Aerial Analytic Triangulation Investigation on Interstate 80 in Wyoming

JESSE R. CHAVES, Aerial Surveys Branch, Highway Standards and Design Division, U.S. Bureau of Public Roads.

Results are reported of an investigation of aerial analytic triangulation along a 20,000-foot segment of Interstate Highway 80 in Wyoming. Eleven aerial photographs taken at a scale of 1:6000 were analytically bridged using x and y coordinates measured on glass plate transparencies of the photographs with a Nistri monocomparator. A Wild PUG point transfer and marking instrument was also utilized.

The mathematical method of analytic triangulation employed is a modified version of the one originally developed by the U.S. Coast and Geodetic Survey. The system consists of four separate parts: plate coordinate refinement, relative orientation, cantilever assembly, and cantilever strip adjustment. This analytic method was programmed in Fortran language and the computations were made with the IBM 1401 and 7010 electronic computer systems.

CONVENTIONAL analog aerial triangulation with first-order bridging instruments and ground control surveys are employed as a means of providing the ground position and elevation of points (supplemental control) needed for the absolute orientation of stereoscopic models in photogrammetric instruments. Large-scale topographic maps with a small contour interval can then be compiled for use by highway location and design engineers. With the introduction of electronic digital computers, analytic (mathematical) bridging became feasible as a means of providing the supplemental control for small-scale mapping purposes. Although well understood and initiated by a few leading engineers before 1960, use of the analytic approach to bridge control for highway engineering mapping has only recently attracted the attention of several highway organizations. There is need, therefore, to evaluate the analytic approach for extending surveyed ground control in order to determine the accuracies which can be attained through use of primarily available photogrammetric equipment and materials in highway organizations.

The investigation of analytic aerial triangulation for highways was justified for these reasons:

1. Conventional ground control surveys are expensive, time consuming, and difficult to accomplish under some circumstances. Ground control extended analytically would minimize the number of points whose position and elevation would otherwise have to be surveyed on the ground.
2. Electronic digital computers are available in most highway organizations.
3. The comparator required to measure x and y photographic plate coordinates for the analytic method can be procured at much less cost than conventional analog optical train bridging instruments.
4. Training requirements for the successful operation of comparators used to measure the x and y coordinates of image points on glass plate transparencies of the aerial

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photographs are much less than they are for operation of the optical train bridging instruments.

5. The analytic method of aerial triangulation has the potential accuracy required to accomplish mapping photogrammetrically for highway location and design and offers greater flexibility than the conventional analog types of bridging instruments.

The mathematical procedures employed for this particular evaluation of analytical photogrammetry were developed by the U.S. Coast and Geodetic Survey (1). [This method of relative orientation and cantilever assembly has now been replaced by a method called "Three-Photo Aerotriangulation" (2).] The method with modifications was programmed in the Fortran language and a preliminary investigation was made in 1964 on an 18,000-ft segment of Interstate Highway 66 in Fairfax County, Virginia (3). Results of this work were reported in which seven photographs, taken with a Wild 6-in. focal length aerial camera at a scale of 1:8400 were analytically bridged (4). A monocular comparator was used to measure x and y coordinates of natural images and targeted points on the photographic glass plate transparencies of the aerial photographs. Second-degree cantilever strip adjustment using three horizontal and six vertical control points yielded root-mean-square errors on test points of 0.41 ft for the horizontal and 0.71 ft for the vertical ground coordinates.

Encouraged by the results of this preliminary work, this investigation was begun with the following major objectives:

1. To analytically bridge 11 photographs (10 stereoscopic models) taken at a scale of 1:6000 (500 ft to 1 in.) with a 6-in. focal length aerial camera;
2. To evaluate the effect of analytically bridging photographs which have been drilled with a Wild PUG point transfer instrument;
3. To develop computer programs written in the Fortran language to reduce coordinates of image points on the photographic glass plates which have side fiducial marks, and apply a polynomial curve-fitting technique to compensate for the effects of radial lens distortion;
4. To determine the density and distribution of ground control needed for adequately adjusting a strip of 10 stereoscopic models;
5. To determine the degree of strip adjustment needed for a strip of 10 stereoscopic models;
6. To analyze photographic materials and photogrammetric instruments, equipment, and methods as sources of error in the analytic system of bridging;
7. To revise an existing cantilever adjustment program originally written for use on an IBM 1401 for the IBM 7010 system; and
8. To make recommendations for improving and implementing the analytic method and to suggest research needed in this field.

INSTRUMENTS, EQUIPMENT, MATERIALS, AND PROCEDURES

Aerial Camera and Photography

Eleven aerial photographs at a scale of 1:6000 were selected from a flight strip taken in July 1963 by Continental Engineers, Inc., for mapping a corridor for Interstate Highway 80 between Green River and Rock Springs in southwest Wyoming. The eleven photograhps utilized covered a strip of topography approximately 4,500 ft wide and 20,000 ft long, having a light-to-moderate brush cover. The photographs were taken from an average flight height of 3,000 ft with a Zeiss RMK A 15/23 aerial camera equipped with a Pleogon lens having a calibrated focal length of 152.45 mm ± 0.02 mm and a maximum aperture of f/5.6. The average value of radial lens distortion based on determinations made on two radii does not exceed ± 5 microns (see Fig. 4). The distortion values have been determined within an accuracy of two microns. The distance between the fiducial marks in both directions is 226.00 mm ± 0.02 mm.

The negative film used had an estar base from which diapositive plates (Kodak Aerographic Positive Plates, Improved, 0.25 in. thick) and photographic prints were made using a LogEtronics CP 18 automatic dodging printer.

Photograph Preparation and Image Selection

The image points used in the triangulation experiment were those for which ground control data were available. These control points had been surveyed for use in compiling topographic maps of a corridor along Interstate 80 in Wyoming. The points measured were images of targets and images of natural objects which were selected in accordance with the mapping needs of the project. Pass points for each stereoscopic model were selected in the usual rectangular pattern in the six classical locations. Two or three additional points were selected in the area of triple overlap of the photographs of each two adjacent stereoscopic models in order to insure that a sufficient number of acceptable points were available for scale adjusting one stereoscopic model to another in the cantilever assembly.

Drilling and Measuring

All image points, targeted and natural, used in the triangulation were predrilled with a Wild PUG3 point transfer instrument equipped with drills having a diameter of 60 microns.

The x and y coordinate measurements were made with the Nistri Monocomparator, Model TA1/P, provided with both digital readout and typewritten outputs (Fig. 1). The comparator had been calibrated a few months before measurements were made. The least reading on this comparator is 1 micron. The diapositive plates were measured with the emulsion side down under a 10 X magnification. The objective lens on this particular instrument was equipped with a 20-micron measuring mark. Measuring marks of other sizes are available from the manufacturer of the instrument. The coordinate output of this particular comparator was in a left-handed system. Provision was made in the coordinate reduction program to change the coordinate system into a right-handed system, whereby all values increased along the y-axis away from the observer and to the right along the x-axis. A simple wiring modification can be made at the factory to produce output directly in the right-handed system. Measurements were made in an air-conditioned room at 72 F. Periodic checks were made for possible instability. The instrument and accessory equipment exhibited excellent stability throughout the measurement operations. It took about one hour to measure an average of 25

Figure 1. Nistri Monocular Comparator, Model TA1/P, and accessory equipment.

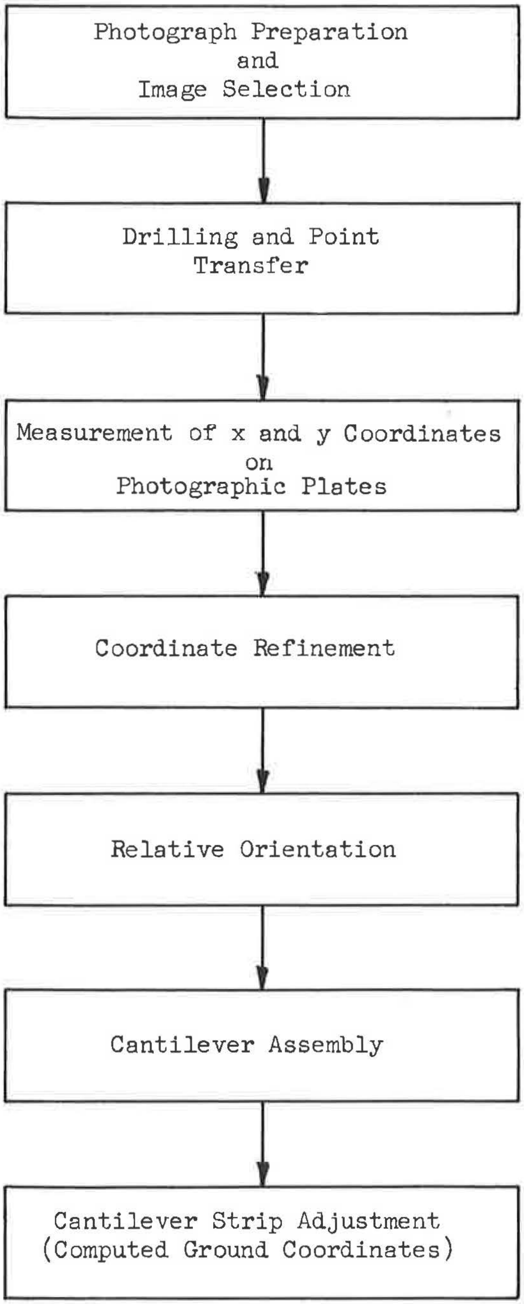

Figure 2. Flow chart of aerial analytic triangulation.

drilled holes per photographic plate and the 4 fiducial marks. Each of the drilled holes was measured 3 times, and each of the fiducial marks, 6 times. The mean of these measurements was accepted as "true" x and y coordinates for each point. The measured points were always approached with the measuring mark from the same direction to avoid the possibility of screw backlash, although screw backlash was found to be only 2 or 3 microns in magnitude.

Computers

Two electronic digital computers were used for making the mathematical computations of the analytic bridge. The cantilever strip adjustment program, which yields the X, Y, and Z ground coordinates of each measured point, was used in an IBM 7010 computer having a 60K digital storage capacity, while all other programs were used in the IBM 1401 with a 12K digital core memory.

Control Survey

Basic ground control was surveyed in a closed traverse approaching second-order accuracy. Points identified in the Appendix with the prefix SW were included in this travese. All other ground-surveyed points are assumed to be of at least third-order accuracy, although no survey closure checks were actually made. The surveying was accomplished using the Electrotape and Tellurometer electronic distance-measuring instruments, and a Wild T-2 Theodolite.

Computations

Figure 2 shows a generalized flow chart of analytic aerial triangulation used in this investigation. The following sections describe the basic computational concepts and procedures used.

Coordinate Refinement

In order to render the measured x and y coordinates suitable for performing the analytic triangulation, certain mathematical operations are required. They are: averaging of each set of coordinate measurements; conversion from a left-handed to a right-handed coordinate system; mathematical translation and rotation of the measured photographic plate coordinates; film deformation compenstaion; and radial lens distortion correction. These computational items are discussed in subsequent sections.

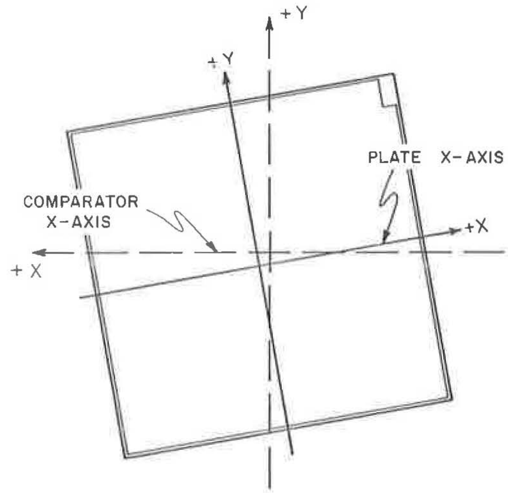

Figure 3. Relationship of diapositive plate to comparator axes at time of measuring.

Plate Coordinate Averages

The measurements made with the monocomparator are recorded in typed form. Card punching was performed directly from the typewritten record of the comparator measurements. Average values for the three measurements made on each of the image points (drilled holes) and six measurements made on each of the fiducial marks were computed with two separate computer programs. In these programs, the comparator measurements of the left-handed coordinate system are converted to a right-handed system by subtracting all x coordinates from an arbitrary constant of sufficient magnitude.

Plate Translation and Rotation

When diapositive plates are placed on the comparator stage for measurement, the x and y axes of the photographic plate must be physically oriented parallel to the respective axes of the comparator, or some means must be provided for mathematical rotation so the two coordinate systems are coincident (Fig. 3). The orientation of a plate so that its axes are precisely parallel to the comparator axes is a time-consuming procedure. Consequently it is expedient to place the plate on the comparator so that their respective axes are more or less parallel, and then mathematically translate and rotate them to coincidence based on the instrument-measured coordinates of the measured fiducials. The mathematical rotations are accomplished by using standard rotation equations from analytic geometry. The resulting translated and rotated coordinates for all points measured on each photographic plate are then referenced to the principal point of the photograph and are ready for further treatment.

Film Deformation Compensation

Plastic films are subject to dimensional change between the time of photographic exposure in the aerial camera and printing the diapositives on optically flat glass plates. Therefore, some means of compensating for the movement of images is necessary. For cameras equipped only with 4 side fiducial marks, the only feasible means of compensation is to compare the distances between the marks in 2 directions on the printed diapositive with those of the aerial camera itself. This distance between fiducial marks in the aerial camera may either be furnished by the manufacturer or measured on a diapositive plate (flash plate) previously exposed directly to the aerial camera. This method of compensation for film deformation was utilized even though it is recognized as being inadequate. Two scale factors were developed for each plate based on the distances between the fiducial marks reported by the manufacturer and those determined for each plate in the x and y directions. The x and y coordinates of all measured points were then multiplied by the respective film deformation correction factors.

Radial Lens Distortion Correction

Figure 4 shows the average radial lens distortion curve for the Zeiss Pleogon lens. Positive values of lens distortion result in the displacement of an image radially outward from the center of the photograph; for negative values the displacement is radially inward. Corrections for this displacement must be made to the x and y coordinates of each measured point on the photograph.

An equation for the lens distortion curve shown in Figure 4 was determined by means of a polynomial curve-fitting program. This program generates an approximating

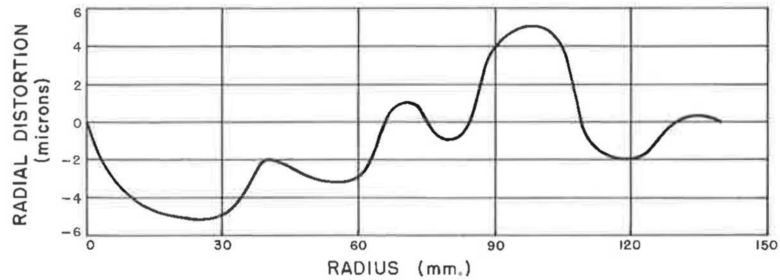

Figure 4. Average radial distortion curve for Zeiss Pleogon lens.

polynomial using the least squares technique. Coefficient terms of the polynomial curve are determined which are used to compute the amount of radial lens distortion for any given radial distance from the center of the photograph. Actual computation of the distortion is accomplished by means of another short program using the appropriate coefficients. The x and y coordinates of each point are then corrected for the effects of image displacement caused by lens distortion. This particular distortion curve had numerous points of inflection. Consequently, the curve was divided into two segments, 0 to 70 mm and 70 to 140 mm, in order to obtain sufficiently accurate polynomials. An equation for each of the two segments of the total curve was determined using radii and distortion data for 15 and 30 points, respectively, as electronic computer input data.

Relative Orientation

Relative orientation may be defined as reconstruction of the perspective conditions existing between a pair of photographs when they were taken (5). It actually consists in determining three rotational (omega, phi, kappa) and two of three translational (bx, by, bz) elements which define the attitude and positions of one photograph with respect to another, providing there is a sufficient common area of overlap in line of flight.

The method of relative orientation (1) used here depends upon enforcing a geometric condition where the photographic image, perspective center, and the object in the stereoscopic model are on a straight line (colinear). For a given stereoscopic model, there are a number of such lines (a pair for each image), but, because of various errors, it is impossible to enforce all colinear conditions completely. Since the left-hand photograph of a pair is assumed to have no tilt in this method of analytic relative orientation, small corrections are allowed to be made to the measured x and y coordinates of the right-hand photograph in a least squares manner. Each stereoscopic model was oriented independently using 12 image points, and required three computer iterations for completion of the relative orientation. The third and usually last iteration yielded x, y, and z coordinates for each point on the stereoscopic model and orientation data, which were utilized in the assembly of independently oriented stereoscopic models to form a strip. The lack of intersection of pairs of lines (corresponding rays) for all the image points in each stereoscopic model were printed out as residual y-parallaxes. These values were reviewed at the completion of the orientation. Substitutions were made for points having unusually high residuals, and a new orientation was performed.

Cantilever Assembly

After completing successively the relative orientation for each of the stereoscopic models, the individual models were "tied together" into a continuous strip. This assembly of models is accomplished by successive mathematical transformation of the model coordinates of each model in the strip. The three transformations, which were performed in order, are rotation, scaling, and translation. These mathematical computations yielded the strip coordinates. (The strip coordinates are analogous to those

TABLE 1

SCALE FACTORS USED TO COMPENSATE FOR FILM DEFORMATION

Photographic Plate No.	X-Axis	Y-Axis
1-40	0.999646	0.999646
1-41	0.999602	0.999611
1-42	0.999690	0.999690
1-43	0.999717	0.999788
1-44	0.999673	0.999805
1-45	0.999712	0.999797
1-46	0.999699	0.999814
1-47	0.999646	0.999735
1-48	0.999717	0.999761
1-49	0.999673	0.999766
1-50	0.999602	0.999797

TABLE 2

AVERAGE ABSOLUTE RESIDUAL Y PARALLAX FROM RELATIVE ORIENTATION

Model Number	Y Parallax (microns)
1	5.9
2	8.8
3	9.6
4	5.0
5	6.0
6	8.2
7	9.4
8	7.7
9	7.7
10	9.7

obtained from an analog triangulation instrument.) The first stereoscopic model in the strip was arbitrarily considered to be at the desired scale and its coordinates in the proper system. Therefore, the mathematical transformations were performed only on the second and succeeding models. Scaling in this cantilever system was accomplished by comparing slope distances between two image points, which occur in adjacent stereoscopic models (the common overlap area of three photographs), and then adjusting the model being attached to another by means of a scale factor.

Adjustment of Cantilever Strip Coordinates

The adjustment of cantilever strip coordinates is the last computational step which yields the X, Y, and Z ground coordinates desired. The mathematical method of strip adjustment used in this investigation is described elsewhere (6). The adjustment is fully applicable to strip coordinates derived from either analog optical train photogrammetric bridging instruments or to coordinates derived from measurements made with comparators. This method of adjustment attempts to correct for curvature of the strip (azimuth), twist or cross tilt, BZ fall off, scale change along the x and y axes of the strip, and local tilt of the strip in the x and y directions. Cumulative errors in a strip tend to be systematic and can be corrected by polynomials. Both second- and third-degree polynomial adjustments were applied to determine their effectiveness.

TABLE 3

SCALE FACTORS USED IN CANTILEVER ASSEMBLY

Models	Scale Factors
1 and 2	0.91743098
2 and 3	0.96043167 0.95981338
3 and 4	0.95736429
4 and 5	0.91459298 0.91491979
5 and 6	0.95755554 0.95778659
6 and 7	0.91712976
7 and 8	0.91668318 0.91630765
8 and 9	0.91052712 0.91084330
9 and 10	0.90574928

The following data are required as input in order to accomplish the vertical and horizontal adjustment of the strip coordinates:

1. A card containing the number of vertical and the number of horizontal ground control points used in the adjustment;
2. One card containing the x and y cantilever strip coordinates of a point near the center of the first stereoscopic model and another point near the center of the last model in the strip;
3. The strip x, y, and z coordinates of all points to be used as a basis for the

TABLE 4

COMPUTED STRIP COORDINATES OF POINTS IN TRIPLE OVERLAP AREAS

Point No.	X	Y	Z
1-40-B	0.98092 0.98092	0.22099 0.22100	-1.60790 -1.60809
1-42-B	0.94122 0.94121	0.71363 0.71364	-1.59301 -1.59322
1-42-L	1.93575 1.93575	0.60043 0.60049	-1.49853 -1.49885
1-42-K	1.94781 1.94780	0.14704 0.14712	-1.57810 -1.57766
1-42-D	1.93855 1.93855	-0.49565 -0.49581	-1.54782 -1.54813
1-42-G	2.88542 2.88542	-0.17437 -0.17442	-1.54318 -1.54327
1-42-J	2.92091 2.92092	0.19175 0.19172	-1.52242 -1.52299
1-44-F	3.81770 3.81770	0.81302 0.81318	-1.51911 -1.51931
42-2	3.82037 3.82037	-0.14599 -0.14601	-1.53610 -1.53600
1-44-D	3.80635 3.80636	-0.64514 -0.64507	-1.53752 -1.53737
1-44-K	4.78011 4.78013	0.59229 0.59277	-1.49192 -1.49278
1-44-G	4.79534 4.79537	-0.01733 -0.01719	-1.51885 -1.51975
41-2	5.67728 5.67728	0.11931 0.11932	-1.49878 -1.49888
1-48-F	7.53923 7.53923	-0.20051 -0.20049	-1.53959 -1.53961
38-1	7.53990 7.53990	0.59161 0.59178	-1.51810 -1.51848
1-50-E	8.45788 8.45788	0.48609 0.48586	-1.48191 -1.48140
1-50-A	8.46795 8.46795	-0.11971 -0.11996	-1.53105 -1.53033

adjustment for which the ground X, Y, and Z coordinates are known;

4. The horizontal and vertical ground control data for the points used in 3; and

5. The cantilever strip coordinates of all points in each model of the strip for which ground coordinates are needed to establish supplemental control.

Three horizontal and five vertical control points are the minimum number required to make a second-degree adjustment, and four horizontal and seven vertical control points are needed to make a third-degree adjustment.

DISCUSSION OF RESULTS

Film Deformation

Results obtained by comparing distances between fiducial marks in both directions on the measured photographic plates with distances between the same marks recorded on the camera calibration certificate showed remarkable uniformity in the deformation of estar base photographic film. The dimensional change in both the x and y photographic plate axes was about the same (Table 1). The computed linear factors for film deformation were in all cases less than unity, indicating the estar base photographic film had expanded rather than shrunk in both directions. The average computed factors from all of the x and y measurements of fiducial marks on the separate photographic plates were 0.999676 and 0.999746, respectively.

This method of correction, based on the distance between side fiducial marks, is known to be less than adequate, since film deformation is random and nonlinear in nature. No better alternative was believed possible for compensating for film deformation when only the four side fiducial marks could be measured. Film deformation represents one of the sources of error in the analytic system of aerial triangulation. Recently developed scale-stable base films, such as estar base, have certainly contributed toward minimizing this source of error.

One possible solution to this problem is the use of glass plates exposed directly in the aerial camera. This would eliminate need for film distortion compensation. No greater accuracy can be expected, however, since glass plate cameras use smaller formats and position accuracy is proportional to the scale of the photograph. The use of reseau equipped cameras offers another possibility (7). There are, however, some practical considerations at present which limit the use of these two techniques. One approach currently being employed is the use of aerial cameras with eight rather than four fiducial marks. This procedure permits more adequate mathematical restitution of points displaced by film movement (8).

It should be noted that the distance between fiducial marks is reported by the manufacturer to an accuracy of only ± 20 microns, and the diameter of the fiducial mark holes is 250 microns. Measuring the precise center of such fiducial marks using a measuring mark only 20 microns in diameter, is, in itself, somewhat uncertain.

Radial Lens Distortion

The polynomial curve-fitting technique was found to give adequate results based on the reliability of the input information provided. Although there were only 15 discrete radii for which lens distortion data were available at intervals of 10 mm on the plate, a smooth curve was plotted through these points (Fig. 4) and accepted as the actual distortion curve. Sufficient values of distortion for specific radii were selected from the curve and used as input data for the curve-fitting electronic computer program. All computed values of distortion, based on the computed curve, fell within less than 0.5 micron of the plotted curve. This technique of radial lens distortion compensation is well within the accuracy tolerances (± 2 microns) given by the manufacturer.

The radial lens distortion compensation program served also as a useful check on erroneous photographic plate coordinate measurements. Two points whose measured coordinates on the photographic plate were in gross error were found to have lens distortion corrections in excess of the maximum values shown in Figure 4.

Relative Orientation

Relative orientation was performed using 12 points for each stereoscopic model (2 points in each of the 6 usual areas of selection). The residual y parallaxes at each of the points were printed out by the computer and reviewed separately. The point with the largest residual was discarded and a point from its immediate vicinity was substituted in its place, then the relative orientation was again computed. This procedure was continued until no residuals larger than 25 microns remained at any given point. Table 2 shows the average absolute values of residual y parallax remaining at the 12 points in each of the stereoscopic models that were oriented. Residual y parallaxes as large as 50 microns were found for some points in the computed stereoscopic models. These larger values of parallax are due largely to errors introduced by the incorrect position of the drilled holes, but the inaccuracy attached to measuring the holes and the method of film deformation compensation are also contributing factors. The method of independent relative orientation of the stereoscopic models employed is dependent upon the intersection of only two rays (lines) from the respective photographs. Thus, no check is possible to determine the accuracy of computed points. Errors in x parallax are reflected as errors in the elevation of points on the ground. Computing the elevations is the final step of the analytic system of aerial triangulation.

Cantilever Assembly

Because of the limited storage capacity of the IBM 1401 computer used in this investigation, only 10 points per stereoscopic model could be accommodated in the cantilever assembly program. For purposes of this investigation, however, 10 points per model were found to be sufficient.

Table 3 contains a listing of scale correlation factors computed from points occurring in the triple overlap area. Wherever enough points were available, two scale factors were computed, and the average value used. Scale factors should be in reasonably close agreement. Points causing anomalies were discarded, and the strip coordinates were then recomputed using a substitute point.

Table 4 contains the strip coordinates of points occurring in triple overlap areas which were computed using data derived from the independently oriented stereoscopic models of the strip. Average values of two sets of coordinates for each point were used as the most acceptable strip coordinates for the point. It should be noted there is slightly greater disparity in computed values of the strip elevations than for the horizontal strip coordinates.

TABLE 5

SUMMARY OF ERRORS FOR FIVE THIRD-DEGREE ADJUSTMENTS

Adjust. No.	No. of Control Points*		No. of Test Points*		RMSE (ft)			Max. Error (ft)			Min. Error (ft)			Algebraic Mean Error (ft)		
	H	V	H	V	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z
(a) Control Points																
1	4	12	—	—	0.05	0.32	0.59	-0.08	-0.49	1.28	-0.01	-0.09	0.00	-0.01	0.00	0.15
2	5	18	—	—	0.09	0.34	0.09	-0.13	-0.56	-1.93	0.02	0.08	-0.03	0.01	-0.12	-0.26
3	4	13	—	—	0.05	0.14	0.75	0.08	0.22	-1.78	0.00	0.02	0.25	0.01	0.30	-0.12
4	4	10	—	—	0.00	0.04	0.34	0.00	-0.19	-0.91	0.00	0.01	0.00	0.00	-0.09	-0.19
5	4	7	—	—	0.10	0.18	0.00	±0.13	-0.27	0.00	-0.02	-0.05	0.00	-0.02	-0.05	0.00
(b) Test Points																
1	4	12	10	40	0.60	0.59	1.52	0.95	1.21	3.60	0.14	-0.01	0.12	-0.07	0.20	0.71
2	5	18	9	34	0.61	0.42	1.42	0.86	0.80	3.18	-0.02	-0.01	0.02	0.06	0.25	0.29
3	4	13	10	39	0.54	0.52	1.67	0.91	-0.97	3.94	0.04	0.08	-0.05	-0.11	-0.15	0.19
4	4	10	10	42	0.55	0.54	1.95	-0.96	-0.93	-4.63	0.02	0.06	0.01	-0.12	-0.09	0.05
5	4	7	10	45	0.69	0.69	1.89	-1.21	-1.22	4.61	-0.14	-0.08	-0.10	-0.48	-0.17	0.39

*H and V = horizontal and vertical.

TABLE 6

SUMMARY OF ERRORS FOR FIVE SECOND-DEGREE ADJUSTMENTS

Adjust. No.	No. of Control Points*		No. of Test Points*		RMSE (ft)			Max. Error (ft)			Min. Error (ft)			Algebraic Mean Error (ft)		
	H	V	H	V	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z
(a) Control Points																
1A	4	12	—	—	0.93	0.10	0.92	1.40	0.44	1.85	0.26	±0.05	0.10	0.00	0.00	0.02
2A	5	18	—	—	0.97	0.33	1.12	1.57	0.45	2.09	0.30	0.14	0.02	-0.01	-0.01	0.00
3A	4	13	—	—	0.41	0.12	1.25	-0.64	-0.17	2.02	-0.05	±0.01	0.07	0.00	-0.08	0.00
4A	4	10	—	—	0.51	0.10	1.21	-0.79	0.16	-1.94	-0.03	0.00	-0.39	0.29	0.00	-0.03
5A	4	7	—	—	0.81	0.23	0.84	1.20	0.38	-1.37	-0.22	0.02	0.17	-0.11	-0.11	-0.01
(b) Test Points																
1A	4	12	10	40	1.26	0.69	1.54	2.01	1.27	4.06	0.35	-0.16	-0.01	0.22	0.55	0.80
2A	5	18	9	34	1.17	0.77	1.51	2.21	1.26	3.89	0.05	0.07	0.01	0.32	0.48	0.27
3A	4	13	10	39	1.71	0.68	1.44	2.86	1.06	3.84	-0.15	-0.19	-0.03	0.58	0.10	0.27
4A	4	10	10	42	1.62	0.60	1.38	2.56	0.94	3.67	-0.36	-0.01	-0.18	0.96	0.19	-0.20
5A	4	7	10	45	1.61	0.61	1.46	-2.55	-1.47	3.84	0.43	0.09	-0.16	-1.17	-0.20	0.33

*H and V = horizontal and vertical.

Point Transfer and Marking

The accuracy with which pass points are transferred to a photographic strip materially affects results obtained. Obviously, precise x and y measurements made for a point inaccurately transferred are of no value. At the present time, there is considerable research being conducted on various aspects of the measurement procedure including evaluation of various types of point transfer and marking instruments (9). At the moment there appears to be no general agreement regarding the efficiency and reliability of the various types of instruments.

When using a monocular comparator it is a practical necessity to use some form of point transfer and marking system. Point transfer should be precise and stereoscopically correct. Measurement of plate coordinates is considerably facilitated and the possibilities of blunders due to misidentification of corresponding images are minimized. Targeted points, on the other hand, can be found and measured readily without being premarked on the diapositive plate.

Largely because of the inexperience in using the PUG point marking and transfer instrument, some errors were introduced into the analytic system on this test project. The magnitude of these errors, however, was impractical to measure. Drilled holes for the most part were nonuniform in shape and size. Some targeted control points were drilled off center. Observation with the Kelsh instrument of each stereoscopic model in the strip of photographs revealed lack of stereoscopic correspondence of the drilled holes with the ground surface for a significant number of the points. Based on this stereoscopic analysis, "digging" or "floating" points were not included as control points for the strip adjustments. A comparison was made between the sign of the errors in the computed elevations and their positions noted in the stereoscopic models as being either on, above, or below the ground surface.

There was a definite correlation between the sign of the errors in elevation and the observed elevation of the holes in the stereoscopic models. No actual elevation measurements were made because a significant number of the drilled holes could not be seen in the stereoscopic model. It should be noted, however, that the transfer of points is only one source of known error in this project.

Computed Ground Coordinates

Tables 5 and 6 summarize the errors in computed ground coordinates using second- and third-degree cantilever strip adjustments. Tables 7 through 9 of the Appendix show the horizontal and vertical errors for ground coordinates computed for 55 separate points in 10 trial adjustments. The distribution of ground control used in each of the adjustments is shown in Figures 5 through 9 of the Appendix.

The third-degree horizontal adjustments were the most accurate. Table 5 shows the root-mean-square errors (RMSE) for the horizontal coordinates of the third-degree adjustment. Values ranged from 0.42 to 0.69 ft and show no significant difference between the X and Y coordinates. The use of 5 rather than 4 horizontal control points did not significantly increase the accuracy of the computed values.

Table 6 shows the RMSE for the horizontal coordinates of second-degree adjustments ranging from 0.60 to 1.71 ft. The magnitude of the error in the X coordinates was about twice that for the Y coordinates. The errors in the computed Y coordinates in both second- and third-degree adjustments were about the same. Errors in the computed X coordinates of the third-degree adjustment were half the size of those for the second-degree adjustments.

Test point 45-1 tended to have slightly more error in horizontal coordinates in 8 of the 10 adjustments because of its position outside the confines of ground control point SW-45, located near the beginning of the flight strip.

A comparison of the vertical RMSE in Tables 5 and 6 for the second- and third-degree adjustments shows no significant difference for the first three sets of trials. Twelve or more vertical control points were used to adjust these flight strips. The second-degree adjustments, however, showed a significant improvement in accuracy over the third-degree in the last two sets of adjustments where vertical control points were not as dense.

It is interesting to note that an increase from 7 to 18 vertical control points had practically no effect on the RMSE of the second-degree adjustments, but it did significantly reduce the RMSE of the third-degree adjustments. The largest vertical errors were found for test points located in areas where no vertical control points were present in their vicinity (adjustments 4, 4A, 5, 5A). The effects of density and distribution of vertical control points on ultimate accuracy are difficult to analyze. It is likely some of the drilled holes for ground control points were not in stereoscopic correspondence with the ground surface. Only those control points observed to "float" or to "dig" in the Kelsh instrument were eliminated as control points for making the strip adjustments.

The algebraic mean errors in Tables 5 and 6 for elevations of test points are positive for all trial adjustments except one (adjustment 4A). The positive sign of these mean errors tends to confirm the stereoscopic observations made of the drilled holes and the existence of systematic errors.

CONCLUSIONS AND RECOMMENDATIONS

Results of this investigation point to at least four principal sources of error in the analytic aerial triangulation system: film deformation, pass point transfer, x and y coordinate measurements of points on the glass plate transparencies, and ground control.

It is recognized that the method of compensation for film deformation is inadequate, but it is the most suitable method which can be employed for cameras equipped with only 4 side fiducial marks. It is, however, impossible to prove the amount of error in ground coordinates due to film deformation. The estar-base film is considerably more stable dimensionally than previous film bases. There are at least three possible alternatives for a more effective treatment of the film deformation problem. The first is equipping existing cameras with 4 additional fiducial marks so that a more effective mathematical restoration of displaced images can be made. A computer program is available to accomplish this. The second alternative is to employ a network of small crosses (reseau) in the focal plane of the camera. The crosses are recorded on the negative film at the instant of exposure. Displacement of images due to film distortion or lack of film flatness at the instant of exposure is compensated by comparing the measured position of the reseau crosses with their calibrated position. The third possible alternative is to use a glass plate aerial camera. This would eliminate the need for compensation due to film effects.

When measuring plate coordinates with a monocular comparator, some form of point transfer and marking technique is a practical necessity. The PUG instrument was used for this project. Largely because of lack of experience with the instrument, some error was introduced from this source. There is a divergence of opinion at present regarding the inherent accuracy of several point transfer and marking instruments. Research and evaluation of these techniques is being done by private firms, universities, and governmental agencies. Ground control and other points which are premarked with photographic targets can be reliably identified and measured without need for point transfer and marking.

The relative size of the measuring mark to that of the drilled hole appears to be a significant factor in the degree of accuracy of measurement. A 20-micron measuring mark and a fiducial hole with a diameter of 250 microns or a drilled hole of 60 microns diameter are not desirable combinations for optimum accuracy of measurement. Greater accuracy could be attained by using a larger measuring mark. There is some doubt about the adequacy of drilled holes that are 60 microns in diameter for use in map compilation with the Kelsh instrument. At a 5:1 projection ratio, a significant number of holes were not discernible in the stereoscopic models. A drilled hole of at least 100 microns in diameter is needed for use with the Kelsh instrument.

Basic ground control surveys on which aerial analytic triangulation is based should be of second-order accuracy (1/10,000 closing error in position) or better, so results of the triangulation will not be degraded because of inferior control. Through such a practice results of the triangulation from the standpoint of accuracy can also be more realistically evaluated. Results of the aerial analytic triangulation cannot be expected to be any better than the ground control survey on which it is based.

The polynomial curve-fitting technique used to determine an equation for the radial lens distortion curve is an acceptable method. It is particularly adapted to determining the equations of curves which are smooth and have only a few points of inflection. Corrections made to the x and y measured coordinates, based on the computed curve, are within the accuracy tolerances for the manufacturer's determination of the lens distortion values.

Corrections for atmospheric refraction and adjustments for earth curvature were not considered in this investigation. Displacement of photographic images due to atmospheric refraction and earth curvature are negligible because of the relatively low flight height from which the photographs were taken and the short length of the flight strip. For bridging photographs taken from higher flight heights and for longer flight strips, appropriate adjustments should be applied for their effects (10, 11).

Use of a small-capacity computer for aerial analytic triangulation requires considerable segmentation of the computer programs and excessive card handling. Computers with larger storage capacity and greater speed must be used in order to perform the analytic operations more efficiently.

Use of mapping-scale photographs (1:6000) for analytical bridging does not appear to be the most accurate or economical approach for securing supplemental ground control data for large-scale topographic mapping. Use of the larger scale photographs requires measurement of x and y for a greater number of stereoscopic models for a given bridged distance. The greater number of intermodel ties needed tends to deform the bridged strip and thus offset the advantages gained from having a larger scale.

Most present-day aerial cameras are not designed or calibrated for analytical photogrammetry. To take full advantage of mathematical techniques to compensate for errors, some slight design changes in aerial cameras and related equipment will undoubtedly be necessary.

Future investigations of aerial analytic triangulation for highway mapping purposes may well consider one or more of the following:

1. Color diapositive plates,
2. Aerial cameras equipped with eight fiducial marks or a reseau grid,
3. Stereocomparator (12),
4. Photography scales of 1:12,000 or smaller,
5. Other mathematical methods of aerial analytic triangulation (2, 13, 14, 15, 16), and
6. Various point transfer and marking devices.

In the writer's opinion, this method of analytic aerial triangulation is capable of providing accurate supplemental control for use in compilation of large-scale maps for location and design of highways. The primary remaining problem lies in refining operational techniques and hardware which now are contributing considerable error to this method of computing the X, Y, and Z coordinates of selected points on the ground.

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REFERENCES

1. Harris, William D., et al. Analytic Aerotriangulation. U.S. Coast and Geodetic Survey Bull. No. 21, July 1963.
2. Keller, M., and Tewinkel, G. C. Three-Photo Aerotriangulation. U.S. Coast and Geodetic Survey Tech. Bull. No. 29, Feb. 1966.
3. Arneson, Clair L. Results of U.S. Forest Service Stereotriangulation Bridging on Virginia Highway Photogrammetric Test Project. Highway Research Record 65, pp. 73-85, 1965.
4. Chaves, Jesse R. Survey Control Extension by Analytic Aerotriangulation for Highways. Thesis, Syracuse University, Sept. 1965.
5. American Society of Photogrammetry. Manual of Photogrammetry. ASP, Washington, 1952, p. 321 (second edition).
6. Keller, M., and Tewinkel, G.C. Aerotriangulation Strip Adjustment. U. S. Coast and Geodetic Survey Bull. No. 23, Aug. 1964.
7. Robinson, G.S. The Reseau as a Means of Detecting Gross Lack of Flatness of Film at the Instant of Exposure. Photogrammetric Record, Vol. 4, No. 22, 1963.
8. Keller, M., and Tewinkel, G.C. Aerotriangulation: Image Coordinate Refinement. U.S. Coast and Geodetic Survey Tech. Bull. No. 25, March 1965.
9. Karara, H.M. Mono Versus Stereo Analytical Photogrammetry—Theoretical Considerations and Experimental Results. Paper presented at the International Symposium on Spatial Aerotriangulation, Urbana, Illinois, 1966.
10. Schut, G.H. Computation of the Height Deformation in Stereoscopic Models Caused by Distortion of the Photographic Image. Canadian Surveyor, Vol. 14, No. 1, 1958.
11. Leyonhufvud, A. On Photogrammetric Refraction. Photogrammetria, Vol. 9, No. 3, 1952-53 (Abstract).
12. Harley, I.A. Some Notes on Stereocomparators. Photogrammetric Record, Vol. 4, No. 21, 1963.
13. Schut, G.H. Analytical Aerial Triangulation. National Research Council of Canada Publ. AP-PR-7, Sept. 1957.
14. Thompson, E.H. A Method of Relative Orientation in Analytical Aerial Triangulation. Photogrammetric Record, Vol. 2, No. 8, 1956.
15. Arthur, D.W.G. Recent Developments in Analytical Aerial Triangulation at the Ordnance Survey. Photogrammetric Record, Vol. 3, No. 14, 1959.
16. Dodge, H.F. Analytical Aerotriangulation by the Direct Geodetic Restraint Methods. Photogrammetric Engineering, Vol. 25, No. 4, 1959.

Appendix

TABLE 7

ERRORS IN HORIZONTAL COORDINATES FROM THIRD-DEGREE ADJUSTMENTS

Point No.	Adjustment Number 1 X (ft)	1 Y (ft)	2 X (ft)	2 Y (ft)	3 X (ft)	3 Y (ft)	4 X (ft)	4 Y (ft)	5 X (ft)	5 Y (ft)
45-1	0.95	-0.68	0.86	-0.74	0.91	-0.58	-0.96	-0.64	*-0.02	-0.05
SW-45	*-0.02	-0.09	*-0.05	-0.18	*-0.04	-0.09	* 0.00	-0.14	0.71	0.53
SW-44	0.14	0.31	-0.02	0.18	-0.05	0.18	* 0.00	-0.19	-0.29	0.74
SW-43	0.31	0.45	* 0.10	-0.29	*-0.03	0.22	0.02	-0.33	-0.14	0.67
42-1	-0.29	-0.61	-0.44	0.47	-0.42	0.45	-0.42	0.52	-0.46	-0.97
42-2	-0.30	1.21	-0.38	-0.01	0.04	0.79	-0.28	0.88	-0.38	-1.22
SW-41	*-0.08	0.28	*-0.13	0.08	* 0.08	0.15	* 0.00	-0.07	*-0.13	0.22
41-2	-0.80	0.10	0.75	-0.09	-0.36	-0.36	-0.49	-0.32	-0.88	-0.22
40-1	-0.73	0.12	-0.69	-0.12	-0.38	-0.47	-0.50	-0.39	-0.89	-0.08
38-1	-0.88	0.91	-0.80	0.80	-0.30	-0.41	-0.42	0.46	-1.21	-0.37
39-1	0.23	0.21	0.29	0.07	0.77	-0.22	0.64	-0.18	*-0.08	-0.27
SW-38	*-0.01	-0.49	* 0.09	-0.56	-0.49	-0.97	0.40	-0.93	-0.49	-0.91
SW-37	* 0.05	0.30	* 0.02	0.37	* 0.00	0.02	* 0.00	0.01	-0.72	0.11
BMC-131	0.68	-0.01	0.71	0.33	-0.85	0.08	0.78	0.06	* 0.13	-0.09

*Horizontal control points used to adjust strips.

TABLE 8

ERRORS IN HORIZONTAL COORDINATES FROM SECOND-DEGREE ADJUSTMENTS

Point No.	Adjustment Number 1A X (ft)	1A Y (ft)	2A X (ft)	2A Y (ft)	3A X (ft)	3A Y (ft)	4A X (ft)	4A Y (ft)	5A X (ft)	5A Y (ft)
45-1	1.89	-0.16	2.21	-0.54	1.72	-0.43	1.92	-0.32	*-0.22	0.02
SW-45	* 0.26	-0.05	* 0.60	-0.35	* 0.25	-0.17	*-0.59	-0.13	-1.18	0.17
SW-44	-1.04	0.25	-0.58	0.07	-0.60	-0.19	*-0.79	0.16	-2.04	0.26
SW-43	-1.54	0.30	*-0.90	0.17	*-0.64	0.01	-0.80	-0.01	-2.29	-0.19
42-1	-2.01	0.30	-1.47	0.24	-1.11	0.35	-1.22	0.22	-2.55	0.09
42-2	-1.39	0.83	-0.81	0.91	-0.15	-0.92	-0.36	0.74	-1.64	0.43
SW-41	*-0.87	0.05	*-0.30	0.14	* 0.44	-0.17	* 0.22	-0.04	*-1.05	-0.38
41-2	-0.35	0.43	0.14	0.54	1.12	0.52	0.87	0.29	-0.44	-0.18
40-1	-0.48	0.39	0.05	-0.23	0.92	-0.34	0.68	-0.56	-0.67	-0.94
38-1	0.70	1.27	1.02	1.26	2.20	1.06	1.94	0.94	0.43	0.22
39-1	1.29	0.95	1.71	1.00	2.86	0.93	2.56	0.74	* 1.20	0.13
SW-38	* 1.40	-0.44	* 1.57	-0.42	-2.69	-0.72	2.51	-0.81	0.81	-1.47
SW-37	*-0.80	0.44	*-0.98	0.45	*-0.05	-0.01	*-0.03	0.00	-2.11	-0.52
BMC-131	0.71	0.97	0.63	1.11	1.55	0.76	1.54	0.68	*-0.38	-0.22

*Horizontal control points used to adjust strips.

TABLE 9

ERRORS IN VERTICAL COORDINATES FROM SECOND- AND THIRD-DEGREE ADJUSTMENTS[1]

Point No.	Adjustment Number									
	1	1A	2	2A	3	3A	4	4A	5	5A
1-40-B	-0.75	-1.36	-0.73	-1.67	-0.36	-0.85	-0.83	-1.84	-0.80	-2.09
1-42-B	-0.82	-0.73	-1.36	-1.00	-1.79	-1.27	-2.51	-1.26	-2.57	-1.26
1-36-D	1.61	1.00	* 0.71	* 0.62	* 0.70	* 0.30	0.31	-0.29	* 0.00	* 0.17
45-1	2.99	1.35	2.94	0.96	3.94	0.65	3.88	0.75	3.68	0.41
1-40-A	-0.50	-0.13	-1.43	-0.40	-1.22	-0.75	-3.02	-0.71	-3.17	-0.75
SW-45	* 0.59	*-0.12	* 0.23	*-0.45	0.47	-0.75	* 0.05	*-0.69	-0.10	-0.91
45-2	3.60	2.67	3.18	3.40	2.83	3.17	2.13	3.17	2.10	3.05
1-42-L	-1.78	*-1.09	-2.19	-1.27	-3.05	-1.42	-3.92	-1.49	3.85	-1.46
1-42-K	*-0.05	0.29	*-0.07	* 0.02	-0.60	-0.09	-1.32	-0.18	-1.23	-0.21
1-42-D	-0.12	-0.06	-0.19	-0.44	0.57	-0.50	0.05	-0.61	0.21	-0.70
1-40-C	1.36	-0.35	2.18	-0.73	3.77	-0.80	3.68	-0.79	3.83	-1.25
SW-44	-1.29	*-1.23	*-0.54	*-1.50	*-1.78	*-1.68	2.47	-1.70	-2.44	-1.83
1-42-C	-2.52	-1.99	-3.01	-2.20	-3.78	-2.41	-4.63	-2.44	-4.61	-2.48
SW-43	*-0.03	*-0.10	*-0.03	*-0.52	* 0.34	*-0.57	*-0.18	*-0.72	* 0.00	*-0.74
1-42-G	-0.54	1.13	0.23	0.81	-0.22	0.76	-0.87	0.57	-0.75	0.75
1-42-J	3.44	4.06	0.16	3.89	2.46	3.84	1.77	3.67	1.87	3.84
3-28-J	-1.03	-0.16	*-1.38	*-0.28	-2.46	-0.38	-3.33	-0.49	-3.27	-0.41
1-42-H	0.20	1.03	-0.06	0.93	-1.12	0.87	-1.91	0.74	-1.84	0.88
1-42-F	1.00	1.18	0.80	0.72	1.02	0.67	0.47	0.47	0.64	0.57
42-1	-0.66	-0.01	0.94	-0.24	-1.67	0.31	-2.35	-0.45	-2.24	-0.40
1-44-F	1.95	2.38	2.05	2.36	1.23	2.42	0.80	-2.26	0.78	2.40
42-2	1.39	1.97	0.02	1.64	-0.50	1.62	0.05	1.36	0.13	1.73
1-44-D	0.92	2.58	*-0.29	* 2.09	* 0.91	* 2.02	*-0.44	* 1.71	0.57	2.21
1-44-A	0.54	1.19	0.29	0.97	-0.40	0.95	-0.99	0.75	-0.93	0.98
3-28-F	1.62	2.23	1.47	2.09	0.68	2.08	0.12	1.90	0.16	2.10
1-44-B	1.88	2.01	1.13	1.92	0.14	1.90	-0.53	1.76	-0.49	1.90
3-28-E	* 1.28	* 1.85	* 1.29	* 1.81	* 0.37	* 1.84	*-0.16	* 1.69	0.16	1.83
1-44-K	1.85	1.75	2.11	1.67	1.79	1.81	1.81	1.59	1.72	1.81
1-44-G	0.89	1.05	0.67	0.73	0.35	0.70	0.27	0.46	0.30	0.94
3-26-D	2.33	2.38	1.62	2.35	2.13	2.48	2.02	2.29	1.94	2.45
SW-41	* 1.04	1.47	*-0.73	* 1.16	*-0.25	* 1.16	*-0.07	* 0.87	* 0.00	* 1.29
41-2	0.69	0.34	0.68	0.01	0.73	0.12	1.05	-0.19	1.02	0.29
3-26-E	1.86	0.39	2.24	1.29	2.25	1.48	2.58	1.25	2.43	1.48
40-1	*-0.20	* 0.13	-0.98	-0.52	-1.24	-0.61	-1.32	-1.03	1.07	-0.16
1-46-E	-0.72	-1.78	-0.21	-1.90	0.42	-1.57	-1.27	-1.79	0.99	-0.68
1-48-H	*-0.47	*-1.01	*-0.79	*-1.61	*-0.50	*-1.55	* 0.00	*-1.94	0.26	-1.19
1-46-F	0.31	0.17	*-0.61	*-0.77	0.60	-0.92	-0.36	-1.44	0.12	-0.20
1-46-C	*-0.85	*-1.47	*-0.44	*-1.58	*-0.29	*-1.36	* 0.16	*-1.59	* 0.00	*-1.37
1-46-H	0.78	0.97	*-0.12	* 0.20	*-0.31	* 0.07	*-0.28	*-0.39	* 0.00	* 0.64
1-46-D	0.56	-1.25	-0.45	-1.58	-0.11	-1.40	0.47	-1.71	0.40	-1.27
1-48-F	0.74	0.11	0.35	-0.60	-0.83	-0.50	1.46	-0.90	1.76	-0.13
1-48-E	* 0.20	*-0.12	-0.84	-1.26	*-0.72	*-1.45	-0.45	-2.02	0.40	-0.58
1-50-E	1.19	0.71	* 0.99	* 0.33	1.79	0.80	2.70	-0.59	2.58	0.60
1-50-A	1.35	1.05	*-0.72	* 0.25	1.19	0.42	1.74	0.05	2.23	0.73
1-48-C	* 0.25	* 0.54	0.25	-0.91	* 1.07	*-0.51	2.02	-0.76	1.89	-0.62
1-50-D	0.13	0.05	*-1.03	*-1.30	-1.00	-1.52	-0.91	-2.00	0.41	-0.54
1-50-G	* 0.00	* 0.72	*-1.93	* 0.23	*-0.55	* 0.80	*-0.03	* 0.62	* 0.00	* 0.48
1-50-C	2.33	2.93	* 0.86	* 1.30	* 0.49	* 1.01	* 0.04	* 0.41	2.08	2.14
BMC-131	*-0.01	* 0.26	-1.38	-1.32	-0.59	-1.61	-1.82	-2.23	* 0.00	*-0.48
1-50-B	0.84	0.98	-0.14	-0.05	-0.05	-0.03	0.30	-0.39	1.21	0.52
1-50-F	0.36	0.47	-0.22	0.03	0.39	0.55	1.12	0.36	1.08	0.29

[1]A indicates second-degree adjustment.

*Vertical control points used to adjust strip elevations.

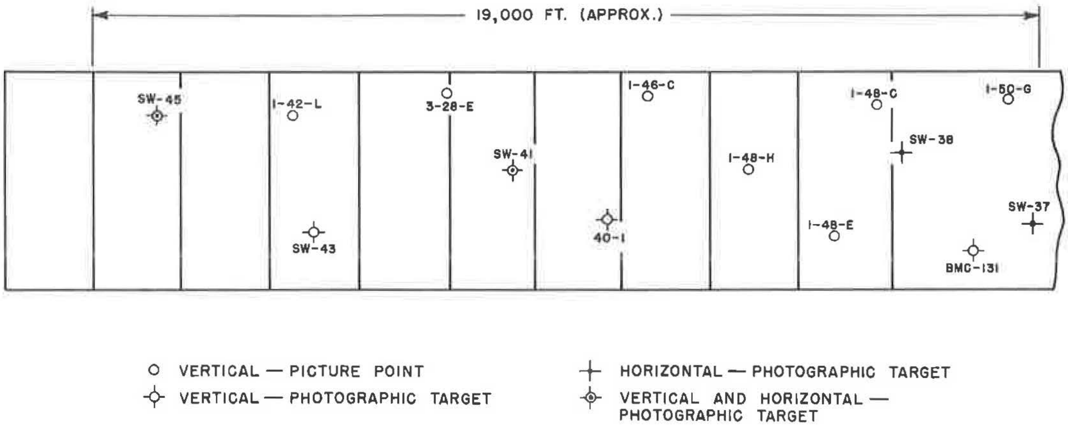

Figure 5. Distribution of ground control used in adjustment No. 1 and 1A.

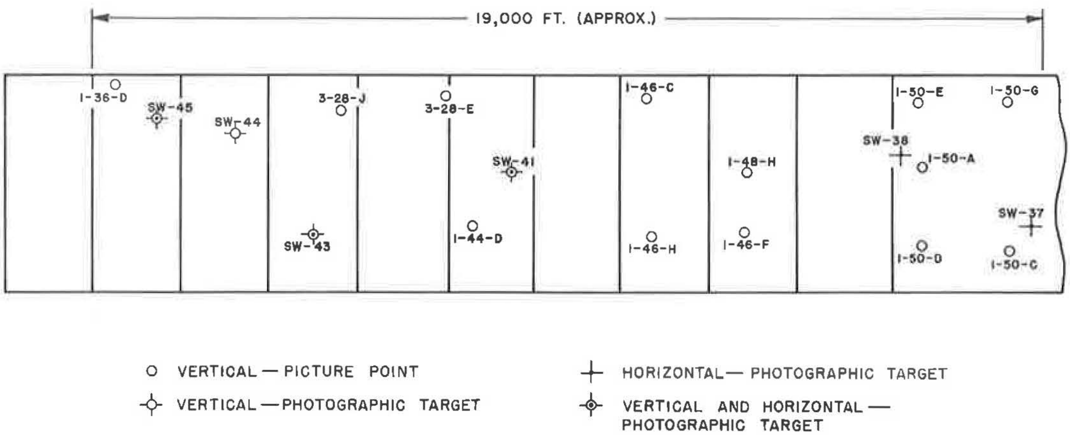

Figure 6. Distribution of ground control used in adjustment No. 2 and 2A.

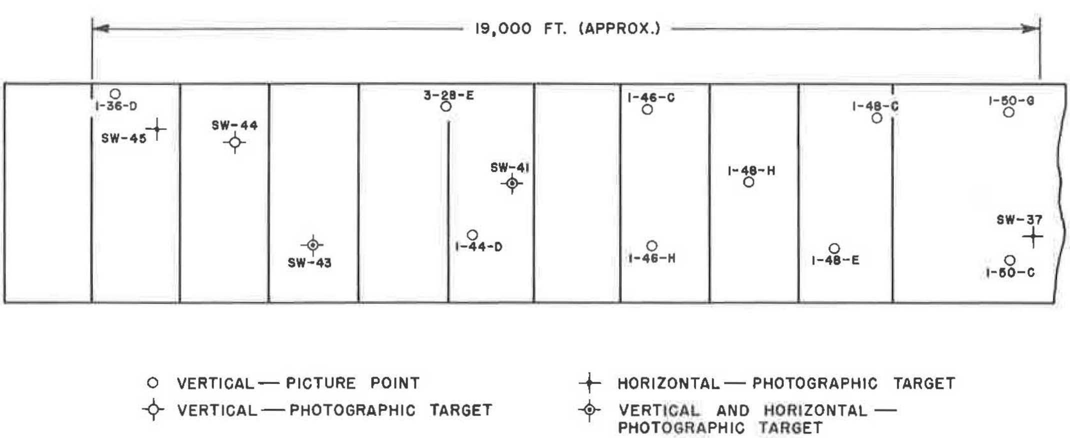

Figure 7. Distribution of ground control used in adjustment No. 3 and 3A.

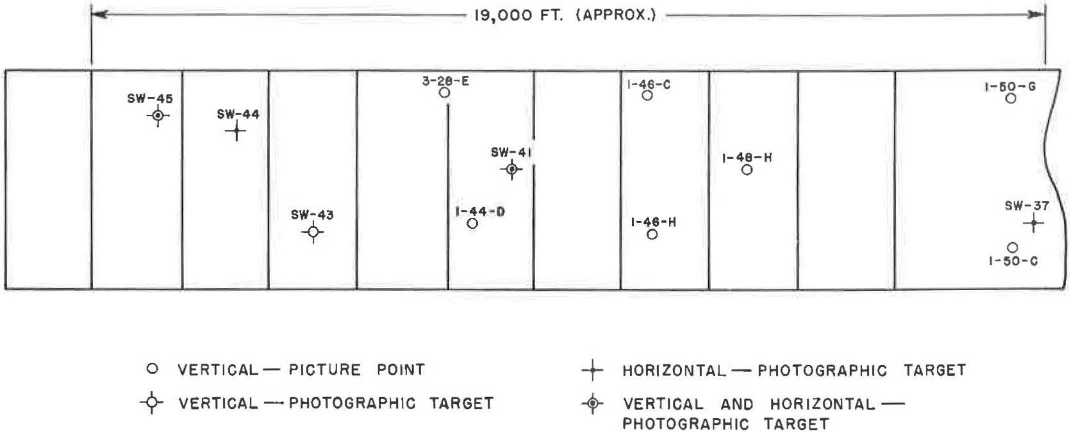

Figure 8. Distribution of ground control used in adjustment No. 4 and 4A.

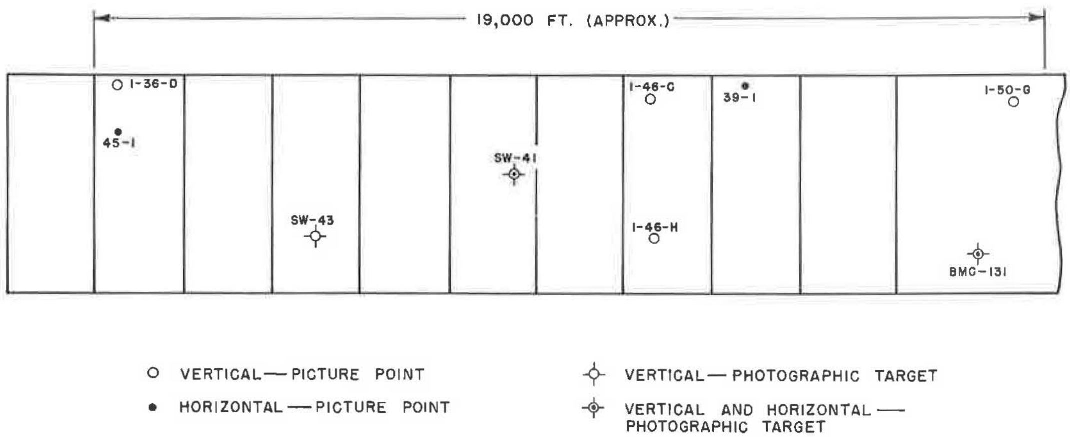

Figure 9. Distribution of ground control used in adjustment No. 5 and 5A.