11 Jun '25 :

Prime factorization to find LCM & GCD :

Write the number as a product of prime numbers.

- $a = p^m \times q^n \times r$, $p, q, r, m, n \in \mathbb{N}$

$b = p^{m-2} \times q^{n+2}$ $\quad m \geq 2, n \geq 0$

$GCD(a,b) = p^{m-2} \times q^n$ [For each prime factor, pick it's smallest power across both a & b]

$LCM(a,b) = p^m \times q^{n+2} \times r$ [For each prime factor, pick it's highest power across a & b]

[For 'r', treat it as r^0 in b]

Eg:-

$a = 2^2 \times 3^2$

$b = 2 \times 3^3$

Factors

1, 2, 3, 4, 6, 9, 12, 18, 36,

1, 2, 3, 6, 9, 18, 27, 54

$GCD = 2^1 \times 3^1$

$LCM = 2^2 \times 3^3$

$\frac{3^3}{3^2} = \frac{3 \times 3 \times 3}{3 \times 3}$

$a \times 3 = LCM$

$b \times 2 = LCM$

→ Find GCD & LCM of (66, 44) using prime factorization.

$66 = 2 \times 33$

$= 2 \times 3 \times 11 = 2^1 \times 3^1 \times 11^1$

$44 = 2 \times 22$

$= 2^2 \times 11 = 2^1 \times 3^0 \times 11^1$

$GCD = 2^1 \times 3^0 \times 11^1 = 22$ [product of smallest powers of all prime factors]

$LCM = 2^2 \times 3^1 \times 11^1 = 132$ [product of largest powers of all prime factors]

HW: Use prime factorization to find LCM, GCD of

(1) 256, 384

(2) 96, 64

(3) 50, 75

(4) 2025, 162

(5) 576, 1024

(6) 256, 384, 1024

(7) 2025, 162, 450

(8) 96, 384, 162

HW: Simplify the following fractions

(1) $\frac{22}{96}$ (2) $\frac{33}{66}$ (3) $\frac{123}{427}$ (4) $\frac{56}{92}$

(5) $\frac{99}{132}$ (6) $\frac{243}{2025}$ (7) $\frac{384}{512}$ (8) $\frac{28}{70}$

→ Simple fraction means numerator & denominator shouldn't have any common factors other than 1. Otherwise just find their GCD and divide them both with GCD to make it into simple fractions.

To find GCD, you can use any of the methods we learnt before.

Eg: $\frac{24}{32}$

GCD (24, 32) = 8

$= \frac{3 \times 8}{4 \times 8}$

$= \frac{3}{4}$