08 Oct '25 | 10 Oct '25

Prime factorization example from last class:

$$a = p^m \times q^n \times r$$

$$b = p^{m-2} \times q^{n+2}$$

$p, q, r, m \in \mathbb{N}$, $n \in W$

$m \geq 3$, $n \geq 1$

p, q, r are prime

$$GCD(a, b) = p^{m-2} \times q^n ;$$

→ Does p^{m-2} divide a?

$$\frac{a}{p^{m-2}} = \frac{p^m \times q^n \times r}{p^{m-2}} = \left(\frac{p^m}{p^{m-2}}\right) \times q^n \times r$$

$$= \left(\frac{p \times p \times \ldots \text{ m times}}{p \times p \times \ldots \text{ m-2 times}}\right) \times q^n \times r$$

$$= \left(\frac{p \times p \times \cdots (m-2)\text{-times}}{p \times p \times \cdots (m-2)\text{-times}} \right) \times (p \times p) \times q^n \times r$$

$$= 1 \times p^2 \times q^n \times r$$

$$= p^2 \times q^n \times r \qquad > 1, \in \mathbb{N}$$

So, yes p^{m-2} divides a

$\longrightarrow$ Does p^{m-2} divide b ? (yes, it's a factor of b)

$$\frac{b}{p^{m-2}} = \frac{p^{m-2} \times q^{n+2}}{p^{m-2}}$$

$$= \left(\frac{p^{m-2}}{p^{m-2}} \right) \times q^{n+2} = 1 \times q^{n+2}$$

$$= q^{n+2}$$

→ Does q^n divide a ? (yes, it's a factor)

$$\frac{a}{q^n} = p^m \times r$$

→ Does q^n divide b ?

$$\frac{b}{q^n} = \frac{p^{m-2} \times q^{n+2}}{q^n}$$

$$= p^{m-2} \times \left(\frac{q^{n+2}}{q^n}\right)$$

$$= p^{m-2} \times q^2$$

$> 1, \in \mathbb{N}$

So, q^n divides b.

$$\frac{q^{n+2}}{q^n}$$

$$= \frac{q \times q \times \ldots (n+2) \text{ times}}{q \times q \times \ldots n \text{ times}}$$

$$= \left(\frac{q \times q \times \ldots n \text{ times}}{q \times q \times \ldots n \text{ times}}\right) \times q \times q$$

$$= 1 \times q \times q$$

$$= q^2$$

HW:→ Does GCD divide a ?

$$\frac{a}{p^{m-2} \times q^{n}} =$$ (Solve step by step)

→ Does GCD divide b?

$$\frac{b}{p^{m-2} \times p^{n}} =$$ (Solve step by step)

→ $LCM(a,b) = p^{m} \times q^{n+2} \times r$

→ Does a divide LCM ?

$$\frac{LCM(a,b)}{a} =$$ (Solve step by step)

→ Does 6 divide LCM (a,b) ?

$\frac{LCM}{6} =$ (solve step by step)

→ Checked previous HWs

→ Checked final exam (AI generated) — grading

→ HW: Create table of contents for everything we learnt in Grad1 - Maths