problem 1:

$(x \text{ and } \sim y) \text{ or } (\sim x \text{ or } y)$

x	y	$\sim y$	x and $\sim y$	$\sim x$	$\sim x$ or y	$(x \text{ and} \sim y)$ or $(\sim x \text{ or } y)$
true	true	false	false	false	true	true
true	false	true	true	false	false	true
false	true	false	false	true	true	true
false	false	true	false	true	true	true

Result is always true: Why?

Let $z = (x \text{ and} \sim y)$; $\sim z = \sim(x \text{ and} \sim y) = (\sim x \text{ or } y)$

So, problem 1 $= z$ or $\sim z$ (one of z, $\sim z$, is always true, so 'or' is always true)

Problem 2: $\sim(x \text{ and } y)$ or $\sim(x \text{ or } y)$

x	y	x and y	$\sim(x \text{ and } y)$	x or y	$\sim(x \text{ or } y)$	$\sim(x \text{ and } y)$ or $\sim(x \text{ or } y)$
true	true	true	false	true	false	false
true	false	false	true	true	false	true
false	true	false	true	true	false	true
false	false	false	true	false	true	true

A (under the $\sim(x \text{ and } y)$ column) B (under the last column)

A & B look the same. Why? Use operator precedence and evaluate the expression.

Problem 2 B $= (\sim x \text{ or } \sim y) \text{ or } (\sim x \text{ and } \sim y)$ [and is like $*$, or is like $+$ in arithmetic]

$= \sim x \text{ or } \sim y \text{ or } (\sim x \text{ and } \sim y)$

$= \sim x \text{ or } ((\sim y \text{ or } \sim x) \text{ and } (\sim y \text{ or } \sim y))$ (and distributes over or, or distributes over and) (but not always).

$= (\sim x) \text{ or } ((\sim y \text{ or } \sim x) \text{ and } (\sim y))$

$= (\sim x \text{ or } (\sim y \text{ or } \sim x)) \text{ and } (\sim x \text{ or } \sim y)$

(Distributive law or over and)

$= (\sim x \text{ or } \sim x \text{ or } \sim y) \text{ and } (\sim x \text{ or } \sim y)$

(Associative & commutative laws for or)

$= (\sim x \text{ or } \sim y) \text{ and } (\sim x \text{ or } \sim y)$

(Since $\sim x$ or $\sim x = \sim x$)

$= \sim x \text{ or } \sim y$ $\quad$ (z or $z = z$, where $z = \sim x$ or $\sim y$)

$= \sim (x \text{ and } y)$

$=$ A

This means $\sim(x \text{ and } y) \text{ or } \sim(x \text{ or } y) = \sim(x \text{ and } y)$.

see operator precedence

Problem 3: $\sim x$ or $\sim(x \text{ and } y)$ and $(x \text{ or } \sim x)$

x	y	$\sim x$	x and y	$\sim(x$ and $y)$	x or $\sim x$	$\sim(x$ and $y)$ and $(x$ or $\sim x)$	$\sim x$ or $\sim(x$ and $y)$ and $(x$ or $\sim x)$
true	true	false	true	false	true	false	false
true	false	false	false	true	true	true	true
false	true	true	false	true	true	true	true
false	false	true	false	true	~~false~~ true	~~false~~ true	true
				(A)			(B)

(A) = (B) why ?

(B) = $\sim x$ or $\sim(x \text{ and } y)$ and $(x \text{ or } \sim x)$

= $\sim x$ or $(\sim(x \text{ and } y)$ and true$)$ [Since x or $\sim x$ is always true]

= $\sim x$ or $(\sim(x \text{ and } y))$ [A and true = A]

= $\sim x$ or $(\sim x$ or $\sim y)$

$= (\sim x \text{ or } \sim x) \text{ or } (\sim y)$ [Associative law for or]

$= \sim x \text{ or } \sim y$

$= \sim(x \text{ and } y)$

$= A$