Project Presentation 1

ME 609 – Optimisation Methods in Engineering

Bracketing Method: ***Bounding Phase Method***

Accurate Method: ***Golden Section Search Method***

Group No : 7

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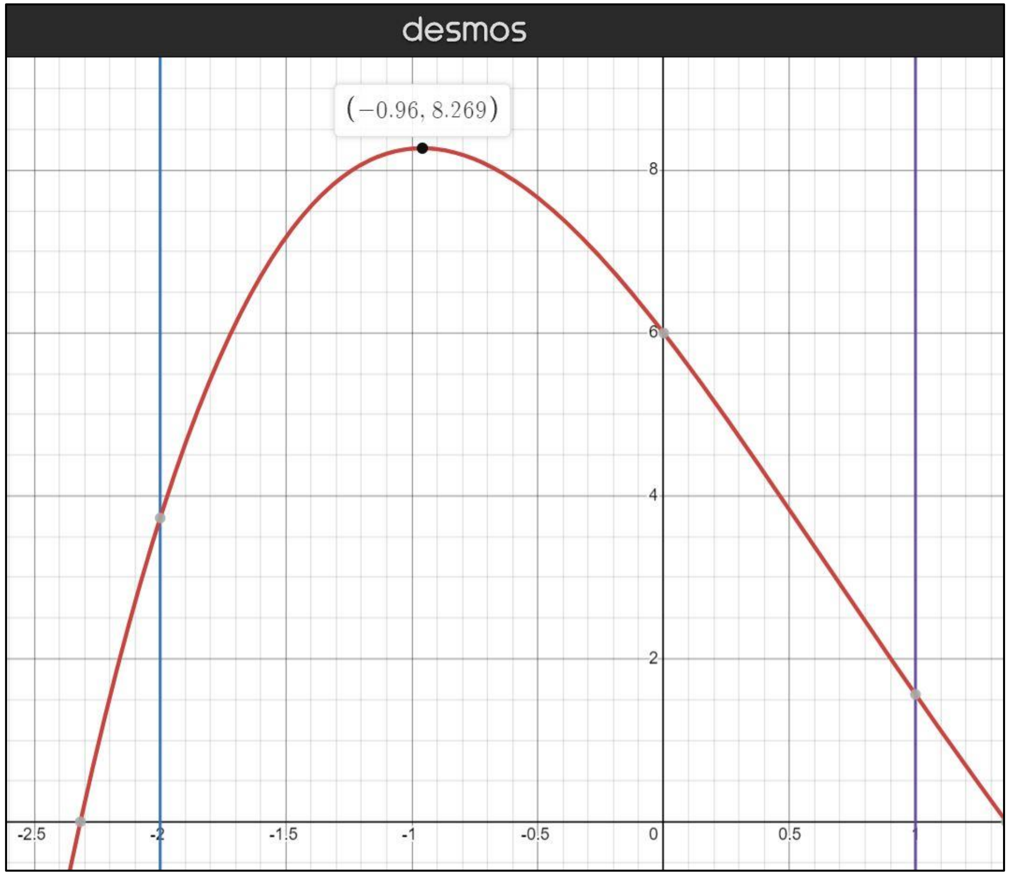
Function: $8 + x^3 - 2x - 2e^x$ in the interval $(-2, 1)$

Plotting the graph on Desmos (online graphing tool) to get an idea of the function we are dealing with.

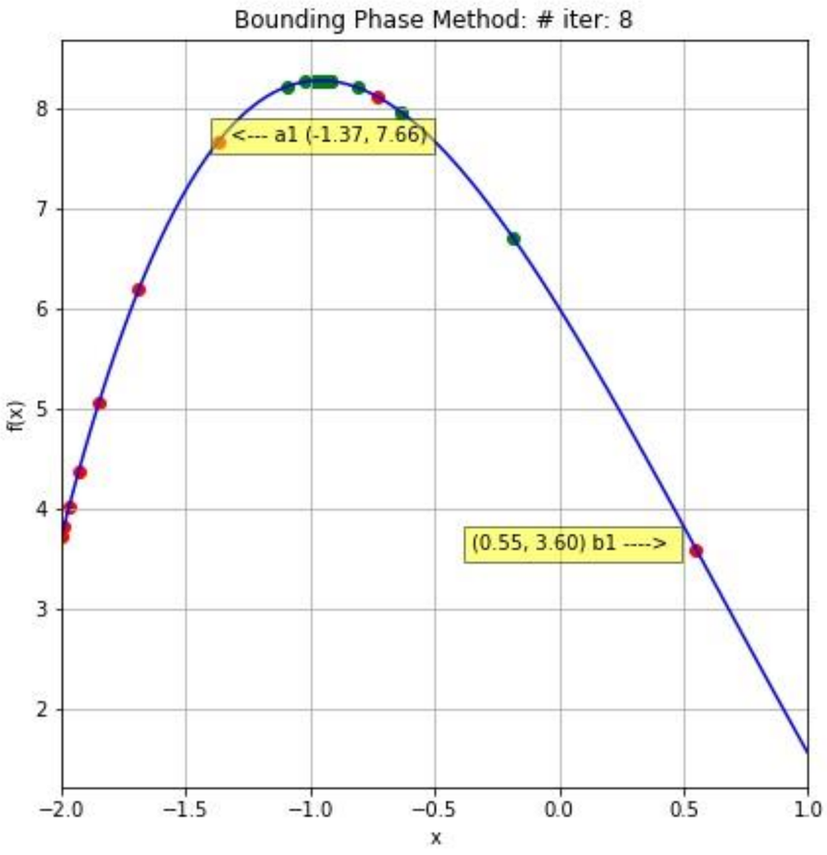


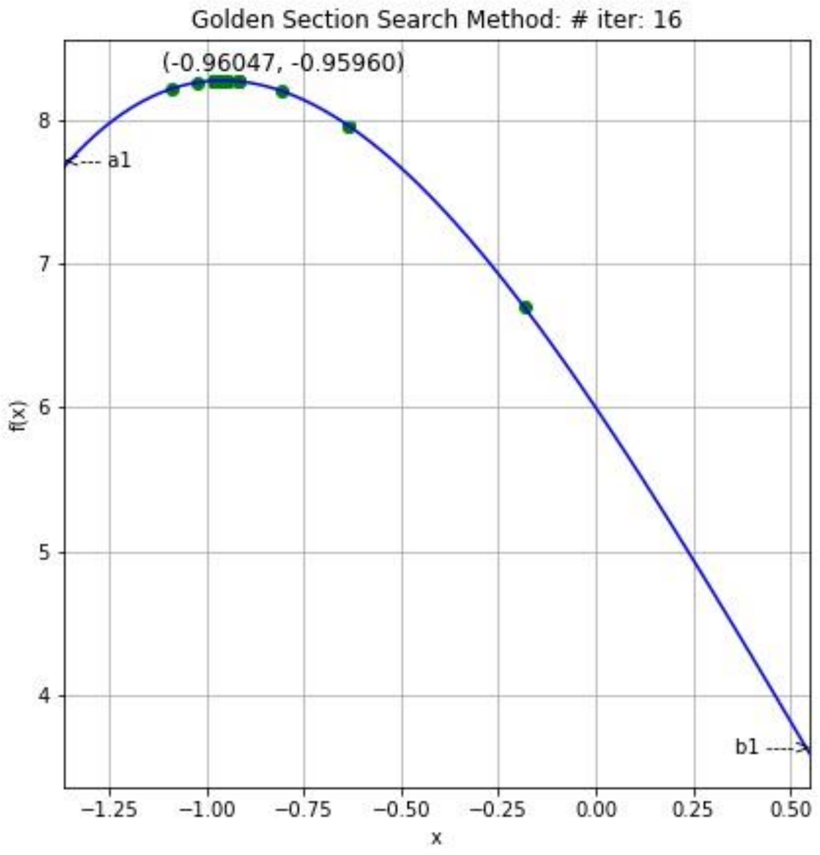
Function: $8 + x^3 - 2x - 2e^x$ in the interval $(-2, 1)$

The bracketing achieved by providing the following inputs:

$(a, b) = (-2, 1)$ $x(0) = -2$ (initial guess)

$delta = 0.01$ $\epsilon = 10^{-3}$

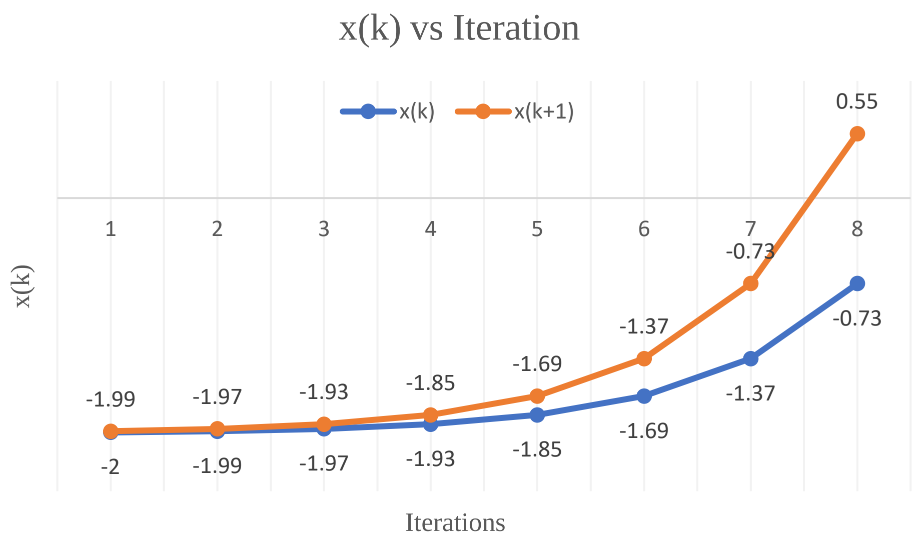


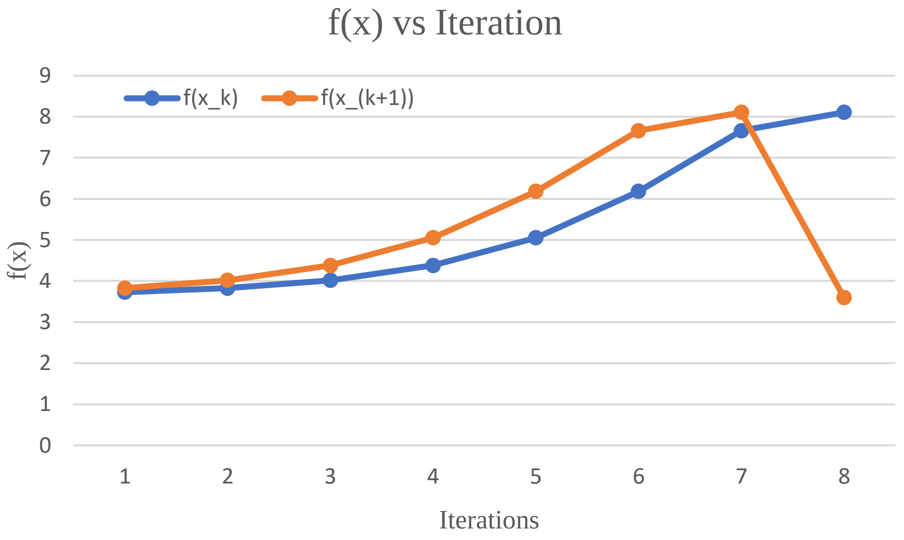


Function: $8 + x^3 - 2x - 2e^x$ in the interval $(-2, 1)$

Bounding Phase Method Iterations:

k	iter	x_k	x_(k+1)	f(x_k)	f(x_(k+1))	Continue/ Terminate
0	1	-2	-1.99	3.729329	3.82601	Continue
1	2	-1.99	-1.97	3.82601	4.015713	Continue
2	3	-1.97	-1.93	4.015713	4.380647	Continue
3	4	-1.93	-1.85	4.380647	5.053901	Continue
4	5	-1.85	-1.69	5.053901	6.184152	Continue
5	6	-1.69	-1.37	6.184152	7.660433	Continue
6	7	-1.37	-0.73	7.660433	8.107165	Continue
7	8	-0.73	0.55	8.107165	3.599869	Terminate

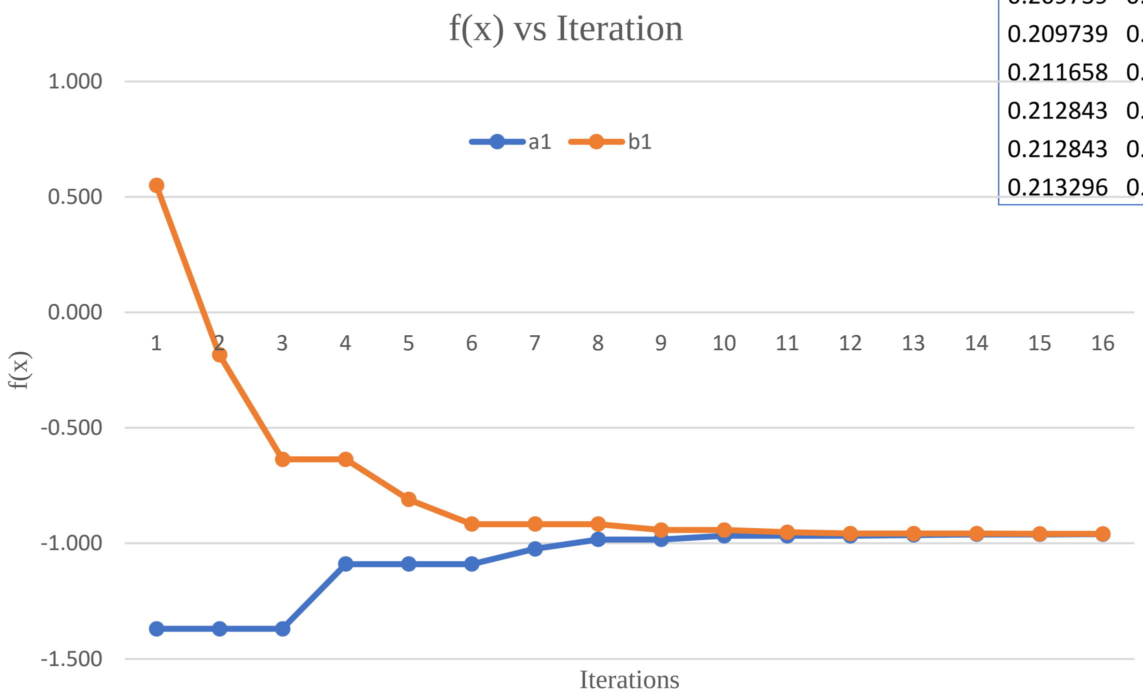




Function: $8 + x^3 - 2x - 2e^x$ in the interval $(-2, 1)$

Golden Section Search Method

aw	bw	lw	w1	w2	f(w1)	f(w2)	Continue/ Terminate	Iter	a1	b1
0	1	1	0.618	0.382	6.695904	7.956962	Continue	**1**	**-1.370**	**0.550**
0	0.618	0.618	0.381924	0.236076	7.95723	8.263398	Continue	**2**	**-1.370**	**-0.183**
0	0.381924	0.381924	0.236029	0.145895	8.263423	8.212644	Continue	**3**	**-1.370**	**-0.637**
0.145895	0.381924	0.236029	0.291761	0.236058	8.198676	8.263408	Continue	**4**	**-1.090**	**-0.637**
0.145895	0.291761	0.145866	0.23604	0.201616	8.263417	8.26778	Continue	**5**	**-1.090**	**-0.810**
0.145895	0.23604	0.090145	0.201605	0.18033	8.267776	8.256047	Continue	**6**	**-1.090**	**-0.917**
0.18033	0.23604	0.05571	0.214759	0.201612	8.269458	8.267778	Continue	**7**	**-1.024**	**-0.917**
0.201612	0.23604	0.034429	0.222888	0.214763	8.268415	8.269458	Continue	**8**	**-0.983**	**-0.917**
0.201612	0.222888	0.021277	0.214761	0.209739	8.269458	8.269311	Continue	**9**	**-0.983**	**-0.942**
0.209739	0.222888	0.013149	0.217865	0.214762	8.269246	8.269458	Continue	**10**	**-0.967**	**-0.942**
0.209739	0.217865	0.008126	0.214761	0.212843	8.269458	8.269474	Continue	**11**	**-0.967**	**-0.952**
0.209739	0.214761	0.005022	0.212843	0.211658	8.269474	8.269439	Continue	**12**	**-0.967**	**-0.958**
0.211658	0.214761	0.003104	0.213576	0.212843	8.269478	8.269474	Continue	**13**	**-0.964**	**-0.958**
0.212843	0.214761	0.001918	0.214029	0.213576	8.269474	8.269478	Continue	**14**	**-0.961**	**-0.958**
0.212843	0.214029	0.001185	0.213576	0.213296	8.269478	8.269478	Continue	**15**	**-0.961**	**-0.959**
0.213296	0.214029	0.000733	0.213749	0.213576	8.269477	8.269478	Terminate	**16**	**-0.960**	**-0.959**



Function: $8 + x^3 - 2x - 2e^x$ in the interval $(-2, 1)$

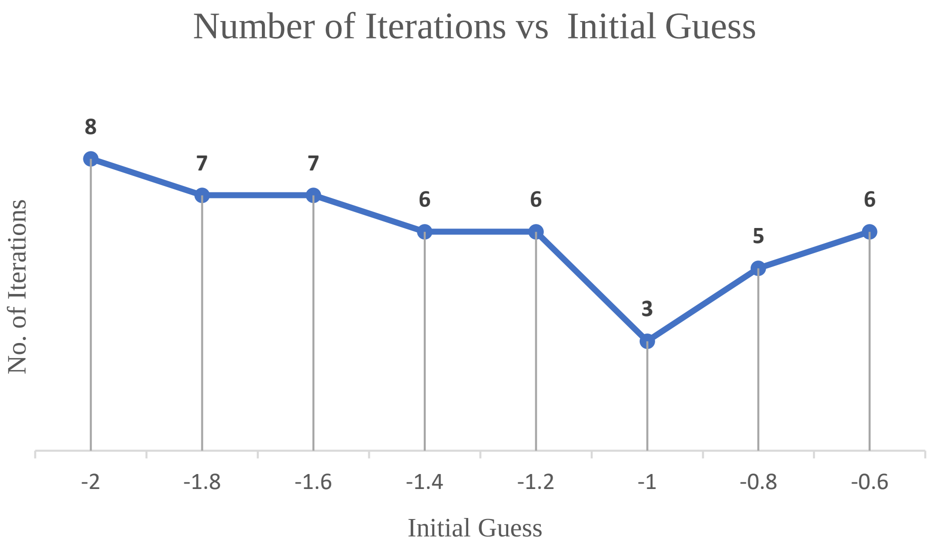
Extracting data by running the program for iterations changing *x(0)*, *epsilon* and *delta*

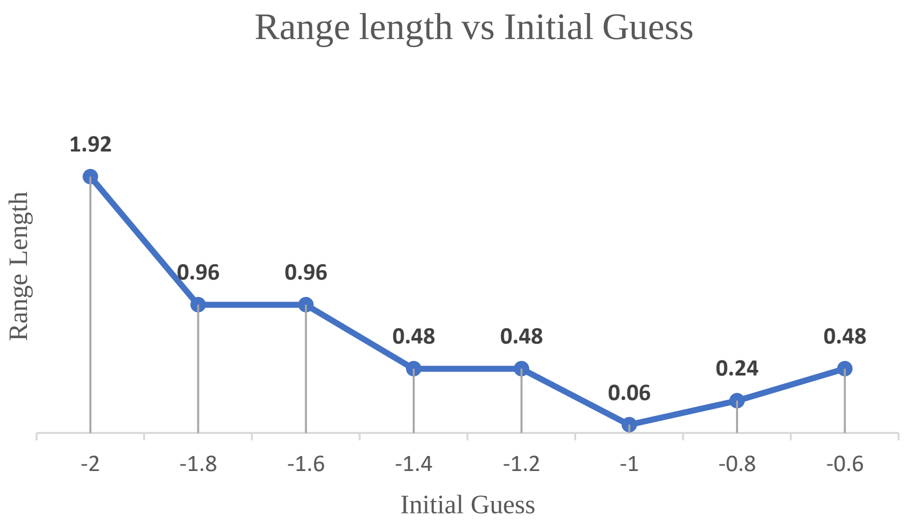
a	b	l	x(0)	delta	Bounding Iterations	a1	b1	l1	epsilon	Golden Section Iterations	final_a	final_b	final_l
-2	1	3	-2	0.01	8	-1.37	0.55	1.92	0.001	16	-0.96047	-0.9596	0.000869
-2	1	3	-1.8	0.01	7	-1.49	-0.53	0.96	0.001	16	-0.96026	-0.95982	0.000435
-2	1	3	-1.6	0.01	7	-1.29	-0.33	0.96	0.001	16	-0.96024	-0.9598	0.000435
-2	1	3	-1.4	0.01	6	-1.25	-0.77	0.48	0.001	16	-0.96022	-0.96	0.000217
-2	1	3	-1.2	0.01	6	-1.05	-0.57	0.48	0.001	16	-0.96025	-0.96003	0.000217
-2	1	3	-1	0.01	3	-0.99	-0.93	0.06	0.001	16	-0.96017	-0.96014	0.0000272
-2	1	3	-0.8	0.01	5	-0.87	-1.11	0.24	0.001	16	-0.96011	-0.96022	0.00011
-2	1	3	-0.6	0.01	6	-0.75	-1.23	0.48	0.001	16	-0.96001	-0.96022	0.00022
-2	1	3	-1.4	0.01	6	-1.25	-0.77	0.48	0.0001	21	-0.96016	-0.96014	1.96E-05
-2	**1**	**3**	**-1.4**	**0.01**	**6**	**-1.25**	**-0.77**	**0.48**	**0.00001**	**25**	**-0.96015**	**-0.96015**	**2.86E-06**
-2	1	3	-1.4	0.01	6	-1.25	-0.77	0.48	0.01	11	-0.96114	-0.95873	0.002411
-2	1	3	-1.6	0.1	4	-1.3	-0.1	1.2	0.001	16	-0.96051	-0.95996	0.000543
-2	1	3	-1.6	0.05	5	-1.25	-0.05	1.2	0.001	16	-0.96038	-0.95984	0.000543
-2	1	3	-1.6	0.005	8	-1.285	-0.325	0.96	0.001	16	-0.96033	-0.9599	0.000435
-2	1	3	-1.6	0.001	10	-1.345	-0.577	0.768	0.001	16	-0.96036	-0.96001	0.000348

Function: $8 + x^3 - 2x - 2e^x$ in the interval $(-2, 1)$

Changing ***initial guess*** $\boldsymbol{x(0)}$ for Bounding Phase Method

a	b	l	x(0)	delta	Bounding Iterations	a1	b1	l1
-2	1	3	**-2**	0.01	**8**	-1.37	0.55	**1.92**
-2	1	3	**-1.8**	0.01	**7**	-1.49	-0.53	**0.96**
-2	1	3	**-1.6**	0.01	**7**	-1.29	-0.33	**0.96**
-2	1	3	**-1.4**	0.01	**6**	-1.25	-0.77	**0.48**
-2	1	3	**-1.2**	0.01	**6**	-1.05	-0.57	**0.48**
-2	1	3	**-1**	0.01	**3**	-0.99	-0.93	**0.06**
-2	1	3	**-0.8**	0.01	**5**	-0.87	-1.11	**0.24**
-2	1	3	**-0.6**	0.01	**6**	-0.75	-1.23	**0.48**

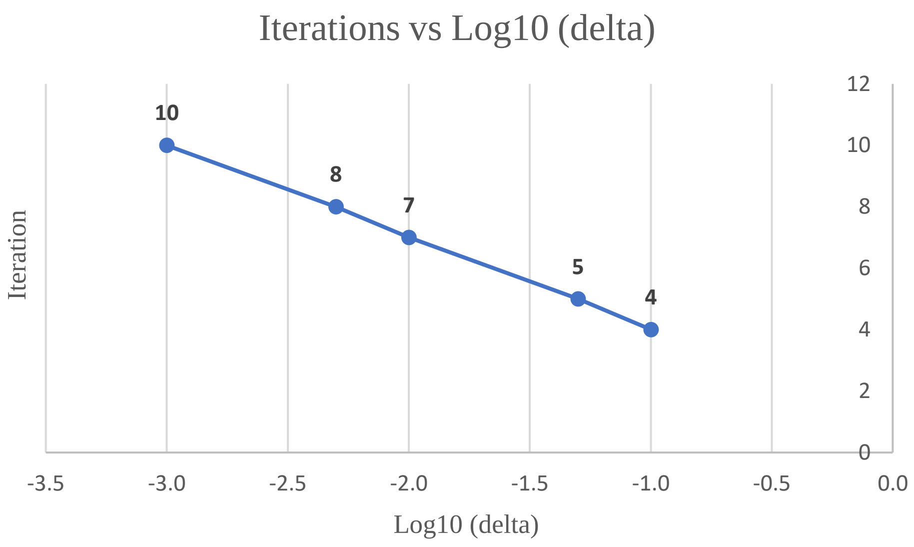


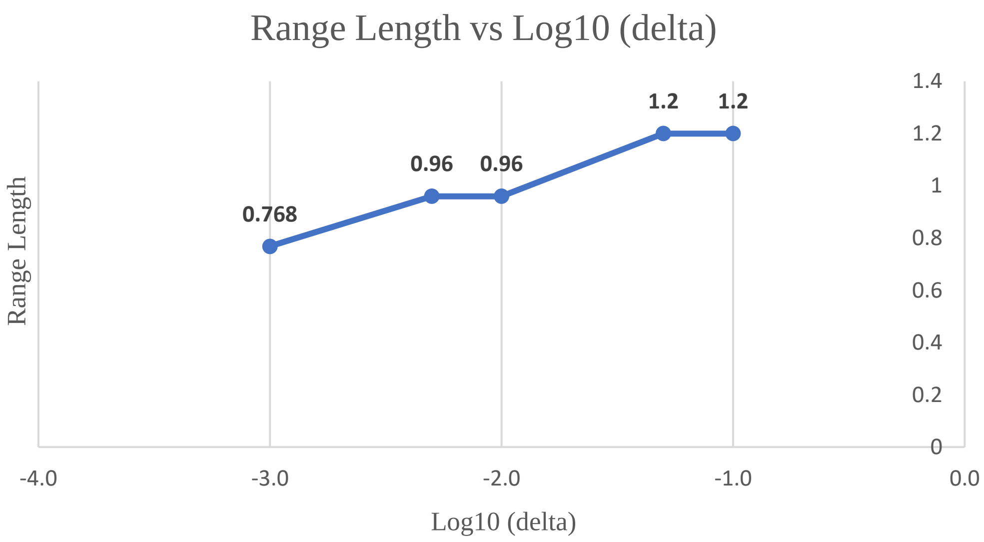


Function: $8 + x^3 - 2x - 2e^x$ in the interval $(-2, 1)$

Changing ***delta*** for
Bounding Phase Method

a	b	l	x(0)	delta	Log Delta	Bounding Iterations	a1	b1	l1
-2	1	3	-1.6	0.1	**-1.0**	**4**	-1.3	-0.1	**1.2**
-2	1	3	-1.6	0.05	**-1.3**	**5**	-1.25	-0.05	**1.2**
-2	1	3	-1.6	0.01	**-2.0**	**7**	-1.29	-0.33	**0.96**
-2	1	3	-1.6	0.005	**-2.3**	**8**	-1.285	-0.325	**0.96**
-2	1	3	-1.6	0.001	**-3.0**	**10**	-1.345	-0.577	**0.768**

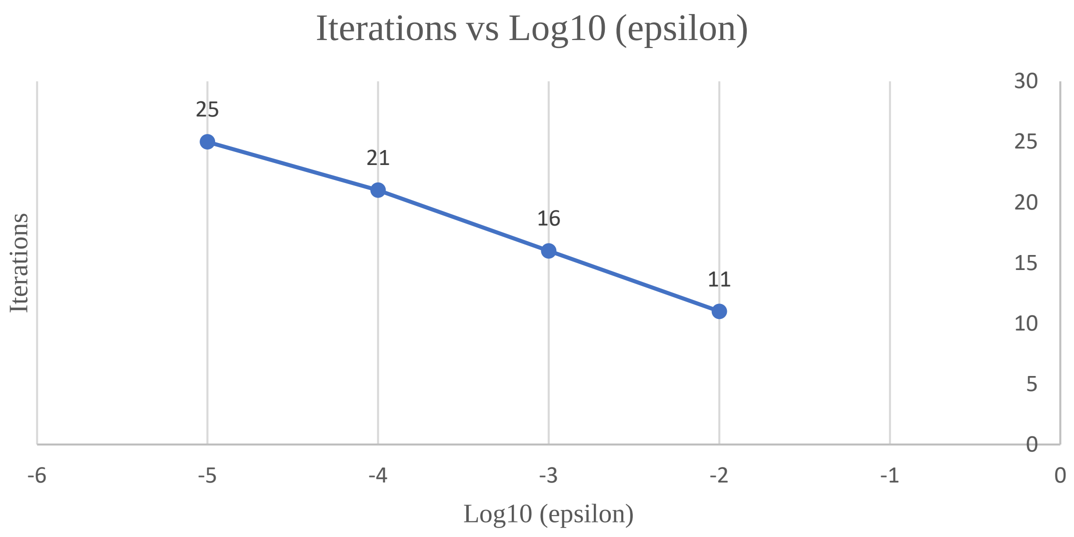


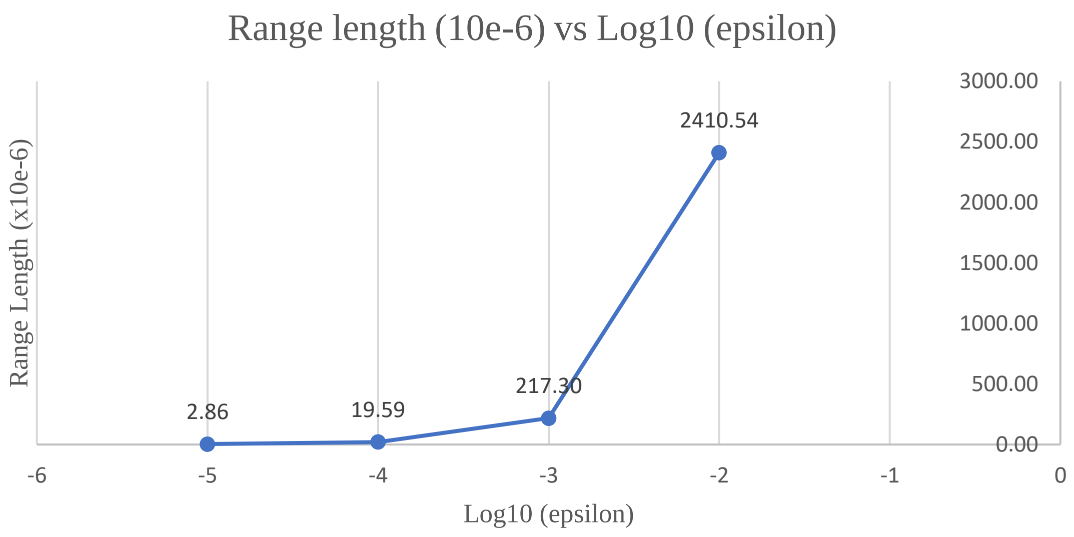


Function: $8 + x^3 - 2x - 2e^x$ in the interval $(-2, 1)$

Changing ***epsilon*** for
Golden Section Search Method

a1	b1	l1	epsilon	Log(epsilon)	Golden Section Iterations	final_a	final_b	final_l	Final L
-1.25	-0.77	0.48	0.01000	**-2**	**11**	-0.96114	-0.95873	0.002411	**2410.54**
-1.25	-0.77	0.48	0.00100	**-3**	**16**	-0.96022	-0.96	0.000217	**217.30**
-1.25	-0.77	0.48	0.00010	**-4**	**21**	-0.96016	-0.96014	0.000020	**19.59**
-1.25	-0.77	0.48	0.00001	**-5**	**25**	-0.96015	-0.96015	0.000003	**2.86**





Results and Observation

Bounding Phase Method	**Golden Section Search Method**
As the *initial guess* $x^{(0)}$ approaches the solution point, the *number of iterations* and reduces to a minimum. With the *reduction in delta*, the *final range decreases*, but the *number of iteration increases.*	The *number of iterations* only depend upon the value of *epsilon.*

Project Presentation 2

ME 609 – Optimisation Methods in Engineering

Multivariable Method: ***Conjugate Direction Method***

Unidirectional Search: ***Bounding Phase & Newton Raphson***

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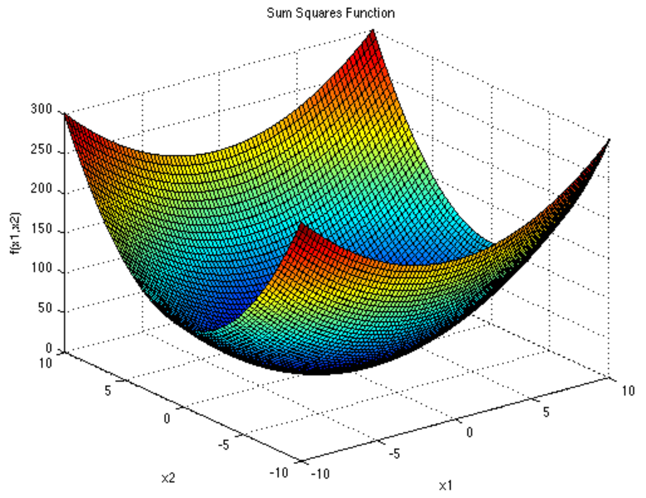
Sum Squares Function



Optimal Solution: $x^* = (0,0,0,0,0); \quad f(x^*) = 0$

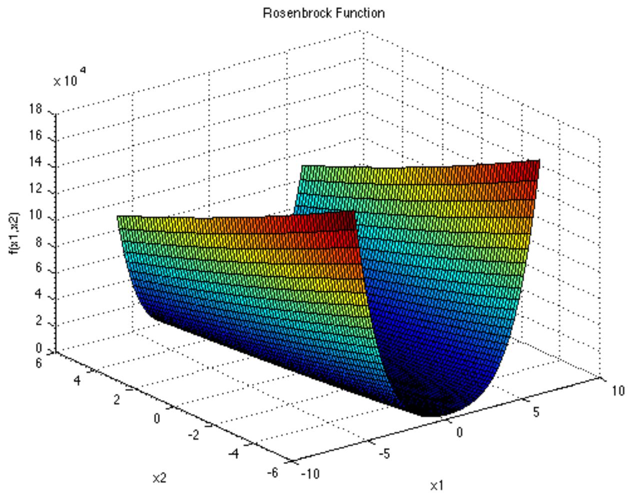
Trial	Initial Point (x0)	Final Point (x*)	Function Value f(x*)	No. of Fn. Eval
1	(5.000, 5.000, 5.000, 4.000, 3.000)	(-0.000, 0.000, 0.000, 0.000, -0.000)	0	203
2	(1.000, 3.000, 2.450, 2.345, 3.345)	(0.000, 0.000, 0.000, 0.000, -0.000)	0	200
3	(-1.000, -3.000, -2.450, -2.345, -3.345)	(-0.000, -0.000, -0.000, -0.000, 0.000)	0	193
4	(5.000, 5.000, 5.000, 5.000, 5.000)	(0.000, -0.000, -0.000, -0.000, 0.000)	0	209
5	(5.000, -5.000, 5.000, -5.000, 5.000)	(0.000, 0.000, -0.000, 0.000, 0.000)	0	209
6	(-5.000, -5.000, -5.000, -5.000, -5.000)	(-0.000, 0.000, 0.000, 0.000, -0.000)	0	202

No.	Point (x)	Fn_val f(x)
0	(5.000, 5.000, 5.000, 4.000, 3.000)	259
1	(-0.000, 5.000, 5.000, 4.000, 3.000)	234
2	(-0.000, 0.000, 5.000, 4.000, 3.000)	184
3	(-0.000, 0.000, -0.000, 4.000, 3.000)	109
4	(-0.000, 0.000, -0.000, -0.000, 3.000)	45
5	(-0.000, 0.000, -0.000, -0.000, -0.000)	0
6	(-0.000, 0.000, -0.000, -0.000, -0.000)	0
7	(-0.000, 0.000, -0.000, 0.000, -0.000)	0
8	(-0.000, 0.000, -0.000, 0.000, -0.000)	0
9	(-0.000, 0.000, -0.000, 0.000, -0.000)	0
10	(-0.000, 0.000, -0.000, 0.000, -0.000)	0
11	(-0.000, 0.000, -0.000, -0.000, -0.000)	0
12	(-0.000, 0.000, 0.000, 0.000, -0.000)	0

Observation:
The point seem to tend toward the Global Minima with any initial value.
The number of function evaluation seem to be about the same for any initial point.

Rosenbrock Function

Optimal Solution: $x^* = (1, 1, 1); \quad f(x^*) = 0$



Trial	Initial Point (x0)	Final Point (x*)	Function Value f(x*)	No. of Fn. Eval
1	(-2.000, 0.000, 2.000)	(0.976, 0.949, 0.901)	0.005	2128
2	(-1.000, 1.000, 0.000)	(-0.758, 0.585, 0.346)	3.276	234
3	(1.020, 1.050, 0.980)	(1.001, 1.002, 1.004)	0.000	193
4	(1.500, 1.500, 0.500)	(0.953, 0.909, 0.825)	0.011	228
5	(-0.700, 0.500, 0.300)	(-0.092, -0.005, 0.009)	2.232	2770
6	(-0.800, 0.600, 0.100)	(-0.372, 0.121, 0.024)	2.697	2478
7	**(-2.000, 0.600, 2.000)**	**(1.106, 1.224, 1.499)**	**0.062**	**312**
8	(-2.000, -2.000, -2.000)	(0.954, 0.908, 0.819)	0.014	1593
9	(-2.000, -2.000, 1.000)	(0.392, 0.136, -0.016)	1.268	631
10	(-2.000, 1.000, 1.000)	(-1.127, 1.280, 1.644)	4.616	388
11	**(1.000, 1.000, 1.000)**	**(1.000, 1.000, 1.000)**	**0.000**	**59**

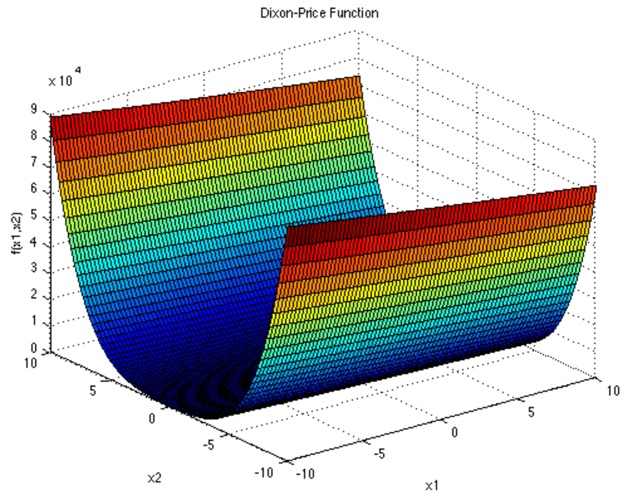
No.	Point (x)	Fn_val f(x)
0	(-2.000, 0.600, 2.000)	1434.120
1	(-0.008, 0.600, 2.000)	306.128
2	(-0.008, 1.224, 2.000)	176.050
3	(-0.008, 1.224, 1.499)	150.961
4	(1.106, 1.224, 1.499)	0.062
5	(1.106, 1.224, 1.499)	0.062
6	(1.106, 1.224, 1.499)	0.062
7	(1.106, 1.224, 1.499)	0.062
8	(1.106, 1.224, 1.499)	0.062

Observation:

There seem to be multiple points at which the conjugate method converges.
When the initial point is near the Global Minima, it converges near it.

Dixon-Price Function

Optimal Solution: $x^* = (1.0,\ 0.707,\ 0.594,\ 0.545);\quad f(x^*) = 0$



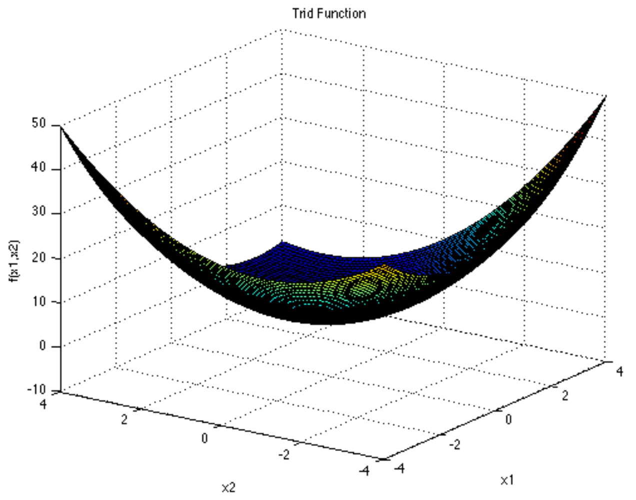
Trial	Initial Point (x0)	Final Point (x*)	Function Value f(x*)	No. of Fn. Eval
1	**(1.000, 0.700, 0.600, 0.500)**	**(1.004, 0.708, 0.596, 0.546)**	**0**	**2282**
2	(3.000, 7.000, 6.000, 5.000)	(-253.848, -0.256, 1.114, -0.733)	193980.9	586
3	(0.000, 0.000, 0.000, 0.000)	(0.778, 0.000, 0.000, 0.001)	1.259	72
4	(1.000, 1.000, 1.000, 1.000)	(0.228, -0.541, -0.104, -0.195)	1.935	4885
5	(0.980, 0.650, 0.550, 0.450)	(0.128, -0.591, 0.055, -0.048)	2.492	2081
6	(-1.000, -0.700, 0.000, -0.500)	(-7.787, 0.176, -0.505, -0.009)	201.754	830
7	(1.000, 0.700, 0.600, -0.500)	(-9.848, -0.202, 0.286, 0.973)	325.62	2109

No.	Point (x)	Fn_val f(x)
0	(1.000, 0.700, 0.600, 0.500)	0.042
1	(1.013, 0.700, 0.600, 0.500)	0.0436
2	(1.013, 0.707, 0.600, 0.500)	0.0411
3	(1.013, 0.707, 0.614, 0.500)	0.059
4	(1.013, 0.707, 0.614, 0.554)	0.0072
5	(1.018, 0.707, 0.614, 0.554)	0.0076
6	(1.018, 0.706, 0.613, 0.551)	0.0074
7	(1.025, 0.706, 0.613, 0.551)	0.0084
8	(1.025, 0.707, 0.613, 0.551)	0.0078
9	(1.025, 0.707, 0.599, 0.551)	0.0024
10	(1.025, 0.707, 0.599, 0.548)	0.0022
30	(1.007, 0.709, 0.596, 0.546)	0.0001
31	(1.006, 0.709, 0.596, 0.546)	0
32	(1.005, 0.709, 0.596, 0.546)	0
33	(1.005, 0.709, 0.596, 0.546)	0
34	(1.005, 0.708, 0.596, 0.546)	0
35	(1.005, 0.708, 0.596, 0.546)	0
36	(1.004, 0.708, 0.596, 0.546)	0
37	(1.004, 0.708, 0.596, 0.546)	0
38	(1.004, 0.708, 0.596, 0.546)	0
39	(1.004, 0.708, 0.596, 0.546)	0
40	(1.004, 0.708, 0.596, 0.546)	0

Observation:
There seem to be multiple points at which the conjugate method converges. When the initial point is near the Global Minima, it converges near it.

Trid Function

Optimal Solution: $x^* = (6, 10, 12, 12, 10, 6); \quad f(x^*) = -50$



Trial	Initial Point (x0)	Final Point (x*)	Function Value f(x*)	No. of Fn. Eval
1	**(6.000, 10.000, 12.000, 12.000, 10.000, 6.000)**	**(6.000, 10.000, 12.000, 12.000, 10.000, 6.000)**	**-50**	**79**
2	**(5.000, 9.000, 10.000, 9.000, 9.000, 3.000)**	**(5.959, 9.816, 11.675, 11.256, 8.992, 5.772)**	**-49.526**	**1381**
3	(-5.000, -9.000, -10.000, -9.000, -9.000, -3.000)	(5.936, 9.910, 11.878, 11.854, 9.761, 5.022)	-49.242	1741
4	(0.000, -10.000, 4.000, 6.000, 10.000, -12.000)	(3.590, 5.373, 5.095, 4.714, 4.313, 2.842)	-32.509	2347
5	(1.000, -1.000, 1.000, 1.000, 1.000, -1.000)	(6.036, 10.055, 12.065, 11.961, 9.977, 6.060)	-49.988	3036
6	(-1.000, -1.000, -1.000, -1.000, -1.000, -1.000)	(5.998, 9.998, 11.998, 11.998, 9.998, 5.997)	-50	1138
7	(0.000, 0.000, 0.000, 0.000, 0.000, 0.000)	(6.001, 10.001, 12.002, 12.001, 10.002, 6.002)	-50	1117
8	(12.000, 12.000, 12.000, 12.000, 12.000, 12.000)	(6.646, 11.260, 13.832, 14.074, 11.800, 7.117)	-48.515	2306
9	(15.000, 15.000, 15.000, 15.000, 15.000, 15.000)	(7.269, 12.548, 14.994, 14.541, 11.609, 6.674)	-47.076	1277
10	(1.000, 1.000, 1.000, 1.000, 1.000, 1.000)	(5.986, 9.971, 11.933, 11.818, 9.349, 6.393)	-49.26	1012

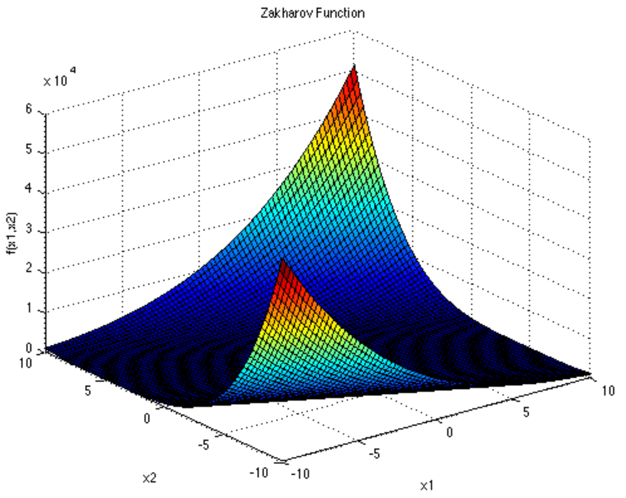
No.	Point (x)	Fn_val f(x)
0	(5.000, 9.000, 10.000, 9.000, 9.000, 3.000)	-40
1	(5.500, 9.000, 10.000, 9.000, 9.000, 3.000)	-40.25
2	(5.500, 8.750, 10.000, 9.000, 9.000, 3.000)	-40.3125
3	(5.500, 8.750, 9.875, 9.000, 9.000, 3.000)	-40.3281
4	(5.500, 8.750, 9.875, 10.437, 9.000, 3.000)	-42.3945
5	(5.500, 8.750, 9.875, 10.437, 7.719, 3.000)	-44.0361
78	(5.959, 9.816, 11.675, 11.257, 8.992, 5.772)	-49.526
79	(5.959, 9.816, 11.675, 11.257, 8.992, 5.772)	-49.526
80	(5.960, 9.816, 11.675, 11.257, 8.992, 5.772)	-49.526
81	(5.960, 9.816, 11.675, 11.257, 8.992, 5.772)	-49.526
82	(5.959, 9.816, 11.675, 11.256, 8.992, 5.772)	-49.526
83	(5.959, 9.816, 11.675, 11.256, 8.992, 5.772)	-49.526
84	(5.959, 9.816, 11.675, 11.256, 8.992, 5.772)	-49.526

Observation:

The point seem to tend toward the Global Minima with almost any initial value.

Zakharov Function

Optimal Solution: $x^* = (0,\ 0);\quad f(x^*) = 0$



Trial	Initial Point (x0)	Final Point (x*)	Function Value f(x*)	No. of Fn. Eval
1	**(0.000, 0.000)**	**(-0.000, 0.000)**	**0**	**33**
2	(1.000, -2.000)	(0.000, 0.000)	0	357
3	**(5.000, -2.000)**	**(0.000, 0.000)**	**0**	**359**
4	(5.000, -10.000)	(0.000, -0.000)	0	336
5	(-15.000, -10.000)	(0.000, -0.000)	0	339

No.	Point (x)	Fn_val f(x)
0	(5.000, -2.000)	29.3125
1	(1.827, -2.000)	9.9119
2	(1.827, -0.518)	3.7855
3	(0.255, -0.518)	0.5097
4	(-0.188, -0.101)	0.0848
5	(0.041, -0.101)	0.0183
6	(-0.041, -0.023)	0.0042
7	(0.001, -0.001)	0
8	(-0.000, -0.000)	0
9	(0.000, -0.000)	0
10	(0.000, 0.000)	0
11	(0.000, 0.000)	0
12	(0.000, 0.000)	0

Observation:

The point seem to tend toward the Global Minima with any initial value.

Thank you

Project Presentation 3

ME 609 – Optimisation Methods in Engineering

Constrained Optimisation Problem

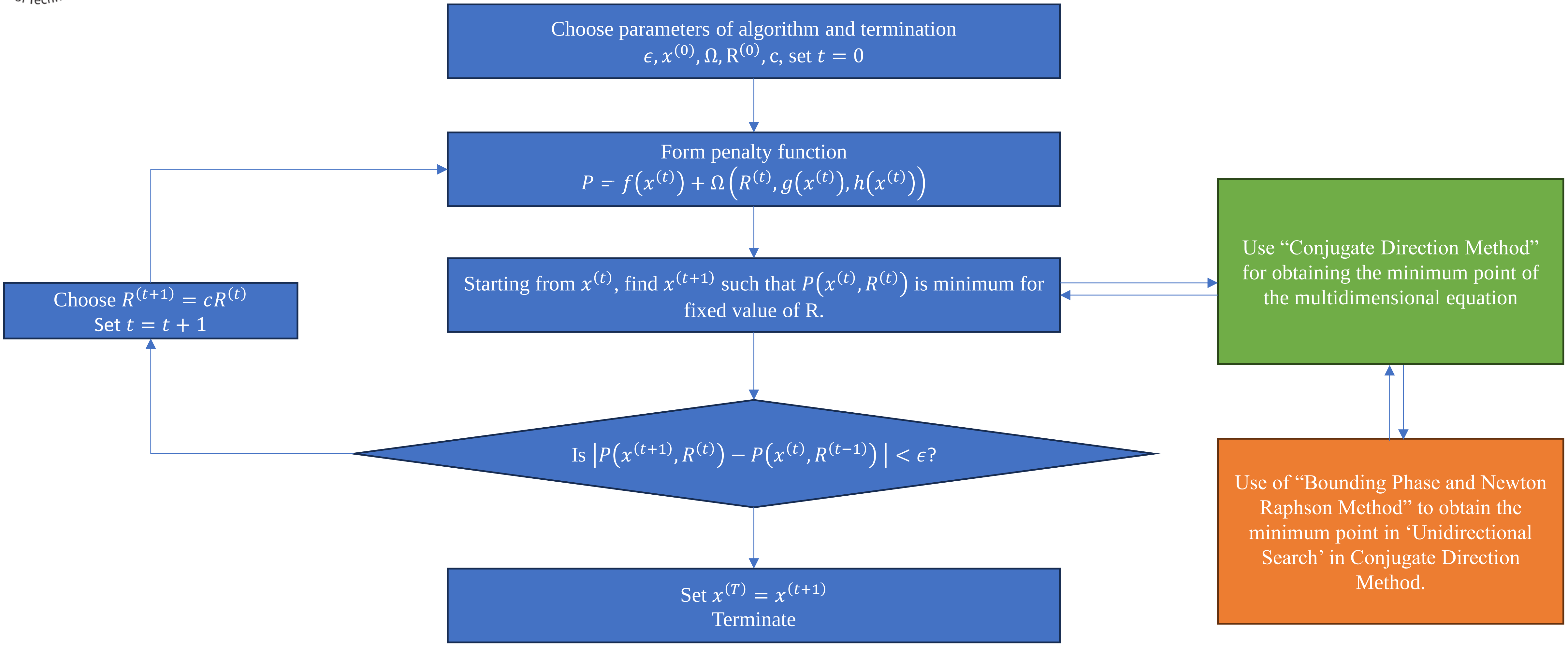
Multivariable Method: ***Conjugate Direction Method***

Unidirectional Search: ***Bounding Phase & Newton Raphson***

Group No : 7

Nirmal S. [234103107]
Rohith Kumar Saragadam [234103109]

Flowchart



Choose parameters of algorithm and termination
$\epsilon, x^{(0)}, \Omega, \mathrm{R}^{(0)}$, c, set $t = 0$
Form penalty function
$P = f\left(x^{(t)}\right) + \Omega\left(R^{(t)}, g\left(x^{(t)}\right), h\left(x^{(t)}\right)\right)$
Starting from $x^{(t)}$, find $x^{(t+1)}$ such that $P\left(x^{(t)}, R^{(t)}\right)$ is minimum for fixed value of R.
Use "Conjugate Direction Method" for obtaining the minimum point of the multidimensional equation
Use of "Bounding Phase and Newton Raphson Method" to obtain the minimum point in 'Unidirectional Search' in Conjugate Direction Method.
Choose $R^{(t+1)} = cR^{(t)}$
Set $t = t + 1$
Is $\left|P\left(x^{(t+1)}, R^{(t)}\right) - P\left(x^{(t)}, R^{(t-1)}\right)\right| < \epsilon$?
Set $x^{(T)} = x^{(t+1)}$
Terminate

Problem 1

$$\min \boldsymbol{f}(\boldsymbol{x}) = (\boldsymbol{x_1} - \mathbf{10})^3 + (\boldsymbol{x_2} - \mathbf{20})^3,$$

$$subject\ to\ \boldsymbol{g_1}(\boldsymbol{x}) = (\boldsymbol{x_1} - \mathbf{5})^2 + (\boldsymbol{x_2} - \mathbf{5})^2 - 100 \geq 0,$$

$$\boldsymbol{g_2}(\boldsymbol{x}) = (\boldsymbol{x_1} - \mathbf{6})^2 + (\boldsymbol{x_2} - \mathbf{5})^2 - 82.81 \leq 0,$$

$$\mathbf{13} \leq \boldsymbol{x_1} \leq \mathbf{20}, \qquad \mathbf{0} \leq \boldsymbol{x_2} \leq \mathbf{4}.$$

Sl. No	Initial Point	x^T	Number of Function Evaluation
1	**(0, 0)**	**(14.06990755, 0.78990993)**	**918**
2	(1, 3)	(14.06990755, 0.78990993)	1020
3	(15, -265)	(14.06990755, 0.78990993)	954
4	(-234, 0.1234)	(14.06990755, 0.78990993)	1013
5	(-23.24, 4853.25)	(14.06990755, 0.78990993)	954
6	(-24, 25)	(14.06990755, 0.78990993)	954
7	(-2, 5)	(14.06990755, 0.78990993)	983
8	(-0.0023, 4.2315)	(14.06990755, 0.78990993)	983
9	(-1046, 1046)	(14.06990755, 0.78990993)	954
10	(-106, 10)	(14.06990755, 0.78990993)	970

Observation:

It takes around 1000 function evaluations for any initial point.

Problem 1

$$\min f(x) = (x_1 - 10)^3 + (x_2 - 20)^3,$$

$$subject\ to\ g_1(x) = (x_1 - 5)^2 + (x_2 - 5)^2 - 100 \geq 0,$$

$$g_2(x) = (x_1 - 6)^2 + (x_2 - 5)^2 - 82.81 \leq 0,$$

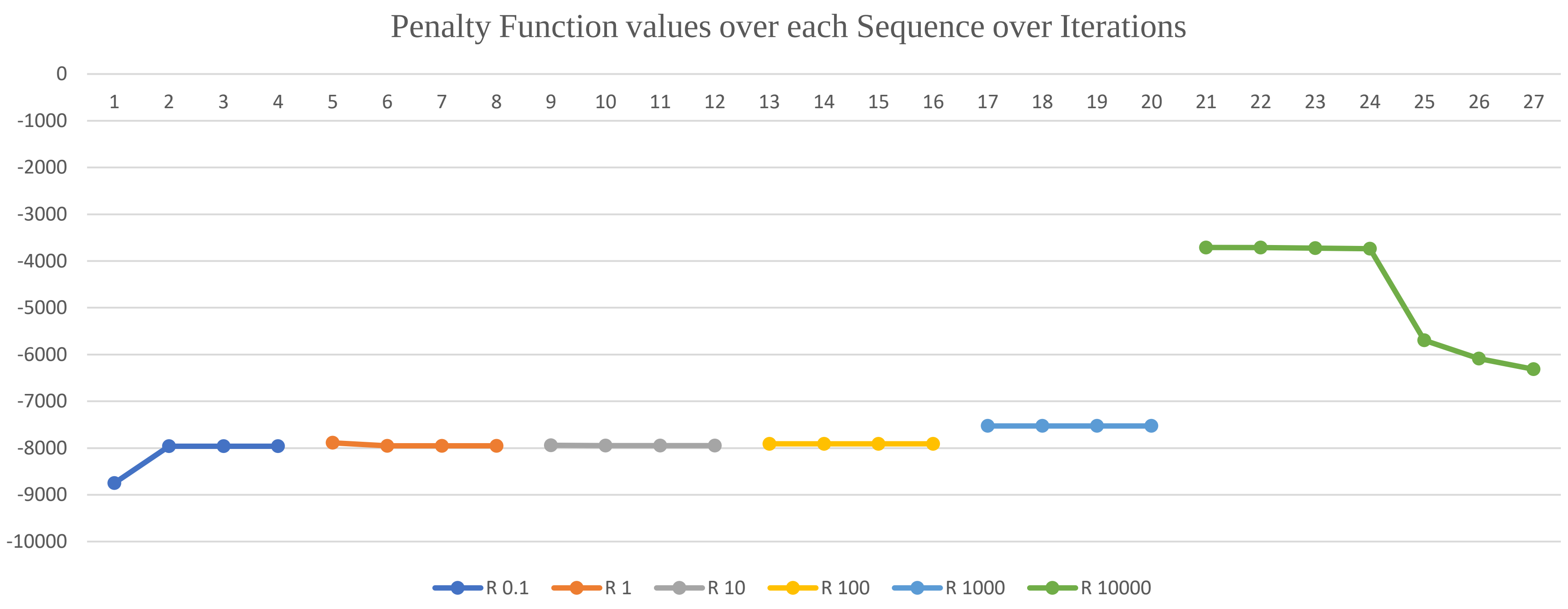
$$13 \leq x_1 \leq 20, \qquad 0 \leq x_2 \leq 4.$$

Sequence(t)	R(t)	x(t)	P(x(t), R(t))
1	0.1	(0.0, 0.0)	-8750
		(13.1236, 0.0)	-7961.41
		…	…
		(13.1236, 0.0)	-7961.41
2	1	(13.1236, 0.0)	-7888.39
		(13.5949, 0.0)	-7952.27
		…	…
		(13.5949, 0.0)	-7952.27
3	10	(13.5949, 0.0)	-7940.82
		(13.6316, 0.0)	-7947.79
		…	…
		(13.6316, 0.0)	-7947.79
4	100	(13.6316, 0.0)	-7908.95
		(13.6349, 0.0)	-7909.54
		…	…
		(13.6349, 0.0)	-7909.54
5	1000	(13.6349, 0.0)	-7527.67
		(13.6353, 0.0)	-7527.73
		…	…
		(13.6353, 0.0)	-7527.73
6	10000	(13.6353, 0.0)	-3709.67
		(13.6353, 0.0)	-3709.68
		…	…
		(13.8762, 0.4074)	-6313.92
7	100000	(13.8762, 0.4074)	4025.336
		(13.8712, 0.4074)	2591.357
		…	…
		(13.0, -1.1524)	1624812
8	1000000	(13.0, -1.1524)	16333059
		(13.0, -1.1524)	16333059
		…	…
		(14.0637, 0.7787)	-5072.91
		(14.0699, 0.7899)	-5415.28

Observation:

At the end of 8th sequence, the point converges near optimum value.

Problem 1

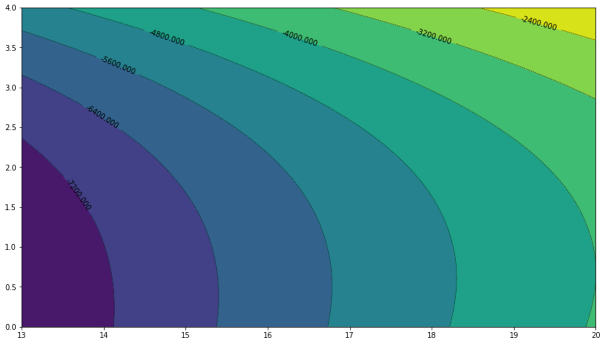


Observation:

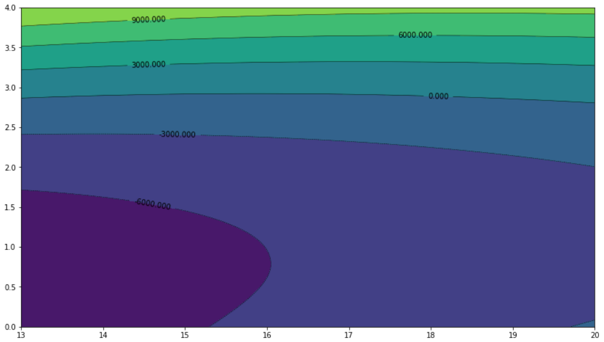
The values of Penalty function changes with each new sequence and then reduces over iterations in the sequence.

Problem 1 Contours

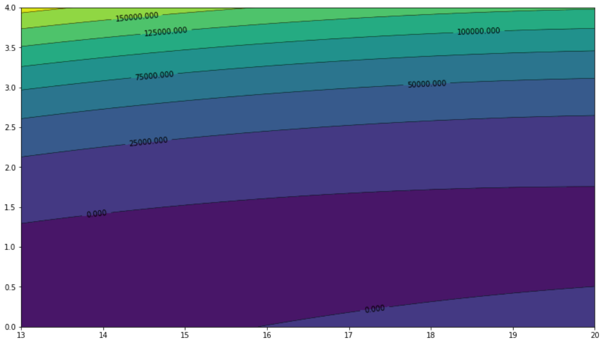
$R = 0.1$



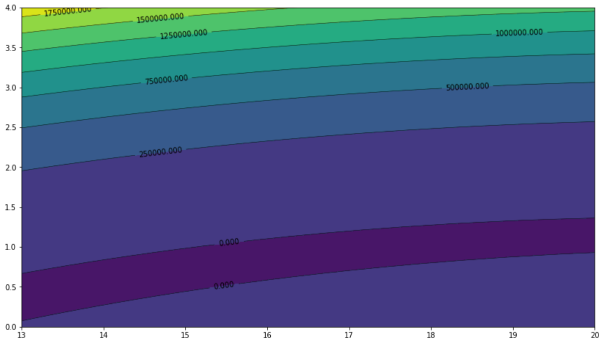
$R = 1$



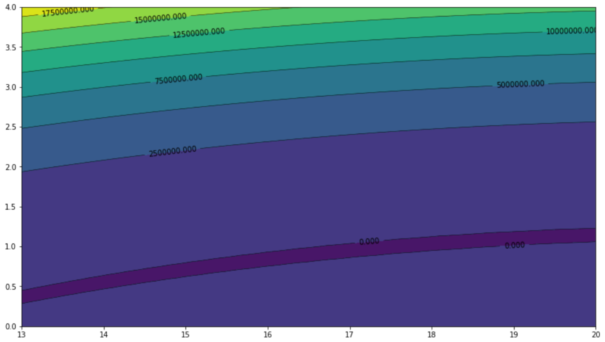
$R = 10$



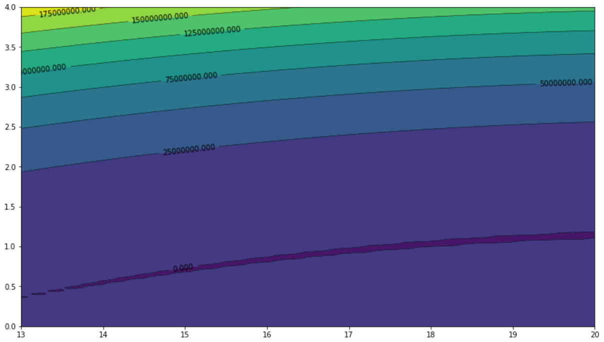
$R = 10^2$



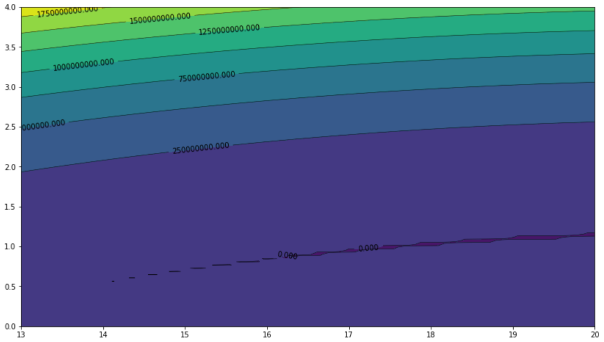
$R = 10^3$



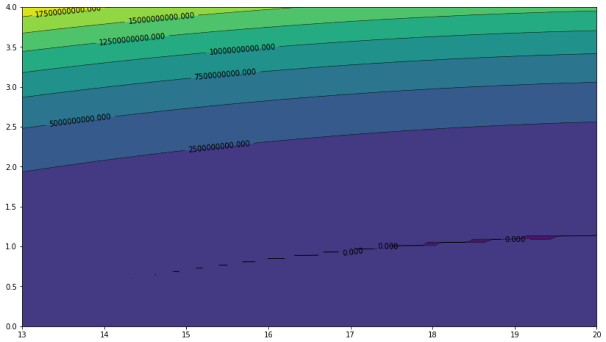
$R = 10^4$



$R = 10^5$



$R = 10^6$



Problem 2, 3

$$\max f(x) = \frac{\sin^3(2\pi x_1)\sin(2\pi x_2)}{x_1^3(x_1+x_2)},$$

$$\text{subject to } g_1(x) = x_1^2 - x_2 + 1 \le 0,$$

$$g_2(x) = 1 - x_1 + (x_2 - 4)^2 \le 0,$$

$$0 \le x_1 \le 10, \quad 0 \le x_2 \le 10$$

$$\min f(x) = x_1 + x_2 + x_3$$

$$\text{subject to } g_1(x) = -1 + 0.0025(x_4 + x_6) \le 0,$$

$$g_2(x) = -1 + 0.0025(-x_4 + x_5 + x_7) \le 0,$$

$$g_3(x) = -1 + 0.01(-x_6 + x_8) \le 0,$$

$$g_4(x) = 100x_1 - x_1x_6 + 833.33252x_4 - 83333.333 \le 0,$$

$$g_5(x) = x_2x_4 - x_2x_7 - 1250x_4 + 1250x_5 \le 0,$$

$$g_6(x) = x_3x_5 - x_3x_8 - 2500x_5 + 1250000 \le 0,$$

$$100 \le x_1 \le 10000$$

$$1000 \le x_i \le 10000, i = 2, 3$$

$$10 \le x_i \le 1000, i = 4, 5, \ldots, 8$$

Result:
Unable to solve the constrained optimization problems due to the values running to large number leading to *nan* error.

Thank you