30 Sep '24

Integer +, - :

$$11 - 3 = 11 + (-3) = (-3) + 11$$

$$-11 + 23 - 46 + 435 - 637 = (-11) + 23 + (-46) + 435 + (-637)$$

$$-11 - 3 = -(11 + 3)$$

I have '-3' apples = I have to give away +3 apples

$$-3 = -(+3) = +(-3)$$

$11 - 3 = 8$ (I am giving 3 apples away out of 11, so 8 are remaining)

$11 - (-3)$ = I am giving away (-3) apples out of 11, so I will have 14 apples

$= 11 + 3 = 14$

$-(-3) = +3$; $+(-3) = -(+3) = -3$;

$-(3+4+5) = -3-4-5$

$+(3+4+5) = +3+4+5$

$-(3-4+5) = -3+4-5$

$+(3-4-5) = +3-4-5$

$-(+) = -$

$-(-) = +$

$+(+) = +$

$+(-) = -$

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$- (-) = +$

Eg:

Krishna	Virata	Together
10 apples	10 apples	$=10+10=20$

→ Krishna gives '3' apples to Virata

Krishna	Virata	Together
$10-3=7$ apples	$10+3=13$ apples	$7+13=20$

→ Krishna gives '−2' apples to Virata

Krishna	Virata	Together
$7-(-2)=9$ apples	$13+(-2)=11$ apples	$9+11=20$

Integer *, /

(+) * (+) = +

(−) * (+) = −

(+) * (−) = −

(−) * (−) = +

(+) / (+) = +

(+) / (−) = −

(−) / (+) = −

(−) / (−) = +

Properties of Integers (laws, rules)

Closure, Associative, Distributive, Commutative properties

Closure property :

If a, b are integers, then

a+b is also integer.

a-b " " "

a*b " " "

a/b may not be integer

This is written as

If $a, b \in \mathbb{Z}$ (belong to) (integers), then

$a+b \in \mathbb{Z}$

$a-b \in \mathbb{Z}$

$a*b \in \mathbb{Z}$

| a/b may or may not $\in \mathbb{Z}$

$+, -, *$ satisfy closure property on 'Z'.

Associative property:

$\overline{a, b, c \in Z}$

$(a+b)+c = a+(b+c)$ | $(3+2)+4 = 3+(2+4)$

$(a*b)*c = a*(b*c)$ | $(3*2)*4 = 3*(2*4)$

$+, *$ satisfy associative property on 'Z'.

$(a/b)/c \neq a/(b/c)$

$(a/b)/c = \frac{a}{(b*c)}$

$(a-b)-c \neq a-(b-c)$

$(a-b)-c = a-b-c = a-(b+c)$

$-, /$ do not satisfy associative property on Z.

Distributive property

$a, b, c \in \mathbb{Z}$

$a * (b + c) = (a * b) + (a * c)$

$a * (b - c) = a * b - a * c$

$4 * (3+2) =$

$(4*3) + (4*2)$

$4 * (3-2)$

$= (4*3) - (4*2)$

Commutative property:

$a, b \in \mathbb{Z}$

$a + b = b + a$ ✓ ; $a * b = b * a$ ✓

$a - b \neq b - a$; $a / b \neq b / a$

$+, *$ are commutative on $\mathbb{Z}$

but $-, /$ are not.

HW: (1) Give two examples each for $+, -, *, /$ to show if they satisfy closure, associative, distributive, commutative laws on $\mathbb{Z}$.

HW: (2) Do the same on 'Q' (rational / floating point numbers)
(Symbol 'R' is for real numbers)

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An operation 'o' (o could be +, −, /, *, %, ** etc.), on the numbers (a,b,c) of type 'K" (K could be $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ etc.) satisfies

(1) Closure property if $a \circ b \in K$, $\forall\, a,b \in K$
(for all)

(2) Commutative property if

$$a \circ b = b \circ a, \quad \forall a, b \in K$$

(3) Associative property if

$$a \circ (b \circ c) = (a \circ b) \circ c, \quad \forall a, b, c \in K$$

(4) Distributive property: For two operations $(*, \circ)$

$$a * (b \circ c) = (a * b) \circ (a * c)$$

Eg: (1) Let $K = \mathbb{Z}$; '$\circ$' is '$+$'; Property = closure;

$2, 3 \in \mathbb{Z}$ $\quad 2+3 = 5 \in \mathbb{Z}$

$3, -2 \in \mathbb{Z}$ $\quad 3+(-2) = 3-2 = 1 \in \mathbb{Z}$

$5, -7 \in \mathbb{Z}$ $\quad 5+(-7) = 5-7 = -2 \in \mathbb{Z}$

$\forall\, a, b \in \mathbb{Z}$, $a+b \in \mathbb{Z}$; So, '$+$' satisfies closure property on $\mathbb{Z}$.

(2) Let $K = \mathbb{Q}$ (rational) ; '0' = division ;
Property = commutative ;

Eg1: $2.3 / 4.2$ $\qquad$ $4.2 / 2.3$

$= 0.5476\ldots$ $\qquad$ $= 1.8260\ldots$

$a, b \in \mathbb{Q}$, $\quad a/b \neq b/a$

So, / doesn't satisfy commutative property on $\mathbb{Q}$.

(3) Let $K = \mathbb{Q}$; '0' = * ; Property = associative

Eg1: $2.3 * (2.4 * 3.0)$ $\qquad$ $(2.3 * 2.4) * 3.0$

$2.3 * (7.2)$ $\qquad$ $5.52 * 3.0$

16.56 $\quad = \quad$ 16.56

Eg 2: $2.5 * (6.2 * 5.1)$ $\qquad$ $(2.5 * 6.2) * 5.1$

$2.5 * 31.62$ $\qquad$ $15.5 * 5.1$

79.05 $\qquad$ 79.05

$\forall a, b, c \in \mathbb{Q}$, $a * (b * c) = (a * b) * c$

So, $*$ satisfies associative property on $\mathbb{Q}$.

$\Rightarrow$ Why do we need these properties?

Eg:

$$\begin{array}{r} 23 \\ \times\ 12 \\ \hline 46 \\ 230 \\ \hline 276 \\ \hline \end{array}$$

$46 = 23 \times 2$

$230 = 23 \times 10$

$23 \times 12 = 23 \times (10 + 2)$

$= 23 \times 10 + 23 \times 2$

$= 230 + 46$

$= 276$

Because natural numbers satisfy distributive property, we can do multiplication like this.

Eg:

$$\begin{array}{r} 231 \\ +\ 193 \\ \hline 424 \\ \hline \end{array}$$

$231 + 193$

$= (200+30+1) + (100+90+3)$

$= (200+100) + (30+90) + (1+3)$

Because + is associative on 'N', we can use this algorithm to perform '+' on bigger numbers.

HW: (1) Finish last homework