29 Apr'25

→ **Finding last digit of a power**

(check previous HW examples)

Let $a, x \in \mathbb{N}$ (i.e., a & x are Natural numbers)

Then, if we want to know the last digit of a^x :

(Here 'a' is called the base
'x' is the exponent/power)

Steps :

(1) Let $b = x \% 4$ and let 'c' be the last digit of 'a' (i.e., $c \in \{0, 1, 2, \ldots 9\}$)

(2) Last digit of a^x = last digit of c^b
(see HW examples to understand why)

$\rightarrow a^x = (d \times 10 + c)^x$

↳ 'c' is last digit of a

If $x = 2$, $a^2 = (d\times 10 + c)^2 = (d\times 10 + c) \times (d \times 10 + c)$

$= d\times 10 \times d \times 10 + d \times 10 \times c + c \times d \times 10 + c \times c$

$= 10(\ldots M \ldots) + c \times c$

Not units digit

↳ units digit will be last digit of $c \times c$

$$\boxed{\begin{aligned}&(a+b)\times(c+d) \quad \hookrightarrow p = (c+d)\\ &= a \times p + b \times p = a \times (c+d) + b \times (c+d)\\ &= a\times c + a \times d + b \times c + b \times d\end{aligned}}$$

If $x = 3$, $a^3 = (10 \times M + c^2)(d \times 10 + c)$

$= 10 \times M \times d \times 10 + 10 \times M \times c + c^2 \times d \times 10 + c^2 \times c$

$= 10 \times (\ldots\ldots) + c^3$

This won't be units place

↳ Last digit of a^3 will be last digit of c^3

And from HW, we know that last digits of all powers repeat in cycles of 4.

So, a^x will have same last digit as $c^{x \% 4}$

HW: Find last digits of

(1) 23767^{45931}

(2) 769321^{673251}

(3) 5643^{989}

(4) 96378^{64397}

(5) 67946^{8932}

HW: Solve 20 AI HW exercises combining divisibility rules and last digits of powers.