divisibility by 2:

$$\frac{abc}{2} = \left(\frac{a \times 100}{2}\right) + \left(\frac{b \times 10}{2}\right) + \left(\frac{c}{2}\right)$$

$$= (a \times 50) + (b \times 5) + \left(\frac{c}{2}\right)$$

We see that we have to check the last digit is divisible by 2. Here, we are using distributive property

divisibility by 3:

$$\frac{abc}{3} = \left(\frac{99a+a}{3}\right) + \left(\frac{9b+b}{3}\right) + \left(\frac{c}{3}\right)$$ [right distributive law for division]

$$= \left(33a + \frac{a}{3}\right) + \left(3b + \frac{b}{3}\right) + \left(\frac{c}{3}\right)$$

$$= \left(\frac{33a}{1} + \frac{a}{3}\right) + \left(\frac{3b}{1} + \frac{b}{3}\right) + \left(\frac{c}{3}\right)$$

$$= \left(\frac{33a}{1} + \frac{3b}{1}\right) + \left(\frac{a}{3} + \frac{b}{3} + \frac{c}{3}\right)$$ [commutative & associative law for addition]

$$= \frac{(33a+3b)}{1} + \left(\frac{a+b+c}{3}\right)$$

So, we have to check if the sum of the digits is divisible by 3. Only then the result will be a whole number.

(33a+3b is always a whole number)

$\frac{a+b+c}{3}$ may or may not be

divisibility by 4:

$$\frac{abcde}{4} = \frac{abc \times 100 + de}{4}$$

$$= \left(\frac{abc \times 100}{4} + \frac{de}{4}\right)$$

$$= (abc \times 25) + \frac{de}{4}$$

[right distributive law for division]

we have to check the last 2 digits for divisibility by 4

divisibility by 5:

$$\frac{abc}{5} = \left(a \times \frac{100}{5}\right) + \left(b \times \frac{10}{5}\right) + \left(\frac{c}{5}\right)$$

$$= (a \times 20) + (b \times 2) + \left(\frac{c}{5}\right)$$

[right distributive]

we have to check the last digit for divisibility by 5.

[$\frac{c}{5}$ may or may not be a whole number; $(a \times 20) + (b \times 2)$ is always a whole number.

divisibility by 6:

it has to be divisible by 3, and 2

divisiblity by 8

$$\frac{abcde}{8} = \frac{(a \times 10000) + (b \times 1000) + (cde)}{8}$$

$$= \left(\frac{a \times 10000}{8}\right) + \left(\frac{b \times 1000}{8}\right) + \left(\frac{cde}{8}\right)$$

$$= (a \times 1250) + (b \times 125) + \left(\frac{cde}{8}\right)$$

We have to check if the last 3 digits are divisible by 8

divisibility by 9:

$$\frac{abc}{9} = \frac{a\times 100 + b\times 10 + c}{9}$$

$$= \left(\frac{a\times 100}{9}\right) + \left(\frac{b\times 10}{9}\right) + \left(\frac{c}{9}\right)$$ [right distributive law over division]

$$= \left(\frac{99a + a}{9}\right) + \left(\frac{9b+b}{9}\right) + \left(\frac{c}{9}\right)$$

$$= \left(\frac{99a}{9} + \frac{a}{9}\right) + \left(\frac{9b}{9} + \frac{b}{9}\right) + \left(\frac{c}{9}\right)$$

$$= \left(11a + \frac{a}{9}\right) + \left(b + \frac{b}{9}\right) + \left(\frac{c}{9}\right)$$

$$= \left(\frac{11a}{1} + \frac{a}{9}\right) + \left(\frac{b}{1} + \frac{b}{9}\right) + \frac{c}{9}$$ [commutative & associative over addition]

$$= \left(\frac{11a}{1} + \frac{1b}{1}\right) + \left(\frac{a}{9} + \frac{b}{9} + \frac{c}{9}\right)$$

$$= \left(\frac{11a}{1} + \frac{1b}{1}\right) + \left(\frac{a+b+c}{9}\right)$$

We have to check the sum of digits for divisibility by 9

divisibility by 10:

$$\frac{abc}{10} = \frac{a\times 100 + b\times 10 + c}{10}$$

$$= \left(\frac{a\times 100}{10}\right) + \left(b\times \frac{10}{10}\right) + \left(\frac{c}{10}\right)$$

(right distributive law)

$$= (a\times 10) + (b\times 1) + \left(\frac{c}{10}\right)$$

we have to check if the last digit is 0

divisibility by 11:

$$\frac{abc}{11} = \frac{a\times 100 + b\times 10 + c}{11}$$

$$= \left(\frac{a\times 100}{11}\right) + \left(\frac{b\times 10}{11}\right) + \left(\frac{c}{11}\right)$$ [left distributive over multiplication]

$$= \left(\frac{a+99a}{11}\right) + \left(\frac{11b-1b}{11}\right) + \left(\frac{c}{11}\right)$$

$$= \left(\frac{a}{11}+\frac{99a}{11}\right) + \left(\frac{11b}{11}-\frac{b}{11}\right) + \left(\frac{c}{11}\right)$$

$$= \left(\frac{a}{11}+\frac{99a}{11}\right) + \left(b-\frac{b}{11}\right) + \left(\frac{c}{11}\right)$$

$$= \left(\frac{a}{11}+\frac{9a}{1}\right) + \left(\frac{b}{1}-\frac{b}{11}\right) + \left(\frac{c}{11}\right)$$

$$= \left(\frac{9a}{1}+\frac{b}{1}\right) + \left(\frac{a}{11}-\frac{b}{11}+\frac{c}{11}\right)$$ [commutative over addition]

$$= (9a+b) + \frac{(a-b+c)}{11}$$

If $(a-b+c)$ is divisible by 11, then abc is divisible by 11

divisibility by 12:

it has to be divisible by 3, and 4