2\omega_{i,j}^{n+1} + \omega_{i,j-1}^{n+1}}{(\Delta y)^2}\right.$$

$$\begin{aligned}\omega_{i,j}^{n+1} = \frac{0.5}{(1+\beta^2)}\Big\{&\left[1 - \frac{\beta\, Re}{4}\left(\psi_{i,j+1}^{n} - \psi_{i,j-1}^{n+1}\right)\right]\omega_{i+1,j}^{n} \\ &+ \left[1 + \frac{\beta\, Re}{4}\left(\psi_{i,j+1}^{n} - \psi_{i,j-1}^{n+1}\right)\right]\omega_{i-1,j}^{n+1} \\ &+ \left[1 + \frac{Re}{4\,\beta}\left(\psi_{i+1,j}^{n} - \psi_{i-1,j}^{n+1}\right)\right]\beta^2\omega_{i,j+1}^{n} \\ &+ \left[1 - \frac{Re}{4\,\beta}\left(\psi_{i+1,j}^{n} - \psi_{i-1,j}^{n+1}\right)\right]\beta^2\omega_{i,j-1}^{n+1}\end{aligned}$$

Using under-relaxation for vorticity equation, we get: (taking $w = 0.2$)

$$\begin{aligned}\omega_{i,j}^{n+1} &= (1-w)\omega_{i,j}^{n} \\ &+ w\frac{0.5}{(1+\beta^2)}\Big\{\left[1 - \frac{\beta\, Re}{4}\left(\psi_{i,j+1}^{n} - \psi_{i,j-1}^{n+1}\right)\right]\omega_{i+1,j}^{n} + \left[1 + \frac{\beta\, Re}{4}\left(\psi_{i,j+1}^{n} - \psi_{i,j-1}^{n+1}\right)\right]\omega_{i-1,j}^{n+1} \\ &\quad + \left[1 + \frac{Re}{4\,\beta}\left(\psi_{i+1,j}^{n} - \psi_{i-1,j}^{n+1}\right)\right]\beta^2\omega_{i,j+1}^{n} + \left[1 - \frac{Re}{4\,\beta}\left(\psi_{i+1,j}^{n} - \psi_{i-1,j}^{n+1}\right)\right]\beta^2\omega_{i,j-1}^{n+1}\end{aligned}$$

Stream Function:

$$\frac{\psi_{i+1,j}^{n} - 2\psi_{i,j}^{n+1} + \psi_{i-1,j}^{n+1}}{(\Delta x)^2} + \frac{\psi_{i,j+1}^{n} - 2\psi_{i,j}^{n+1} + \psi_{i,j-1}^{n+1}}{(\Delta y)^2} = -\omega_{i,j}^{n+1}$$

$$\psi_{i,j}^{n+1} = \frac{0.5}{(1+\beta^2)}\left[\psi_{i+1,j}^{n} + \psi_{i-1,j}^{n+1} + \beta^2\left(\psi_{i,j+1}^{n} + \psi_{i,j-1}^{n+1}\right) + (\Delta x)^2\omega_{i,j}^{n+1}\right]$$

Boundary Conditions:

Left Boundary:

For $\boldsymbol{y \in [0,1]}$

$$\psi_{1,j} = 0$$

$$\omega_{1,j} = -\frac{2}{(\Delta x)^2}\left[\psi_{2,j} - \psi_{1,j}\right]$$

For $\boldsymbol{y \in [1,2]}$

$$\psi_{1,j} = -2(\Delta y \times j)^3 + 9(\Delta y \times j)^2 - 12(\Delta y \times j) + 5$$

$$\omega_{1,j} = 12(\Delta y \times j - 1.5)$$

Right Boundary:

$$\psi_{M,j} = \psi_{M-1,j}$$

$$\omega_{M,j} = \omega_{M-1,j}$$

Top Boundary:

$$\psi_{i,N} = 1$$

$$\omega_{i,N} = -\frac{2}{(\Delta x)^2}\left[\psi_{i,N-1} - \psi_{i,N}\right]$$

Bottom Boundary:

$$\psi_{i,1} = 0$$

$$\omega_{i,1} = -\frac{2}{(\Delta x)^2}\left[\psi_{i,2} - \psi_{i,1}\right]$$

Streamlines:

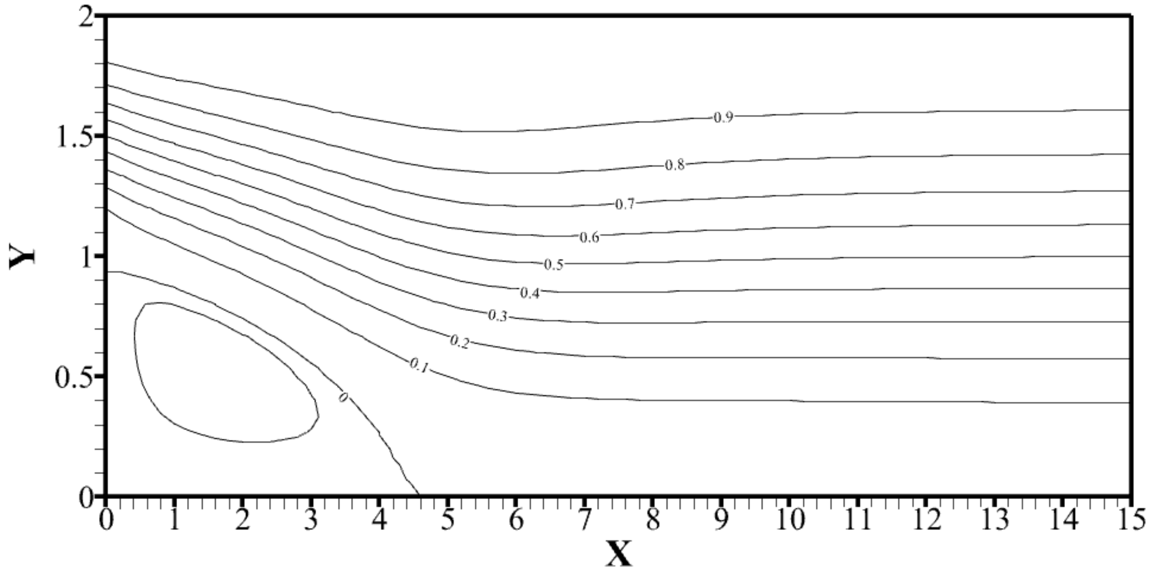

Fig 2: Streamlines of the flow.

Velocity Vectors:

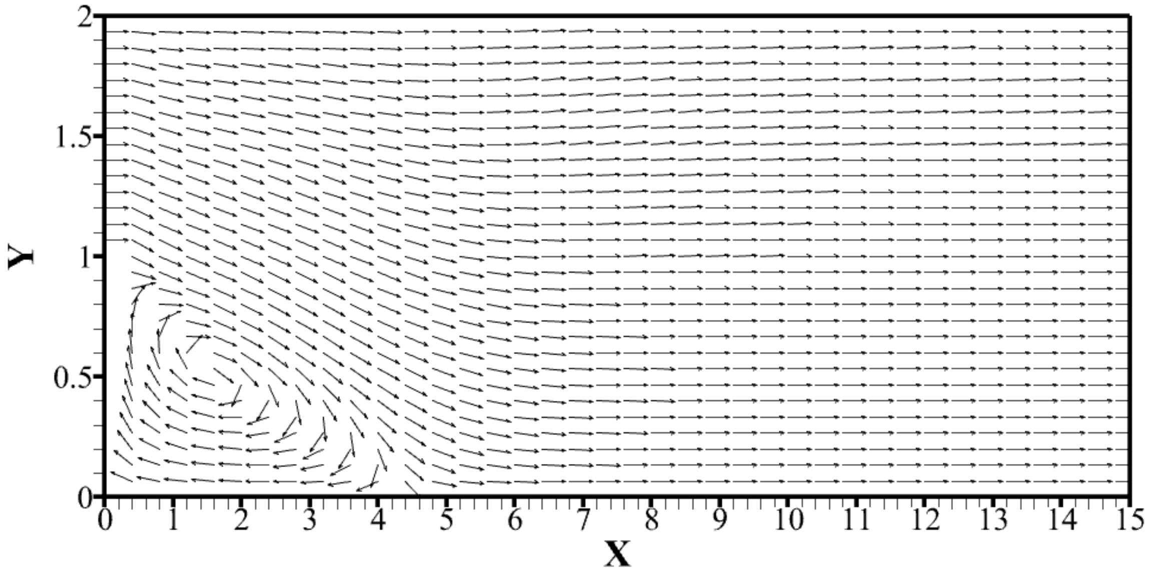

Fig 3: Velocity Vectors of the flow.

u velocity profile at x=2&10:

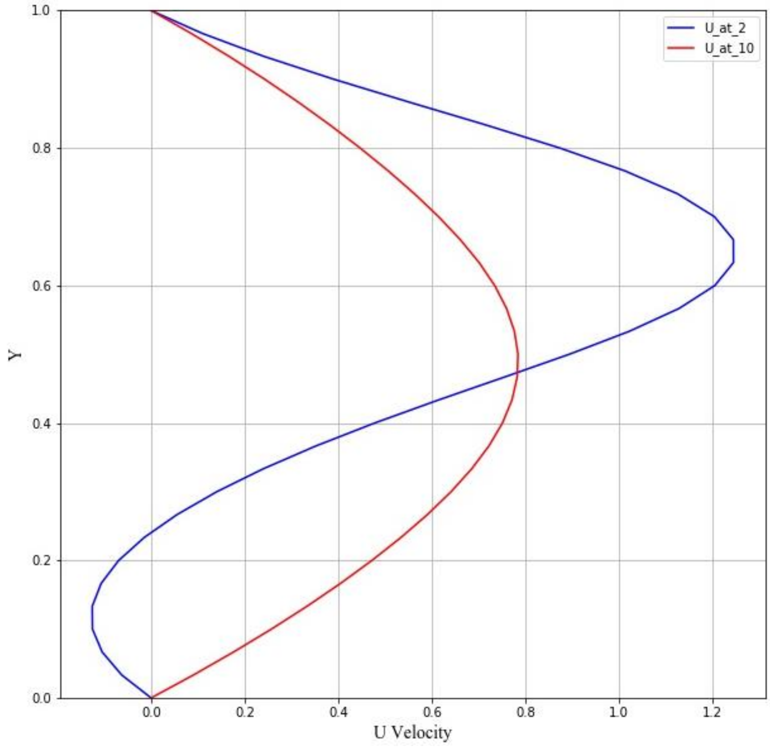

Fig 4: Horizontal velocity (u) profile at x=2 and x=10.

Recirculation length $(l_r) = 4.6$ units

Time taken and Number of Iterations

Re	Time Taken	Number of Iterations
100	2m: 32s	3,393