

ME 543: Computational Fluid Dynamics



COMPUTER ASSIGNMENT - 3

Study of Lid Driven Cavity using Finite Difference Method.

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Lid Driven Cavity:

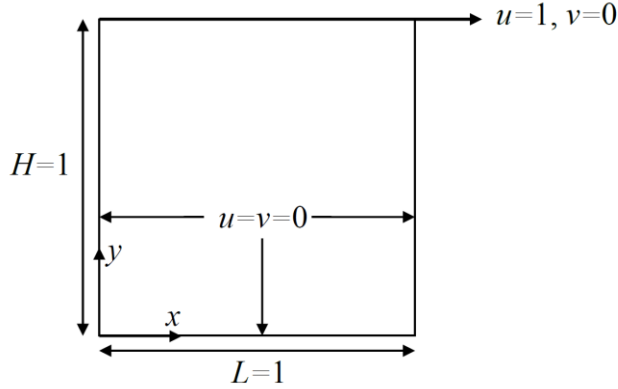


Fig A: Lid driving the fluid on top boundary

Governing Equations (Differential Equations):

$$u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \frac{1}{Re} \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

Discretized Equations: n : current iteration, $n + 1$: next iteration

Vorticity Equation:

$$\frac{\psi_{i,j+1}^n - \psi_{i,j-1}^{n+1}}{2\Delta y} \left[\frac{\omega_{i+1,j}^n - \omega_{i-1,j}^{n+1}}{2\Delta x} \right] - \frac{\psi_{i+1,j}^n - \psi_{i-1,j}^{n+1}}{2\Delta x} \left[\frac{\omega_{i,j+1}^n - \omega_{i,j-1}^{n+1}}{2\Delta y} \right]$$

$$= \frac{1}{Re} \left[\frac{\omega_{i+1,j}^n - 2\omega_{i,j}^{n+1} + \omega_{i-1,j}^{n+1}}{(\Delta x)^2} + \frac{\omega_{i,j+1}^n - 2\omega_{i,j}^{n+1} + \omega_{i,j-1}^{n+1}}{(\Delta y)^2} \right]$$

$$\omega_{i,j}^{n+1} = \frac{0.5}{(1 + \beta^2)} \left\{ \left[1 - \frac{\beta Re}{4} (\psi_{i,j+1}^n - \psi_{i,j-1}^{n+1}) \right] \omega_{i+1,j}^n \right.$$

$$+ \left[1 + \frac{\beta Re}{4} (\psi_{i,j+1}^n - \psi_{i,j-1}^{n+1}) \right] \omega_{i-1,j}^{n+1}$$

$$+ \left[1 + \frac{Re}{4\beta} (\psi_{i+1,j}^n - \psi_{i-1,j}^{n+1}) \right] \beta^2 \omega_{i,j+1}^n$$

$$+ \left[1 - \frac{Re}{4\beta} (\psi_{i+1,j}^n - \psi_{i-1,j}^{n+1}) \right] \beta^2 \omega_{i,j-1}^{n+1}$$

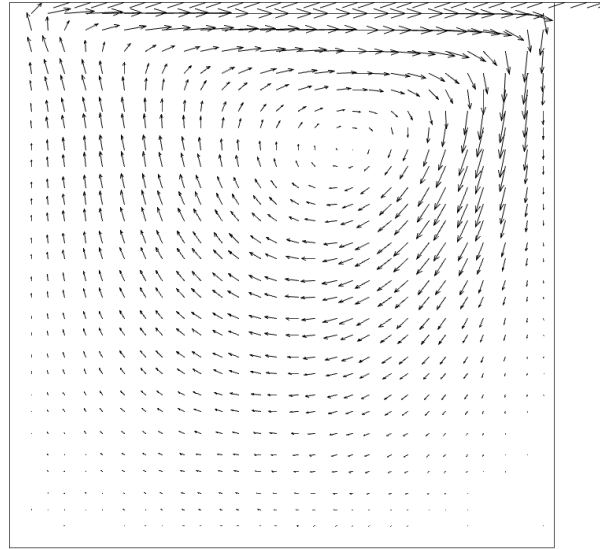
Stream Function:

$$\frac{\psi_{i+1,j}^n - 2\psi_{i,j}^{n+1} + \psi_{i-1,j}^{n+1}}{(\Delta x)^2} + \frac{\psi_{i,j+1}^n - 2\psi_{i,j}^{n+1} + \psi_{i,j-1}^{n+1}}{(\Delta y)^2} = -\omega_{i,j}^{n+1}$$

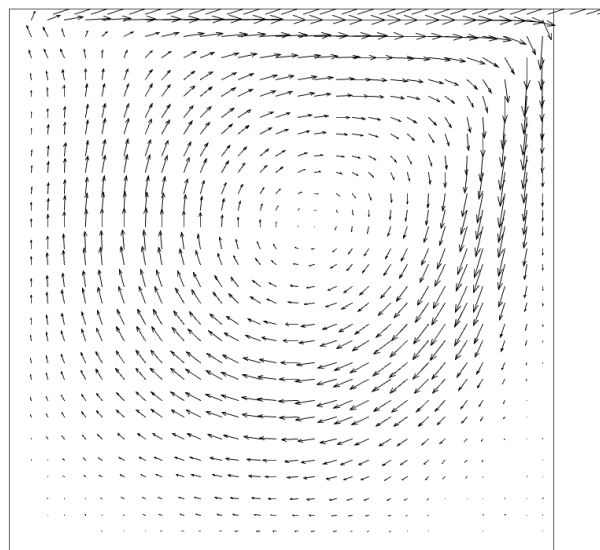
$$\psi_{i,j}^{n+1} = \frac{0.5}{(1 + \beta^2)} [\psi_{i+1,j}^n + \psi_{i-1,j}^{n+1} + \beta^2(\psi_{i,j+1}^n + \psi_{i,j-1}^{n+1}) + (\Delta x)^2 \omega_{i,j}^{n+1}]$$

Comparison of Velocity Vectors, Streamlines and Vorticity Contours at Re values of 100 and 400.

Velocity Vectors:

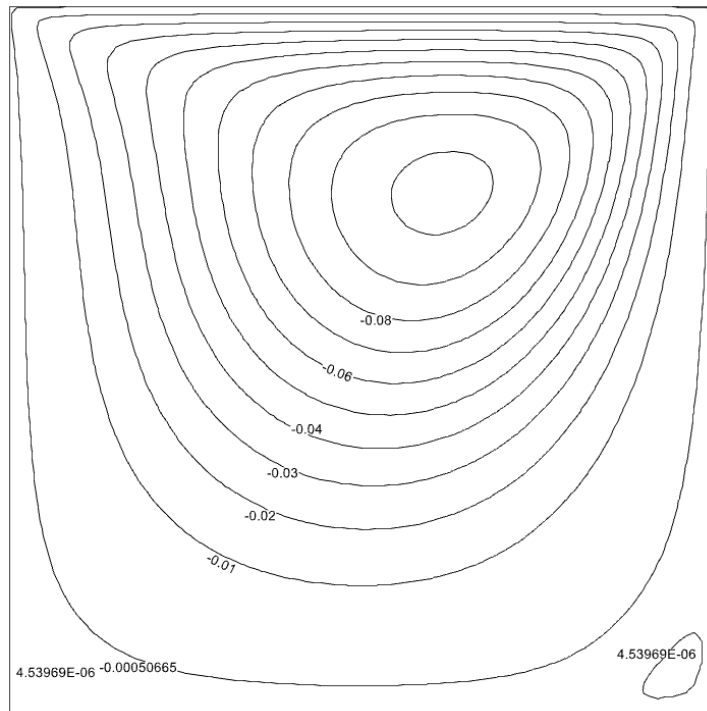


Re=100

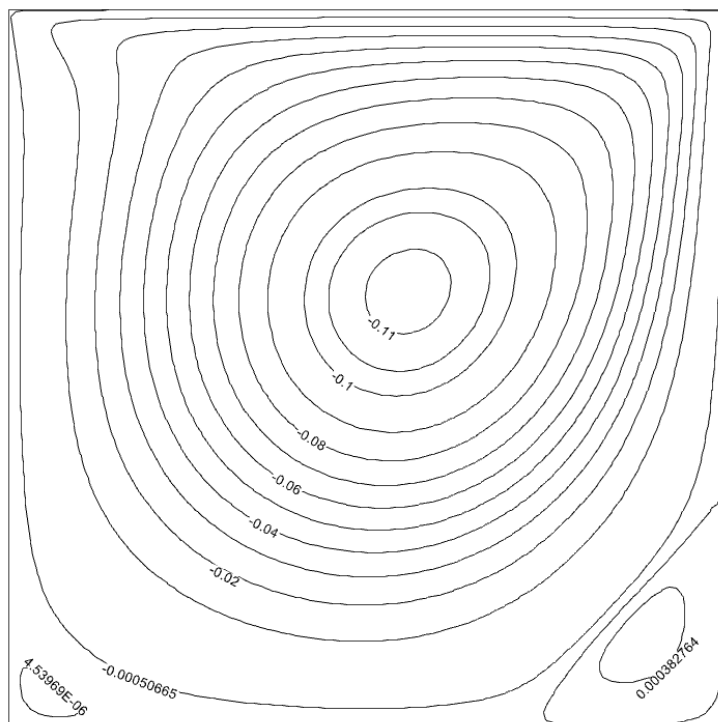


Re=400

Streamlines:

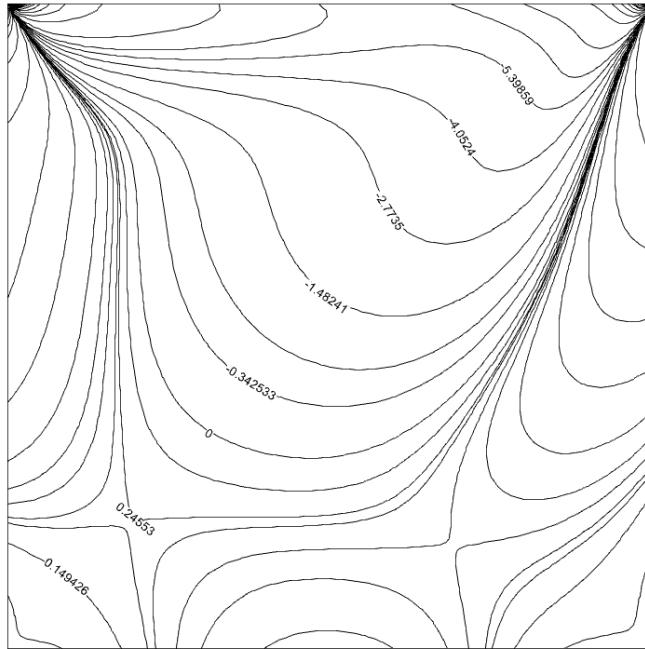


Re=400

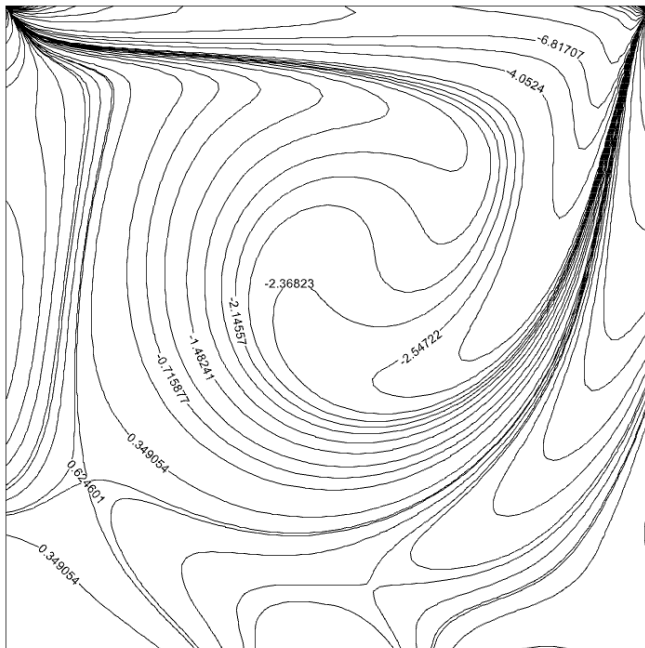


Re=100

Vorticity Contours:

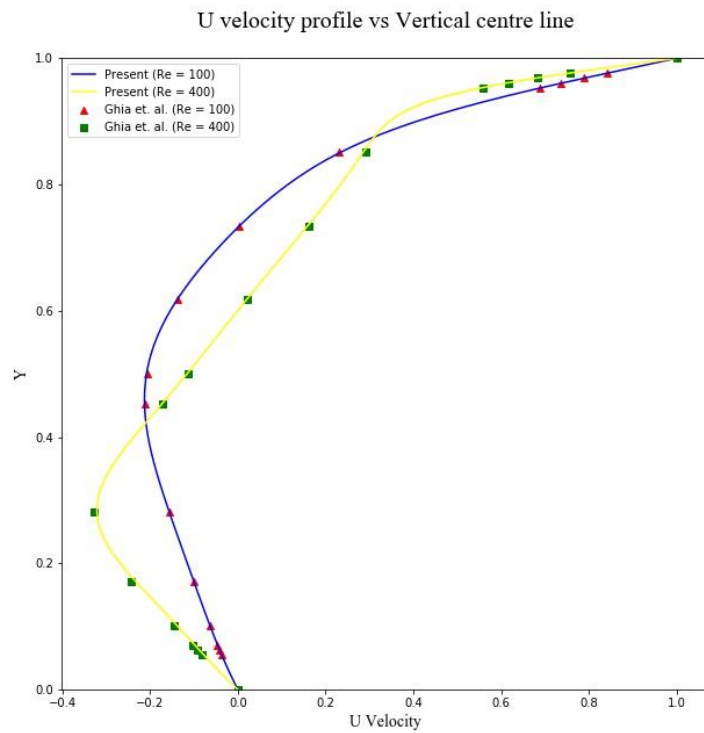


Re=100

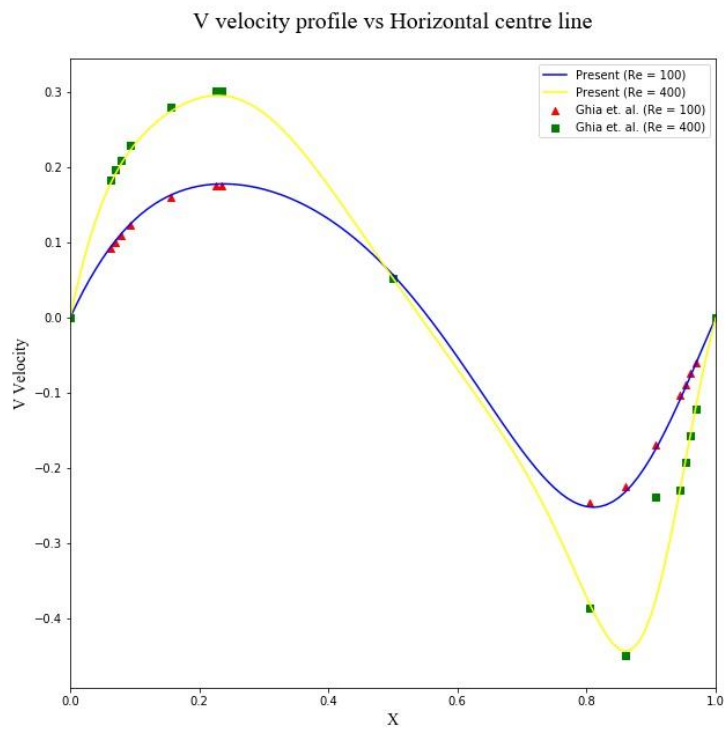


Re=400

U Velocity vs Vertical Center Line:



V Velocity vs Horizontal Center Line:



Time taken and Number of Iterations

Re	Time Taken	Number of Iterations
100	16m: 39s	6,536
400	35m: 30s	9,219