Section 1: Simplify powers (1-20)

1. $6^3 \times 6^5$

$6^3 \times 6^5$

$= 6^{3+5}$

$= 6^8$ (Because $A^m \times A^n = A^{m+n}$) ✓

(use small letters for variables)

2. $10^9 \times 10^1 = 10^{10}$

$10^9 \times 10^1$

$= 10^{9+1}$

$= 10^{10}$ (Because $A^m \times A^n = A^{m+n}$) ✓

3. $y^7 \times y^2 = y^9$ ($A^m \times A^n = A^{m+n}$)

$y^7 \times y^2$

$= y^{7+2}$ ✓

$= y^9$ (Because $A^m \times A^n = A^{m+n}$)

4. $2^4 \times 2^{-1}$

$2^4 \times 2^{-1}$

$= 2^{4+-1}$

$= 2^{4-1}$

$= 2^3$ (Because $A^m \times A^{-n} = A^{m+n} = A^{m-n}$) ✓ (You can write $a^m \times a^n = a^{m+n}$, $m, n \in \mathbb{Z}$ can be +ve/-ve)

5. $(-3)^2 \times (-3)^3$

$-3^2 \times -3^3$

$= \frac{1}{3^2} \times \frac{1}{3^3}$

$= \frac{1 \times 1}{3^2 \times 3^3}$

$= \frac{1}{3^5}$

$= 3^{-5}$ (Because $\frac{1}{x^m} = x^{-m}$)

$a^2$

$= a \times a$

$= (-3) \times (-3)$

$= 9$

Don't remove brackets!

$a^{-n} = \frac{1}{a^n}$ ; $a^{\frac{1}{n}} \neq -a^n$

6. $\frac{8^7}{8^4}$

$8^{7-4}$

$= 8^3$ (Because $\frac{A^m}{A^n} = A^{m-n}$) ✓

7. $\frac{x^{10}}{x^3}$

$x^{10-3}$

$= x^7$ (Because $\frac{A^m}{A^n} = A^{m-n}$) ✓

8. $\frac{5^3}{5^5}$ ✗ $\frac{5^2}{5^5}$ ✗

$5^{3-5}$

$= 5^{-2}$ (Because $\frac{A^m}{A^n} = A^{m-n}$)

9. $\frac{11^2}{11^0} \times \frac{11^0}{11^2}$

$\frac{11^2}{11^0}$

$= 11^{2-0}$

$= 11^2$ (Because $x^0 = 1$)

10. $\frac{9^4}{9^4}$

$\frac{9^4}{9^4}$

$= 9^{4-4}$

$= 9^0$

$= 1$ (1. Because $\frac{A^m}{A^n} = A^{m-n}$ 2. $x^0 = 1$)

11. $(2^3)^5$

$= (2^3)^5$

$= 2^{3 \times 5}$

$= 2^{15}$ (Because $(x^y)^z = x^{y \times z}$) ✓

12. $\frac{(4^{-2})^3}{(4^{-2})^3}$

$= 4^{-2 \times 3}$

$= 4^{-6}$ (Because $(x^y)^z = x^{y \times z}$) ✓

13. $(a^6)^0$

$= (a^6)^0$

$= a^{6 \times 0}$

$= a^0$

$= 1$ (Because $(x^y)^z = x^{y \times z}$) ✓ only if $a \neq 0$

14. $((-7)^3)^2$

$(-7^3)^2$

$=\left(\frac{1}{7^3}\right)^2$

$=(7^{-3})^2$

$=-343^2$

$=\frac{1}{54729}\left(A^{-b}=\frac{1}{A^b}\right)$

$((-7)^3)^2=(-7)^{3\times2}$

$=(-7)^6$

$=(-1\times7)^6=(-1)^6\times7^6$

$=1\times7^6=7^6$

15. $(10^{1/2})^4$

$(10^{1/2})^4$

$=(\sqrt[2]{10})^4$

$=(\sqrt{10})^4$ (Because $A^{\frac{1}{n}}=\sqrt[n]{A}$, and $\sqrt[2]{A}=\sqrt{A}$)

$=10^{\frac{1}{2}\times4}=10^2=100$

16. $(3\times5)^3$

$(3\times5)^3$

$=3^3\times5^3$ (Because $(A\times B)^n=A^n\times B^n$) ✓

17. $(4a)^2$

$(4a)^2$

$=(4\times a)^2$

$=4^2\times a^2$ (Because $(A\times B)^n=A^n\times B^n$) ✓

18. $(x^2y^3)^4$

$(x^2y^3)^4$

$=(x^2\times y^3)^4$ (Because $xy=x\times y$) $=(x^2)^4\times(y^3)^4$

$=x^8\times y^{12}$

19. $9^0$

$9^0$

$= 1$ (Because $x^0 = 1$) ✓

20. $7^{-3}$

$7^{-3}$

$= \frac{1}{7^3}$ (Because $x^{-y} = \frac{1}{x^y}$) ✓

Section 2 : Find the actual answer (21-35):

21. $4^3$

$= 4 \times 4 \times 4$

$= 16 \times 4$

$= 64$ ✓

22. $1^9$

$= 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

$= 1$ ✓

23. $10^5$

$= 10 \times 10 \times 10 \times 10 \times 10$

$= 100000$ ✓

24. $(-2)^4$

$= -2^4$ ✗

$= (-2)(-2)(-2)(-2)$

$= 8$ ✗

$(-2)^4 \neq -2^4$

$\downarrow$ $\quad\quad$ $\downarrow$

$16 \quad\quad -16$

25. $\frac{6^5}{6^3}$

$= 6^{5-3}$

$= 6^2$ ✓

$= 36$ $\left(\frac{n^m}{n^x} = n^{m-n}\right)$

26. $\frac{3^2}{3^4}$

$\frac{3^2}{3^4} = 3^{2-4} = 3^{-2} = 0.03$ $\left(\frac{n^m}{n^x} = n^{m-n}\right)$ ✗

$3^{-2} = \frac{1}{3^2} = \frac{1}{9} = 0.\dot{1}$

$27.\ 5^{-2}$

$5^{-2}$

$= \frac{1}{5^2}$

$= \frac{1}{25}$

$= 0.04$ ✓

$28.\ 2^3 \times 2^1$

$2^3 \times 2^1$

$= 2^{3+1}$

$= 2^4$

$= 16$ (Because $A^m \times A^n = A^{m+n}$)

29. $\sqrt{49}$

$\sqrt{49}$

$= \sqrt{7 \times 7}$

$= 7$

30. $\sqrt[3]{27}$

$= \sqrt[3]{3 \times 3 \times 3}$

$= 3$

31. $\sqrt[4]{81}$

$\sqrt[4]{81}$

$= \sqrt{3 \times 3 \times 3 \times 3}$

$= 3$

32. $\sqrt{100} + \sqrt[3]{1}$

$= \sqrt{10 \times 10} + \sqrt[3]{1 \times 1 \times 1}$

$= 10 + 1$

$= 11$ ✓

33. $4^{\frac{1}{2}}$

$4^{\frac{1}{2}}$

$= \sqrt[2]{4}$

$= \sqrt{4}$ ✓

$= \sqrt{2 \times 2}$

$= 2$ (Because $m^{\frac{1}{n}} = \sqrt[n]{m}$

34. $8^{\frac{1}{3}}$

$8^{\frac{1}{3}}$

$= \sqrt[3]{8}$

$= \sqrt[3]{2 \times 2 \times 2}$ ✓

$= 2$

35. $25^{\frac{3}{2}}$

$25^{\frac{3}{2}}$

$= (25^{\frac{1}{2}})^3$

$= (\sqrt{25})^3$

$= 5^3$

$= 125$ ✓

Section 3: write as a fractional exponent, or radical (36-50):

36. $\sqrt[3]{5^2}$

$\sqrt[3]{5^2}$

$= (5^2)^{\frac{1}{3}} = 5^{2/3}$

37. $\sqrt{a^7}$

$\sqrt{a^7}$

$= (a^7)^{\frac{1}{2}} = a^{7/2}$

38. $\sqrt[5]{12}$

$\sqrt[5]{12}$

$= 12^{\frac{1}{5}}$ ✓

39. $\sqrt[4]{x}$

$\sqrt[4]{x}$

$= x^{\frac{1}{4}}$ ✓

40. $y^{\frac{1}{5}}$

$y^{\frac{1}{5}}$

$= \sqrt[5]{y}$ ✓

41. $10^{\frac{1}{2}}$

$10^{\frac{1}{1}}$

$= \sqrt{10}$ ✓

42. $7^{\frac{3}{4}}$

$7^{\frac{3}{4}}$

$= (7^{\frac{1}{4}})^3$

$= (\sqrt[4]{7})^3 = \sqrt[4]{7^3} = \sqrt[4]{343}$

43. $z^{\frac{5}{3}}$

$z^{\frac{5}{3}} = (z^{\frac{1}{3}})^5 = (\sqrt[3]{z})^5$ $= z^{5/3} = z^{5 \times \frac{1}{3}} = (z^5)^{\frac{1}{3}}$

$= \sqrt[3]{z^5}$

44. $\sqrt[5]{-32}$

$\sqrt[5]{-32}$

$= \sqrt[5]{-2 \times -2 \times -2 \times -2 \times -2}$ ✓

$= -2$

$\sqrt[4]{-16} \neq -2$

$\sqrt{-25} \neq -5$ ; These are not real numbers!

45. $x^{1/2} \times x^{\frac{1}{2}}$ $= x^{\frac{1}{2}+\frac{1}{2}}$

$= x^1 = x$

$= \sqrt{x} \times \sqrt{x}$

$= \sqrt{x \times x}$ $= x$

$x^2 = -25$

Let $x \geq 0 \Rightarrow x^2 \geq 0$

$x < 0 \Rightarrow x^2 > 0$

So, $x$ can't be $\pm$ Real

(They are maginary)

$\sqrt{-25} = 5i$ , $(i^2 = -1.)$

$\sqrt{-25} = \sqrt{25 \times -1}$

$= \sqrt{25} \times \sqrt{-1}$

$= \pm 5i$

$\sqrt{25} = \pm 5$

46. $\frac{y^2}{y^{\frac{1}{2}}}$ $= y^{2-\frac{1}{2}}$

$\frac{y^2}{y^{\frac{1}{2}}}$ $= y^{\frac{2}{1}-\frac{1}{2}}$

$= y$

$= y^{\frac{2}{1}\times\frac{2}{2}-\frac{1}{2}\times\frac{1}{1}}$

$= y^{\frac{4}{2}-\frac{1}{2}} = y^{3/2}$

$= \sqrt{y^3}$

$= \frac{y \times y}{\sqrt{y}}$

$= 1 \times 2$

$= 4$

47. $\sqrt{\frac{1}{9}}$

$= \sqrt{\frac{1}{3\times3}}$

$\sqrt{\frac{1}{9}}$

$= \sqrt{\left(\frac{1}{3}\right)\times\left(\frac{1}{3}\right)}$

$= \sqrt{0.111}$

$= \frac{1}{3}$

48. $\sqrt[3]{64}$

$\sqrt[3]{64}$

$= \sqrt[3]{4\times4\times4}$

$= 4$ ✓

$\frac{2}{1} - \frac{1}{2}$

$= \frac{2}{1}\times\frac{2}{2} - \frac{1}{2}\times\frac{1}{1}$

$2\ \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$

49. $\sqrt{121}$

$\sqrt{121}$

$= \sqrt{11\times11}$ ✓

$= 11$

50. $(4^2)^{\frac{1}{2}} = 4^{\frac{1}{1}} = \sqrt{4} = \sqrt{2\times2} = 2$ ✓

$2^{2\times\frac{1}{2}} = 2^1 = 2$