

02 Dec'24

Decimal Divisions :

$$\frac{1}{3} = 0.3333\ldots = 0.\overline{3}$$

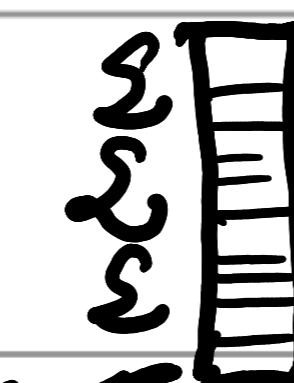
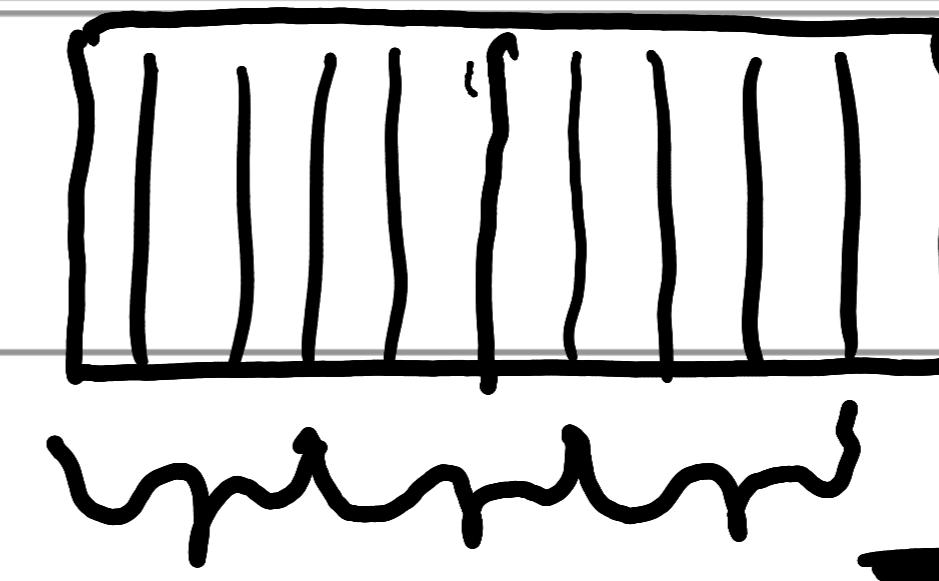
We are dividing one chocolate among 3 people.

In decimal number system, everything is divided into powers of 10.

$$\begin{array}{r} 0.3 \\ \hline 3 \) 1.0 \\ \hline 9 \\ \hline 1 \end{array}$$

We break the chocolate into 10 pieces first (10^{th} parts) and each person gets 3 10^{th} parts, 1 10^{th} is remainder.

$$3 \overline{)1.0} \quad \begin{array}{c} 0.333 \\ \hline 1.0 \\ -9 \\ \hline 10 \\ -9 \\ \hline 1 \end{array}$$



$$= 0.1 = 0.10$$

remainder

1

1 tenth
part

1

10 hundredth
parts

10 (thousandth
parts)

After 3 steps, each person gets 0.333

chocolates (= 333 thousandth parts) and remainder

is 0.001 (= 1 thousandth part)

→ Supposing we have a floating point number with repeating decimals like

$$0.\overline{3} = 0.333\dots$$

This can be converted into a fraction

(division of integers) like $\frac{a}{b}$ where $a, b \in \mathbb{Z}$.

Process for converting repeating decimal
to a fraction:

$$\text{Let } x = 0.\overline{3}$$

$$x = 0.3333\dots$$

$$10x = 3.3333\dots$$

$$10 \times x = 3.3333 \dots$$

$$x = 0.3333 \dots$$

$$10 \times x - x = 3.000000000$$

$$(10-1) \times x = 3$$

$$9 \times x = 3$$

$$x = \frac{3}{9} = \frac{3 \times 1}{3 \times 3} = \frac{3}{3} \times \frac{1}{3} = 1 \times \frac{1}{3} = \frac{1}{3}$$

$$\left\{ \frac{a \times b}{c \times d} = \frac{a}{c} \times \frac{b}{d} \right\}$$

To convert a repeating decimal to a fraction, we multiply the number with a power of 10 (based on the periodicity of the repeating decimal).

$$\rightarrow 0.\overline{343434343\dots}$$

$$= 0.\overline{343}$$

period = 43 = repeating digits

periodicity = '2' = no of digits in period

$$\rightarrow 32.\underline{2312312312\dots}$$

$$= 32.\overline{2312}$$

period = 312

periodicity = 3

To convert a repeating decimal with periodicity 'k' into a fraction, we multiply with $10^k = 10^{**k}$, to subtract and make the repeating parts zeros.

$$x = 3.2424242\dots$$

$$100x = 324.2424242\dots \quad \begin{matrix} \text{period} = 24 \\ \text{periodicity} = 2 \end{matrix}$$

$$x = 3.2424242\dots$$

$$99x = 321.000000\dots$$

$$x = \frac{321}{99} = \frac{107 \times 3}{33 \times 3} = \frac{107}{33} \times \frac{3}{3} = \frac{107}{33}$$

HW: Convert the following repeating decimals into fractions. Verify by converting the fraction back to decimals by division (also in Python).

(1) $3.0232323\dots$

(2) $49.11111\dots$

(3) $0.9131313\dots$

(4) $7.632163216321\dots$

(5) $0.142857142857142857\dots$

HW: Evaluate the following expressions
manually and also verify in Python

$$(1) \frac{3.5 + 2.75 * (6.2 - 4.8)}{1.5 - 0.75}$$

$$(2) \frac{5.25}{[1.25 + 0.75 * (2.4 - 1.6)]^2}$$

$$(3) 0.125 * [8.4 - 2 * (3.15 + 1.2)] + 3.5$$

$$(4) 6.75 - 2.25 * \{3.5 / [1.75 + 0.25 * (4.8 - 3.2)]\}$$

$$(5) 12.5 / \{2.5 + [3.75 * (5.6 - 4.4) - 1.5]^2\}$$