21 Nov' 25

→ More practice on Roman numerals

HW: (1) Answer the new problem set (AI generated)

(2) Verify your answers (Solve all problems first; Then come back and convert your answers back without looking at the problems then check that it matches.)

Eg:

39	XXX IX	39
2043	MMXLIII	2043
452	CDLII	452

→ Exponents and Roots

See AI notes.

→ $a^m \times a^n = (a \times a \times \cdots \text{ m times}) \times (a \times a \times \cdots \text{ n times})$

$= (a \times a \times a \times \cdots \text{ (m+n) times})$

$= a^{m+n}$

→ $\frac{a^m}{a^n} = \frac{a \times a \times \cdots \text{ m times}}{a \times a \times \cdots \text{ n times}}$

If $m > n$

$= \left[\frac{a}{a} \times \frac{a}{a} \times \cdots \text{ (n times)}\right] \times (a \times a \cdots \text{ (m-n) times})$

$= [1 \times 1 \times \cdots] \times a^{m-n}$

$= a^{m-n}$

If $m < n$

$$= \left[\frac{a}{a} \times \frac{a}{a} \cdots \text{m times}\right] \times \frac{1}{a^{n-m}}$$

$$= \frac{1}{a^{n-m}}$$

$$= a^{m-n} \qquad \left[\because a^{-p} = \frac{1}{a^p}\right]$$

$$\rightarrow \frac{a^m}{a^n} = a^{m-n}$$

Let $m = n$.

$$\frac{a^m}{a^m} = a^{m-m} = a^0$$

$$\Rightarrow 1 = a^0 \; ; \; \text{So } a^0 = 1 ;$$

Eg: $a^0 = a^{5-5} = \frac{a^5}{a^5} = 1.$

$a^0 = 1$, if $a \neq 0$

If $a = 0$, $\frac{a^3}{a^3} = \frac{0}{0}$

Division with '0' is not defined.

$0 \times 1 = 0$

$0 \times 2 = 0$

$0 \times n = 0$

$\frac{0}{0} = ?$ 1? 2? 3? - - .

⟶ If $a^m = p$

then $a = p^{\frac{1}{m}} = \sqrt[m]{p}$

$a^m = p$

$\Rightarrow$ $a \times a \times \cdots = p$

$a^m = p$

$(a^m)^{\frac{1}{m}} = p^{\frac{1}{m}}$

$\left[\because (a^m)^n = a^{m \times n}\right]$

$\Rightarrow$ $a^{m \times \frac{1}{m}} = p^{\frac{1}{m}}$

$\Leftrightarrow$ $a = p^{\frac{1}{m}}$

This is also written as $\sqrt[m]{p}$ (m^{th} root of p)

Eg:→ If $a^3 = p \Rightarrow \sqrt[3]{p} = a = p^{\frac{1}{3}}$

(cube root of p)

→ If $a^2 = p \Rightarrow \sqrt{p} = a = \sqrt{a} = p^{\frac{1}{2}}$

(sq. root of p)

Eg: What is $\sqrt[4]{21}$?

Let $\sqrt[4]{21} = x$

$\Rightarrow 21^{\frac{1}{4}} = x$

$\Rightarrow x^4 = 21$

$2^4 = 16 < x^4 = 21 < 3^4 = 81$

$2 < x < 3$

$x = 2.\ldots\ldots$ something.

If you want more digits in x,
you can compute
2.1^4 2.2^4 etc.
and find when it crosses the
boundary of 21.

HW: Write a python program to find n^{th} root of a number N this way.

HW: Solve AI exercises on powers/roots.
At each step you have to write which formula you are using.

Eg:- $16^{9/2} = 16^{9 \times \frac{1}{2}} = 16^{\frac{1}{2} \times 9}$

$= \left(16^{\frac{1}{2}}\right)^9$ [$\because a^{m \times n} = (a^m)^n$]

$= \left((4^2)^{\frac{1}{2}}\right)^9$ $\left(\sqrt{16}\right)^9$

$= \left(4^{2 \times \frac{1}{2}}\right)^9$ $= 4^9$

$= (4)^9$

$= - - -$