

closure property:

$$\begin{array}{l|l} + \left. \begin{array}{l} 5+2=7 \\ 5, 2, 7 \in \mathbb{Z} \end{array} \right\} & \begin{array}{l} 9+8=17 \\ 9, 8, 17 \in \mathbb{Z} \end{array} \end{array}$$

$$5, 2 \in \mathbb{Z}; 5+2=7 \in \mathbb{Z}$$

$$9, 8 \in \mathbb{Z}; 9+8=17 \in \mathbb{Z}$$

closure property is satisfied for addition on ' \mathbb{Z} '

$$9-7=2$$

$$4-3=1$$

$$9, 7, 2 \in \mathbb{Z}$$

$$4, 3, 1 \in \mathbb{Z}$$

$$9, 7 \in \mathbb{Z}; 9-7=2 \in \mathbb{Z}$$

$$4, 3 \in \mathbb{Z}; 4-3=1 \in \mathbb{Z}$$

$$5, 7 \in \mathbb{Z}; 5-7=-2 \in \mathbb{Z}$$

closure property is satisfied for subtraction on ' \mathbb{Z} '.

$$5 \times 2 = 10$$

$$6 \times 3 = 18$$

$$5, 2 \in \mathbb{Z}, 10 \text{ also } \in \mathbb{Z}. \quad 6, 3, 18 \in \mathbb{Z}$$

closure property satisfies for multiplication.

$$\div \left. \begin{array}{l} 5 \div 2 = 2 \text{ quotient is '2', remainder is not '0'} \\ \text{closure property does not } \in \text{ to Division on } \mathbb{Z} \end{array} \right\}$$

$$5, 2 \in \mathbb{Z} \text{ but } \frac{5}{2} \notin \mathbb{Z}$$

associative property:

$$(2+5)+1 = 2+(5+1)$$

$$8$$

$$=8$$

$$(1+2)+3 = \underbrace{(3+1)+2}_{6} \quad \text{with } 1+(2+3) \text{ above}$$

addition is associative on \mathbb{Z} .

$$\left. \begin{array}{l} (5-1)-2 \\ \quad 4-2 \\ \quad \quad 2 \end{array} \right\} \neq \left. \begin{array}{l} 5-(1-2) \\ 5-(-1) \\ \quad 6 \end{array} \right\} \begin{array}{l} 5-(2-1) \\ \quad 5-1=4 \end{array} \quad \left. \begin{array}{l} (6-3)-4 \\ 3-4 \\ \quad -1 \end{array} \right\} \neq \left. \begin{array}{l} 6-(3-4) \\ 6-(-1) \\ \quad +7 \end{array} \right\}$$

subtraction is not associative on \mathbb{Z} .

$$\left(\begin{array}{l} (2 \times 3) \times 4 \\ 6 \times 4 \\ 24 \end{array} \right) = \left(\begin{array}{l} 2 \times (3 \times 4) \\ 2 \times 12 \\ 24 \end{array} \right) \quad \left(\begin{array}{l} (2 \times 4) \times 1 \\ 8 \times 1 \\ 8 \end{array} \right) = \left(\begin{array}{l} 2 \times (4 \times 1) \\ 2 \times 4 \\ 8 \end{array} \right)$$

multiplication is associative on \mathbb{Z} ✓

$$\div) (6 \div 2) \div 2 \neq 6 \div (2 \div 2)$$

$$\begin{array}{l} 3 \div 2 \\ \text{quotient} = 1 \\ \text{remainder} = 1 \end{array} \neq \begin{array}{l} 6 \div 1 \\ 6 \end{array}$$

$$(9 \div 3) \div 2 \neq 9 \div (3 \div 2)$$

$$\begin{array}{l} 3 \div 2 \\ 1 = 1 \\ R = 1 \end{array} \quad \begin{array}{l} 9 \div 1 (R = 1; 2 = 1) \\ 9 \end{array}$$

division is not associative in \mathbb{Z} .

Distributive property:

$$+) 3 \times (1 + 2) = (3 \times 1) + (3 \times 2)$$

$$\begin{array}{l} 3 \times 3 \\ 9 \end{array} = \begin{array}{l} 3 \\ 9 \end{array} + \begin{array}{l} 6 \end{array}$$

$$6 \times (2 + 3) = (6 \times 2) + (6 \times 3)$$

$$\begin{array}{l} 6 \times 5 \\ 30 \end{array} = \begin{array}{l} 12 \\ 30 \end{array} + \begin{array}{l} 18 \end{array}$$

$+$ is Distributive in \mathbb{Z} .

$$-) 3 \times (4 - 5) \mid (3 \times 4) - (3 \times 5)$$

$$\begin{array}{l} 3 \times (-1) \\ -3 \end{array} \mid \begin{array}{l} 12 - 15 \\ -3 \end{array}$$

$-$ is distributive in \mathbb{Z} .

$$2 \times (6 - 3) \mid (2 \times 6) - (2 \times 3)$$

$$\begin{array}{l} 2 \times 3 \\ 6 \end{array} \mid \begin{array}{l} 12 - 6 \\ 6 \end{array}$$

commutative property:

$$\begin{array}{lcl} +) & 3+2=2+3 & 3+9=9+3 \\ & 5=5 & 12=12 \end{array}$$

$+$ is commutative in \mathbb{Z} .

$$\begin{array}{lcl} -) & 2-3 \neq 3-2 & 9-2 \neq 2-9 \\ & -1 \quad 1 & 7 \quad -7 \end{array}$$

$-$ is not commutative in \mathbb{Z} .

$$\begin{array}{lcl} \times) & 4 \times 3 = 3 \times 4 & 9 \times 2 = 2 \times 9 \\ & 12 \quad 12 & 18 \quad 18 \end{array}$$

\times is commutative in \mathbb{Z} .

$$\begin{array}{lcl} \div) & 4 \div 2 \neq 2 \div 4 & 9 \div 2 \neq 2 \div 9 \\ & 2 \quad 0 & 9=4 \quad 2=0 \end{array}$$

\div is not commutative R/ on \mathbb{Z} .

closure property on \mathbb{Q} :

$$\begin{array}{l|l} +) \begin{array}{l} 6.2 + 9.2 \\ 15.4 \end{array} & \begin{array}{l} 1.3 + 4.5 \\ 5.8 \end{array} \end{array} \quad \begin{array}{l} 6.2, 9.2 \in \mathbb{Q} \\ 6.2 + 9.2 \in \mathbb{Q}. \\ 1.3, 4.5 \in \mathbb{Q} \\ 1.3 + 4.5 \in \mathbb{Q}. \end{array}$$

+ is closure on \mathbb{Q} .

$$\begin{array}{l|l} -) \begin{array}{l} 2.3 - 6.1 \\ -3.8 \end{array} & \begin{array}{l} 6.2 - 4.1 \\ 2.1 \end{array} \end{array} \quad \begin{array}{l} 2.3, 6.1 \in \mathbb{Q} \\ 2.3 - 6.1 \in \mathbb{Q}. \\ 6.2, 4.1 \in \mathbb{Q} \\ 6.2 - 4.1 \in \mathbb{Q}. \end{array}$$

- is closure on \mathbb{Q}

$$x) \begin{array}{l} 1.1 \times 6.2 \\ 6.82 \end{array} / \begin{array}{l} 2.3 \times 1.6 \\ 3.68 \end{array}$$

x is closure on \mathbb{Q} .

$$\div) \begin{array}{l} 6.2 \div 1.1 \\ 5.6363636363... \end{array}$$

$$6.2, 1.1 \in \mathbb{Q}.$$

$$6.2 \div 1.1 \in \mathbb{Q}$$

\div is closure on \mathbb{Q} .

$$1.1, 6.2 \in \mathbb{Q}$$

$$1.1 \times 6.2 \in \mathbb{Q}.$$

$$2.3, 1.6 \in \mathbb{Q}$$

$$2.3 \times 1.6 \in \mathbb{Q}.$$

$$9.1 \div 6.3 = 1.444444...$$

$$9.1, 6.3 \in \mathbb{Q}$$

$$9.1 \div 6.3 \in \mathbb{Q}$$

associative property:

$$\begin{array}{l} + \left\{ \begin{array}{l|l} (6.2 + 9.8) + 2.1 & 6.2 + (9.8 + 2.1) \\ 16 + 2.1 & 6.2 + 11.9 \\ \hline 18.1 & = 18.1 \end{array} \right. \\ \\ \left\{ \begin{array}{l|l} (1.2 + 6.1) + 6.0 & 1.2 + (6.1 + 6.0) \\ 7.3 + 6.0 & 1.2 + 12.1 \\ \hline 13.3 & = 13.3 \end{array} \right. \end{array}$$

$+$ is associative in \mathbb{Q} .

$$(9.9 - 2.3) - 1.1 \neq 9.9 - (2.3 - 1.1)$$

$$7.6 - 1.1 \neq 9.9 - 1.2$$

$$6.5 \neq 8.7$$

$$(5.5 - 9.2) - 6.6 \neq 5.5 - (9.2 - 6.6)$$

$$-3.7 - 6.6 \neq 5.5 - 2.6$$

$$-10.3 \neq 2.9$$

- is not associative in \mathbb{Q} .

$$\begin{array}{l|l} x) (9.6 \times 2.1) \times 11.2 & 9.6 \times (2.1 \times 11.2) \\ 20.16 \times 11.2 & 9.6 \times 23.52 \\ 225.792 & 225.792 \end{array}$$

$$\begin{array}{ccc}
 (55.22 \times 11.2) \times 1.1 & | & 55.22 \times (11.2 \times 1.1) \\
 618.464 \times 1.1 & & 55.22 \times 12.432 \\
 686.49504 & = & 686.49504
 \end{array}$$

\times is associative on \mathbb{Q} .

$$\begin{array}{ccc}
 (5.5 / 2.2) / 9.1 & | & 5.5 / (2.2 / 9.1) \\
 2.5 / 9.1 & & 5.5 / 0.241758... \\
 0.274725... \neq & & 21.7500227500...
 \end{array}$$

\div is not associative on \mathbb{Q} .

Distributive property: $\underline{a} \times (b - c) = a \times b - a \times c$

$$6.2 \times (9.1 + 6.3)$$

$$6.2 \times 15.4$$

$$95.48$$

$$6.2 \times 9.1 + 6.2 \times 6.3$$

$$56.42 + 39.06$$

$$95.48$$

$$9.1 \times (6.1 + 999.999)$$

$$9.1 \times 1006.099$$

$$9155.5009$$

$$9.1 \times 6.1 + 9.1 \times 999.999$$

$$55.51 + 9099.9909$$

$$9155.5009$$

+ is distribute on \mathbb{Q} .

$$9.26 \times (9.99 - 8.1)$$

$$9.26 \times 1.89$$

$$17.5014$$

$$9.26 \times 9.99 - 9.26 \times 8.1$$

$$92.5074 - 9.26 \times 8.1$$

$$17.5014$$

$$\begin{aligned}
 3.05 \times (9.9 - 2.1) &= 3.05 \times 9.9 - 3.05 \times 2.1 \\
 3.05 \times 7.8 &= 30.195 - 3.05 \times 2.1 \\
 23.79 &= 23.79
 \end{aligned}$$

\times is distributive on \mathbb{Q} .

Commutative; $\forall x, y \in \mathbb{Q}$

$$\begin{array}{l} + \end{array} \left(\begin{array}{cc|cc} 9.1 + 2.1 & 2.1 + 9.1 & 5.1 + 6.2 & 6.2 + 5.1 \\ \hline 11.2 & = 11.2 & 11.3 & = 11.3 \end{array} \right)$$

$+$ is commutative on \mathbb{Q} .

$$\begin{array}{l} - \end{array} \left(\begin{array}{cc|cc} 9.1 - 8.1 & 8.1 - 9.1 & 9.9 - 8.6 & 8.6 - 9.9 \\ \hline 1.0 & \neq -1.0 & 1.3 & \neq -1.3 \end{array} \right)$$

$-$ is not commutative on \mathbb{Q} .

$$\begin{array}{l|l}
 \times \left. \begin{array}{l} 9.99 \times 8.1 = 8.1 \times 9.99 \\ 80.919 = 80.919 \end{array} \right\} & \begin{array}{l} 6.1 \times 9.1 = 9.1 \times 6.1 \\ 55.51 = 55.51 \end{array}
 \end{array}$$

\times is commutative on \mathbb{Q} .

$$\begin{array}{l|l}
 \div \left. \begin{array}{l} 5.5 \div 2.1 \neq 2.1 \div 5.5 \\ 2.6190... \neq 0.381818... \end{array} \right\} & \begin{array}{l} 9.99 \div 1.1 \neq 1.1 \div 9.99 \\ 9.0 \neq 0.11111111... \end{array}
 \end{array}$$

\div is not commutative on \mathbb{Q} .