ME 543: Computational Fluid Dynamics

COMPUTER ASSIGNMENT - 2

Study of Couette Flow using Finite Difference Method using Implicit and Explicit methods

Explicit Methods:

Forward Time Central Space (FTCS)

Implicit Methods:

Backward Time Central Space (BTCS) – Gauss-Seidel
Backward Time Central Space (BTCS) – Tridiagonal Matrix Algorithm
Crank Nicolson (CN) – Tridiagonal Matrix Algorithm

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Couette Flow:

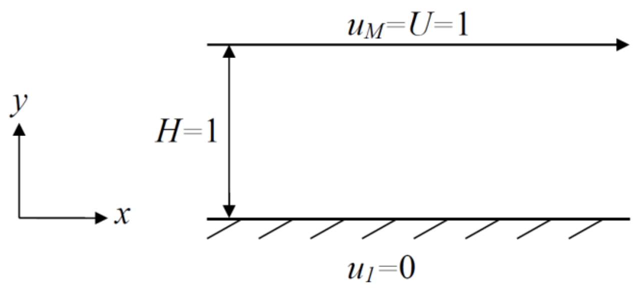

Fig A: Initial state of flow (at time t=0)

Differential Equation: $\frac{\partial u}{\partial t} = \frac{1}{Re_H}\frac{\partial^2 u}{\partial y^2}$; $\quad where\ Re_H = \frac{UH}{v}$

Inputs to the code:

$M = 101$
$N = 101$
$Re_H = 100$

$\Delta t = 5 \times 10^{-3}\ (for\ Explicit\ Methods)$
$\Delta t = 1 \times 10^{-2}\ (for\ Implicit\ Methods)$

1. Explicit Method:

1.1. Forward Time Central Space (FTCS)

Discretized Equation:

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{1}{Re_H} \frac{u_{j-1}^n - 2u_j^n + u_{j+1}^n}{(\Delta x)^2}$$

$$u_j^{n+1} = u_j^n + \Gamma\left(u_{j-1}^n - 2u_j^n + u_{j+1}^n\right)$$

$$where, \Gamma = \frac{\Delta t}{(\Delta x)^2 \, Re_H}$$

Taking $\Delta x = 0.01,\ \Delta t = 5 \times 10^{-3}\ and\ Re_H = 100$:
$\therefore \Gamma = 0.5$

Velocity Profiles at different times:

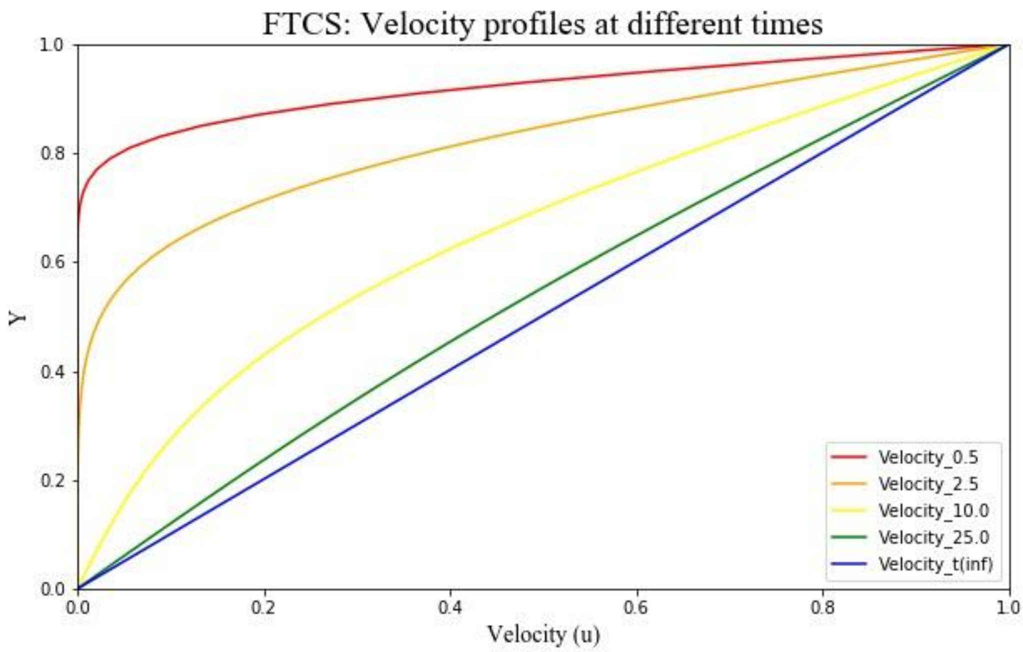

Fig 1: Velocity Profiles at different times for FTCS iterative Method

2. Implicit Method:

2.1. BTCS: Gauss Seidel Iterative Method

Discretized Equation:

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{1}{Re_H} \frac{u_{j-1}^{n+1} - 2u_j^{n+1} + u_{j+1}^{n+1}}{(\Delta x)^2}$$

$$\Gamma u_{j-1}^{n+1} - (1 + 2\Gamma) u_j^{n+1} + \Gamma u_{j+1}^{n+1} = -u_j^n$$

$$where, \Gamma = \frac{\Delta t}{(\Delta x)^2 \, Re_H}$$

Taking $\Delta x = 0.01,\ \Delta t = 10^{-2}\ and\ Re_H = 100$:

$\therefore \Gamma = 1$

2.2. BTCS: Tridiagonal Matrix Algorithm

Discretized Equation:

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{1}{Re_H} \frac{u_{j-1}^{n+1} - 2u_j^{n+1} + u_{j+1}^{n+1}}{(\Delta x)^2}$$

$$\Gamma u_{j-1}^{n+1} - (1 + 2\Gamma) u_j^{n+1} + \Gamma u_{j+1}^{n+1} = -u_j^n$$

$$where, \Gamma = \frac{\Delta t}{(\Delta x)^2 \, Re_H}$$

Taking $\Delta x = 0.01,\ \Delta t = 10^{-2}\ and\ Re_H = 100$:

$\therefore \Gamma = 1$

2.3. Crank Nicolson: Tridiagonal Matrix Algorithm

Discretized Equation:

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{1}{2\,Re_H}\left[\frac{u_{j-1}^{n+1} - 2u_j^{n+1} + u_{j+1}^{n+1}}{(\Delta x)^2} + \frac{u_{j-1}^n - 2u_j^n + u_{j+1}^n}{(\Delta x)^2}\right]$$

$$\frac{\Gamma}{2}u_{j-1}^{n+1} - (1+\Gamma)u_j^{n+1} + \frac{\Gamma}{2}u_{j+1}^{n+1} = -\frac{\Gamma}{2}u_{j-1}^n - (1-\Gamma)u_j^n - \frac{\Gamma}{2}u_{j+1}^n$$

$$where, \Gamma = \frac{\Delta t}{(\Delta x)^2\,Re_H}$$

Taking $\Delta x = 0.01,\ \Delta t = 10^{-2}\ \ and\ Re_H = 100$:
$\therefore \Gamma = 1$

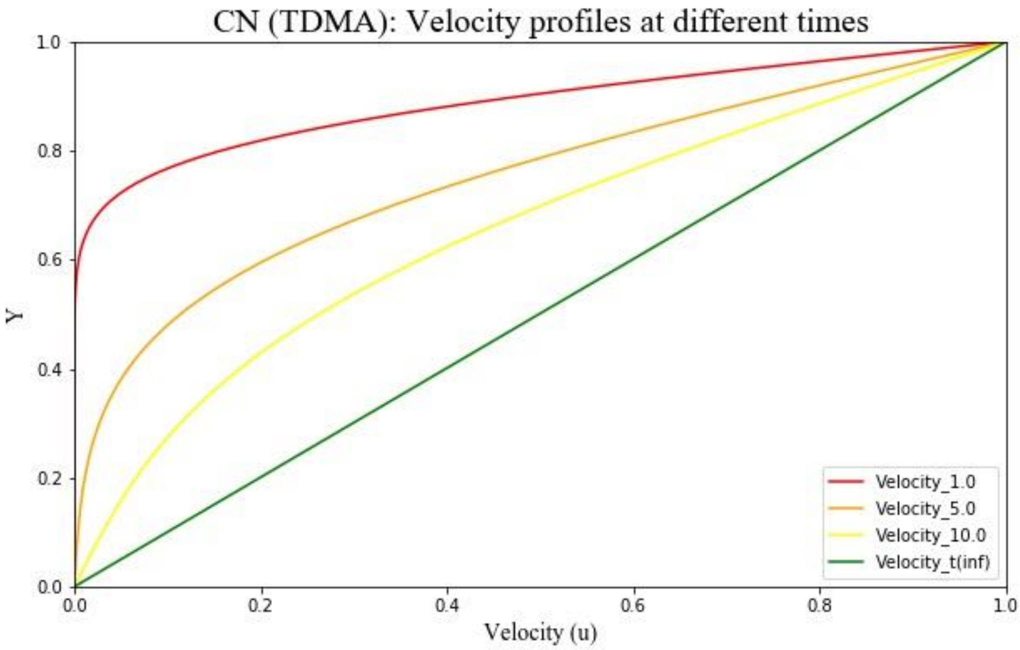

Fig 2: Velocity Profiles at different times for FTCS iterative Method

Convergence History ($\epsilon\ vs\ T$) for all Schemes:

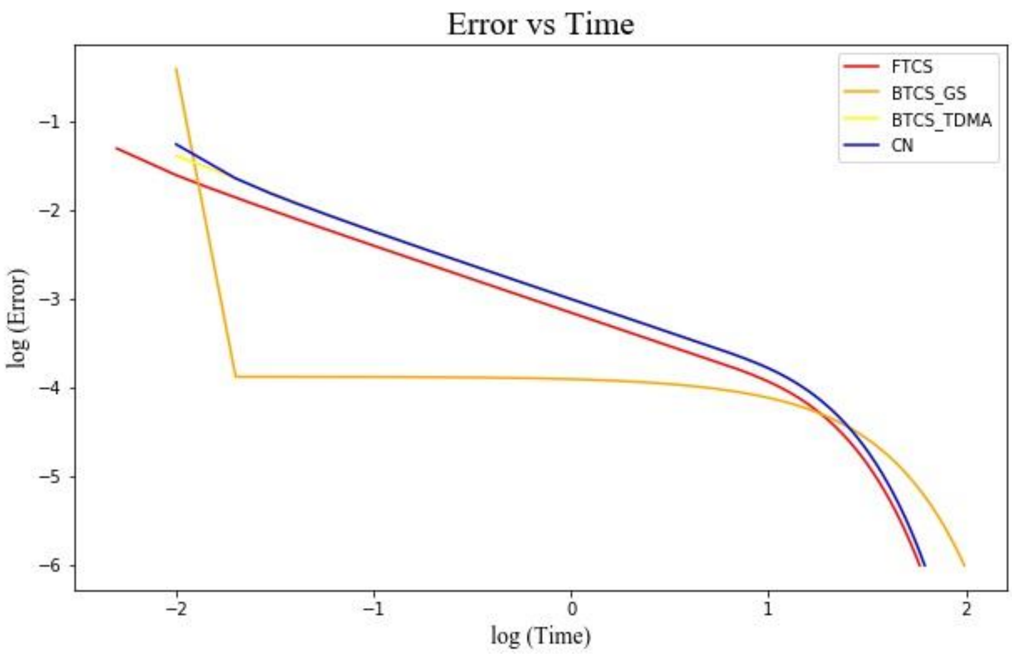

Fig 3: Convergence History of Error vs Time for all Schemes

Comparison Study of number of time iterations and physical time taken to converge up to $\epsilon = 10^{-6}$.

Table 1: Number of Time Iterations and Physical time taken for each iteration to converge up to $\epsilon = 10^{-6}$

Iterative Method	No. of Time Iterations	Physical Time taken (msec)
FTCS	58.205	6016.210
BTCS Gauss Seidel	98.02	9267.148
BTCS TDMA	61.76	4610.940
Crank Nicolson TDMA	61.73	6015.469