A Comprehensive Method for Multipath Performance Analysis of GNSS Navigation Signals

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Abstract*—Multipath error is the most important unavoidable source of error contributing to the User Equivalent Range Error (UERE). It is difficult to model, but it is very necessary for us to describe signal multipath performance of Global Navigation Satellite System (GNSS). Multipath error envelope based on non-coherent early-late code tracking loop has become a common approach to describe the effects of specular multipath, which is widely used in navigation signal performance evaluation. A more practical and reliable way for multipath performance assessment is the computation of the running average of multipath error envelope. For this purpose, only the absolute envelope values are considered and their cumulative sum is used to compute the running average for the multipath error envelope. The mean value of the absolute values of the in-phase and the anti-phase shift component of the multipath error envelopes has been used to compute the running average of an envelope. Considering the numerical method can not reveal the direct relation between the navigation signal and multipath error envelope for the assessment of the navigation signal, theoretical approximation explicit expression for multipath error envelope is derived through first order Taylor expansion. The explicit expression method has its limitations. The computation of multipath error envelopes is based on the assumptions that the direct signal component is always available and no shadowing effects, only one multipath signal is obtained and the multipath signal undergoes specific amplitude attenuation relative to the direct signal. Furthermore, a static environment is assumed. To overcome these limitations and get a more realistic view on the actual multipath performance, a simple model that considers statistical distributions of geometric path delays and multipath relative amplitudes is introduced, allowing the computation of weighted multipath error envelopes which are valid for different multipath environments such as open, rural, suburban and urban scenarios. The paper aims at deriving multipath error envelope explicit expression from simplified and practical statistical channel model with considering open, rural, suburban and urban scenarios. The results indicate that the comprehensive method is effective. This multipath error envelope analytical method may be used in future navigation signal design.

Keywords- Taylor expansion; explicit expression method; signal to multipath ratio; correlator spacing; weighted running average multipath error envelope; statistical channel models

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I. INTRODUCTION

GNSS pseudo-range and carrier phase measurements suffer from a variety of systematic biases. The sources of these are:

- Satellite orbital prediction errors
- Satellite clock bias
- Ionospheric refraction
- Tropospheric refraction
- Signal multipath
- Receiver clock offset
- Receiver noise error

The satellite orbit, satellite clock, ionospheric, and tropospheric errors can be removed by differencing techniques or significantly reduced by modeling. The receiver clock offset can also be removed by differencing but is often solved for as an unknown in the position solution. The measurement bias caused by signal multipath acts different. Unlike the other error sources, this bias is heavily dependent on the environment surrounding the receiver antenna, but with little relevance in space, so it is difficult to eliminate through differential method. Therefore, the multipath performance of navigation signals in the design process must be considered as an important attribute. A key point in the choice of the signals for the future GNSS System is the multipath performance. Studies have clearly shown that the synchronization performance of a specific navigation signal strongly depends on reflections from the environment. Especially, short delayed reflections significantly decrease the performance of the receiver. The positioning error becomes even worse if these reflections are strong and varying slowly over time [1].

There are two kinds of traditional multipath error envelope analysis methods, one is analytic method based on theory of non-band-limited assumption, and the other is numerical simulation method based on numerical calculation [2] [3]. Obviously, analytic method is only fit for the circumstance that signal bandwidth is much larger than code rate, so it can not satisfy most applications. Numerical simulation method, which is commonly employed nowadays, is able to derive reliable multipath error envelope curve. However, the numerical method can not reveal the direct relation between the navigation signal and multipath error envelope for the assessment of the navigation signal, so this method can not provide direct and theoretical guidance for navigation signal design. In order to get the explicit expression of multipath error envelope and provide direct theoretical guidance for navigation signal design, a theoretical analysis method for code tracking multipath error under band-limited condition by the use of linear model under small error assumption and consideration of simplifying the calculation is presented.

Apart from the computation of multipath error envelopes, there are also other, more complex models that consider more than one multipath signal in different multipath environment. The most realistic but also the most complex approach to assess the multipath performance of a given signal/receiver combination is to consider not only the signal characteristics and the receiver architecture but also different multipath environments and different elevation angles.

As a conclusion, this paper will present a comprehensive analytical method for multipath performance of GNSS signal under simplified and practical statistical channel model, and all simulation results are given.

II. MULTIPATH PERFORMANCE ANALYSIS BASED ON NUMERICAL METHOD

A. Signals and Parameters Selection

As presented in the foregoing part, multipath inevitably cause Pseudo-code and carrier phase systematic error. To which extent these multipath signals contribute to ranging or carrier phase errors depend on various signal and receiver parameters [4]

- Signal type namely modulation scheme (e.g. BPSK, BOC, MBOC (CBOC, TMBOC) …)
- Pre-correlation bandwidth and filter characteristics
- Chipping rate of code
- Relative power levels of multipath signals (signal attenuation due to reflection)
- Actual number of multipath signals
- Geometric path delay of multipath signal
- Chip spacing between correlators used for tracking
- Type of discriminator
- Carrier frequency (carrier multipath)

In order to visualize the difference criteria of multipath performance assessment considering all the parameters above, four sample signals will be used, namely the BPSK-R (1) representing the current GNSS widely used coarse range code, the BPSK-R (10) representing the current GNSS precise range code signal, the BOC (10, 5) and the MBOC (6, 1, 1/11), adopted signals for future optimization of GNSS new military and civil signals [5] [6] [7]. The power spectral density functions and the normalized autocorrelation functions of these four signals are illustrated in Figure 1 and Figure 2 respectively. They base on a pre-correlation bandwidth of 24MHz and ideal low pass filter.

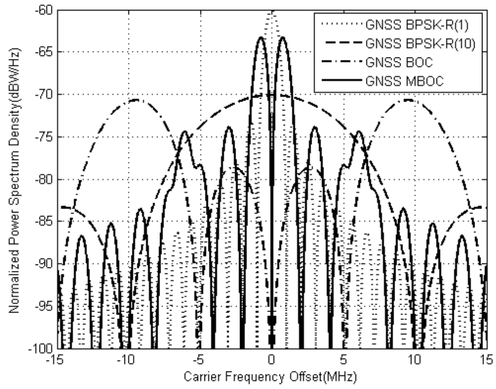

Figure 1. GNSS carrier signals power spectrum density.

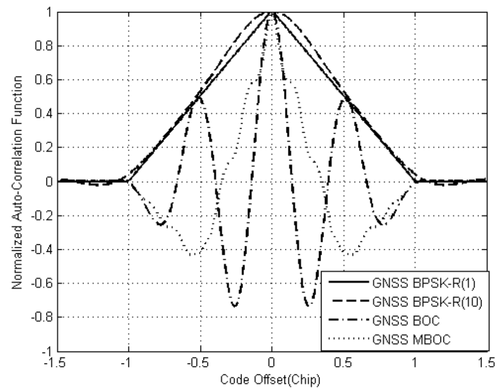

Figure 2. GNSS carrier signals normalized auto-correlation function.

B. Multipath Signal Model

The signals considered in the present study are GNSS modulated signals [2] [3]. Concerning the multipath model, the presence of N specular reflected signals affecting the direct Line of Sight (LOS) signal will be assumed. The low-pass equivalent complex envelope of received GNSS modulated signals can be expressed by

$$r(t) = a_0 e^{j\varphi_0} x(t-\tau_0) + \sum_{n=1}^{N} a_n e^{j\varphi_n} x(t-\tau_n) \tag{1}$$

where $x(t)$ is the complex envelope of transmitted signal, τ_0 is the propagation delay of LOS signal, a_0 is the amplitude of LOS signal, φ_0 is the phase of LOS signal. N denotes the numbers of multipath signals, τ_n , a_n , φ_n are respectively the propagation delay, amplitude, phase of multipath signals.

A very typical receiver architecture using Early minus Late Power (EMLP) delay lock loop is displayed on figure 3. In presence of multipath signals, the outputs of the correlators have the expressions

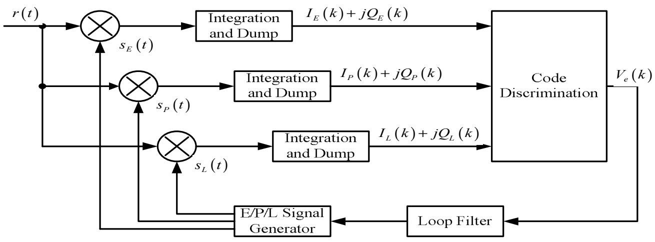

Figure 3. Diagram of code tracking loop.

$$s_E(t)=e^{-j\hat{\varphi}_0}x^*(t-\hat{\tau}_0+d/2) \tag{2}$$

$$s_P(t)=e^{-j\hat{\varphi}_0}x^*(t-\hat{\tau}_0) \tag{3}$$

$$s_L(t)=e^{-j\hat{\varphi}_0}x^*(t-\hat{\tau}_0-d/2) \tag{4}$$

where $s_E(t)$, $s_P(t)$, $s_L(t)$ denote respectively early, prompt and late reference signals. $\hat{\tau}_0$ is the propagation delay estimation value of LOS signal, $\hat{\varphi}_0$ is the phase estimation of LOS signal, d denotes early late chip spacing.

$$R_E=a_0e^{j\varepsilon_\varphi}R(\varepsilon_\tau-d/2)+\sum_{n=1}^{N}a_ne^{j(\varepsilon_\varphi+\tilde{\varphi}_n)}R(\varepsilon_\tau-\tilde{\tau}_n-d/2) \tag{5}$$

$$R_P=a_0e^{j\varepsilon_\varphi}R(\varepsilon_\tau)+\sum_{n=1}^{N}a_ne^{j(\varepsilon_\varphi+\tilde{\varphi}_n)}R(\varepsilon_\tau-\tilde{\tau}_n) \tag{6}$$

$$R_E=a_0e^{j\varepsilon_\varphi}R(\varepsilon_\tau+d/2)+\sum_{n=1}^{N}a_ne^{j(\varepsilon_\varphi+\tilde{\varphi}_n)}R(\varepsilon_\tau-\tilde{\tau}_n+d/2) \tag{7}$$

where $R(\tau)$ is the auto-correlation function, ε_φ is the phase estimation error of LOS signal, $\tilde{\varphi}_n$ denotes phase error of LOS signal relative to multipath signal, ε_τ is the propagation delay estimation of LOS signal, $\tilde{\tau}_n$ denotes extra propagation delay of multipath signal relative to LOS signal.

$$\begin{aligned}I_E=\mathrm{Re}[R_E]&=a_0\cos(\varepsilon_\varphi)R(\varepsilon_\tau-d/2)\\&+\sum_{n=1}^{N}a_n\cos(\varepsilon_\varphi+\tilde{\varphi}_n)R(\varepsilon_\tau-\tilde{\tau}_n-d/2)\end{aligned} \tag{8}$$

$$\begin{aligned}Q_E=\mathrm{Im}[R_E]&=a_0\sin(\varepsilon_\varphi)R(\varepsilon_\tau-d/2)\\&+\sum_{n=1}^{N}a_n\sin(\varepsilon_\varphi+\tilde{\varphi}_n)R(\varepsilon_\tau-\tilde{\tau}_n-d/2)\end{aligned} \tag{9}$$

$$\begin{aligned}I_P=\mathrm{Re}[R_P]&=a_0\cos(\varepsilon_\varphi)R(\varepsilon_\tau)\\&+\sum_{n=1}^{N}a_n\cos(\varepsilon_\varphi+\tilde{\varphi}_n)R(\varepsilon_\tau-\tilde{\tau}_n)\end{aligned} \tag{10}$$

$$\begin{aligned}Q_P=\mathrm{Im}[R_P]&=a_0\sin(\varepsilon_\varphi)R(\varepsilon_\tau)\\&+\sum_{n=1}^{N}a_n\sin(\varepsilon_\varphi+\tilde{\varphi}_n)R(\varepsilon_\tau-\tilde{\tau}_n)\end{aligned} \tag{11}$$

$$\begin{aligned}I_L=\mathrm{Re}[R_L]&=a_0\cos(\varepsilon_\varphi)R(\varepsilon_\tau+d/2)\\&+\sum_{n=1}^{N}a_n\cos(\varepsilon_\varphi+\tilde{\varphi}_n)R(\varepsilon_\tau-\tilde{\tau}_n+d/2)\end{aligned} \tag{12}$$

$$\begin{aligned}Q_L=\mathrm{Im}[R_L]&=a_0\sin(\varepsilon_\varphi)R(\varepsilon_\tau+d/2)\\&+\sum_{n=1}^{N}a_n\sin(\varepsilon_\varphi+\tilde{\varphi}_n)R(\varepsilon_\tau-\tilde{\tau}_n+d/2)\end{aligned} \tag{13}$$

where I_E , I_P , I_L , Q_E , Q_P , Q_L denote respectively the In-phase component and quadrature component output of early, prompt and late correlator arm. The code discriminator output of Early minus Late Power (EMLP) delay lock loop can be expressed as following:

$$D_{EMLP}=(I_E{}^2+Q_E{}^2)-(I_L{}^2+Q_L{}^2) \tag{14}$$

C. Multipath Error Envelope for Band-Limited Signals

From the equation (8) to (14), we can see that give a general multipath expression is very difficult if considering all the affected factors. In order to simplify the multipath analysis process, some hypotheses are inevitable. Here we just concern only one multipath signal; what is more, carrier phase error is viewed as 0. As a result a useful operational single reflection model is derived; this will not change the analytical results.

$$\begin{aligned}D_{EMLP}&=\left[a_0R(\varepsilon_\tau-d/2)+a_1\cos(\tilde{\varphi}_1)R(\varepsilon_\tau-\tilde{\tau}_1-d/2)\right]^2\\&+\left[a_1\sin(\tilde{\varphi}_1)R(\varepsilon_\tau-\tilde{\tau}_1-d/2)\right]^2\\&-\left[a_0R(\varepsilon_\tau+d/2)+a_1\cos(\tilde{\varphi}_1)R(\varepsilon_\tau-\tilde{\tau}_1+d/2)\right]^2\\&-\left[a_1\sin(\tilde{\varphi}_1)R(\varepsilon_\tau-\tilde{\tau}_1+d/2)\right]^2\end{aligned} \tag{15}$$

$$\begin{aligned}D_{EMLP}&=a_0\left[R(\varepsilon_\tau-d/2)-R(\varepsilon_\tau+d/2)\right]\\&\pm a_1\left[R(\varepsilon_\tau-\tilde{\tau}_1-d/2)-R(\varepsilon_\tau-\tilde{\tau}_1+d/2)\right]\end{aligned} \tag{16}$$

$$D(\varepsilon_\tau)=0 \tag{17}$$

It is obvious that the output of code discriminator is nonlinear, but we can get linear approximation of $D(\varepsilon_\tau)$ around $\varepsilon_\tau=0$ under the circumstance of the error is tiny. So we can derive approximation explicit expression for multipath error envelope through first order Taylor expansion near 0.

$$D(\varepsilon_\tau) = D(0) + D^{'}(0)\times\varepsilon_\tau + o(\varepsilon_\tau^{\ 2}) \tag{18}$$

$o(\varepsilon_\tau^{\ 2})$ is higher-order indefinitely small quantity.

$$\varepsilon_\tau \approx -D(0)/D^{'}(0) \tag{19}$$

$$\varepsilon_\tau \approx \frac{\pm\tilde{a}_1\int_{-\beta_r/2}^{\beta_r/2} s(f)\sin(2\pi f\tilde{\tau}_1)\sin(\pi f d)df}{2\pi\int_{-\beta_r/2}^{\beta_r/2} fs(f)\sin(2\pi f\tilde{\tau}_1)\left[1\pm\tilde{a}_1\cos(2\pi f\tilde{\tau}_1)\right]df} \tag{20}$$

Where $\tilde{a}_1$ denotes the amplitude ration of multipath signal to LOS signal. The sign ± is selected + when the phase difference between multipath and LOS signal is 0 degree, – while phase difference between multipath and LOS is 180 degrees. Figure 4 shows multipath error envelope of GNSS signals which are defined above.

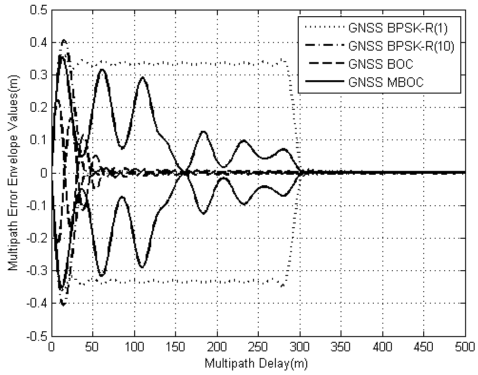

Figure 4. Multipath error envelope of GNSS signals.

Theoretical approximation explicit expression for multipath error envelope is derived through first order Taylor expansion. Simulation results show that the multipath error envelope difference between theoretical approximation and numerical simulation is less than 9% when signal to multipath ratio (SMR) is 6dB and the running average of multipath error envelope difference between these two methods is less than 6%, what's more important is that the higher the SMR, the less the difference.

III. Multipath Error Analysis Method Based on Statistical Channel Models

A. *Statistical Channel Models*

Apart from the computation of multipath error envelopes, there are also other, more complex models that consider more than one multipath signal in different multipath environment. The most realistic but also the most complex approach to assess the multipath performance of a given signal/receiver combination is to consider not only the signal characteristics and the receiver architecture but also different multipath environments and different elevation angles. However, this requires exact knowledge of the multipath propagation channel under various conditions. Based on extensive measurement campaigns, such channel models have been derived in the past, or are currently being developed [8] [9]. The model parameters have been determined using least square fits with measured data. The Gaussian Newton algorithm was adopted for computation. Numeric values of the parameters for the direct, near echo and far echo path are derived. The parameter sets are distinguished by environments. Figure 5 illustrates the typical distribution of path delays and amplitudes for a urban environment (based on 1000 simulation runs) as obtained from the statistical channel model.

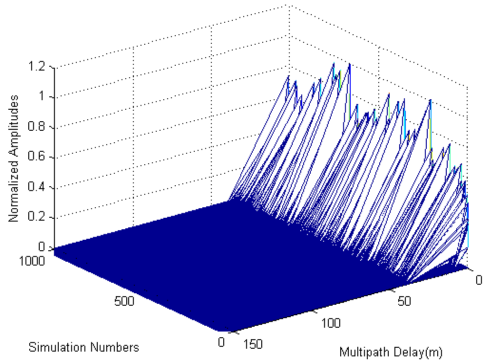

Figure 5. Distribution of multipath delay and amplitude. (urban, elevation=25 degrees)

B. *Running Average Multipath Error Envelope*

A more meaningful representation of performance in multipath is to portray the running average of an envelope. For this purpose, only the absolute envelope values are considered and their cumulative sum is used to compute average ranging errors. Within the framework of this paper, the mean value of the in-phase and the 180 degree phase shift component of the multipath error envelope are used to compute the running averages [10]. Then define the running average multipath error envelope to be [3]

$$\varepsilon_a(\tilde{\tau}_1^{'}) = \frac{1}{\tilde{\tau}_1^{'}}\int_0^{\tilde{\tau}_1^{'}}\left[\frac{abs\left(\varepsilon_\tau(\tilde{\tau}_1)|_{\tilde{\varphi}_1=0}\right) + abs\left(\varepsilon_\tau(\tilde{\tau}_1)|_{\tilde{\varphi}_1=180}\right)}{2}\right]d\tilde{\tau}_1^{'} \tag{21}$$

Figure 6 illustrate the result of such a computation for the four sample signals based on the running average multipath error envelope for the suburban environment.

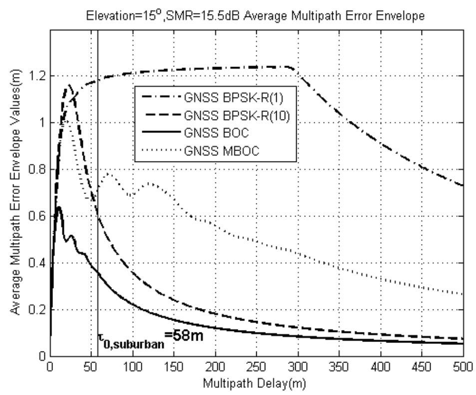

Figure 6. Comparison of runing averages of multipath error envelopes. (suburban environment, elevation=15 degrees)

Since running averages are also computed directly from the error envelopes, the results also reflect the differences between the difference signal structures.

C. Weighted Multipath Error Envelope

In order to get a more realistic view on the actual multipath performance, a statistical channel model can be used to scale the obtained multipath error envelopes [10]. This model takes into account that short-delay multipath occurs more frequently than long-delay multipath and that long geometrical path delays result in relatively weak multipath signals. It can be mathematically expressed by

$$\rho(\tau)=\frac{1}{\tau_0}e^{-\frac{\tau}{\tau_0}} \quad \text{and} \quad A(\tau)=\alpha_0 e^{-\frac{\tau}{2\tau_0}} \tag{22}$$

where $\rho(\tau)$ models the multipath delays and $A(\tau)$ the multipath amplitudes, using the coefficient of reflection α_0 and a path delay τ_0 which characterizes a typical multipath environment. By using equation (21), the combined distribution $D(\tau)$ of multipath delays and amplitudes can be written as followings

$$D(\tau)=\rho(\tau)A(\tau)=\frac{\alpha_0}{\tau_0}e^{-\frac{3\tau}{2\tau_0}} \tag{23}$$

This model can now be used to scale the common multipath error $E(\tau)$. Since the coefficient of reflection α_0 has already been considered during the computation of the multipath error envelope, we can use the modified combined distribution for the further computation.

$$D^{'}(\tau)=\frac{1}{\tau_0}e^{-\frac{3\tau}{2\tau_0}} \tag{24}$$

In order to obtain scaled envelopes where the resulting ranging errors are expressed in meter, equation (23) has to be normalized such that $D^{'}(\tau=0)=1$. This approach results in weighted multipath error envelopes $E^{'}(\tau)$ which can be expressed as followings

$$E^{'}(\tau)=E(\tau)e^{-\frac{3\tau}{2\tau_0}} \tag{25}$$

This approach allows a more realistic view on the actual multipath performance of a given signal receiver combination. Figure 7 shows the weighted multipath error envelope of typical four signals defined above.

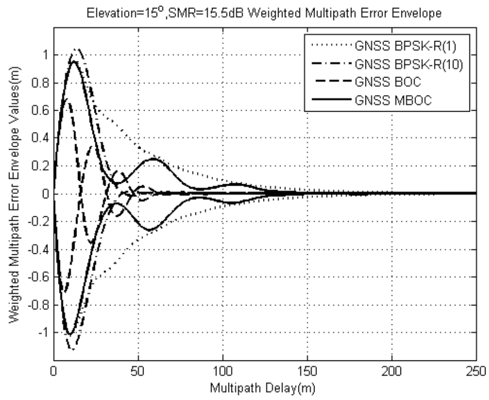

Figure 7. Comparison of weighted multipath error envelopes. (SMR=15.5dB, elevation=15 degrees)

IV. SIMULATION RESULTS AND ANALYSIS

The code multipath performance of typical four GNSS signals is analyzed in detail. It is illustrated by means of weighted average multipath error envelopes, where the resulting ranging errors are plotted as a function of the geometric path delay, different environment and signal to multipath ratio. When using the weighted average multipath error envelopes for performance assessment, a statistical channel model can be used to scale the obtained multipath error envelopes. This model takes account of that short-delay multipath and that long geometrical path delay result in relatively weak multipath signals. The main benefit of using statistical channel models is that they allow the computation of multipath errors under realistic conditions by integrating information about the multipath environment and by considering different elevation angles. As a result, both qualitative and quantitative statements about the expected multipath performance can be obtained.

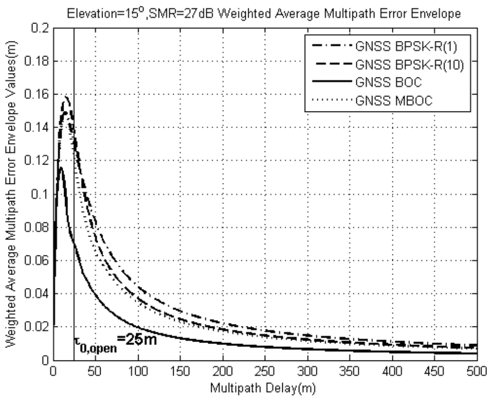

Figure 8. Weigthed average multipath error envelope. (elevation=15 degrees, SMR=27dB, open environment)

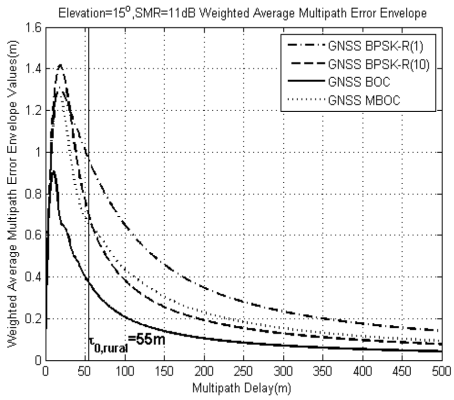

Figure 9. Weigthed average multipath error envelope. (elevation=15 degrees, SMR=11dB, rural environment)

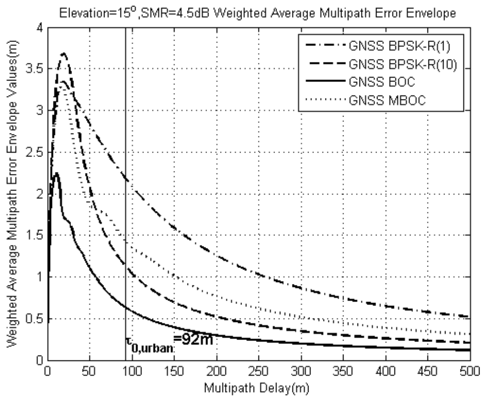

Figure 10. Weigthed average multipath error envelope. (elevation=15 degrees, SMR=4.5dB, urban environment)

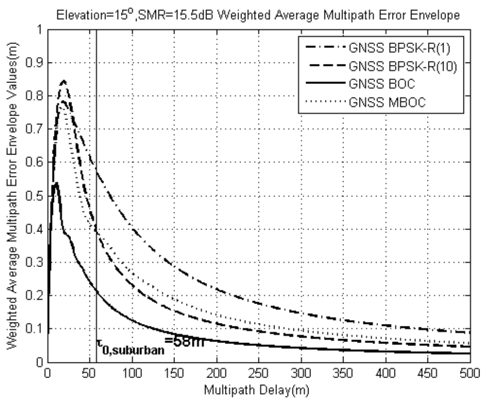

Figure 11. Weighted average multipath error envelope. (elevation=15 degrees, SMR=15.5dB, suburban environment)

$\tau_{0,open}$, $\tau_{0,rural}$, $\tau_{0,urban}$ and $\tau_{0,suburban}$ which are displayed in the four figures (figure8, figure 9, figure 10 and figure 11) are respectively the typical multipath delay of open, rural, urban and suburban environment, their values are derived from statistical channel model. In general, a good multipath performance is characterized by a small maximum average value and a rapid decrease towards zero. Based on this criterion, from the four figures, the GNSS BOC shows the best overall multipath performance with respect to code multipath, followed by the MBOC, BPSK-R (10) and BPSK-R (1).

V. CONCLUSIONS

Several models that characterize GNSS multipath propagation have been discussed and analyzed with respect to their potential of providing meaningful and realistic values for occurring multipath errors. All models are suitable for comparing different GNSS signals with respect to their multipath performance. This paper have derived multipath error envelope explicit expression from simplified and practical statistical channel model with considering open, rural, suburban and urban scenarios. The results indicate that the comprehensive method is effective. This multipath error envelope analytical method may be used in future navigation signal design.

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REFERENCES

[1] Fantino M, Marucco G, Mulassano P, et al. Performance Analysis of MBOC, AltBOC and BOC Modulations in Terms of Multipath Effects on the Carrier Tracking Loop within GNSS Receivers. 2008 Ieee/Ion Position, Location and Navigation Symposium, Vols 1-3, 2008:1075-1082.

[2] Irsigler M, Eissfeller B. Comparison of multipath mitigation techniques with consideration of future signal structures. ION GPS/GNSS 2003, 9-12 september 2003, Portland, OR. 2584-2592.

[3] Tang Zuping, Hu Xiulin, Huang Xufang. Analysis of multipath rejection performance in GNSS signal design. The journal of HuaZhong University of Science and Technology (Natural Science Edition). Vol.37 NO .5, 2009.

[4] Tang Zuping, Hu Xiulin. Research on relevant theory for GNSS signal design and assessment. Dissertation for the degree of Doctor of Philosophy in Engineering. HuaZhong University of Science and Technology,2009.

[5] Betz J W. The Offset Carrier Modulation for GPS Modernization. 1999.639-648.

[6] Hein G W, Avila-Rodriguez J A, Wallner S, et al. . MBOC: The New Optimized Spreading Modulation Recommended for Galileo L1 OS and GPS L1C. 2006 IEEE/ION Position, Location and Navigation Symposium, Vols 1-3, 2006:883-892.

[7] Avila-Rodriguez J, Hein G W, Wallner S, et al. The MBOC Modulation: The Final Touch to the Galileo Frequency and Signal Plan. in:ION GNSS 20th International Technical Meeting of the Satellite Division. Fort Worth, TX, 2007.1515-1529.

[8] Jahn, A. et al. (1996): Channel Characterization for Spred Spectrum Satellite Communications, Proceedings of the IEEE 4th International Symposium on Spread Spectrum Techniques & Applications (ISSSTA'96), 1996.

[9] Steingaß, A. and Lehner, A. (2003): Land Mobile Satellite Navigation – Characteristics of the Multipath Channel, Proceedings of the ION GNSS 2003, 9-12 September 2004, Portland,Oregon, USA.

[10] Irsigler M, Rodriguez J, Hein G. Criteria for GNSS Multipath Performance Assessment. in:ION GNSS 18th International Technical Meeting of the Satellite Division. Long Beach, CA, 2005.2166-2177.