26 May '25

→ Factors of a number

→ Check until it's square root as factors come in pairs (a,b) such that $a \times b = n$

If $a < \sqrt{n}$, then $b > \sqrt{n}$

$a = \sqrt{n}$, then $b = \sqrt{n}$

$a > \sqrt{n}$, then $b < \sqrt{n}$

→ To find whether a number is prime, we only have to check if any of $2, 3 \ldots, \sqrt{n}$ divides 'n'.

HW: Write a programming function to check if a number 'n' is prime.

(1) Do it as before

(2) Do it using sqrt(n)

HW: Last digits HW (previous) – correct it – show the steps – question at the top – answers at the bottom after all the steps

HW: Factors HW (previous) – show all the steps

Algorithm to find GCD: Instead of finding all the factors of a, b to find their GCD, there is an easier way.

Let 'x' be the GCD of a, b. $(x, a, b \in N)$ $(a > b)$

So, $x \mid a$ (x divides a not x/a) & $x \mid b$.

Let $\frac{a}{x} = p$ and $\frac{b}{x} = q$; $p, q \in N$; $p > q$

since $x \mid a$ & $x \mid b$

$$\frac{a-b}{x} = \frac{a}{x} - \frac{b}{x}$$

(Division is right distributive over subtraction)

$$= p - q$$

$$= r \qquad (r \in N)$$

So, $x \mid (a-b)$

So, $GCD(a, b) = GCD(b, a-b)$

since 'x' is the largest number that divides a & b, and it also divides $(a-b)$.

Suppose $y > x$ & $y \mid \underline{(a-b)}$ & $y \mid \underline{b}$

$$\Rightarrow \frac{a-b+b}{y} = \frac{a-b}{y} + \frac{b}{y}$$

$$\Rightarrow \frac{a}{y} = m + n$$

$\Rightarrow y \mid a$ So, y will be GCD of a, b but that's not true.

So, there is no $y > x$ which divides b & $a-b$

Eg: $GCD(120, 96)$

$= GCD(96, 120-96)$

$= GCD(96, 24) = 24.$ [$\because 24 \times 4 = 96$]

$GCD(a, b) = GCD(b, a \% b)$ $(a > b)$

Why?

$GCD(a, b) = GCD(b, a-b)$ (if $a-b > b$)

$= GCD(b, a-b-b)$

$$= GCD(b, a - l \times b)$$

$$= GCD(b, r)$$

where 'l' is quotient of $\frac{a}{b}$

'r' is remainder

$(r < b)$ $a = l \times b + r$

So $GCD(a, b) = GCD(b, a \% b)$

Steps to find GCD

$$\begin{array}{l} \overset{b}{96} \,)\, \overset{a}{120} \,(\, 1 \\ \quad\quad\ \ 96 \\ \hline r1 = 24 \,)\, 96 \,(\, 4 \\ \quad\quad\quad\quad\ 96 \\ \hline \quad\quad\ r2 = 0 \end{array}$$

Steps: To find GCD (a,b) where a > b

(1) Divide a with b to find remainder 'r1'

(2) Divide 'b' with r1 to find remainder r2

(3) Repeat in loop until remainder is zero. Then the divisor is the GCD.

Eg: $GCD(96, 108)$

$$
\begin{array}{l}
96)\,108\,(1 \\
\quad\;\; 96 \\
\hline
\quad\;\; 12)\,96\,(8 \\
\qquad\quad\;\; 96 \\
\hline
\qquad\quad\;\;\; 0 \\
\hline
\end{array}
$$

12 → GCD

Eg: $GCD(150, 95)$

$$
\begin{array}{l}
95)\,150\,(1 \\
\quad\;\; 95 \\
\hline
\quad\;\; 55)\,95\,(1 \\
\qquad\;\; 55 \\
\hline
\qquad\;\; 40)\,55\,(1 \\
\qquad\qquad 40 \\
\hline
\qquad\qquad 15)\,40\,(2 \\
\qquad\qquad\quad 30 \\
\hline
\qquad\qquad\quad 10)\,15\,(1 \\
\qquad\qquad\qquad 10 \\
\hline
\qquad\qquad\qquad 5)\,10\,(2 \\
\qquad\qquad\qquad\;\; 10 \\
\hline
\qquad\qquad\qquad\;\;\; 0 \\
\hline
\end{array}
$$

5 → GCD

Eg: GCD(97, 100)

$$
\begin{array}{l}
97\,)\,100\,(\,1 \\
\quad\ \ 97 \\
\quad\ \overline{\ \ 3\,)\,97\,(\,32} \\
\qquad\quad 96 \\
\qquad\ \overline{\ \ 1\,)\,3\,(\,3} \\
\qquad\qquad 3 \\
\qquad\qquad \overline{\ 0\ }
\end{array}
$$

GCD

HW: Find GCDs of (72, 42), (93, 62), (144, 88), (64, 52) using the algorithm

LCM : Least common multiple of two numbers (a, b).

Eg: LCM(4, 3)

Multiples of 4 = [4, 8, 12, 16, 20, 24, ...]

Multiples of 3 = [3, 6, 9, 12, 15, 18, 21, 24 ...]

Common multiples = [12, 24 ...]

LCM = 12.

If '12' is LCM then multiples of 12 are also common multiples of 3 & 4.

How to find LCM(a, b)

$(a, b, x, p, q \in \mathbb{N})$

Let $GCD(a,b) = x$.

So, $a = x \times p$; $b = x \times q$

and p & q don't have any common factors, so $LCM(p,q) = p \times q$

Multiples of a : $x \times p \times 1, x \times p \times 2, \ldots, \underline{x \times p \times q}, \ldots$

Multiples of b : $x \times q \times 1, x \times q \times 2, \ldots x \times q \times p, \ldots$

$LCM(a,b) = x \times p \times q$ $\quad (\because x \times p \times q = x \times q \times p$ commutative law$)$

$= \dfrac{(x \times p) \times (q \times x)}{x}$

$LCM(a,b) = \dfrac{a \times b}{x} = \dfrac{a \times b}{GCD(a,b)}$

$\Rightarrow \quad LCM(a,b) \times GCD(a,b) = a \times b$

So, to find $LCM(a,b)$, we can compute $\dfrac{a \times b}{GCD(a,b)}$ after finding $GCD(a,b)$ using the previous algorithm.

Eg: $LCM(4,3)$

$$= \frac{4 \times 3}{GCD(4,3)} = \frac{4 \times 3}{1} = 12$$

Eg: $LCM(96,120) = \dfrac{96 \times 120}{GCD(96,120)} = \dfrac{\cancel{96}^{\,4} \times 120}{\cancel{24}}$

$$= 480$$

Eg: $\dfrac{4}{96} + \dfrac{5}{120} = \dfrac{4}{96} \times \dfrac{120}{120} + \dfrac{5}{120} \times \dfrac{96}{96}$

(old way)

$$\frac{4}{96} + \frac{5}{120} = \frac{4}{96} \times \frac{5}{5} + \frac{5}{120} \times \frac{4}{4}$$

$$= \frac{20}{480} + \frac{20}{480} = \frac{4\cancel{0}}{48\cancel{0}} = \frac{1}{12}$$

→ AI notes on GCD algorithm, LCM, fractions

HW: Complete 10 exercises on fraction sum using LCM

→ AI notes on prime factorization for LCM, GCD.

HW: Read it and correct any mistakes.